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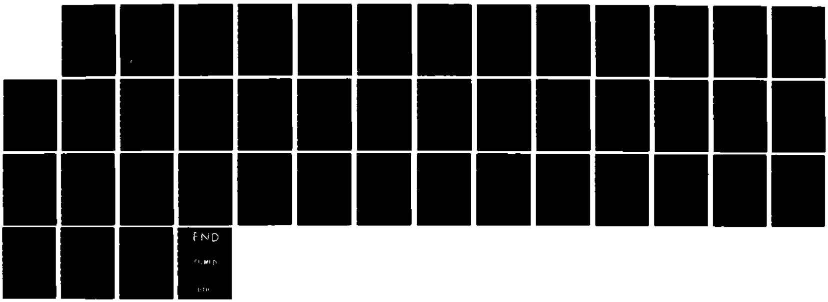

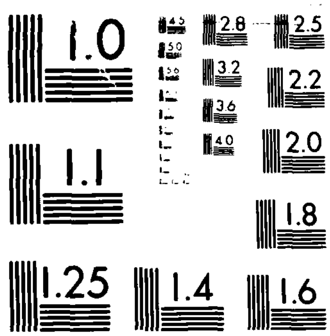

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LINEAR PROPAGATION MODEL FOR LASER-INDUCED ULTRA-SOUND IN WATER

BY ROBERT CAWLEY

RESEARCH AND TECHNOLOGY DEPARTMENT

AUGUST 1982

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FOREWORD

The long-range propagation of acoustic transients produced experimentally from absorption of laser energy by water is examined theoretically. The linear theory, which it is argued, applies for ranges $r \gg r_0$ where viscosity effects will dominate, results in a diffusion equation for the transient pulse form. The solutions exhibit "solitonic" behavior, the linear 1/r peak pressure falloff law going over eventually to a $1/r^2$ law rather than the much more severe exponential law which applies to pure tone (harmonic) waves. The scale distance for the cross-over to r^{-2} behavior (fresh water), $r_1' = 0.33\ \text{km} \bullet (2t_1)^2$, where $2t_1$ is the transient pulse width (one-half the period) in μsec, is predicted to be 14 meters for CO_2 laser induced pulses observed below threshold for vaporization. Above the vaporization threshold by a modest amount, $2t_1 = 1\mu\text{sec}$ and $r_1' = 330$ meters; calculated pressure levels at 1.5 km in the r^{-2} region, are 40 db re: 1 μbar and the pulse width has spread to 2.5 μsec. Necessary generalizations are described briefly for taking account of salinity in sea water, and of non-linear acoustic effects.

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A-1

CONTENTS

ILLUSTRATIONS

CHAPTER 1

INTRODUCTION

Laser energy absorption in water results in the production of high frequency pressure transients. This was observed experimentally, long ago,[1,2] and is expected theoretically. The mechanism of coupling depends upon the laser wavelength, and the light intensity in the absorption region. CO_2-laser light, $\lambda = 10.6\ \mu m$, is absorbed in a very thin surface layer whose thickness scale at the lower end, set by Beer's law, is of order 10 μm. Illuminated spot sizes are set by beam quality and focussing geometry. Assuming direct thermal absorption by the water, surface vaporization from 20°C requires a local energy density $u_0 = 2.63\ KJ/cm^3$ (=629 cal/g). This corresponds to a laser energy threshold of order 2.6 j in order to vaporize a 10 μm surface layer of 1 cm^2 cross section. In recent experiments at NSWC, using a 10 joule (rated) TEA multimode CO_2 laser, varying conditions have been examined, for energies ranging from about a tenth of a joule up to 6 joules, and for spot sizes from 0.1 cm^2 to a few cm^2. These input conditions easily straddle the threshold energy density u_0, and the induced pressure pulse-form characteristics reflect a threshold effect. Higher frequency transients, 100 - 400 ns oscillations, have been observed for lower laser intensities while lower frequency pulse-forms, 2 μs oscillations, are produced from high laser intensities. The latter have somewhat larger peak pressures, and for intermediate laser intensities both kinds of pulse are often present together; these pulse-forms frequently show a high frequency, relatively low amplitude "leader oscillation," followed by a somewhat stronger low frequency main pulse oscillation.

Observed pulse-forms are broad-band, however, since only a single pressure oscillation cycle, or two, of the main pulse bodies occur. (See Figure 1). Even at rather close ranges the peak pressure amplitudes are generally not too high for the application of linear acoustics (e.g., 25 bar at a distance 2 cm vertically below the laser spot, in one instance. See below, Section 2, for acoustic approximation criteria). At certain longer ranges, $r > r_0$ say, where r is the distance of a field observation point from the illuminated spot, and r_0 is a scale-length for assumed dominance of linear absorption over nonlinear effects (see below), approximately linear propagation and absorption of the pulses is expected. In this report, I present the application of linear theory to pulse-form, or, in

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[1] Carome, E. F., Clark, N. A. and Moeller, C. E., Applied Physics Letter, 4, 1964, p. 95.

[2] Carome, E. F., Moeller, C. E. and Clark, A. N., J. Applied Physics, 40, 1969, p. 1462.

other words, wave packet, propagation and absorption at long ranges. The predictions of linear theory alone, without need for invoking possible nonlinear mechanisms, are qualitatively different from the case of a single frequency, or pure tone (harmonic) pulse. In the latter case, the sound wave aplitude is attenuated exponentially, with an extinction coefficient β proportional to the square of the frequency, for high frequencies: $\beta = \kappa\omega^2$, where $\omega = 2\pi f$ is in radians/sec. But the radial falloff law for a wave packet turns out to be a much less severe, power law. The reason is that the above harmonic exponential attenuation law implies a diffusion equation for wave packet pulse-forms, with radial distance in the role of "time"; that results in pressure pulses at long ranges which are convolutions of short range forms with the corresponding Green function. To determine the law of long-range propagation and absorption for wave packet pulse-forms, it is necessary therefore merely to evaluate solutions to that equation.

The simple diffusion equation that results from $\beta = \kappa\omega^2$ in linear acoustics must be generalized if relaxation effects are also present, as in the case of sea water for frequencies below 100 - 200 KHz, or so. Calculations from that important generalization are not given here.

The key assumption I have made is the dominance, asymptotically, of linear dissipation over nonlinear advection. Pulse propagation including nonlinear advection effects as well as viscous damping can be investigated to better approximation with an acoustic Burgers' equation treatment. A recent study of this equation by Crighton and Scott[3] has yielded an asymptotic form in agreement with that obtained herein. (See Eq. (4.8).) However, experiments readily yield the pulse-forms for $r \sim r_0$, which can supply initial data for linear analysis, and further theoretical work is planned to model close-in regions, as well as to compute effects of relaxation mentioned earlier for sea water. Finally, of great importance also will be the very close-in coupling region of the first mm, or so, because it is here that the dynamical arena for main pulse formation is located.

The organization of the report is as follows. In Chapter 2, I review the idealizations of acoustic theory and identify the scale length r_0 for dominance of linear viscosity in pulse propagation, absorption and distortion. In Chapter 3, I derive the diffusion equation for the pulse forms. In Chapter 4, I examine general properties of the solution for the propagating pulses. In Chapter 5, I work out a representative model example exactly and display general features derived in Chapter 4. In Chapter 6, I generalize to a complete class of examples, illustrating again broad propagation features identified earlier in Chapter 4, such as "pure tone" rapid attenuation, and long-range power law peak pressure falloff (r^{-1} and r^{-2}) for wave packet pulse forms. In Chapter 7, I discuss generalizations and extensions of the present work, including the "solitonic" framework for understanding results of the earlier linear theory developments. In Chapter 8, I conclude the report, and a mathematical appendix is attached at the end. (See Appendix A.)

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[3] Crighton, D. G., and Scott, J. F., Phil. Trans. Roy Soc. (London) A292, 13, 1979.

CHAPTER 2

REVIEW OF ACOUSTIC SCALES

The acoustic approximation is that of linearized hydrodynamics, with the fluid particle velocity v small in magnitude compared to the sound speed, defined as

$$c = \sqrt{(\partial p/\partial \rho)_s} \quad , \tag{2.1}$$

where it is assumed that the fluid particles undergo adiabatic processes.[4] In equation (2.1), ρ, p and s are particle pressure, density and specific entropy. The Navier-Stokes equation is

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \bullet \nabla\right) \vec{v} = - \frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{v} \; , \tag{2.2}$$

where $\nu = \mu/\rho$ is the kinematic viscosity, with $\mu \equiv 4/3\ \eta + \zeta$ the dynamic viscosity, and η, ζ the shear and bulk viscosities, respectively; for water,* $\nu = 0.045\ cm^2/sec$. Equation (2.2) neglects x,t dependence of μ arising from that of local pressure and temperature (T) through $\mu = \mu(p,T)$. Finally, the flow is assumed compressible, so that $\nabla \bullet \vec{v} \neq 0$, but irrotational, so that $\nabla \times \vec{v} = 0$ implies existence of a velocity potential ϕ,

$$\vec{v} = \nabla \phi \; . \tag{2.3}$$

For small density deviations $\rho' = \rho - \rho_0$, from the ambient value ρ_0, the pressure difference, $p' = p - p_0$, from the corresponding ambient value p_0 can be computed from equation (2.1) as

$$p' = c_0^2 \rho' \; , \tag{2.4}$$

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[4] Landau, L. D. and Lifshitz, E. M., Fluid Mechanics, (Reading, MA:Addison-Wesley (Pergamon), 1959), Chapter 8.

*At 20°C.

where c_0 is the acoustic (linearized) sound speed. Equation (2.4) holds to good approximation in the presence of irreversibility due to general viscous (and thermally induced*) damping, except for the extreme high frequency case of acoustic over-damping, which can be safely ignored.** Note that p' does not have to be small compared to p_0 in equation (2.4); only $\rho'/\rho_0 << 1$ is assumed, whence, from equation (2.4), $p'/\rho_0 C_0^2 << 1$. For water, $p' << 22.5$ Kbar. A second relation, assuming also $\rho'/\rho_0 << 1$, viz.

$$\frac{\partial \phi}{\partial t} = - \frac{1}{\rho_0} p' + \nu_0 \nabla^2 \phi \doteq - \frac{1}{\rho_0} p' , \qquad (2.5)$$

follows from equations (2.2) and (2.3) if the nonlinear advection term $(\vec{v} \bullet \nabla)\vec{v}$ is ignored, and if p_0, ρ_0 have vanishing gradients. The neglect of the viscosity term in comparison to the dominant first and second terms of equation (2.5) is justified again in the absence of acoustic over-damping.**

The equation of continuity,

$$\frac{\partial \rho}{\partial t} + \left(\vec{v} \bullet \nabla\right)\rho = - \rho \nabla \bullet \vec{v} , \qquad (2.6)$$

again to the assumptions of small ρ' and $\vec{v}$, becomes

$$\frac{\partial \rho'}{\partial t} = - \rho_0 \nabla \bullet \vec{v} . \qquad (2.7)$$

*The sound attenuation caused by thermal conduction can be included[4] by replacing ν in the expression for β (see below) by $\nu_{eff} = \nu + \kappa T \rho_0 c_0^2 (\tilde{\beta}/\rho_0 c_p)^2$, where for water (AIP Handbook, 1972), the thermal conductivity $\kappa = 5.8 \times 10^4$ erg/cm - sec - K and the volume coefficient of thermal expansion $\tilde{\beta} = 2.07 \times 10^{-4} K^{-1}$, both at 20°C, and where c_p is the specific heat at constant pressure. The increase this causes is only 0.02%, i.e., 10^{-5} cm^2/sec over $\nu = 4.5 \times 10^{-2}$ cm^2/sec.

**This neglect corresponds to the condition $\beta^{-1} >> \bar{\lambda}$, where $\bar{\lambda}$ is the reduced wavelength for a given Fourier component. Using the form $\beta = \frac{1}{2} c_0^{-3} \omega^2 \nu = \frac{1}{2} c_0^{-1} k^2 \nu$, the condition becomes $\lambda >> \pi \nu / c_0 \sim 90$ Å for $\nu = 0.045$ cm^2/sec. For comparison, at 1 MHz, $\lambda = 1.5$ mm, five orders of magnitude greater than 90 Å.

Combining equations (2.3), (2.4), (2.5), and (2.7), one gets

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0. \tag{2.8}$$

The smallness condition on $\vec{v}$ can be made precise for solutions ϕ obeying equation (2.8). For outgoing spherical waves, $r\phi = f(t - r/c_0)$ satisfies equation (2.8), so that

$$\frac{1}{c_0}\frac{\partial\phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r\phi) = \frac{\partial\phi}{\partial r} + \frac{\phi}{r} . \tag{2.9}$$

For a pulse disturbance having scale-length $\bar{\lambda}$, the last term is of order $\bar{\lambda}/r$ in comparison to $\partial\phi/\partial r$, and is negligible for sufficiently large r. Combining equations (2.3), (2.5), and (2.9) one gets

$$v_r/c_0 = - p'/\rho_0 c_0^2 \tag{2.10}$$

and the condition on fluid particle speed becomes $v/c_0 << 1$.

To include the <u>simplest</u> effects of damping, which is the purpose of the present report, consider the relative magnitudes of the advection and viscous (second and fourth) terms in equation (2.2). These are of order

$|(\vec{v} \bullet \nabla)\vec{v}| \sim v^2/\bar{\lambda}$, nonlinear term,

$|\nu\nabla^2\vec{v}| \sim \nu v/\bar{\lambda}^2$, viscous terms.

The ratio of the two is

$$\left|\frac{\text{nonlinear term}}{\text{viscous term}}\right| \sim \frac{v\bar{\lambda}}{\nu} \sim \frac{p'\bar{\lambda}}{\nu\rho_0 c_0} \sim \frac{p'}{\nu\rho_0\omega} , \tag{2.11}$$

by equation (2.10), where $\omega = c_0/\bar{\lambda}$ is the frequency corresponding to the time scale for the pulse. The condition that the viscous term dominate is thus

$$p' \leq 2\pi\nu\rho_0 f = 0.285f(\text{water}). \tag{2.12}$$

If f is in MHz, then p' is in bars; $f = 0.5$ MHz gives $p' \approx 142$ mbar for water; in this case, $p'/\rho_0 c_0^2 = 0.7 \times 10^{-5}$. Note here also that $c_0 = 1.5$ mm/μs gives $\lambda = 3$ mm, so the neglect of $\bar{\lambda}/r$ in equation (2.9) is justified for $r \gg 3$ mm.

The dominant ideal acoustic terms in equation (2.2) are of order

$$\left|\frac{\partial \vec{v}}{\partial t}\right| \sim \omega v \sim \omega p'/\rho_0 c_0 \ , \qquad \text{first term,}$$

$$\left|\frac{1}{\rho} \nabla p'\right| \sim p'/\rho_0 \bar{\lambda} \sim \omega p'/\rho_0 c_0 \ , \qquad \text{third term.}$$

In relation to these, the nonlinear advection term is

$$\left|\frac{\text{nonlinear term}}{\text{acoustic terms}}\right| \sim \frac{v^2/\bar{\lambda}}{\omega v} \sim \frac{v}{c_0} \sim \frac{p'}{\rho_0 c_0^2} \ . \tag{2.13}$$

So, for a moderately strong $p' = 22.5$ bar, the nonlinear term is only $(22.5/22.5 \times 10^5) = 10^{-5}$ times the dominant ideal acoustic terms.

When it can be assumed the peak pressure disturbances do satisfy the condition for the acoustic approximation, then (2.12) identifies a scale distance for validity of a linear absorption and propagation model, by the requirement that the peak pressure disturbances at r_0 satisfy

$$p'_{pk}(r_0) = 2\pi\nu\rho_0 f \ , \tag{2.14}$$

where f is a suitably chosen frequency scale value (Fourier component) for the disturbance at $r = r_0$. For smaller values of r the propagation may be acoustic to good approximation, but the viscous damping deviations from "ideal" behavior should be less then the nonlinear deviations. For $r > r_0$, however, linear damping should dominate the nonlinear effect unless a self-sustained equilibrium of viscous and advection effects conspires to yield a sustained Taylorlike shock.

I will examine the propagation of pulses from $r = r_0$ to longer ranges assuming that only the viscous effects need be considered. The results will be seen to agree with the asymptotic form found by Crighton and Scott,[5] in their analysis of the acoustic Burgers equation (see Section 7 herein).

[5] Crighton and Scott, 13, 1979.

CHAPTER 3

LINEAR ABSORPTION AND PROPAGATION ($r > r_o$)

A dispersion relation can be derived for the linearized equations. Using equation (2.4) to eliminate p´, equations (2.2) and (2.7) when linearized are

$$\frac{\partial \vec{v}}{\partial t} + \frac{1}{\rho_0}\nabla p' - \nu\nabla^2\vec{v} = 0 \tag{3.1}$$

$$\frac{\partial p'}{\partial t} + \rho_0 c_0^2 \nabla\cdot\vec{v} = 0 \tag{3.2}$$

Assuming a waveform exp $i(\vec{k}\cdot\vec{r} - \omega t)$,

$$(-i\omega + \nu\vec{k}^2)\vec{v} + i\rho_0^{-1}\vec{k}p' = 0 \tag{3.3}$$

$$i\rho_0 c_0^2 \vec{k}\cdot\vec{v} - i\omega p' = 0 \tag{3.4}$$

In the far field, $kr >> 1$, $\vec{k} = k\hat{r}$ and $\vec{v} = v\hat{r}$, where r is a unit radial vector; equations (3.3) and (3.4) become

$$(-i\omega+\nu k^2)v + i\rho_0^{-1}kp' = 0 \tag{3.5}$$

$$i\rho_0 c_0^2 kv - i\omega p' = 0 \tag{3.6}$$

which give the dispersion relation,

$$\det \begin{Vmatrix} -i\omega+\nu k^2 & i\rho_0^{-1}k \\ i\rho_0 c_0^2 k & -i\omega \end{Vmatrix} = 0, \tag{3.7}$$

having the solution

$$k = \frac{\omega}{c_0}(1 - i\nu\omega/c_0^2)^{-1/2} \tag{3.8}$$

$$\doteq \frac{\omega}{c_0} + i\,\frac{\omega^2}{2c_0^3}\nu \equiv \frac{\omega}{c_0} + i\kappa\omega^2. \tag{3.9}$$

This dispersion relation implies a diffusion equation for travelling wave pulse forms.

To derive this, notice first that ϕ, $v = v_r$, p', ρ' , at large r, all have the form $r^{-1}f(t - r/c_0)$; e.g.

$$p' = \frac{A}{r} f\ (t - r/c_0),\ kr>>1, \tag{3.10}$$

for the case of no damping. (See equations (2.9) and (2.10).) The case $\kappa = 0$ in equation (3.9) gives $k = \omega/c_0$, which implies the equation

$$\frac{\partial f}{\partial r} = - \frac{1}{c_0} \frac{\partial f}{\partial t}\ , \tag{3.11}$$

since it holds for every Fourier component; equation (3.11) is obeyed by $rp' = A\ f(t - r/c_0)$. For $\kappa \neq 0$, equation (3.9) gives

$$\frac{\partial f}{\partial r} = - \frac{1}{c_0} \frac{\partial f}{\partial t} + \kappa \frac{\partial^2 f}{\partial t^2}, \tag{3.12}$$

while the variable change $(r, t) \rightarrow (r', \tau')$, where $r' = r$ and $t' \equiv \tau = t - r/c_0$, so that

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial r'} - \frac{1}{c_0} \frac{\partial}{\partial t'}\ ,\ \frac{\partial}{\partial t} = \frac{\partial}{\partial t'}\ , \tag{3.13}$$

gives

$$\frac{\partial f}{\partial r} = \kappa \frac{\partial^2 f}{\partial \tau^2}\ , \tag{3.14}$$

where I dropped the (formal) prime on r in the last line.

Equation (3.14) is a diffusion equation for the acoustic amplitude, with r in the role of the "time" and with "spatial" variable $\tau \equiv t - r/c_0$ being the physical time variable for the pulse <u>form</u> itself. Thus, asymptotically,

$$p' = \frac{A}{r} f\ (r,\tau), \tag{3.15}$$

where f satisfies equation (3.14). Typically, r_0 is already in the far-field, so near-field "corrections" to equation (3.15) are physically uninteresting.

Let $f = f(r_0,\tau) \equiv f_0(\tau)$ be the pulse-form at $r = r_0$; the solution to the initial value problem $f^0 = f_0$ for $f(\tau,r)$, the pulse form at $r > r_0$, is given from equation (3.14) to be

$$f(r,\tau) = (4\pi\kappa p)^{-1/2} \int d\tau' f_0(\tau') \exp\{-(\tau' - \tau)^2/4\kappa\rho\}, \tag{3.16}$$

where $\rho = r - r_0$. In evaluating equation (3.16) it is important to remember that spherical sound waves have zero impulse;[4] this follows from equation (2.5),

$$\int_{-\infty}^{+\infty} d\tau\, p' = -\rho_0 \int_{-\infty}^{+\infty} d\tau\, \partial\phi/\partial\tau = 0 \tag{3.17}$$

To check this, we can substitute equations (3.15) and (3.16) into equation (3.17),

$$\int_{-\infty}^{+\infty} d\tau\, p' = A r^{-1} \int_{-\infty}^{+\infty} d\tau' f_0(\tau')\,(4\pi\kappa\rho)^{-1/2} \int_{-\infty}^{+\infty} d\tau\, \exp\{-(\tau'-\tau)^2/4\kappa\rho\}$$

$$A r^{-1} \int_{-\infty}^{+\infty} d\tau' f_0(\tau') \tag{3.18}$$

So equation (3.17) will be met if it is met at r_0, i.e., if

$$\int_{-\infty}^{+\infty} d\tau' f_0(\tau') = 0 \tag{3.19}$$

CHAPTER 4

GENERAL PROPERTIES OF THE SOLUTION

For pulse forms $f_0(\tau)$ having finite width τ_0, equation (3.16) implies another scale length $r_1 > r_0$, defined by the condition that τ_0 equal the width of the Gaussian propagator, which is $2 \times (2\kappa\rho)^{1/2}$, i.e.,

$$r_1 - r_0 \equiv \rho_1 = (8\kappa)^{-1} \tau_0^2 \qquad (4.1)$$

This can be written also as

$$r_1 - r_0 = \rho_1 = \frac{1}{8} \beta_0^{-1} , \qquad (4.2)$$

where

$$\beta_0 = \kappa \tau_0^{-2} \approx \kappa (\Delta\omega)^2 , \qquad (4.3)$$

in which $\Delta\omega/2\pi$ is the pulse bandwidth. The three spatial regions, divided by r_0 and r_1, are shown in Figure 2.

If $r << r_1$, the Gaussian is narrow compared to $f_0(\tau)$ and behaves like a δ-function in equation (3.16) if $f_0(\tau)$ is smooth enough,

$$(4\pi\kappa\rho)^{-1/2} \exp \left[-(\tau' - \tau)^2 / 4\kappa\rho \right] \longrightarrow \delta(\tau - \tau'), \qquad (4.4)$$

giving

$$f(r,\tau) \doteq f_0(\tau), \qquad r << r_1, \qquad (4.5)$$

a result corresponding to no distortion. From equation (3.15),

$$\phi(r,t) \approx \frac{A}{r} f_0(\tau), \qquad r << r_1 . \qquad (4.6)$$

Where $r >> r_1$, the Gaussian is broad compared to f_0, and may be expanded about $\tau' = 0$. Now equation (3.16) gives

$$f(r,\tau) = (4\pi\kappa\rho)^{-1/2}\int d\tau'\ f_0(\tau')\left\{\exp(-\tau^2/4\kappa\rho) - \tau'\ \frac{d}{d\tau}\left[\exp(-\tau^2/4\kappa\rho)\right] + \ldots\right\}$$

$$= (4\pi\kappa\rho)^{-1/2}\ \frac{d}{d\tau}\left[\exp(-\tau^2/4\kappa\rho)\right]\bullet\left(-\int_{-\infty}^{+\infty} d\tau'\ \tau'\ f_0(\tau')\right), \quad (4.7)$$

neglecting higher terms, and where I have made use also of the zero-impulse property, equation (3.19). Evaluating equation (4.7),

$$f(r,\tau) = \frac{2}{\sqrt{\pi\kappa^3}}\left(\int_{-\infty}^{+\infty} d\tau'\ \tau'\ f_0(\tau')\right)\rho^{-3/2}\ \tau\ \exp(-\tau^2/4\kappa\rho),\quad r >> r_1, \quad (4.8)$$

which agrees with the form found by Crighton and Scott.[6] The two peaks of the pulse are at $\tau = \pm (2\kappa\rho)^{1/2}$, for an overall width of $2(2\kappa\rho)^{1/2}$, while the peak amplitude is

$$f_{pk} = f(r,(2\kappa\rho)^{1/2}) = 2\kappa^{-1}\sqrt{\frac{2}{e\pi}}\left(\int_{-\infty}^{+\infty} d\tau'\ \tau'\ f_0(\tau')\right)\rho^{-1},\quad r >> r_1. \quad (4.9)$$

From equation (3.15), taking $\rho = r - r_0 \doteq r$, one finds

$$p'_{pk} \approx 2A\ \kappa^{-1}\sqrt{\frac{2}{e\pi}}\left(\int_{-\infty}^{+\infty} d\tau'\ \tau'\ f_0(\tau')\right) r^{-2},\quad r >> r_1, \quad (4.10)$$

so the very long range peak pressure falloff law is r^{-2}. The energy in the pulse involves the integral of p'^2 over all τ, and, owing to the spreading of the packet in equation (4.8), goes like $r^{-7/2}$.

If $f_0(\tau)$ were a pure harmonic, $f_0 = C \sin \omega_S\tau$, $-\infty < \tau < +\infty$, $\Delta\omega$ would be zero and equations (4.2) and (4.3) would give $r_1 = \infty$. In this case, $r >> r_1$ is impossible. Although $r << r_1$, accordingly, is well met, the Gaussian does not act like a δ-function and equation (4.6) doesn't apply either. Equation (3.16) can be evaluated exactly, and one gets

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[6]Crighton and Scott 13, 1979.

$$f(r,\tau) = (4\pi\kappa\rho)^{1/2} \int_{-\infty}^{+\infty} d\tau' \, C\sin \omega_S\tau' \exp\left[-(\tau' - \tau)^2/4\kappa\rho\right]$$

$$= (4\pi\kappa\rho)^{1/2} \, \mathrm{Im}\left[C\exp i\omega_S\tau \int_{-\infty}^{+\infty} d\tau'' \exp\left(i\omega_S\tau'' - (4\kappa\rho)^{-1}\tau''^2\right)\right] \quad (4.11)$$

$$= \mathrm{Im}\left[C\exp\left(i\omega_S\tau - \kappa\omega_S^2\rho\right)\right] = \exp(-\beta_S\rho)\, f_0(\tau), \quad \beta_S = \kappa\omega_S^2$$

which is the correct pure tone attenuation law.

From a practical standpoint, to produce large values of r_1 in equation (4.2), which gives sound levels like those in equation (4.6) for longer ranges, equation (4.3) shows that what is needed is small pulse bandwidth $\Delta\omega$. Equation (4.11) then suggests further that a "pure tone carrier" on the pulse will be attenuated expotentially at a rate appropriate to its frequency.

I will exhibit these qualitative features explicitly in exactly soluble cases in the next two sections.

CHAPTER 5

REPRESENTATIVE EXAMPLE

A convenient analytical model of the pulses observed at long ranges experimentally* is (C < 0)

$$f_0(\tau) = C\tau \exp(-\tau^2/2t_1^2), \quad -\infty < \tau < +\infty, \tag{5.1}$$

for which equation (3.16) can be evaluated exactly (see Figure 3). We have

$$f(r,\tau) = (4\pi\kappa\rho)^{-1/2} \, C \int_{-\infty}^{+\infty} d\tau' \tau' \exp\left[-\frac{1}{2t_1^2}\tau'^2 - \frac{1}{4\kappa\rho}(\tau' - \tau)^2\right]$$

$$= C\tau \bullet \left(1 + \frac{2\kappa\rho}{t_1^2}\right)^{-3/2} \exp\left[-\tau^2 \Big/ 2t_1^2 \left(1 + \frac{2\kappa\rho}{t_1^2}\right)\right] \quad . \tag{5.2}$$

The width of $f_0(\tau)$ is $2t_1$, the distance between peaks at $\tau = \pm t_1$ ($2t_1$ is half the period of the single-cycle oscillation shown in Figure 1), so equation (4.1) gives $\rho_1 \sim (8\kappa)^{-1}(2t_1)^2 = t_1^2/2\kappa$, which conveniently represents (but somewhat underestimates**) the radial distance for transition of the form in equation (5.2) between short and long range limits. Evidently, equation (5.2) displays the limiting behavior predicted in Chapter 4 for $r \ll r_1$ (equation (4.5)) and $r \gg r_1$ (equation (4.8)). Over the intermediate ranges the pulse form (5.1) is preserved here, and the pulse spreads, having width

$$\tau_0 = \tau_0(r) = 2t_1 \bullet \left(1 + \frac{2\kappa\rho}{t_1^2}\right)^{1/2} = 2t_1 \bullet \left(1 + \frac{\rho}{\rho_1}\right)^{1/2} \quad . \tag{5.3}$$

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*At shorter ranges, the pulse looks like a sawtooth. See also Reference 7.

**See below, page 16 (eq. (5.8)).

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[7] Crighton, D. G., and Scott, J. F., Phil. Trans. Roy. Soc., (London) A292, 13, 1979.

The pulse bandwidth, $\Delta f = \Delta\omega/2\pi$, is a function of ρ,

$$\Delta f = 1/2\pi\tau_0 = \left(4\pi\tau_1\sqrt{1 + 2\kappa\rho/t_1^2}\right)^{-1} . \qquad (5.4)$$

For one unfocussed beam condition, $2t_1 \doteq 0.2$ μsec, and peak pressures are typically 1 bar at $r \approx 0.1$m, vertically beneath the laser spot. For certain focussed beam conditions, $2t_1 \approx 1$ μsec, while 1 bar peak pressures are seen around $r \approx 0.5$m, again vertically beneath the spot. From equation (2.14) with $f \sim (4t_1)^{-1}$, these values correspond to r_0(unfoc.) ~ 0.14m and r_0(foc.) ~ 3.5m. To get r_1, take $\tau_0 = 2t_1$ in equation (4.1) so that $\rho_1 = t_1^2/2\kappa$, and use $\kappa = 0.6 \times 10^{-17}$ sec^2/cm; this gives r_1(unfoc.) $\approx \rho_1 = 8$m and r_1(foc.) $\approx \rho_1 = 200$m. Summarizing, for the pulse in equation (5.1), linear theory predicts $1/r$ peak pressure falloff out to r_1, roughly, and $1/r^2$ falloff at longer ranges beyond-- where $r_1 \sim 8$m under unfocussed conditions, and $r_1 \sim 200$m under certain typical focussed conditions.

Combining equations (3.15), (4.1) and (5.2) (with C = - 1), the expression of the acoustic pressure is

$$p' = \frac{A}{r} \bullet \frac{-\tau}{(1 + \rho/\rho_1)^{3/2}} \bullet \exp\left[-\frac{1}{2}\tau^2/t_1^2\,(1 + \rho/\rho_1)\right] \qquad (5.5)$$

$$= \frac{p_0' r_0}{r(1 + \rho/\rho_1)^{3/2}} \bullet \frac{-\tau}{t_1} \bullet \exp\frac{1}{2}\left[1 - \tau^2/t_1^2\,(1 + \rho/\rho_1)\right] , \qquad (5.6)$$

where p_0' is the peak pressure at $r = r_0$. The peak pressure for $r > r_0$ is

$$p_{pk}' = \frac{p_0'\, r_0}{r(1 + \rho/\rho_1)} \qquad (5.7)$$

Equation (5.7) is plotted in Figure 4 for the focussed beam example above: $r_0 = 3.5$m, $p_0' = 142$ mbar, $\rho_1 = 200$m. The cross-over point, of the intersection of r^{-1} and r^{-2} asymptotes, is at $r_1' = \sqrt{e} \bullet \rho_1 = 1.649\,\rho_1 \doteq 330$m.

Since ρ_1 is proportional to t_1^2, it can be extended considerably if t_1 is increased experimentally. Numerically,

$$r_1' = \sqrt{e} \bullet \rho_1 = 0.33 \text{ km} \bullet (2t_1)^2, \; t_1 \text{ in μsec.} \qquad (5.8)$$

Conversely, ρ_1 can be reduced considerably by reducing t_1 experimentally.

This identifies the practical problems of (i) pulse forming in the very close in coupling region, at the spot, and (ii) what pulse forms might be best for desired pressures at long range. In particular smaller period pulses can be studied more conveniently experimentally due to the smaller scale required by ρ_1. Equation (5.7) is plotted again in Figure 5 for the higher frequency, unfocussed beam example above: r_0 = 14 cm. p_0' = 7.4 mbar, and ρ_1 = 8m. In this way, the theoretical predictions of the asymptotic linear theory described in the present report can be confronted.

CHAPTER 6

MORE GENERAL PULSE FORMS

The example equation (5.1) is a special case of a general (and mathematically complete) class, which I will display in this section, in part for possible later utility. In preparation for this, consider again equation (3.16), but in the frequency domain now $f(\rho,\tau)$ is a convolution, so its Fourier transform is a product. Let

$$f(\rho,\tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, \tilde{f}(\rho,\omega)\, e^{-i\omega\tau} \qquad (6.1)$$

stand to define the Fourier transform conventions, as well as $\tilde{f}(\rho,\omega)$. Inverting equation (3.16) gives immediately

$$\tilde{f}(r,\omega) = \tilde{f}_0(\omega)\, e^{-\kappa\rho\omega^2} , \qquad (6.2)$$

which, by the way, constitutes a recovery of the harmonic exponential attenuation law. Combining equations (6.1) and (6.2), we get another formula for $f(\rho,\tau)$, equivalent to equation (3.16), and which can also be derived directly from the harmonic attenuation law by the superposition principle,

$$f(\rho,\tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, \tilde{f}_0(\omega)\, e^{i\omega\tau-\kappa\rho\omega^2} . \qquad (6.3)$$

The zero-impulse condition, equation (3.19), requires, finally,

$$\tilde{f}_0(0) = 0 . \qquad (6.4)$$

For the example in the last section, equation (5.1), one has

$$\tilde{f}_0(\omega) = i\sqrt{2\pi}\, C t_1^3\, \omega \exp\left(-\frac{1}{2} t_1^2 \omega^2\right) . \qquad (6.5)$$

Evidently, $f_0(\tau)$ and $\tilde{f}_0(\omega)$ are exactly the same functional forms. This is a property shared by all the eigen-functions of the quantum mechanical harmonic oscillator owing to the symmetry of the Hamiltonian under interchange of position and momentum, viz. of $H = 1/2(p^2 + x^2)$. The eigen-functions are products of a common Gaussian factor, $\exp(-\tau^2/2t_1^2)$ in the present notation, and Hermite polynomials, $H_m(\tau/t_1)$, m = 0, 1, ... It can be shown that the zero-impulse condition selects the odd-parity oscillator eigen-functions, allowing only m = 1, 3, 5, ... Equations (5.1) and (6.5) are the lowest, m = 1, eigen-functions. In fact, any moderately well-behaved function $f_0(\tau)$, tending to zero at $\pm\infty$ sufficiently rapidly, and satisfying equations (5.19), can be expanded,

$$f_0(\tau) = \sum_{m:odd} c_m H_m(\tau/t_1) \exp\left(-\frac{1}{2}\tau^2/t_1^2\right), \tag{6.6}$$

where, explicity,[8]

$$c_m = \frac{1}{2^m m! \sqrt{\pi} t_1} \int_{-\infty}^{+\infty} d\tau f_0(\tau) H_m(\tau/t_1) \exp\left(-\frac{1}{2}\tau^2/t_1^2\right). \tag{6.7}$$

For single component, "Hermite-pulse" forms, having $c_m \neq 0$, for all $m \neq n$, viz. for

$$f_0 = f_{0n}(\tau) = c_n H_n(\tau/t_1) \exp\left(-\frac{1}{2}\tau^2/t_1^2\right), \tag{6.8}$$

the function f_0 possesses exactly $1/2\bullet(n + 1)$ cycles. For example, $f_{01}(\tau)$ in equation (5.1) goes through a single full oscillation cycle; for n = 3, the Hermite polynomial is an alternating cubic and $f_{03}(\tau)$ exhibits two oscillation cycles, the Gaussian factor pulling the polynomials down toward zero at infinity. The cases n = 3 and 5 are shown in Figure 6. The n = 3 case is not a bad representative of the lower curve in Figure 1; but for large n, the curves will tend towards symmetry about the mid-point, where the peaks will be smallest, and these are rather "unlikely" individual shapes. Nevertheless, the large n curves are pulse-carriers of increasing overall (pulse) width, of order $2n^{1/2}t_1$ so they are useful for a brief propagation study here. (The carrier frequency is of order $1/2\bullet(n + 1)/2n^{1/2}t_1 \doteq \sqrt{n}/4t_1$.)

The propagated Hermite-pulse forms, from equation (6.8), can be evaluated in closed form from equation (6.3). I do this in the Appendix; the result, equation (A.15) with the constant c_n re-inserted, is

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[8] See, for instance, Eugen Merzbacher, <u>Quantum Mechanics</u>, (New York: John Wiley & Sons Inc., 1961), pg. 56 et seq., for properties of the oscillator eigen-functions.

$$f_n(\rho,\tau) = c_n \frac{t_1}{t_\rho}\left(\frac{2t_1^2}{t_\rho^2} - 1\right)^{n/2} H_n\left[\left(\frac{2t_1^2}{t_\rho^2} - 1\right)^{-1/2} \bullet \frac{t_1\tau}{t_\rho^2}\right] \exp\left(-\frac{1}{2}\tau^2/t_\rho^2\right), \qquad (6.9)$$

where

$$t_\rho = \left(t_1^2 + 2\kappa\rho\right)^{1/2}. \qquad (6.10)$$

Equation (6.9) for $f_n(\rho,\tau)$ reduces to $f_{on}(\tau)$ for $\rho = 0$, and displays $1/2 \bullet (n + 1)$ oscillation cycles, accordingly. For $n > 1$, the oscillations disappear completely beyond $\rho = t_1^2/2\kappa$ since this is the value for which the common agrument of the square roots in equation (6.9) changes sign. At $\rho = t_1^2/2\kappa$, we have

$$f_n = c_n \frac{1}{\sqrt{2}} 2^n \left(\frac{\tau}{2t_1}\right)^n \exp\left(-\tau^2/4t_1^2\right) \qquad (6.11)$$

$$= \frac{c_n}{\sqrt{2}} \left(\tau/t_1\right)^n \exp\left(-\tau^2/4t_1^2\right),$$

where I have used $H_n(\xi) = 2^n\xi^n + O(\xi^{n-1})$ for large ξ. Peak values of this pulse-form expression occur at $\tau = \pm(2n)^{1/2}t_1$. Beyond $\rho = t_1^2/2\kappa$, the argument of H_n is imaginary, and all the coefficients of the polynomial take the same sign (after extracting a factor i.) See equations (A.17).) So, all that is left of the Hermite-pulse is two endpoint transient-pulse oscillations; the (n - 1) cycle "Hermite-carrier" center of the pulse is gone! See Figure 7.

CHAPTER 7

GENERALIZATION AND EXTENSIONS

The observation that acoustic pulses undergoing linear attenuation with an extinction coefficient proportional to ω^2 should obey a diffusion equation is not new, and was made long ago by Landau and Lifshitz.[9]

However, in the literature of subsequent acoustical theory, Burgers' equation,

$$\frac{\partial u}{\partial x} + u\,\frac{\partial u}{\partial \tau} = \delta \cdot \frac{\partial^2 u}{\partial \tau^2}\ , \qquad (7.1)$$

has been used to represent combined nonlinear, and linear viscosity propagation effects.[10] The advantage to this is that equation (7.1) can be solved exactly.[11] For one-dimensional geometry, Blackstock has derived*

$$\frac{\partial u}{\partial x} - c_0^{-2}\left(1 + \frac{B}{2A}\right) u\,\frac{\partial u}{\partial \tau} = \kappa\,\frac{\partial^2 u}{\partial \tau^2}\ , \qquad (7.2)$$

where B/A is the nonlinearity parameter arising from the equation of state $p = p(\rho, s)$,

$$p = p_0 + A\rho_0^{-1}\rho' + \frac{1}{2}B\rho_0^{-2}\rho'^2 + \ .\ .\ .) \qquad (7.3)$$

and where u is the fluid particle velocity. As previously, $\tau = t - c_0^{-1}x$ is the retarded time. Notice equation (7.2) reduces to equation (3.14) if the nonlinear term is dropped.

For spherical waves, a more complicated form taking into account additional nonlinearity effects associated with spherical spreading, and neglected in the

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[9] See Reference 3, page 301.

[10] Crighton and Scott,13, 1979.

[11] Whitham, G. B., Linear and Nonlinear Waves, (New York: John Wiley & Sons, Inc., 1974), Chapter 4.

* The most readable source, with primary references, is Beyer's "Nonlinear Acoustics."

analysis I have given, was also derived by Naugol'nykh and co-workers[12,13] and by Blackstock.[14] Starting from

$$\frac{\partial u}{\partial r} + r^{-1} u - c_0^{-1} \left(1 + \frac{B}{2A}\right) u \frac{\partial u}{\partial \tau} = \kappa \frac{\partial^2 u}{\partial \tau^2} , \tag{7.4}$$

from which again equation (3.14) for f = ru follows if the nonlinear term is dropped, a modified Burgers' equation was derived for the quantity $W = u \bullet \log r/r_0$, where r_0 is a "source radius," and where the independent variable r is replaced by a "stretched coordinate," proportional to $\log r/r_0$. In the modified form, δ in equation (7.1) is replaced by a function of r, and the resulting "Burgers' equation" has not been solved exactly.[15]

These generalizations can be undertaken for the present problem, but the extension to the more general form of $\beta(\omega)$, to take account of the knees associated with onset of chemical relaxation, of $MnSO_4$ disassociation and recombination primarily, in sea water is also of interest. See Figure 8. The dispersion relation equation (3.9) is replaced by

$$k \doteq \frac{\omega}{c_0} + i\omega^2 \left(\kappa + \kappa' \frac{1}{1 + (\omega\tau_s)^2}\right) \left(1 - \mu P\right), \tag{7.5}$$

where τ_s is a relaxation time, and κ' is proportional to the salinity[12] S;

$$\frac{\kappa'}{\kappa} = \frac{AS}{B} = 0.692S, \tag{7.6a}$$

with A and B (new) constants, and S in ppt; in Eq. (7.5), also, P is the ambient pressure and $\mu = 1.654 \times 10^{-3} (\text{bar})^{-1}$. In addition, τ_s is temperature dependent

$$2\pi\tau_s^{-1} \quad f_s = 219 \times 10^{5-1520/(T + 273)} \text{ KHz}, \tag{7.6b}$$

where T is in Celsius degrees. For T = 20°, f_s = 142 KHz, and the point of inflection of $\log \alpha$ vs. $\log f$ is at $f_i = (1 + \kappa'/\kappa)^{1/4} f_s$ = 310 KHz for S = 35. This marks the location of the knee in Figure 8. Below 100 KHz, or so, the second term tends increasingly to dominate equation (7.5) and one has

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[12] Naugol'nykh, K., Sov. Phys.-Acoustics, 5, 1959, p.79.

[13] Naugol'nykh, K., Soluyan, S. I., and Khokhlov, R. V., Sov. Phys.-Acoustics, 9, 1963, p. 42.

[14] Blackstock, D. T., JASA, 36, 1964, p.217.

[15] Crighton and Scott, 1979, p. 13.

$$k \doteq \frac{\omega}{c_0} + i\omega^2\kappa' \tag{7.7}$$

since now $(\omega\tau_s)^2 = (f/f_s)^2 << 1$. For T = 25°C and S = 35 again, $\kappa' \sim 25\kappa$.

In the linear regime, equation (6.3) is readily adapted to sea water application, without finding the differential equation to which (7.5) corresponds. In fact, one has simply, from linearity,

$$f(\rho,\tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\ \tilde{f}_0(\omega) e^{i\omega\tau - \beta(\omega)\rho} , \tag{7.8}$$

where for $\beta(\omega)$ one inserts the imaginary part of k from equation (7.5). For the nonlinear viscous case, however, this procedure is too simple, and can not be generally applicable.[16]

Returning to the simple Burgers' equation, it is fundamentally related to the linear diffusion equation because the equations,[17]

$$\frac{\partial U}{\partial \tau} = - \frac{1}{2}\delta^{-1}\ uU \tag{7.9}$$

$$\frac{\partial U}{\partial x} = - \left(2\delta \frac{\partial u}{\partial \tau} - u^2 \right) \frac{U}{4\delta} , \tag{7.10}$$

comprise a Bäcklund transformation[18] implying

$$\frac{\partial u}{\partial x} + u \frac{\partial u}{\partial \tau} = \delta \frac{\partial^2 u}{\partial \tau^2} \tag{7.11}$$

$$\frac{\partial U}{\partial x} = \delta \frac{\partial^2 U}{\partial \tau^2} \tag{7.12}$$

Equation (7.11) is Burgers' equation, of course, and its solutions are constructed from those of equation (7.12) by means of either of equations (7.9) and (7.10).

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[16] Crighton and Scott 15, 1979.

[17] Whitham, G. B., "Linear," Chapter 4.

[18] Ablowitz, Mark J. and Segur, Harvey, Solutions and the Inverse Scattering Transform, (Philadelphia:SIAM, 1981), Chapter 3.

The solitonic behaviour commonly observed in problems admitting this kind of treatment is exemplified in a theorem applying to completely integrable systems:[19] if a solution of an initial value problem on $-\infty < \tau < +\infty$ contains "solitons" then as $r \to \infty$ the "solitons" remain $O(1)$. For acoustic transients, $f = rp$ is $O(r^{-1})$ asymptotically which is in between $O(1)$, which holds in the absence dissipation, and $O(\exp - \beta r)$, which holds for pure-tone pulses undergoing linear dissipation.

[19] Ablowitz and Segur, "Solutions," Chapter 3, p.68

CHAPTER 8

CONCLUSION

Laser energy absorption in water produces experimentally observed ultrasonic transients, which are predicted from acoustical theory to exhibit "solitonic" propagation behaviour at long ranges. Beyond a scale range r_0, at which viscous effects become comparable to nonlinear effects in governing the propagation, the peak pressure falloff at first obeys a $1/r$ law, but eventually, for $r >> r_1$, a second scale range, it goes over to a $1/r^2$ law. This result agrees with that of a recent theoretical analysis of the asymptotic solution to the acoustic Burgers' equation for spherical geometry.[20] Pressure transient pulse forms obey a diffusion equation in the linear regime, with r in the role of the "time" and diffusion coefficient $\kappa = \beta/\omega^2$ for fresh water, where β is the harmonic attenuation constant. When nonlinearities are included, the diffusion equation gives way to a Burgers' equation, which will apply for $r < r_0$. When $MgSO_4$ relaxation in sea water is important, the propagation can be determined for long ranges, $r >> r_0$, from linear theory.

In the present paper, I have examined the propagation of pulse-forms very roughly representative of those seen experimentally, and described their properties in detail for the linear regime, $r > r_0$, for fresh water.

[20] Crighton and Scott, 13, 1979.

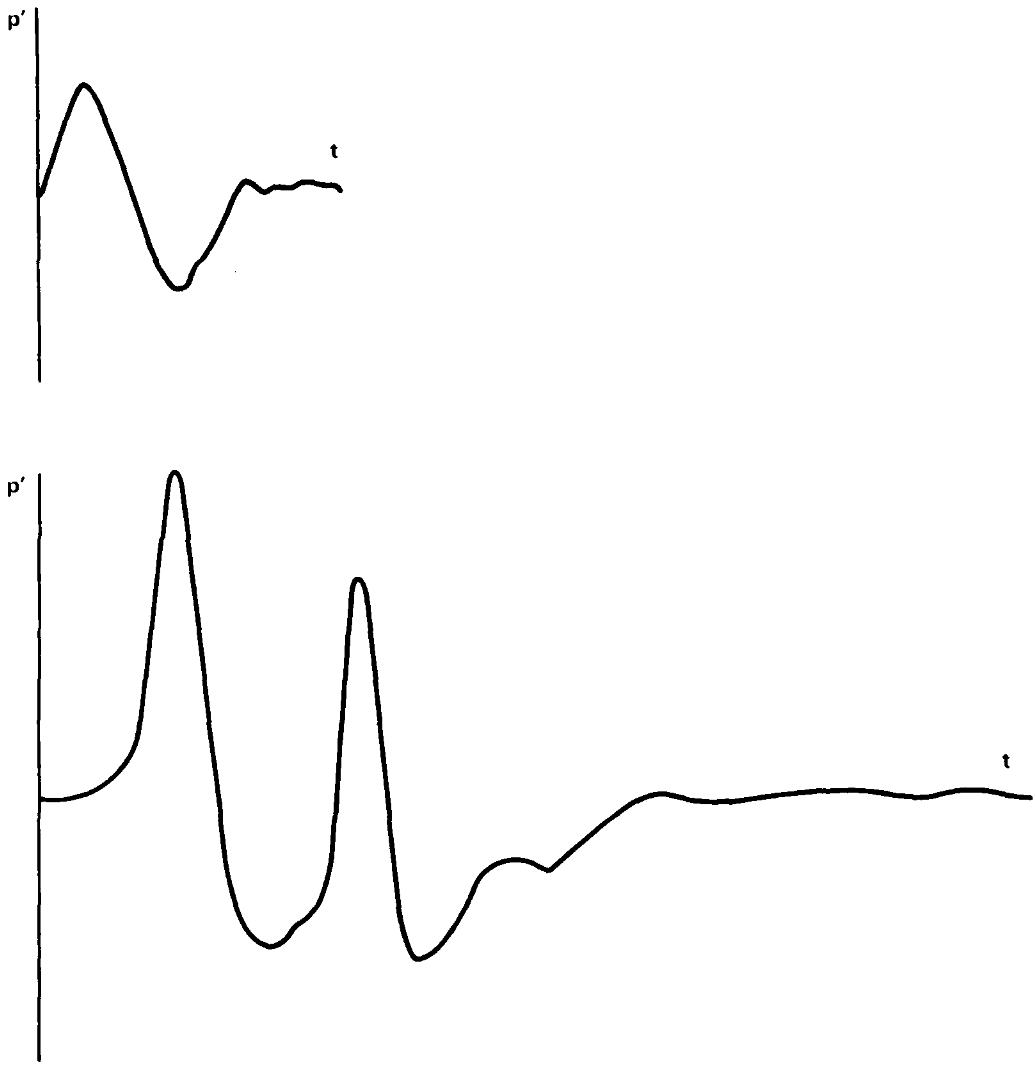

FIGURE 1. TYPICAL PRESSURE TRANSIENT PULSE-FORMS IN WATER INDUCED BY CO_2 LASER ABSORPTION AT THE SURFACE

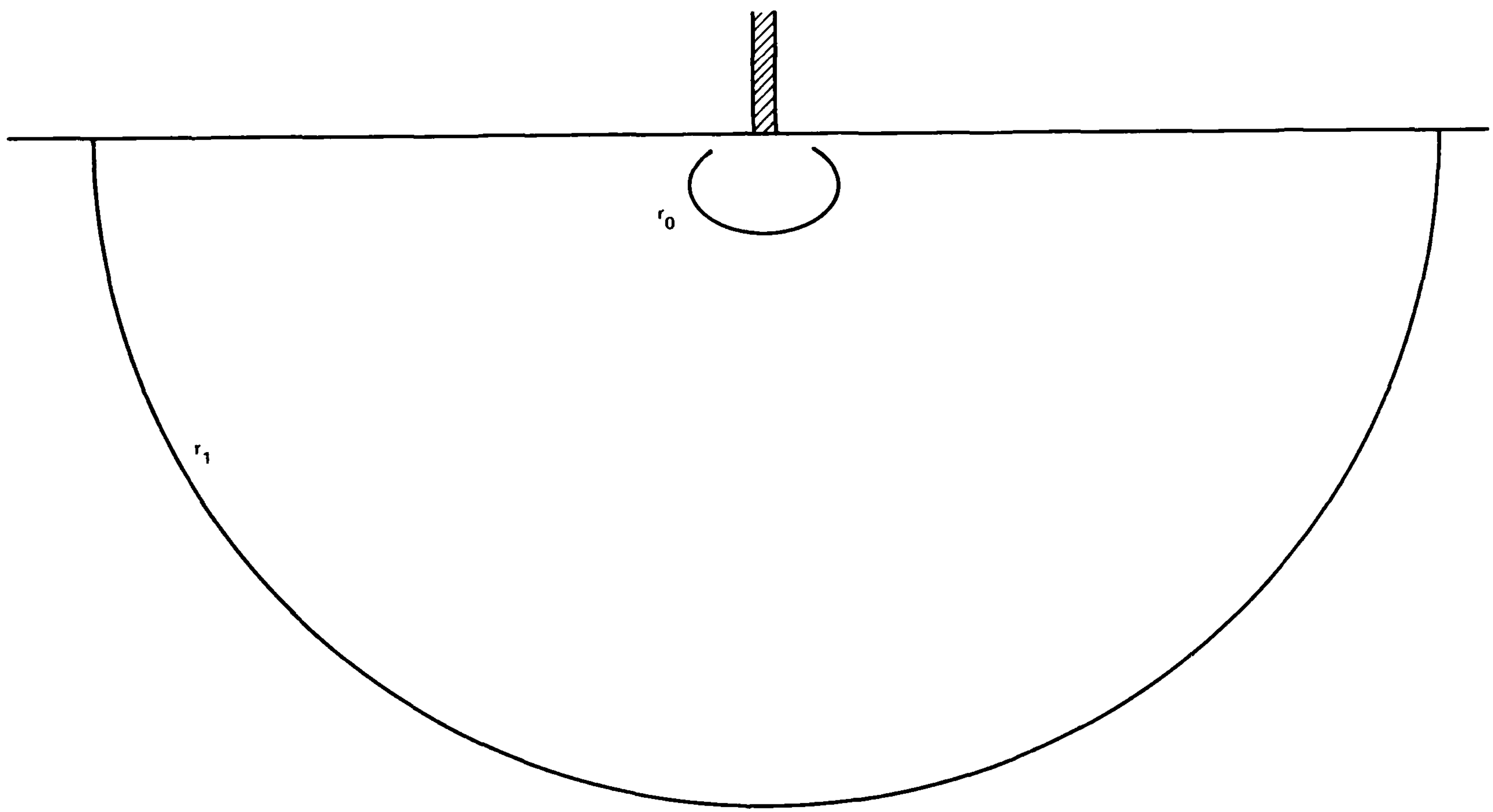

FIGURE 2. SCALE RADII r_0, r_1, DIVIDE THE ACOUSTIC FIELD INTO THREE DOMAINS (r_0 IS DEFINED AS THE RADIAL DISTANCE FROM THE LASER SPOT FOR WHICH (cgs) $p \approx \nu f/2\pi = (2/7)f$, WHERE $f = T^{-1}$, WITH T THE PERIOD OF THE PULSE. $r_1 \cong r_0 + \tau_0^2/8x$, WHERE τ_0 IS THE WIDTH OF THE PULSE ($\tau_0 \approx \frac{1}{2}$ T FOR A SINGLE OSCILLATION PULSE).)

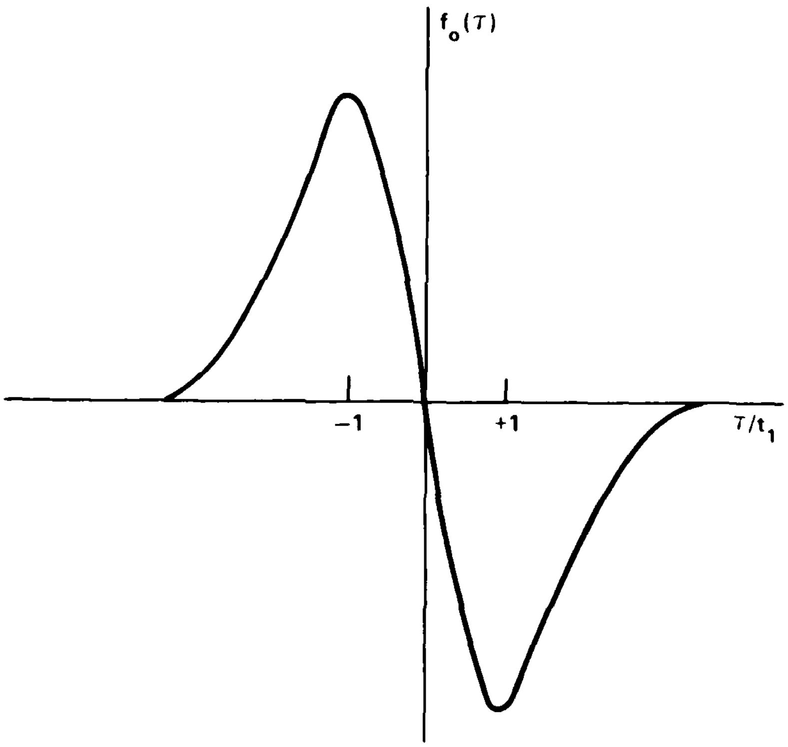

FIGURE 3. PLOT OF $F_0(\tau)$, FOR C = 1 (EQUATION (5.1)), USED TO MODEL THE UPPER PULSE SHOWN IN FIGURE 1

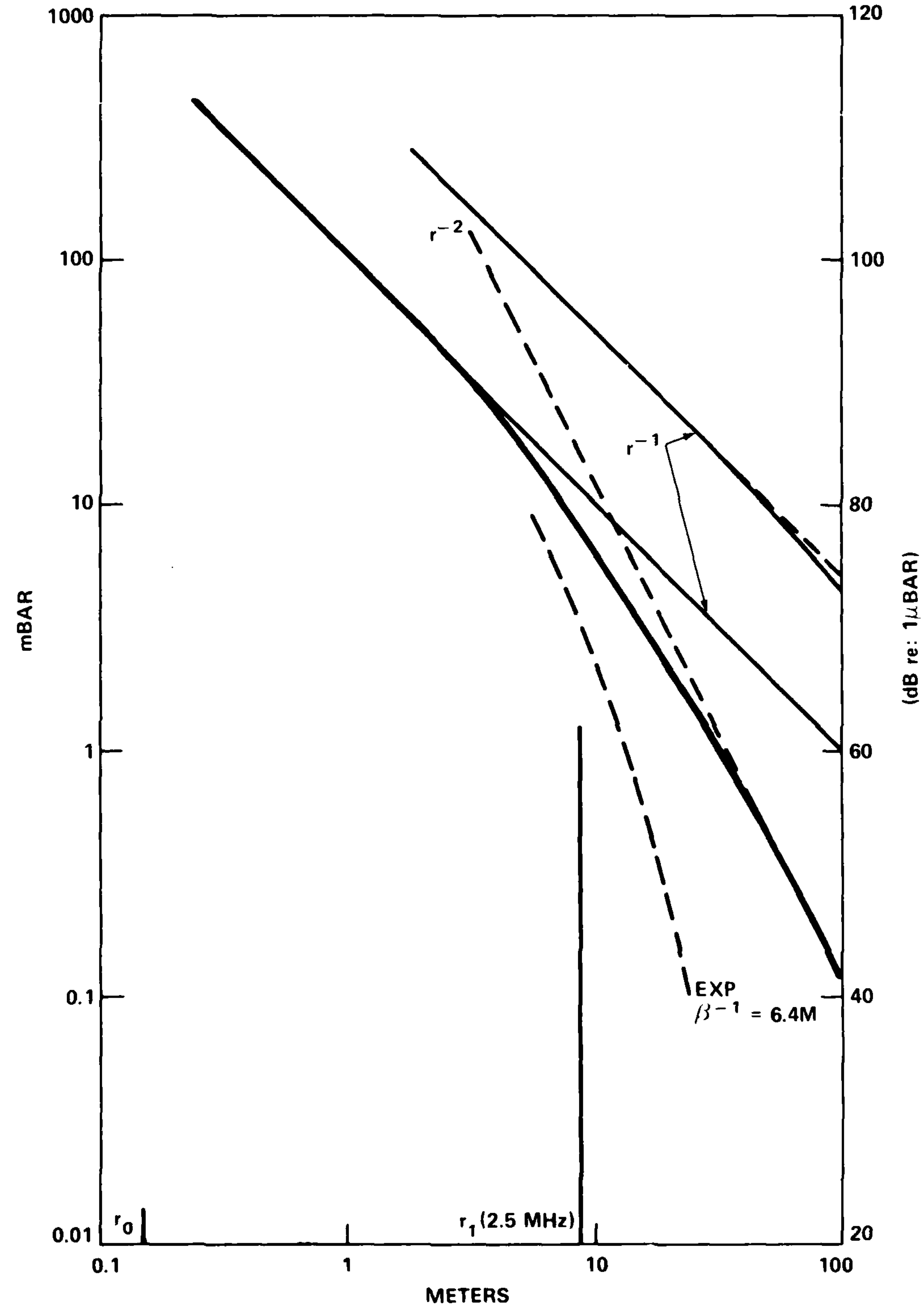

FIGURE 4. PEAK PRESSURE AS A FUNCTION OF r FROM EQUATION (5.7), FOR REPRESENTATIVE OBSERVED PULSE PARAMETER VALUES (DARK CURVE) (r_1 IS THE SCALE LENGTH FOR CROSS-OVER to r^{-2} BEHAVIOR.)

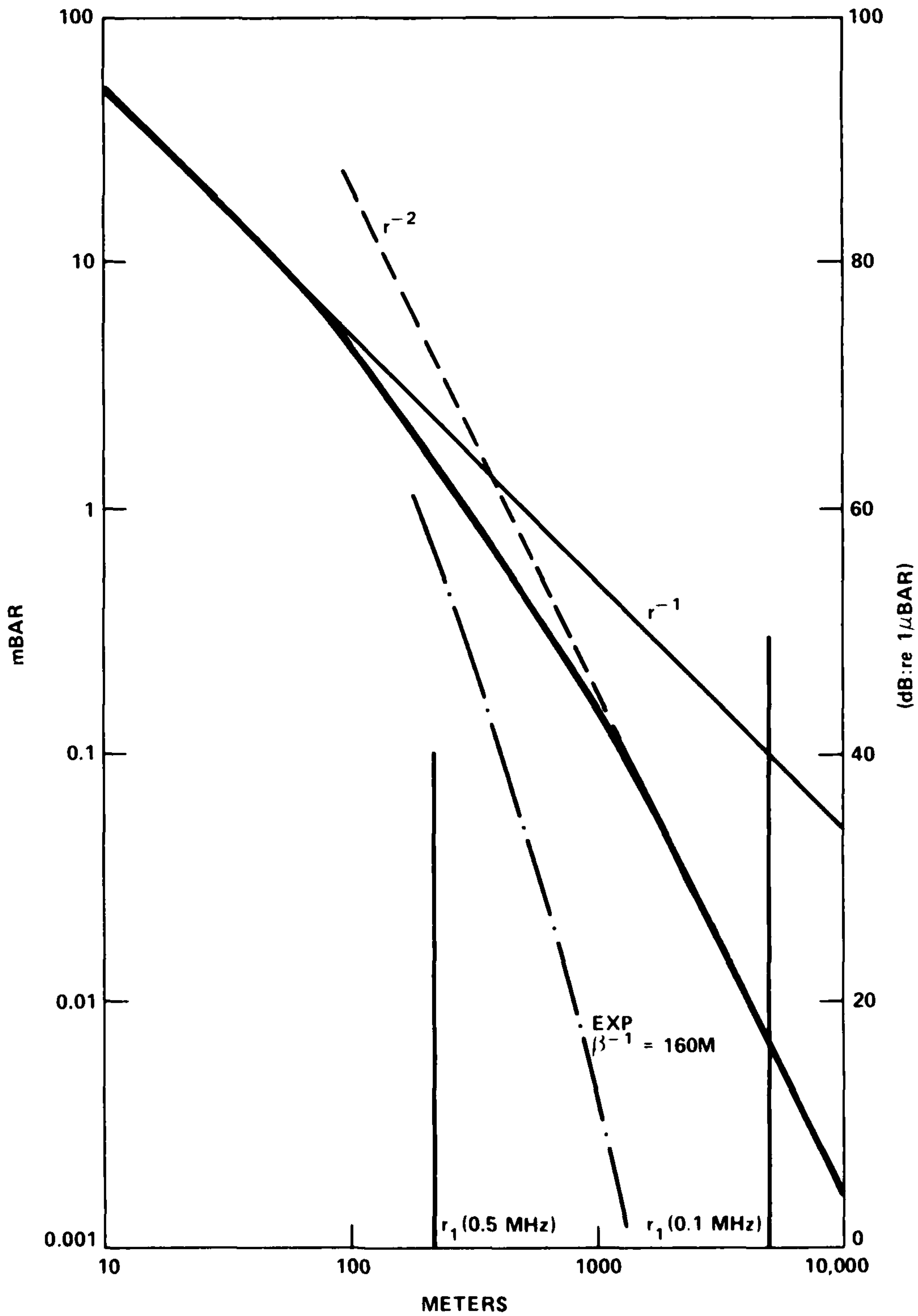

FIGURE 5. PEAK PRESSURE AS A FUNCTION OF r FROM EQUATION (5.7), USING A TYPICAL, LOWER AMPLITUDE, FASTER PULSE (DARK CURVE) (THE UPPER CURVE SHOWS A PORTION OF THE PLOT FROM FIGURE 4.)

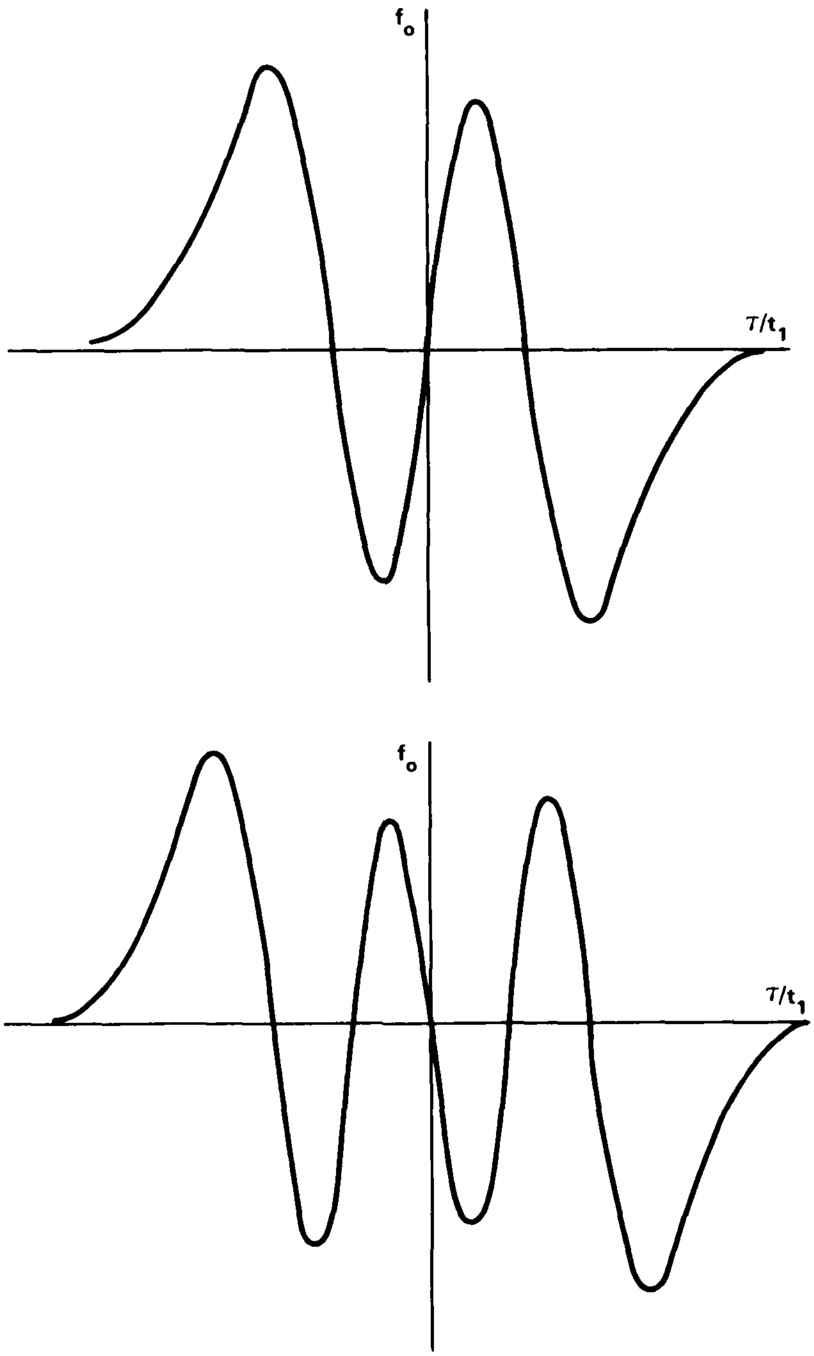

FIGURE 6. PLOTS OF $-H_n(\xi)\exp{-1/2\xi^2}$ FOR n = 3,5 (H_n IS THE HERMITE POLYNOMIAL OF ORDER n.)

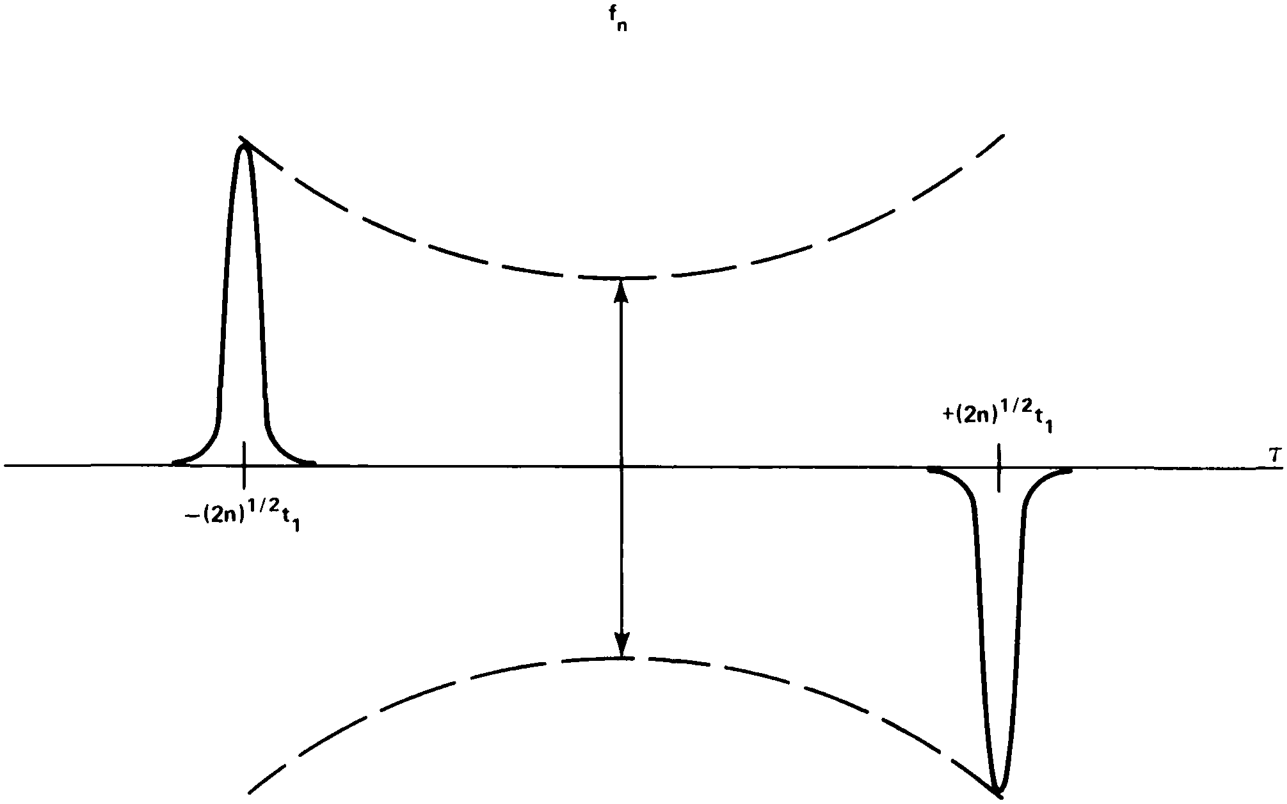

FIGURE 7. PULSE-FORM AT $\rho = \rho_1 = (2t_1)^2/8\kappa$, FOR THE CASE $f_0(\tau) = -H_n(\tau/t_1)$ EXP $-\tau^2/2t_1$. (THE DASHED CURVES INDICATE THE ENVELOPE OF "CARRIER" OSCILLATION PEAKS THAT WOULD BE PRESENT HAD THEY SURVIVED PROPAGATION.)

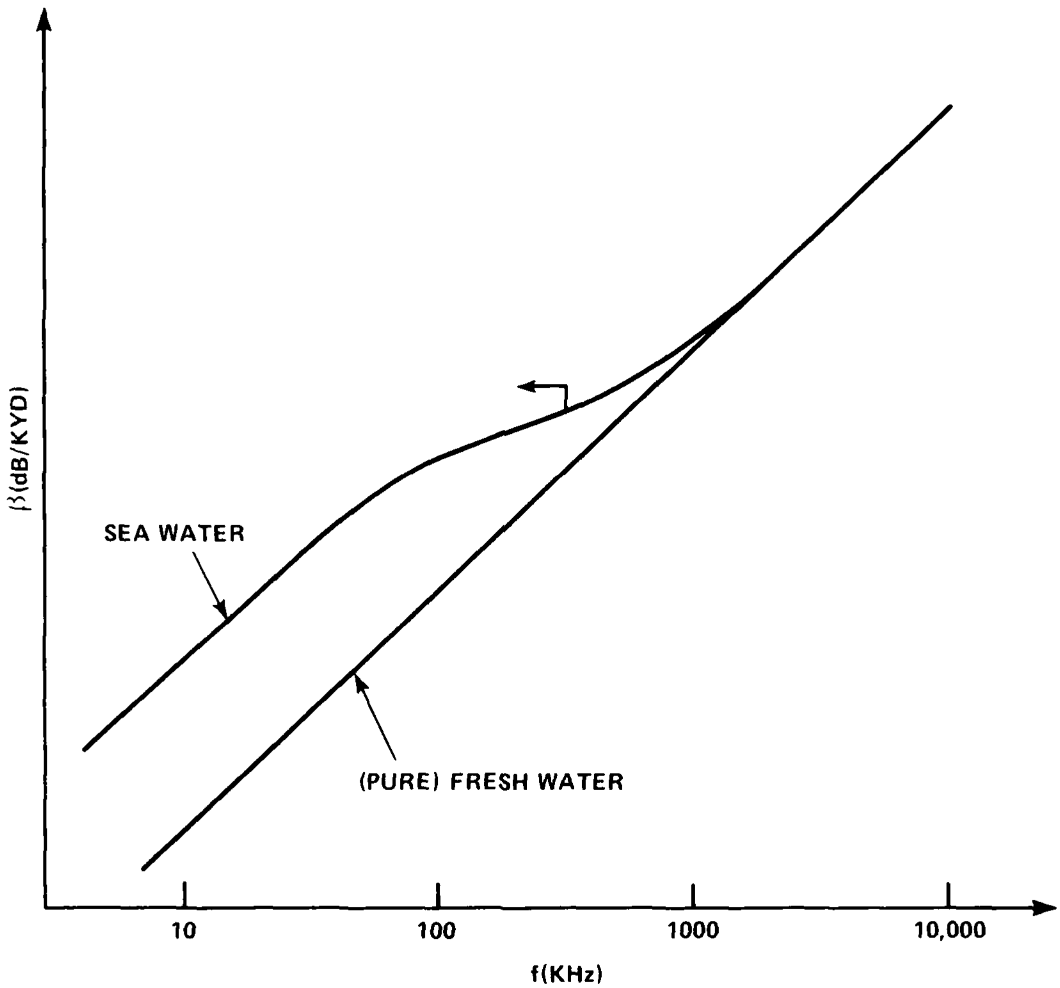

FIGURE 8. SCHEMATIC PLOT OF THE SPECTRAL SOUND EXTINCTION COEFFICIENT β AS A FUNCTION OF FREQUENCY $f = \omega/2\pi$ FOR WATER (THE DASHED LINE SHOWS THE FRESH WATER FORM ASSUMED IN THIS REPORT)

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APPENDIX A

"H_n-PULSE" PROPAGATION

The generating function for Hermite polynomials is[5]

$$F(s,\xi) = e^{-s^2+2s\xi} = \sum_{n=0}^{\infty} \frac{H_n(\xi)}{n!} s^n, \tag{A.1}$$

so the Fourier transform of $f_{on}(\tau)$ (equation (6.9), with $c_n = 1$),

$$\tilde{f}_{0n}(\omega) = \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} H_n(\tau/t_1)\exp\left(-\tfrac{1}{2}\tau^2/t_1^2\right), \tag{A.2}$$

obeys

$$\sum_{n=0}^{\infty} \frac{\tilde{f}_{0n}(\omega)}{n!} s^n = \int_{-\infty}^{+\infty} d\tau\, e^{i\omega\tau - \frac{1}{2}\tau^2/t_1^2} e^{-s^2 + 2s\tau/t_1} \tag{A.3}$$

$$= \sqrt{2\pi}\, t_1 \exp\left(-\tfrac{1}{2}t_1^2\omega^2\right) e^{s^2 + 2t_1\omega i s} \tag{A.4}$$

$$= \sqrt{2\pi}\, t_1 \exp\left(-\tfrac{1}{2} t_1^2\omega^2\right) \sum_{n=0}^{\infty} \frac{i^n H_n(t_1\omega)}{n!} s^n, \tag{A.5}$$

by equation (A.1), whereupon,

$$\tilde{f}_{0n}(\omega) = \sqrt{2\pi}\, t_1\, i^n H_n(t_1\omega)\exp\left(-\tfrac{1}{2}t_1^2\omega^2\right) \tag{A.6}$$

Equation (A.6) verifies the claim in Chapter 6, that f_{0n} and its Fourier transform are the same functions.

To propagate $f_{0n}(\tau)$ I have to evaluate equation (6.3),

$$f_n(\rho,\tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega\tau-\kappa\rho\omega^2} \sqrt{2\pi}\, t_1\, i^n\, H_n(t_1\omega)\exp\left(-\tfrac{1}{2}t_1^2\omega^2\right). \tag{A.7}$$

This can be managed in the same way. We have

$$\sum_{n=0}^{\infty} \frac{(-i)^n s^n f_n(\rho,\tau)}{n!} = \frac{t_1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\omega\, e^{-i\omega\tau-\kappa\rho\omega^2-\frac{1}{2}t_1^2\omega^2} \sum_{n=0}^{\infty} \frac{H_n(t_1\omega)}{n!}\, s^n \tag{A.8}$$

$$= \frac{t_1}{\sqrt{2\pi}}\, e^{-s^2} \int_{-\infty}^{+\infty} d\omega\, e^{-\frac{1}{2}t_\rho^2\omega^2 + (2st_1 - i\tau)\omega}, \tag{A.9}$$

where

$$t_\rho^2 = t_1^2 + 2\kappa\rho . \tag{A.10}$$

Proceeding, the right side of equation (A.9) is, with some re-arranging,

$$\frac{t_1}{t_p}\exp\left[-s^2 + \frac{1}{2t_p^2}(2st_1 - i\tau)^2\right] = \frac{t_1}{t_\rho}\, e^{-\frac{1}{2}\tau^2/t_\rho^2}\, F(\tau,\rho), \tag{A.11}$$

where

$$\sigma = -is\left(\frac{2t_1^2}{t_\rho^2} - 1\right)^{\frac{1}{2}} = -is\left(\frac{t_1^2 - 2\kappa\rho}{t_1^2 + 2\kappa\rho}\right)^{\frac{1}{2}}, \tag{A.12}$$

and

$$\xi = \left(\frac{2t_1^2}{t_\rho^2} - 1\right)^{-\frac{1}{2}} \frac{t_1\tau}{t_\rho^2} = \left(t_1^4 - (2\kappa\rho)^2\right)^{\frac{1}{2}} t_1\tau. \tag{A.13}$$

Combining the expansion of $F(\sigma,\xi)$ from equation (A.1) with equations (A.9) and (A.11)-(A.13),

$$\sum_{n=0}^{\infty} \frac{(-i)^n s^n f_n(\rho,\tau)}{n!} = \frac{t_1}{t_\rho} e^{-\frac{1}{2}\tau^2/t_\rho^2} \sum_{n=0}^{\infty} \frac{(-i)^n s^n}{n!} \left(\frac{2t_1^2}{t_\rho^2} - 1\right)^{n/2}$$

$$H_n\left[\left(\frac{2t_1^2}{t_\rho^2} - 1\right)^{-\frac{1}{2}} \frac{t_1\tau}{t_\rho^2}\right], \tag{A.14}$$

whence

$$f_n(\rho,\tau) = \frac{t_1}{t_\rho}\left(\frac{2t_1^2}{t_\rho^2} - 1\right)^{n/2} H_n\left[\left(\frac{2t_1^2}{t_\rho^2} - 1\right)^{-\frac{1}{2}} \frac{t_1\tau}{t_\rho^2}\right]$$

$$\exp(-\tfrac{1}{2}\tau^2/t_\rho^2). \tag{A.15}$$

For the general form, equation (6.8), one has then also

$$f(\rho,\tau) = \sum_{n:\text{odd}} c_n f_n(\rho,\tau), \tag{A.16}$$

where $f_n(\rho,\tau)$ is given by equation (A.15). The first few odd order Hermite polynominals are

$$H_1 = 2\xi \quad , H_3 = - 12\xi + 8\xi^3 ,$$

$$H_5 = 120\xi - 160\xi^3 + 32\xi^5 . \tag{A.17}$$

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