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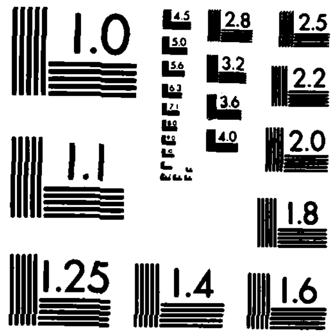

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FAULT TOLERANT STATISTICAL SIGNAL PROCESSING ALGORITHMS FOR PARALLEL ARCHITECTURES

FINAL REPORT

Gerard G. L. Meyer and Howard L. Weinert

ELECTRICAL ENGINEERING & COMPUTER SCIENCE

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FAULT TOLERANT STATISTICAL SIGNAL PROCESSING ALGORITHMS FOR PARALLEL ARCHITECTURES

FINAL REPORT

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ABSTRACT

The goal of the research effort supported by the Office of Naval Research under Contract N00014-81-K-0813 was to analyze signal processing algorithms in terms of speed and reliability. In this final report we analyze the effects of hardware faults on the performance of computer-implemented signal detectors, as measured by the probability of detection and the probability of false alarm. We then use these results to design fault-tolerant detectors using hardware redundancy.

I. INTRODUCTION AND PRELIMINARIES

When a signal detector designed on some theoretical basis is actually put to use, its performance is often not as good as the theory predicted. The two principal causes of this phenomenon are inaccuracies in the underlying statistical model, and inaccuracies due to the computing device which processes the observations. Methods to deal with the former problem come under the heading of robust detection. The latter problem really has two parts, one due to the inherent finite precision of the computer, which necessitates the use of a numerically stable algorithm, and one due to actual hardware faults in the computing device. In this paper we will determine the effects of hardware faults on detector performance, and we will propose and analyze some methods for masking these effects. The result will be a fault-tolerant detector: one which will satisfy desired performance criteria even when implemented on an imperfect computing device.

In this paper we will be concerned with the non-sequential binary detection problem in which the hypotheses are "signal present" and "signal absent". Whatever decision criterion is adopted, the relevant hypothesis test results in a decision variable d^*. If $d^* = 1$ the signal is declared present, and if $d^* = 0$ the signal is declared absent. Let S be the event that the signal is present, let S^c be the event that the signal is absent, and suppose P(S) is unknown. We will measure the performance of the theoretical detector by the probability of false alarm P_F^* and the probability of detection P_D^*, where

$$P_F^* = P(d^*=1|S^c)$$
$$P_D^* = P(d^*=1|S).$$

The receiver operating characteristic (ROC) relates P_D^* to P_F^*. The ROC will be expressed in functional form as

$$P_D^* = f(P_F^*). \tag{1.1}$$

Any point (P_F^*, P_D^*) on the ROC will be called an operating point of the theoretical detector.

Suppose that performance bounds P_F^0 and P_D^0 are given on the probabilities of false alarm and detection, respectively. An operating point (P_F^*, P_D^*) will be called <u>feasible</u> if $P_F^* \leq P_F^0$ and $P_D^* \geq P_D^0$. To simplify the exposition of our results we shall restrict attention to the case of practical interest stated in the following hypothesis.

<u>Hypothesis 1</u>:

(i) $P_F^0 < 0.5$

(ii) $P_D^0 > 0.5$

(iii) The theoretical detector has at least one feasible operating point.

In Section II we show how the performance of the theoretical detector is affected when it is implemented on an imperfect computing device. We characterize the minimum device reliability that is necessary to guarantee that the implemented detector has at least one feasible operating point. In Sections III and IV we show how to decrease this minimum device reliability using either unconditional or conditional masking schemes based on hardware redundancy. An example is presented in Section V.

We note that the use of hardware redundancy for reliability purposes is not new. The idea goes back to work done in the Fifties by von Neumann [8, pp. 322-378] and Moore and Shannon [6]. This early work is summarized in [1, Ch. 7]. Since then, many researchers have continued to investigate these techniques; see for example [2], [3], [4], [5].

The work cited above is concerned only with the evaluation of the reliability of unconditional masking schemes, without reference to any specific problem in which the computing devices might be used. On the contrary, our work analyzes the impact of hardware faults and associated masking schemes on the specific problem of signal detection.

II. PERFORMANCE OF THE IMPLEMENTED DETECTOR

Suppose that the theoretical detector described in the previous section is implemented on a computing device which carries out the hypothesis test and produces a decision variable d. If d = 1 the signal is declared present, and if d = 0 the signal is declared absent. The probabilities of false alarm and detection for the implemented detector are

$$P_F = P(d=1|S^c)$$

$$P_D = P(d=1|S).$$

One would like the implemented detector to meet the same performance constraints as the theoretical detector; that is, $P_F \leq P_F^0$ and $P_D \geq P_D^0$. This will obviously be the case if the computed decision variable d is always equal to the theoretical decision variable d^*. In general, however, d and d^* may be unequal due to hardware failures in the computing device. We will now relate the performance of the implemented detector to that of the theoretical detector under the physically reasonable assumption that the presence of the signal and the value of the theoretical decision variable are each independent of the correctness of the computations. Therefore we shall adopt the following hypothesis:

Hypothesis 2: The events S and $(d^*=1)$ are each independent of the event $(d=d^*)$.

With this hypothesis the following theorem can be established.

<u>Theorem 1</u>: If Hypothesis 2 is satisfied,

$$P_F = KP_F^* + (1-K)(1-P_F^*) \tag{2.1}$$

$$P_D = KP_D^* + (1-K)(1-P_D^*) \tag{2.2}$$

where $K = P(d=d^*)$.

<u>Proof</u>: Let B be the event $(d=d^*)$ and let B^c be its complement. The definition of P_F implies

$$P_F = \frac{P(d=1 \text{ and } S^c)}{P(S^c)} = \frac{X + Y}{P(S^c)}$$

where

$$X = P(d=1 \text{ and } S^c \text{ and } B)$$

$$Y = P(d=1 \text{ and } S^c \text{ and } B^c).$$

Since the events (d=1 and B) and (d^*=1 and B) are identical, and the events (d=1 and B^c) and (d^*=0 and B^c) are identical,

$$X = P(d^*=1 \text{ and } S^c \text{ and } B)$$

$$Y = P(d^*=0 \text{ and } S^c \text{ and } B^c).$$

Then using Hypothesis 2,

$$X = P(d^*=1 \text{ and } S^c)P(B) = P(d^*=1|S^c)P(S^c)P(B)$$

$$Y = P(d^*=0 \text{ and } S^c)P(B^c) = P(d^*=0|S^c)P(S^c)P(B^c) = (1 - P(d^*=1|S^c))P(S^c)(1 - P(B))$$

and Eq. (2.1) follows. The proof of Eq. (2.2) proceeds along the same lines. ∎

The ROC of the implemented detector may be obtained from the ROC of the theoretical detector and the value K that characterizes the accuracy of the implementation. Without loss of generality we can take $K \geq 0.5$. If $K = 0.5$ the ROC of the implemented detector is the single point (0.5,0.5). If $K > 0.5$ then Eq. (2.1) implies $1 - K \leq P_F \leq K$ and

$$P_F^* = \frac{P_F - (1-K)}{2K-1}. \tag{2.3}$$

Substitution into Eq. (1.1) and then into Eq. (2.2) yields

$$P_D = (2K-1) f\left[\frac{P_F - (1-K)}{2K-1}\right] + 1-K = f_K(P_F),\ 1-K \leq P_F \leq K,\ K > 0.5. \tag{2.4}$$

Note that the functional form for the implemented ROC satisfies $f_1(.) = f(.)$, $f_K(1-K) = 1-K$ and $f_K(K) = K$.

Any point (P_F,P_D) on the implemented ROC will be called an operating point of the implemented detector. As before, an operating point (P_F,P_D) is feasible if $P_F \leq P_F^0$ and $P_D \geq P_D^0$. An examination of Eqs. (2.1) - (2.2) shows that as K decreases from 1, the operating point (P_F,P_D) of the implemented detector moves on a straight line from (P_F^*,P_D^*) toward $(1-P_F^*,1-P_D^*)$. This fact immediately gives the following corollary.

<u>Corollary 1</u>: Under Hypotheses 1 and 2, given a feasible theoretical operating point (P_F^*,P_D^*), the corresponding operating point of the implemented detector is also feasible if and only if

$$K \geq \max\left[\frac{P_F^*+P_F^0-1}{2P_F^*-1}, \frac{P_D^*+P_D^0-1}{2P_D^*-1}\right] = K(P_F^*,P_D^*). \qquad (2.5)$$

If (P_F^{**},P_D^{**}) is a feasible theoretical operating point that satisfies

$$K(P_F^{**},P_D^{**}) \leq K(P_F^*,P_D^*)$$

among all feasible theoretical operating points (P_F^*,P_D^*), and if

$$K_{min} = K(P_F^{**},P_D^{**}),$$

then the next corollary is obtained.

<u>Corollary 2</u>: Under Hypotheses 1 and 2, the implemented detector will have at least one feasible operating point if and only if $K \geq K_{min}$.

When $K = K_{min}$ the implemented detector has exactly one feasible operating point, namely (P_F^0,P_D^0). Therefore K_{min} can be obtained from the equation

$$P_D^0 = f_{K_{min}}(P_F^0).$$

Alternatively, we can use the fact that (P_F^{**},P_D^{**}) lies at the intersection of the theoretical ROC and the line that passes through the points (P_F^0,P_D^0) and $(1-P_F^0,1-P_D^0)$, and thus

$$K_{min} = \frac{1 - P_F^0 - P_F^{**}}{1 - 2P_F^{**}}.$$

Note that Eq. (2.4) implies

$$K_{min} \geq 1 - P_F^0.$$

Equations (2.1) and (2.2) express the performance of the implemented detector as a function of the performance of the theoretical detector and K, the probability of coincidence between the implemented and theoretical decision variables. The next step is to express K in terms of hardware reliability parameters.

We characterize the reliability of the computing device on which the detector is implemented by the probability q that it remains fault-free during the mission time of the detector; that is, during the time required for the computing device to produce the decision variable d. If A is the event that the computing device is fault-free during the mission time of the detector, then q = P(A). We can now find the probability of coincidence p between the theoretical and implemented decision variables d^* and d. If we let

$$r = P(d=d^* | A^c)$$

then

$$p = P(d=d^*) = q + r(1-q). \qquad (2.6)$$

Although it is reasonable to assume that q can be determined experimentally, one cannot make the same statement concerning r. Therefore r will be considered a parameter to be chosen by the designer based on his level of optimism. In the most pessimistic case every hardware fault would cause $d \neq d^*$ and thus one would choose $r = 0$. The case $r = 1$, in which hardware faults can never affect the value of d, is of no interest, and thus we shall always assume $r < 1$.

We can now relate K to p. In view of the definition of K given in Theorem 1, and the definition of p given in Eq. (2.6),

$$K = g_1(p)$$

where $g_1(p) = p$.

In light of Corollary 2 and Eq. (2.6), the implemented detector will have at least one feasible operating point if and only if

$$g_1(p) \geq K_{min}$$

that is

$$q + r(1-q) \geq K_{min}.$$

For a fixed r there is clearly a smallest value of q for which the above inequality is satisfied. As a result the following corollary is obtained.

Corollary 3: Under Hypotheses 1 and 2, for a given confidence level $r < 1$, at least one operating point of the implemented detector is feasible if and only if

$$q \geq \max\left\{0, \frac{K_{min}-r}{1-r}\right\} = q_1. \qquad (2.7)$$

III. PERFORMANCE USING UNCONDITIONAL MASKING

The minimum computing device reliability q_1 obtained in the preceding section may exceed the reliability of any available device. In this case some action must be taken to decrease the required minimum reliability. As we shall now show, one way to accomplish this goal is to use an unconditional masking scheme based on hardware redundancy. To implement such a scheme we send the original data sample to α identical computing devices which are instructed to carry out identical operations on the data, and which produce decision variables $d_1, d_2, \ldots, d_\alpha$. These decision variables are then sent to the masker which produces a final decision variable d_m whose value is equal to that of the majority of the d_i's. If $d_m = 1$ the signal is declared present; if $d_m = 0$ the signal is declared absent.

We shall adopt the following hypothesis concerning the computing devices and the masker.

<u>Hypothesis 3</u>:

(i) The events $(d_i = d^*)$, $i = 1, 2, \ldots, \alpha$, are mutually independent and have identical probabilities of occurrence.

(ii) The events S and $(d^* = 1)$ are each independent of the events $(d_i = d^*)$, $i = 1, 2, \ldots, \alpha$.

(iii) The masker generates the quantity d_m as follows:

$$d_m = 1 \text{ if } \sum_{i=1}^{\alpha} d_i > \alpha/2$$

$$d_m = 0 \text{ if } \sum_{i=1}^{\alpha} d_i < \alpha/2$$

$d_m = 1$ or 0 with probability 0.5 each, otherwise.

Note that (ii) is just a restatement of Hypothesis 2 for each computing device.

When an unconditional masker is used, the probabilities of false alarm and detection are given by

$$P_F = P(d_m=1|S^c)$$

$$P_D = P(d_m=1|S).$$

The following theorem relates the performance of the detector implemented with an unconditional masker to that of the theoretical detector.

Theorem 2: If Hypothesis 3 is satisfied,

$$P_F = KP_F^* + (1-K)(1-P_F^*) \tag{3.1}$$

$$P_D = KP_D^* + (1-K)(1-P_D^*) \tag{3.2}$$

where $K = P(d_m=d^*)$.

Proof: Once one recognizes that parts (ii) and (iii) of Hypothesis 3 imply that the events S and $(d^*=1)$ are each independent of the event $(d_m=d^*)$, one can proceed in analogy to the proof of Theorem 1. ∎

It is clear from a comparison of Theorems 1 and 2 that the use of the unconditional masking scheme does not change the form of the relationship between the pairs (P_F,P_D) and (P_F^*,P_D^*). Therefore if Hypotheses 1 and 3 are satisfied, the detector implemented with the masker will have at least one feasible operating point if and only if $K \geq K_{min}$. The next step is to relate K to the reliability of the computing devices.

As in Section II we characterize the reliability of the computing devices that generate the quantities $d_1, d_2, \ldots, d_\alpha$ by the probability q that each remains fault-free during the mission time of the detector, and thus if we let

$$p = P(d_i = d^*), \quad i = 1, 2, \ldots, \alpha$$

then

$$p = q + r(1-q) \tag{3.3}$$

where r is the confidence level parameter also discussed in Section II.

Next let $G(.,.,.)$ be the real-valued map defined for all positive integers α, non-negative integers $\gamma \leq \alpha$, and scalars ρ in the interval [0,1], by

$$G(\alpha,\gamma,\rho) = \sum_{j=0}^{\gamma} \frac{\alpha!}{j!(\alpha-j)!}(1-\rho)^j \rho^{\alpha-j}.$$

It is clear that $G(\alpha,\gamma,p)$ is the probability that at most γ of the d_i's do not equal d^*. This fact immediately leads to the following lemma, in which K is expressed in terms of α and p.

<u>Lemma 1</u>: If Hypothesis 3 is satisfied,

$$K = P(d_m = d^*) = g_2(\alpha,p)$$

where

$$g_2(\alpha,p) = G(\alpha,\frac{\alpha-1}{2},p), \ \alpha \text{ odd} \tag{3.4}$$

$$g_2(\alpha,p) = G(\alpha,\frac{\alpha-2}{2},p) + 0.5\frac{\alpha!}{(\alpha/2)!(\alpha/2)!}(1-p)^{\alpha/2}p^{\alpha/2}, \ \alpha \text{ even.} \tag{3.5}$$

In light of Lemma 1 and the discussion following Theorem 2, it is clear that the detector implemented with an unconditional masker will have at least one feasible operating point if and only if

$$g_2(\alpha,p) \geq K_{min}. \tag{3.6}$$

In order to write Eq. (3.6) as a constraint on q, some well known facts about $g_2(.,.)$ need to be stated.

<u>Lemma</u> <u>2</u>: (i) For a fixed positive integer α, $g_2(\alpha,p)$ is a continuous, strictly increasing function of p with $g_2(\alpha,0) = 0$ and $g_2(\alpha,1) = 1$. (ii) If α is odd, $g_2(\alpha+1,p) = g_2(\alpha,p)$; if in addition $p > 0.5$, then $g_2(\alpha+2,p) > g_2(\alpha,p)$ and $\lim_{\alpha\to\infty} g_2(\alpha,p) = 1$.

The next theorem follows from Lemma 2(i).

<u>Theorem</u> <u>3</u>: Under Hypotheses 1 and 3, for a given confidence level $r < 1$ and a given redundancy level α, the detector implemented with an unconditional masker has at least one feasible operating point if and only if

$$q \geq \max\left\{0, \frac{p_2(\alpha)-r}{1-r}\right\} = q_2(\alpha) \tag{3.7}$$

where $p_2(\alpha)$ satisfies

$$g_2(\alpha,p_2(\alpha)) = K_{min}. \tag{3.8}$$

It is important to determine whether the use of unconditional masking leads to a weakening of the reliability requirement relative to that obtained in Section II. The following corollary gives the affirmative answer.

<u>Corollary 4</u>: If $\alpha = 1$ or 2, then $q_2(\alpha) = q_1$. If $\alpha \geq 3$ and $q_1 = 0$, then $q_2(\alpha) = 0$. If $\alpha \geq 3$ and $q_1 > 0$, then $q_2(\alpha) < q_1$.

<u>Proof</u>: Since $g_2(1,p) = g_2(2,p) = p$, $q_2(1) = q_2(2) = q_1$. If $\alpha \geq 3$, since $K_{min} > 0.5$, $g_2(\alpha,K_{min}) > K_{min}$ and thus Eq. (3.8) implies $p_2(\alpha) < K_{min}$. If $q_1 = 0$ then $K_{min} \leq r$ and therefore $p_2(\alpha) < r$ and $q_2(\alpha) = 0$. If $q_1 > 0$ then $K_{min} > r$ and

$$q_2(\alpha) = \max\left\{0, \frac{p_2(\alpha)-r}{1-r}\right\} < \frac{K_{min}-r}{1-r} = q_1. \blacksquare$$

In many situations the type of computing device is given, and thus q is fixed. The next theorem, which follows from Lemma 2(ii), characterizes the minimum redundancy level which will ensure that at least one feasible operating point exists.

<u>Theorem 4</u>: Under Hypotheses 1 and 3, for a given confidence level $r < 1$, a given q such that $q+r(1-q) > 0.5$, and $K_{min} < 1$, the minimum redundancy level necessary to guarantee that the detector implemented with an unconditional masker has at least one feasible operating point is $\alpha_2(q)$, where $\alpha_2(q)$ is the smallest (necessarily odd) integer such that

$$q \geq q_2(\alpha_2(q)).$$

IV. PERFORMANCE USING CONDITIONAL MASKING

It may be the case that unconditional masking is not sufficient. That is, for a given computing device, the minimum redundancy level specified by Theorem 4 may be too costly; or for a given redundancy level, the minimum device reliability stated in Theorem 3 may exceed that of any available device. In this case the use of conditional masking may be appropriate. With conditional masking the final decision variable d_m is produced as in Section III, but d_m is used to make a decision concerning the presence of the signal only when the number of identical d_i's is sufficiently high. Thus the operation of the conditional masker will be characterized by a positive integer $\xi \leq \alpha$, and a boolean variable b that will be set at 0 if at least ξ of the d_i's are identical, and set at 1 otherwise. If b = 0, we make a decision concerning the presence of the signal using the decision variable d_m. If b = 1, we make no decision concerning the signal. Note that if $\xi < \left\lceil \frac{\alpha+2}{2} \right\rceil$, the quantity b is always equal to 0 and conditional masking reduces to unconditional masking. The notation $\lceil x \rceil$ indicates the smallest integer greater than or equal to x.

We shall therefore adopt, in addition to Hypothesis 3, the following hypothesis concerning the masker.

<u>Hypothesis 4</u>: Given a positive integer $\xi \leq \alpha$, the masker generates the quantity b as follows:

$$b = 0 \text{ if } \sum_{i=1}^{\alpha} d_i \geq \xi \text{ or } \sum_{i=1}^{\alpha} d_i \leq \alpha-\xi$$

$$b = 1, \text{ otherwise.}$$

When a conditional masker is used, the probabilities of false alarm and detection are given by

$$P_F = P(d_m=1 | S^c \text{ and } b=0)$$

$$P_D = P(d_m=1 | S \text{ and } b=0).$$

The penalty that must be paid when using a conditional masker is the probability P_R of making no decision concerning the signal given that the final decision variable is equal to the theoretical decision variable; that is,

$$P_R = P(b=1 | d_m=d^*).$$

Therefore we must specify an upper bound P_R^0 on the penalty P_R in addition to the already existing bounds on the probabilities of false alarm and detection, and the detector implemented with a conditional masker must satisfy the constraints $P_F \leq P_F^0$, $P_D \geq P_D^0$, and $P_R \leq P_R^0$.

The following theorem relates the performance measures P_F and P_D of the detector implemented with a conditional masker to those of the theoretical detector.

<u>Theorem 5</u>: If Hypotheses 3 and 4 are satisfied,

$$P_F = KP_F^* + (1-K)(1-P_F^*) \tag{4.1}$$

$$P_D = KP_D^* + (1-K)(1-P_D^*) \tag{4.2}$$

where $K = P(d_m = d^* | b=0)$.

<u>Proof</u>: Since Hypotheses 3 and 4 imply that the events S and $(d^*=1)$ are each independent of the event $(b=0)$, one can proceed in analogy to the proof of Theorem 1. ∎

Once again the form of the relationship between the pairs (P_F, P_D) and (P_F^*, P_D^*) is the same as it was in Theorems 1 and 2. Therefore if Hypotheses 1, 3 and 4 are satisfied, the detector implemented with the conditional masker will have at least one feasible operating point (that is, it will satisfy $P_F \leq P_F^0$ and $P_D \geq P_D^0$) if and only if $K \geq K_{min}$.

The next step is to relate K and P_R to the reliability of the computing devices.

<u>Lemma 3</u>: If Hypotheses 3 and 4 are satisfied, then for $\xi \geq \lceil \frac{\alpha+2}{2} \rceil$

$$K = P(d_m = d^* | b=0) = g_3(\alpha, \xi, p)$$

$$P_R = P(b=1 | d_m = d^*) = g_4(\alpha, \xi, p)$$

where

$$g_3(\alpha, \xi, p) = \frac{G(\alpha, \alpha-\xi, p)}{1 + G(\alpha, \alpha-\xi, p) - G(\alpha, \xi-1, p)} \tag{4.3}$$

$$g_4(\alpha, \xi, p) = 1 - \frac{G(\alpha, \alpha-\xi, p)}{g_2(\alpha, p)}. \tag{4.4}$$

<u>Proof</u>: If $\xi \geq \lceil \frac{\alpha+2}{2} \rceil$, the events ($d_m = d^*$ and $b=0$), (at least ξ of the d_i's equal d^*) and (at most $\alpha-\xi$ of the d_i's do not equal d^*) are equivalent. Also, the event $(b=0)$ is equal to the union of the two disjoint events

(at least ξ of the d_i's equal d^*) and (at least ξ of the d_i's do not equal d^*). It follows that

$$P(d_m=d^* | b=0) = \frac{P(d_m=d^* \text{ and } b=0)}{P(b=0)}$$

$$= \frac{G(\alpha,\alpha-\xi,p)}{1 + G(\alpha,\alpha-\xi,p) - G(\alpha,\xi-1,p)}.$$

Furthermore,

$$P(b=1 | d_m=d^*) = 1 - \frac{P(d_m=d^* \text{ and } b=0)}{P(d_m=d^*)}$$

$$= 1 - \frac{G(\alpha,\alpha-\xi,p)}{g_2(\alpha,p)}. \quad \square$$

The following facts about $g_3(.,.,.)$ and $g_4(.,.,.)$ are immediate consequences of their probabilistic interpretations.

<u>Lemma</u> 4: For any two fixed positive integers α and ξ satisfying $\lceil\frac{\alpha+2}{2}\rceil \leq \xi \leq \alpha$, $g_3(\alpha,\xi,p)$ is a continuous, strictly increasing function of p, $g_3(\alpha,\xi,0) = 0$ and $g_3(\alpha,\xi,1) = 1$.

<u>Lemma</u> 5: For any two fixed positive integers α and ξ satisfying $\lceil\frac{\alpha+2}{2}\rceil \leq \xi \leq \alpha$, $g_4(\alpha,\xi,p)$ is a continuous, strictly decreasing function of p, $g_4(\alpha,\xi,0) = 1$ and $g_4(\alpha,\xi,1) = 0$.

The following two theorems are consequences of Lemmas 4 and 5, respectively.

<u>Theorem</u> <u>6</u>: Under Hypotheses 1, 3 and 4, for a given confidence level $r < 1$, a given redundancy level α, and a condition level $\xi \geq \left\lceil \frac{\alpha+2}{2} \right\rceil$, the detector implemented with a conditional masker has at least one feasible operating point if and only if

$$q \geq \max \left\{ 0, \frac{p_3(\alpha,\xi)-r}{1-r}, \right\} = q_3(\alpha,\xi)$$

where $p_3(\alpha,\xi)$ satisfies

$$g_3(\alpha,\xi,p_3(\alpha,\xi)) = K_{min}. \tag{4.5}$$

<u>Theorem</u> <u>7</u>: Under Hypotheses 1, 3 and 4, for a given confidence level $r < 1$, a given redundancy level α, and a condition level $\xi \geq \left\lceil \frac{\alpha+2}{2} \right\rceil$, the detector implemented with a conditional masker satisfies $P_R \leq P_R^0$ if and only if

$$q \geq \max \left\{ 0, \frac{p_4(\alpha,\xi)-r}{1-r} \right\} = q_4(\alpha,\xi)$$

where $p_4(\alpha,\xi)$ satisfies

$$g_4(\alpha,\xi,p_4(\alpha,\xi)) = P_R^0. \tag{4.6}$$

It is clear that if $\xi < \left\lceil \frac{\alpha+2}{2} \right\rceil$ then conditional masking reduces to unconditional masking. In this case we will define $q_3(\alpha,\xi) = q_2(\alpha)$ and $q_4(\alpha,\xi) = 0$.

In view of Theorems 6 and 7, the minimum device reliability q needed to meet all the constraints must satisfy

$$q \geq \max \{q_3(\alpha,\xi), q_4(\alpha,\xi)\}.$$

The condition level ξ can be chosen by the designer to optimize performance. Given α, let $\xi_*(\alpha)$ be a condition level that satisfies

$$\max\{q_3(\alpha,\xi_*(\alpha)), q_4(\alpha,\xi_*(\alpha))\} \leq \max\{q_3(\alpha,\xi), q_4(\alpha,\xi)\}$$

for every ξ in the interval $[1,\alpha]$. We are then led immediately to the following theorem.

Theorem 8: Under Hypotheses 1, 3 and 4, for a given confidence level $r < 1$ and a given redundancy level α, the detector implemented with a conditional masker has at least one feasible operating point and satisfies $P_R \leq P_R^0$ if and only if

$$q \geq \max \{q_3(\alpha,\xi_*(\alpha)), q_4(\alpha,\xi_*(\alpha))\} = q_5(\alpha). \tag{4.7}$$

One must determine on a case by case basis whether the use of conditional masking leads to a weakening of the reliability requirement relative to that obtained for unconditional masking in Section III. Although it is clear that $q_5(\alpha) \leq q_2(\alpha)$, a strict improvement is achieved only when the penalty bound P_R^0 is sufficiently large.

If we consider $q_5(\alpha)$ as a function of α, we can directly determine the minimum redundancy level necessary to meet all the constraints for a given device reliability q.

Theorem 9: Under Hypotheses 1, 3 and 4, for a given confidence level $r < 1$, a given device reliability q such that $q+r(1-q) > 0.5$, and $K_{min} < 1$, the minimum redundancy level necessary to guarantee that the detector implemented with a conditional masker has at least one feasible operating point and satisfies $P_R \leq P_R^0$ is $\alpha_5(q)$, where $\alpha_5(q)$ is the smallest integer such that

$$q \geq q_5(\alpha_5(q)). \tag{4.8}$$

It is clear that $\alpha_5(q) \leq \alpha_2(q)$, but strict inequality will hold only if P_R^0 is sufficiently large. Once again this issue must be decided on a case by case basis.

V. CONCLUSION AND EXAMPLE

In this paper we have analyzed the effects of hardware faults on signal detector implementations. We have shown that, under reasonable assumptions, the design of the theoretical detector and the design of the implementation are coupled through a single quantity K_{min}. As a result, we have been able to determine the minimum device reliability needed to ensure that the performance constraints are satisfied, whether we use no masking, unconditional masking or conditional masking.

The results of this paper can be used to synthesize a step-by-step design procedure for fault tolerant signal detectors.

Step 1: Given the theoretical ROC, P_F^0 and P_D^0, compute K_{min}.

Step 2: Using the paper's results, choose the type and number of computing devices, and the type of masking scheme.

Step 3: Compute the value of K that corresponds to the choices in Step 2.

Step 4: At this stage, we have one or more feasible operating points on the implemented ROC. Choose one of these points, say (P_F,P_D), according to some optimality criterion. For example, for the Neyman-Pearson operating point choose $P_F = P_F^0$ and $P_D = f_K(P_F^0)$.

Step 5: Using K, P_F and P_D, compute the corresponding theoretical operating point (P_F^*,P_D^*). It is this operating point that is used in the instructions of the computing devices.

We now present a simple example to demonstrate the improvement on minimum device reliability provided by the masking schemes. Consider the ROC that occurs in the context of discriminating between two zero-mean Gaussian processes with variances σ_0^2 and σ_1^2 [7, p. 41]:

$$P_D^* = P_F^{*(\sigma_0/\sigma_1)^2}, \sigma_0 < \sigma_1.$$

If we choose a confidence level $r = 0.5$, $\sigma_0/\sigma_1 = 1/3$, $P_F^0 = 0.1$, $P_D^0 = 0.7$ and $P_R^0 = 0$, then $K_{min} = 0.949$ and $q_1 = 0.898$. This is the minimum required device reliability when only a single device is used. If we now use unconditional masking, then for $\alpha = 1$ or 2, $q_2(\alpha) = q_1 = 0.898$; if $\alpha = 3$ or 4, $q_2(\alpha) = 0.728$; if $\alpha = 5$ or 6, $q_2(\alpha) = 0.620$. Note that we cannot use conditional masking since $P_R^0 = 0$.

Suppose now that $P_R^0 = 0.25$. Obviously, we can use unconditional masking to obtain the same results given above, but we can try to improve on those results by using conditional masking. In fact, if $\alpha = 2$, then $\xi_*(\alpha) = 2$ and $q_5(\alpha) = 0.624$; if $\alpha = 3$, then conditional masking offers no improvement over unconditional masking; if $\alpha = 4$, then $\xi_*(\alpha) = 3$ and $q_5(\alpha) = 0.536$; if $\alpha = 5$, then $\xi_*(\alpha) = 4$ and $q_5(\alpha) = 0.558$; if $\alpha = 6$, then $\xi_*(\alpha) = 4$ and $q_5(\alpha) = 0.478$.

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