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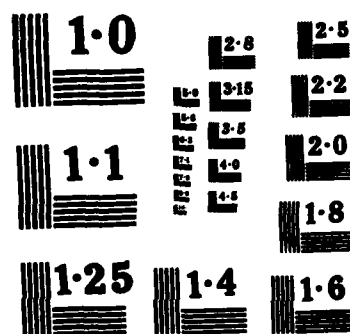
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A REVIEW OF DEVELOPMENTS IN AGRICULTURAL SCIENCE APPLICABLE TO MILITARY SOIL MOISTURE PREDICTION REQUIREMENTS

by

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A number of investigators have made a concerted effort during the last 20 years in agricultural science in soil moisture modeling and prediction. A parallel effort by soil physicists over the last three decades has improved their capability to estimate input data values for more complex soil moisture predic- tion models, which is particularly significant in worldwide applications and in applications involving inaccessible areas. Advances are also being made in treating problems caused by spatial variability of soil properties. (Continued)		

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20. ABSTRACT (Continued).

These developments in the prediction of soil moisture content by the civilian community are potentially important to many military operations because of the negative effects of excessive soil moisture (reduction of ground vehicle mobility and enhancement of flooding in field areas, for example). Current soil moisture prediction procedures for mobility are based on a military model formulated over 20 years ago. While excellent for its time and purpose, this model does not predict several critical soil moisture conditions with desirable accuracy. Additionally, only simplistic estimates of antecedent soil moisture are used in military hydrology models.

Some aspects of improved soil moisture prediction capabilities developed in agricultural science and in soil physics are now fully applicable to military requirements. Other aspects will require adaption to specific military applications. This report provides a comprehensive literature survey and review of mathematical models for predicting soil moisture that were developed in agricultural science and soil physics and that are potentially adaptable to military modeling of ground vehicle mobility and hydrology. Further, the report presents recommendations (a) for comparing the accuracy of soil moisture predictions made by the current military model and by agricultural science and soil physics models for soil and moisture conditions important to the military, and (b) for developing the soil moisture prediction model(s) most useful for the military based on results of this comparison and on an evaluation of characteristics of all prediction models included in the comparison.

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PREFACE

The work reported herein was sponsored by the Office, Chief of Engineers (OCE), US Army through the US Army Corps of Engineers Waterways Experiment Station (WES) as a pilot study in the reevaluation and possible updating of the system currently used for predicting soil trafficability through the estimation of soil moisture content. The study was performed July 1981 through July 1982 and was authorized by WES Work Order No. DACW-39-81-M-3793, dated 13 July 1981. Funding for the study was provided by Research and Development Project No. 4A161102AT24, Task Area E3, Work Unit 004, "Weather and Climate Influence on Mobility." The work was performed by the Department of Soil Science and Biometerology, Agricultural Experiment Station, Utah State University; Dr. L. F. Hall was the Principal Investigator.

The work was monitored by Mr. C. J. Nuttall, Jr., Chief, Mobility Systems Division (MSD), Geotechnical Laboratory (GL), WES, and by Mr. G. W. Turnage, MSD, under the general supervision of Dr. W. F. Marcuson III, Chief, GL.

COL Tilford C. Creel, CE, and COL Robert C. Lee, CE, were Commanders and Directors of WES during the conduct of this study and preparation of this report. COL Creel was also the Contracting Officer. Mr. Fred R. Brown and Dr. Robert W. Whalin were Technical Directors.

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A REVIEW OF DEVELOPMENTS IN AGRICULTURAL SCIENCE APPLICABLE
TO MILITARY SOIL MOISTURE PREDICTION REQUIREMENTS

PART I: INTRODUCTION

Background

1. Soil moisture content is critical to military hydrology and mobility. Excessive soil moisture reduces infiltration of rain or snowmelt water and decreases soil strength. These effects lead to high stream levels, flooding, reduced soil trafficability, and general impairment of activity. Prediction of the moisture content of soils is therefore important to effective planning of field operations.

2. The soil moisture prediction methods that are currently used by the US Army for hydrology and mobility models are based on work performed in the 1930's and 1950's, respectively. While age is not of itself a condemnation of a model, witness the Pythagorean theorem, and while some improvements in the models have been made since their formulation, the present soil moisture models for military applications are distinctly inferior to the state of the art.

3. Soil moisture prediction models for entire watersheds and individual fields have been developed in the civilian community since the current military models were formulated. The more advanced models directly solve the differential equations related to soil water movement by numerical integration with digital computers. These newer physically based models, therefore, are in dramatic contrast with the correlation methods used in the military models. The newer models are possible because of overwhelming increases in the computing power of modern machines, compared to past decades, and because of fundamental advances in evapotranspiration and soil physics theory. Modern models have also benefited from improvements in computational methods, and from both improved field measurement techniques and field data sets.

4. Recent reviews of the military soil moisture prediction models have noted the wealth of knowledge which exists in the civilian community on this subject, e.g., Proceedings of the Military Hydrology Workshop, 17-19 May

1978, Vicksburg, Mississippi (US Army Engineer Waterways Experiment Station (WES), 1979). Evaluation of civilian methods, improved technology transfer, and consideration of military model updating have generally been recommended by the review panels. Because much of the recent modeling work has been performed in connection with agricultural problems, such as irrigation scheduling, yield prediction, and runoff estimation, the present review is a large step toward those goals.

Objectives

5. The objectives of the research discussed herein were to review the soil moisture-weather-time relations developed in agricultural science, and to report on the applicability of such relations to military trafficability problems. Emphasis was to be placed on the effects of soil water--its movement, replenishment, and depletion by evapotranspiration.

Scope

6. The project began with a thorough review of the current soil moisture prediction model used for military trafficability problems, and the historical development of the model. The review included interviews with WES personnel familiar with the model and its history, and a review of WES documents written by both WES and US Forest Service personnel during model development (see Meyer, 1976, for Bibliography).

7. It became clear early in the project that numerical modeling of soil moisture was the activity which would provide the best route through the literature, and was the most significant development in the last three decades. Other theoretical and experimental developments were identified and reviewed as they related to modeling. Several approaches to the literature were included in a concerted effort to ensure that each major modeling group was discovered and represented:

- a. A review paper on crop yield models (Kanemasu et al., 1980) and a review paper on soil moisture determination methods (Schmugge, Jackson, and McKim, 1979) were scanned for appropriate references. The latter included a review paper on soil moisture modeling (Hildreth, 1978) that provided additional references. (A journal paper version of the second reference (Schmugge, Jackson, and McKim, 1980) was discovered during subsequent searching.)

- b. Recent indices for the Soil Science Society of America Journal (formerly Proceedings), Soil Science, Transactions of the American Society of Agricultural Engineers, and Soviet Soil Science were reviewed under title and subject categories for relevant publications.
- c. The table of contents for each journal from which a paper was obtained was read in search of relevant titles or work by authors known to be involved in soil moisture modeling. Several papers not otherwise referenced were thereby discovered. While not comprehensive, this technique included most of the recent work (to December 1980) published in Soil Science, Soil Science Society of America Journal, Water Resources Research, and Transactions of the American Society of Agricultural Engineers.
- d. Relevant referenced books which were available in the WES library were reviewed.
- e. The Defense Technical Information Center literature search capability was accessed via a library computer terminal through a number of key words related to soil moisture modeling.

8. The scope of this literature review was therefore large, but not exhaustive. Work published since December 1980 was generally not included. Modeling effort which has not been revealed through publications or reference in the materials searched may well exist. Also, some unobtainable work has been noted, particularly work of the modeling group at Wageningen, The Netherlands. These caveats notwithstanding, the literature search conducted was fully adequate to the purpose of the project in that it has identified a number of approaches to soil moisture modeling which establish the applicability of these developments to military problems.

9. The materials identified as relevant to this project included the following:

- a. All references to numerical modeling of soil moisture, water infiltration, redistribution, drainage, and evapotranspiration.
- b. All references to numerical modeling of crop yields as due to soil moisture availability that also included a significant treatment of soil moisture.
- c. References to analytic solutions of soil moisture problems that were referenced as numerical modeling check values.
- d. References to procedures for the calculation of hydraulic conductivity or moisture diffusivity from particle size distributions, pore size distributions, or the soil characteristics curve of moisture content versus matric potential (soil moisture tension).

- e. References to characteristic curves that are typical for soils identified by type (in contrast to specific samples).
- f. Principal references to measurement and estimation of soil property heterogeneity within single fields or soil types.
- g. Survey papers on soil moisture determination via remote sensing.
- h. References identifying field data that may be used for model validation or that have been so used.
- i. References to soil moisture modeling with respect to soil survey maps, and large area modeling.
- j. Principal references to modeling of water tables, ground water, and saturated flow.

10. These reference materials have been reviewed under the headings, "Soil Physics," "Evapotranspiration," and "Numerical Modeling of Soil Moisture" in Appendices A, B, and C, as well as being utilized in preparation of the body of this report.

Definitions

11. Several terms used in this report are given here with general definitions for the convenience of the reader. Appropriate texts should be consulted for more complete and precise definitions of technical terms.

Accretion. Addition of moisture to a soil column.

Algorithm. A set of steps to be performed to solve a problem, used for mathematical steps in a numerical solution.

Analytic solution. An exact mathematical expression for the state of a system throughout space and time. It is bounded in accuracy by the validity of assumptions needed to derive it and the accuracy of input variables and parameters used in the expression.

Catenary process. A total process made up of a series of partial processes in which the total process rate is limited by the slowest partial process rate.

Correlation relations. Empirical solutions derived by regression of assumed dependent variable data against assumed independent variable data.

Crop coefficient. A multiplier of calculated potential evapotranspiration used to account for crop characteristics which differ from short grass, including crop height and stage of growth.

Depletion. Removal of moisture from a soil column.

Desorption. The movement of water out of a soil sample due to soil moisture tension (matric potential).

Drainage. The movement of water out of a soil column due to gravity.

Drying curve. The soil characteristic curve derived during drying of a saturated soil sample.

Empirical solution. An approximate expression derived from experimental data for the relationship between variables.

Evaporation. The loss of soil moisture or ponded water at or through the soil or water surface after vaporization, including vaporization of dew or intercepted precipitation from plant surfaces.

Evapotranspiration. Evaporation plus transpiration.

Explicit numerical method. Dependent variables at a given time step of a numerical solution are calculated from known values of dependent variables that were calculated at a prior time step.

Field capacity. The moisture content which prevails in a soil column after a period of drainage from saturated conditions, nominally two days with no evapotranspiration.

Field heterogeneity. Variability in space of a soil property.

Hydraulic conductivity. The ratio of water flux density to hydraulic potential gradient.

Hydraulic head. See hydraulic potential.

Hydraulic potential. Sum of matric and gravitational potentials in unsaturated systems.

Hysteresis. A system property for which the state of a system and changes to that state depend on the direction of change and on previous states of the system.

Implicit numerical method. Dependent variables at a given time step of a numerical solution are calculated simultaneously with other dependent variable values at other spatial points for the same time step. The algorithm may include known values of dependent variables from prior time steps, also.

Infiltration. The entry of water into a soil column through the soil surface.

Integro-differential equation. An equation in which terms of both differential and integral form appear.

Latent evaporation. The evaporation rate from an evaporimeter, an instrument with a saturated porous surface which is exposed to sun and wind.

Lidar. Light detection and ranging. Active remote sensing system using laser illumination and telescopic receiver to measure distance to and reflection by dust, clouds, or precipitation events.

Matric potential. The pressure (negative) required to hold water in equilibrium with soil moisture across a porous membrane due to adsorptive forces on the soil water. (See soil physics text for details.)

Moisture diffusivity. The ratio of water flux density to the gradient of soil moisture content.

Nonlinear. An adjective used to denote systems and their mathematical descriptions which involve products of variables and their derivatives or other functions of those variables, e.g., when conductivity is a function of potential, then the product of conductivity and the gradient of potential is nonlinear.

Numerical solution. An approximate algorithm through which the state of a system may be calculated for discrete points in space and time via stepwise representation of temporal and spatial gradients. Normally, compared to analytic solutions, numerical solutions are less bounded by system simplifications, are equally bounded by input variable and parameter accuracy, and are more bounded by computational expense required for stability and equivalent accuracy.

Permanent wilting point. The moisture content which prevails in a column of soil when plants are observed to wilt permanently.

Porosity. The ratio of total pore volume to bulk soil volume.

Potential evapotranspiration. The rate of evapotranspiration that would occur at a well watered surface, i.e., limited only by solar, atmospheric, and submedium influences which control the supply of latent heat for vaporization and remove water vapor.

Pressure head. See matric potential.

Redistribution. The movement of soil water within a soil column in response to hydraulic gradients, normally applied to the period following infiltration.

Remote sensing. Noncontact measurement using properties of electromagnetic radiation and its emission or reflection by surfaces.

Scanning curve. One of an unbounded set of soil characteristic curves derived during rewetting of a partially dried soil sample, or drying of an initially unsaturated soil sample. Scanning curves run between wetting and drying curves of the sample.

Soil characteristic curve. Graph of matric potential against soil moisture content.

Soil moisture content. Water present in the soil matrix. May be expressed as volume of water per volume of soil, equivalent depths of water and soil, or mass of water per mass of soil.

Soil moisture model. A set of relationships, normally mathematical expressions, describing soil moisture content in space and time as a consequence of water input, water movement, water storage, and water loss. The solution technique is considered part of the model, particularly in the case of numerical solutions.

considered constant during calculation of matric potential for one time step, a new hydraulic conductivity is computed with the calculated matric potential for the time step, and the loop is continued until two values of matric potential for the time step are sufficiently close. Calculation then proceeds to the next time step after all grid points have been resolved. This method has been used, though not extensively, in soil moisture models.

49. The more common approach to the problem of nonlinearity is to remove the nonlinearity by use of an estimated hydraulic conductivity that is considered constant for the time step. This may be done explicitly or implicitly. Explicit linearization of the equation uses conductivity (or diffusivity or specific water capacity, depending on the formulation of the basic equation--see Appendix A) that is calculated via known values of matric potential or soil moisture content. Usually, a spatial average value across one or two grid intervals is used with improved results in comparison to a single point value. Alternately, a predicted value of matric potential for one-half step forward in time can be calculated using an explicitly linearized equation (predictor step). This predicted value is used to obtain the half-step value of hydraulic conductivity, which is then considered constant for the whole step and used to calculate a corrected value of matric potential for a whole time step (corrector step). In addition to this predictor-corrector approach, a number of other estimates of the matric potential after a time step have been used by various investigators to calculate the coefficients for use over the time step. The method used to deal with nonlinearity should be noted for each model.

50. Numerical models may use a method of removing the nonlinearity of the equations because each time step of the calculations is essentially independent. The results of prior steps may be considered initial conditions for a discrete change. Analytic models, on the other hand, treat continuous changes via functional relationships which do not permit introduction of temporarily constant coefficients. The discrete stepping of numerical solutions is therefore of distinct benefit in treating nonlinear systems, in compensation for less accurate representation of continuous changes. Also, hysteresis is easily treated by numerical models, because history can be considered for a step and the proper values of conductivity, matric potential, etc., can be selected from functional relations or tables. Analytic models (and, generally, empirical models) do not possess this flexibility.

as central versus one-sided differencing, handling of initial and boundary conditions, and discussion of theoretical accuracy of various methods of differencing, are not treated here. The unfamiliar reader is referred to Remson, Hornberger, and Molz (1971), or to general texts on numerical methods.

45. Explicit finite differencing results in an equation for the value of a dependent variable at a succeeding time step and spatial grid point in terms of known values of dependent and independent variables and coefficients. The equation can be directly solved for the dependent variable. Explicit finite differencing is the most straightforward approach and the simplest to program and may be used to resolve the entire spatial grid one point at a time. However, small time steps must be used based on spatial grid size and process physics, to avoid instability.

46. Implicit finite differencing results in a system of equations for the values of a dependent variable at all spatial grid points at a succeeding time step in terms of the dependent variable at adjacent grid points. The system of equations results because spatial gradients of the dependent variable are written using future values of the variable. While not as simple to program or visualize as an explicit method, an implicit method has advantages. It is more stable and more economical for a single time step resolution of the entire spatial grid, and its stability permits further economy through larger permissible time steps.

47. One implicit finite difference approach that has been used extensively is the Crank-Nicolson scheme. In this scheme, spatial gradients are written as averages of gradient terms involving known and unknown values of the dependent variable, i.e., explicit and implicit terms. While more complex in formulation than a fully implicit form, the Crank-Nicolson scheme results in improved accuracy due to better control of problems which arise within the numerical integration itself. The integration over a time step is performed using a gradient appropriate to the step midpoint due to the averaging of explicit and implicit terms.

48. The Richards equation (and each of its equivalent forms) is a strongly nonlinear partial differential equation. The nonlinearity arises since Darcy's law is applied to unsaturated media for soil moisture prediction, and hydraulic conductivity (coefficient) is a strong function of matric potential (gradient term) in unsaturated soils. The nonlinear equation may be solved by iteration, where an estimated hydraulic conductivity is

42. Matric potential and volumetric moisture content are both unknowns for models of the unsaturated zone, and the Richards equation contains both (see Appendix C, paragraph 6). One of the two is normally eliminated from the equation mathematically, and then obtained from the soil characteristic curve after the other has been calculated by the numerical solution. Elimination of volumetric moisture content enables more general application of the model, as it is then applicable to cases with ponded water on the surface. Further, when total potential is used as the unknown, the equation also applies to saturated flow. On the other hand, elimination of matric potential leads to a diffusion equation with advantages in some applications. Both approaches have been used with good results for appropriate initial and boundary conditions, although for general modeling, it is better to use the matric potential form of the equation (Philip, 1958).

43. Given the Richards equation (or the equivalent in either matric potential or volumetric moisture content) as the basic mathematical description of the physical process of soil moisture movement and storage, or the principle of conservation with Darcy's law flow into and out of soil volumes, the equations must be rewritten in a form suitable for computer solution. The most commonly used approach is to rewrite gradients in space and time in terms of finite spatial and temporal steps, i.e., as finite differences. The solution is then calculated for the points of the discrete spatial grid by stepping through time. Step sizes must be selected to achieve an optimum balance between solution accuracy and computational expense and time for each situation, with additional consideration required to ensure that the solution is stable and converges to the "true" result. An alternate method coming into more frequent use is to use principles of variational calculus to formulate the numerical algorithm, frequently resolving the spatial variation in terms of finite elements rather than at points of a grid. This approach is well covered in Remson, Hornberger, and Molz (1971) and Guymon, Scott, and Herrmann (1970). At this time, it appears most suitable to areas with complex boundaries. Because most agricultural applications have involved only the finite difference approach, it has been emphasized in this report.

44. Two other major decisions complete the outline of a specific model, namely, explicit versus implicit finite differencing and the method of treating nonlinearity of the equations. Other details of solutions, such

39. Judgements of various models given in the above-cited references must be viewed in the context of the respective texts or reports, however. Both crop yield models and remote sensing techniques are too coarse to be able to use the information from the more sophisticated models; thus they tend to emphasize the economy of budget models for soil moisture prediction. While runoff estimation may fall in a similar category with respect to resolution in depth and area as yield models and remote sensing techniques, the depth and areal resolution for soil strength prediction must be greater. This is particularly true when mobility is considered beyond a simple GO-NOGO prediction, or when hours count in tactical operation, or when the trafficability predictions apply for vehicles covering a broad range of trafficability capabilities.

40. Numerical models for soil moisture prediction are based on Darcy's law (moisture flow is proportional to hydraulic potential gradient) and the principle of the conservation of mass (equation of continuity). Models have been developed which are based on the numerical solution of a single equation of continuity (see Appendix A), and also based on the change of water substance in a soil layer (or volume) from the sum of flows across layer surfaces. Generally, the former approach has been used by modelers who did their own programming in FORTRAN, particularly modelers using an implicit numerical method of solution, while the latter approach has been used by most modelers working with CSMP and using explicit numerical methods. Neither representation of process physics is intrinsically superior to the other for model applications.

41. Flow in the unsaturated zone of the soil is generally transient in situations of interest, including temporary saturation during infiltration. Soil moisture prediction models calculate flow and soil moisture content for a future time by stepwise temporal change from specified initial conditions, with specified boundary conditions on the volume for which prediction is being made. Flow in the saturated zone is generally treated as steady-state movement in response to specified boundary conditions, although water table movement (transient) is treated in a few models. Some form of iterative solution is used in the saturated flow case, such that successive approximations to the flow converge to steady-state. Models which treat both unsaturated and saturated zones require coupling between basically transient and steady-state solutions for the respective zones.

36. In all, the concepts of potential evapotranspiration, catenary processes for water movement through plants, and stages of drying of a soil have aided understanding of the complex process of evapotranspiration. Most soil moisture prediction models include these concepts and their mathematical implementation to some degree at present.

Numerical Models

37. Fleming (1975) includes a short summary of the development of computing machines in the beginning of his book on watershed modeling, including the dramatic increases in their computational speed and memory capacity during the three decades since the introduction of semiconductor devices. The increases in both speed and memory have made possible models of physical phenomena that were unthinkable in the early 1950's, at least outside science fiction. Parallel software development has produced computer languages, such as FORTRAN, and higher order software, such as CSMP (Continuous System Modeling Program), which greatly simplify communication with the computers. Numerous models have been developed for soil moisture prediction which utilize these increased computational capabilities, ranging from simple mass balance models similar to the WES soil moisture prediction model to complex three-dimensional watershed models with extensive consideration of process physics. The general features of these models are discussed here, while Appendix C includes details of the approaches used by several active modeling groups.

38. Several references are particularly recommended for additional study; for an overview of soil moisture prediction models with emphasis on budget (mass balance) approaches--Schmugge, Jackson, and McKim (1980); for an excellent discussion of numerical techniques in hydrological applications, including soil moisture prediction models--Remson, Hornberger, and Molz (1971); for general information on CSMP and a discussion of its application to a broad range of specific soil moisture prediction problems, including a two-dimensional watershed model--Hillel (1977); for a rigorous comparison of six numerical modeling approaches to both analytic solutions and laboratory data for the se of infiltration into sand--Havercamp et al. (1977); for further details on a number of budget models--Hildreth (1978); and for a review of soil moisture modeling as it relates to crop yield models--Kanemasu et al. (1980).

the rate of evapotranspiration itself, as it exceeds the rate at which water can be supplied. Several developments have recently aided organization of this complex of factors with resulting progress in evapotranspiration prediction.

33. The most important concept with respect to evapotranspiration was put forth in the mid-1940's by Thornthwaite et al. (1944), namely, potential evapotranspiration. Potential evapotranspiration is that evapotranspiration rate that is not limited by moisture supply, but only by energy supply and ventilation. Thornthwaite (1948) was able to formulate useful regression relations based on mean temperature and day length for the calculation of potential evapotranspiration, while Penman (1956) and Van Bavel (1966) modeled potential evapotranspiration in terms of the energy balance at the surface and ventilation. This concept has proven of value in the separation of limits to evapotranspiration and more detailed study of the atmospheric contributions. Soil and plant limits to evapotranspiration are then seen in terms of the difference between potential and actual rates.

34. A second organizing concept of great importance was put forth by Van den Honert in 1948 and extended by Cowan (1965), namely, to treat the flow of moisture from the soil to the atmosphere through a plant as a catenary process, i.e., limited by the flow through the portion of the path with highest resistance. Gardner (1960) modeled the flow of moisture to a root with consideration of the transpiration demand rate, an approach extended by Cowan (1965) to include varying demand. These results (and others discussed in Appendix B) established that the concept of permanent wilting point is questionable because plant wilting depends on both demand and average soil moisture content.

35. An excellent discussion of three stages of drying of a soil is given in Heller (1968), while research reported by Idso et al. (1974) and Jackson et al. (1973) describes surface color variations and soil moisture measurements related to drying of an irrigated soil. Theoretical developments relative to water movement in drying soils have been accomplished by Philip and de Vries (1957) and by Gardner (1959). An interesting approach was taken by Staple (1974) who used relative vapor pressure of a partially dry surface in the Penman equation for potential evapotranspiration to form the upper boundary condition for flow of moisture within the soil.

soil characteristic curves, and results in an unbounded set of transition curves, called scanning curves, for partially wetted or dried soils which are then subjected to change in the opposite direction. Hysteresis imposes enormous complexities for analytic solutions to problems of alternate wetting and drying. It is handled in numerical solutions by using separate data sets and checking the progress of the solution to see what curve to use for the next step. New opportunities to deal with hysteresis have been opened by Golden (1980) using percolation theory, but like spatial variability, hysteresis is a problem yet to be fully resolved.

31. Advances in soil moisture measurement techniques have been well covered in a review paper by Schmugge, Jackson, and McKim (1979 and 1980), including discussion of both in situ methods and remote sensing methods with a listing of the relative advantages and disadvantages of each method. Little would be gained by a repetition of their material here, but some comments are in order. In situ measurement is labor intensive, particularly when field heterogeneity is considered, while the remote sensing techniques are generally limited to the surface layer of the soil, 1 to 5 cm (1/2 to 2 in.). Remote sensing of vegetation temperature may sense deeper layers since the vegetation temperature is dependent in part on transpirational cooling, but serious problems remain in relating temperature to moisture for a broad spectrum of plant species and environmental conditions. One is led to the natural conclusion that remote sensing and in situ methods will be useful for soil moisture prediction purposes only if coupled to models which more efficiently treat depth and spatial variations beneath and between the measured soil volumes.

Evapotranspiration

32. Evaporation and transpiration are primary depletion mechanisms for soil moisture and must be considered in any soil moisture prediction scheme. Evapotranspiration is basically a limited process, where the evapotranspiration rate is limited by the supply of energy for vaporization, by ventilation to remove water vapor, or by the water supply. It involves solar energy, atmospheric radiation, sensible heat from the atmosphere, near-surface winds, conduction of heat from the soil, moisture movement within the soil, plant transmission of water to leaves, root growth, and the

characteristic curve that was adjusted to conform to limited data and derive representative curves of both matric potential and hydraulic conductivity for all moisture contents with sufficient accuracy. Even without measurement specific to a sample, moderately accurate curves could be generated. This approach has been used by several modelers (see Appendix C) and is consistent in concept with the development of tentative average relations for sand, silt, and clay as done for the current WES soil moisture prediction model. Examples of typical soil characteristic curves for USDA system soil types are given in Hanks and Ashcroft (1980) after work reported in Taylor and Ashcroft (1972). Unfortunately, none are known for USCS soil types, except by translation from USDA system soil types as in Meyer and Knight (1961), although typical USCS soil curves could be generated.

28. A number of analytic solutions to relatively simple problems of soil moisture prediction have been developed in the past three decades. The majority of the solutions apply to problems of infiltration into dry soil, as hysteresis does not then complicate the physics of water movement. While of limited value for extensive soil moisture prediction in the field, these analytic solutions are of great value in testing numerical models, as the correspondence of analytic solution and numerical solution should be high for identical system assumptions.

29. Field heterogeneity of soil properties has been studied with some success in the treatment of spatical variability. Field heterogeneity is a problem of massive proportions for military trafficability and hydrology applications, as well as agricultural applications, because it reduces the spatial scale which must be considered to the order of metres, and thereby increases the computational requirements by orders of magnitude relative to consideration of the spatial scale of soil types. The principal success to date has been in the treatment of soils which have similar physical characteristics; i.e., they differ only in scale at the microscopic level, but much remains to be done. Measured matric potentials and/or hydraulic conductivities have proven useful for the estimation of the microscopic scaling parameter, while research on its spatial variation is continuing.

30. In addition to the spatial variation of soil properties, there are temporal variations in the relations between soil moisture content and both matric potential and soil moisture diffusivity due to the phenomenon of hysteresis. Hysteresis requires the determination of both wetting and drying

to military problems and recommendations for improvement of the present military approaches are presented in following parts. Additional details on developments in soil physics, evapotranspiration, and numerical modeling are given in Appendices A, B, and C, along with a more ordered review of the literature of the last three decades.

Soil Physics

25. Hydraulic conductivity is a critical parameter of the soil with respect to soil moisture movement and its prediction. Water flux density is given by the product of hydraulic conductivity and the gradient of hydraulic potential (Darcy's law), and enters all models which deal with the process physics of soil moisture changes. Hydraulic conductivity is a difficult factor to measure directly; thus research leading to methods for calculating it from the desorption soil characteristic curve (matric potential versus soil moisture content) of a sample has simplified the data acquisition burden for soil moisture prediction.

26. The recent approaches to the calculation of hydraulic conductivity from a soil characteristic curve are revisions of the original effort of Childs and Collis-George (1950). The method utilizes the soil characteristic curve as a measure of the distribution of passages for water movement within the soil matrix, in contrast to prior efforts to model flow blockage due to soil grains using the grain size distribution. Because the soil characteristic curve is more readily measured than hydraulic conductivity and may be measured using an essentially undisturbed soil sample in contrast to soil grain size distributions, the approach is superior on two counts. Numerous investigators have extended the method, including consideration of soil moisture diffusivity (see citations in Appendix A). Results of the method have been found sufficiently accurate.

27. Another development of significance in reducing the data acquisition burden has been the identification of soil characteristic curves for typical soils representing the soil types in the USDA system. Given knowledge of soil type and a limited set of site-specific measurements of matric potential versus soil moisture content, say at -15 , -0.06 , and -0.005 atmospheres matric potential corresponding roughly to permanent wilting point, field capacity, and saturation, respectively, one could use a typical soil

PART II: SOIL MOISTURE PREDICTION IN AGRICULTURAL SCIENCE

22. Military or civilian applications of trafficability and hydrology require knowledge of soil moisture content as a function of depth for extensive areas in order to estimate soil strength without in situ measurement and to predict surface runoff. Prediction of soil moisture content in turn requires knowledge of precipitation, surface flow, drainage, evapotranspiration, and soil type as a function of depth for the same extensive areas, and for a substantial period prior to the time for which a prediction is being made.

23. Severe limitations are imposed on soil moisture prediction by the areal extent for which predictions are needed, the spatial and temporal variability of precipitation and evapotranspiration, insufficient data on soil characteristics with depth, and inaccessibility of some areas critical to military operations. Even nominally homogeneous areas of a single soil type have been shown to have variations of soil properties which must be considered in soil moisture prediction; thus prediction must be made for fairly small areas or must account for variability within larger areas. Subsurface soil type also varies widely within nominally homogeneous areas of a specific surface soil type, and its determination presents very serious difficulties due to the large areas involved. A measure of spatial variability of precipitation from cyclonic storms and of evapotranspiration may be obtained from local topography, including slope orientation, while evapotranspiration is also influenced by vegetation type. Convective storm precipitation is more randomly variable, however, and cannot be adequately measured by surface networks of rain gages at commonly used spacings.

24. A coordinated program of measurement and computer modeling is required to deal with the problem of soil moisture prediction. The program must be further adapted to treat extensive areas with a useful degree of accuracy. Research in agricultural applications has led to greatly improved understanding and definition of the important factors for soil moisture prediction and to vastly improved models and computational methods for their solution over the last three decades. Advances in remote sensing of near-surface soil moisture have also occurred. The recent developments in agricultural science related to soil moisture prediction are discussed in this part of the report. A discussion of the applicability of these developments

correlations of data in forming the model. This historical point has been emphasized because model improvement is likely to require reconsideration of processes which were lumped into the regression relationships, and because subsequent reports have not communicated the early attempts to build the model from a sound scientific base, leading to inappropriate criticism by some model reviewers.

21. The WES soil moisture prediction model in current use is therefore an updated version of an approach developed almost three decades ago. For the time, the approach was formulated with awareness of the known physics of the extremely complex problem. For the present, the model has incorporated some increased knowledge, but it has been overtaken by models based on more complete analysis of problem details. Comparison tests may establish that the current WES model is still as good as others for trafficability and mobility predictions worldwide. However, in the opinion of the author, this cannot last, and a theoretically based model would meet with much broader acceptance.

actual moisture content of the two layers just prior to the precipitation event. Precipitation in excess of accretion for the two layers is not explicitly treated.

18. Seasonally dependent depletion rates are used in the model via depletion relations for winter, summer, and transition (spring and fall) seasons. Separate rates are also required for each soil type; sand, silt, and clay, and for each layer, resulting in 18 depletion relations for the model. The depletion relations for the USDA soil types; sand, silt, and clay, have been correlated with soil types under the Unified Soil Classification System (USCS), also. The depletion relations were derived by visual averaging of a large number of superimposed graphs of moisture content versus time, first to derive relations for specific sites, then for several sites with a common soil type. The present model uses polynomials of up to sixth order that were curve-fitted to the averaged graphs to generate functions for computer programming of the model. These depletion relations are scaled to the range of soil moisture contents between field maximum and field minimum for each application.

19. The model was found to be reasonably accurate when tested against field data from over 100 sites with fine-grained soils, but as noted above (paragraph 14), it was less accurate for wet soils and soils with high organic content and clay. Additional details of the model and comparison to field data may be found in Smith and Meyer (1973) and Carlson and Horton (1959).

20. The early effort leading to the present WES soil moisture model was based on knowledge of many atmospheric and soil factors that affect the temporal and spatial variation of soil moisture, particularly the work of the Forest Service personnel. Lull (1953) reviewed the literature of soil physics, plant physics, and evapotranspiration with quoted excerpts which demonstrate an awareness of the state of knowledge at that time. Two progress reports prepared by the Forest Service (US Forest Service, 1951 and 1952) also include discussion of physical and biological factors known to be relevant to the problem of predicting soil moisture. Unfortunately, these factors became blended into brute force regression relations and are no longer explicitly treated. Soil moisture prediction proved to be intractable as a problem in physics at that time, and it was found necessary and sufficient to rely on

change from specific relationships derived for prediction sites to general relations based on soil type, this step was necessary for model applications to extensive areas without data. The accuracy retained indicates the basic validity of the relations derived with respect to dominant physical processes, but the excessive errors (judgement from Carlson and Horton (1959)) in the prediction of soil moisture for poorly drained and wet sites, and for soils high in organic content and clay, indicated model limitations. It is also notable that the overall accuracy level achieved was largely due to good predictions for low moisture contents, which are not critical for trafficability or mobility.

15. Additional studies in tropical areas met with similar results; i.e., the prediction accuracy associated with USDA soil types rather than with soil data for particular sites was reduced; however, the results from the model were the best predictions available, and model adjustments for improved predictions could be made with the enlarged data base. After a trafficability and mobility program review by a selected board of consultants was held at WES in 1966 (WES, 1967), principal research emphasis was shifted to other problem areas. The model was refined as time, data, and support allowed, with some proposed improvements incorporated in the computer program of the model, written around 1970 (Smith and Meyer, 1973).

16. The WES soil moisture prediction model is a budget model in which a simple bookkeeping procedure is used to account for daily moisture changes in two 15-cm (6-in.) layers in the upper 30 cm (1 ft) of soil. (The model has recently been extended to 90 cm (3 ft) with tentative relations subject to validation.) Soil moisture is increased by accretion due to precipitation and reduced by depletion between precipitation events. The physical processes of depletion are not explicitly treated. The range of soil moisture variation is held between field maximum and field minimum moisture contents that have been determined by site-specific measurement or by expressions derived from multiple linear regressions against soil properties for similar soils.

17. Soil moisture accretion for each layer is determined by expressions derived by linear regressions of accretion data against precipitation or available moisture storage in the layers. The former form is used for cases where precipitation is less than available storage, while the latter form is used when precipitation exceeds available storage. Available storage is defined as the difference between field maximum moisture content and

Soil property. A physical characteristic of the soil matrix, such as porosity, soil particle types, or moisture diffusivity.

Tensiometer. An instrument for measuring matric potential.

Trafficability. The capacity of a soil to withstand traffic of vehicles.

Transpiration. The loss of soil moisture or plant moisture by vaporization at plant surfaces after passage through the plant.

Tridiagonal matrix. A square matrix in which the only nonzero elements are on the main diagonal and on the diagonals immediately above and below the main diagonal.

Weighting lysimeter. A device for weighting a physically isolated block of soil in nominally natural surroundings to measure evapotranspiration.

Wetting curve. The soil characteristic curve derived during wetting of a dry soil sample.

Wetting front. The spatial position of the sharp gradient of soil moisture between dry and saturated portions of a soil column during wetting.

WES Soil Moisture Prediction Model

12. A brief review of the WES soil moisture prediction model is included in this section to provide a reference for readers who are not familiar with the model, and as a guide to the author's viewpoint for those who are.

13. Pilot studies were conducted during the period from 1948 to 1953 by WES and US Forest Service personnel to identify the scope of the problem of predicting soil moisture and to formulate a preliminary prediction model and research plan for its improvement. Two preliminary models were developed independently by the WES Trafficability Section and the US Forest Service during this period. As the Forest Service model afforded greater potential for future development, it was selected for continued effort, although each model had predicted soil moisture about as well for the data available (WES and US Forest Service, 1954, Vol 3).

14. Data collection was extended via cooperation with other groups and additional sampling by WES and Forest Service personnel. The enlarged data base enabled formulation of the model for the general soil types; sand, silt, and clay, from the US Department of Agriculture (USDA) soil classification system (Carlson and Horton, 1959). While some accuracy was lost in the

51. The choice of a particular set of model characteristics is not self-evident. Models have been formulated with various combinations of methods of finite differencing, nonlinearity treatment, hysteresis treatment, and step size selection. An optimum combination requires consideration of the physical process treated, the intended use of the results, resources, and modelers' points of view concerning accuracy, stability, economy, and convenience. Direct comparisons of model performance, such as that of Haverkamp et al. (1977) for infiltration, are required as an objective guide to model selection or model development for new applications.

52. In addition to models which treat the movement of water in the soil matrix in one-, two-, and three-dimensional systems, several models have been developed which include plant roots in the soil, and a few include explicit consideration of the entire plant. Plant considerations are important in determining the depth to which soil moisture is strongly affected by evapotranspiration, and some redistribution of moisture within the soil profile is effected by root transfer of moisture from more moist to dryer layers.

53. Two basic representations of roots within the soil volume are used. A microscopic root model treats flow of moisture to individual roots that are considered straight, infinitely long, and isolated from other roots. The moisture uptake of the roots per unit length is calculated, and then multiplied by a measure of total root length in a volume to obtain total water extraction. This representation of root-soil water interaction is necessary for study of the effects of moisture gradients in the vicinity of roots. A macroscopic root model treats flow of moisture to roots via a bulk measure of rooting, such as rooting density. The process physics of flow to the roots is not explicitly treated in these models, with consequent loss of resolution and reduction of computational costs. Each approach has been used with success (see Appendix C).

54. Currently available soil moisture prediction models, even the simpler mass balance models to be discussed in the following section, are limited by available input data. Several papers have reported differences between model output and laboratory data which may best be explained as data errors, for instance, the assumption of uniform packing of soil in a laboratory test column. Moisture movement occurs in response to actual soil packing density, while the model must use what the experimenter inputs about

packing density. Field data of sufficient vertical and horizontal resolution for specification of initial moisture condition and soil characteristics important to soil moisture prediction are extremely sparse.

55. Field and laboratory data obtained at Agricultural Experiment Stations and Agricultural Research Service offices throughout the nation in connection with various experiments and modeling efforts are frequently sufficient for modeling purposes; however, no collection of such data sets was noted during the literature review. A set of desorption data for over 1800 soils on experimental watersheds was published by Holtan et al. (1968, see also, Holtan et al., 1967). The data include textural class. These data were used by Clapp and Hornberger (1978) to evaluate their power curve representations of soil hydraulic properties. The only larger body of data noted is that gathered during the development of the WES soil moisture prediction model and reported with that effort.

56. Model data requirements vary greatly, as the application for which each model was developed and the modeler's anticipation of data availability vary greatly. On the one hand, models have been used with estimated characteristics for typical soils and schematic (sinusoidal) variation of evapotranspiration, while on the other hand, some models require detailed curves of matric potential and hydraulic conductivity or moisture diffusivity versus soil moisture content, detailed soil profile data, and full meteorological data for calculation of potential evapotranspiration, as well as precipitation on site. A separate research project would be required to detail data required for each model and assess the significance of each datum. While this has not been attempted, inputs for several of the models reviewed in Appendix C are noted along with results of sensitivity studies when reported. Further comments on data for models are included in sections on applicability and recommendations.

57. A measure of cost for numerical modeling of soil moisture was given by Wind and Van Doorne (1975). Their one-dimensional model cost about 38¢ per simulation day to run. It produced results appropriate to trafficability applications, including depth to the water table, for the top 1 m (3 ft) of soil. Programming convenience may result in costs an order of

magnitude higher (Haverkamp et al., 1977; Richter, 1980), while some cost reduction may result from tailoring the model to specific applications. Higher cost will accrue for two- and three-dimensional models with similar vertical resolution. Cost will also increase with the number of sites and simulation days required.

Mass Balance Models

58. Mass balance (budget) models are based on the law of conservation of matter. They generally calculate soil moisture content of a soil layer(s) from a prior moisture content by simple addition and subtraction of moisture fluxes due to such processes as precipitation, evapotranspiration, runoff, and drainage. Measured, estimated, or otherwise forecast precipitation and irrigation are used as inputs, while outputs are generally determined by statistical methods, including linear regressions based on field data. Since process physics is not influenced by the character of a model, the seemingly simple expressions used for moisture fluxes in mass balance models must still incorporate physical processes via parameterization or in the regression coefficients. Darcy's law is not used in these models, thus rendering them tractable without numerical integration schemes, but many of the models do explicitly treat evapotranspiration and other aspects of the total process physics with separate expressions. Most are adapted to computer applications.

59. Relatively simple one-layer budget models were developed by Thornthwaite and associates (Thornthwaite and Mather, 1954) and by Jensen, Robb, and Franzoy (1970). The Thornthwaite model was used for estimating soil moisture for prediction of traction capability. Moisture input was given by measured precipitation, while both gravitational drainage (above field capacity) and evapotranspiration were considered for moisture losses. Potential evapotranspiration via the Thornthwaite method was adjusted for soil moisture content below field capacity. The Jensen et al. model was developed for irrigation scheduling. It uses measured and forecast precipitation plus irrigation in soil moisture prediction. Potential evapotranspiration via the Penman method is adjusted by means of crop coefficients to determine actual evapotranspiration (the only loss) through the growing season.

60. The two-layer model developed at WES has been discussed in the Introduction of this report. It is a mass balance model with extensive use of data correlation in the model parameterization.

61. Multilayer mass balance models have been developed by Baier and Robertson (1965, 1966), Jones and Verma (1971), and Stuff and Dale (1978). Several additional multilayer models related primarily to crop yield modeling are also noted by Hildreth (1978) with a listing of model equations. Baier and Robertson use their model for latent evaporation to estimate potential evapotranspiration and then combine potential evapotranspiration with available soil moisture in each layer to calculate actual evapotranspiration. Their calculation includes an empirical coefficient to account for soil and plant characteristics of each layer. Precipitation, runoff, and percolation through soil layers are considered in expressions used to calculate soil moisture change in each layer each simulation day. Variable layer thickness is allowed in the model.

62. Hildreth (1978) compared calculations by the Baier and Robertson model with field data for one month under a wheat crop. He found the model results to be within 10 percent of measured values in six 30-cm (1-ft) layers at the end of that period, with the total moisture content calculated within 2 percent of the measured amount. He also found, however, that several combinations of model parameters resulted in similarly accurate results, but one set of parameters resulted in poor model performance in an independent test. Hildreth's sensitivity tests of the model indicated no amplification of errors in input values, but output error varied one-to-one with input error of 10 percent in soil moisture capacity, available soil moisture, rooting coefficient, and a soil dryness coefficient. Further, he found errors to be additive, leading to potentially large error in model calculations.

63. The model of Stuff and Dale (1978) was developed to treat soil moisture prediction with high water tables under corn. It was based on empirical relations developed from two years' field data, including measured water table depth and capillary rise from the water table. The model had seven layers each 15 cm (6 in.) thick. Evapotranspiration was based on

measured pan evaporation with adjustment factors for crop development and moisture stress factors. Further model development would be required for application to other areas and to extend the range of soil moisture used for parameterization, but this model is one of the few mass balance models with consideration of water table effects.

64. Bouma et al. (1980a, 1980b) also considered the effects of water table levels using several levels of detail in their model input data that were based on soil surveys and field sampling. They found strong influence on predicted soil moisture from subsoil characteristics and also found that using a single field boring to determine subsoil type for a given homogeneous area on a soil map was detrimental to predicting accuracy. Either field estimates of subsoil type made during soil surveys or multiple borings yielded superior predictions. Their work dramatizes the importance of subsoil data, as even the relatively simple prediction model they used was limited in prediction accuracy by input soil data.

65. Schmugge, Jackson, and McKim (1980), Kanemasu et al. (1980), and Hildreth (1978) should be consulted for additional details and references to individual models of the mass balance type. Further information is also provided in Appendix C.

PART III: DISCUSSION

66. Numerous advances in the areas of soil physics, evapotranspiration, and modeling of soil moisture have been made in agricultural science since the currently applied model for military soil moisture predictions was developed. During that period there have also been several advances in the state of the WES soil moisture model as additional data and research have been used to improve the regression relations used; to improve estimation of field maximum and field minimum moisture contents; and to treat problems of spatial variability of soil properties, water table influences, and problem soils of high organic content. The WES model has been adapted for computer applications with some revision of the accretion and depletion relations, fitted polynomial curves for these relations, and, in addition to soil type, consideration of site conditions (by means of a wetness index used to describe the influence of water table and drainage conditions on potential maximum soil moisture content). The model has also been tentatively applied to depths of 1 m (3 ft) for C5A transport aircraft taxiways.

67. The question of applicability of the developments in agricultural science to military problems of trafficability and hydrology is therefore not simple, because the current system is useful, tuned over many years to specific military considerations, and in place. In one sense, perhaps the dominant sense in view of limited financial and manpower resources and competing opportunities, the applicability of developments in agricultural science requires determination that their implementation would result in improvement to the present system worth the effort. In a less pragmatic sense, applicability judgement would be based on potential usefulness of a specific development without evaluation of competitive stature with respect to current military methods. The following discussion strives for moderation and addresses opportunities in preference to certainties.

68. In keeping with recent workshops on soil moisture prediction (Heilman et al., 1978) and military hydrology (WES, 1979) it is appropriate to consider short- and long-term research needs and to consider amendments to the present model versus development of a new approach.

Short-term Research Applications

WES soil moisture prediction model

69. The WES soil moisture prediction model uses soil classification, field maximum and field minimum soil moisture contents, accretion and depletion relations, seasonal transition dates, initial conditions of soil moisture, and precipitation data.

70. Soil classification. The present model uses only three soil types, sand, silt, and clay, combining several separately recognized soil types in both the USDA and USCS classification systems into these three categories. Revision of field moisture content factors and accretion and depletion relations to incorporate additional classes could be done with the guidance of typical soil characteristic curves, moisture capacities, model outputs for typical soils, and the guidance of process physics. Advances in soil physics and modeling have provided bases for such revision.

71. Typical soils have been identified and described only for USDA types, while the USCS system has been emphasized in military applications. The above-noted possibility, however, in conjunction with the association of USCS soils with USDA soil types on the USDA textural triangle in Meyer and Knight (1961), may be productive in the definition of model parameters for USCS-specific soil types. Characteristics of typical soils and model simulations to test the proposed characteristics of USCS typical soils against field data are direct applications of recent developments to improvement of the WES model.

72. Research on field heterogeneity of soil properties has established that variations within nominally homogeneous areas can be sufficiently large to critically affect predicted soil moisture content. On the one hand, this variation creates great difficulty for soil moisture prediction models because the input soil characteristics become uncertain. On the other hand, the variability of soil properties places bounds on the precision with which soil properties may defensibly be specified in field applications to large areas. Judicious application of the results of field heterogeneity research to military problems may well simplify the modeling of soil moisture by justifying use of typical soils and moderate accuracy models. The results of modeling would, of necessity, be recognized as estimates with bounds of

probable error stated as part of the simulation output. In either case, limitations on potential accuracy or justification of model simplification, field heterogeneity is applicable to military problems.

73. The modeling work of Bouma and associates is applicable to this issue, in that they have tested a relatively simple soil moisture prediction model against several levels of input data detail, and have related this work to soil types defined by soil survey maps. Military applications of a soil moisture prediction model to large areas will necessarily involve use of soil survey data, and both horizontal and vertical heterogeneity will require treatment. This research direction may lead to use of different soil types for the two layers of the WES model.

74. Field moisture contents. Field maximum moisture content can now be simply calculated for a specified soil profile using one of many numerical models of infiltration. The definition of field maximum moisture content may be used to specify boundary conditions, including water input. Summation of soil moisture content in the appropriate soil volume after steady-state infiltration has been reached gives the desired value. With little effort an existing model may be adapted for this calculation, or values for typical soils may be precalculated for use in specific applications. The cost of such an approach would be small, while these models are so accurate that significant differences between measured and calculated values would clearly indicate that the field situation was not well known.

75. Field minimum moisture content is not as simple because the influence of plants and the rate of potential evapotranspiration will affect the field minimum moisture content for a specified soil. Research in evaporation and transpiration may be used to define the gradient of moisture content which would prevail for specified boundary conditions at the surface and deeper in the soil than the volume of interest, but the potential for significant improvement in modeling of subsequent high moisture contents in the soil that are critical to military operations is small. This statement is not true in cases of high water tables or significant subsurface inflow to a region of interest. In such cases, subsurface soil and flow data are required for use in two- and three-dimensional numerical models. See the discussion of long-term research for further comments.

76. Accretion and depletion relations. The accretion and depletion relations used in the WES model embody the model process physics. These relations incorporate the processes of infiltration, redistribution, drainage, and evapotranspiration. Since correlation relations such as these frequently contain implicit error compensation in the regression coefficients, there is a high probability that accurate, explicit introduction of one of the processes, say evapotranspiration, will not lead to improved results, even after new regression relationships are derived for the residuals. Short-term research effort would be better spent in other areas, such as identifying and researching soil and subsurface flow conditions which violate the assumption of a well-drained homogeneous soil that is implicit in the present model.

77. Seasonal transition dates. If the possibility of frozen soil is set aside, transition dates are probably dominated by changes in potential evapotranspiration from season to season. Several models for calculation of potential evapotranspiration are available, and additional steps to convert potential to actual evapotranspiration have also been worked out. Short-term research with a high potential for success would involve correlations of calculated potential and actual evapotranspiration with accepted transition dates for cases in the WES data files. Strong correlation is anticipated between WES model transition dates and periods of rapid transition in calculated potential evapotranspiration.

78. Initial soil moisture conditions and precipitation. When measurement is not possible or cost effective, initial conditions may readily be calculated by starting one to three months prior to the required initial time, using nominal initial conditions, and running the model for that period. WES in-house research has been done on this problem, and many agricultural scientists have performed this test on their models. It would simply remain to apply these previous results to identify an optimum lead time for given soil and precipitation conditions.

79. Precipitation timing and magnitude, particularly in forecasting soil moisture contents, is a significant unknown. Even instrumented areas are seldom covered well enough to define the spatial pattern of precipitation from each event. Recent advances in weather radar and the use of lidar systems for rainfall measurement for calibration is frequently required.

Because most of the modeling work and other research related to soil moisture prediction in agricultural science has been related to rainfall deficiency and irrigation requirements, no major advances toward a solution to the problem of precipitation were noted in this literature review.

New modeling approaches

80. Several mass balance models have been published which could be tested against the current WES model to determine their potential for improved calculation of soil moisture. Several of the models are multilayer models and may be better able to handle organic silts and moist areas than the present model. Possibilities include replacement of the present model, model selection for different soil moisture conditions, and/or development of a hybrid model with features of two or more models included. Implementation of these models would be relatively easy due to their simplicity, especially if supported by the developers' experience. Testing them to the degree that the WES model has been tested would be a larger effort.

81. Preliminary investigations of numerical models, particularly models written in CSMP or another simulation language, would also fall under the category of short-term research. These models work and are directly applicable to soil moisture prediction problems for trafficability and hydrology, and their utility for prediction of conditions which are poorly handled by the WES model needs to be tested. This investigation would also permit a critical evaluation of the WES model against the state of the art. Tests would be most meaningful if limited to cases critical to trafficability or hydrology, as accuracy at soil moisture contents so low that mobility is not impaired and runoff does not occur is generally of minimum value for military applications.

82. These newer models that could be implemented with relatively little investment may also prove valuable in a two-stage modeling approach. The first stage would utilize the current model to screen-specific applications for potential problems, e.g., rating cone indices between 15 and 125, where it is assured that impossibility of movement or certainty of movement have been established at these extremes. Cases between these limits would then be treated with a more complex model to improve the precision with which potential mobility could be predicted. Numerous models have been developed that are applicable to this approach. Tests are required to determine whether the approach would be a cost-effective improvement to the present system.

Long-term Research Applications

WES soil moisture prediction model

83. This section discusses the applicability of recent developments in agricultural science to evolutionary changes in the present modeling approach, which is characterized by precipitation-dependent accretion relations and moisture content-dependent depletion relations in a mass balance approach. The following section discusses revolutionary changes in approach to soil moisture modeling.

84. The problems due to field heterogeneity of soil properties need to be addressed with field experiments and theoretical developments. The short-term approach outlined above may be extended to incorporate into area-wide modeling both developments to the present and future research advances in this area. Serious definition of the implications of field heterogeneity and bounds on prediction must be addressed for multiple-vehicle operations as well as along a single-vehicle path. Adaption of the current approach of prediction of both soil moisture and soil strength to area-wide predictions of mobility for straight and curved vehicle paths will require extensive research just to exhaust present knowledge of the field heterogeneity problem.

85. Adaption of the model to enable effective use of present and developing capabilities in remote sensing of soil moisture requires consideration of a thinner surface layer. The physics of radiant emission as a function of water content of the soil prevent sensing the moisture content below the first 1 to 5 cm (0.5 to 2 in.). The present top layer thickness of 15 cm (6 in.) in the WES model is too great to allow reliable application of remotely sensed soil moisture contents for initial conditions or modeling accuracy checks. Further, model predictions for heavier vehicles will require additional layers beneath the 30-cm (12-in.) layer for which the model was developed, and to which the present data base applies for the most part.

86. Acquisition of sufficient data to increase the near surface depth resolution and overall depth of the present model would entail significant expense. It would be far more cost-effective to use numerical models to generate simulated data for development of accretion and depletion relations.

While this appears unwise on the surface, and is likely to meet with resistance unless part of a well planned program which includes experimental data, it is a fundamentally sound approach for several reasons. First, many numerical models are very accurate, frequently more accurate than field conditions are known for specification of initial and boundary conditions. Since assumptions as to field conditions are involved in the WES model, they may be used for initial and boundary condition specification for the numerical model. Second, numerical experiments are far less expensive than field experiments per simulated data point (versus "real" data point). Initial and boundary conditions for the model may be varied to simulate the scatter of field data and to broaden the simulated results prior to performing correlations that would produce simulated accretion and depletion relations for additional soil layers. Third, thin surface layers may be modeled to correspond to remote-sensing limitations in depth, and several numerical model layers may be combined to simulate the 15-cm (6-in.) layers of the WES model or the limited resolution of neutron sampling methods of soil measurement. Direct comparison of simulation results with field data and WES model predictions may therefore be made to ensure reliability of the data set used to increase the vertical resolutions of the WES model. Fourth, anomalous and unknown field conditions do not enter the process, thus avoiding contamination of the correlations with data due to high water tables, vertical variation of soil properties, or subsurface flow. Fifth, the present extensive data set upon which the present model was based, which contains soil strength measurements in addition to soil moisture measurement, could be used in expanding the depth resolution of the model.

87. The present author's preference would be to directly implement a numerical modeling approach to soil moisture prediction. However, it is recognized that the present model has several benefits related to execution time and simplicity, as well as being part of the present mobility model. Limits to defensible resolution and accuracy and the advantages of evolutionary change mediate in favor of a higher resolution model using the present approach, but developed with the assistance of the state of the art in soil moisture prediction.

88. The WES model does not do well in cases of organic soils, high water tables, or subsurface water movement. Specific field cases for which

WES model performance has proven poor could be examined via simulation modeling. Model inputs could be changed to test hypotheses as to the cause of WES model error and/or inaccurate simulation by a more complex model. Existing numerical models are sufficient to this task, and improved definition of the WES model in response to identified error sources would be the result.

89. Topographic influences on soil moisture content are strong. Not only is overland flow influenced by slope aspect and orientation, but also evapotranspiration is strongly affected by slope due to the importance of solar energy supply of latent heat for vaporization. Studies have shown that subsurface movement of water frequently leads to nonuniform moisture distributions which are not directly related to surface relief, also. Potential evapotranspiration estimates using energy incident on a sloping surface and models of infiltration which consider runoff in two dimensions may be applied to this problem, but unknown subsurface soil layering and flow channeling pose a more difficult problem to solve, as specific applications require extensive specific site data. Long-term research in the similar problems created by field heterogeneity and topography is indicated.

New modeling approaches

90. Given a decision to implement a new modeling approach for either trafficability or hydrology applications of soil moisture prediction, a large selection of both mass balance and process-oriented models are available. Many of these models may be directly applied in their present form to both watershed and soil strength applications, while long-term research would doubtless lead to specific model adaption(s) for optimum use in strategic and tactical military activities. A two-level approach which involved initial screening by an economical, fast model to be followed by more thorough modeling of situations that could pose problems for military operations is feasible with present capabilities. Superior performance would be possible after optimization research based on prior model development.

91. The most difficult aspect of military applications of soil moisture prediction is the magnitude of the area involved. Implementation of models which can use remotely sensed data (thin layer sampling and poor spatial resolution) and typical soil characteristics appears necessary.

Standard model treatment of variability of soil properties in all three dimensions in a way that produces usable predictions is also necessary. Research results have been published in each of these areas, but a concerted, long-term research effort will be needed to bring these results to bear on military problems in an efficient, effective model. Research has also been conducted on soil moisture prediction based on soil survey data. This research direction is also necessary to deal with the area problem, since it bears directly on specification of typical soils for model use, and represents the most probable approach to military field applications of the model.

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- f. Contract one or two modeling groups to work with WES personnel in the development of the two submodels noted above. Strong emphasis should be placed on technology transfer in this step to build state-of-the-art expertise at WES during model development.
- g. Revise the current WES model as defensible, using methods discussed in Part III of this report, to increase soil moisture prediction capabilities during new model development. This recommendation would be particularly germane, if the basic approach of the WES model is selected for the screening submodel.
- h. It is strongly recommended that the model development include generating model capability to exploit remote sensing for initial condition specification, operational boundary condition adjustment, and simulation validation. This will require treatment of a thin surface layer and spatial averaging to simulate low spatial resolution.
- i. It is also strongly recommended that model development emphasize the use of typical soils, including a standard procedure that involves supplemental calculations for alternate submedium soil types that are likely to occur with a given surface soil type.
- j. In recognition of field heterogeneity, uncertainty of sub-medium soil type, unknown subsurface flow, and precipitation variability, it is recommended that some form of probability forecasting be used, either for the predicted soil moisture content or in the form of error estimates on the prediction.

94. Numerous details have been left out of the steps relating to new model development. Several possible steps and details of approaches have been discussed at several places in this report. It is recognized that the results of a serious test of the WES model against other models, on cases where a model is likely to be used and the results are important, would be the best guide to further effort.

PART IV: RECOMMENDATIONS

92. It is recommended that WES undertake an applied research project to develop an improved model for soil moisture prediction for use in military hydrology and trafficability problems as soon as resources permit. It is further recommended that the soil moisture prediction model consist of two basic submodels which are compatible but individually applicable to specific problem areas. One submodel is envisioned as a screening model that efficiently identifies soil moisture contents that may lead to excessive runoff or may impair mobility in military operations. The second submodel is envisioned as a more complex, but more accurate, numerical model to be used for problem cases identified by the screening model.

93. The following steps are recommended as one orderly approach to an improved soil moisture prediction model that includes staged development and optimization based on earlier results:

- a. Create a computer file of the extensive soil moisture/soil strength data acquired in previous projects for use in model development.
- b. Supplement WES data with data obtained from agricultural experiment stations, Agricultural Research Service offices, and civil engineering departments engaged in soil moisture/soil strength research.
- c. Select a limited set of cases for which field data are available that represent potential applications of a military soil moisture prediction model.
- d. Test the WES model on the cases selected in c using the full input data set and data subsets representing probable data limits in real military operations.
- e. Contract representative soil moisture modeling groups to test their models with the same data set and subsets, and to report and discuss their modeling results in a conference format, including comparisons of accuracy, cost, and model versatility against the WES model.

This sequence of steps will provide a factual basis for decision on whether to proceed with a serious attempt to modify or replace the current model. It will also provide a basis for assessment of the modeling approaches most likely to be optimum for screening and detailed simulation submodels, if continuation is indicated. Assuming further model development is supported by the facts collected in the above steps:

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APPENDIX A: SOME RECENT DEVELOPMENTS IN SOIL PHYSICS

Introduction

1. A number of developments in soil physics which have been cited in connection with numerical modeling of soil moisture are briefly discussed in this appendix. Other developments which are considered particularly germane to problems of trafficability and soil moisture modeling for large and/or diverse areas are also included, such as field heterogeneity of soil properties and methods of calculating soil characteristics curves. The latter developments would be of value, either to improve the basis and procedures of the current WES soil moisture model, or to create a new model. Recent development of soil moisture simulation models are discussed in separate appendices.

Fundamentals

2. The fundamental equations describing the movement of water in soils had been derived by 1952, about the same time as the basic concepts of the WES model were being formulated and implemented. Effective solutions of the equations required another couple decades for additional analytical work for special cases (for instance, by Philip and by Parlange) and for numerical techniques to be developed (see Appendix C).

3. Soil water exists in gaseous, liquid, and solid states, depending on temperature and water content. The frozen case has not been considered in this report. Water vapor movement is important during the final stages of soil drying and during periods of strong temperature gradients in unsaturated soils (Hanks and Ashcroft, 1980;* Taylor and Ashcroft, 1972; Nielsen et al., 1972; Philip, 1969; Hadas, 1968; Cary, 1966; Philip and De Vries, 1957; and many others cited in these). Temperature gradients also are noted to affect liquid flow to a lesser extent than vapor flow. Neither factor is considered further herein except as it has been included in a model discussed.

* References are collected at the end of the body of this report.

4. Movement of soil moisture as a liquid has been described by means of Darcy's law and the equation of continuity (see Klute, 1952b or Philip, 1969 for good general discussions, or any soil physics text). Darcy (1856) published an empirical relationship in which the rate of steady state water movement in saturated soils was proportional to the gradient of soil water potential, or

$$q = K \nabla \phi \quad (A1)$$

where:

q = the one-dimensional flow velocity [cm sec^{-1}] (volume per unit time per unit area)

K = the factor of proportionality known as coefficient of permeability or hydraulic conductivity [cm sec^{-1}]

∇ = the gradient operator [$\text{grad} () = \nabla () = i \frac{\partial ()}{\partial x} + j \frac{\partial ()}{\partial y} + k \frac{\partial ()}{\partial z}$ for Cartesian coordinates]

ϕ = the hydraulic potential [cm] forcing the flow.

If hydraulic potentials due to osmotic pressures or solute concentrations and gradients are ignored, the hydraulic potential, ϕ , may be written as the sum of matric potential (soil water tension, or suction), ψ , and gravitational potential, z , where z is the vertical distance from a prescribed datum level (arbitrary for system) with units [cm] for each.

5. The equation of continuity for this system may be written simply,

$$\frac{\partial \theta}{\partial t} = -\nabla \cdot q \quad (A2)$$

where θ is the volumetric water content [$\text{cm}^3 \text{ cm}^{-3}$] (to correspond with WES soil moisture model usage, note that inches of water per six inches of soil corresponds to 60). In Equation A2, convergence into a volume increases the water content with time, while divergence of the water flow reduces water content. Equation A1 is now substituted for q in Equation A2 to produce

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K \nabla \phi) = \nabla \cdot (K \nabla \psi) + \frac{\partial K}{\partial z} \quad (A3)$$

or rewritten for one-dimensional flow in the vertical,

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(K \frac{\partial \phi}{\partial z} \right) = \frac{\partial}{\partial z} \left(K \frac{\partial \Psi}{\partial z} \right) + \frac{\partial K}{\partial z} \quad (\text{A4})$$

where $\phi = \Psi + z$ has been substituted and simplified in the second equality. Equations A3 and A4 are quite general; they apply to homogeneous or heterogeneous soils and pose no special requirements on relations among θ , ϕ , and K .

6. It is convenient to introduce the concept of diffusivity (Childs and Collis-George, 1950; and Klute, 1952a and b) in order to eliminate matric potential from Equations A3 and A4. Thus, defining the diffusivity, D , as

$$D = K \frac{\partial \Psi}{\partial \theta} \quad (\text{A5})$$

and introducing D into Equations A3 and A4 produces, respectively,

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (D \nabla \theta) + \frac{\partial K}{\partial \theta} \cdot \frac{\partial \theta}{\partial z} \quad (\text{A6})$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(D \frac{\partial \theta}{\partial z} \right) + \frac{\partial K}{\partial \theta} \cdot \frac{\partial \theta}{\partial z} \quad (\text{A7})$$

where the chain rule has been used in each equation. It is necessary for this transformation that the soil be homogeneous over the domain of application of the equation, and that K and Ψ be single-valued functions of θ . D is then also a single-valued function of θ . Another transformation of value in many problems may be made when K and θ are single-valued functions of Ψ . It may then be written that

$$\frac{\partial \theta}{\partial \Psi} \cdot \frac{\partial \Psi}{\partial t} = \nabla \cdot (K \nabla \Psi) + \frac{\partial K}{\partial \Psi} \cdot \frac{\partial \Psi}{\partial z} \quad (\text{A8})$$

$$\frac{\partial \theta}{\partial \Psi} \cdot \frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial z} \left(K \frac{\partial \Psi}{\partial z} \right) + \frac{\partial K}{\partial \Psi} \cdot \frac{\partial \Psi}{\partial z} \quad (\text{A9})$$

While Equations A6 and A7 are generally more tractable than Equations A8 and A9, the latter may be used with ponded water on the surface, and are used in some of the models discussed in Appendix C. According to Philip (1969) and others, Richards (1931) first set down formal equivalents of Equations A3 and A8, while Equation A6 was first formulated by Klute (1952a and b).

7. The cautious reader will be concerned by such extensive development based on an empirical relation, Darcy's law, by the application of this law to unsaturated soils (it was derived from saturated flow data), and by the requirement for single-valued relationships among K , θ , and Ψ . Each concern is treated more fully in the following paragraphs.

8. The basic saturated form of Darcy's law has been deduced analytically by Hubbert (1940) for conditions of negligible inertial forces in the flow (low Reynold's number in soil pores), and Fancher, Lewis, and Barnes (1933) present data for sands which indicate that the law holds at Reynold's numbers less than unity. Poulovassilis (1977) has shown that even lower soil pore Reynold's numbers may be necessary to maintain Darcian flow at low water contents. Departures from Darcy's law have also been observed for nearly impermeable clay materials (Swartzendruber, 1968), and due to inertial effects during some transient flows (Philip, 1969). Such details are not generally important for soil moisture modeling for trafficability or military hydrology, however. For the present, and for the present purpose, Darcy's law may be considered valid.

9. It was assumed by Richards (1931) during his derivation of Equations A3 and A8 that within the context of soil-moisture models Darcy's law holds for unsaturated media. This was confirmed by Childs and Collis-George (1950), and by several others since then. This finding may be applied to swelling soils, also, when flow is related to the local soil particles and continuity of the total soil-water matrix is considered (Philip, 1969).

10. The requirement for single-valued functions among volumetric water content, θ , matric potential, Ψ , and water conductivity, K , is not normally met in general soil water flows. It is well met in several specific cases, however, such as infiltration of water into a profile initially at equilibrium with a water table or with uniform initial water content, and rainage of a saturated soil. It is also met during periods of wetting or drying of a local soil volume, even when other processes are occurring elsewhere in the soil column. In these cases, the requirements are met locally, and computations merely require consideration of whether the soil has been wetting, drying, or unchanging. The requirement is more restrictive for analytic solutions of the equations than for numerical solutions; as in the latter case, each computation for steps in space or time is relatively independent.

11. The principal developments since 1952 have been in methods of determining the essential $K-\theta$ and $\Psi-\theta$ relations for use of the above relations, formulation of analytical solutions for special cases, and in studies of heterogeneity of soil properties in nominally uniform areas. This category is covered more fully in the following paragraphs.

Methods for Estimation of Soil Hydraulic Properties

12. Childs and Collis-George (1950) opened the way to an entirely new set of opportunities in soil moisture physics by proposing a method for calculation of hydraulic conductivity, K , from the pore size distribution of a soil. Previously, hydraulic conductivity (or permeability) had been calculated from the particle size distribution via Kozeny's (1927) equation, or by equations derived by others (noted by Childs and Collis-George). The view was that of a distribution of passages for flow (pores) rather than a distribution of blockages to flow (particles). The subsequent development by Millington and Quirk (1961), Brutsaert (1967), Green and Corey (1971), Dapp and Hornberger (1978), and Ahuja, Green, and Chong (1980) is recommended reading, along with the original paper. While each of the method development papers includes data for comparison to calculations, particular emphasis on measurement and test of the method was reported by Jackson, Reginato, and Van Bavel (1965); Kunze, Uehara, and Graham (1968); Bruce (1972); and Elzeftawy and Mansell (1975). Additional contributions have been made by Marshall (1958), Millington and Quirk (1959), Jackson and Isler (1970), Rogowski (1971, 1972a and b), Jackson (1972), and Parkes and Peters (1980). Emphasis on soil moisture diffusivity, D , was discussed by White (1952b), Bruce and Klute (1956), and Pouloussis (1977), for both calculation of soil moisture movement and determination of soil moisture characteristics.

13. The method uses the curve of matric potential, Ψ (soil water tension, or suction), against volumetric moisture content, θ , as a measure of soil porosity at each water content. This approach is much easier than attempting to measure porosity directly, and possesses the advantage of including water-pore interactions in the primary data, Ψ versus θ . The Ψ versus θ data are often taken only at selected values of Ψ , but typical

ht of site-specific data, in particular the stages of crop growth (crop efficient), the methods of determining radiant energy fluxes, and the tilation factor. However, while it is clearly not a total answer to the blem, potential evapotranspiration has succeeded in bringing greater ceptual order to solutions.

10. It is interesting to quote from a final report on a project titled "Estimating Soil Tractionability from Climatic Data" (Thornthwaite Mather, 1954, p 401) in the context of this report:

Similar computations for other places and other years support the conclusion that soil moisture can be determined with all needed precision from climatological data. It is apparent from the agreement found between measured and computed values that the climatologic approach will permit the accurate determination of the movement of water through soils and the amount of storage in any selected layer in the soil. . . .

tory has shown this to be too strong a statement.

11. Another general approach to the problem of evapotranspiration was eloped by Baier and Robertson (1965), who used single and multiple ressions of evaporation measured by evaporimeter against several atmos-ric inputs to derive an expression for latent evaporation. This approach examined in the early work on the WES model, with similar correlation fficients (Carlson and Horton, 1959 (Appendix E); Dortignac and Lull, 1). While correlations of moderate accuracy may be derived, they must be pled to a soil moisture model for prediction. This approach was not lowed for the WES model, as it requires detailed modeling of soil sture movement. The depletion relations developed were deemed accurate ugh, but were not readily adapted to incorporate evaporation as a arate depletion mechanism. Further, the correlation coefficients were high enough to inspire confidence in the approach.

12. The Baier and Robertson model has been related to potential potranspiration by Baier (1971). This model has been used for both soil sture modeling (Baier and Robertson, 1966; Chieng, Broughton, and Foroud, 8) and crop models (Baier, 1973; Feyerherm, 1977). It was reviewed and ted for sensitivity by Hildreth (1977).

13. Another approach of personal interest to the author, which has n shown to be valuable objectively as well, is that of Lettau, called apotranspiration climatology." The approach (Lettau, 1969; Lettau and

ρ = the air density

ϵ = the ratio of molecular weights of water vapor and air

k = the Von Karman constant

p = air pressure

u_a = the windspeed at the level z_a

z_a = the specified height above the surface

z_o = the surface roughness parameter of aerodynamic theory

ing periods of strong surface heating or cooling, the atmosphere is not rally stratified, and corrections must be made for stability in Equa-B13.

7. To calculate PET it is necessary to measure or estimate H , p , z_o , T_a , and d_a . The other factors in the equations are er constants or dependent on T_a , the air temperature at z_a . This ntially reduces to determining the long- and short-wave radiant energy es (other contributions to H generally being small), wind speed, erature, and humidity, as ρ and z_o may be estimated with sufficient racy if necessary.

8. The Thornthwaite and Penman approaches have been compared for opical area by Brutsaert (1965). He found that the temperature-based l was insensitive there, due to a small annual variation of monthly eratures. The Penman equation produced evapotranspiration values ing from 0.71 to 1.16 times the measured values (weighing lysimeter). s et al., (1973) compared evaporation measured via lysimeter with that icted by the Penman and van Bavel equations under conditions of uently strong advection of sensible heat. They found that, during a e-week period when water was not limiting, the Penman equation predicted times actual evapotranspiration, while the Van Bavel equation predicted times actual. Daily plots of hourly values indicate that the latter estimate is due to excessive response to strong late-afternoon winds, as Van Bavel results follow the actual evapotranspiration much better in the ing hours.

9. The concept of potential evapotranspiration is particularly able because it enables partial separation of the atmospheric contri-on from the plant and soil contributions to the limits on evapotrans- tion. It has generally been found beneficial to revise the equations in

If it is assumed that the exchange processes of heat and water vapor are similar, LE and A may be written as

$$LE = -LB_v(e_o - e_a) \quad (B5)$$

and

$$A = -\gamma LB_v(T_o - T_a) \quad (B6)$$

where

B_v = the water vapor exchange coefficient

γ = the psychrometric constant, required to maintain proper units in Equation B6.

Defining Δ as the slope of the e' versus T curve, it follows that

$$(T_o - T_a) = (e'_o - e'_a)/\Delta \quad (B7)$$

approximately. Equation B7 is introduced into Equation B6 with the result

$$A = -(\gamma/\Delta)LB_v(e'_o - e'_a) \quad (B8)$$

Since $(e'_o - e'_a) = (e'_o - e_a) + (e_a - e'_a)$, it follows that Equation B8 may be written

$$A = -(\gamma/\Delta)LB_v(e'_o - e_a) + (\gamma/\Delta)LB_v(e'_a - e_a) \quad (B9)$$

and since potential evapotranspiration occurs from a well watered surface, it is appropriate to consider that surface saturated, thus, $e_o = e'_o$, and

$$A = (\gamma/\Delta)L(PET) + (\gamma/\Delta)LB_v d_a \quad (B10)$$

where $d_a = (e'_a - e_a)$, the vapor pressure deficit at the specified height above the surface noted earlier. When A from Equation B10 is substituted into Equation B4 with $LE = L(PET)$

$$L(PET) + H + (\gamma/\Delta)L(PET) + (\gamma/\Delta)LB_v d_a = 0 \quad (B11)$$

and solving for $L(PET)$ produces

$$L(PET) = \frac{(\gamma/\Delta)H + LB_v d_a}{(\gamma/\Delta) + 1} \quad (B12)$$

Penman used an empirical relationship for B_v , while Van Bavel used a relationship based on neutral atmosphere boundary layer theory given by

$$B_v = \frac{\rho \epsilon k^2}{p} \frac{u_a}{[\ln(z_a/z_o)]^2} \quad (B13)$$

id

$$m = 6.75 \times 10^{-7} I^3 - 7.71 \times 10^{-5} I^2 + 0.01792 I + 0.49239$$

ET* was then adjusted for a specific month length (d) and average day length in the month (h) by

$$PET = PET^*(h/12)(d/30) \quad (B2)$$

to derive the potential evapotranspiration, PET. Thornthwaite certainly knew better than anyone that heat supply and ventilation are important to evapotranspiration, but T is more available than any other usable data, and this empirical expression worked reasonably well.

5. The approach of Penman (1948, 1956, and 1963) requires less readily available data for the computation of potential evapotranspiration, but it is more physically based, and has been extended by Van Bavel (1966) to remove the empiricism used by Penman. The Van Bavel development will be shown with note taken of the differences.

6. The development begins with an expression for evaporation based on work of Dalton (1802), where

$$E = f(u)(e'_o - e_a) \quad (B3)$$

where

$f(u)$ = a function of windspeed at some height above the surface
(Dalton used $u^{1/2}$)

e'_o = the saturation vapor pressure of water at the temperature of the surface, T_o

e_a = the vapor pressure of the air at that height

While the saturation vapor pressure at T_o may be found in tables, determination of T_o is more difficult. Penman and Van Bavel thus sought elimination of e'_o from Equation B3. To do so, it is first necessary to invoke the surface energy budget,

$$LE + H + A = 0 \quad (B4)$$

where

LE = the evaporation rate multiplied by the latent heat of vaporization

H = all other energy fluxes at the surface, such as solar radiation, reflected solar energy, longwave radiation to and from the surface, sensible heat exchange with the ground, and energy used in photosynthesis

A = sensible heat exchange with the atmosphere

radiation and heat exchange with air and ground)." The value of the concept was affirmed by Penman (1948, 1956, and 1963) in his formulation of a potential transpiration equation. Nearly all models of evapotranspiration used in current numerical models for soil water depletion derive from either Thornthwaite's or Penman's approach, often with modifications, but using an estimate of potential evapotranspiration as a basis for calculating an actual transpiration.

4. Potential evapotranspiration is a function of meteorological factors with small contribution from soil heat flux, thus it would appear that this relationship could be exploited to derive a means of calculating the former from the latter. Realizing that data for a straightforward approach to the problem would frequently be lacking, Thornthwaite (1948) formulated his equations for potential evapotranspiration in terms of station monthly average temperature and day length only. Penman, on the other hand, chose to use a combination method using a Dalton (1802) style equation of evaporation in terms of ventilation and vapor pressure gradients, and the balance of energy fluxes at the evaporating (or transpiring) surface. These models are presented in greater detail in the following paragraphs. While the input data requirements are greater for the Penman equation, they are not excessive. The Penman equation and numerous revised equations derived from it are generally more accurate (Pelton, King, and Tanner, 1960; Tanner and Pelton, 1960; Van Bavel, 1966 - good revised equation; Hanks et al., 1973; and many others). Budyko (1956) has developed a similar approach. Thornthwaite (1948) formulated his theory empirically in terms of station temperature. He defined potential evapotranspiration for a standard 12-hour daylight period and 30-day month, PET* in units of cm of water, as

$$PET^* = 1.6 (10 T/I)^m \quad (B1)$$

where

T = the monthly mean station air temperature, °C

I = a heat index for the station given by

$$I = \sum_{i=1}^{12} i; i = (T/5)^{1.514}$$

APPENDIX B: SOME RECENT EVAPOTRANSPIRATION DEVELOPMENTS

Introduction

1. A number of recent developments in the theory and modeling of evaporation and transpiration are discussed in this appendix. Work was selected for review in large part on the basis of its use in numerical modeling of soil moisture. Theoretical and experimental developments are emphasized here, as treatment of evaporatranspiration in numerical models is discussed in Appendix C.

2. Symon's (1867)* has said that "evaporation is the most desperate branch of the desperate science of meteorology." While some progress has been made in the ensuing century, our understanding of evapotranspiration is still insufficient. The difficulties encountered in the treatment of evapotranspiration accrue, in part, from the fact that it is not simply a meteorological problem. There are distinct contributions to actual evapotranspiration from soil and plant physics as well. Basically, evapotranspiration may be viewed as a limited process, in the sense that the rate of water loss is limited by available energy and/or the rate of water movement to the ground surface and/or the rate of water movement through plants and/or the rate of vapor removal by the atmosphere. The virtual intractability of the problem of evapotranspiration derives from the complexity of each of these limiting processes and their interaction.

Atmospheric Limits to Evapotranspiration

3. Perhaps the most valuable concept put forth in the field of evapotranspiration is that of potential evapotranspiration, set down by Thornthwaite in the Report of the Committee on Transpiration and Evaporation, 1943-44 (Thornthwaite et al., 1944). Potential evapotranspiration has been defined in several slightly different ways, but an acceptable definition is: "that evapotranspiration rate which occurs from a well watered surface, i.e., the rate limited only by meteorological factors (including

* References are collected at the end of the body of this report.

by Warrick, Mullen, and Nielsen (1977) with good results in both cases. The spatial variation of l_x is presently under study to find measures of the spatial applicability about a measured point. Autocorrelation functions and other statistical techniques appear to be important in this effort (personal communication, D. R. Nielsen and J. Wagenet, 1981).

32. Vertical heterogeneity has been studied analytically in the case of evaporation from a water table through nonhomogeneous soils by Hadas and Hillel (1972) and is discussed with a particular example of stochastic heterogeneity by Philip (1980). Study of infiltration into layered soils was the object of one of the earlier numerical models by Hanks and Bowers (1962), and many of the models discussed in Appendix C have been applied to the vertically heterogeneous case.

33. Other papers related to spatial variability discovered during the literature search are: Cassel and Bauer (1975) - a study of spatial variability below tillage depths in fields; Cameron (1978) - a study of variations of the soil water characteristic curves and calculated hydraulic conductivity from five sites at six depths in a 225 m^2 (2400 ft^2) plot; Bell et al. (1980) - a study of spatial variability of surface moisture from 58 large field sites of $400 \times 400 \text{ m}$, 16 hectares each ($1310 \times 1310 \text{ ft}$, 40 acres each) for use in evaluation of remote sensing techniques; and two studies of watersheds by Rogowski (1972b) and Peck, Luxmoore, and Stolzy (1977). These references, and the others cited earlier, should be consulted for additional information and work cited in their references.

34. A corollary problem, that of predicting soil moisture from soil survey or soil map data, is discussed in Appendix C. It is likely that heterogeneity, like hysteresis, is a problem which must be treated in any model which is widely applied, as it also is simply a fact of life in the soil-water domain.

29. In addition to horizontal and vertical spatial heterogeneity of soil properties, the prediction of soil moisture is also affected by spatial variations of precipitation, other weather events, ground-water movement, and the temporal variation of the same influences. The WES model requires measured local precipitation (as do other models) for useful accuracy, for instance. Fortunately, some progress is being made on the problems.

30. Philip (1980) describes two basic forms of heterogeneity, deterministic and stochastic (see also the discussion by Freeze (1975)). Deterministic heterogeneity refers to cases where soil properties vary in a known way; whereas, stochastic heterogeneity refers to random variability. Deterministic heterogeneity includes the case of scale-heterogeneity, where similar media are involved and the variation may be identified in variation of a local microscopic length scale. According to Warrick, Mullen, and Nielsen (1977), similar media have identical porosities and the same relative particle and pore size distributions. Stochastic heterogeneity also includes a simpler form in which the statistics of variation are independent of location or time. The simpler forms are yielding to research to some extent. The following discussion focuses on developments for similar media.

31. Similarity theory for porous media was introduced by Miller and Miller (1956). Experimental work by Klute and Wilkinson (1958) and Wilkinson and Klute (1959) supported the concept with data for soil water characteristic curves (Ψ versus θ), hydraulic conductivity values, and infiltration flow into similar media. Philip (1967, also 1980) proposed that, if two media differ geometrically only in their characteristic length scales, say l_1 and l_2 , then matric potential and hydraulic conductivities of the two media may be related by

$$l_1 \Psi_1 = l_2 \Psi_2 \quad \text{and} \quad K_1 / l_1^2 = K_2 / l_2^2 . \quad (\text{A11})$$

The values of l_x for several soils may be determined from either measured matric potentials or measured hydraulic conductivities. The method was tested on several soils ranging from a clay to a fine sand by Reichardt, Nielsen, and Biggar (1972a) and found to hold. This texture range is extreme, compared to the assumptions, and is encouraging for more general application of the approach. The method was also tested by Russo and Bresler (1980) and applied to three data sets containing several hundred observations

some approximation, many have achieved excellent correspondence with field or laboratory measurements. Recent exploration of the applicability of a branch of abstract probability theory, percolation theory, to soil moisture problems by Golden (1980) has opened new opportunities to deal with hysteresis; however, much more work is still to be done.

Field Heterogeneity of Soil Properties

27. Variation of hydraulic and physical soil properties throughout a field of nominally identical soil type (field heterogeneity) places often severe restrictions on the accuracy of any prediction technique applied to the entire area. Philip (1980), in an excellent overview of the subject, refers to the "enormity" of the problem, meaning not "enormousness," but rather "monstrous wickedness, deviation from normal type, that which is abnormal, a monstrous offense." Field heterogeneity generates staggering problems in the general case, but several approaches to solution of real problems have been partially successful because the basic physical character of a nominally homogeneous field (one soil type, for instance) is similar from point to point.

28. Spatial variation of soil properties has been noted throughout the development of the WES soil moisture prediction model (WES, 1951 and 1952; WES and US Forest Service, 1954; Carlon and Horton, 1957 and 1959; Collins, 1971; and Broadfoot and Burke, 1958), and a special study was conducted in the area near WES during 1958-1960 by US Forest Service and WES personnel to measure variability for guidance in model development and application (Carlson and McDaniel, 1967). For four loess-derived soil series, they found greater variability within series than between series, and concluded that minor topographical variations were important. Nielsen, Biggar, and Erh (1973) measured several soil hydraulic and physical properties for six depths at twenty locations in a 150-hectare (370-acre) field in California. The field had been graded some time earlier for improved irrigation efficiency, and had been disturbed in the upper 60 cm (2 ft) by farming operations. They found variations of steady infiltration rate from 0.5 to 45.7 cm day⁻¹ (0.2 to 18.0 in. day⁻¹) and variation of steady hydraulic conductivities from about 10⁻¹ to roughly 10² cm day⁻¹ (0.04 to 40 in. day⁻¹). Other strong variations of properties are noted in the reference cited.

and other porous media by Gardner (1959); to the relations of external conditions to drying of soils by Gardner and Hillel (1962); to bare soil evaporation, drainage, and storage by Black, Gardner, and Thurtell (1969); to redistribution of irrigation water by Gardner, Hillel, and Benyamini (1970); and to power series solutions of the flow equation by Scott et al. (1962). Equations for calculation of matric potential from volumetric water content and hydraulic conductivity from water content used by Campbell (1974) and Clapp and Hornberger (1978) (cited above) were based on the Gardner, Hillel, and Benyamini's (1970) developments.

24. One of the powerful analytical techniques in solving the diffusion equation is to reduce the number of variables by integration of an assumed function of one of the variables which reasonably approximates the real situation. This assumed shape may be iterated to provide more accurate expressions. This approach has been taken by Parlange (1972), Aylor and Parlange (1973), and Parlange and Braddock (1980) for problems of one-dimensional infiltration, infiltration into layered soils, and an accurate approximate solution of the diffusion equation.

25. The method of Green and Ampt (1911) has been described by Philip (1974) as a primitive integral method using a step function for the advancing moisture profile. As with the Boltzman transformation, integral methods reduce the partial differential equation to an ordinary differential equation, which is easier to solve. The Green and Ampt approach has been used with varying modifications by Bouwer (1969) for infiltration into nonuniform soil, Raats (1973) for the analysis of unstable wetting fronts, James and Larson (1976) for modeling infiltration and distribution of intermittent water applications, and Jarrett and Fritton (1978) for the analysis of entrapped air effects on infiltration.

26. Various other authors' analytical solutions or techniques have been used in development of numerical models, including the study of one-dimensional infiltration into homogeneous soil by Fok and Hansen (1966), infiltration and runoff for small plots by Swartzendruber (1974) and Swartzendruber and Hillel (1975), vertical infiltration by Brutsaert (1977), infiltration in layered soils by Takagi (1960) and by Fok (1970), infiltration into crusted soils by Hillel and Gardner (1969) and by Ahuja (1973), and even flow in deformable porous media by Narasimhan and Witherspoon (1977). While each analytical solution to a "real" problem has involved

problems and exact solutions which may be used to evaluate numerical techniques. In this regard, analytical solutions are preferable to field data, as the required answer is precise. Both analytical and numerical techniques must be checked against field data in the final analysis, however.

21. The analytical approach has been led by E. C. Childs, J. R. Philip, W. R. Gardner, and more recently, J. Parlange during the last three decades. Only work related to the numerical modeling effort has been referenced in this report.

22. The work of Childs on the problem of predicting hydraulic conductivity for soils has been noted above. Philip (1957a and b) opened a series of papers on the theory of infiltration, which is particularly suited to analytical treatment since problems may be formulated with uniform antecedent conditions and the soil is monotonically wetted. Many problems of interest may be treated as horizontal infiltration, thus converting Equation A6 to the simpler diffusion equation. Philip (1969) has written a thorough review of the theory of infiltration, while Philip (1975a and b) has presented an analysis of stability during infiltration. He has also reviewed progress in the solution of nonlinear diffusion equations (Philip, 1974), which is a valuable confirmation of the fact that analytical solutions may be developed for important problems. Philip also contributed to the early application of numerical techniques to soil moisture problems, as noted in Appendix C.

23. Gardner and colleagues have utilized a Boltzman transformation, $y = x/(D_0 t)^{1/2}$, to reduce the horizontal diffusion equation, $\partial\theta/\partial t = \partial(D \cdot \partial\theta/\partial x)\partial x$, from a partial differential equation to an ordinary differential equation,

$$-\frac{y}{2} \frac{d\theta}{dy} = \frac{d}{dy} \frac{D}{D_0} \frac{d\theta}{dy} \quad (A10)$$

with consequent simplification of the problem. They have also written the diffusivity in terms of the volumetric moisture content θ as $D = D_0 \exp \beta(\theta - \theta_0)$ on the basis of reasonable fit to measurements and earlier work by Wagner, which also enables analytic solution for some problems. In these transformations, x is the horizontal distance, D_0 is the diffusivity at θ_0 , t is time, and β is a constant. This approach was applied to one-dimensional infiltration by Gardner and Mayhugh (1958); to drying of soils

Ψ versus K and θ versus K relations has been found inadequate for glass bead media (Topp and Miller, 1966), for sandy loam soil (Topp, 1969), and for fine sand (Vachaud and Thony, 1971). Hysteresis calculation remains important, nonetheless, as it is imperative that it be considered in application of the equations of soil moisture movement, and it is not readily measured. The relative magnitude of $\Psi - \theta$ variations due to hysteresis and field variability has been studied by Royer and Vachaud (1975). They conclude hysteresis must be considered in field-wide applications, as $\Psi - \theta$ variations due to hysteresis are greater than standard deviations due to spatial variability reported by Nielsen, Biggar, and Erh (1973). The above authors who have measured Ψ versus K hysteresis conclude that it is small enough to ignore in most applications.

18. Numerous analytical and numerical models of soil moisture flow have been formulated with consideration of hysteresis. This consideration increases the computation time and input data requirements of the models, but hysteresis is simply a fact of life in the soil-water domain.

Analytical Solutions for Special Cases

19. Analytical solutions to the soil moisture flow equations are of great value, but they are rare because of the difficulty of solving nonlinear partial differential equations, such as Equations A3, A6, and A8. The value of analytical solutions derives from the exactness of the solution (within the limits of the solution), the general simplicity of calculations for specific times or locations, and the potential of learning about the fundamental structure of the equation solution. Numerical methods (discussed in Appendix C) are capable of treating more general problems, but each application is, in a sense, separate from the others.

20. The complexity of most applications of soil moisture flow equations for soil moisture prediction related to trafficability or hydrology far outstrip the current analytical solutions of the equations. The need for answers can largely be met by numerical methods by application of brute force solutions via digital computers, and this is likely to remain true for some time. The fundamental equations are simply too difficult to solve in general under the conditions of the water-soil system. The analytic solution contribution will be greatest in the provision of precisely stated

were required. Such applications are feasible using typical curves for soil types, while increased accuracy should follow from shifting the curves to correspond to site-specific measurements while retaining their characteristic shapes. This approach is similar to that developed using tentative average depletion relations for sand, silt, and clay in the current WES model (Carlson and Horton, 1959).

16. A phenomenon of importance in the determination of typical curves and use of any soil hydraulic property data is hysteresis. Hysteresis is a system property in which the state of the system and changes of that state depend on the system history. The relation between matric potential and volumetric water content exhibits this property, which is also well known from magnetism studies. During the process of drying, water is removed from pores in the soil, but the removal is resisted by capillary forces at the small neck adjacent to an air-filled pore. When the matric potential decreases below a critical value equal (but opposite sign) to the capillary force, the pore suddenly drains completely because capillary forces are lower in the larger diameter pore. During wetting, however, the pore gradually fills up to its maximum cross section at less negative matric potentials. When the water reaches decreasing cross sections, the pore suddenly fills completely due to increasing capillary forces for smaller cross sections. Thus, the pore may range from full to empty at a given matric potential, depending on whether wetting or drying is occurring. This point was noted earlier in the discussion of single-valued relations among Ψ , θ , and K . Any soil physics text may be referenced for further discussion and figures. Nielsen et al. (1972) and Hanks and Ashcroft (1980) were referenced for the above.

17. Hysteresis is known to be important in the Ψ versus θ relations, and it has been shown to exist in the Ψ versus K relations. While numerous experimental determinations of hysteresis in soil hydraulic properties have been made, they are far less common than determinations of the drying curves of Ψ versus θ due to experimental difficulties. Studies of hysteresis in undisturbed field samples, or in the field, are even less frequent. An attempt to formulate a model for calculation of hysteretic Ψ versus θ curves by Poulouvasilis (1962) and basically used by Poulouvasilis and Tzimas (1974 and 1975) for study of hysteresis in the

curves of Ψ versus θ have been determined for all soil types in the USDA system (none are known for USCS soil types, except by translation from the USDA type data as in Meyer and Knight (1961). Examples are given in Hanks and Ashcroft (1980) after work reported in Taylor and Ashcroft (1972). These typical curves may be anchored for a particular soil at the measured values of Ψ versus θ , and thereby provide continuous data for the soil. Moisture tension data at 0.005 atm, 0.06 atm, and 15 atm matric potential were reported for the soils used in development of the WES soil moisture prediction model, and additional data at 0.33 atm and 3 atm matric potential are also reported for some soils (Meyer, 1976). Soil studies conducted at Agricultural Experiment Stations located at Land Grant universities throughout the nation will provide an extensive data base for particular soils in addition to examples published in various journal papers. Empirical expressions for the Ψ versus θ relation have also been derived by Gardner, Hillel, and Benyamini (1970), Campbell (1974), and Clapp and Hornberger (1978). The latter paper reports comparison of soil hydraulic properties calculated via their expressions with data for 1,800 soils reported by Holtan (1967). Regression models for soil hydraulic characteristics in terms of soil physical properties have been developed by Gupta and Larson (1979); the use of physical methods to improve soil designation with respect to drainage properties has been discussed by Bouma (1973).

14. Several equations of generally similar structure, but with differing assumptions and parameter values, have been presented for the calculation of hydraulic conductivity versus moisture content from curves of matric potential versus moisture content in the above-cited references. Brutsaert (1967), Green and Corey (1971), Jackson (1972), Rogowski (1972a), and Gardner (1974) each discuss the various approaches and should be consulted in lieu of further discussion herein.

15. The significance of these development is perhaps obvious, but it will be stated for emphasis. The application of the equations for soil water relations developed in the above section requires knowledge of Ψ versus θ , K versus θ , and/or K versus Ψ at each point in the soil modeled. Application of these equations in analytical or numerical models to the extensive areas of water sheds or areas of military operations requiring mobility predictions would be impossible if site-specific data

Baradas, 1973; K. Lettau, 1974; Hall 1977; Lettau, Lettau, and Molion, 1979) uses a simplified measure of soil water content and includes both atmospheric and soil-plant effects through parameterization (see also Appendix C). It also utilizes both absorbed global radiation (calculated via shortwave radiation climatology) (Lettau and Lettau, 1969) and precipitation as joint forcing functions for evapotranspiration. The concept of potential evapotranspiration is therefore not necessary, nor are the many adjustments (such as crop coefficient), although this physically based model must incorporate these realities in the parameterization. This specific approach is not yet widely accepted.

14. An evapotranspiration model developed by Ritchie (1972) has received relatively wide acceptance and is frequently referenced. It is based on the Penman approach, but incorporates revisions for treating a growing row crop. Kanemasu, Stone, and Powers (1976) tested the Ritchie model with field data from crops of soybean and sorghum, finding that daily and seasonal estimates of evapotranspiration agreed with lysimetric measurements. Another relatively well used approach is that of Shaw (1963), a model for estimation of soil moisture under corn. He uses pan evaporation data for Iowa as a measure of evapotranspiration, which is then adjusted with a crop growth factor and water stress factor for plant response to high evaporation demand. These factors are taken from figures in the paper. Approximately 80 percent of the estimates of June and August soil moisture were within 0.5 in./ft over the 5-ft-deep profile. This model was partially an outgrowth of earlier work (Denmead and Shaw, 1959). In later work (Saxton, Johnson, and Shaw, 1974a and b) emphasis was shifted toward the combination method of Penman. The Penman approach with modifications for the specific problems of data availability and objectives of the application has been used by Jensen, Wright, and Pratt (1971); Hanson (1976); and Morton (1975). In the latter paper, specific attention has been paid to a higher estimate of potential evaporation that would occur in a large area where water was limiting (thus reducing evapotranspiration and increasing air temperature and heat advection).

15. The Penman equation has been demonstrated to be reasonably valid for periods of a day or hour, but the Thornthwaite equation must not be used for periods much shorter than a month. Martin, Worm, and Wilson (1979)

noted the failure of the Thornthwaite method to follow weather events over even periods of one week in their comparison of several evaporation pan types, but inexplicably conclude that it may be used for a daily moisture balance technique for irrigation scheduling.

16. Several micrometeorologically oriented methods of estimating evapotranspiration have been developed during the last three decades which do not use utilize the concept of potential evapotranspiration. They are formulated to calculate evapotranspiration from measurements directly, using the energy balance, the energy balance in conjunction with the Bowen ratio (sensible heat flux to atmosphere/latent heat flux [evaporation]), or aerodynamic properties of the atmospheric boundary layer. These methods are discussed by Tanner (1967), as well as several other authors. As they generally require much more detailed data than the above techniques and are basically unsuited to potential trafficability and hydrology models, they will not be discussed further herein.

Soil Limits to Evaporation

17. The atmospheric properties control evapotranspiration under many conditions when water is not limiting, as discussed above. When water is limiting, however, the evaporation rate from the surface and transpiration rate from vegetative surfaces is limited by soil and plant physical (and chemical) properties. Much of the progress in dealing with soil limitations to evapotranspiration has come from numerical modeling of the process. Several papers dealing with experiment or analysis without numerical techniques have been selected for brief discussion here, as they emphasize evapotranspiration limiting by soil water.

18. Three stages of the drying of soils have been identified with respect to water availability (Idso et al., 1974). The first stage is that of unlimited water for evaporation, the second stage is a falling rate stage where evaporation is limited by vapor diffusion from relatively free water beneath the air-dry surface, and the third stage is limited by adsorptive forces holding water to individual soil particles. These three stages have been discussed qualitatively for decades, including by US Forest Service and WES personnel involved in development of the WES soil moisture prediction model, but they had been demonstrated experimentally only in laboratory

studies before 1974. The approach used by Idso et al. included measurement of the surface reflectivity for shortwave radiation (albedo) as it changed with surface soil moisture content. The change from the first to the second stage of drying was particularly evident via this technique. Generally, the surface would become air dry during the afternoon of one day, but became remoistened via upward transport of water during the night. It would then become air dry again, but at an earlier hour of the day. Clearly, the division between stages one and two is dependent on the rate of water loss, as well as soil factors.

19. The above results were obtained during a series of intensive soil measurements, as reported by Jackson et al. (1973). They sampled at depths of 0.5, 1, 2, 3, 4, 5, 7, and 9 cm (six replicates) every half hour for the full 24 hours of sixteen days during the first 38 days after field irrigation. Data for surface flux was obtained via a weighing lysimeter. The data for four days are plotted in three-dimensional perspective figures, which clearly demonstrate the complexity of near-surface water fluxes. While many previous experiments had been conducted under laboratory conditions, very few had been conducted to show diurnal variation in the field. In a following paper, Jackson et al. (1974) compare their measured soil water fluxes with values calculated by the theory of Philip and De Vries (1957). They found the theory predicts water fluxes best at intermediate water contents, but some serious deviations between measurement and theory occur at high or very low water contents. One point raised was the need for very accurate values of the moisture diffusivity for application of the theory. This paper brings out the mutual limits to accuracy created by approximations required for analytical solutions and experimental difficulties in precisely measuring important soil physical properties.

20. A discussion of the three stages of drying and the movement of the bone dry front in soils is presented by Heller (1968) with an excellent overview of the physics of saturated and unsaturated moisture flow. It is recommended reading, particularly for its thoroughness and conceptual clarity. The influence of surface residue and evaporation potential (provided by ventilation and infrared lamps) was studied in laboratory soil columns by Bond and Willis (1970) with emphasis on the effect on the first stage of drying, while Hanks, Gardner, and Fairbourn (1967) studied separate

effects of wind and radiation on evaporation rate in laboratory columns via a similar exposure to heat lamps and fans. Their results show strong surface cooling under ventilation and surface heating under radiation. The total evaporation from the surface was similar, with slightly greater totals under ventilation for two of the three soil types tested. The first stage of drying is also stressed in an experimental technique developed by Arkin, Ritchie, and Adams (1974) for measurement of the effect of surface mulches on evaporation from the surface. Their technique has further utility, in that it may be used to measure evaporation from the bare surface of cropped fields to provide data for separating evaporation from transpiration when total evapotranspiration has been measured or estimated. This separation is important in crop yield modeling, and may be important in soil moisture modeling to separate deep depletion by roots from more shallow depletion via surface evaporation.

21. The control of evaporation rates by water movement in the soil has been studied theoretically and experimentally by W. R. Gardner and colleagues (Gardner, 1959; Gardner and Hillel, 1962; Black, Gardner, and Thurtell, 1969). The theoretical work was based on the diffusion form of the basic nonlinear partial differential equation of moisture flow using an exponential approximation for the soil water diffusivity (see discussion in Appendix A). They have aided formation of a sound theoretical basis for the above discussion of soil limited evaporation. Soil limited evaporation enables one to calculate evaporation from soil moisture data under a limited set of conditions, but the approach cannot handle the case of fully wet surfaces or shortcircuiting of the soil moisture path by plant roots. This approach was also used by H. R. Gardner (1973) to analyze experimental data on evaporation from laboratory columns of water additions in several amounts with differing evaporation intervals. The total water added was constant in three separate tests, as was the total time for evaporation. Departures of calculated total evaporation from measured under three conditions of water amount and evaporation interval were small, but errors of up to 18 percent were noted for individual daily values. The effect of crust formation by rain versus flooding applications of water was studied by Bresler and Kemper (1970) to clarify the importance of surface crusts on infiltration and subsequent evaporation. It was found that the crust formed by rain reduces total evaporation by approximately 25 percent. A combined

application of the developments in the study of soil limits on evaporation is reported by Staple (1974). He modified Penman's equation by including the relative vapor pressure of a partially dry surface soil, then used this modified equation to provide the upper boundary condition for solving the flow equation within the soil. He cites several investigators who had previously used atmospheric vapor pressure as the upper boundary condition, but his mating of atmosphere and soil water is far better. He found reasonable correspondence between measurements from small cylinders of soil imbedded in a fallow plot and calculations, but again problems were encountered with both the limiting assumptions and data requirements of the theory, and difficulties in field sampling of the soil.

Plant Limits to Transpiration

22. The third component of the evapotranspiration system is the plant. The plant exerts both passive and active influences on transpiration in systems of interest for trafficability or hydrology. The passive influence derives from the plant's presence in both the soil and the lower atmosphere through distributed root, branch, and leaf systems. The roots access water from deeper soil layers than is available for surface evaporation, while the branches and leaves enhance the exposure of evaporating surfaces. The plant also exerts active influence on the stream of transpiration water at both the roots and the leaves. Additional supplies of soil moisture for transpiration are made available by root growth, both deeper into the soil, and also horizontally into untapped soil volumes. Resistance to vapor diffusion from leaf stomata is increased as moisture stress increases due to a change of shape by stomate guard cells, thereby reducing transpiration. These passive and active plant roles increase survival potential for the plant under varying moisture conditions, but they also greatly increase the complexity of the processes of soil moisture depletion.

23. It is desirable in the applied disciplines of engineering and agriculture to simplify the problem of soil water removal by plants by setting upper and lower limits of soil moisture which is available for transpiration. The upper limit, field capacity, is set by gravity drainage of soil water, while the lower limit, permanent wilting point, is set by permanent wilting of the plant and the consequent cessation of transpiration.

The quantity of water available between these two soil moisture contents is defined as available, or extractable, water. With these approach, transpiration occurs until the available water is used up, then ceases. Unfortunately, the natural system is not that simple, and departures of the real system from the simplistic system cannot be ignored in most cases.

24. Three dominant process details that prevent use of the simplistic field capacity/permanent wilting point model of soil water depletion by plants are:

- a. Soil water moves under the influence of moisture potential gradients, thus the moisture content at the surface of a root is lower than in the bulk of the soil.
- b. The rate of water loss varies strongly in time in response to both atmospheric demand (potential evapotranspiration) and the state of growth of the plant.
- c. Gravity drainage of water from many soils continues several days beyond the 2-day period normally used in the definition of field capacity; thus water which would have drained from the soil (or evaporated) in the absence of vegetation has been partially utilized for transpiration.

25. A valuable organizing concept for the study of transpiration was put forth by Van den Honert in 1948. He treated the movement of water for transpiration through the plant/soil system as a catenary, or chainlike, process. In a catenary process the overall rate is limited to the rate of the slowest partial process; that is, it is a series system. This organizing concept permits the study of individual partial processes under the assumption that other partial processes are not limiting. In this way it is similar to the study of potential evapotranspiration under the assumption that water is not limiting at the soil surface.

26. Gardner (1960) analyzed the flow of soil water to individual roots under different soil moisture conditions and transpiration rates, with emphasis on dynamic effects. His analytic solution to the flow equation for water movement to a root in cylindrical coordinates enabled calculation of soil moisture gradients in the vicinity of the root for different average moisture contents and for different transpiration rates. The bulk soil moisture content at the wilting point for the plant based on the soil matric potential at the root surface varied significantly as the transpiration rate was varied. Thus, wilting would occur at higher bulk moisture content under strong transpiration demand when steep moisture gradients formed in the vicinity of the root.

27. Gardner and Ehlig (1963) carried the analysis further with more accurate assumptions about the plant response to moisture stress, based on recent measurements. They found the transpiration rate to be nearly a linear function of soil moisture percentage when below a threshold value. Their analysis led also to the conclusion that soil water remains available for transpiration at matric potentials well beyond the 15-bar value normally used as permanent wilting point. The limits to transpiration rates due to soil water movement to the roots under steady evaporative demand when water movement occurred between soil layers and regions of high versus low rooting density were also considered.

28. Cowan (1965) extended the prior work with a more complete treatment of the transpiration stream. He noted first that the catenary process description put forth by van den Honert did not consider parallel paths of flow through the system. The principal parallel path for soil water transfer to the atmosphere consists of soil surface evaporation combined with the through-plant transpiration stream. Even within the latter, the multiplicity of roots, xylem vessels, branches, and leaves leads to a complex series/parallel network through which the flow occurs. Cowan therefore formulated an analytical model of transpiration which included series/parallel paths in the plant environment and the plant itself.

29. Cowan also noted that diurnal variations of potential transpiration strongly affect the plant-soil moisture response, and that consideration of steady evaporative demand was not sufficient to analyze the entire problem of transpiration limits. He therefore introduced diurnal variation to his model and analyzed the response of soil moisture content and matric potential over several diurnal periods. His results indicate reduced transpiration during diurnal periods due to soil moisture gradients near the roots, despite adequate bulk soil moisture content.

30. Denmead and Shaw (1962) measured the transpiration rates of corn plants in a field experiment to investigate the effects of varying potential transpiration under a range of soil moisture conditions. They found that actual transpiration fell below potential transpiration despite "adequate" bulk soil moisture content under conditions of high potential transpiration. At potential rates of 6 to 7 mm/day (0.24 to 0.28 in./day), the actual transpiration began to fall relative to potential transpiration at soil

APPENDIX C: NUMERICAL MODELING OF SOIL MOISTURE

Introduction

1. This appendix presents an ordered review of the majority of soil moisture modeling efforts over the past three decades. The discussion is organized along a combination of historical and modeling group lines, where the set of publications for each coherent group of investigators is briefly reviewed in the order of their initial publication. As neither library resources nor available time were unbounded in this project, the following review could not be exhaustive, but it is more complete than many others discovered during the course of the effort.

2. While this appendix can provide an ordered guide to the literature, an outline of model characteristics, and identification of laboratory and field data used for model verification, it cannot prove the superiority of one model over another, detail all models published, nor detail the published comparisons of simulation results with data. Moreover, since the characteristics of a best model are contradictory, e.g., convenience versus computational speed and cost, any attempt at absolute judgement would be absurd.

3. Each of the soil moisture models discussed in this appendix has been published in a research journal after peer review. As a result of this prior filtering process, each of the approaches is "good" in some significant sense, and poor correspondence with data is seldom noted. In fact, the model limitations seldom result from inaccuracy of the numerical method used, as precise analytical solutions can be reproduced to any accuracy for which time and money are available. The model limitations derive, rather, from the completeness and accuracy with which all relevant processes have been included (or are known). Similarly, validation of the models is complicated by the lack of certain knowledge of the physical processes relevant to particular measurements. In many respects the models more accurately reflect their input assumptions and data than do the results of laboratory and field experiments. Departures of simulated results from real data are therefore often indicators of omission, and are often clues by which improved understanding through additional study is provoked.

4. An excellent discussion of numerical modeling appropriate to the papers reviewed in this appendix is contained in the book by Remson, Hornberger, and Molz (1971). Hillel (1977) has compiled a summary of modeling effort which indicates the range of possibilities for model applications, although with nearly exclusive emphasis on the approach he uses. Excellent comparisons of analytic and numerical models as applied to one-dimensional infiltration were conducted and reported by Haverkamp et al. (1977) and by Vauclin et al. (1977). In these papers, several different basic approaches to the problem were used and their simulations were compared to an analytic solution and laboratory data. Freeze (1969) also reviewed numerical models for soil moisture flow which had been published. This paper contains a large table of model characteristics, enabling ready comparison of the models. Each of these works is recommended reading.

Physical Process Modeling of Soil Moisture

5. Several terms are used in the following model discussions which have been defined in the main text of this report and discussed in Appendix A. A general discussion of their significance is presented here, as an introduction to the model reviews.

6. Richards (1931) combined the equation of continuity of water substance with Darcy's law to derive the Richards equation used by most of the model developers as a starting point. Darcy's law states that moisture flux is proportional to the gradient of hydraulic potential, while the equation of continuity simply relates the change of water content of a volume due to net flow across the volume boundaries. The Richards equation is written in one dimension (vertical) as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(K \frac{\partial \Psi}{\partial z} \right) + \frac{\partial K}{\partial z} \quad (C1)$$

where

θ = volumetric moisture content

t = independent variable of time

z = independent variable of depth

K = hydraulic conductivity, the coefficient of proportionality in Darcy's law

Ψ = matric potential, with hydraulic potential $H = \Psi + z$

7. This equation poses several inconveniences for direct solution, and the specific approaches used by various modelers to deal with the equation lead to several different models. First, the equation is written with two dependent variables, θ and Ψ . This complication is normally removed by writing $K(\partial\Psi/\partial z)$ as $D(d\theta/dz)$, where $D = K(\partial\Psi/\partial\theta)$ is defined as the soil moisture diffusivity, or by writing $\partial\theta/\partial t$ as $C(\partial\Psi/\partial t)$ where $C = \partial\theta/\partial\Psi$ is defined as the soil specific moisture capacity. The former is referred to in the following as the Richards equation in diffusivity formulation, while the latter is referred to as the equation in pressure head formulation. Either formulation may be used for most applications, but if ponded water on the surface is considered, the latter form is required.

8. The second complication is that the coefficients of the partial derivatives in either formulation are strong functions of the dependent variables, making Equation C1 a nonlinear partial differential equation. This complication is enriched by the presence of hysteresis and by nonlinear variation of the partial derivative coefficients over a range of 5 to 7 orders of magnitude in cases of interest. The nonlinearity may be treated by using coefficients at each time step which are derived from estimates of the dependent variables at the next time step, or from coefficients appropriate to the dependent variable value at the present time, or from some average value, or from a preliminary calculation of the new dependent variable value which is used to calculate coefficients for use in calculating the corrected dependent variable value, or by iteration in which new coefficients are calculated from preliminary calculations of the dependent variable values until two iterations result in values which are sufficiently close. The method selected by each modeler is generally noted in the following model discussions.

9. Where the model is formulated in such a way that hysteresis cannot be ignored, model algorithms generally test the direction of soil moisture change to select from a set of drying, wetting and scanning curves the one which is appropriate for a given moisture content and direction of change. This curve is then used in a nonlinearity treatment to derive the proper coefficient value (or value of matric potential from moisture content and vice versa).

10. The third complication arises because, as a partial differential equation, Equation C1 is valid only for infinitesimal changes in time and space. Digital computers are incapable of infinitesimal increments, even if time or monetary resources are available; thus selections of increments of the time and space variables for realistically accurate computations are required. In most of the models, the space variable increments are fixed, and the time increment is adjusted by the program to meet some specification, often the magnitude of change of a dependent variable during a single time step. In some models, the space variable increments are also controlled. These controls enable simulations of good accuracy without excessive expenditure of resources. This accuracy-related control of increment size is not directly related to step size controls for stability of the numerical scheme which are discussed below.

11. The solution of Equation C1 by numerical methods also involves several selections among alternate formulations. With respect to time, the value of the dependent variable may be written in terms of values known from the previous time step, or in terms of unknown values at the time step for which the calculation is being made. In the former case, the unknown value is an explicit function of known values, while in the latter case, the unknown value is an implicit function of itself and other unknown values. These formulations are appropriately identified as explicit and implicit, respectively. Explicit methods are the most straightforward approach and result in directly solvable expressions for each time-space grid point, but require small time increments to avoid instability. On the other hand, implicit methods result in a system of equations which must be solved simultaneously for all grid points in space for each time, although they are absolutely stable in linearized systems. In addition to absolute stability, the implicit methods reward the greater programming effort required by actually requiring fewer operations to resolve the entire spatial grid each time step. They are thus doubly efficient in comparison to explicit methods by permitting larger time steps (bounded by accuracy requirements only) and simulating each step more rapidly.

12. Several of the models use a matrix formulation of the system of simultaneous equations which results in a tridiagonal coefficient matrix

the implicit formulation in one dimension. This matrix equation may be efficiently solved using a method reported by Richtmyer (1957). Iterations have also been used when the coefficients were not linearized.

13. For spatial derivatives, the finite difference approximations are usually written in terms of the values of the dependent variable at adjacent grid points on either side of a point, or above and below it.

are known as central differences. Central differences cannot be used at the boundaries of a region unless imaginary grid points are introduced outside the boundary; thus one-sided spatial differences are generally used at the boundaries. Boundary conditions, such as constant matrix potential, specified gradient of matrix potential, a no-flow condition, are specified for model applications to suit the physical process under consideration. Original papers must be consulted for such detail. Further general discussion is provided in the above-cited references.

14. L. F. Richardson has been credited with being the first investigator to seriously apply finite difference approximations to partial differential equations for the solution of flow problems. He applied the method to a problem of seepage through an earth dam prior to 1910. He is noted for his use of numerical methods for weather forecasting during the following decade. However, before the advent of electronic digital computers, such pioneering effort was severely limited.

15. A. Klute (1952a and b) is generally credited with developing the first numerical model for solution of the soil moisture flow equation by digital computer. In keeping with a suggestion sketched out by Childs and Collins-George (1950), Klute formulated the moisture flow equation for matrix moisture content with a diffusivity coefficient as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left(D \frac{\partial \theta}{\partial x} \right) \quad (C2)$$

where x is horizontal distance and the other terms are defined for Equation C1. A Boltzmann transformation, $y = xt^{-1/2}$, was used to rewrite the linear partial differential equation as a nonlinear ordinary equation in Boltzmann time-distance variable, y . An iteration scheme was then used to solve for the soil moisture content as a function of y . The numerical simulation of flow into an horizontal soil column successfully produced a sharp spatial gradient at the leading edge of the wetting column

ing front), in contrast to earlier theoretical models of the phenomenon. Though the front appeared to move too rapidly, and weakened with time, the defects were attributed to insufficient input data leading to an analysis problem. Despite these defects, a significant advance had been

16. Bruce and Klute (1956) undertook the measurement of diffusivity using the above model. They were able to calculate moisture diffusivity as a function of soil moisture content by using measured values of moisture content against distance in an experimental soil column.

17. Ashcroft et al. (1962) published a model for the solution of Darcy's diffusion equation in volumetric moisture content, θ . They used an implicit technique. The equations were linearized by use of diffusivity values for each time step calculated by the method of Bruce and Klute (1956) with an estimated θ . The estimated value was derived by adding a small increment to the previous θ value. The implicit method resulted in a system of simultaneous equations which were solved by a Gaussian elimination scheme. The model was applied to horizontal diffusion of water into a homogeneous, semi-infinite medium with good results in comparison to experimental data, and to analytical solutions utilizing the Boltzmann transformation. While this model has not had a significant impact on succeeding work, the paper is notable because the authors included rationale and discussion of choices made for the model which are of benefit to understanding their work and the work of many others who have been quite delinquent in this regard.

18. Whisler and Klute (1965) retained the iterative characteristics of the earlier Klute model in a new formulation of the problem, but their model was revised in many important respects. The revised model was formulated to calculate the time rate of change of pressure head, $h(=\psi)$, rather than volumetric moisture content, θ , as

$$C(h,z) \frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \left[K(h,z) \frac{\partial h}{\partial z} \right] + \frac{\partial K(h,z)}{\partial z} \quad (C3)$$

where all symbols have been defined above. This equation may be solved for general vertical flow problems with appropriate initial and boundary conditions, and was used in several of the following applications with little modification.

19. The Whisler-Klute model utilizes nondimensional variables and coefficients, such as nondimensional pressure head, given by $\phi = h/L$, L is the length of a soil column under study, in formulation of the equations to be finite differenced and solved. Implicit finite differencing is used for both the partial derivative approximations and the coefficient approximations at each modeled time step. Since the values of both dimensionless pressure head and the equation coefficients are thus formulated in terms of the values at adjacent spatial points, a system of nonlinear, simultaneous equations is generated for the values at the given time step. A set of algebraic equations cannot be solved by direct elimination, or deterministic techniques, as neither the coefficients nor the pressure head values are known, but they are both present as product factors in the terms of the equation.

20. The Whisler-Klute model system of algebraic, nonlinear, simultaneous equations is solved for the dimensionless pressure head and the coefficients by iteration. In this procedure, values for the coefficients at a given time are first approximated by calculation of their values using pressure head values from the prior time step. These approximate coefficients are then used as constant coefficients in the set of simultaneous equations, which are solved for pressure head at each spatial grid point by an implicit technique. The improved head estimates were then used to obtain new approximations of the coefficients, and the process continued until the new head estimates were within a specified amount of the head estimates to obtain them. The final set of coefficient and head values were used after this convergence and used to calculate the coefficient and head values for the succeeding time step by repeating the iteration process. The entire time and space dependence of pressure head was thus resolved.

21. The theories of Childs and Collis-George (1950) and Millington and Kirk (1961) were used for computer calculation of the necessary variation of hydraulic conductivity versus head from curves of volumetric water content versus head. The latter curves were arbitrarily drawn by the authors for typical soil types as relationships which appeared reasonable to them. The combined typical soil-dimensionless analysis approach was

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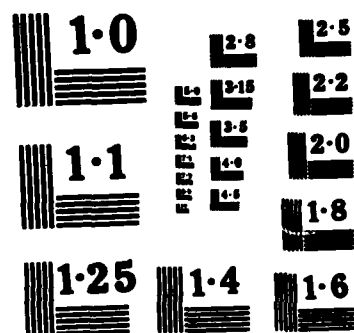
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used to enhance the generality of the results of model simulations, as specific cases of soil type and flow geometry with matching dimensionless variables produce identical distributions of pressure head in time and space.

22. Whisler and Klute (1965) apply the model to the analysis of infiltration into a column of soil which has been drained to equilibrium with a water table from a saturated condition. Whisler, Klute, and Millington (1968) use a similar model to analyze steady-state evaporation from a soil column, including water uptake and redistribution by roots. In this application, the iteration is performed for the single time representing the steady-state condition, but with the additional complication of a space varying source term due to the roots. A macroscopic approach to root water uptake is used, where only bulk characteristics are treated, contrary to the approaches of Gardner (1960) and Cowan (1965).

23. Whisler and Klute (1969) used their 1965 model to study additional cases of infiltration, including infiltration from rainfall in addition to ponded water on the surface, and layered soils. Infiltration and outflow rates were also calculated with the 1969 model, and cumulative infiltration was included. However, the finite differencing and the iterative model solution algorithm were essentially unchanged.

24. Whisler and Watson (1969) modified the 1965 Whisler-Klute model to include provision for a variable time step in a simulation of infiltration into a draining porous medium. The variable time step enabled optimum computational efficiency while ensuring stable, accurate calculations. Their time step was not set arbitrarily, however, but rather was adjusted on the basis of the magnitude of change in pressure head for the previous time step. If the head change was too large, the step was repeated with a smaller time increment.

25. Watson and Whisler (1972) used the Whisler-Watson 1969 model to treat drainage of heterogeneous media from saturation with a water table below, but no surface input flux. The media were assumed to exhibit scale heterogeneity (see Appendix A). While field heterogeneity required modifications in the coefficient expressions of the mathematical model, the method of solution of the equations during simulation was little changed from the earlier paper. Whisler, Watson, and Perrens (1972) used the

Whisler-Waston 1969 model to simulate infiltration of ponded water into two soil columns with different heterogeneity distributions and one homogeneous column for comparison. Hysteresis was avoided in this study by specifying an initial moisture content that was constant with depth.

26. The 1965 Whisler-Klute version of the model was used by Klute and Heermann (1974) to simulate soil-water profile response to periodic surface boundary conditions, with and without a water table. A high degree of harmonic distortion was simulated as the periodic disturbance propagated into the soil, due to the nonlinearity of the soil-water flow system.

27. Bruce, Thomas, and Whisler (1976) used the 1969 Whisler-Watson model version to predict infiltration into layered field soils to study the effects of various distributions of hydraulic characteristics with depth. The soils used in this study were characteristic of field soils, contrary to the more general soil types studied in earlier model applications.

28. This sequence of papers indicates the range of problems which may be treated with a relatively consistent modeling approach. Similar sequences of work by several authors will be outlined in following paragraphs for several different approaches.

29. Philip (1955, 1957a) published numerical solution models for the horizontal diffusion equation and for vertical flow (summarized in Philip, 1957b). His solutions also utilized the Boltzmann transformation, $\phi = \eta t^{-1/2}$, to reduce the partial differential equations to ordinary differential equations, as did Klute (1952a), but Philip then made volumetric moisture content, θ , the independent variable and formed an integro-differential equation in ϕ and θ to be solved. This equation was solved by forward integration with an initial condition derived by iteration. The vertical flow case required an approximation of distance and an initial estimate by a solution to the equation without gravity. A series of partial solutions was formed which converged to the final solution. As this approach had little influence on succeeding numerical modeling effort, it will not be discussed further.

30. Another early soil moisture modeling effort which did not exert great influence on following work, but is of historical interest, was published by Day and Luthin (1956). Their numerical solution was applied to a drainage problem for which they had obtained laboratory experiment data. For each time step (treated as the dependent variable) a vertical distribution

of pressure head was estimated. The estimates were used to specify K and θ for each level, and to calculate the water loss required by the θ changes from the preceding time step. An explicit centered difference approximation of the flow equation with explicit linearization of K using previous estimates was used at each level to correct the head value. The cycle was iterated until the distribution of head values did not change significantly, and the time lapse consistent with the top head value changed (not iterated) was calculated from the total desorption in the column and the calculated outflow rate. Steeping through time was accomplished by a new (lower) head estimate at the surface, repeating the above procedure.

31. While the introduction of the high-speed digital computer rendered this procedure obsolete, several of the more sophisticated models still used elements of the method, enhancing its historical significance. The model predicted higher drainage rates than measured, and therefore a more rapid decrease of pressure head, although it reproduced some vertical gradients caused by uneven packing of the soil column moderately well. Departures were attributed by the authors to both uncertainties in the $K - \theta$ relationship, determined experimentally, and the large time steps dictated by available computational means.

32. Hanks and Bowers (1962) were the first to combine the Crank-Nicholson finite difference scheme, a rapid algorithm for tridiagonal matrix solution, and the computing power of a digital computer in a numerical model for solution of the soil moisture flow equation. Klute (1952b) had suggested that the method of Crank and Nicholson (1947) might be adaptable to the soil moisture equation for vertical flow, but after working out the necessary procedure, Philip (1957a) found considerable labor would be required even for limited accuracy. However, by programming the solution for processing by an IBM 650 computer with an algorithm for solution of the tridiagonal matrix generated in the Crank-Nicholson method that was published by Richtmyer (1957), both accuracy and labor problems were brought under control. While this model has been extended in many ways since its inception, and while other numerical methods have now been used with good results, even this early numerical model was able to reproduce moisture profiles predicted by state-of-the-art analytical methods with good accuracy; then it was used to produce profiles for layered soils which were well beyond any other computational capability.

33. The Hanks-Bowers numerical model solves the soil moisture flow equation in one dimension in the form

$$\frac{\partial h}{\partial t} = \frac{1}{C} \frac{\partial}{\partial z} \left(K \frac{\partial H}{\partial z} \right) \quad (C4)$$

where

h = pressure or tension head

t = time

C = specific moisture capacity ($C = \partial\theta/\partial h$)

z = vertical distance

K = conductivity

H = hydraulic head ($H = h + z$)

θ = volumetric moisture content

34. Specifics of the finite differencing of Equation C4 are published in Hanks and Bowers (1962), which should be consulted for details. Basically, the method steps forward from the previous to the present time using pressure head values from both previous and present time levels. It is therefore an implicit method (explicit methods use only past values to step to the present). A set of simultaneous equations results from an implicit method at each time step, as unknown values of pressure head at adjacent spatial points are required to calculate the pressure head at each point. While explicit methods are conceptually more direct, the implicit method has better stability characteristics. Further, the system of simultaneous equations can be written in matrix form with a tridiagonal coefficient matrix; that is, only the main diagonal and the one just above and below it have nonzero entries. Using efficient algorithms, the system of simultaneous equations can actually be solved with fewer operations per time step than are required for the apparently simpler explicit method (Remson, Hornberger, and Molz, 1971).

35. The need for iteration to resolve both coefficients and head values was eliminated in the Hanks-Bowers model by writing C and K in explicit form at half-time steps. A pseudo-implicit form is actually used for C , where an extrapolation of the present value of θ is made from prior values, and C is calculated using this estimated value. Superior results were obtained when K was written as a somewhat complicated function of soil moisture diffusivity, rather than simpler forms also tried.

Both K and C were considered constant over a time interval, therefore linearizing the flow equation and enabling the system of simultaneous equations to be solved without iteration.

36. The time interval was made variable in order to achieve both good temporal resolution during the rapidly changing infiltration periods and computational efficiency during later periods of slow change. The interval was defined as the time for a specified quantity of water to enter the soil, and was adjusted after each time step. Cumulative infiltration was computed from the change of θ , while infiltration rate was calculated from the pressure head gradient at the surface and the conductivity in the first soil layer.

37. The depth increments were defined for the layered soil cases such that the change of soil type occurred at a half-space level. Pressure head was required to be continuous across the boundary, as was flow; thus volumetric moisture content changed abruptly at the boundary of the differing soils. The conductivity at the half-step was defined as an average of conductivities above and below the boundary.

38. The model requires initial and boundary conditions and known relations between moisture content and both conductivity and diffusivity. A series of computations are made at each time step, and the model is stepped through time for a specified period to effect the solution desired.

39. The model was used by Hanks and Bowers (1962) to calculate infiltration into homogeneous and layered media. The homogeneous media results for two soils were compared to solutions for the same soils derived analytically after Scott et al. (1962) and Philip (1955). In these comparisons the same restrictive assumptions required by the analytical methods were applied to soil characteristics input to the model. Very good correspondence between analytical and numerical results was found, except for somewhat erratic simulated infiltration rates at midtime of the simulations. The model was then applied to layered soil cases (fine over coarse soil and coarse over fine soil). The results were reasonable, but data for comparison were not available.

40. Hanks and Bowers (1963) applied the model to evaluate the influence of variations in the diffusivity-water content relations on infiltration, while Hanks and Gardner (1965) used it to evaluate the

influence of the variations on surface evaporation. In the latter case, the model was modified to handle surface drying due to evaporation. Simulated model results were found to be consistent with other research results, and indicated that accuracy in the relation between diffusivity and water content is most important at high water contents.

41. Jensen and Hanks (1967) applied the model to the problem of nonsteady-state drainage from porous media, with comparison to laboratory data. The method of determining K was modified for this application, but the model was otherwise unchanged. Very good correspondence with three laboratory experiments on different soils was achieved, except at very short times. Some discrepancy between data and simulation could also be attributed to imperfections in the soil packing for the experiment, etc.; however, as these details were unknown, they could not be included in the model input.

42. The complication of hysteresis in the pressure head-water content relationship was avoided in the above model applications by starting from a moisture content constant with depth and only wetting (infiltration) or drying (evaporation and drainage) the soil. This restriction on the model was removed in 1969 (Hanks, Klute, and Bresler, 1969), and the model was applied to a complex case of infiltration, redistribution, drainage, and evaporation from the soil. While the $K-\theta$ relationship was assumed to be without hysteresis (generally valid), both $C-\theta$ and $h-\theta$ relations were treated as hysteretic. The method used was similar to that of Rubin (1967). This required a model modification to evaluate whether wetting or drying was occurring during a time step and whether or not this change was the same as the prior time step. The appropriate scanning curve of the hysteretic relationship was then selected to derive θ and C from computed values of h . With this modification and specification of appropriate initial and boundary conditions, the model was applied to the complex case. Model simulation results were compared to experimental data from laboratory soil columns wetted at three different rates, then allowed to drain with and without evaporation from the surface (Bresler, Kemper, and Hanks, 1969). Evaporation was underestimated somewhat, while drainage during evaporation was somewhat overestimated, but the results of the comparison were generally very good.

43. The 1969 version of the model was used by Bresler and Hanks (1969), with an added algorithm for treatment of salt, to estimate simultaneous movement of water and salt in unsaturated soils. The same problem was treated by Warrick, Biggar, and Nielsen (1971), using the 1962 Hanks-Bowers model with a different salt algorithm. In each case, field data or laboratory data was used for comparison with simulations by the model. (In a subsequent paper, Reichardt, Neilsen, and Biggar (1972b) used a new explicit model, with C and K definitions similar to those of Hanks and Bowers, to study horizontal infiltration into layered soils.) The 1969 version of the model was also used by Bresler (1973), although with an extensively revised algorithm for treatment of salt movement without numerical dispersion. Again, these modeling efforts were compared to laboratory and field data with good correlation.

44. While modeling of soil moisture under bare fields is of value in many applications and aids the understanding of soil water movement under those conditions, it is not sufficient for agriculture or for more general concerns of trafficability and hydrology. As noted in the discussion of evapotranspiration (Appendix B), soil water depletion via evaporation and transpiration is a complex interrelated set of processes, which include plant influences in both the soil and the atmospheric boundary layer. Effective soil moisture modeling must include the influences of plants, either implicitly in the determination of parameters, or explicitly.

45. Nimah and Hanks (1973a and b) applied the Hanks-Bowers 1962 model to the problem of soil moisture prediction under an alfalfa crop. The model was expanded for this application by including a root extraction function for soil water, partitioning potential evapotranspiration into potential evaporation and potential transpiration to treat the differing depletion mechanisms and surface boundary conditions, and including an iteration procedure to solve for the changing surface boundary conditions. Exchange of moisture with a water table was included in the model, but hysteresis effects were ignored. Potential evapotranspiration was determined with a modified Penman approach. The numerical procedure was also modified to include a variable depth increment to improve efficient resolution of gradients, while the variable time increment was based on the total change of water content in the column for this model application. Brief comparison with field data is made in the first of the two papers, while more

extensive comparison is made in Nimah and Hanks (1973b) against data from a field experiment. Model simulations compared well in both time and depth, except for immediately after irrigation or rainfall.

46. Feddes, Bresler, and Neuman (1974) used a modified form of the 1973 Nimah-Hanks model to simulate water balances in comparison to field data on root uptake of soil water by red cabbage. Their modifications included the surface layer under dry surface conditions, rather than a fixed air-dry water content, use of a potential evapotranspiration model which included crop structure characteristics, and use of a revised root extraction function. They found that the changed root extraction function and the use of an equilibrium surface moisture content made little difference on simulated values of evaporation, transpiration, and soil moisture, but that the changes of the potential evapotranspiration estimation were of significant value.

47. Childs and Hanks (1975) added diurnal variation of potential evapotranspiration and seasonally varying partitioning of potential evaporation and potential transpiration on the basis of crop development to the Nimah-Hanks model. The approach of the latter modification was extended by Childs, Gilley, and Splinter (1977) in a simple model which included the above-ground plant functions, as well as root extraction. The Childs-Hanks model was used by Watts and Hanks (1978) with the Bresler (1973) formulation of solute transfer to study soil-water-nitrogen interactions. Comparison with field data was made in each case with good correlation, but neither perfect nor completely consistent correlation, as in the other cases. Tillotson et al. (1980) report the current state of the model of soil moisture, plant root interaction, and solute flow. This publication provides a more complete exposition of the model than do the journal papers.

48. Liakopoulos (1966) used a revised formulation of Equation C4 to model unsteady, unsaturated soil moisture flow. He expanded hydraulic head $H = h + z$, wrote out the derivative on the right side of Equation C4, used the standard definition of soil moisture diffusivity, $D = K/C = K(\partial h/\partial \theta)$, defined $B = \partial K/\partial \theta$, and derived the equation

$$\frac{\partial h}{\partial t} = D \frac{\partial^2 h}{\partial z^2} + B \frac{\partial h}{\partial z} \left(\frac{\partial h}{\partial z} + 1 \right) \quad (C5)$$

where

h = pressure head

t = time

z = vertical coordinate

θ = volumetric moisture content

The time derivative was written as a finite forward difference, while the second derivative was written as an implicit centered difference, and the first derivatives of the second term on the right were written as explicit centered differences. The equation as nonlinearity was removed by writing the coefficients D and B in explicit form, evaluated at the beginning of the time step.

49. The implicit finite difference formulation results in a system of linear equations to be solved for the values of pressure head at all grid locations each time step. This system was solved by Gaussian elimination using the same matrix formulation as used by Hanks and Bowers (1962) and others, but Liakopoulos fully described the solution scheme, which has otherwise been covered only in textbooks. The derivation of the tridiagonal coefficient matrix and the recursive method for determination of coefficients of a reduced matrix are clearly presented, and are recommended reading, as this scheme is quite frequently used in soil moisture modeling.

50. Liakopoulos claims satisfactory comparison of results from the model with actual experimental data, but presents only theoretical results for gravity drainage, evaporation, infiltration, and capillary rise. Initial conditions were formulated in each case to avoid hysteresis.

51. Staple (1966) solved the moisture flow equation using explicit finite differences in both the original form derived by Richards (1931), namely,

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(K \frac{\partial \Psi}{\partial z} \right) + \frac{\partial K}{\partial z} \quad (C6)$$

where Ψ = pressure head or matric potential (h above) and the other symbols are defined as above; and in the form derived by Klute (1952a), namely,

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(D \frac{\partial \theta}{\partial z} \right) + \frac{\partial K}{\partial z} \quad (C7)$$

where $D = K(\partial\psi/\partial\theta) = K/C$, as above. Staple used Equation C6 for the upper profile of the soil during drying, and he used Equation C7 during infiltration of the soil column and wetting of the lower soil column. Staple (1966) cites an early work, Staple and Lehane (1954), in which Equation C6 was solved by explicit finite difference. Historically, this would place his work just after that of Klute. The 1954 paper is seldom cited by others, however, and authors have generally chosen to eliminate either θ or $\psi(h)$ from the Richards equation before proceeding to finite differencing and numerical solution.

52. Staple (1966) used average values of conductivity, K , over adjacent grid points and the method of Staple and Lehane (1954) and Hanks and Bowers (1962) to derive mean values of diffusivity, D , for use in the solution. Laboratory data was used for the $K-\theta$, $K-\psi$, and $D-\theta$ relations required, obtained from earlier work with emphasis on hysteresis (Staple, 1962 and 1965). Tabulated input data were used in the computer program.

53. Staple (1966) applied his model to simulation of moisture profiles during infiltration and subsequent redistribution in a soil column. The results were found to be of the right magnitude in comparison to related measurements. Staple (1969) applied the same model to measured laboratory data (with an improved algorithm for treatment of the transition from wetting to drying) and found the agreement satisfactory. In this latter application, Staple used implicit formulation of the flow equation in some of the tests, since it was faster.

54. Rubin (1969) (publication of 1966 Symposium paper in Rijtema and Wassink, 1969) presented another approach to numerical solution of the moisture flow equation. He reduced the Richards equation in two variables, θ and ψ , to an equation in a single dependent variable, v , via a Kirchhoff transformation:

$$v = v(H) = \frac{1}{V} \int_{H_{\max}}^H K(h) dh \quad (C8)$$

where

$$V = \int_{H_{\max}}^{H_i} K(h) dh \quad (C9)$$

and where both H and h represent the pressure head here, with $H_{\max}$ and H_1 the upper and lower bounds of pressure heads in the medium under consideration. Assuming the inverse of $v(H)$ exists and $\partial H / \partial v = V / K(H)$, the Richards equation may be written in the single dependent variable, v , as

$$Y(v) \frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial z^2} - Z(v) \frac{\partial v}{\partial z} \quad (C10)$$

where

$$Y(v) = C / K(H(v))$$

$$Z(v) = \partial \ln K(H) / \partial H$$

$$C = \partial \theta / \partial H$$

(See Rubin's paper or a general text discussion of the Kirchhoff transformation for more detail.)

55. Rubin's justification of use of the Kirchhoff transformation instead of the pressure head formulation is not confirmed by the comparison evaluations performed by Haverkamp et al. (1977), but his justification of its use instead of the moisture content formulation has been noted elsewhere in this report. Actually, it is a viable alternate approach which may be preferred under certain circumstances; it needs no further justification.

56. The numerical solution is effected by means of the Crank-Nicholson implicit scheme, which is also used by several other investigators. Actually, the Crank-Nicholson scheme involves an averaging of both implicit and explicit terms, but is generally referred to as implicit without qualification. Nonlinearity due to the coefficients Y and Z is removed by an explicit determination of their values at the midpoint of the time step from known values of v , Y , and Z at the beginning of the time step. First, v is calculated for the half-step, then $Y(v)$ and $Z(v)$ are computed. K is also required in the finite difference expression for $\partial v / \partial z$, and is calculated with an explicit method for the entire time step. The specific method of solving the resulting system of linear simultaneous equations is not noted in the paper, but is likely to have been the efficient tridiagonal matrix algorithm in Richtmyer (1957), used by others.

57. The program uses tabulated values of $Y(v)$ and $Z(v)$ derived from functions fitted to data for the soils under consideration. These functions are also used to derive h output values from the computed v values.

58. This numerical model was applied to calculation of the soil moisture profiles during preponding infiltration of rainfall and infiltration from rain ponds on the surface. The results were found both reasonable and enlightening, although only profiles under flooding were available for comparison.

59. Rubin (1967) used a different model for analysis of postinfiltration redistribution of soil moisture. In this model Equation C7 (in θ) was solved using implicit finite difference formulation, based on earlier work by Rubin and Steinhardt (1963). The equation was linearized by extrapolation of θ from prior time steps and calculation of D and K from the extrapolated values. The particular method of solving the system of linear simultaneous equations is not noted.

60. This paper is notable for the method of hysteresis treatment. As the K - θ and D - θ relationships are functions of whether drying or wetting is occurring and the moisture content at the time, the condition and direction of change of soil moisture must be considered in calculation of K and D from θ . To this end, a second grid is defined with a one-to-one correspondence with the soil moisture grid in which this information is held. The decision as to use of a wetting, drying, or scanning curve is made on the basis of this information each time D or K is determined, and when moisture contents are converted to pressure head values. The required information is simplified by the assumption that once drying begins at a grid point under redistribution it will continue. Thus the coded information is: use wetting relationship or use drying or scanning curve with parameter stored in second grid point. Details may be found in Rubin (1967).

61. Rubin (1963) formulated a numerical model for use in two-dimensional cases in unsaturated and partly saturated soils. The Kirchhoff transformation used in Rubin (1967) was used to model a case of two-dimensional horizontal infiltration, while

$$C \frac{\partial H}{\partial t} = \frac{\partial}{\partial x} \left(K \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial z} \left(K \frac{\partial H}{\partial z} \right) \quad (C11)$$

was used for a falling-water-table, ditch-drainage case, where $H = h + z$, $C = \partial\theta/\partial h$, the specific water capacity, K is the hydraulic conductivity, and t , x , and z are independent variables of time, horizontal distance, and vertical distance, respectively.

with set (2) data supplemented by field examination of a single boring randomly placed in the nodal area, including rooting depth; and (4) simulations were performed with additional data available through field and laboratory measurements of the soil from the single boring of set (3).

103. They found simulation of the water table depths to be poor for data sets (1)-(3), but to be greatly improved using data set (4). This improvement proved to be due, primarily, to better characterization of the subsoil properties, however, as results with data sets (1)-(3) were greatly improved by the single addition of subsoil data from set (4). These results may in part be due to the two-layer model used, also, as was noted for the results of simulation of evapotranspiration. In this latter case, the model was quite sensitive to rooting depth. To further check on the representativeness of the results, and in particular to check on the data set (2) and data set (4) differences in simulated evapotranspiration, several additional borings were made in the five nodal areas. It was found that the single boring used for data set (4) was generally less representative of the field conditions of the nodal area than the estimates made from the soil survey information for data set (2). It was thus recommended not to perform any soil borings in the field unless several were possible.

104. It is notable that the principal difficulty encountered in this simulation effort was caused by subsoil characteristics which were poorly represented by data available from the soil surveys, and even more poorly represented by a single soil boring. Present computer hardware and software capabilities for simulation of soil moisture regimes have so far outstripped the available data, that even this greatly simplified model is limited by available inputs.

Mass Balance Modeling of Soil Moisture

105. Mass balance models of soil moisture have an appeal which derives from their simplicity of concept and generally uncomplicated application. In such a model, one simply accounts for the accretion and depletion from a soil column or soil layer by addition or subtraction of an appropriate quantity of water, and the moisture volume remaining after each step is the proper moisture content. This simplicity is very misleading, however, as

with field data due to Jackson (1972) was obtained, however, indicating to the authors that the model equations are reasonable representations of the physical processes modeled. One obvious problem may be noted, however. They used the simplified expressions of Clapp and Hornberger (1978) for the relations of pressure head and hydraulic conductivity to volumetric moisture content, and their Figure 3-7 shows significant departures of these expressions from the field data at high water contents. Since Hanks and Bowers (1963), Hanks and Gardner (1965) and Van Keulen and Hillel (1974) had clearly demonstrated that accuracy in these quantities is particularly important at high water contents, some of the model error is likely to be from this source.

101. Finally, a numerical model for soil moisture simulation by De Laat (1976) was used by Bouma et al. (1980a) to treat the effects of lowering water tables on grass production using soil survey identification of soils, and by Bouma et al. (1980b) to simulate regional soil moisture regimes using soil survey data. The model of De Laat (1976) is characterized more by its design intent of great computational efficiency than by its sophistication, being a pseudo-steady state, two-layer model using Penman method estimation of evapotranspiration, but this application is of great interest for predicting trafficability in inaccessible areas.

102. The former paper by Bouma et al. (1980a) includes mapping via computer of simulated soil sensitivity to water table drawdown. They emphasize the advantages of computer storage of soil survey and simulation information which may be mapped by the computer in response to specific concerns, in comparison to the traditional mapping of interpretations. The latter paper by Bouma et al. (1980b) discusses a sequence of simulations for soils identified by a soil survey map. An area of 6 km by 6 km (3.7 miles by 3.7 miles) was gridded for computer modeling, and data were presented for five selected nodal areas of 25 hectares (62 acres) each. Simulations were performed for comparison with field data using four data sets of increasing specificity, namely: (1) Simulations were performed for each of the five nodes using soil physical characteristics reported in the literature for the soil type most common in the 6 km by 6 km area; (2) simulations were performed for the most common soil type in each 25 ha nodal area using data from sources as in set 1; (3) simulations were performed for each area

simulated transpiration from a corn plant quite well, once the initially assumed plant resistance was adjusted. The difficulty of plant resistance determination was noted as a model limitation. This model is not unique in this regard.

98. Two papers treating ground water flow in relation to flow in the unsaturated zone were also noted, despite an extremely thin connection with agricultural science, and are briefly included here. There is a large body of literature related to watershed modeling which has not been included because of the focus of this work on agricultural developments, and these two papers provide a "window" into the region.

99. Green et al. (1970) modeled the movement of water under a shallow pond using an implicit-iterative technique. They considered movement of both air and water in a two-phase flow. They obtained good correlation between experimental data and model simulation, but only after some modification of the originally assumed values of the porous media properties. As in so many other cases, the numerical solution to the partial differential equations used to model the complex water movement process is far less uncertain than knowledge of the actual physical conditions in a field experiment. Freeze (1971) extended his one-dimensional modeling effort (Freeze, 1969) to three dimensions. The resulting equations are solved by a line successive overrelaxation technique, and the model is quite general in treatment of a small watershed.

100. A program considering the simultaneous flow of heat and moisture in soils, reminiscent of the work of Van Bavel and Hillel (1976), was developed by Camillo and Schmugge (1981) for use in conjunction with remote-sensing techniques for soil moisture. They formulated the equations of heat, liquid water, and water vapor flow in terms of respective diffusivities, including the movement of water vapor through the atmospheric boundary layer. The general solution was accomplished via an Adams-Bashford finite difference approach, while the nonlinear dependence on surface temperature, created by terms of the surface energy budget formulation, was treated via iteration. This publication is notable for the completeness of their description of the numerical solution. Simulated results compare well to analytic and quasi-analytic solutions, where the precise boundary conditions and soil properties are identical for the two approaches, but problems again develop in application to field conditions. Qualitative correspondence

The Millington and Quirk (1961) method was used to calculate conductivity and diffusivity from field-measured relations between moisture content and matric potential. Model results compared favorably with data.

95. The successive overrelaxation (SOR) technique was used by Amerman (1976) and Reisenauer (1963) to treat two-dimensional and multidimensional soil water movement, respectively, while the finite element approach was used by Neuman and Witherspoon (1970); Guymon, Scott, and Herrmann (1970); Neuman, Feddes, and Bresler (1975), theory; Feddes, Neuman, and Bresler (1975), field application; and Parkes and O'Callaghan (1980). A numerical method using the flow velocity equation was used with success by Wind and van Doorne (1975), and Richter (1980). Each of these methods has value in special applications, such as use of the finite element technique for problems with complex boundaries. However, they have not been in the mainstream of development and will not be discussed further here. Remsom, Hornberger, and Molz (1971) provide a thorough discussion of the SOR and finite element methods, which should be consulted for details.

96. Two additional papers relating to the modeling of water uptake by roots should be noted. Feddes et al. (1976) use an implicit finite difference model of the flow equation in diffusivity formulation with volumetric moisture content as the dependent variable and an added root uptake source term. The method of linearization of the equations during calculation is not specified, but appears to be linear averaging of the coefficients. The root effectiveness function used was a function of soil moisture content and critical moisture contents for root activity. Root growth was considered. While cumulative evaporation and transpiration were well simulated in comparison to field data, the vertical distribution of soil water content was not. This simpler model did compare favorably in simulation of field measurements with the more complex model of Feddes, Bresler, and Neuman (1974), although the two models frequently produced opposite departures from the data.

97. The second paper was published by Slack, Haan, and Wells (1977). They used a microscopic approach to evaluate the root extraction function of depth, then a macroscopic model to solve the flow equation with the added sink term. An implicit method was used, but details were given by reference to a Ph.D. dissertation which was unavailable for this review. The model

values by Van Bavel and Ahmed (1976) revealed higher-than-normal evapotranspiration. However, as they had modeled a sequence of fair-weather June days, the excess may have been due to that choice.

92. A different continuous simulation language, DYNAMO II (Pugh, 1970), was utilized by Hansen (1975) to formulate a numerical model of the water state and transportation in the soil-plant-atmosphere system. While an appendix contains a complete model listing, no discussion of the relative merits of DYNAMO II, CSMP, and modeler-written computer algorithms is included. It is noted that the basic integration follows Euler's methods, thus explicit methods are used. The model addresses the problem of the state and flow of water in the soil and growing crop, and is thus time dependent. It was partially based on experimental data, but discussion of comparison to data was deferred to a subsequent paper (not reviewed).

93. Several other modeling approaches to the problem of soil moisture have been used, but do not fall naturally into the above groups. One such approach was taken by Wang and Lakshminarayana (1968) to simulate water movement in nonhomogenous soils. The Richards equation (Equation C1) was used, but the vertical derivatives were first written after the fashion of Liakopoulos (Equation C5) with further consideration of the spatial variation of hydraulic conductivity, due to the application to heterogeneous soils. Both explicit and implicit finite difference formulations were used, while the nonlinearity of the equation was addressed by use of iteration. While hysteresis was neglected in this model formulation, calculated and field-measured data compare favorably for vertical drainage and infiltration.

94. Giesel, Renger, and Strebel (1973) treated unsaturated vertical flow with hysteresis using the Crank-Nicholson method and Jacobi iteration to deal with the nonlinearity of the equation for soil moisture movement. The alternating direction implicit (ADI) method was used by Bresler (1975) and by Busscher (1979) to model nonsteady infiltration from surface and subsurface sources. Comparison of the former model with laboratory data is made by Bresler and Russo (1975). Two-dimensional flow was considered in each model, as implied by use of the ADI method. De Jong and Cameron (1979) used an explicit difference formulation of the diffusion form of the soil moisture equation with explicit linearization of the diffusivity and conductivity to study the water movement through soils with a field crop. Root extraction and interception of precipitation by vegetation were included.

and soil water dynamics in layered soils (Hillel and Talpaz, 1977). These four papers are a tribute to the potential of models for meaningful numerical experiments. While much of the simulation output is simply reasonable, quantitative comparisons of the effects of single parameter variation are possible. Such single parameter variation is essentially impossible in physical experiments, but it is demonstrably necessary, as recognized by most modelers, to tie the calculations down as frequently as possible to laboratory and field data. The "typical soils" approach of this series of papers, however, may prove to be the only rational way to treat global soil moisture problems.

90. Hillel, Van Beek, and Talpaz (1975) discuss the relative characteristics and merits of microscopic (single root) and macroscopic (bulk) models of water extraction from the soil by roots and then develop a microscopic model of the phenomenon. Their CSMP model includes solute movement. The model is formulated in cylindrical coordinates, as were the analytical models of Gardner (1960) and Cowan (1965). Root water extraction under two transpiration demand rates was included. Soil water potentials were calculated for a basic case, and for different initial moisture contents, different transpirational demand, and soil resistance values. The effects of rooting depth and density on soil moisture were simulated with a macroscopic model by Hillel, Talpaz, and Van Keulen (1976), while the latter model was modified by Hillel and Talpaz (1976) to include effects of root growth and death. Growth was treated as root extension and root proliferation within a volume. The model was compared to laboratory data, along with the model of Molz and Remson (1970), by Belmans, Feyen, and Hillel (1979) and by Feyen, Belmans, and Hillel (1980). The results of the comparison were generally satisfactory for both models. Details are given in the cited papers.

91. Lambert and Penning de Vries (1973) and Van Bavel and Ahmed (1976) published models using CSMP to treat the entire soil-plant-atmosphere system. The former model used the microscopic approach, while the latter used the macroscopic approach, to soil water extraction by roots. Each model considers details of the canopy, including heat balances for the leaves to better treat transpiration. Comparison of maximum evapotranspiration rates for their simulated sorghum crop with generally accepted

hydraulic conductivity via constant flux procedures. Calculations were made with wetting, drying, and scanning curves to consider hysteresis in both hydraulic conductivity and soil water tension relations to moisture content. Correspondence of model results with measurements were improved when hysteresis effects were considered, as opposed to single-valued relations from only wetting or drying of the soil.

87. Van Keulen and Hillel (1974) used a CSMP-based model to evaluate the effects of vapor diffusion at very low soil moisture contents, applying the technique of Hanks and Gardner (1965). Hillel (1975) simulated the soil moisture and evaporation rate response to cyclic variation of potential evaporation (called "evaporativity" by Hillel), calculating larger soil moisture contents relative to steady potential evaporation after several days. Recovery of near-surface soil moisture during nighttime suspension of evaporation was modeled. (Most of the results of these and following papers authored or coauthored by Hillel are included in Hillel (1977).)

88. Van Bavel and Hillel (1976) developed a comprehensive treatment of evaporation, considering both water and heat transfer in the soil, as well as energy forcing and aerodynamic transfer in the near-surface atmosphere. They found that bare soil does not support the concept of potential evaporation very well, as evaporation continued to fall due to a number of feedback effects (such as albedo, emissivity, and temperature), the change in the first four days amounting to 12 percent of the first day's evaporation. They also found that calculations with the Van Bavel formula for potential evaporation (Van Bavel, 1966) were sufficiently similar to model calculations as to challenge the wisdom of the more complex approach. (If one is satisfied with potential evapotranspiration, this is true, but compare Hanks et al. (1973) and Appendix B.) Van Bavel and Hillel applied their model to seven locations throughout the continental United States using June data, and discussed variations due to the differing diurnal atmospheric conditions. This paper is recommended reading as an excellent discussion of bare soil evaporation.

89. Hillel, Van Bavel, and Talpaz (1975) simulated evaporation from a soil covered by a mulch of hydrophobic aggregates. This model was later used without the surface mulch, but with three typical soils labeled sand, loam, and clay to evaluate profile water storage (Hillel and Van Bavel, 1976) hysteresis effects with cyclic potential evaporation (Hillel, 1976),

84. Van der Ploeg (1974) simulated one-dimensional infiltration into soils using CSMP. He extolled the virtues of the language for soil scientists unfamiliar with advanced mathematics or computer programming. He provides a clear discussion of the formulation of the equations for computer application, which is essentially a calculation of the vertical moisture flux divergence with layer-averaged conductivities, and potential gradients between layers using Darcy's law. The flux divergence results in a change in moisture content in each layer. He finds good correspondence between model output and the calculations of Philip (1957a) and Parlange (1971) for Yolo light clay. Van der Ploeg and Benecke (1974) simulated one-, two-, and three-dimensional infiltration in soils. They used an alternating-direction procedure (explicit) for the two-dimensional case, while the three-dimensional case was reduced to one dimension (radial). They found good correspondence with results by Philip and Parlange, and noted that their model represented field data as well as the model of Bresler et al. (1971).

85. Beese, Van der Ploeg, and Richter (1977) tested their model against field data, a 218-day experiment on fallow loess soil. Field data included tensiometer measurements at 11 depths to 2 m (6 ft) in the field and in a soil monolith, which had been removed from the field and transported to a lysimeter station a few kilometers (miles) away. Evaporation was estimated from a correlation of potential evaporation/actual evaporation (lysimeter) and matric potential at 5-cm (2-in.) depth. Precipitation was measured daily. The soil capillary conductivity was measured as a function of depth. Field data was plotted on graphs from a CSMP-based model output for depths of 20, 40, 60, 100, and 140 cm (8, 16, 24, 40, and 56 in.). Model accuracy increases with depth, while the average departures of calculated from measured values for all depths were less than 15 percent (matric potential units). The departure may have been due to the values of evapotranspiration and capillary conductivity determined as inputs, as well as model errors, according to the authors.

86. Dane and Wierenga (1975) modeled the effect of hysteresis on infiltration, redistribution, and drainage in a layered soil with CSMP. They compared model output with laboratory data for moisture contents in Glendale clay loam over river sand. Soil water tension was measured via tensiometers, soil moisture content via neutron scatter techniques, and

This equation was also solved via the Douglas-Jones method. The equations were linked via use of the water table location as a lower boundary condition on the unsaturated zone, and definition of S as soil water in the unsaturated zone which was unavailable for evaporative loss. Iteration was not necessary for small time steps when the saturated zone equation was solved for the water table height using values of S and w from the prior time step, but it could be used when necessary or to achieve larger time increments.

80. Comments on the Pikul, Street, and Remson paper were published by Vachaud and Vauclin (1975) with a reply by Pikul, Street, and Remson (1975). In comparison to laboratory experiment data, the former authors found larger horizontal water movement in the unsaturated zone than allowed by the assumptions of the model and strongly challenged the concept of the storage coefficient, S . The reply was essentially "application to a watershed cannot be made with the same approach used in the laboratory because of sheer size." S was claimed to be useful, while lateral flow in the unsaturated zone would likely be far less than in the laboratory experiment.

81. A number of models have been formulated using the IBM CSMP, which was developed for use by individuals generally unfamiliar with computer programming (IBM Corporation, 1972). These models are reviewed as a group in the following paragraphs, as the papers share a common modeling technique, albeit developed by IBM rather than originated by an individual soil scientist with programming skills.

82. All CSMP models discussed in the following paragraphs use explicit integration procedures, using either the Milne method or a fourth-order Runge-Kutta scheme. The user of these methods is protected from instability in the explicit integrations by software checks, including reference to a user-specified accuracy requirement. Complete programs have been published for most of the models, as they are quite short because of the amount of detail left up to the software.

83. Van Keulen and Van Beek (1971) simulated water movement in layered soils with CSMP to evaluate the influence of a plow layer and tillage hardpan. They included appendices which discuss the influence of layer thickness selected for the vertical model resolution and the magnitude of the time step used in integration. Bhuiyan et al. (1971) used CSMP to model vertical infiltration into unsaturated soils. They found fairly good agreement between their model and the power series solution due to Philip (1957a) for Yolo light clay.

This problem is solved with both a Taylor series expansion and the method of Douglas and Jones, described above. They found that the Taylor series required too high an accuracy in higher order derivatives of the soil moisture characteristics curves, which are difficult to determine, and that it was not acceptable for this reason. Solution with the Douglas-Jones predictor-corrector method of finite differencing was satisfactory in comparison with experimental data from gravity drainage of a column of soil.

77. Pikul, Street, and Remson (1974) published a numerical model for two-dimensional applications based on coupled one-dimensional models for the unsaturated and saturated zones. While known limitations to this approach were cited, the potential benefits of a relatively efficient application to watershed-size problems were cited as justification. Further, at the watershed scale, many of the assumptions are reliable.

78. The equation of soil moisture flow in the unsaturated zone was written in terms of pressure head using the specific moisture capacity, C , as

$$\frac{\partial}{\partial z} = \left[K \left(\frac{\partial \Psi}{\partial z} + 1 \right) \right] = C \frac{\partial \Psi}{\partial t} \quad (C13)$$

where

- z = vertical coordinate
- K = hydraulic conductivity
- Ψ = pressure head (matric potential)
- C = specific moisture capacity ($=\partial\theta/\partial\Psi$)
- t = time

This equation was solved using the Douglas-Jones method discussed in the earlier papers.

79. The saturated flow zone was modeled with the equation

$$K_o = \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) = S(x,t) \frac{\partial h}{\partial t} - w(x,t) \quad (C14)$$

where

- K_o = saturated hydraulic conductivity
- x = horizontal independent variable
- h = height of the water table above datum
- S = a storage coefficient used to link the saturated and unsaturated zones
- t = temporal independent variable
- w = a sink or source term

73. The extraction term was first written as a function of only depth and transpiration rate. The resulting equation was solved with an Adams predictor-corrector method started by a Runge-Kutta procedure. Reasonable agreement was found with measured moisture flux. A second, more realistic model with the extraction term as a function of moisture content, as well as transpiration rate and depth was then formulated. Expressions for both the rooting density function and the resulting root extraction function with depth are given in the cited reference. The Douglas-Jones predictor-corrector method was used to solve the resulting mathematical model. Good results were found for the first several days of simulation in comparison to data from a field experiment, but the simulation and field data departed strongly after that period. There was evidence in the data that maximum extraction was shifting to depths of lower root uptake capacity as the soil dried, a feature not included in the model (see Belmans, Feyen, and Hillel (1979) for additional comparison to data).

74. The Douglas-Jones predictor-corrector method is outlined in an Appendix to the paper (see also Remson, Hornberger, and Molz, 1971). In the Douglas-Jones method a predictor equation is used to evaluate the unknowns at one-half step forward in time using an implicit formulation and solution via the tridiagonal matrix method. These values are then used to derive the coefficients at the half-time step for use in a corrector equation. The latter is again implicit, but it is applied over the entire time step. Solution is again by the tridiagonal matrix method. The nonlinearity of the equations during the corrector step is therefore removed by use of known values from the predictor equation solution, while for the predictor step the coefficients are taken as appropriate to the beginning of the step (explicitly). First derivatives are written explicitly for each step, also. The method is convenient, efficient, and accurate.

75. Molz and Remson (1971) apply the 1970 model to a number of cases of soil moisture extraction by roots, but unfortunately without adequate comparison to experimental data.

76. Hornberger and Remson (1970) model the one-dimensional saturated-unsaturated transient flow problem with a formulation of the basic equation in pressure head. They depart from the approaches of Rubin (1968) and Freeze (1969) discussed above, however, in that they model a discontinuity in pressure head at the water table. Their rationale is given in the paper.

70. Remson, Fungaroli, and Hornberger (1967) present still another approach to numerical modeling of soil moisture. They formulated a three-dimensional model based on flow divergence, $V V = \partial\theta/\partial t$, in which the six components of flow toward and away from a point in three dimensions were first written out in terms of head, conductivity, and flow area. Because discrete differences between adjacent spatial grid points were used in writing the volumetric flow rate components, the finite difference approximation was complete with the addition of an explicit, backward-difference term for the time rate of change of volumetric moisture content. Tabulated values of K versus θ and h versus θ were used with an iteration scheme to compute the solution, i.e., the spatial variation of θ at each time step.

71. The model was applied to a problem of a draining soil with evaporation, but in only one dimension. Comparison with computed results for a similar case by Remson et al. (1965) was good without hysteresis, but computed results with hysteresis had a problem near the region of transition between wetting and drying, which was attributed to use of an insufficient number of hysteretic scanning curves between the wetting and drying curves.

72. Molz and Remson (1970) formulated a mathematical model for soil water extraction by roots. Contrary to the approach of an earlier paper--Molz et al. (1968)--in which flow to a single root was modeled (microscopic model), they used the macroscopic approach. In this case, the bulk characteristics of the roots and their extraction of water with depth are treated. A negative source term was added to the equation of continuity, which was then combined with Darcy's law to derive an equivalent of the Richards equation, but with the added source. The diffusivity modification due to Klute was then used to derive the final equation:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z} \left(D \frac{\partial\theta}{\partial z} \right) - \frac{\partial K}{\partial z} - S \quad (C12)$$

where

θ = volumetric moisture content

t = time

z = the vertical coordinate (positive down)

D = soil moisture diffusivity

K = soil moisture conductivity

S = the root extraction source term

After an informative discussion of the problem of incorporating treatment of both saturated and unsaturated domains in a single model, and of the physics of ground water recharge and discharge relative to water table depths, Freeze formulates his model in terms of pressure head (matric potential) using a Crank-Nicholson implicit method with linearization of coefficients via extrapolation after Rubin and Steinhardt (1963). The system of simultaneous equations is solved by the tridiagonal matrix approach frequently used (Richtmyer, 1957).

67. Freeze (1969) applied the model to several hypothetical situations using experimental data for characteristic curves of three soils. A subsequent paper was noted in which comparison of results with field data would be made, but it was not available for review. Nonetheless, this paper is highly recommended reading for its development of the problem.

68. Schneider and Luthin (1978) utilized the Rubin (1968) formulation of the saturated-unsaturated flow problem to study perched water tables. Their problem required addition of a source term, which was included in the finite difference equations implicitly. The solution was obtained by iteration as used by Rubin. Model results were compared with data from a laboratory experiment. The results indicated that the model was calculating too large an unsaturated horizontal flow, but qualitative comparison of both perched and ground-water tables was valuable.

69. In an earlier paper, Taylor and Luthin (1969) had approached the combined unsaturated-saturated zone problem by first solving for the two zones separately, then adjusting the boundary condition between the zones. Because they applied their numerical model to drawdown of an aquifer by a well, the equations were formulated in cylindrical coordinates. Expressions for both the saturated and unsaturated zones were written in terms of hydraulic head $\phi = h + z (=H)$, with the difference being the absence of a time derivative in the saturated zone. Explicit finite difference formulation was used and solved by iteration. Variable spacing was used in the vertical and radial directions to balance computational accuracy and efficiency. Good correspondence was found with laboratory data for sand.

62. The alternating-direction implicit (ADI) method of Peaceman and Rachford (1955) was used for the solution of the two-dimensional infiltration problem. In this method (see also Remson, Hornberger, and Molz, 1971) the two-dimensional equation is solved in two steps per advance in time, which may be either one-half time increments or whole time increments. In the latter case, two time increments are required for complete application of the method. Basically, the approach is to first write the equation implicitly in one variable, say x , with the variation of the second variable, say z , denoted explicitly. This results in a system of equations for H at x -grid points which is solved via tridiagonal matrix techniques. The second time step is then accomplished by reversing the implicit-explicit application to the variables, so that x variations are written explicitly and z variations implicitly. The new system of equations is again solved via a tridiagonal technique for the z -grid points, and one cycle is complete. This is an efficient, stable technique for solution of problems in two space dimensions.

63. Rubin (1968) uses the extrapolation technique of Rubin and Steinhardt (1963) to linearize the equations by explicit calculation of the $Y(v)$ and $Z(v)$ coefficients obtained with the Kirchhoff transformation.

64. The falling-water-table, ditch-drainage problem is also treated by an ADI scheme, but because the saturated region at the base results in an elliptic equation (no time variation), iteration is required for a solution. An iteration term of the form $I_m K \Delta H$ is added to the left side of the finite difference forms (x -implicit and z -implicit) of Equation C11 and the calculations are repeated for each complete time step until two consecutive iterations are sufficiently close. The iteration parameter is cycled during iteration to improve convergence. It is given by $I_m = R^s$, where $s = 0, 1, 2, \dots, S$, and R and S are constants. The values used in Rubin (1968) were $R = 0.22$ and $S = 6$. The cited reference should be consulted for details and complete expressions of the finite difference equations.

65. The results of this model were again found to be reasonable and enlightening, but experimental data was not available for evaluation.

66. Freeze (1969) used a model similar to that of Rubin and Steinhardt (1963) to study one-dimensional, vertical, unsteady, unsaturated flow above a recharging or discharging ground-water flow system. His table of prior work and discussion thereof is frequently cited by later authors.

the method of computation cannot change the process physics. Details are marked, and frequently merged, by use of correlation relations for the unknown physics of the processes of moisture increase and decrease. The process is nonetheless complex, and simple models are simply wrong in cases which depart in too large measures from the data used to define the model relations.

106. On the other hand, despite great strides during the past several decades, many important elements of the physics of moisture exchange processes remain unknown, or require inaccessible detail or extensive computer time to treat. Thus, the models of the foregoing section may not produce superior results in comparison to a simple budget model in real cases which are bounded by temporal and financial resources. Except for the quantity of data required in some applications, the budget models may be implemented with far less computing power than the physically more accurate numerical models.

107. Thornthwaite and Mather (1954) published an updated version of a budget (mass balance) model for soil moisture prediction with application to soil tractionability. Water is input to the soil column via measured precipitation, while it is removed by both gravitational drainage and evapotranspiration at moisture contents above field capacity, and by only evapotranspiration below this amount. The Thornthwaite method of calculating potential evapotranspiration is used (see Appendix B), while actual evapotranspiration is adjusted to reflect the soil moisture remaining via a linear relationship. Soil type is considered in determination of field capacity, but other properties such as conductivity are not used. They claim accurate determination of moisture content and flow through soils, and compare to field data.

108. A budget model developed by personnel of the US Forest Service and US Army Corps of Engineer Waterways Experiment Station is discussed by Burke and Turnbull (1959). This model is discussed more fully in the body of this report, but, briefly, it utilizes precipitation as a model input which is distributed into two 15-cm (6-in.) layers as determined by statistically derived accretion relations, while depletion of the layers is effected by moisture-content-dependent depletion relations, also statistically derived from a large data set. The relations are different for sand, silt, and clay soils, and for winter, transition, and summer seasons.

The model avoids the questionable concept of field capacity by use of a field maximum water content, which is again derived from the data. Good results were obtained for well drained, fine-grained soils in locales similar to those of the original data; however, poorer correspondence was obtained between measurement and model calculations in poorly drained soils, soils with high organic content, and tropical regions.

109. Baier and Robertson (1965 and 1966) developed a six-layer soil moisture budget model in which primary depletion was effected via evapotranspiration. Evapotranspiration was determined by means of correlations using up to six input variables, namely, maximum temperature, temperature range, solar energy at the top of the atmosphere, total solar energy at the surface, wind run, and vapor deficit. Depletion from each level was allowed through a factor accounting for soil and plant root characteristics, and a linear relation between actual evapotranspiration and layer moisture content, as in the Thornthwaite approach. Runoff was estimated via a relation which included rainfall and soil moisture in the upper layer. Daily evapotranspiration was calculated for days with rain before adding the appropriate precipitation amount. Baier (1971) found the Baier and Robertson Versatile Soil Moisture Budget (VSMB) to produce results closer to the output of the Penman equation for potential evapotranspiration than did the Thornthwaite approach. The VSMB has been used in several other models. Hildreth (1977) provides a discussion and error analysis of this model in connection with outlines of several other mass balance approaches. Hildreth also outlines several models for evapotranspiration and potential evapotranspiration, also discussed in Appendix B.

110. A relatively simple budget model for irrigation scheduling was published by Jensen, Robb, and Franzoy (1970). Essentially only rainfall and evapotranspiration are considered, although the Penman approach with active consideration of varying crop coefficients through the growing season was used. Both prior weather and forecasts were used over a period of several days in model applications to estimate moisture deficits and required irrigation.

111. A combination budget-physical process model was developed by Jones and Verma (1971). Each model day, rainfall minus potential evaporation was simply distributed into a layered soil, bringing each layer to saturation in turn, until the water was exhausted. The soil moisture was

then linearly redistributed into the soil layers while holding either the lowest layer moisture content constant or the surface layer at saturation, depending on the depth of penetration of the wetting front. As long as the surface layer remained above air-dry moisture content, all evaporation moisture was taken from the uppermost layer and was set equal to potential evaporation (determined as 0.7 times evaporation measured by US Weather Bureau Class A evaporation pan). When the surface moisture content fell below the air-dry value, evaporation was calculated with the general approach of Hanks and Bowers (1962), but applied to the flow equation written in terms of moisture content and moisture diffusivity. Good correspondence was found between model simulation and field data. The importance of accuracy in diffusivity values near saturation was noted, in keeping with prior work noted above. Predicted values of soil moisture content were generally within 10 percent of measurements over a period of 43 days of natural weather.

112. Stuff and Dale (1978) used a budget model to account for shallow water table influences on soil moisture under corn. Actual evapotranspiration was estimated from Class A pan data which was adjusted for both a crop factor dependent on stage of growth and nonmoisture stress factors related to demand rate and moisture deficits. Empirical relations were developed for water table depth and capillary rise from two years of field data and then utilized to test the resulting model against two different years' data. The general performance of the model was considered adequate, with concern expressed about the validity of the data used in deriving the empirical relationships used, including the range of moisture contents represented in the original data set.

113. Lettau (1969) divided precipitation input to a surface into two main components in the formulation of a budget-process hybrid model. That portion of the precipitation which ran off or was evaporated from the surface during the input data interval was parameterized in a budget model, and subtracted from the total precipitation, resulting in a reduced forcing function for trends of soil moisture. The balance of actual runoff and evapotranspiration for a given data interval was assumed to come from water which had been stored in the soil during previous intervals. The immediate fluxes were modeled in terms of the dual forcing functions of precipitation

and solar energy, while the delayed fluxes were set proportional to soil moisture content. This approach resulted in a simple differential equation for the trends of soil moisture, which was solved by an unspecified numerical integration technique. The model and parameterization were intended for watershed application over periods of several months, and results for the North American Midwest were found to be superior to other large-scale models. The model approach was used by Lettau and Baradas (1973) for a small watershed in the Phillipine Islands, by K. Lettau (1974) in a steppe environment, by Lettau and Lettau (1975) for the tundra and boreal forests of Canada, and by Hall (1977) for the North American Great Plains. In the latter two applications, the evapotranspiration model was used in conjunction with solar irradiance and surface energy budget models to address both input and distribution of energy at the surface of the earth.

114. Each of the mass balance approaches has had to rely on parameterization or empirical relations to incorporate process physics into the model equations. An attempt to avoid this was made by Lund and Needleman (1974), who developed straightforward regression relations between meteorological data and measured soil moisture in the layer from 15 to 30 cm (6 to 12 in.) for a tropical location. Even with consideration of time lags, they were only able to distinguish wet and dry seasons, with the possibility of determining rates of transition noted. It is of value to eliminate an unworkable method, as these authors have done, so effort can be directed to less simple methods without hope for a raw statistical approach continually surfacing. Soil moisture modeling has proven to be a strongly nonlinear problem of much greater complexity than one which could be solved so simply.

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