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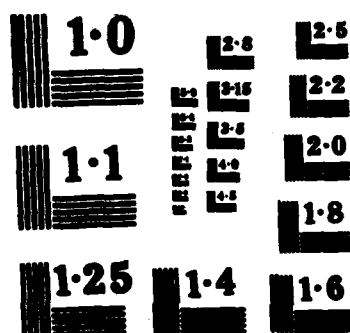
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A FLEET MAINTENANCE SYSTEM DESIGN MODEL

by

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A FLEET MAINTENANCE SYSTEM DESIGN MODEL

by

Ronald W. Boling^{*} and Frederick S. Hillier^{**}

ABSTRACT

A general model for the design of fleet maintenance systems involving several crews working in sequence is developed. Emphasis is placed on optimizing the system for scheduled periodic maintenance activities. Results are given for the case where crew service times are exponentially distributed. When mean service times for all crews are equal, an optimal system configuration is determined as a function of the number of crews, cost of crew activities and cost of a fleet unit's idle time.

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A FLEET MAINTENANCE SYSTEM DESIGN MODEL

INTRODUCTION

Fleet maintenance systems typically are designed to service fleet units, such as trucks, airplanes, cars, etc., on a scheduled maintenance program as well as emergency repair basis. Basic system design variables include the division of work between service crews, level of service provided by such crews and a limit on the number of fleet units idle at one time.

A general model for the design of such systems is developed and described in this article. Since in a given fleet environment the cost and effect of emergency repairs tend to be independent of system design, emphasis is placed on optimizing the system for scheduled maintenance activities.

Typical fleet maintenance systems are made up of multiple crews and skills which must be scheduled in a fixed sequence to complete specific maintenance tasks on each unit. Task times usually vary considerably from one unit to the next. Studying such systems is important because of the large number of organizations confronted with this problem and the magnitude of investment and expense consumed by such systems.

The particular model developed here considers a fleet of arbitrary size and a service facility consisting of a finite number of crews working in a fixed sequence on each fleet unit removed from operation. A limit is placed on the number of units which can be removed from the operating fleet at any one time. Once the limit is reached, the next scheduled fleet unit must wait until another unit is released from the maintenance system before it can enter. In its general form, system costs are a function of scheduled maintenance, emergency repairs and opportunity or capital costs associated

with idle operating units. Capital costs and operating expenses associated with housing the system are not considered. In the general model, crew service times are considered to be independent random variables with a general distribution. However, the analysis here is restricted to the case where service times are exponentially distributed. When mean service times for all crews are equal, an optimal system configuration is determined as a function of the number of crews, the cost of crew activities, and the cost of a fleet unit's idle time. Such configurations are described in terms of the limit on the number of units which can be removed from the operating fleet at any one time and the level of service rate necessary for each crew.

GENERAL COST MODEL

Consider the following model.

$$Z = B[F(R(\underline{\mu}, \gamma))] + Y[L(\underline{\mu}, \gamma)] + \sum_{k=1}^N C(\mu_k)$$

where

Z = expected total system cost per unit time (in steady state)

$B(\cdot)$ = expected cost associated with emergency repairs per unit time

$Y(\cdot)$ = expected cost associated with fleet units idle during their removal from the operating fleet per unit time

$C(\mu_k)$ = expected cost associated with maintenance crew k per unit time

and

$F(\cdot)$ = expected number of units requiring emergency repairs per unit time

$R(\cdot)$ = expected production rate (rate of completing maintenance of units) of maintenance system

$\mu_k = (E[\delta_k])^{-1}$, the service rate for crew k .

where

$E[\delta_k]$ = the expected service time for crew k

$\underline{\mu}$ = the vector whose components are the $\mu_k (k = 1, 2, 3, \dots, N)$

N = the number of crews in system

γ = maximum number of fleet units removed from operating fleet at any point in time which are available to maintenance crews

$L(\cdot)$ = expected number of fleet units in the maintenance system.

Components of cost considered here relate to emergency repair, opportunity costs of idle units and crew costs associated with scheduled maintenance activities. It is recognized that actions can be taken and, where possible, should be taken, to reduce $E(\cdot)$; however, for our purposes $E(\cdot)$ is considered to be constant and so does not influence the design of the maintenance system required to service scheduled maintenance activities.

SEQUENTIAL CREW MODEL

The problem now becomes that of solving for the values of $\mu_1, \mu_2, \dots, \mu_N$ and γ that minimize total system cost, where the chosen value of each μ_k then would be (approximately) realized by adjusting the size, composition, and mode of operation of crew k appropriately. For those systems which are kept operating at as close to capacity as possible, given the values of these design variables, the Sequential Crew Model [1] can be used to provide system production rates as a function of the number of crews, crew service rates, and the limit on the number of fleet units available to the maintenance system.

The assumptions made by this model are the following:

- (1) The system of N crews is available continuously.
- (2) These N crews always perform their tasks on each unit in the same fixed order, where the next crew begins (if not already occupied on another unit) as soon as the preceding crew ends.
- (3) For crew k , the service times for the respective units are independent and identically distributed according to an exponential distribution with parameter μ_k .
- (4) No more than γ units can be "in the system" (already begun by at least the first crew but not yet finished by the last crew) at one time.
- (5) Except when the system is at this upper limit, the first crew always starts work on another unit as soon as it completes its work on its current unit.

The following results then are obtained from the Sequential Crew Model.

$$R(\underline{\mu}, \gamma) = \mu_1 \sum_{i \in S} P_i$$

and

$$L(\underline{\mu}, \gamma) = \sum_{i=1}^N V_i P_i$$

where

P_i = stationary probability of being in state i of the underlying Markov chain.

V_i = number of units in the system in state i .

S = the set of P_i with the first crew busy.

A more complete description of this model and its characteristic behavior can be found in [2].

It can be readily observed that this model can be formulated as a continuous time parameter Markov chain; hence, the P_i can be obtained from the solution of a set of linear equations. Since most practical systems require large sets of equations, the reader may wish to refer to [5] and [6] for procedures for simplifying solution methods.

Means are now available to determine an optimal design by calculating $R(\cdot)$ and $L(\cdot)$ and using an appropriate numerical search routine. An example using a direct numerical approach is given in Appendix A for illustrative purposes.

BEHAVIOR OF BALANCED SYSTEMS

A very useful and frequently observed case of this system occurs when expected service times of all crews are equal, i.e., $\mu_k = \mu$ for all k . An examination of the transition intensity matrix for this case reveals that it is doubly stochastic. Hence,

$$P_i = \frac{1}{m} \text{ for all } i ,$$

where m is the number of states in the Markov Chain and P_i is the stationary probability of the system being in state i (see [4]).

This fortunate relationship makes it possible to calculate P_i by counting the number of states in the Markov Chain representing a particular system. Such systems and respective Markov Chains are defined by a given number of crews, crew service rates, and a limit on the number of units allowed in the system.

OPTIMIZING BALANCED SYSTEMS

Derivations of $R(\cdot)$ and $L(\cdot)$ for balanced systems are given in Appendix B. For such systems

$$R(\cdot) = \mu \cdot \left(\frac{\gamma}{\gamma + N - 1} \right)$$

$$L(\cdot) = \gamma - \frac{\gamma(\gamma - 1)}{N(\gamma + N - 1)} ,$$

where μ is the common value of the μ_k ($k = 1, 2, \dots, N$).

Now assume that $C(\underline{\mu}) = \sum_{k=1}^N C(\mu_k)$ is an increasing linear function of $\underline{\mu}$,

$$C(\underline{\mu}) = c\mu \quad \text{where } c > 0 ,$$

and that $Y(L(\cdot))$ is an increasing linear function of $L(\cdot)$,

$$Y(L(\cdot)) = ML(\cdot) \quad \text{where } M > 0 .$$

Further assume that, for a fixed integer $N > 1$, the objective is to select a (positive integer) value of γ so as to

$$\text{minimize } Z = c\mu + ML(\underline{\mu}, \gamma)$$

(P) subject to

$$R(\underline{\mu}, \gamma) = R^* .$$

Thus, R^* is the preestablished mean rate at which routine maintenance of fleet units would be scheduled. (Note that R^* is the mean rate rather than the actual rate over any short period of time, since the actual rate at which units enter the system is μ when the number already in the system is less than γ , whereas this rate becomes zero while this number equals γ .) Given

the number of units in the fleet, the value chosen for R^* normally would be based on the desired maintenance cycle (time between scheduled maintenance events) for the individual units.

For any given choice of γ , satisfying the constraint of the problem amounts to choosing the value of μ that equates $R(\underline{\mu}, \gamma)$ with R^* , namely,

$$(*) \quad \mu = R^* \frac{\gamma + N - 1}{\gamma}$$

Therefore, the problem (P) can be restated as selecting a (positive integer) value of γ so as to

$$(P') \quad \text{minimize } Z = cR^* \left[\frac{\gamma + N - 1}{\gamma} \right] + M \left[\gamma - \frac{\gamma(\gamma - 1)}{N(\gamma + N - 1)} \right] .$$

To begin solving this problem (P'), the integer restriction shall be dropped temporarily in order to treat γ as a continuous variable. Call this relaxed problem ($\bar{P}$). After considerable algebra, the first two derivatives can be expressed as

$$\frac{\delta Z}{\delta \gamma} = - \frac{cR^*(N-1)}{\gamma^2} + M - \frac{M}{N} + \frac{(N-1)M}{(\gamma + N - 1)^2} ,$$

$$\frac{\delta^2 Z}{\delta \gamma^2} = 2(N-1) \left[\frac{cR^*}{\gamma^3} - \frac{M}{(\gamma + N - 1)^3} \right] .$$

The common approach now to finding a global minimum would be to set the first derivative to zero and solve. However, since Z is not a convex function of γ in general (note that the second derivative becomes negative for sufficiently large γ if M is large relative to cR^*), further analysis is needed to show that this necessary condition for optimality also is sufficient. This is done by proving the following theorem.

Theorem 1: For problem $(\bar{P})$, there exists a unique positive $\gamma = \gamma^*$ such that

$$\frac{\delta Z}{\delta \gamma} \begin{cases} < 0, & \text{if } 0 < \gamma < \gamma^* \\ = 0, & \text{if } \gamma = \gamma^* \\ > 0, & \text{if } \gamma > \gamma^* \end{cases} ,$$

so $\gamma = \gamma^*$ achieves the global minimum.

Proof: Since $N \geq 2$, it is clear that

$$\frac{\delta Z}{\delta \gamma} < 0, \frac{\delta^2 Z}{\delta \gamma^2} > 0 \text{ for sufficiently small } \gamma > 0 .$$

Also note that

$$\lim_{\gamma \rightarrow \infty} \frac{\delta Z}{\delta \gamma} = M - \frac{M}{N} > 0 .$$

Since $\frac{\delta Z}{\delta \gamma}$ is a continuous function, it thereby assumes the value zero at

some point by the Intermediate Value Theorem. The proof now reduces to

showing that

$$\frac{\delta Z}{\delta \gamma} = 0 \Rightarrow \frac{\delta^2 Z}{\delta \gamma^2} > 0 ,$$

since this implies that $\frac{\delta Z}{\delta \gamma}$ can never return to zero (or less) as γ

increases beyond the first zero point of $\frac{\delta Z}{\delta \gamma}$, because $\frac{\delta^2 Z}{\delta \gamma^2}$ is a

continuous function and so must be strictly positive when $\frac{\delta Z}{\delta \gamma}$ is strictly positive but sufficiently close to zero.

More generally, note that

$$\begin{aligned}
 \frac{\delta Z}{\delta \gamma} \leq 0 &\Rightarrow -\frac{cR^*(N-1)}{\gamma^2} + \frac{(N-1)M}{(\gamma+N-1)^2} < 0 \\
 &\Rightarrow \frac{2}{\gamma} \left[\frac{cR^*(N-1)}{\gamma^2} - \frac{(N-1)M}{(\gamma+N-1)^2} \right] > 0 \\
 &\Rightarrow \frac{2cR^*(N-1)}{\gamma^3} - \frac{(N-1)M}{(\gamma+N-1)^3} > 0 \\
 &\Rightarrow \frac{\delta^2 Z}{\delta \gamma^2} > 0 ,
 \end{aligned}$$

which completes the proof.

Corollary: If γ^* is integer, it is the optimal solution for (P') or (P). If γ^* is not integer, then this optimal solution is obtained by rounding γ^* up or down to an integer and selecting the one with the smaller value of Z for (P') .

Unfortunately, solving directly for γ^* is not easy because it involves solving a quartic equation. However, the following theorem gives an upper bound on γ^* that is relatively tight when M is small relative to cR^* . This bound then provides the basis for an efficient algorithm for solving (P') or (P) , as summarized in the corollary to Theorem 2.

Theorem 2: The value $\gamma = \gamma^*$ identified in Theorem 1 satisfies the inequality,

$$\gamma^* < A ,$$

where

$$A = \sqrt{\frac{cR^*N}{M}} = N \sqrt{\frac{cR^*}{MN}} .$$

Proof: Since

$$\frac{\delta Z}{\delta \gamma} > (N-1) \left[\frac{M}{N} - \frac{cR^*}{\gamma^2} \right] \quad \text{for all } \gamma > 0 ,$$

it immediately follows from Theorem 1 that γ^* is bounded above by the positive root of the equation,

$$\frac{M}{N} - \frac{cR^*}{\gamma^2} = 0 ,$$

which yields the desired result.

Corollary: Let A^* be the least integer greater than or equal to A , and let $Z(n)$ be the value of Z for (P') when $\gamma = n$. An optimal solution for (P') or (P) then is identified by the following simple algorithm:

- (0) Set $n = A^*$.
- (1) If $n = 1$, then $\gamma = 1$ is optimal so stop.
- (2) Calculate $Z(n)$ and $Z(n-1)$.
- (3) If $Z(n) \leq Z(n-1)$, then $\gamma = n$ is optimal so stop. Otherwise, reset $n = n-1$ and return to Step 1 for another iteration.

Once the optimal value of γ has been obtained, the corresponding optimal value of μ is obtained by plugging this value of γ into equation (*).

A numerical example illustrating this algorithm is presented in Appendix A.

It has been assumed so far that the number of crews N has been fixed in advance. If N actually is a design variable in addition to γ , then an overall optimal solution for both N and γ can be obtained by repeating the above algorithm for each of the several possible values of N , and then choosing the one (along with its optimal γ) that yields the overall minimum for Z .

Conclusions

The use of the sequential crew model makes it possible to analyze and evaluate systems performing maintenance on fleet units. When such sequential crew systems are balanced, then it is relatively straightforward to calculate optimal values for the limit on the number of units allowed in the maintenance system at any one time and optimal crew service rates.

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APPENDIX A

Example Illustrating Solution Procedure for Optimizing Balanced Systems

Consider a fleet of 200 units, each requiring programmed maintenance every six months. This establishes R^* at $33\frac{1}{3}$ units to be scheduled per month on the average. Assume it has been determined that the cost for keeping a fleet unit idle would be \$600 per month, and experience or work standards indicate that total crew costs would be \$10,000 per month if the service rate for each crew were $33\frac{1}{3}$ units per month and each crew could work independently full time and without interference from other crews. Also consider that the skills involved necessitate three crews working in sequence.

Thus, using notation defined previously and one month as the unit of time,

$$N = 3, R^* = 33\frac{1}{3}, c = \$300, M = \$600 .$$

Applying Theorem 2,

$$\gamma^* \leq A = \sqrt{\frac{300(33\frac{1}{3})(3)}{600}} = \sqrt{50} ,$$

so that $A^* = 8$. Now applying the algorithm outlined in the corollary to Theorem 2 yields the following results:

Iteration 1: $Z(8) = \$16,180$ and $Z(7) = \$16,124$.

Since $Z(8) > Z(7)$, do another iteration.

Iteration 2: $Z(6) = \$16,183$.

Since $Z(7) < Z(6)$, stop.

$\gamma = 7$ is optimal.

The numerical calculation of these $Z(\gamma)$ is summarized below.

γ	μ	$L(\mu, \gamma)$	$c\mu$	$ML(\mu, \gamma)$	$Z(\gamma)$
8	$41\frac{2}{3}$	$6\frac{4}{30}$	\$12,500	\$3,680	\$16,180
7	$42\frac{6}{7}$	$5\frac{4}{7}$	\$12,857	\$3,267	\$16,124
6	$44\frac{4}{9}$	$4\frac{3}{4}$	\$13,333	\$2,850	\$16,183

Hence, the optimal design allows a maximum of 7 fleet units idle at any one time ($\gamma = 7$) and provides sufficient manpower, etc., in each crew to achieve a crew service rate of $42\frac{6}{7}$ units per month. Such a system will process an average of $33\frac{1}{3}$ fleet units per month at the lowest expected total cost.

APPENDIX B

Derivation of $R(\cdot)$ and $L(\cdot)$

for Balanced Systems

To determine the expected system service rate, $R(\cdot)$, for a balanced system, one must first determine m , the total number of states in the Markov chain. These states indicate how many of the γ units are with each of the respective crews, where any number larger than one for the first crew indicates that these extra units are not yet in the system because the first crew hasn't started on them yet. Therefore, m is equal to the number of ways γ indistinguishable objects can be partitioned among N cells, which elementary combinatorial analysis (e.g., see [3, p. 38]) gives as

$$m = \binom{\gamma + N - 1}{\gamma} .$$

(Note that having more than one object at the first cell corresponds to having those units not yet in the system because the first crew hasn't started work on them yet.) The number of states in which the first crew is busy is equal to the number of ways $(\gamma - 1)$ indistinguishable objects can be partitioned among N cells (since one additional object has been preassigned to the first cell), or

$$m_1 = \binom{\gamma + N - 2}{\gamma - 1} .$$

Since the Markov chain is doubly stochastic, the expected system service rate then can be calculated as

$$R(\cdot) = \mu \left(\frac{N}{N} \right) = \mu \left(\frac{\gamma}{\gamma + N - 1} \right) .$$

The expected number of fleet units in the maintenance system, $L(\cdot)$, can be determined in the following manner. Trivially, this expected number would be γ if γ units were in the system at all times. However, there are times when "empty spaces" (less than γ units) exist in the system. From the cyclic queues equivalent of the sequential crew model (see [7, pp. 174-177]), it can be shown that the total number of empty spaces which occur in all the states of the Markov chain is equal to the number of ways N indistinguishable objects can be partitioned among $(\gamma - 1)$ cells, or

$$E = \binom{N + \gamma - 2}{N} .$$

Hence,

$$L(\cdot) = \gamma - \frac{E}{N} = \gamma - \frac{\gamma(\gamma - 1)}{N(\gamma + N - 1)} .$$

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