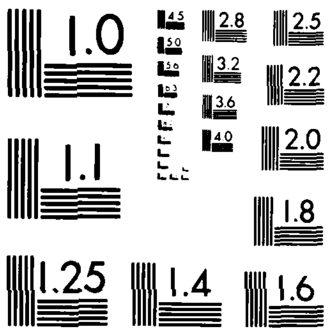
AD-A157 776 APPLICATION OF PASTERNAK MODEL TO SOME SOIL-STRUCTURE INTERACTION PROBLEM. (U) KERR (ARNOLD D) WILMINGTON DE A D KERR MAY 85 WES/TR/K-85-1-VOL-1 DACW39-85-M-1533 1/1

UNCLASSIFIED F/G 13/13 NL

END
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART

US Army Corps of Engineers

AD-A157 776

TECHNICAL REPORT K-85-1

APPLICATION OF PASTERNAK MODEL TO SOME SOIL-STRUCTURE INTERACTION PROBLEMS

Volume I

SOLUTIONS FOR PLATES CONTINUOUSLY SUPPORTED ON A PASTERNAK BASE

by

Arnold D. Kerr
903 Overbrook Road
Wilmington, Delaware 19807

US ARMY ENGINEER WATERWAYS EXPERIMENT STATION

DTIC
ELECTE
JUL 1 9 1985
S D
G

May 1985

Final Report

Approved For Public Release; Distribution Unlimited

DTIC FILE COPY

Prepared for DEPARTMENT OF THE ARMY
US Army Corps of Engineers
Washington, DC 20314-1000

Under Contract No. DACW 39-83-M-15[illegible]

Monitored by Automation Technology Center
US Army Engineer Waterways Experiment Station
PO Box 631, Vicksburg, Mississippi 39180-0631

AUTOMATION TECHNOLOGY CENTER

85 [illegible] 034

Destroy this report when no longer needed. Do not return it to the originator.

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

The contents of this report are not to be used for advertising, publication, or promotional purposes. Citation of trade names does not constitute an official endorsement or approval of the use of such commercial products.

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER Technical Report K-85-1	2. GOVT ACCESSION NO. A157 [redacted] 776	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) APPLICATION OF PASTERNAK MODEL TO SOME SOIL-STRUCTURE INTERACTION PROBLEMS; Volume I: Solutions for Plates Continuously Supported on a Pasternak Base		5. TYPE OF REPORT & PERIOD COVERED Final Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Arnold D. Kerr		8. CONTRACT OR GRANT NUMBER(s) Contract No. DACW 39-83-M-1533
9. PERFORMING ORGANIZATION NAME AND ADDRESS Dr. Arnold D. Kerr 903 Overbrook Road Wilmington, Delaware 19807		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS DEPARTMENT OF THE ARMY US Army Corps of Engineers Washington, DC 20314-1000		12. REPORT DATE May 1985 13. NUMBER OF PAGES 27
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) US Army Engineer Waterways Experiment Station Automation Technology Center PO Box 631, Vicksburg, Mississippi 39180-0631		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Available from National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia 22161.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Foundations (LC)
Pasternak foundation model (WES)
Plates (Engineering) (LC)
Soil mechnaics (LC)
Structural engineering (LC)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This study is reported in two volumes concerning the analyses of continuously supported structures and their foundation responses.

Volume I presents the research for the Pasternak Base support and utilizes it for solutions in situations arising with structures. Solutions are offered for a variety of plates subjected to the different loading conditions occurring in the design of such structures.

(Continued)

20. ABSTRACT (Continued).

Volume II similarly gives methods of parameter determination of foundation models for use in supported-structure analyses. Four methods for determining these parameters are discussed in detail.

PREFACE

This study developed into a report of two volumes dealing with soil-structure interaction models. Volume I presents a Pasternak Base model and its use for design of hydrotechnical structures. Solutions are given for a variety of plates with loadings that are common with structures designed by the Corps of Engineers. The Pasternak foundation model is labeled a two-parameter model since two independent parameters control the behavior of the model. The two parameters are dependent on the physical properties of the foundation.

Volume II discusses criteria for selecting suitable foundation models. It presents thoughts on different procedures for choosing the independent parameters for any soil-structure interaction model.

Dr. Arnold D. Kerr, P.E., Wilmington, Delaware, prepared the report under Contract No. DACW 39-83-M-1533. The work in producing this report was accomplished with funds provided to the US Army Engineer Waterways Experiment Station (WES), Vicksburg, Miss., by the Civil Works Research and Development Program of the Office, Chief of Engineers (OCE), under the Soil-Structure Interaction (SSI) Studies Project of the Structural Engineering Research Program Work Unit.

Dr. N. Radhakrishnan, Chief, Automation Technology Center (ATC), coordinated and provided objectives for the work. Dr. Robert L. Hall, Research Group, Scientific and Engineering Application Division, ATC, monitored the work and Mr. Donald R. Dressler was the point of contact in OCE.

Commanders and Directors of WES during the research and publication of this report were COL Tilford C. Creel, CE, and COL Robert C. Lee, CE. Technical Director was Mr. F. R. Brown.

CONTENTS

APPLICATION OF PASTERNAK MODEL TO SOME SOIL-STRUCTURE INTERACTION PROBLEMS

SOLUTIONS FOR PLATES CONTINUOUSLY SUPPORTED* ON A PASTERNAK BASE

PART I: INTRODUCTION

Background

1. Hydrotechnical structure design plate problems are the basis for this study, and their solutions are presented and discussed. The analysis of continuously supported structures (for example, beams, plates, and shells) requires the inclusions of the foundation response. The simplest representation of the foundation response was suggested in 1867 by E. Winkler,[1] when he assumed that the contact pressure and the deflection at a point on the base surface are proportional. Thus, for a two-dimensional plane surface represented by the x,y coordinate system, the response expression is

$$p(x,y) = k\ w(x,y) \tag{1}$$

where k, the proportionality coefficient, is often referred to as the foundation modulus.

2. This simple model, although useful for many structural and geotechnical engineering analyses, exhibits shortcomings, especially along the "free" boundaries of the structure. The situation created a need for the development of more accurate pressure-displacement relations. A number of these models were discussed by the author[2] in 1964 and more recently by A. P. S. Selvadurai.[3]

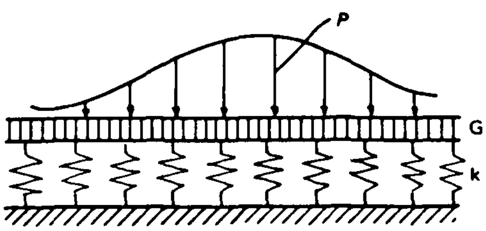


Figure 1

3. At present there is general agreement that the Pasternak[4] foundation model, shown in Figure 1,** is the next

---

* Research supported by the US Army Engineer Waterways Experiment Station, Vicksburg, Miss. 39180.

** The Pasternak foundation model, consisting of a shearing layer and a spring layer, was introduced by the author in 1964 to facilitate the derivations. The original model presented by Pasternak (1954) does not contain a shearing "layer"; the derivation presented is of questionable validity but the obtained response expression is correct.

order generalization of the Winkler model. Its response expression is

$$p(x,y) = k\, w(x,y) - G\, \nabla^2 w(x,y) \tag{2}$$

where $\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$ is the Laplace operator.

4. A continuously supported thin elastic plate (Figure 2) is governed by the partial differential equation

$$D\, \nabla^4 w + p(x,y) = q(x,y) \tag{3}$$

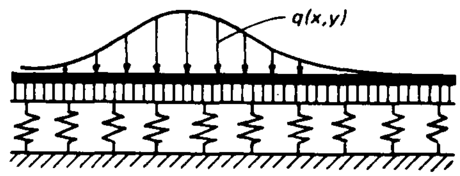


Figure 2

When the base response is represented by the Pasternak model, Equation 2, Equation 3 becomes

$$D\, \nabla^4 w - G\, \nabla^2 w + kw = q \tag{4}$$

where $D = Eh^3/[12(1 - \nu^2)]$.

This differential equation is identical to the response equation of a plate on a Winkler foundation that is stretched by a uniform force field $N = G$.

Purpose and Scope

5. In the following parts, solutions for plate problems governed by Equations 2 and 4 will be presented. Emphasis is placed on problems that occur in the design of hydrotechnical structures, beginning with a brief discussion of the analytical features of the Pasternak foundation model. These features will be needed for the solution of some of the plate problems.

PART II: ANALYTICAL FEATURES OF THE PASTERNAK FOUNDATION MODEL

Line Load

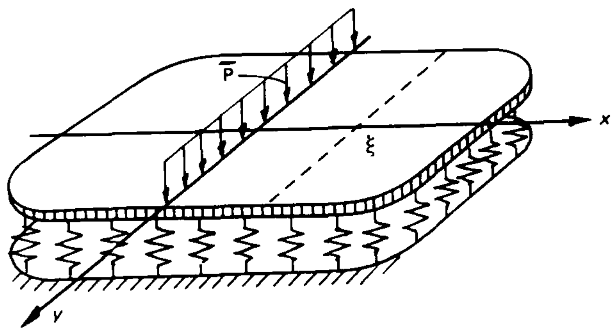


Figure 3

6. When $p(x,y)$ represents a line load $\overline{P}$ along the y-axis, as shown in Figure 3, $w = w(x)$ and Equation 2 reduces to

$$\frac{d^2w}{dx^2} - \alpha^2 w = 0 \qquad (5)$$

where

$$\alpha^2 = \frac{k}{G} \qquad (6)$$

its solution is, noting the regularity conditions for $w(x)$ as $x \to \pm\infty$,

$$w(x) = \frac{\overline{P}\alpha}{2k} e^{-\alpha|x|} \qquad -\infty < x < +\infty \qquad (7)$$

Thus, the Green's function for any position ξ of the line load $\overline{P}$ is

$$K(x;\xi) = \frac{\alpha}{2k} e^{-\alpha|x-\xi|} \qquad (8)$$

Note that the obtained deflection of the foundation surface is nonoscillatory.

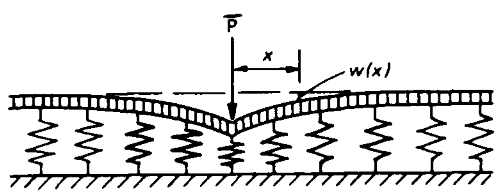


Figure 4

The line load $\overline{P}$ causes a discontinuity in the slope of the shear layer, dw/dx , along $\overline{P}$, as shown in Figure 4. This phenomenon, which does not occur in the classical plate *bending* theory, will be of importance when considering plates with free edges.

Distributed Load

7. Superposition can be used to obtain deflections of the foundation

surface that are caused by a distributed load $p(x)$. Setting $\bar{P} = p(\xi)\Delta\xi$ and integrating over the loaded interval, as shown in Figure 5, we obtain

$$w(x) = \frac{\alpha}{2k} \int_{-a}^{+b} p(\xi)\, e^{-\alpha\left|x-\xi\right|} d\xi \quad (9)$$

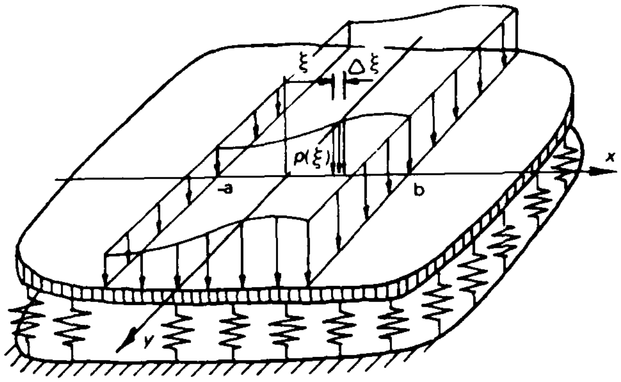


Figure 5

Arbitrary Load

8. For an arbitrary load distribution on the foundation surface, the necessary Green's function is obtained from the solution of a concentrated force P acting at the origin of the coordinate system (Figure 6). Because of the expected rotational symmetry $w = w(r)$, Equation 2 reduces to

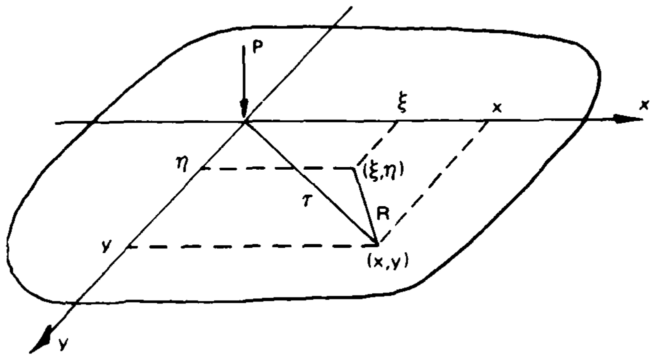


Figure 6

$$r^2 \frac{d^2w}{dr^2} + r \frac{dw}{dr} - \alpha^2 r^2 w = 0 \qquad (10)$$

9. The general solution of Equation 10 is

$$w(r) = A_1 K_o(\alpha r) + A_2 I_o(\alpha r) \qquad (11)$$

where K_o and I_o are modified Bessel functions. Noting the regularity conditions at $r \to \infty$, and the vertical equilibrium at P , we obtain

$$w(r) = \frac{P}{2\pi G} K_o\,(\alpha r) \qquad r > o \qquad (12)$$

Thus, the Green's function for the three-dimensional foundation is

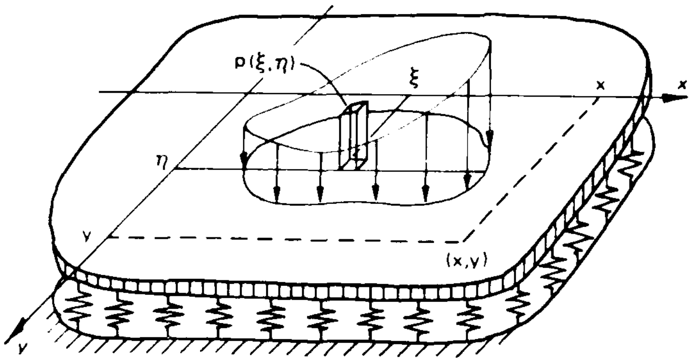


Figure 7

$$K(x,y;\xi,\eta) = \frac{1}{2\pi G} K_o (\alpha R) \quad (13)$$

where, as shown in Figure 6,

$$R = \sqrt{(x - \xi)^2 + (y - \eta)^2} \quad (14)$$

10. For a distributed load $p(x,y)$ over an area A, shown in Figure 7, the deflection of the foundation surface at a point (x,y) is

$$w(x,y) = \frac{1}{2\pi G} \int_A \int p(\xi,\eta) K_o (\alpha R) \, d\xi \, d\eta \quad (15)$$

PART III: PLATES AND STRIPS

Long "Rigid" Plate Strip; Symmetrically Loaded

11. The load on the plate strip consists of the uniform distribution q_o (for example, own weight and weight of water above it) and two line loads $\bar{F}$ along the edges (representing the weight of walls), as shown in Figure 8.

12. Because of the symmetry of the load we assume that the strip is subjected to a central line load

$$\bar{P} = q_o\, 2L + 2\bar{F} \tag{16}$$

per unit length of strip axis, as shown in Figure 9.

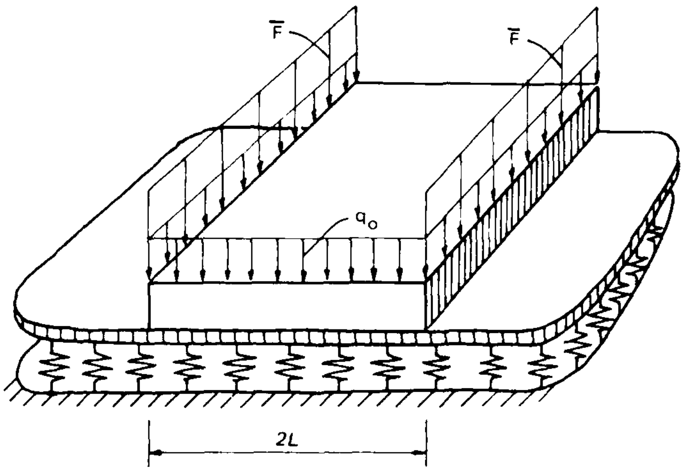


Figure 8

13. The deformed state of the foundation is shown in Figure 9. Under the strip, $-L \leq \xi \leq L$,

$$w = w_s = \text{constant} \tag{17}$$

Outside the strip the unloaded foundation is governed by the differential equation

$$-G\, w'' + kw = 0 \qquad 0 \leq x \leq \infty \tag{18}$$

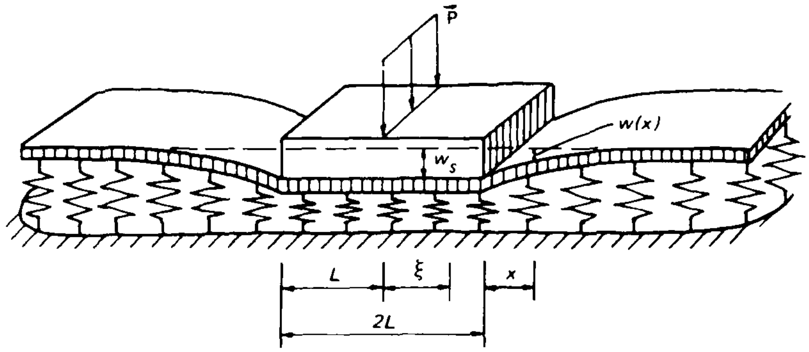


Figure 9

and the boundary conditions

$$\left.\begin{array}{l} w(0) = w_s \\ \lim_{x \to \infty}\{w\} \to \text{finite} \end{array}\right\} \tag{19}$$

41. The contact forces at the plate-foundation interface ($x \geqq 0$) consist of the distributed pressure

$$\dot{p}(x) = kw_2(x) - Gw_2''(x)$$

$$= q_o \left\{ 1 - \frac{\alpha G}{D} \frac{\{2\kappa \cos \rho x + [(\kappa^2 - \rho^2)/\rho - (\kappa^2 + \rho^2)^2/(\rho\alpha^2)] \sin \rho x\} e^{-\kappa x}}{(\kappa^2 + \rho^2)^2 + (\kappa^2 + \rho^2 + 2\kappa\alpha) G/D} \right\} \qquad 0 \leqq x \leqq \infty \tag{84}$$

and the line reaction force along the free edge ($x = 0$)

$$\bar{R} = G\left[w_1'(0) - w_2'(0)\right]$$

$$= \frac{q_o}{\alpha} \left\{ 1 - \frac{(\kappa^2 + \rho^2 + 2\kappa\alpha) G/D}{(\kappa^2 + \rho^2)^2 + (\kappa^2 + \rho^2 + 2\kappa\alpha) G/D} \right\} \tag{85}$$

The bending moments in the plate are

$$M_x(x) = -Dw''(x) = \frac{q_o}{\alpha} \frac{{\kappa^2 + \rho^2}^{2} \sin \rho x \, e^{-\kappa x}}{\rho\left[\left(\kappa^2 + \rho^2\right)^2 + \left(\kappa^2 + \rho^2 + 2\kappa\alpha\right) G/D\right]} \tag{86}$$

$$M_y(x) = \nu \, M_x(x) \tag{86'}$$

42. The distribution of the reaction forces and bending moments is shown, schematically, in Figure 24.

43. Note, that if base is modeled as a Winkler foundation the corresponding results are:

$$w(x) = \frac{q_o}{k} = \text{constant}$$

$$p(x) = q_o = \text{constant}$$

$$M_x(x) \equiv 0 \; ; \quad M_y(x) \equiv 0$$

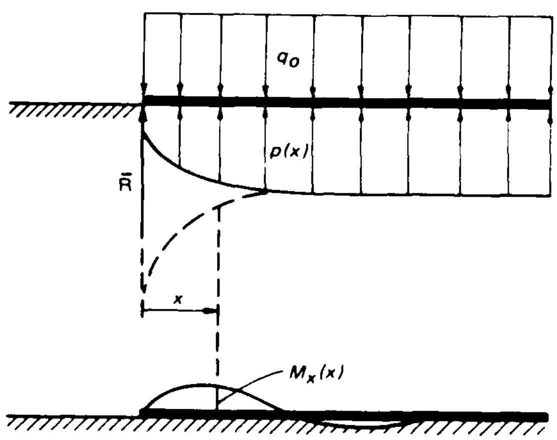


Figure 24

$$\left.\begin{aligned} w_1(x) &= A_1 e^{-\alpha x} + A_2 e^{\alpha x} && -\infty \leqq x \leqq 0 \\ w_2(x) &= e^{-\kappa x}(A_3 \cos \rho x + A_4 \sin \rho x) + e^{\kappa x}(A_5 \cos \rho x + A_6 \sin \rho x) + \frac{q_o}{k} \\ & && 0 \leqq x \leqq \infty \end{aligned}\right\} \quad (78)$$

where, according to Equation 66,

$$\left.\begin{matrix} \kappa \\ \rho \end{matrix}\right\} = \sqrt{\sqrt{\frac{k}{4D}} \pm \frac{G}{4D}} \quad (79)$$

Because of the regularity conditions in Equation 77

$$A_1 = A_5 = A_6 = 0 \quad (80)$$

It then follows from the conditions in Equation 76 that

$$\left.\begin{aligned} A_2 &= A_3 + \frac{q_o}{k} \\ A_3 &= \frac{-2\kappa\alpha \frac{q_o}{k} \frac{G}{D}}{(\kappa^2 + \rho^2)^2 + \frac{G}{D}(\kappa^2 + \rho^2 + 2\kappa\alpha)} \\ A_4 &= \frac{-(\kappa^2 - \rho^2)\, \alpha \frac{q_o}{k} \frac{G}{D}}{\rho\left[(\rho^2 + \kappa^2)^2 + \frac{G}{D}(\kappa^2 + \rho^2 + 2\kappa\alpha)\right]} \end{aligned}\right\} \quad (81)$$

thus,

$$w_1(x) = \frac{q_o}{k}\left[1 - \frac{\alpha G}{D} \frac{2\kappa}{(\kappa^2 + \rho^2)^2 + (\kappa^2 + \rho^2 + 2\kappa\alpha)\, G/D}\right] e^{\alpha x} \quad (82)$$

$$w_2(x) = \frac{q_o}{k}\left\{1 - \frac{\alpha G}{D} \frac{\{2\kappa \cos \rho x + [(\kappa^2 - \rho^2)/\rho] \sin \rho x\} e^{-\kappa x}}{(\kappa^2 + \rho^2)^2 + (\kappa^2 + \rho^2 + 2\kappa\alpha)\, G/D}\right\} \quad \begin{matrix} -\infty \leqq x \leqq 0 \\ 0 \leqq x \leqq +\infty \end{matrix} \quad (83)$$

Semi-infinite Plate Subjected to a Uniform Load

38. Consider the semi-infinite plate problem shown in Figure 23. Of special interest is the distribution of the contact pressure and the plate bending moment in the vicinity of the free edge (for example, along the free edge of a highway or airport pavement).

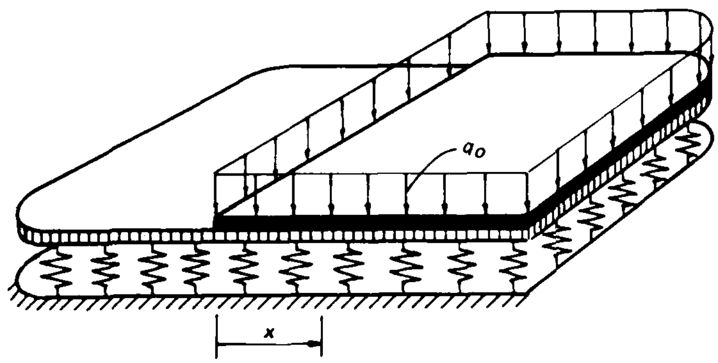


Figure 23

39. The formulation of this problem consists, noting that $w = w(x)$, of the differential equations

$$\left.\begin{array}{lr} -G\, w_1'' + k\, w_1 = 0 & -\infty \leqq x \leqq 0 \\ D w_{2}^{IV} - G\, w_2'' + k\, w_2 = q_o & 0 \leqq x \leqq \infty \end{array}\right\} \quad (75)$$

the boundary conditions

$$\left.\begin{array}{l} w_1(0) = w_2(0) \\ w_2''(0) = 0 \\ D w_2'''(0) + G\left[w_1'(0) - w_2'(0)\right] = 0 \end{array}\right\} \quad (76)$$

and the regularity conditions

$$\left.\begin{array}{l} \lim_{x \to -\infty}\{w_1\} \to \text{finite} \\ \lim_{x \to \infty}\{w_2\} \to \text{finite} \end{array}\right\} \quad (77)$$

40. The general solution is

35. When the plate is subjected to a distributed load $q(x)$, as shown in Figure 21, the deflection may be obtained using superposition. Namely,

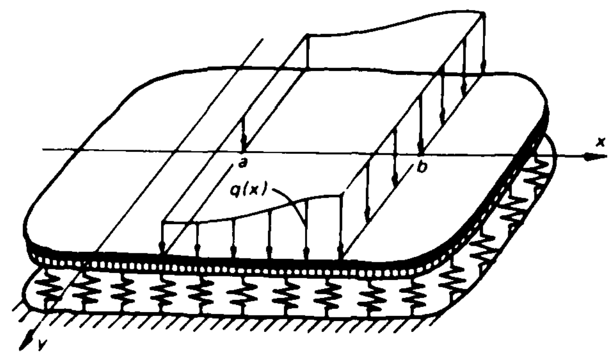


Figure 21

$$w(x) = \int_a^b q(\xi)K(x;\xi)d\xi \qquad (72)$$

where

$$K(x;\xi) = \frac{1}{4\kappa\rho\sqrt{kD}} e^{-\kappa|x-\xi|} \left[\rho \cos(\rho|x - \xi|) + \kappa \sin(\rho|x - \xi|)\right] \qquad (73)$$

The corresponding contact pressure distribution and the plate bending moment are obtained from Equations 70 and 71.

Infinite Plate Subjected to a Uniform Load

36. Consider an infinite plate of constant thickness subjected to a uniformly distributed load, q_o = constant, as shown in Figure 22. The governing equation

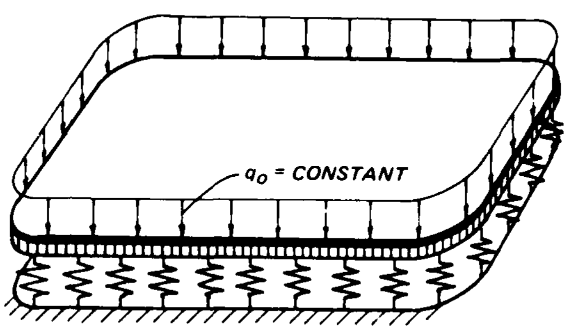


Figure 22

$$D\nabla^4 w - G\nabla^2 w + kw = q_o \qquad (4 \text{ bis})$$

is valid over the entire x,y domain.

37. The problem suggests that w = constant throughout. Thus,

$$w = q_o/k \qquad (74)$$

Note that in this case the shearing layer has no effect on the solution. The contact pressure at the interface of plate and foundation is $p = q_o$. The bending moment in the plate, due to q_o, is equal zero throughout the plate.

$$m_{1,2,3,4} = \pm\sqrt{2\left[\frac{G}{4D} \pm \sqrt{\left(\frac{G}{4D}\right)^2 - \frac{k}{4D}}\right]} \qquad (65)$$

Noting that $(G/4D)^2 - k/4D = (G/4D - \sqrt{k/4D})\,(G/4D + \sqrt{k/4D})$ we distinguish three cases when $G \lesseqgtr 2\sqrt{kD}$. For plates resting on a soil foundation the case $G < 2\sqrt{kD}$ is usually of interest. For this case

$$m_{1,2,3,4} = \pm(\kappa \pm i\rho) \qquad (65')$$

where

$$\left.\begin{matrix}\kappa \\ \rho\end{matrix}\right\} = \sqrt{\sqrt{\frac{k}{4D}} \pm \frac{G}{4D}} \qquad (66)$$

and the general solution is

$$w(x) = e^{-\kappa x}(A_1 \cos \rho x + A_2 \sin \rho x) + e^{\kappa x}(A_3 \cos \rho x + A_4 \sin \rho x) \qquad (67)$$

33. Because of the regularity condition, Equation 62, it follows that

$$A_3 = A_4 = 0 \qquad (68)$$

Determining A_1 and A_2 from the boundary conditions in Equation 61 and substituting them into $w(x)$, we obtain the plate deflection

$$w(x) = \frac{P}{4\kappa\rho\sqrt{kD}} e^{-\kappa x}(\rho \cos \rho x + \kappa \sin \rho x) \qquad (69)$$

for $x \geqq 0$.

34. The contact pressure between plate and foundation is obtained as

$$p(x) = kw(x) = G\frac{d^2w}{dx^2} \qquad (70)$$

The corresponding bending moment distribution in the plate is

$$M(x) = -D\frac{d^2w}{dx^2} \qquad (71)$$

$$w(r,\theta) = w_o \frac{K_1(\alpha r)}{K_1(\alpha a)} \cos \theta \tag{59}$$

The procedure for determination of the contact reactions is similar to the one used in the previous section.

Infinite Plate Subjected to a Line Load

31. The problem shown in Figure 20 is symmetrical. Thus, the formulation consists of the differential equation for $w(x)$

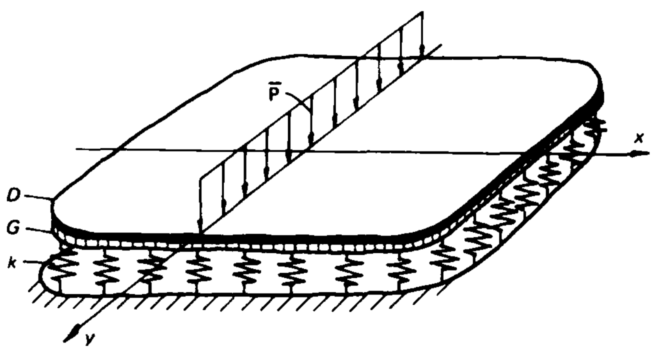


Figure 20

$$Dw^{IV} - G\,w'' + kw = 0 \tag{60}$$

in $0 \leqq x \leqq \infty$,

the boundary conditions

$$\left.\begin{aligned} W'(0) &= 0 \\ w'''\ (0) &= \frac{P}{2D} \end{aligned}\right\} \tag{61}$$

and the regularity condition

$$\lim_{x\to\infty}\{w,w'\} \rightarrow \text{finite} \tag{62}$$

32. The solution of Equation 60 is in the form

$$w(x) = Ae^{mx} \tag{63}$$

Substituting this into Equation 60, it follows that m has to satisfy

$$m^4 - m^2 \frac{G}{D} + \frac{k}{D} = 0 \tag{64}$$

The four roots of this equation are

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \tag{52}$$

the boundary condition

$$w(a,\theta) = w_o \cos\theta \tag{53}$$

and the regularity condition

$$\lim_{r\to\infty}\{w\} \to \text{finite} \tag{54}$$

29. Noting Equation 53, the solution is assumed to be of the form

$$w(r,\theta) = W(r)\cos\theta \tag{55}$$

Substituting it into the differential Equation 51, it follows that this equation will be satisfied when W is a solution of

$$r^2 \frac{d^2W}{dr^2} + r\frac{dW}{dr} - (1 + \alpha^2 r^2)\,W = 0 \tag{56}$$

in $a \leqq r \leqq \infty$. Setting $\alpha r = \rho$ the above equation becomes

$$\rho^2 \frac{d^2W}{d\rho^2} + \rho\frac{dW}{d\rho} - (1 + \rho^2)\,W = 0 \tag{57}$$

This is a Bessel equation. Its solution is

$$W(r) = A_1\, I_1(\alpha r) + A_2 K_1(\alpha r) \tag{58}$$

where I_1 and K_1 are modified Bessel functions of order one.

30. For large r values, $I_1 \sim e^{\alpha r}/\sqrt{2\pi\alpha r}$. Therefore, according to Equation 54, $A_1 = 0$. Boundary condition Equation 53 yields $A_2 = w_o/K_1(\alpha a)$. Thus,

and

$$\bar{R} = \frac{(1 + \alpha L)\bar{M}}{2L\left[1 + \alpha L + (\alpha L)^2/3\right]} \tag{49}$$

where $\bar{M} = \bar{P}e$. Thus, the rotation of the strip and the contact reactions are expressed in terms of the moment $\bar{M}$ and the strip and base parameters.

Circular "Rigid" Plate; Eccentrically Loaded

27. The plate problem for consideration is illustrated in Figure 18. It is assumed that the eccentricity is sufficiently small so that there will be no separation between the plate and base. Because the resulting formulation is linear the solution consists of two parts, as shown in Figure 14. Since specifics of the centrally loaded plate were presented previously, the solution for the plate subjected to a moment $M = Pe$ is detailed below.

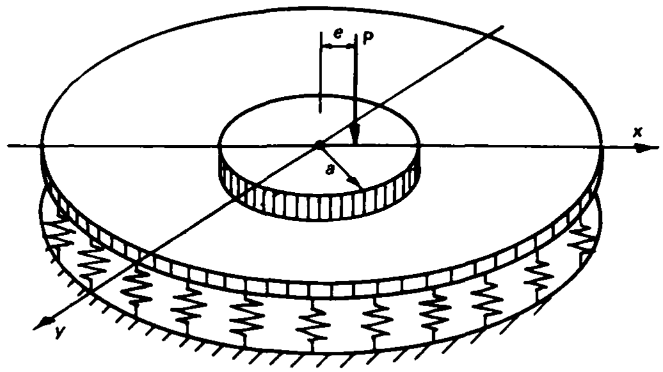


Figure 18

28. Considering the deformed state shown in Figure 19, from the shaded triangle it follows that

$$w(r,\theta) = w_o \cos\theta \tag{50}$$

for $0 < r \leqq a$ and $0 \leqq \theta \leqq 2\pi$. The unloaded base region is governed by the differential equation

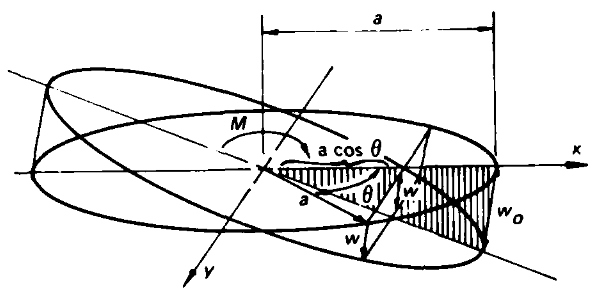


Figure 19

$$-G\,\nabla^2 w + kw = 0 \qquad a \leqq r \leqq \infty \tag{51}$$

where

$$\alpha = \sqrt{\frac{k}{G}} \tag{44}$$

24. The contact pressure under the strip is

$$p(\xi) = k\ w_s - G\ \frac{d^2 w_s}{d\xi^2} = w_o k\ \xi/L \tag{45}$$

Considering the vertical equilibrium of the shearing layer in the vicinity of $\xi = L$, as shown in Figure 16, it follows that the line reactions along the free edges are

$$\bar{R} = G\left[w_s'(L) - w'(0)\right]$$

or

$$\bar{R} = G\ w_o(1 + \alpha L)/L \tag{46}$$

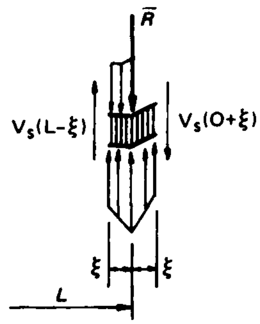


Figure 16

25. In order to express w_o in terms of $\bar{M}$, we consider the moment equilibrium (per unit length of strip axis), as shown in Figure 17. It is

$$\bar{M} - 2\bar{R}L - 2\int_o^L p(\xi)\ \xi\ d\xi = 0$$

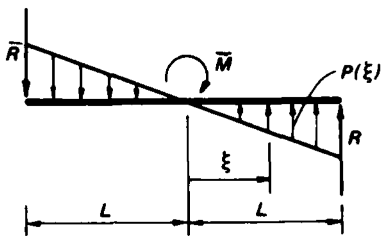


Figure 17

Noting Equations 45 and 46 and performing the indicated integration, we obtain

$$w_o = \frac{\bar{M}}{2G\left[1 + \alpha L + (\alpha L)^2/3\right]} \tag{47}$$

26. Substitution of the above expression into Equations 45 and 46 yields

$$p(\xi) = \frac{\alpha^2 \bar{M}}{2\left[1 + \alpha L + (\alpha L)^2/3\right]}\ \frac{\xi}{L} \tag{48}$$

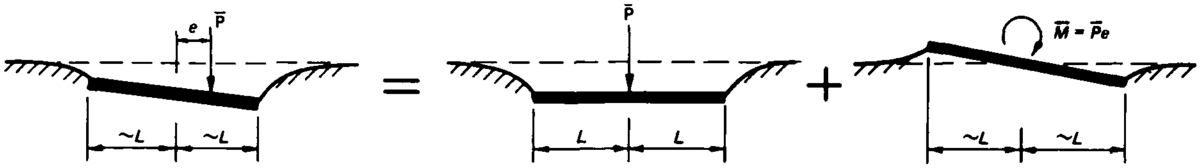


Figure 14

It is assumed that the eccentrically loaded strip stays in contact with the base. Thus, there is no partial lift-off from the base. Since the solution for the centrally loaded strip has been presented, only the solution for the line moment $\bar{M}$ will be dealt with in the following text.

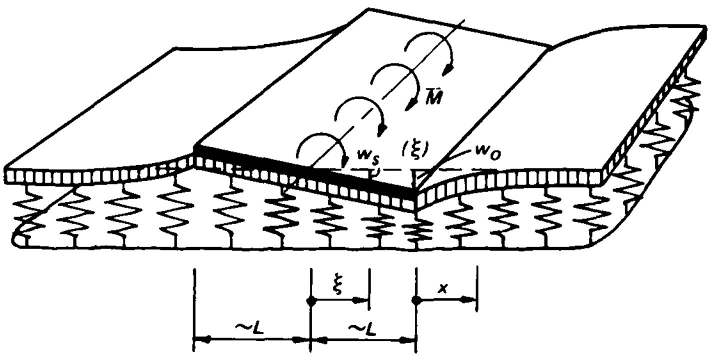


Figure 15

22. Consider a very long strip subjected to a line momemt $\bar{M}$. The corresponding deformed state is shown in Figure 15. Because of the anticipated asymmetrical deformations only the right half will be analyzed.

23. Under the strip, the deflections are

$$w_s\ (\xi) = w_o\ \frac{\xi}{L} \tag{40}$$

The deflections outside the strip are governed by the boundary value problem

$$-G\ \frac{d^2w}{dx^2} + kw = 0 \qquad 0 \leqq x \leqq \infty \tag{41}$$

$$\left.\begin{array}{l} w(0) = w_o \\ \lim\limits_{x \to \infty} \{w\} \to \text{finite} \end{array}\right\} \tag{42}$$

The solution is

$$w(x) = w_o\ e^{-\alpha x} \qquad 0 \leqq x \leqq \infty \tag{43}$$

$$w_s = \frac{P}{\pi a\left[2\sqrt{kG}\, K_1(\alpha a)/K_o(\alpha a) + ka\right]} \qquad (36)$$

Thus, in terms of P, the uniform pressure is

$$p = \frac{P}{\pi a^2 \left\{2 K_1(\alpha a)/[\alpha a\, K_o(\alpha a)] + 1\right\}} \qquad (37)$$

and the line reaction becomes

$$\bar{R} = \frac{P}{\pi a\left[2 + \alpha a\, K_o(\alpha a)/K_1(\alpha a)\right]} \qquad (38)$$

where $\alpha^2 = k/G$ and K_o and K_1 are modified Bessel functions.

Long "Rigid" Strip; Eccentrically Loaded

20. The load on the rigid strip consists of a uniform distribution q_o (own weight and weight of water) and two unequal line loads $\bar{F}_1$ and $\bar{F}_2$ along the edges (weight of walls), as shown in Figure 13. Thus, the rigid strip may be considered as being subjected to an eccentrically located line load

$$\bar{P} = q_o\, 2L + \bar{F}_1 + \bar{F}_2 \qquad (39)$$

per unit length of strip axis, as shown in Figure 14.

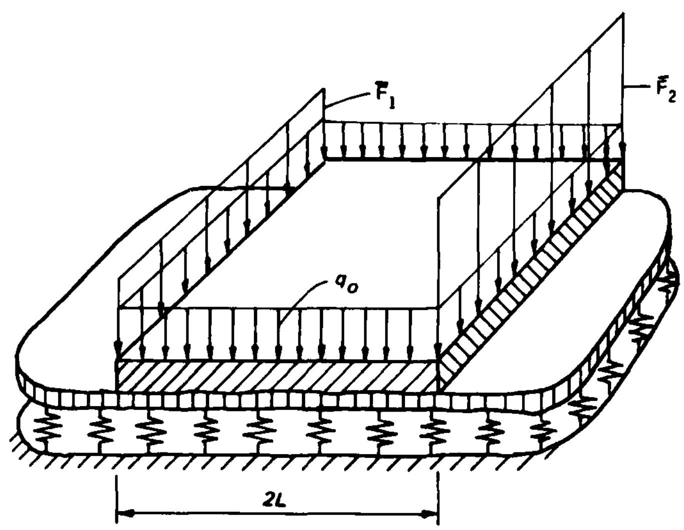


Figure 13

21. Because of the linearity of the formulation, the answer for the eccentrically loaded strip consists of the combined solution due to a central load $\bar{P}$ and a line moment $\bar{M} = \bar{P}_e$ as shown, schematically, in Figure 14.

$$\lim_{x \to \infty} \{w, w'\} \to \text{finite} \tag{29}$$

18. The general solution of Equation 10 is

$$w(r) = A_1 K_o(\alpha r) + A_2 I_o(\alpha r) \tag{11 bis}$$

because of the regularity condition (Equation 29), $A_2 = 0$. Substituting the above expression into the boundary condition (Equation 28), we obtain $A_1 = w_s / K_o(\alpha a)$. Thus,

$$w(r) = w_s \frac{K_o(\alpha r)}{K_o(\alpha a)} \qquad a \leqq r \leqq \infty \tag{30}$$

The expression for the contact pressure is

$$p(r) = k\, w_s = \text{constant} \qquad 0 \leqq r \leqq a \tag{31}$$

since under the rigid plate $dw/dr \equiv 0$. Because of the radial slope discontinuity in the shearing layer across $r = a$, there appears to be a concentrated line reaction force $\bar{R}$ along the free edge of plate. Considering the vertical equilibrium of the shearing layer in the vicinity of $\bar{R}$ we obtain

$$\bar{R} = -G \left.\frac{dw}{dr}\right|_{r=a} \tag{32}$$

Noting that

$$\frac{dK_o(\alpha r)}{dr} = -\alpha K_1(\alpha r) \tag{33}$$

it follows that

$$\bar{R} = w_s \sqrt{kG}\; \frac{K_1(\alpha a)}{K_o(\alpha a)} \tag{34}$$

19. The relation between w_s and P is obtained from the vertical equilibrium of the circular plate. It is

$$P - 2\pi a \bar{R} - \pi a^2 k\, w_s = 0 \tag{35}$$

Solving it for w_s , and noting Equation 34, we obtain and the concentrated reactions along the free edges are

$$\bar{R} = \sqrt{kG}\; w_s = \frac{1}{2(1 + \sqrt{k/G}\; L)}\, \bar{P} \qquad (26)$$

Note that the concentrated reactions $\bar{R}$ in Figure 10 represent in an actual continuum base a strong increase of the reaction pressure at the free ends, as shown by the dashed line.

"Rigid" Circular Plate; Centrally Loaded

16. The next consideration is a "rigid" circular plate being subjected to a central load P, as shown in Figure 11.

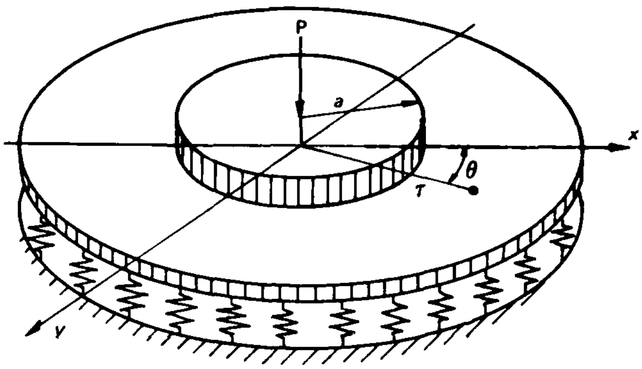


Figure 11

17. The deformed state and the notation used are shown in Figure 12. Under the plate, $0 \leqq r \leqq a$,

$$w = w_s = \text{constant} \qquad (27)$$

Since the problem is rotationally symmetrical, outside the plate region the unloaded foundation is governed by the differential equation

$$r^2 \frac{d^2w}{dr^2} + r\frac{dw}{dr} - \alpha^2 r^2 w = 0 \qquad (10 \text{ bis})$$

in $a \leqq r \leqq \infty$, the boundary condition

$$w(a) = w_s \qquad (28)$$

and the regularity condition

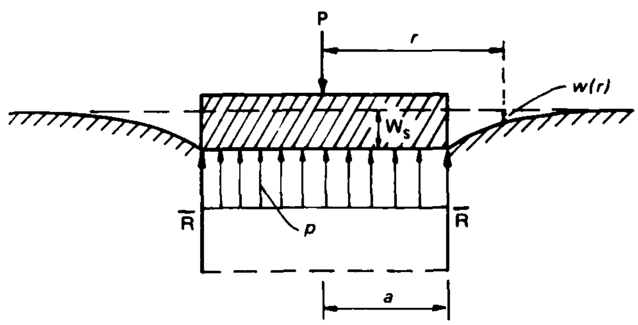


Figure 12

Because of symmetry, it is sufficient to consider only one side. The solution for the above boundary value problem is

$$w(x) = w_s e^{-\alpha x} \qquad 0 \leqq x \leqq \infty \tag{20}$$

14. For design purposes, the pressure distribution under the strip is of interest. Since $w(\xi) = \text{constant}$, it follows that $d^2w/d\xi^2 \equiv 0$ and thus

$$p(\xi) = k\, w_s = \text{constant} \tag{21}$$

in $-L \leqq \xi \leqq L$. Because of the slope discontinuity along the free edges of the strip, at $\xi = \pm L$, there occur concentrated reaction forces $\bar{R}$, as shown in Figure 10.

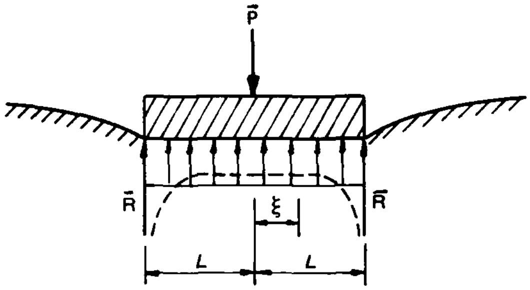


Figure 10

15. Considering the vertical equilibrium of the shearing layer in the vicinity of $\bar{R}$, we obtain

$$\bar{R} = -G\, w'\, (0) \tag{22}$$

Noting that $w(x) = w_s e^{-\alpha x}$, it follows that

$$R = \sqrt{kG}\; w_s \tag{23}$$

The relation between w_s and P is obtained by considering the vertical equilibrium of the strip (Figure 10) per unit length of strip axis. It is $\bar{P} - 2\bar{R} - k\, w_s\, 2L = 0$. Noting Equation 23, it yields

$$w_s = \frac{\bar{P}}{2\left(\sqrt{kG} + kL\right)} \tag{24}$$

Thus, in terms of $\bar{P}$, the reaction pressure in $-L < \xi < L$ is

$$p(\xi) = k\, w_s = \frac{1}{2\left(\sqrt{G/k} + L\right)}\,\bar{P} \tag{25}$$

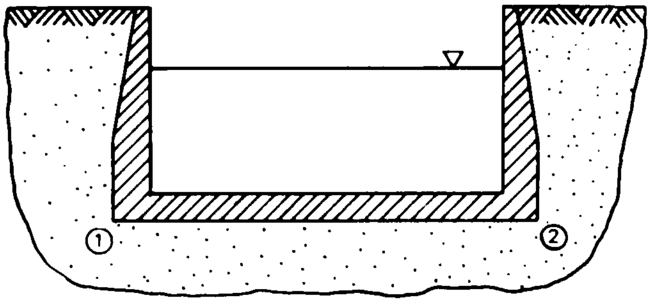


Figure 25

44. In designing dock structures and navigation locks, structures as shown in Figure 25 are encountered.

45. One approach for analyzing structures of this type is to estimate the largest pressures that can be expected on the finished walls, subject the structures to a variety of these anticipated loads, then analyze.

46. In the past, the Winkler foundation was used to represent the response of the base. A major shortcoming of this model is that it does not correctly represent the vertical pressures near the edges 1 and 2, thus, the resulting bending moments in the floor plate may strongly deviate from the actual ones.

47. To eliminate part of this shortcoming, it is suggested that the Pasternak foundation be used rather than the Winkler. The corresponding model of the structure to be analyzed is shown in Figure 26.

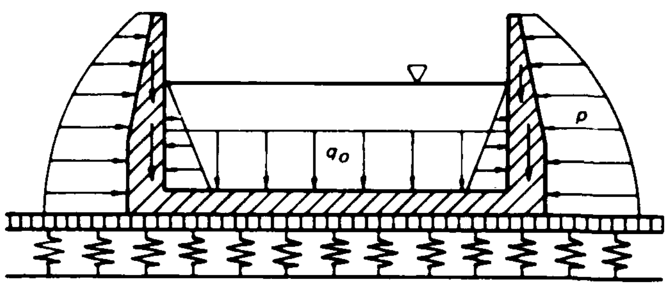


Figure 26

48. Since the pressures and the wall weights are assumed to be known, the problem reduces to the analysis of the structure shown in Figure 27. Note that the problem is similar to the one shown in Figure 8, except that in Figure 27, the floor plate is flexible.

49. Noting that $w_1 = w_1(x)$ and $w_2 = w_2(\xi)$, and utilizing symmetry, we can formulate this problem by use of the differential equations

$$\begin{aligned} Dw_1^{IV} - Gw_1'' + kw_1 &= q_o \qquad && 0 \leqq x \leqq L \\ - Gw_2'' + kw_2 &= 0 \qquad && 0 \leqq \xi \leqq \infty \end{aligned} \tag{87}$$

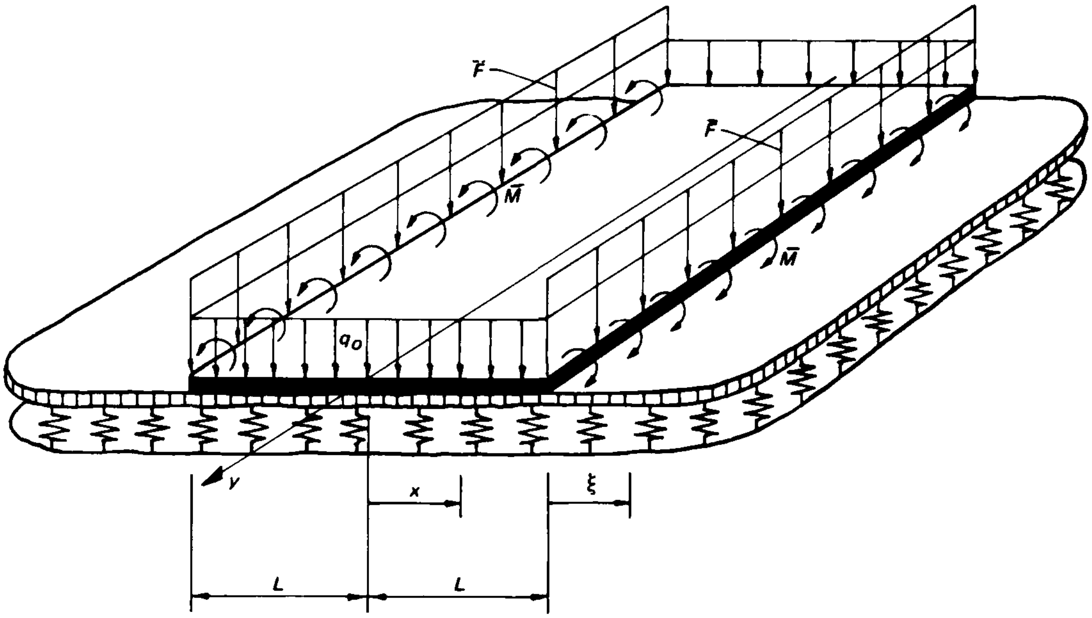


Figure 27

the boundary conditions

$$w_1'(0) = 0 \quad ; \quad w_1'''(0) = 0 \tag{88}$$

$$\left.\begin{aligned} w_1(L) &= w_2(0) \\ Dw_1''(L) &= \bar{M} \\ -Dw_1'''(L) + G\left[w_1'(L) - w_2'(0)\right] &= \bar{F} \end{aligned}\right\} \tag{89}$$

and the regularity conditions

$$\lim_{x\to\infty} \{w_2, w_2'\} \to \text{finite} \tag{90}$$

The general solution is

$$\begin{aligned} w_1(x) = {} & A_1 \cos \rho x \cosh \kappa x + A_2 \cos \rho x \sinh \kappa x \\ & + A_3 \sin \rho x \cosh \kappa x + A_4 \sin \rho x \sinh \kappa x + \frac{q_o}{k} \end{aligned} \tag{91}$$

where

$$w_2(x) = A_5 e^{\alpha\xi} + A_6 e^{-\alpha\xi} \tag{92}$$

$$\alpha = \sqrt{\frac{k}{G}} \tag{91'}$$

and

$$\left.\begin{matrix}\kappa \\ \rho\end{matrix}\right\} = \sqrt{\sqrt{\frac{k}{4D}} \pm \frac{G}{4D}} \tag{92'}$$

for $G < 2\sqrt{kD}$.

50. Note that in the above formulation the axial compression forces N_x in the floor plate are neglected in the determination of $p(x)$ and $M(x)$. They may be easily included, however, by replacing G by $(G - N_x)$ in the first equation of Equation 87 and in Equation 92', and by the corresponding modification of the third equation in Equation 89.

51. From the boundary conditions in Equation 88 it follows that $A_2 = A_3 = 0$. From the regularity condition, Equation 90, it follows that $A_5 = 0$. The remaining three constants A_1, A_4, A_6 are determined from the three boundary conditions in Equation 89.

52. The corresponding pressure distribution is

$$\begin{aligned} p(x) &= kw_1 - Gw_1'' \\ &= q_o + k\left[(A_1 \cos\rho x \cosh\kappa x) + (A_4 \sin\rho x \sinh\kappa x)\right] \\ &\quad - G\left[(\kappa^2 - \rho^2) A_1 + 2\kappa\rho A_4\right] \cos\rho x \cosh\kappa x \\ &\quad - G\left[(\kappa^2 - \rho^2) A_4 - 2\kappa\rho A_1\right] \sin\rho x \sinh\kappa x \end{aligned} \tag{93}$$

The reactions along the edges are

$$\begin{aligned} R = G\left[w_1'(L) - w_2'(0)\right] = G\Big[&(\kappa A_4 - \rho A_1) \sin\rho L \cosh\kappa L \\ &+ (\rho A_4 + \kappa A_1) \cos\rho L \sinh\kappa L + A_6\alpha\Big] \end{aligned} \tag{94}$$

and the bending moments in the floor plate are

$$\begin{aligned} M(x) &= -Dw_1''(x) \\ &= -D\Big\{\left[(\kappa^2 - \rho^2) A_1 + 2\kappa\rho A_4\right] \cos\rho x \cosh\kappa x \\ &\qquad + \left[(\kappa^2 - \rho^2) A_4 - 2\rho\kappa A_1\right] \sin\rho x \sinh\kappa x\Big\} \end{aligned} \tag{95}$$

Other Related Problems

53. A number of related problems not discussed in this report are described by Selvadurai.[3]

REFERENCES

1. Winkler, E., "Die Lehre von der Elasticitaet and Festigkeit," Dominicus, Prag, 1867.

2. Kerr, A. D., "Elastic and Viscoelastic Foundation Models," Journal of Applied Mechanics, Vol 31, 1964, pp 491-498.

3. Selvadurai, A. P. S., Elastic Analysis of Soil-Foundation Interaction, Elsevier, 1979, pp 142-161.

4. Pasternak, P. L., "Osnovy Novogo Metoda Rascheta Fundamentov na Uprugom Osnovanii pri Pomoshchi Dvukh Koeffitsientov Posteli," ("Foundations of a New Method of Analysis of Footings on an Elastic Foundation by Means of Two Base Parameters," in Russian,) 1954, Gos. Izdat. Lit. po Stroit. i Arkhitekture, Moscow, USSR.

END

FILMED

9-85

DTIC