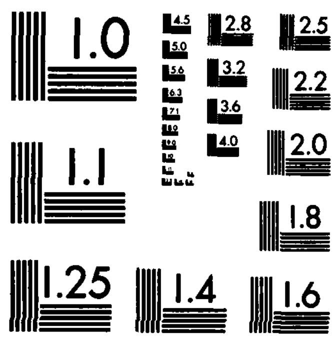
AD-A157 796 ITERATIVE AND PADE SOLUTIONS FOR THE WATER-WAVE DISPERSION RELATION(U) COASTAL ENGINEERING RESEARCH CENTER VICKSBURG MS H S CHEN ET AL. APR 85 CERC-85-4 1/1

UNCLASSIFIED F/G 20/4 NL

END

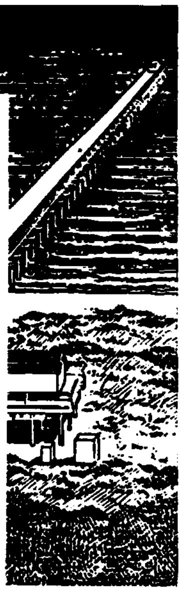
FILMED

DTIC


1.0
4.5
5.0
5.6
6.3
2.8
3.2
3.6
4.0
2.5
2.2
2.0
1.1
1.8
1.25
1.4
1.6

US Army Corps
of Engineers

AD-A157 796



MISCELLANEOUS PAPER CERC-85-4

ITERATIVE AND PADE SOLUTIONS FOR THE WATER-WAVE DISPERSION RELATION

by

H. S. Chen and E. F. Thompson

Coastal Engineering Research Center

DEPARTMENT OF THE ARMY
Waterways Experiment Station, Corps of Engineers
PO Box 631
Vicksburg, Mississippi 39180-0631

April 1985

Final Report

Approved For Public Release; Distribution Unlimited

Prepared for DEPARTMENT OF THE ARMY
US Army Corps of Engineers
Washington, DC 20314-1000

Under Waves at Entrances Work Unit 31673

85 6 28 122

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Miscellaneous Paper CERC-85-4	2. GOVT ACCESSION NO. A157796	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ITERATIVE AND PADÉ SOLUTIONS FOR THE WATER-WAVE DISPERSION RELATION		5. TYPE OF REPORT & PERIOD COVERED Final report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) H. S. Chen E. F. Thompson		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Engineer Waterways Experiment Station Coastal Engineering Research Center PO Box 631, Vicksburg, Mississippi 39180-0631		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Waves at Entrances Work Unit 31673
11. CONTROLLING OFFICE NAME AND ADDRESS DEPARTMENT OF THE ARMY US Army Corps of Engineers Washington, DC 20314-1000		12. REPORT DATE April 1985
		13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Available from National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia 22161.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Water waves--Dynamics (LC)
Dispersion relations (LC)
Pade' approximant (LC)
Iterative methods (Mathematics) (LC)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report gives → The iterative and Padé solutions of the propagating and evanescent wave modes of the dispersion relation for the linear water-wave theory are given in this report. Their corresponding FORTRAN subroutines are also provided. The Padé solution is generally more efficient than the iterative one in most engineering applications. Keywords

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

A

PREFACE

This report is a by-product of the Waves at Entrances Work Unit, Harbor Entrances and Coastal Channels Program, Civil Works Research and Development, at the Coastal Engineering Research Center (CERC) of the US Army Engineer Waterways Experiment Station (WES).

The report was prepared by Drs. H. S. Chen and E. F. Thompson, Coastal Oceanography Branch (COB), CERC, under direct supervision of Dr. J. R. Houston, Chief, Research Division, CERC, and under general supervision of Dr. R. W. Whalin, Chief, CERC.

The authors wish to acknowledge the assistance of Ms. Willie A. Brown, Physicist, CERC, in performing some of the calculations.

Commander and Director of WES during the publication of this report was COL Robert C. Lee, CE. Technical Director was Mr. F. R. Brown.

CONTENTS

ITERATIVE AND PADÉ SOLUTIONS FOR THE WATER-WAVE DISPERSION RELATION

PART I: INTRODUCTION

1. In linear water-wave theory, the dispersion relation

$$\omega^2 = gk \tanh kh \qquad (1)$$

describes the relationship between wave frequency $\omega = 2\pi/T$ and wave number $k = 2\pi/L$ for gravity waves on water depth h where g is the gravitational acceleration, T the wave period, and L the wavelength. It has been shown (Mei 1983) that for a given ω and h Equation 1 has a pair of real roots, $k = \pm k_o$, representing the propagating wave mode and an infinite number of discrete imaginary roots, $k = \pm i\alpha_n$ $(n = 1,2,\ldots,\infty)$, representing the evanescent wave mode. The terms k_o and α_n are real and $i = \sqrt{-1}$. The propagating wave mode represents progressive waves, while the evanescent wave mode represents local disturbances or standing waves which decay out in the far field. The latter is particularly important in the study of wave-structure interaction which involves measurement or calculation in the near field, such as wave forces on a structure, flow field and free surface elevation near a structure, wave maker design, etc. Since there is no exact explicit solution of Equation 1 for k, the wave number is usually obtained by an iterative method. Hunt (1979) derived an approximate solution for the propagating wave mode k_o by the Padé approximation. Hunt's solution was given also in a Coastal Engineering Technical Note (CETN) (US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center 1982) which is currently being revised. The approximate solution is very accurate, even in the short wave range. In this study a Padé solution for the evanescent wave mode α_n is also developed. Additionally, comparisons of accuracy and central processing unit (CPU) time for the iterative and Padé solutions are presented as well as FORTRAN subroutines for calculating wave number by both schemes.

PART II: APPROXIMATE SOLUTIONS OF THE DISPERSION RELATION

Propagating Wave Mode

2. Equation 1 is rewritten in the following form:

$$y = kh = \frac{\sigma}{\tanh kh} \tag{2}$$

where $\sigma = \omega^2 h/g$. Then a pair of real roots $\pm k_o$ exists as solutions of Equation 1. The solutions can be represented graphically as the intersection points of the two curves, $y = kh$ and $y = \sigma/\tanh kh$, as shown in Figure 1.

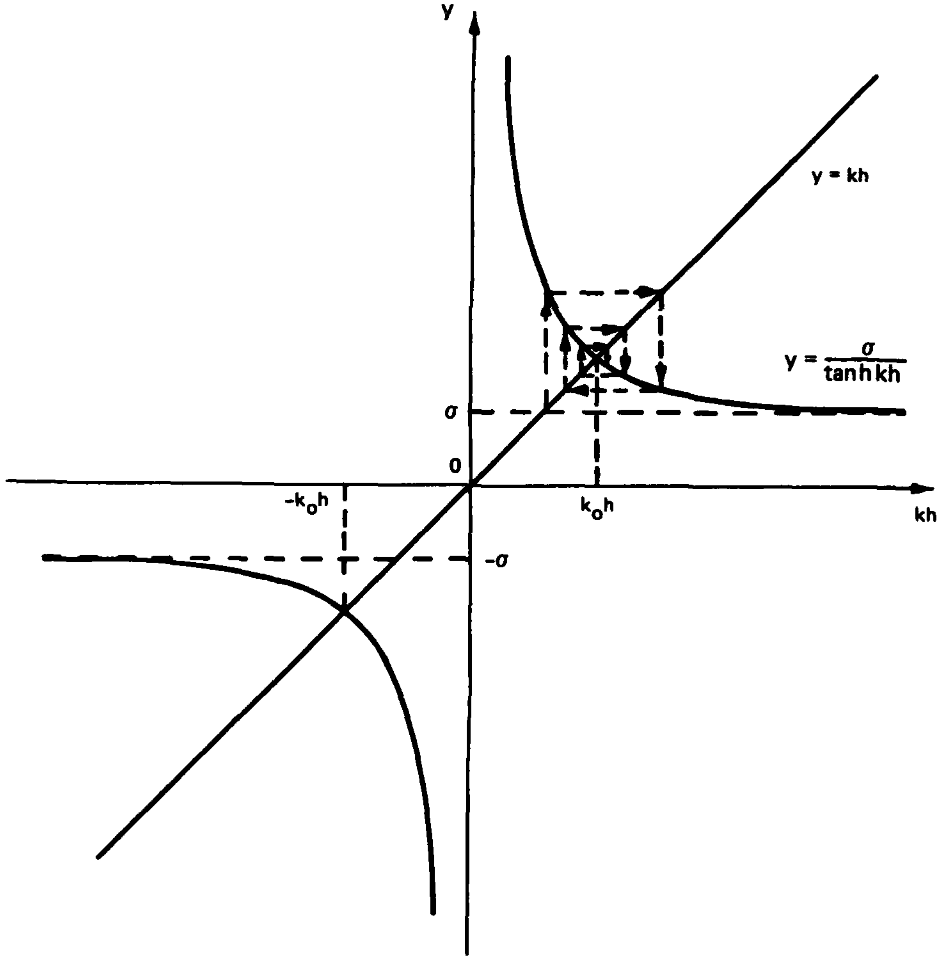


Figure 1. Real roots of the dispersion relation

Iterative approximation

3. An iterative algorithm for calculating k_o is easily established by

using Equation 2 and following the dashed arrow lines to the intersection point as illustrated in Figure 1. A FORTRAN subroutine, PKITER, for calculating k_o using the iterative approximation is provided in Appendix A.

<u>Padé approximation</u>

4. By employing the Padé scheme, Hunt (1979) obtained an approximate solution for Equation 1 in the form of the following rational function:

$$k_o h = \left(\sigma^2 + \frac{\sigma}{1 + \sum_{m=1}^{M} d_m \sigma^m} \right)^{1/2} \tag{3}$$

5. The first nine values of d_m are: $d_1 = 0.66667$, $d_2 = 0.35550$, $d_3 = 0.16084$, $d_4 = 0.06320$, $d_5 = 0.02174$, $d_6 = 0.00654$, $d_7 = 0.00171$, $d_8 = 0.00039$, $d_9 = 0.00011$. A FORTRAN subroutine, PKPADE, for calculating k_o using the Padé approximation is provided in Appendix A.

6. Calculated results of k_o for various values of σ by the iterative scheme with the truncation errors of 10^{-8} and 10^{-6} and the Padé scheme with $M = 6$ (six terms) and $M = 9$ (nine terms) are given in Table 1. It

Table 1

Comparison of the Results of the Iterative and Padé Approximations for the Propagating Wave Modes

Calculation Technique	σ	$k_o h$	Relative Error
PKPA6*	0.001	3.16280480E-02	1.58E-07
PKPA9*		3.16280480E-02	1.58E-07
PKIT6**		3.16275484E-02	-1.56E-05
PKIT8**		3.16280430E-02	0.00
PKPA6	0.010	1.00166971E-01	2.99E-08
PKPA9		1.00166971E-01	2.99E-08
PKIT6		1.00167469E-01	5.00E-06
PKIT8		1.00166968E-01	0.00

(Continued)

* PKPA6 and PKPA9 are the Padé solutions of M = 6 and M = 9, respectively.

** PKIT6 and PKIT8 are the iterative solutions with truncation errors of 10^{-6} and 10^{-8}, respectively. The values of PKIT8 are used as reference values for calculating the relative errors.

Table 1 (Concluded)

Calculation Technique	σ	$k_o h$	Relative Error
PKPA6	0.050	2.25487360E-01	-3.10E-08
PKPA9		2.25487360E-01	-3.10E-08
PKIT6		2.25487841E-01	2.10E-06
PKIT8		2.25487367E-01	0.00
PKPA6	0.100	3.21595936E-01	8.40E-08
PKPA9		3.21595936E-01	8.40E-08
PKIT6		3.21596366E-01	1.42E-06
PKIT8		3.21595909E-01	0.00
PKPA6	0.500	7.71706799E-01	5.81E-06
PKPA9		7.71704463E-01	2.78E-06
PKIT6		7.71702707E-01	5.07E-07
PKIT8		7.71702316E-01	0.00
PKPA6	1.000	1.19985791E+00	1.49E-04
PKPA9		1.19968005E+00	1.18E-06
PKIT6		1.19967884E+00	1.67E-07
PKIT8		1.19967864E+00	0.00
PKPA6	2.000	2.06858341E+00	1.57E-03
PKPA9		2.06522702E+00	-5.38E-05
PKIT6		2.06533819E+00	2.42E-08
PKIT8		2.06533814E+00	0.00
PKPA6	3.000	3.01938365E+00	1.63E-03
PKPA9		3.01459484E+00	3.72D-05
PKIT6		3.01448277E+00	-3.32E-09
PKIT8		3.01448278E+00	0.00
PKPA6	3.500	3.51050861E+00	1.20E-03
PKPA9		3.50657600E+00	7.62E-05
PKIT6		3.50630882E+00	0.00
PKIT8		3.50630882E+00	0.00
PKPA6	4.000	4.00588653E+00	8.04E-04
PKPA9		4.00298773E+00	7.93E-05
PKIT6		4.00267030E+00	0.00
PKIT8		4.00267030D+00	0.00
PKPA6	5.000	5.00205761E+00	3.21E-04
PKPA9		5.00067224E+00	4.37E-05
PKIT6		5.00045361E+00	0.00
PKIT8		5.00045361E+00	0.00
PKPA6	6.000	6.00082146E+00	1.25E-04
PKPA9		6.00017538+00	1.69E-05
PKIT6		6.00007372E+00	0.00
PKIT8		6.00007372E+00	0.00

indicates that the Padé solutions of $M = 6$ and $M = 9$ are within accuracies of 0.1 percent and 0.01 percent, respectively, in comparison with the iterative solutions with truncation error of 10^{-8}. Besides the convenience of an explicit form, the Padé solution in Equation 3 also saves CPU time. For calculating the twelve values of σ, the amount of CPU time saved is shown below:

PKPA6*	PKPA9	PKIT6	PKIT8
less than 0.0012	0.0012	0.9302	0.9998

* Time in seconds.

Evanescent Wave Mode

7. In Equation 1 there are imaginary roots $k = i\alpha$ where α is real. Inserting this expression for k into Equation 1 leads to the following equation:

$$\omega^2 = -g\alpha \tan \alpha h \qquad (4)$$

8. The real roots of Equation 4 correspond to the imaginary roots of Equation 1. Equation 4 can be rewritten as

$$y = -\frac{\sigma}{\alpha h} = \tan \alpha h \qquad (5)$$

where $\sigma = (\omega^2 h)/g$ as before.

9. These roots are the intersection points of the curves $y = -\sigma/\alpha h$ and $y = \tan \alpha h$. Since $\tan \alpha h$ has an infinite number of branches, there is an infinite number of discrete roots, $\alpha = \pm\alpha_n$ $(n = 1,2,\ldots,\infty)$, as shown graphically in Figure 2.

Iterative approximation

10. An iterative algorithm for calculating α_n is established by using Equation 5 and following the dashed arrow lines to the intersection point as illustrated in Figure 2. A FORTRAN subroutine, EKITER, for calculating α_n using the iterative approximation is provided in Appendix A. Calculated results of α_n $(n = 1,2,\ldots,10)$ with truncation errors of 10^{-8} and 10^{-4} are given in Tables 2 and 3, respectively.

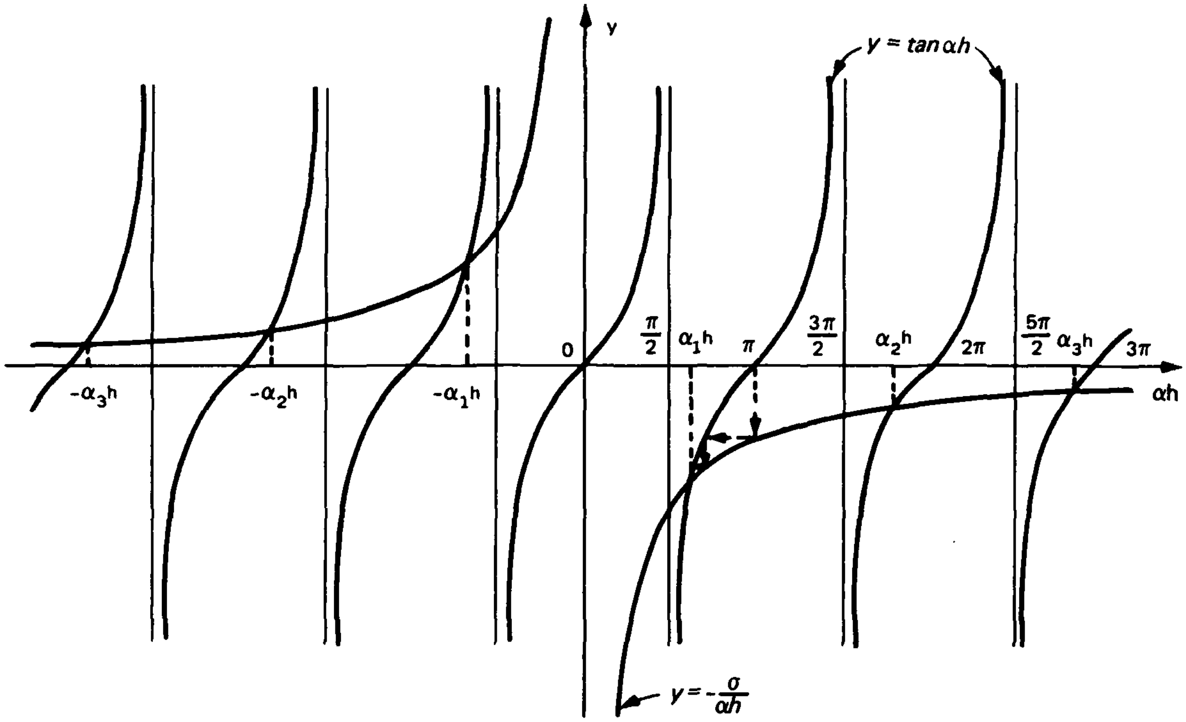


Figure 2. Imaginary roots of the dispersion relation

Padé approximation

11. For brevity, Equation 4 can be rewritten as

$$\sigma = -x \tan x \tag{6}$$

where $x = \alpha h$. In order to take appropriate account of the multiplicity of roots, let

$$x = n\pi - x' \ , \quad -\frac{\pi}{2} < x' \leq 0 \quad \text{and} \quad n = 1,2,\ldots,\infty \tag{7}$$

Substituting Equation 7 into Equation 6 gives

$$\sigma = (n\pi - x') \tan x' \tag{8}$$

From Equation 8, we have

$$x' \rightarrow \frac{\sigma}{n\pi} \quad \text{as } \sigma \rightarrow 0 \tag{9a}$$

$$x' \rightarrow \frac{\pi}{2} \quad \text{as } \sigma \rightarrow \infty \tag{9b}$$

Table 2

Iterative Solution with Truncation Error of 10^{-8} for the Evanescent Wave Mode, α_n

σ	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
0.001	3.141274	6.283026	9.424672	12.56629	15.70790	18.84950	21.99110	25.13270	28.27429	31.41589
0.010	3.125597	6.275218	9.419470	12.56239	15.70478	18.84690	21.98887	25.13075	28.27256	31.41433
0.050	3.138406	6.281593	9.423717	12.56557	15.70732	18.84902	21.99069	25.13234	28.27398	31.41560
0.100	3.109444	6.267231	9.414156	12.55840	15.70159	18.84424	21.98660	25.12876	28.27079	31.41274
0.500	2.975086	6.202750	9.371475	12.52647	15.67607	18.82299	21.96839	25.11283	28.25664	31.40000
1.000	2.798386	6.121250	9.317866	12.48645	15.64412	18.79640	21.94561	25.09291	28.23893	31.38407
2.000	2.458714	5.959392	9.210964	12.40654	15.58029	18.74325	21.90007	25.05308	28.20353	31.35222
3.000	2.204525	5.806282	9.106541	12.32765	15.51698	18.69040	21.85473	25.01337	28.16823	31.32043
3.500	2.114192	5.735255	9.055976	12.28890	15.48568	18.66418	21.83218	24.99361	28.15063	31.30458
4.000	2.043009	5.668691	9.006835	12.25077	15.45470	18.63814	21.80976	24.97392	28.13309	31.28877
5.000	1.941108	5.549860	8.913573	12.17674	15.39390	18.58676	21.76534	24.93484	28.09823	31.25730
6.000	1.873445	5.449768	8.827828	12.10623	15.33501	18.53651	21.72164	24.89625	28.06370	31.22609

Table 3

Iterative Solution with Truncation Error of 10^{-4} for the Evanescent Wave Mode, α_n

σ	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
0.001	3.141274	6.283026	9.424672	12.56629	15.70790	18.84950	21.99110	25.13270	28.27429	31.41589
0.010	3.125597	6.275218	9.419470	12.56239	15.70478	18.84690	21.98887	25.13075	28.27256	31.41433
0.050	3.138406	6.281593	9.423717	12.56557	15.70732	18.84902	21.99069	25.13234	28.27398	31.41560
0.100	3.109444	6.267231	9.414156	12.55840	15.70159	18.84424	21.98660	25.12876	28.27079	31.41274
0.500	2.975088	6.202750	9.371477	12.52647	15.67607	18.82299	21.96839	25.11283	28.25664	31.40000
1.000	2.798392	6.121253	9.317867	12.48645	15.64412	18.79640	21.94561	25.09291	28.23893	31.38407
2.000	2.458749	5.959394	9.210967	12.40654	15.58030	18.74325	21.90007	25.05308	28.20354	31.35222
3.000	2.204543	5.806292	9.106552	12.32765	15.51698	18.69041	21.85473	25.01337	28.16823	31.32043
3.500	2.114207	5.735274	9.055995	12.28891	15.48568	18.66420	21.83219	24.99361	28.15064	31.30458
4.000	2.043020	5.668718	9.006863	12.25077	15.45470	18.63814	21.80977	24.97392	28.13310	31.28877
5.000	1.941138	5.549904	8.913575	12.17675	15.39391	18.58676	21.76534	24.93485	28.09823	31.25731
6.000	1.873459	5.449822	8.827832	12.10625	15.33501	18.53651	21.72164	24.89627	28.06371	31.22610

12. Therefore, for convenience the following rational function for approximation is chosen:

$$x' = \frac{\pi}{2}\left(1 - \frac{1}{1 + \sum_{m=1}^{M} b_m \sigma^m}\right) \quad (10)$$

13. Note that Equation 10 is by no means the only form for the approximate solution of Equation 8, as one can readily understand from the Padé approximation (Baker 1975 and Bender and Orszag 1978). Expanding the functions of Equation 8 and Equation 10 into a series and matching the coefficients as in the Padé approximation, we obtain the following values of b_m (m = 1,2,3, 4,5):

	b_1	b_2	b_3	b_4	b_5
n = 1	0.20264	0.06159	0.01395	0.00182	-0.00065
n = 2	0.10132	0.01283	0.00083	-0.00003	-0.00001
n = 3	0.06755	0.00532	0.00017	-0.00001	0.0
n = 4	0.05066	0.00289	0.00006	0.0	0.0
n = 5	0.04053	0.00181	0.00003	0.0	0.0
n = 6	0.03377	0.00124	0.00001	0.0	0.0
n = 7	0.02895	0.00090	0.00001	0.0	0.0
n = 8	0.02533	0.00068	0.00001	0.0	0.0
n = 9	0.02252	0.00054	0.0	0.0	0.0
n = 10	0.02026	0.00043	0.0	0.0	0.0

and the solution of the evanescent wave number is

$$\alpha_n h = n\pi - \frac{\pi}{2}\left(1 - \frac{1}{1 + \sum_{m=1}^{M} b_m^{(n)} \sigma^m}\right) \quad (11)$$

14. We should note that computational experiments in this study indicate that an increase in M (such as M = 6, 7, 8, 9, or 10) does not improve accuracy in the short wave range at $\sigma = 4$, 5 , and 6 . Therefore, we choose M = 5 in the calculations. A FORTRAN subroutine, EKPADE, for calculating α_n for n up to 10 using Equation 11 is provided in Appendix A. Calculated

results of α_n (n = 1,2,...,10) are given in Table 4.

15. Comparisons of accuracy of the results in Tables 2, 3, and 4 are shown in Table 5. It indicates that the Padé solution in Equation 11 gives an approximation with maximum error of 1.5 percent at $\sigma = 6$ in comparison with the iterative solution with a truncation error of 10^{-8}. The error becomes smaller as σ becomes smaller. We should note that the large value of σ corresponds to short waves; for example, T is about 3 ~ 1 sec for $\sigma = 4 \sim 6$, at water depth h = 10 m. In this short wave range, most engineering applications are relatively insignificant. The CPU time savings is about threefold or sixfold that of the iterative solutions, depending on the truncation errors accepted. The CPU time (in seconds) for calculating the ten modes (M = 1,2,...,10) of the twelve values of σ is shown below:

EKIT 4*	EKIT 8*	EKPA**
0.03662	0.07324	0.01221

* EKIT8 and EKIT4 stand for the iterative solutions with truncation errors of 10^{-8} and 10^{-4}, respectively. The values of EKIT8 are used as reference values for calculating the relative errors.

** EKPA stands for the solution of Equation 11.

16. All calculations were conducted on a Control Data Cyber 170 Model 760 computer. Without loss of generality, the water depth h was taken to be 1.0 in the calculations.

Table 4

Padé Solution (of Equation 11) with $M = 5$ for the Evanescent Wave Mode, α_n

σ	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
0.001	3.141274	6.283026	9.424672	12.56629	15.70790	18.84950	21.99110	25.13270	28.27429	31.41589
0.010	3.125597	6.275218	9.419470	12.56239	15.70477	18.84690	21.98887	25.13075	28.27256	31.41433
0.050	3.138406	6.281593	9.423717	12.56557	15.70732	18.84902	21.99069	25.13234	28.27398	31.41560
0.100	3.109444	6.267231	9.414156	12.55840	15.70159	18.84425	21.98660	25.12876	28.27079	31.41274
0.500	2.975105	6.202752	9.371475	12.52647	15.67607	18.82300	21.96839	25.11283	28.25663	31.40000
1.000	2.798604	6.121251	9.317870	12.48644	15.64411	18.79640	21.94560	25.09290	28.23892	31.38408
2.000	2.457470	5.959251	9.210947	12.40643	15.58019	18.74326	21.90003	25.05302	28.20353	31.35226
3.000	2.192070	5.805512	9.106301	12.32719	15.51664	18.69040	21.85458	25.01318	28.16825	31.32054
3.500	2.095524	5.733966	9.055463	12.28810	15.48512	18.66416	21.83195	24.99330	28.15069	31.30473
4.000	2.020995	5.666839	9.005896	12.24948	15.45384	18.63809	21.80940	24.97346	28.13319	31.28898
5.000	1.927556	5.547430	8.911223	12.17392	15.39214	18.58655	21.76463	24.93393	28.09843	31.25765
6.000	1.901449	5.449008	8.823262	12.10102	15.33185	18.53598	21.72040	24.89468	28.06405	31.22660

Table 5

Comparisons of Accuracy of the Iterative and Padé Approximations for the Evanescent Wave Mode

σ	Relative Error EKIT4	EKIT8	EKPA
0.001	0	0	3.2E-09
0.010	1.3E-07	0	6.7E-08
0.050	3.2E-09	0	1.3E-08
0.100	1.3E-08	0	1.5E-07
0.500	4.8E-07	0	6.4E-06
1.000	2.0E-06	0	7.8E-05
2.000	1.4E-05	0	-5.1E-04
3.000	7.9E-06	0	-5.7E-03
3.500	7.3E-06	0	-8.8E-03
4.000	5.6E-06	0	-1.1E-02
5.000	1.5E-05	0	-7.0E-03
6.000	7.7E-06	0	1.5E-02

PART III: CONCLUSION

17. The Padé solutions of the dispersion relation in Equation 1, given by Equations 3 and 11, are rational functions which are preferable for handy calculation. Their accuracies--being within 0.1 percent for Equation 3 and 1.5 percent for Equation 11--and CPU time savings--being tremendous for Equation 3 and at least threefold for Equation 11--should enhance wave-scattering calculations in most engineering applications. Four FORTRAN subroutines calculating wave numbers for the propagating and evanescent wave modes by iterative and Padé schemes are documented in Appendix A.

REFERENCES

Baker, G. A., Jr. 1975. Essentials of Padé Approximants, Academic Press, New York, N. Y.

Bender, C. M. and Orszag, S. A. 1978. Advanced Mathematical Methods for Scientists and Engineers, McGraw-Hill, New York, N. Y.

Hunt, J. N. 1979. "Direct Solution of Wave Dispersion Equation," Journal of the Waterway, Port, Coastal and Ocean Division, ASCE, Vol 105, No. WW4, pp 457-459.

Mei, C. C. 1983. The Applied Dynamics of Ocean Surface Waves, Wiley, New York, N. Y.

US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center. 1982 (Jan). "A Direct Method for Calculating Wavelength," Coastal Engineering Technical Note I-17, Vicksburg, Miss.

APPENDIX A: LISTING OF SUBROUTINES

1. In this appendix, the following FORTRAN subroutines for calculating wave number are documented:

   a. PKITER: calculating the propagating wave mode by the iterative approximation.

   b. PKPADE: calculating the propagating wave mode by the Padé approximation in Equation 3 with $M = 9$.

   c. EKITER: calculating the evanescent wave mode by the iterative approximation.

   d. EKPADE: calculating the evanescent wave mode, $n = 1$ to 10, by the Padé approximation in Equation 11 with $M = 5$.

2. In the subroutines, the inputs are as follows: OMGG is ω^2/g ; H is water depth h ; TOLR is truncation error; N is the n^{th} evanescent wave mode; and PI = π = 3.141592654. The outputs are PK , the propagating wave number k_o , and EK , the evanescent wave number α_n .

3. The listing of the FORTRAN programs is as follows:

```
      SUBROUTINE PKITER(OMGG,H,PK,TOLR)
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C  OMGG=OMGA**2/G, H=WATER DEPTH,  PK=WAVE NO.
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      Y=OMGG*H
      XJ=Y
   10 XI=XJ
      XJ=Y/TANH(XI)
      IF(ABS(XI-XJ).GT.TOLR) GOTO 10
      PKH=XJ
      PK=PKH/H
      RETURN
      END
      SUBROUTINE PKPADE(OMGG,H,PK)
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C  OMGG=OMGA**2/G, H=WATER DEPTH, PK=WAVE NO.
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      Y=OMGG*H
      XX=Y*(Y+1./(1.+Y*(0.66667+Y*(0.35550+Y*(0.16084+Y*(0.06320+Y*
     1   (0.02174+Y*(0.00654+Y*(0.00171+Y*(0.00039+Y*0.00011)))))))))))
      X=SQRT(XX)
      PK=X/H
      RETURN
      END
      SUBROUTINE EKITER(OMGG,H,EK,N,PI,TOLR)
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C  OMGG=OMGA**2/G, H=WATER DEPTH, EK=EVANESCENT WAVE NO., N=NTH MODE.
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      Y=OMGG*H
      XJ=FLOAT(N)*PI
      DX=XJ
   10 XI=XJ
      XJ=ATAN(-Y/XI)+DX
      IF(ABS(XI-XJ).GT.TOLR*XJ) GOTO 10
      EKH=XJ
      EK=EKH/H
      RETURN
      END
```

```
      SUBROUTINE EKPADE(OMGG,H,EK,N,PI)
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C  OMGG=OMGA**2/G, H=WATER DEPTH, EK=EVANESCENT WAVE NO., N=NTH MODE.
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      DIMENSION B(5,10)
      DATA B/0.20264,0.06159,0.01395,0.00182,-0.00065,0.10132,0.01283,
     1 0.00083,-0.00003,-0.00001,0.06755,0.00532,0.00017,-0.00001,0.0,
     2 0.05066,0.00289,0.00006,0.0,0.0,0.04053,0.00181,0.00003,0.0,0.0,
     3 0.03377,0.00124,0.00001,0.0,0.0,0.02895,0.00090,0.00001,0.0,0.0,
     4 0.02533,0.00068,0.00001,0.0,0.0,0.02252,0.00054,0.0,0.0,0.0,
     5 0.02026,0.00043,0.0,0.0,0.0/
      PI2=PI/2.
      Y=OMGG*H
      EKH=FLOAT(N)*PI
      EKH=EKH-PI2*(1.-1./(1.+Y*(B(1,N)+Y*(B(2,N)+Y*(B(3,N)+Y*(B(4,N)+
     1             Y*B(5,N)))))))
      EK=EKH/H
      RETURN
      END
```

END

FILMED

9-85

DTIC