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FOREWORD

Because of their simplicity, polytropic models have historically been used to provide approximate physical representations for a variety of classical astrophysical objects such as stars, planets, and globular clusters. Since closed-form solutions exist only for three values of the polytropic index, i.e.,  $n = 0, 1,$  and  $5,$  much attention has been devoted to providing analytic representations that are valid for a range of polytropic index. This work was conducted in order to develop convergent power series solutions for the radial Emden functions  $\psi_0$  and  $\psi_2$  found in the theory of rotating polytropes. The practicality of such an approach is that when used in astrophysical applications, mathematical operations such as differentiation and integration may be analytically performed upon these functions within the bounds of their radii of convergence. This report was reviewed and approved by Dr. R. J. Anderle and Mr. R. W. Hill.

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## 1. INTRODUCTION

In recent years there has been a renewed interest in the study of polytropic gas spheres and their astrophysical applications. New and refined numerical solutions to the Lane-Emden equation<sup>1</sup> have been discussed by Seidov and Kuzakhmedov,<sup>2</sup> and by Suleiman.<sup>3</sup> Approximate representation formulae for these solutions have also been developed by Service<sup>4</sup> and by Pascual.<sup>5</sup> Several numerical studies associated with the characteristics of rotating and tidally distorted polytropes have been reported by Naylor and Anand,<sup>6</sup> Hachisu and Eriguchi,<sup>7</sup> Jabbar,<sup>8</sup> and Singh and Singh.<sup>9</sup> Convergent power series solutions to the Lane-Emden equation have recently been developed by Seidov and Kuzakhmedov,<sup>10</sup> and by Mohan and Al-Bayaty.<sup>11</sup>

It is the purpose of the present investigation to apply the method of Frobenius<sup>12</sup> to the rotating polytrope problem in order to obtain power series solutions that originate at the center of a polytropic model for the associated radial Emden functions  $\psi_0$  and  $\psi_2$ . In section 2 the equations describing the structure of a slowly rotating polytrope are formulated. The power series solutions for the radial Emden functions  $\psi_0$  and  $\psi_2$  are developed in section 3, and their convergence characteristics are discussed in section 4. Conclusions drawn concerning the present study are presented in section 5.

## 2. MATHEMATICAL FORMULATION

Consider the equation of hydrostatic equilibrium for a rotating spheroid:

$$\vec{\nabla}P = \rho \vec{\nabla}(V + V') \quad (1)$$

where  $P$  is the pressure,  $\rho$  the density,  $V$  the gravitational potential, and  $V'$  the rotational potential given by

$$V' = \frac{1}{3} \omega^2 r^2 [1 - P_2(\mu)] \quad (2)$$

Here  $\omega$  is the angular rotation rate,  $r$  is the radial distance from center,  $P_2(\mu)$  is a Legendre polynomial, and

$$\mu = \cos z \quad (3)$$

where  $z$  is the colatitude in spherical polar coordinates. Taking the gradient of Equation (1) gives

$$\vec{\nabla}\left[\frac{1}{\rho} \vec{\nabla}P\right] = \nabla^2(V + V') \quad (4)$$

Using the Poisson equation

$$\nabla^2V = -4\pi G\rho \quad (5)$$

where  $G$  is the gravitational constant, and the fact that

$$\nabla^2V' = 2\omega^2 \quad (6)$$

in Equation (4) gives

$$\vec{\nabla}\left[\frac{1}{\rho} \vec{\nabla}P\right] = -4\pi G\rho + 2\omega^2 \quad (7)$$

Assume that the spheroid's internal structure can be represented by a polytrope of index  $n$  so that

$$P = K\rho^{1+1/n} \quad (8)$$

and

$$\rho = \lambda\sigma^n \quad (9)$$

where  $K$  is a constant related to the mass  $M$  and radius  $R$  of the configuration, and  $\lambda$  is its central density. Since

$$\vec{\nabla}\left[\frac{1}{\rho} \vec{\nabla}P\right] = (n + 1)K\lambda^{1/n}\nabla^2\sigma \quad (10)$$

then

$$\left[ \frac{(n+1)K\lambda^{1/n-1}}{4\pi G} \right] \nabla^2 \sigma = -\sigma^n + \frac{\omega^2}{2\pi G\lambda} \quad (11)$$

Define

$$\beta = \left[ \frac{(n+1)K\lambda^{1/n-1}}{4\pi G} \right]^{-1/2} \quad (12)$$

$$\alpha = \frac{\omega^2}{2\pi G\lambda} \quad (13)$$

and

$$\xi = \beta r. \quad (14)$$

Then Equation (11) becomes

$$\nabla^2 \sigma = -\sigma^n + \alpha \quad (15)$$

where the Laplacian is now taken with respect to  $\xi$  instead of  $r$ .

Let us now assume that a solution to the last equation to first order in the small parameter  $\alpha$  is of the form

$$\sigma = \theta(\xi) + \alpha \Psi(\xi, \mu) \quad (16)$$

where  $\theta$  is a solution to the classical Lane-Emden equation

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\theta}{d\xi} \right) + \theta^n = 0 \quad (17)$$

and is subject to the boundary conditions

$$\theta(0) = 1; \quad \theta'(0) = 0. \quad (18)$$

After substituting the relations

$$\nabla^2 \sigma = \nabla^2 \theta + \alpha \nabla^2 \psi \quad (19)$$

and

$$\sigma^n = \sum_{j=0}^n \alpha^j \binom{n}{j} \psi^j \theta^{n-j} \approx \theta^n + \alpha n \theta^{n-1} \psi \quad (20)$$

into Equation (15) and equating orders of the parameter  $\alpha$ , one obtains

$$\nabla^2 \psi = -n \theta^{n-1} \psi + 1. \quad (21)$$

Adopt a solution to the last equation of the form

$$\psi = \psi_0(\xi) + \sum_{k=1}^{\infty} A_k \psi_k(\xi) P_k(\mu). \quad (22)$$

The  $A_k$  coefficients may be determined from an examination of the associated interior and exterior gravitational potentials. Upon integration of the equation of hydrostatic equilibrium

$$K \lambda^{1/n} (n+1) \vec{\nabla} \sigma = \vec{\nabla} (V + V') \quad (23)$$

one obtains

$$\phi_{\text{int}} = \sigma - \frac{1}{6} \alpha \xi^2 [1 - P_2(\mu)] + \text{constant} \quad (24)$$

where

$$\phi_{\text{int}} = \frac{V}{K(n+1)\lambda^{1/n}} \quad (25)$$

is the interior potential. The external potential is the Laplace solution:

$$\phi_{\text{ext}} = \frac{\gamma_0}{\xi} + \alpha \sum_{j=1}^{\infty} \gamma_j \xi^{-(j+1)} P_j(\mu). \quad (26)$$

These potentials and their gradients may be matched at the surface  $\xi = \xi_1$ , i.e.

$$\left. \begin{aligned} \phi_{\text{int}}(\xi_1) &= \phi_{\text{ext}}(\xi_1) \\ \phi'_{\text{int}}(\xi_1) &= \phi'_{\text{ext}}(\xi_1) \end{aligned} \right\} \quad (27)$$

to give

$$\left. \begin{aligned} \theta(\xi_1) + \alpha[\psi_0(\xi_1) - \frac{1}{6} \xi_1^2] &= \gamma_0/\xi_1 + \text{constant} \\ \theta'(\xi_1) + \alpha[\psi'_0(\xi_1) - \frac{1}{3} \xi_1] &= -\gamma_0/\xi_1^2 \end{aligned} \right\} \quad (28)$$

$$\left. \begin{aligned} A_1 \psi_1(\xi_1) &= \gamma_1/\xi_1^2 \\ A_1 \psi'_1(\xi_1) &= -2\gamma_1/\xi_1^3 \end{aligned} \right\} \quad (29)$$

$$\left. \begin{aligned} A_2 \psi_2(\xi_1) + \frac{1}{6} \xi_1^2 &= \gamma_2/\xi_1^3 \\ A_2 \psi'_2(\xi_1) + \frac{1}{3} \xi_1 &= -3\gamma_2/\xi_1^4 \end{aligned} \right\} \quad (30)$$

and for  $k > 2$

$$\left. \begin{aligned} A_k \psi_k(\xi_1) &= \gamma_k \xi_1^{-(k+1)} \\ A_k \psi'_k(\xi_1) &= -(k+1)\gamma_k \xi_1^{-(k+2)} \end{aligned} \right\} \quad (31)$$

The conditions of Equations (30) define  $\gamma_0$ , and those of Equations (29) and (31) imply that  $A_k = 0$  ( $k \neq 2$ ).  $A_2$  is obtained from Equations (30), which yield

$$A_2 = -\frac{5}{6} \left\{ \frac{\xi_1^2}{3\psi_2(\xi_1) + \xi_1\psi_2'(\xi_1)} \right\}. \quad (32)$$

Thus

$$\Psi = \psi_0(\xi) + A_2\psi_2(\xi)P_2(\mu) \quad (33)$$

where

$$\psi_0(0) = \psi_0'(0) = 0; \quad \psi_2(0) = \psi_2'(0) = 0. \quad (34)$$

Substitution of Equation (33) into Equation (21) provides

$$\nabla^2 \psi_0 = -n\theta^{n-1} \psi_0 + 1 \quad (35)$$

and

$$\nabla^2 (\psi_2 P_2) = -n\theta^{n-1} \psi_2 P_2. \quad (36)$$

Since

$$\nabla^2 = \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left( \xi^2 \frac{\partial}{\partial \xi} \right) + \frac{1}{\xi^2} \frac{\partial}{\partial \mu} \left[ (1 - \mu^2) \frac{\partial}{\partial \mu} \right] \quad (37)$$

then one may finally rewrite Equations (35) and (36) as

$$\frac{\partial^2 \psi_0}{\partial \xi^2} + \frac{2}{\xi} \frac{\partial \psi_0}{\partial \xi} + n\theta^{n-1} \psi_0 = 1 \quad (38)$$

and

$$\frac{\partial^2 \psi_2}{\partial \xi^2} + \frac{2}{\xi} \frac{\partial \psi_2}{\partial \xi} - \frac{6}{\xi^2} \psi_2 + n\theta^{n-1} \psi_2 = 0 \quad (39)$$

3. POWER SERIES SOLUTIONS FOR  $\psi_0(\xi)$  AND  $\psi_2(\xi)$ 

In this section, the method of Frobenius is applied to Equations (38) and (39) in order to obtain power series solutions for the radial Emden functions  $\psi_0$  and  $\psi_2$ . Development of the series from the center will only be considered here. Seidov and Kuzakhmedov<sup>10</sup> have provided such a development for the Lane-Emden function  $\theta$ . Since their results are used in this work, pertinent expressions are repeated here:

$$\theta^{n-1} = 1 + \sum_{j=1}^{\infty} c_j \xi^{2j} \quad (40)$$

where

$$c_j = \frac{1}{j} \sum_{i=1}^j (in-j) a_i c_{j-i} \quad (41)$$

$$a_{j+1} = \frac{1}{j(j+1)(2j+3)} \sum_{i=1}^j (in+i-j)(j-i+1)[3 + 2(j-i)] a_i a_{j+1-i} \quad (42)$$

$$c_0 = 1; \quad a_1 = -1/6 \quad (43)$$

and  $j \geq 1$ .

Consider first the solution for Equation (38). Let

$$\psi_0 = \sum_{j=1}^{\infty} b_j \xi^{2j} \quad (44)$$

so that

$$\frac{\partial \psi_0}{\partial \xi} = \sum_{j=1}^{\infty} 2j b_j \xi^{2j-1} \quad (45)$$

and

$$\frac{\partial^2 \psi_0}{\partial \xi^2} = \sum_{j=1}^{\infty} 2j(2j-1) b_j \xi^{2j-2}. \quad (46)$$

Substituting the last three equations along with Equation (40) into Equation (38) gives

$$\sum_{j=1}^{\infty} b_j \left\{ 2j(2j+1) \xi^{2j-2} + n \left[ 1 + \sum_{m=1}^{\infty} c_m \xi^{2m} \right] \xi^{2j} \right\} = 1. \quad (47)$$

The lowest power of  $\xi$  occurs when  $j=1$  yielding

$$b_1 = 1/6 \quad (48)$$

The recursion relation for successive values of  $b_j$  may be obtained by letting  $j \rightarrow j+1$  and  $j \rightarrow j-m$  in the first and last terms of Equation (47), respectively, giving

$$\sum_{j=1}^{\infty} \left\{ 2b_{j+1} (j+1)(2j+3) + nb_j + n \sum_{m=1}^{j-1} b_{j-m} c_m \right\} \xi^{2j} = 0. \quad (49)$$

This expression has been equated to zero since there are no vanishing exponents of  $\xi$  for all  $j \geq 1$ . It should also be noted that a finite upper limit has been placed upon the summation index  $m$  since  $j-m \geq 1$ . Thus for a given polytropic index  $n$ , one has

$$b_{j+1} = \frac{-n}{2(j+1)(2j+3)} \left[ b_j + \sum_{m=1}^{j-1} b_{j-m} c_m \right]. \quad (50)$$

The cases when  $n=0$  and  $n=1$  are of particular interest. Note that when  $n=0$  one sees from Equation (50) that  $b_j=0$  for  $j > 1$ , so that

$$\psi_0 = b_1 \xi^2 = \frac{1}{6} \xi^2. \quad (51)$$

When  $n=1$ ,  $\theta^{n-1} = 1$  and the summation in Equation (50) vanishes giving

$$\psi_0 = \frac{1}{6} \xi^2 - \frac{1}{120} \xi^4 + \frac{1}{5040} \xi^6 - \dots \quad (52)$$

or

$$\psi_0 = \sum_{j=1}^{\infty} \frac{(-1)^{j-1}}{(2j+1)!} \xi^{2j} = 1 - \frac{\sin \xi}{\xi}. \quad (53)$$

These results thus reduce identically to the known closed form solutions for  $\psi_0$  given by Jabbar<sup>8</sup> when  $n=0$  and  $n=1$ .

In order to obtain a solution to Equation (39), let

$$\psi_2 = \sum_{j=1}^{\infty} d_j \xi^{2j}. \quad (54)$$

Then

$$\frac{\partial \psi_2}{\partial \xi} = \sum_{j=1}^{\infty} 2j d_j \xi^{2j-1} \quad (55)$$

and

$$\frac{\partial^2 \psi_2}{\partial \xi^2} = \sum_{j=1}^{\infty} 2j(2j-1) d_j \xi^{2j-2}. \quad (56)$$

After substitution of these three expressions into Equation (39) and using Equation (40), one has

$$\sum_{j=1}^{\infty} d_j \left\{ 2(2j+3)(j-1)\xi^{2j-2} + n \left[ 1 + \sum_{m=1}^{\infty} c_m \xi^{2m} \right] \xi^{2j} \right\} = 0. \quad (57)$$

The recursion relation for  $d_j$  may be obtained by letting  $j \rightarrow j+1$  in the first term and  $j \rightarrow j-m$  in the last term. After rearranging terms the following expression is obtained:

$$\sum_{j=1}^{\infty} \left\{ 2j(2j+5)d_{j+1} + nd_j + n \sum_{m=1}^{j-1} d_{j-m} c_m \right\} \xi^{2j} = 0 \quad (58)$$

or

$$d_{j+1} = \frac{-n}{2j(2j+5)} \left[ d_j + \sum_{m=1}^{j-1} d_{j-m} c_m \right] \quad (59)$$

for a given polytropic index  $n$ . Since  $d_1$  is arbitrary, one may choose

$$d_1 = 1. \quad (60)$$

#### 4. CONVERGENCE CHARACTERISTICS OF THE $\psi_0$ AND $\psi_2$ SERIES

The convergence properties of the power series representations for  $\psi_0$  and  $\psi_2$  will be examined in this section. Specifically, approximate radii of convergence will be established for these series by comparing results obtained from them with those found in the literature that were computed using numerical integration techniques. Additional comparisons will be made where possible between results for the first derivatives of the series and their numerically derived counterparts.

Table 1 is a comparison between the solutions obtained by Jabbar<sup>8</sup> from numerical integration of Equation (38) and those computed from the series given by Equations (44) and (50) using the Naval Surface Weapons Center CDC 6700 computer. The first 100 terms were used in the series evaluation, and the polytropic index  $n$  was allowed to range in increments of 0.1 between 0 and 1.8. Both sets of results were evaluated at  $\xi = \xi_1$ , where the  $\xi_1$  are the  $n$  dependent first zeros of the Emden functions  $\theta$ . Power series evaluations have not been included in this table for  $n \geq 1.9$ , because the series diverge for this range of polytropic index when  $\xi = \xi_1$ .

As can be seen from the values in Table 1, the agreement between the numerically integrated results and those obtained from the 100 term series evaluations is quite good, with the level of agreement generally being through three to four decimal places. Increasing the number of terms in the series by 50% produces negligible improvement for all  $n$  considered, whereas reducing the number of terms by 50% tends to reduce the level of agreement with the numerically integrated results by one decimal place for  $0.1 \leq n \leq 0.9$ .

It should be mentioned that the polytropic index convergence range of  $n \leq 1.8$  for the  $\psi_0(\xi_1)$  series is consistent with the convergence range of  $n \leq 1.5$  when  $\xi \leq \xi_1$  determined by Mohan and Al-Bayaty<sup>11</sup> for the Seidov and Kuzakhmedov<sup>10</sup> series representation for  $\theta$ . This result is expected, since use is made of their  $a_i$  and  $c_i$  coefficients in forming the  $b_i$  coefficients. In fact, based upon recent evaluations of the  $\theta$  series made by the author, the polytropic convergence range for  $\theta$  should also be  $n \leq 1.8$  when  $\xi \leq \xi_1$ .

Additional comparisons can be made between numerically integrated results obtained by Naylor and Anand<sup>6</sup> and Singh and Singh<sup>9</sup> for  $\psi_0$ ,  $\psi_2$ , and their first derivatives and those obtained from evaluation of the first 100 terms of Equations (44), (45), (54), and (55). These results are presented in Table 2 where it can be seen that the agreement is quite good for both series and their derivatives. The level of agreement is somewhat better for the  $n = 1.5$  cases (especially so with the Naylor and Anand<sup>6</sup> data) than for the  $n = 2.0$  cases. This might be expected, since the entire interior of the polytrope is contained within the radius of convergence for  $n = 1.5$ , whereas the values of  $\xi$  used for series evaluation in the  $n = 2.0$  cases are nearly the same as the radius of convergence.

Estimates of the radii of convergence for the  $\psi_0$  and  $\psi_2$  series were made by evaluating these series at increments of  $0.1\xi_1$  for  $0 \leq \xi \leq \xi_1$  and increments of

TABLE 1. COMPARISON OF NUMERICALLY INTEGRATED AND  
 SERIES GENERATED VALUES FOR  $\psi_0(\xi_1)$

n	$\xi_1$	$\psi_0(\xi_1)$ -Integrated*	$\psi_0(\xi_1)$ -Series**
0.0	2.44949	1.00000	1.00000
0.1	2.50454	0.96500	0.96586
0.2	2.56222	0.93938	0.94037
0.3	2.62268	0.92205	0.92288
0.4	2.68610	0.91229	0.91288
0.5	2.75270	0.90963	0.91002
0.6	2.82268	0.91384	0.91407
0.7	2.89628	0.92484	0.92496
0.8	2.97376	0.94270	0.94276
0.9	3.05543	0.96764	0.96768
1.0	3.14159	1.00000	1.00000
1.1	3.23261	1.04024	1.04021
1.2	3.32887	1.08897	1.08894
1.3	3.43081	1.14693	1.14690
1.4	3.53893	1.21504	1.21502
1.5	3.65375	1.29440	1.29438
1.6	3.77590	1.38630	1.38628
1.7	3.90606	1.49231	1.49230
1.8	4.04501	1.61427	1.61427

\*Jabbar's results presented here have been rounded in the fifth decimal place

\*\*The first 100 terms have been used to evaluate the series

TABLE 2. COMPARISON OF NUMERICALLY INTEGRATED AND SERIES GENERATED VALUES FOR  $\psi_0$ ,  $\psi_2$ , AND THEIR FIRST DERIVATIVES

Source	n	$\xi$	$\psi_0$	$\frac{\partial \psi_0}{\partial \xi}$	$\psi_2$	$\frac{\partial \psi_2}{\partial \xi}$
	1.5	3.025				
Series <sup>†</sup>			0.95793	0.46940	4.12962	1.08133
S-S*			0.95783	0.46440	4.12860	1.08108
		3.20				
Series <sup>†</sup>			1.04232	0.49659	4.31342	1.02551
N-A**			1.0423	0.49660	4.3134	1.0255
	2.0	3.47				
Series <sup>†</sup>			1.21098	0.63453	4.37464	1.22236
S-S*			1.20729	0.63147	4.37357	1.22206
		3.60				
Series <sup>†</sup>			1.29604	0.67434	4.53587	1.25675
N-A**			1.2961	0.67495	4.5360	1.2617

<sup>†</sup>The first 100 terms used to evaluate series

\*Results from Singh and Singh<sup>9</sup>

\*\*Results from Naylor and Anand<sup>6</sup>

0.1 for  $0 \leq n \leq 3.3$ . That value of  $\xi$  after which an obvious and abrupt discontinuity in the value of the first 100 terms of the series occurred was assumed to correspond to the associated radius of convergence for the series. Identical radii of convergence were obtained in this manner for  $\psi_0$  and  $\psi_2$  and they are presented in Figure 1. These results are consistent with the findings derived for  $\psi_0$  in the Table 1 numerical comparisons, i.e. the radii of convergence are greater than the polytropic radii  $\xi_1$  for  $n \leq 1.8$ . Estimates made in this fashion track quite well the radii of convergence obtained for the Seidov and Kuzakhmedov<sup>10</sup>  $\theta$  series by Mohan and Al-Bayaty<sup>11</sup>, also shown on Figure 1.

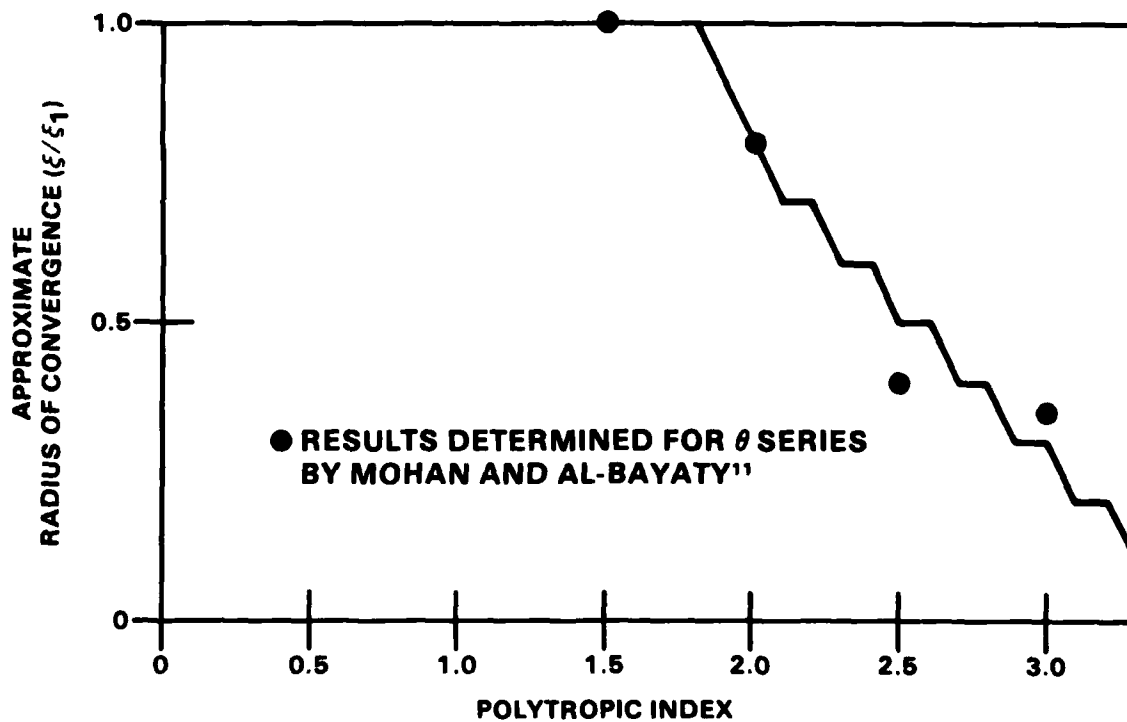


FIGURE 1. APPROXIMATE RADII OF CONVERGENCE AS A FUNCTION OF POLYTROPIC INDEX FOR  $\psi_0(\xi)$  AND  $\psi_2(\xi)$

## 5. CONCLUSIONS

Power series solutions for the radial Emden functions  $\psi_0$  and  $\psi_2$  have been developed at the polytropic center which are convergent in the whole interior of a rotating polytrope for values of the polytropic index  $n \leq 1.8$ . Three to four decimal places of accuracy can be obtained when the first 100 terms are used to evaluate these series in this region. For values of the polytropic index  $n > 1.8$ , the radii of convergence for both series gradually diminishes until  $n = 3.3$  where the radius of convergence is approximately  $0.1\xi_1$ .

The obvious advantages of such a representation are that within the radii of convergence: (i) numerical integration of Equations (38) and (39) is no longer required to evaluate  $\psi_0$  and  $\psi_2$ , and (ii) mathematical operations, such as differentiation and integration, can be performed analytically.

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