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DYNAMIC RESPONSE IN AN ELASTIC-PLASTIC PROJECTILE DUE TO NORMAL IMPACT

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**US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
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20. ABSTRACT (CONT'D)

analytical solution in order to evaluate accuracy and stability. Then, additional numerical results for perfectly-plastic materials are discussed in order to show the effect of strain-hardening. Finally, some results for a multi-linear material model based on two-dimensional elements are presented in order to show the lateral effect.

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INTRODUCTION

The propagation of elastic-plastic waves in long rods has been treated extensively in the literature (refs 1-3) since the pioneering works of Donell, Karman, Taylor, and Rakhmatulin. The study of plastic wave propagation is important because it attempts to explain the response of materials to intense dynamic loading and serves also as a basis for determining dynamic material properties.

Analytical solutions can be obtained for only a few idealized situations; hence, many impact studies have been performed using numerical methods. Many computer codes using either finite-element or finite-difference approaches have been developed. The computer simulation of impact phenomena in solids is still quite involved and it depends critically on the impact velocity. For high velocity impact and penetration problems, a good review was given by Zukas et al (ref 3). For low velocity contact-impact problems, many structural response codes were reviewed by Noor (ref 4).

A numerical study of the dynamic response of an elastic-plastic projectile due to normal impact is reported here using the finite element structural response code ADINA (ref 5). The projectile is a finite length

¹Cristescu, N., Dynamic Plasticity, John Wiley & Sons, Inc., New York, 1967.

²Nowacki, W. K., Stress Waves in Non-Elastic Solids, Pergamon Press, Oxford, 1978.

³Zukas, J. A., Nicholas, T., Swift, H. F., Greszczak, L. B., and Curran, D. R., Impact Dynamics, John Wiley & Sons, Inc., New York, 1982.

⁴Noor, A. K., "Survey of Computer Programs for Solution of Nonlinear Structural and Solid Mechanics Problems," Computer and Structures, Vol. 13, 1981, pp. 425-465.

⁵Bathe, K. J., "ADINA Users' Manual," Report AE-81-1, ADINA Engineering, Inc., Watertown, MA, 1981.

cylindrical bar made of a high strength steel. The bar is long and travels with velocity $V = 75$ m/s before it strikes a rigid target. First, three direct integration schemes have been used for the uniaxial stress wave problem in a linear-hardening material, and the results are compared with an exact analytical solution in order to evaluate the accuracy and stability. Then, additional numerical results for perfectly-plastic materials are discussed in order to show the effect of strain-hardening for a multi-linear material model. Finally, some results based on two-dimensional elements are presented in order to show the lateral effect.

ANALYTICAL SOLUTION

The problem of the normal impact of a rod against a rigid target has been considered by many authors. Various schemes have been used for different kinds of initial conditions and various material properties. For a linear work-hardening material due to sudden impact, an analytical solution for the uniaxial stress wave problem is available (ref 1) and it is presented here for comparison with the corresponding ADINA results. Thus, consider a bar of length L and diameter D that is moving with velocity V in the negative Z direction. At time 0 the bar strikes a rigid wall $Z = 0$ (Figure 1a). Guided by the stress-strain curve for a high strength steel supplied to us (ref 6), the following material data will be used:

$$\begin{aligned} E &= 208 \text{ GPa}, & \rho &= 0.783 \text{ g/cc}, & \nu &= 0.293 \\ \sigma_y &= 1.3 \text{ GPa}, & E_p &= 4 \text{ GPa}, \end{aligned} \tag{1}$$

¹Cristescu, N., *Dynamic Plasticity*, John Wiley & Sons, Inc., New York, 1967.
⁶Wright T. W., Private Communication, 1982.

where E , ρ , ν , σ_y , and E_p are Young's modulus, density, Poisson's ratio, yield stress, and plastic modulus, respectively. The elastic and plastic one-dimensional wave speeds will then be $c_e = \sqrt{E/\rho} = 5154$ m/s and $c_p = \sqrt{E_p/\rho} = 715$ m/s, respectively. The velocity V_y corresponding to the yield stress σ_y of the material is

$$V_y = \sigma_y/(\rho c_e) = c_e \epsilon_y = 32.21 \text{ m/s} \quad (2)$$

Both elastic and plastic waves will be generated if the impact velocity $V > V_y$. Let us consider the case $V = 75$ m/s, $L = 1.1$ m, $D = 0.1$ m. After impact, two shock wave fronts delimit three distinct regions in the bar (Figures 1b and 1c). The analytical solutions for the particle velocity, strain, and stress in these three regions are

$$\begin{aligned} V_0 &= -V = -75 \text{ m/s}, & \epsilon_0 &= 0, & \sigma_0 &= 0, \\ V_1 &= -V + V_y = -42.79 \text{ m/s}, & \epsilon_1 &= -\epsilon_y = -0.625\%, & \sigma_0 &= -\sigma_y = -1.3 \text{ GPa}, \\ V_2 &= 0, & \epsilon_2 &= -\epsilon_y + (V_y - V)/c_p = -6.610\% \\ \sigma_2 &= -\sigma_y + E_p(\epsilon_2 + \epsilon_y) = -1.539 \text{ GPa} . \end{aligned} \quad (3)$$

After time $t = L/c_e = 213$ μ s, the elastic wave front is reflected from the free end. Behind this front (Figure 1d), $\sigma_3 = \epsilon_3 = 0$, $V_3 = 2V_y - V$. At time $t_S = 2L/(c_e + c_p) = 374.85$ μ s, the wave fronts of the plastic and of the elastic unloading waves meet at the section S. The stress and velocity are continuous but the strain is discontinuous across this section S, a nonpropagable discontinuity surface. Since the inequality

$$2V_y < V < \left(1 + \frac{2c_e}{c_e + c_p}\right) V_y = 88.79 \text{ m/s} \quad (4)$$

is satisfied, the plastic wave stops at S and elastic waves again propagate from S in both directions (Figure 1e). The analytical solutions in regions 4

and 5 are

$$V_4 = V_5 = \frac{1}{2} \left(3 - \frac{c_e}{c_p} \right) (V_y - V) + V = 13.79 \text{ m/s}$$

$$\epsilon_4 = \frac{1}{2} (1 + c_p/c_e) (\epsilon_y - V/c_e) = -0.473\%$$

$$\epsilon_5 = - \left(2 \frac{c_e}{c_p} + 1 - \frac{c_p}{c_e} \right) (\epsilon_y - V/c_e) = -6.342\%$$

$$\sigma_4 = \sigma_5 = -0.983 \text{ GPa} \quad (5)$$

Figures 1b through 1e show the locations of the wave fronts at time $t = 100, 200, 300, 400 \text{ } \mu\text{s}$, respectively. The analysis can be continued until the contact between the bar and the target ceases at $t_c = 4L/(c_e + c_p) = 749.7 \text{ } \mu\text{s}$. More detailed information about the analytical solution can be found in the book by Cristescu (ref 1).

ADINA SOLUTION

The ADINA code, developed by K. J. Bathe, is a general purpose finite element program for Automatic Dynamic Incremental Nonlinear Analysis (ref 5). In nonlinear analysis the incremental finite element equations of motion used are, in implicit time integration,

$$\underline{M} \ddot{\underline{U}}_{t+\Delta t} + \underline{C} \dot{\underline{U}}_{t+\Delta t} + \underline{t_K} \underline{U} = \underline{t+\Delta t R} - \underline{t_F} \quad (6)$$

and in explicit time integration,

$$\underline{M} \ddot{\underline{U}} + \underline{C} \dot{\underline{U}} + \underline{t_K} \underline{U} = \underline{t_R} - \underline{t_F} \quad (7)$$

where $\underline{M}$, $\underline{C}$, $\underline{t_K}$, $\underline{t_R}$, $\underline{t+\Delta t R}$, $\underline{t_F}$ are constant mass matrix, constant damping matrix, tangent stiffness matrix at time t , external load vector applied at

¹Cristescu, N., Dynamic Plasticity, John Wiley & Sons, Inc., New York, 1967.

⁵Bathe, K. J., "ADINA Users' Manual," Report AE-81-1, ADINA Engineering, Inc., Watertown, MA, 1981.

time t , $t+\Delta t$, nodal point force vector equivalent to the element stresses at time t , respectively, and $\underline{U}$ is the vector of nodal point displacement increments from time t to time $t+\Delta t$, i.e., $\underline{U} = {}^{t+\Delta t}\underline{U} - {}^t\underline{U}$. The solution of Eq. (6) yields, in general, an approximate displacement increment $\underline{U}$. To improve the solution accuracy and in some cases to prevent the development of numerical instabilities, it may be necessary to use equilibrium iteration in each or preselected time steps.

In ADINA, the central difference method is employed for explicit time integration and either the Newmark method or Wilson method is employed for implicit time integration. The integration schemes (ref 7) are given by:

$${}^t\ddot{U} = \frac{1}{(\Delta t)^2} ({}^{t+\Delta t}U - 2{}^tU + {}^{t-\Delta t}U) \quad (8)$$

$${}^t\dot{U} = \frac{1}{2\Delta t} ({}^{t+\Delta t}U - {}^{t-\Delta t}U) \quad (9)$$

for the central difference method,

$${}^{t+\Delta t}\dot{U} = {}^t\dot{U} + [(1-\delta){}^t\ddot{U} + \delta{}^{t+\Delta t}\ddot{U}]\Delta t \quad (10)$$

$${}^{t+\Delta t}U = {}^tU + {}^t\dot{U}\Delta t + \left[\left(\frac{1}{2} - \alpha\right){}^t\ddot{U} + \alpha{}^{t+\Delta t}\ddot{U}\right](\Delta t)^2 \quad (11)$$

for the Newmark method, and

$${}^{t+\Delta t}\ddot{U} = {}^t\ddot{U} + \left(\frac{\tau}{\theta\Delta t}\right)({}^{t+\theta\Delta t}\ddot{U} - {}^t\ddot{U}) \quad (12)$$

$$\theta > 1, \quad 0 \leq \tau \leq \theta\Delta t$$

for the Wilson method.

⁷Bathe, K. J., Numerical Methods in Finite Element Analysis, Prentice-Hall, Inc., New Jersey, 1976.

The Wilson and Newmark methods are unconditionally stable if $\theta > 1.37$ or $\alpha > 1/4(1/2+\delta)$, $\delta > 1/2$. In our numerical study, we have chosen $\theta = 1.4$, $\alpha = 1/4$, $\delta = 1/2$. In using the central difference method, the time step, Δt , has to satisfy the Courant condition

$$\Delta t = \Delta t_{cr} = K\Delta l/c \quad \text{or} \quad \frac{2}{\omega} \quad (13)$$

where Δl is the minimum mesh size, ω is the maximum natural frequency, c is the local sound speed, and $K < 1$.

NUMERICAL COMPARISON

Consider a bar with the following geometrical and material data: $L = 1.1$ m, $D = 0.1$ m, $E = 208$ GPa, $\rho = 0.783$ g/cc, $\sigma_y = 1.3$ GPa, $E_p = 4$ GPa, subjected to two values of impact velocity: $V = 25$ m/s or 75 m/s. Since the velocity corresponding to the yield stress σ_y of the material is $V_y = 32.2$ m/s, the impact is elastic for the first case and elastic-plastic for the second case. Analytical solutions are known for both cases. We used 100 one-dimensional truss elements to simulate this uniaxial stress wave problem. In order to satisfy the stability criterion for explicit integration by the central difference method, we have chosen the time step $\Delta t = 2 \mu s$ which is less than the critical time step

$$\Delta t_{cr} = \Delta Z/c_e = 2.13 \mu s$$

Our study compares three integration schemes with the same time step. The computations are all stable, and the numerical results for the axial stress and velocity when $V = 25$ m/s are shown in Figures 2 through 5. Figure 2 shows results for the particle velocity and stress along the rod at $t = 100, 200, 300, 400, 500 \mu s$ when the Wilson method was used. Figure 3 shows the

similar results for the Newmark method. The numerical results based on these two methods are less accurate when compared with the results based on the central difference method. Figure 4 shows a comparison of the results for the particle velocity along the length of the bar based on the central difference method, the Newmark method, and the analytical solution at $t = 100, 200, 300, 400 \mu s$. A similar comparison between the central difference method, and the analytic solution for the axial stress along the bar is shown in Figure 5. It should be noted that the computations were performed under the assumption that the rod and target remain in contact after impact while the theoretical interval of contact is $t_c = 2L/c_e = 426 \mu s$.

Calculations were also performed for elastic-plastic impact with $V = 75$ m/s using the three integration schemes with the same mesh size and time step. A comparison of numerical results for the axial stress and velocity using the central difference and Newmark methods is shown in Figure 6 at $t = 200 \mu s$. The solid and dotted curves represent the results based on the central difference method and Newmark method, respectively. Similar results are shown in Figure 7 at $t = 400 \mu s$. As can be seen from these two figures, the central difference method gives more accurate results than the Newmark method. The numerical results based on the Wilson method are compared with those based on the central difference method in Figure 8. This also demonstrates that the numerical results based on the central difference method are more accurate. We may then conclude from this study on elastic as well as elastic-plastic impact that the central difference method will give more accurate results than the other two integration schemes.

HARDENING AND LATERAL EFFECTS

After reaching the above conclusion, we used the central difference method for the rest of this study. In order to show hardening effects, we obtained numerical results for displacement, velocity, strain, and stress in a long rod of an elastic-perfectly-plastic material. Figures 9 and 10 show the results of the particle velocity and stress for a linear work-hardening as well as a perfectly-plastic material at $t = 300$ and $400 \mu\text{s}$, respectively. It can be seen from these comparisons that the effect of strain-hardening on the particle velocity and stress is quite significant even though the plastic modulus $E_p = 4 \text{ GPa}$ is small when compared with the elastic modulus $E = 208 \text{ GPa}$.

In order to study lateral effects, we have used two-dimensional four-node quadrilateral ring elements to obtain numerical results. We chose the same mesh size $\Delta r = \Delta z = 0.011 \text{ m}$ and used 50 elements along the length of the bar with $L/D = 25$. In this arrangement, the new length of the bar is only half of the original. We have used the same time step $\Delta t = 2 \mu\text{s}$ as in the one-dimensional truss elements. This time step yields stable computations for one-dimensional truss elements but not for two-dimensional quadrilateral ring elements. This seems due to the lateral effect such as the Poisson's ratio. Including the effect of Poisson's ratio ($\nu = 0.293$), the speed of longitudinal elastic wave is $c_d = [(E/\rho)((1-\nu)/(1+\nu))/(1-2\nu)]^{1/2} = 5923 \text{ m/s}$, which reduces to $c_e = (E/\rho)^{1/2} = 5154 \text{ m/s}$ in case of $\nu = 0$. Guided by the stability criterion for linear elastic problems, we thus chose $\Delta t < \Delta t_{cr} = \Delta z/c_d = 0.011/5923 \text{ sec} = 1.857 \mu\text{s}$. For this reason, we carried out the computations for 250 time steps using $\Delta t = 1 \mu\text{s}$. The total number of time steps is the

same for both the one and two-dimensional problems. The length of the bar and time increment for the two-dimensional case are only half of the one-dimensional case. For the two-dimensional case, we have carried out the computations for impact velocities of $V = 25$ m/s and 75 m/s. The results for the elastic impact ($V = 25$ m/s) are not shown here. For elastic-plastic impact ($V = 75$ m/s), we have carried out the computations for a bilinear as well as a multi-linear material model. Seven points are used to represent the stress-strain curve, i.e., $(\sigma \text{ in GPa}, \epsilon \text{ in } \%) = (1.3, 0.63), (1.355, 1.03), (1.38, 1.83), (1.394, 2.63), (1.415, 4.23), (1.43, 5.83), (1.45, 8.63)$.

The numerical results for the case of a multi-linear material are shown in Figures 11 through 14. Figure 11 shows the effective stress and axial velocity along the length of the bar at the end of 50 and 100 time steps. Curves 1 and 2 represent the results at $t = 50$ and 100 μs , respectively. Figure 12 shows the axial stresses along the length of the bar. We have 2×2 stations to carry out the numerical integration in the spatial direction in each element. The results for the stresses are calculated at four integration stations. As can be expected from a long rod ($L/D = 25$), the differences in the results along the stations near the centerline or outside are very small. Figures 13 and 14 show the similar results for the effective stress, particle velocity, and axial stress along the length of the bar at $t = 150$ and 200 μs . As can be seen in these two figures, the axial stress in the plastic zone shows bigger oscillations than corresponding effective stress. Comparing the results shown in Figures 11 through 14 with the one-dimensional results shown in Figures 6 through 8, we conclude that the transition near the wave front is not as steep as the one-dimensional case and dispersion behind the wave front

can be observed. Some of the oscillations are real due to lateral effects such as radial inertia, radial shear, etc., but some are due to numerical errors such as truncation error in the finite element system, approximations in time integration schemes, numerical integration in the spatial directions, etc. It is difficult to identify how many of the oscillations are real and how many are due to numerical error. We hope to develop a numerical model to minimize the numerical error for this purpose while trying to improve the theoretical model.

CONCLUSION

Based on our numerical study of the uniaxial stress wave problem in a linear-hardening material, the central difference method gives more accurate results than the Wilson and Newmark methods. The effect of hardening is significant and lateral effects due to radial motion need further study. We plan to improve the theoretical model and to develop a numerical scheme. We hope to compare our numerical results with experiments involving normal impact of cylindrical rods.

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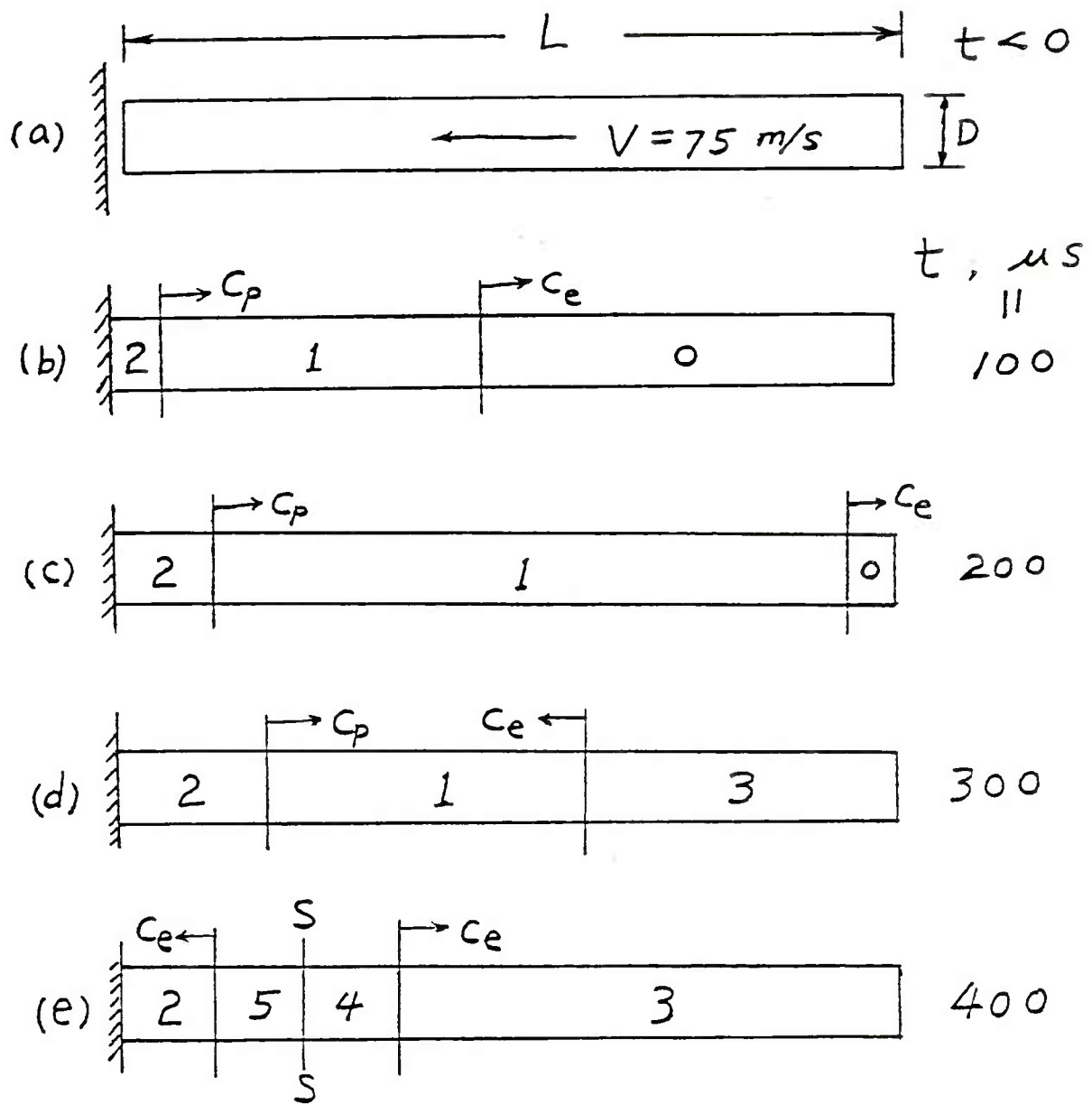


Figure 1. Analytical solution to a projectile striking a rigid target.

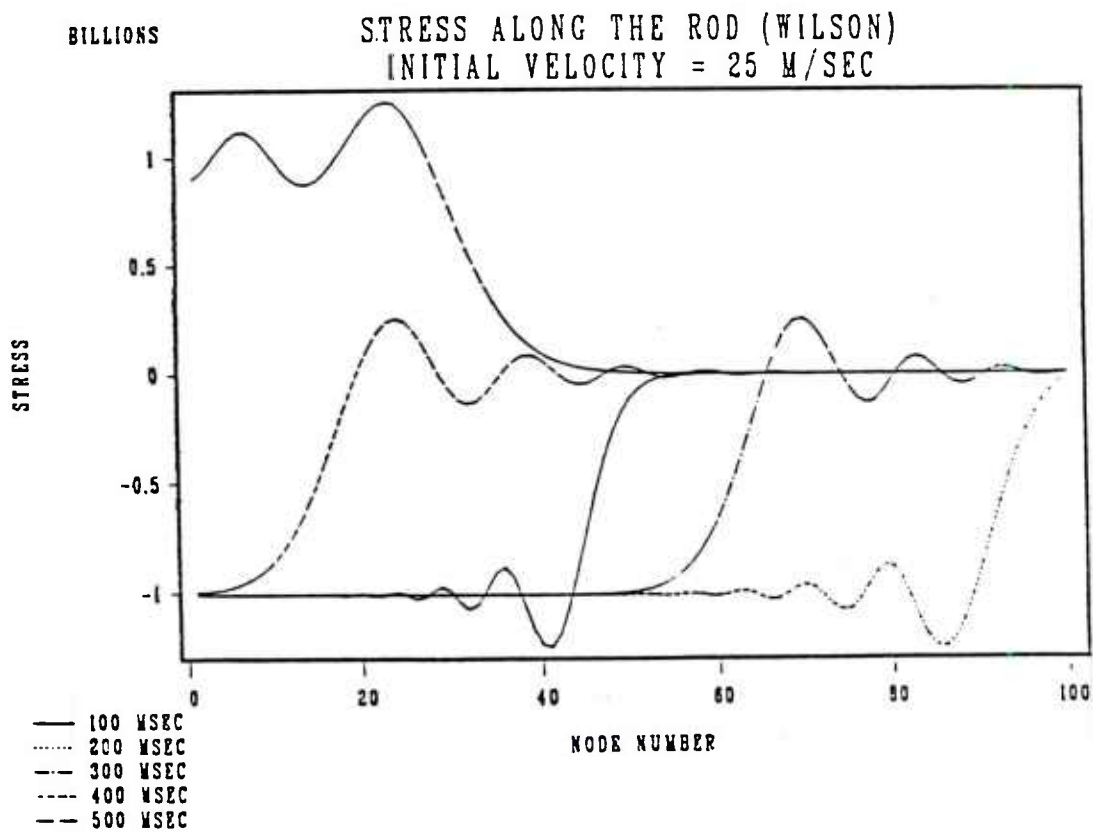
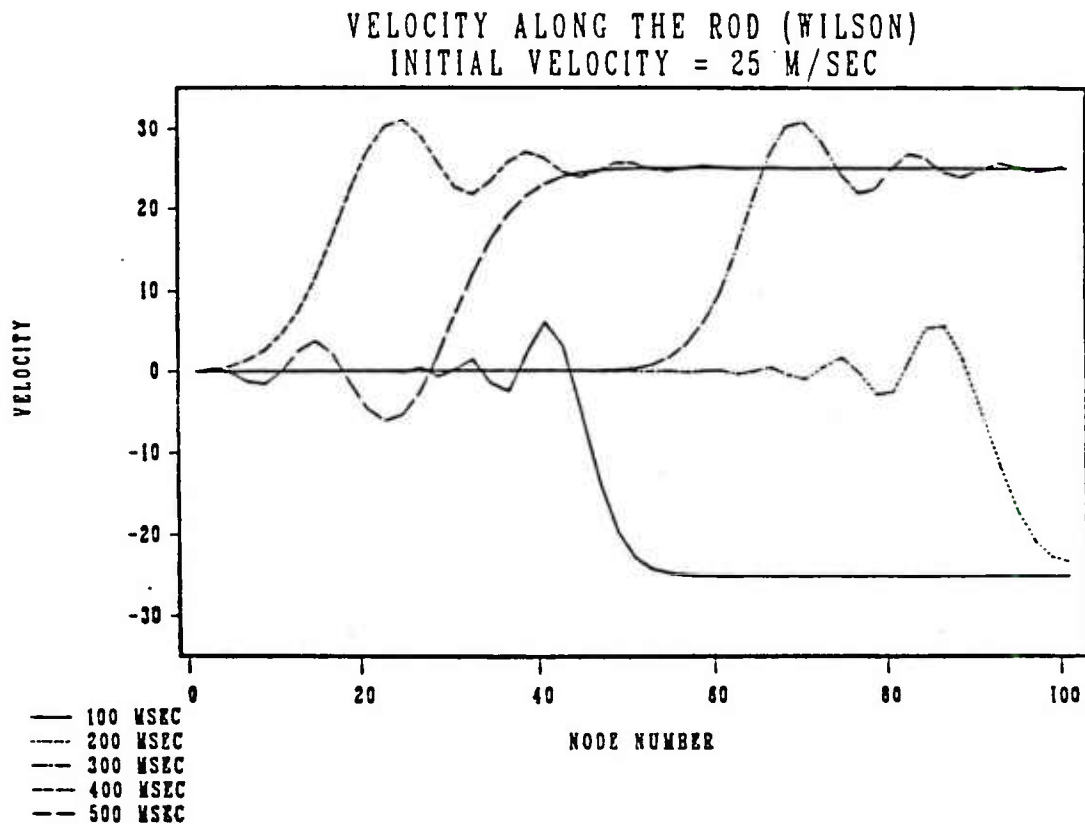
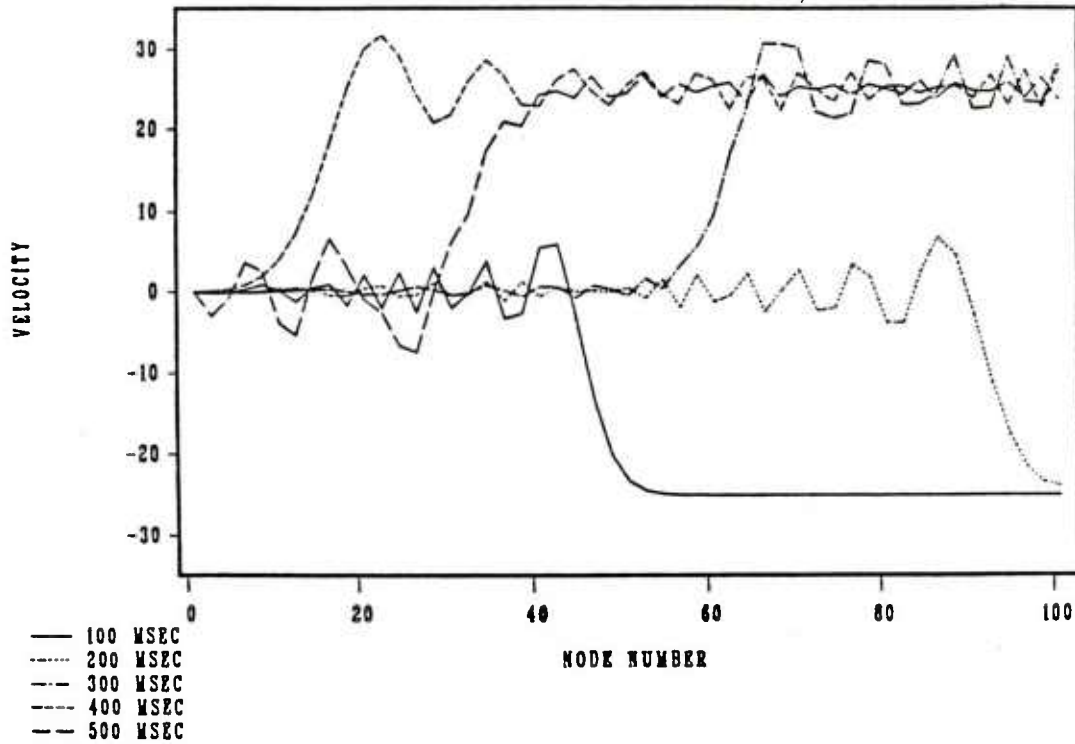


Figure 2. Axial velocity and stress based on the Wilson method ($V = 25$ m/s).

VELOCITY ALONG THE ROD (NEWMARK)
INITIAL VELOCITY = 25 M/SEC



BILLIONS

STRESS ALONG THE ROD (NEWMARK)
INITIAL VELOCITY = 25 M/SEC

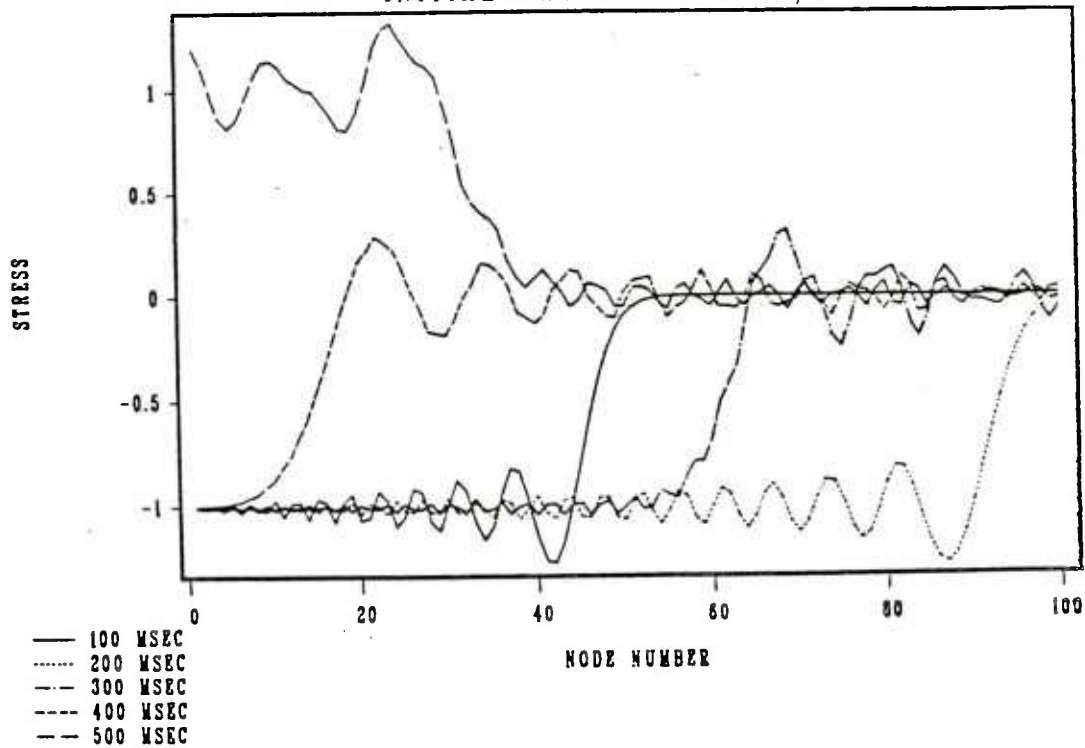


Figure 3. Axial velocity and stress based on the Newmark method ($V = 25$ m/s).

———— Central - - - - - Newmark

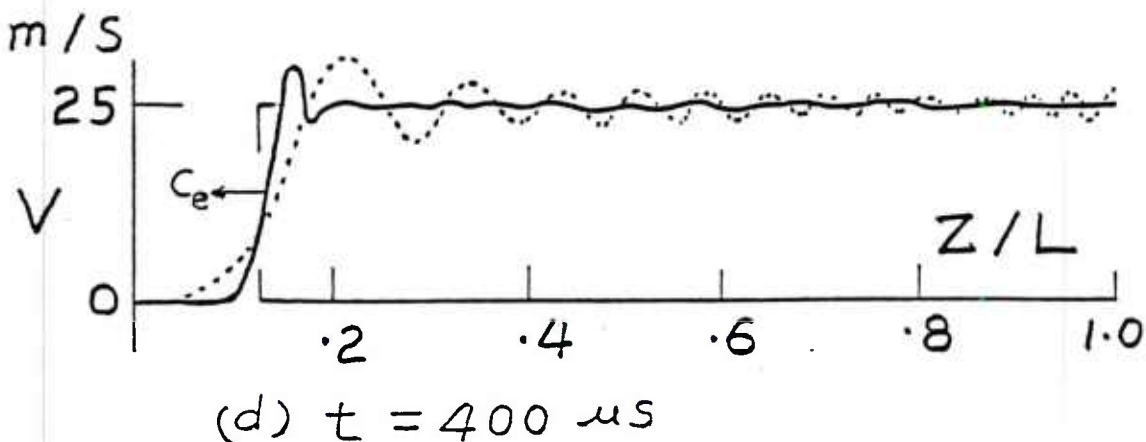
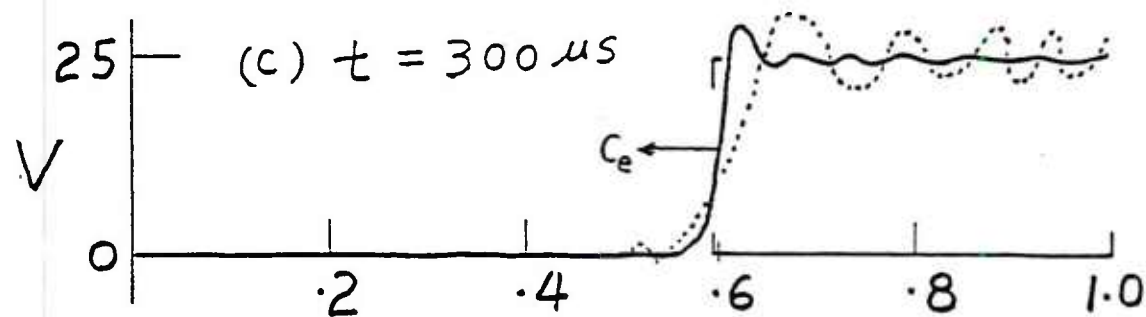
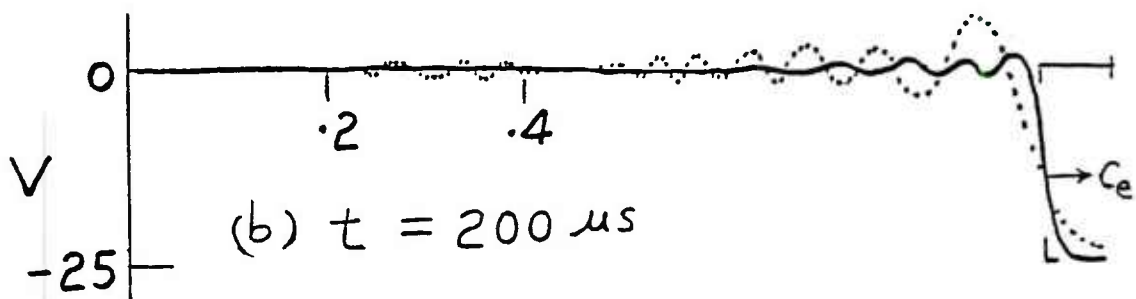
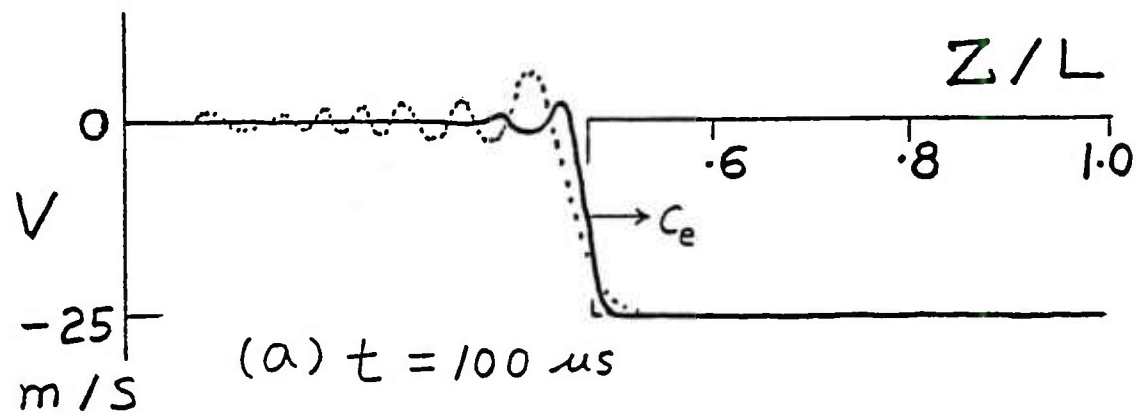


Figure 4. A comparison of the central difference method, the Newmark method, and the analytical solution for the particle velocity ($V = 25 \text{ m/s}$).

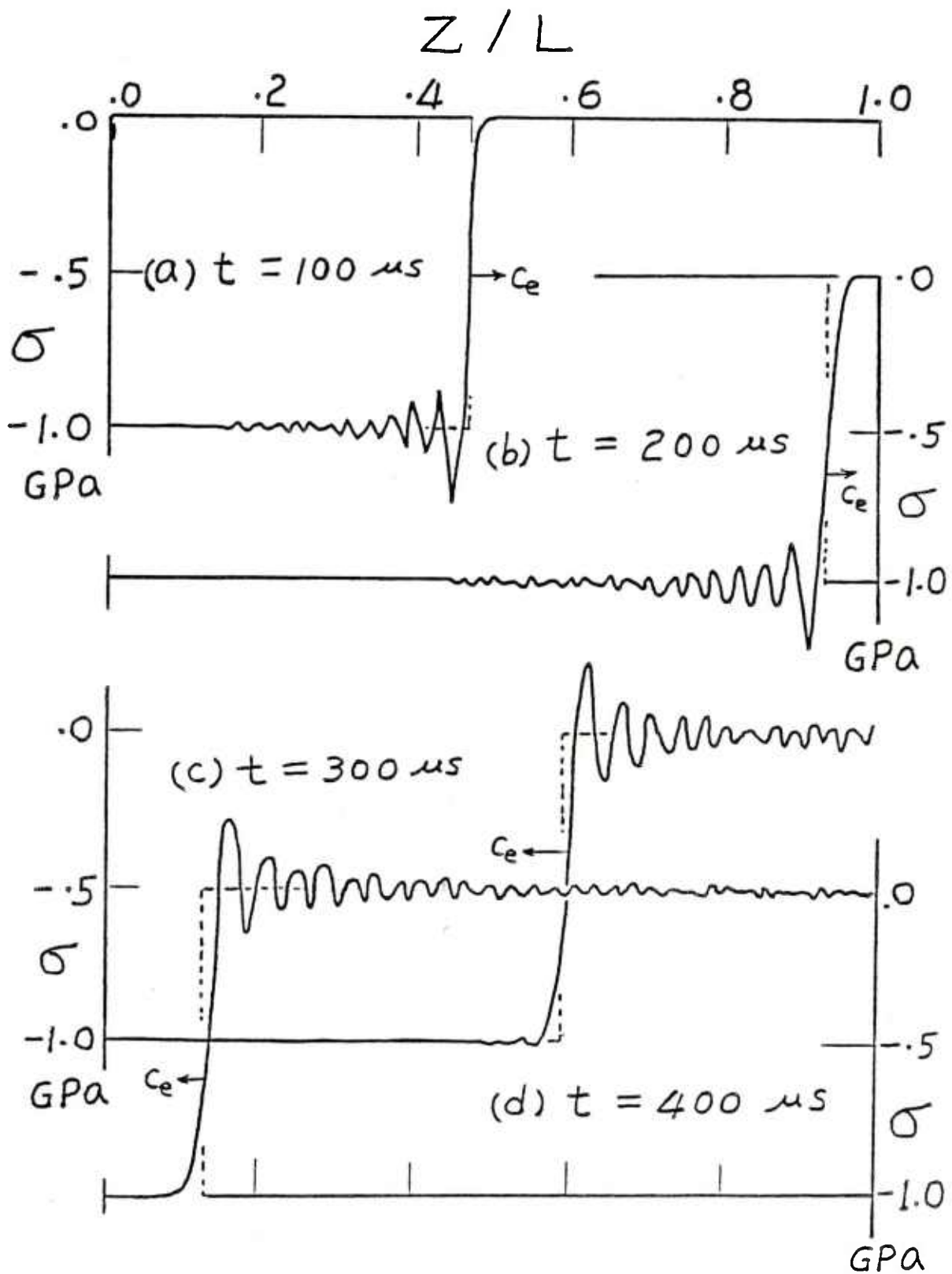


Figure 5. A comparison of the central difference method and the analytical solution for the axial stress ($V = 25$ m/s).

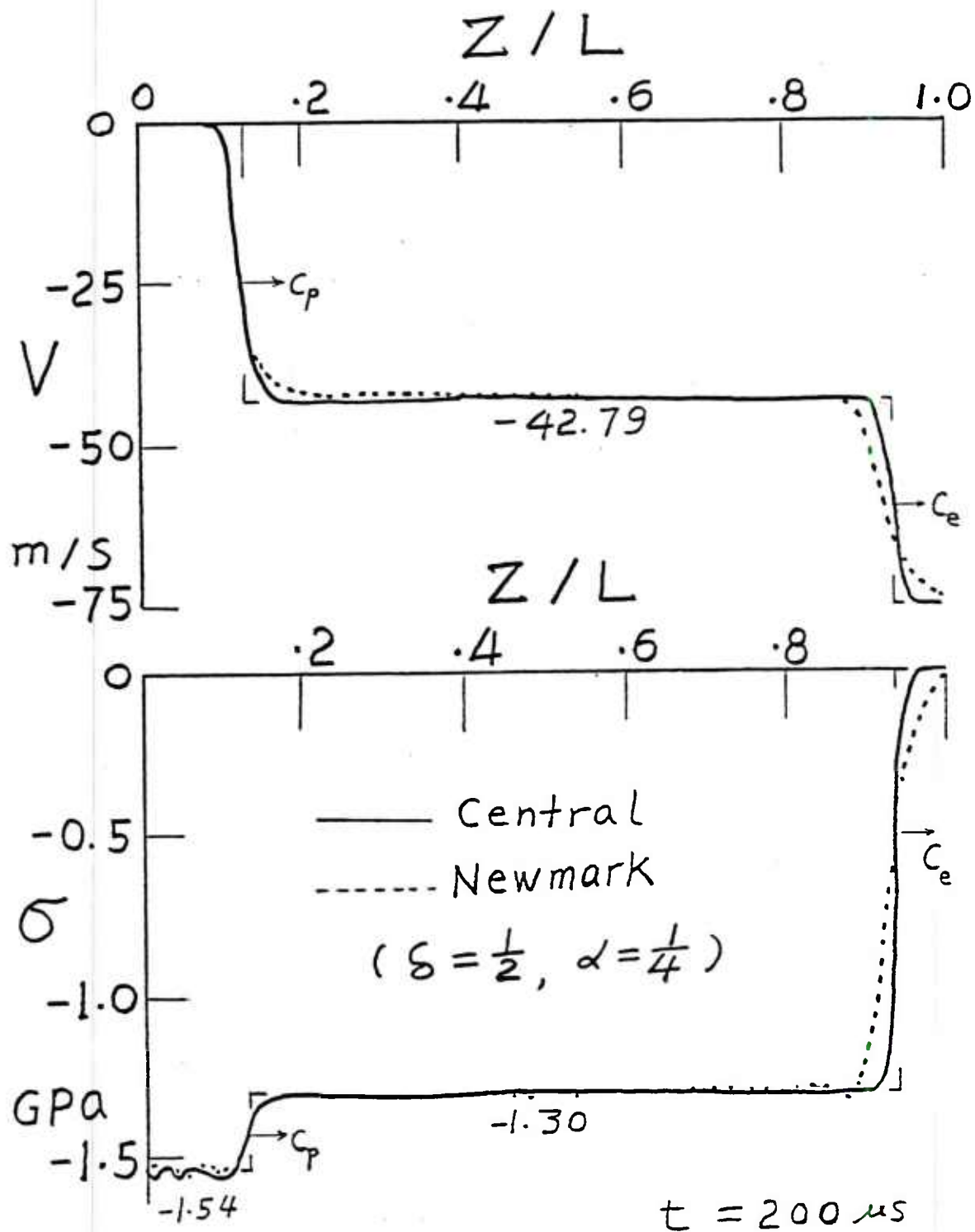


Figure 6. A comparison of the central difference method and the Newmark method for V and σ at $t = 200 \mu s$ ($V = 75 \text{ m/s}$).

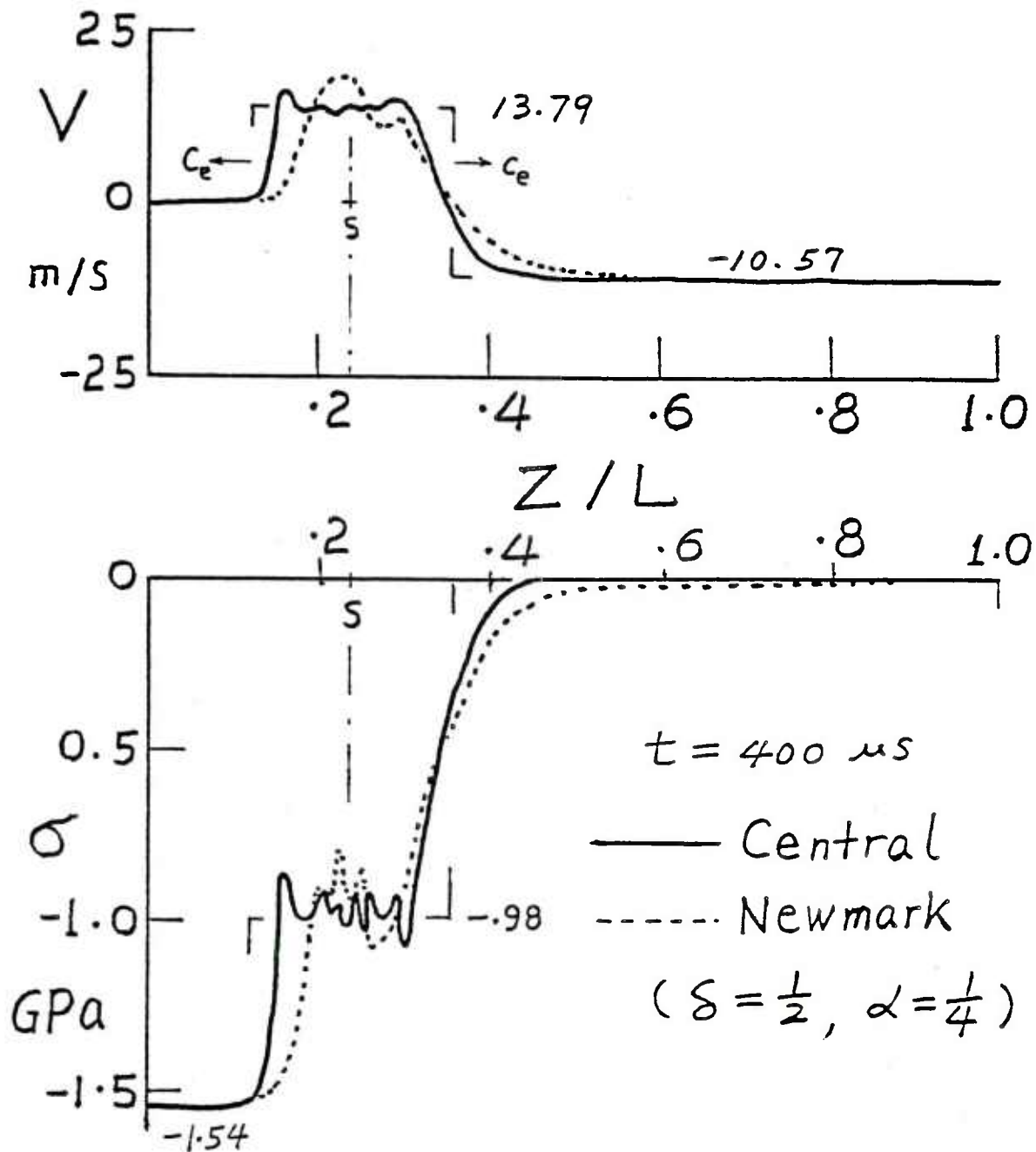


Figure 7. A comparison of the central difference method and the Newmark method for V and σ at $t = 400 \mu s$ ($V = 75 \text{ m/s}$).

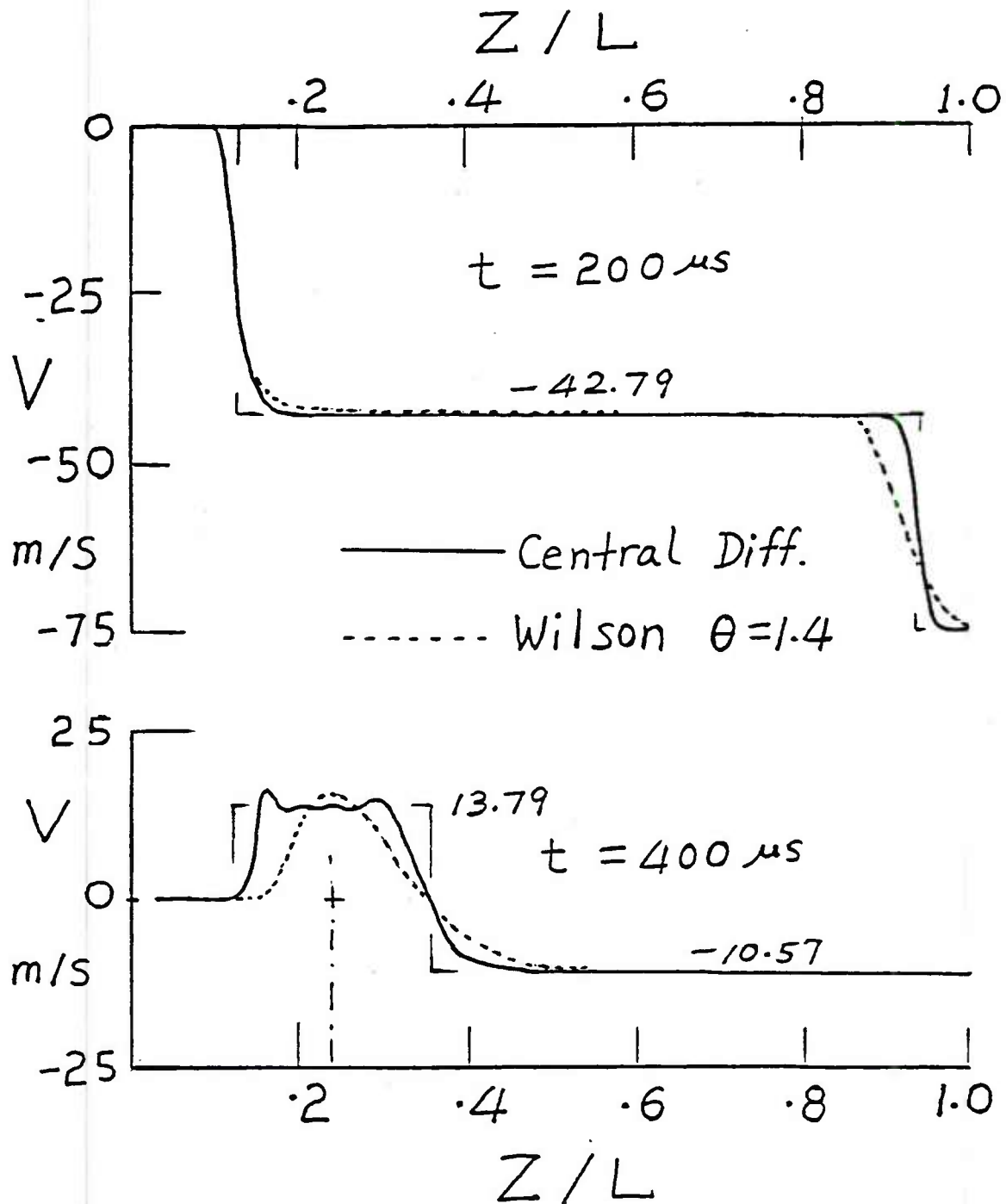


Figure 8. A comparison of the central difference method and the Wilson method for V at $t = 200$ and $400 \mu s$ ($V = 75$ m/s).

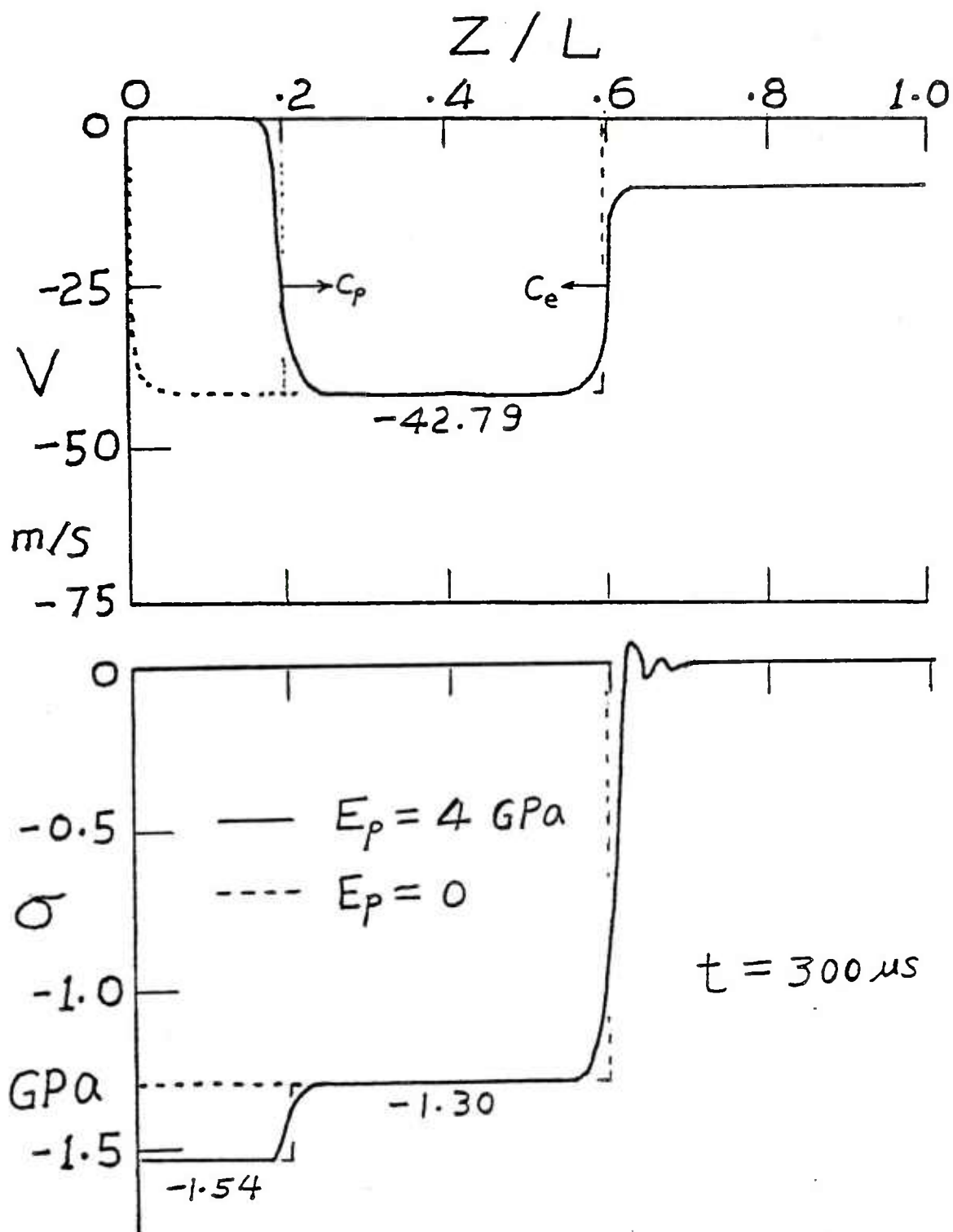


Figure 9. The effect of hardening on V and σ at $t = 300 \mu s$.

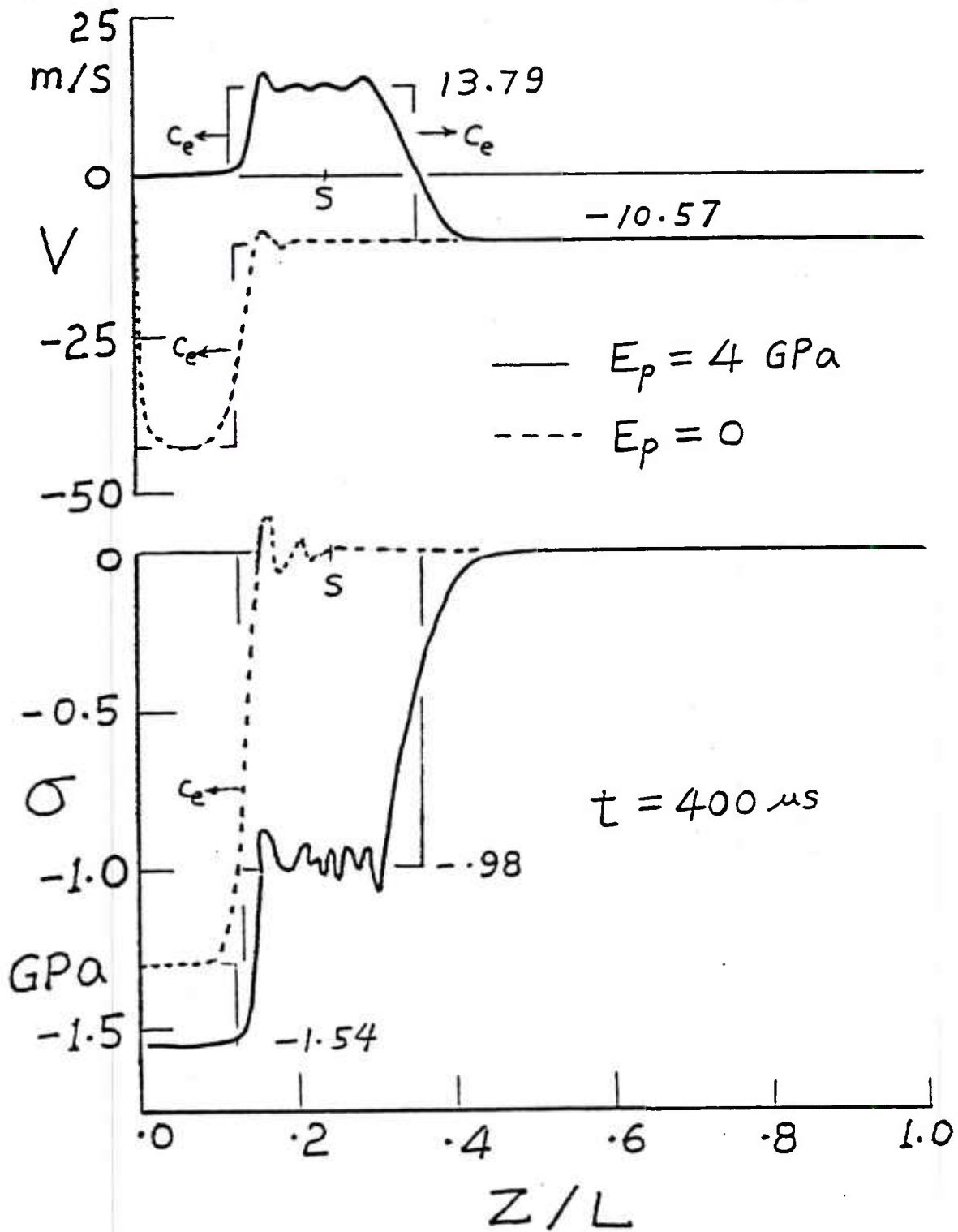


Figure 10. The effect of hardening on V and σ at $t = 400 \mu s$.

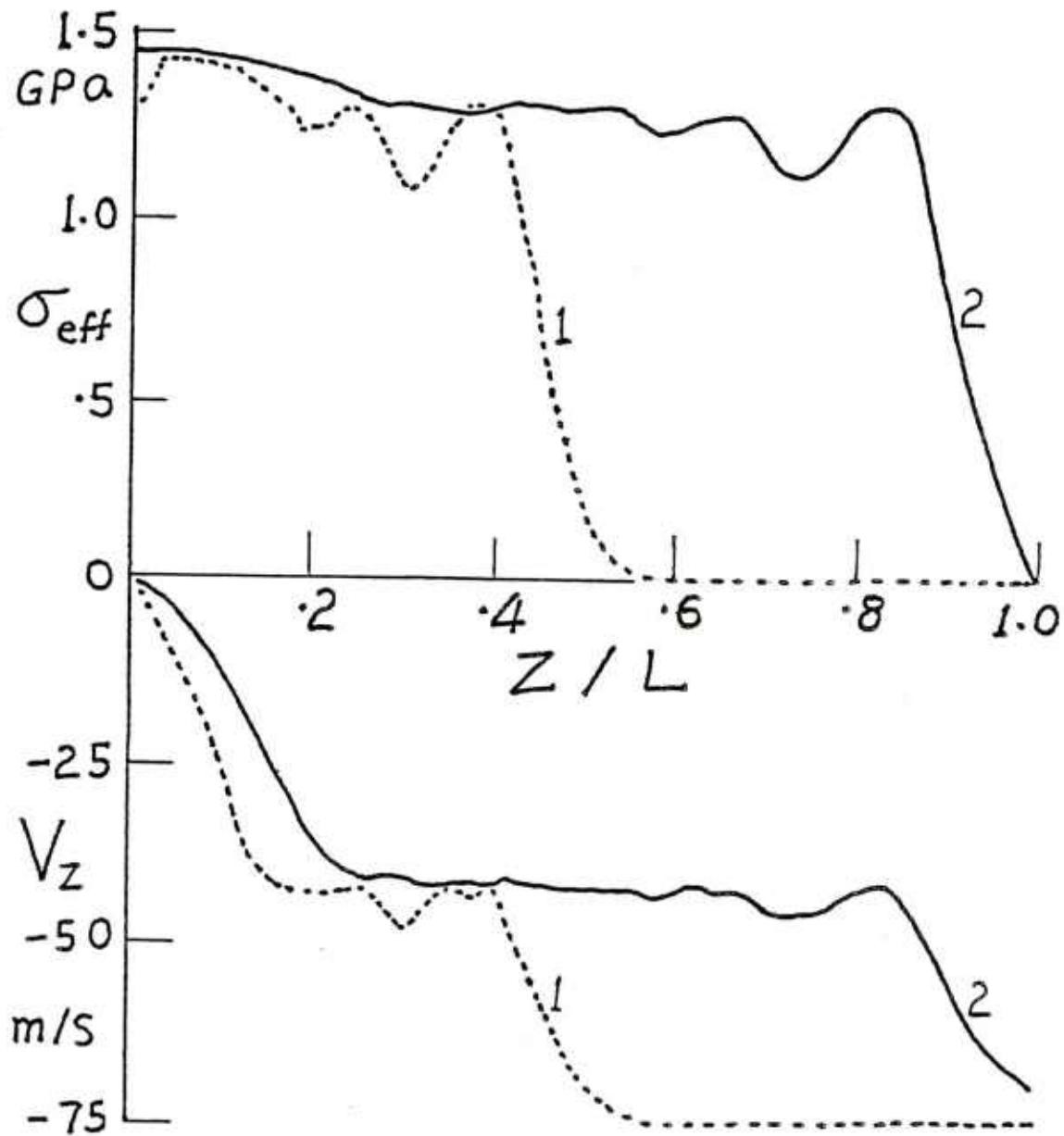


Figure 11. The effective stress and axial velocity based on 2-D elements at $t = 50$ and $100 \mu s$.

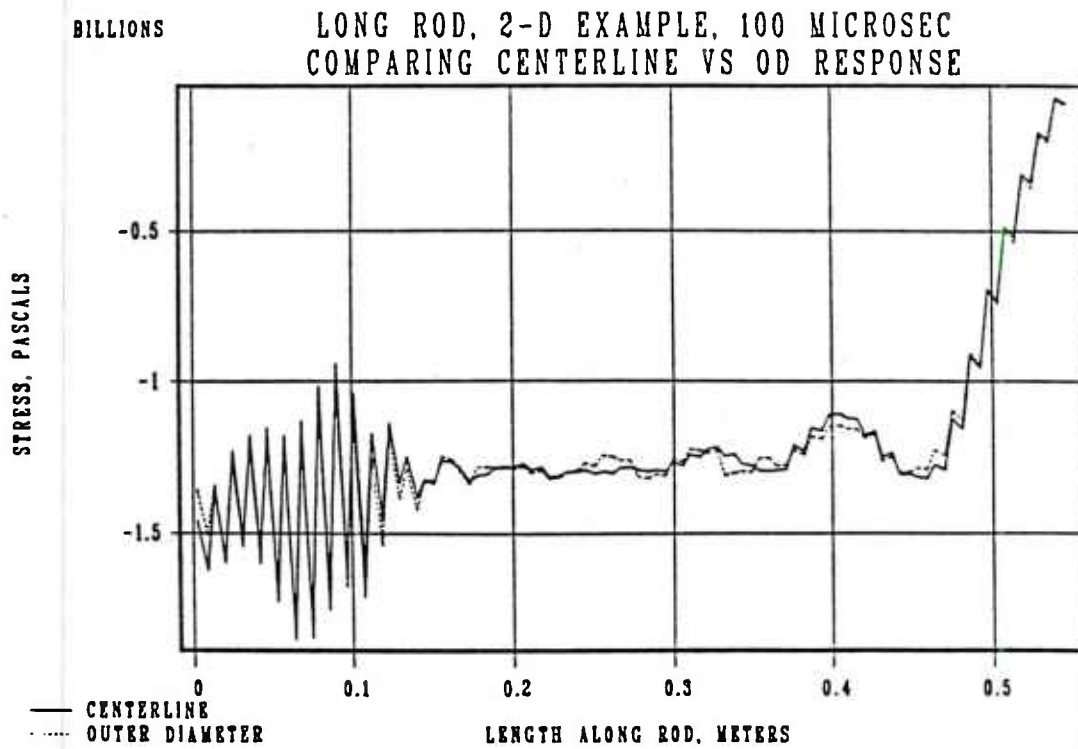
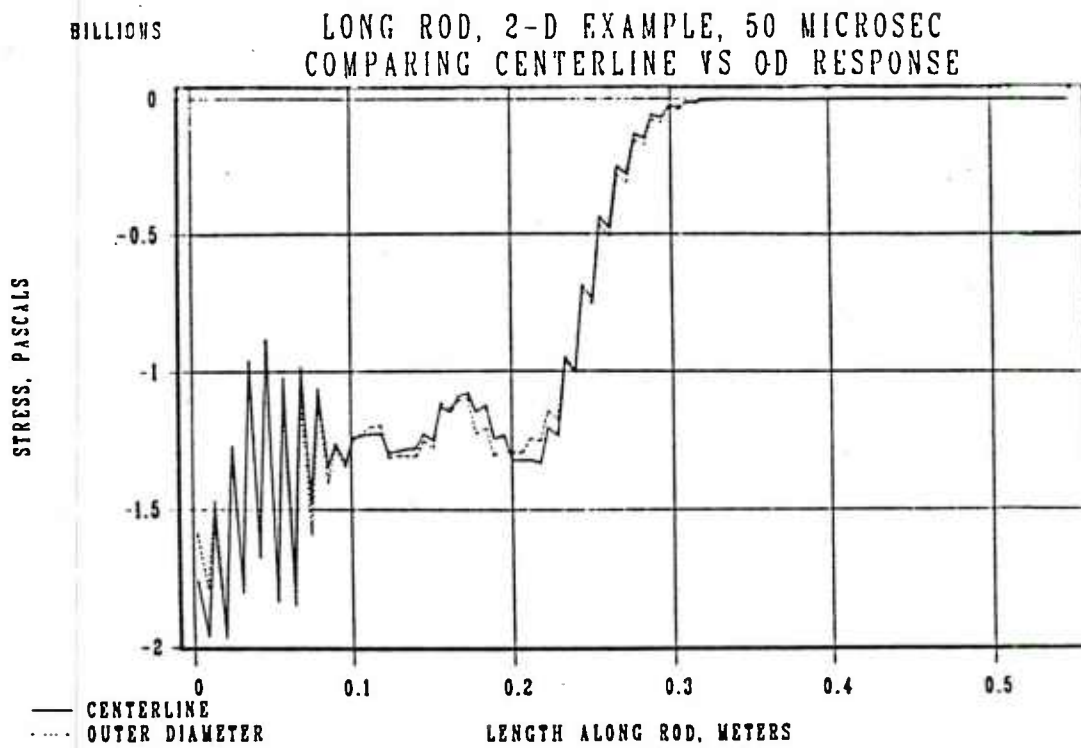


Figure 12. Axial stress (σ_z) based on 2-D elements at $t = 50$ and $100 \mu\text{s}$.

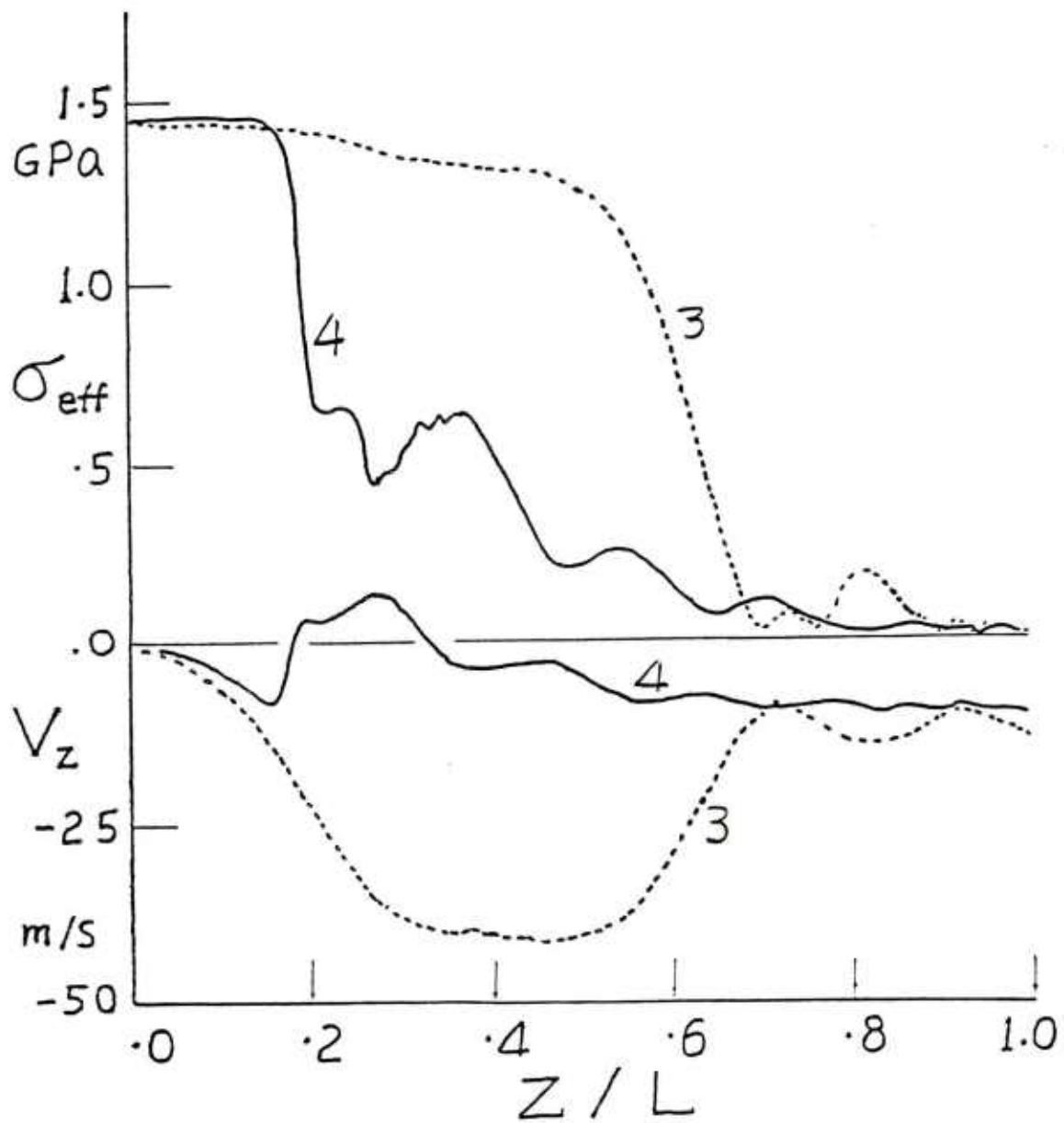


Figure 13. The effective stress and axial velocity based on 2-D elements at $t = 150$ and $200 \mu\text{s}$.

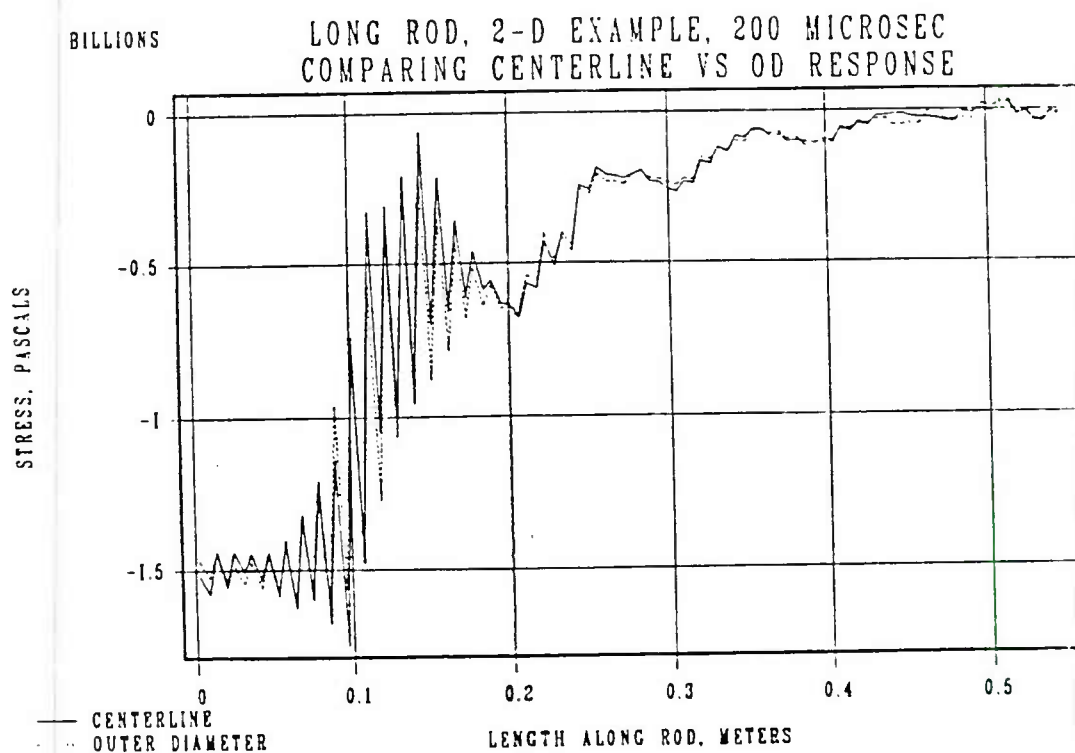
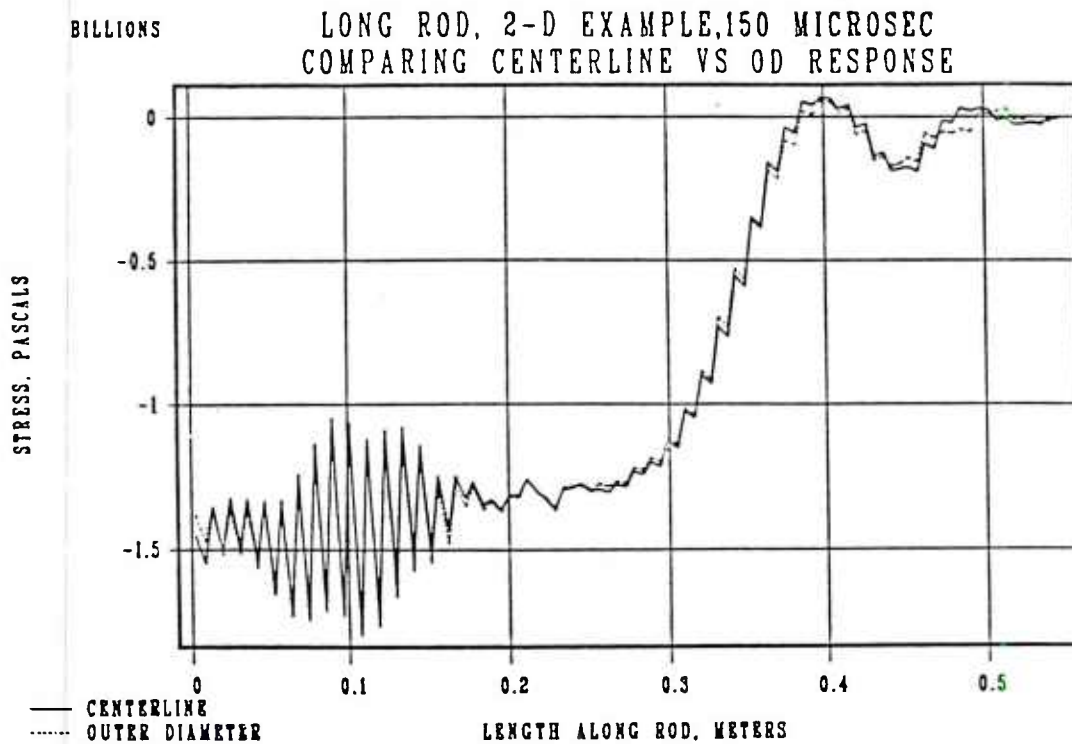


Figure 14. Axial stress (σ_z) based on 2-D elements at $t = 150$ and $200 \mu\text{s}$.

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