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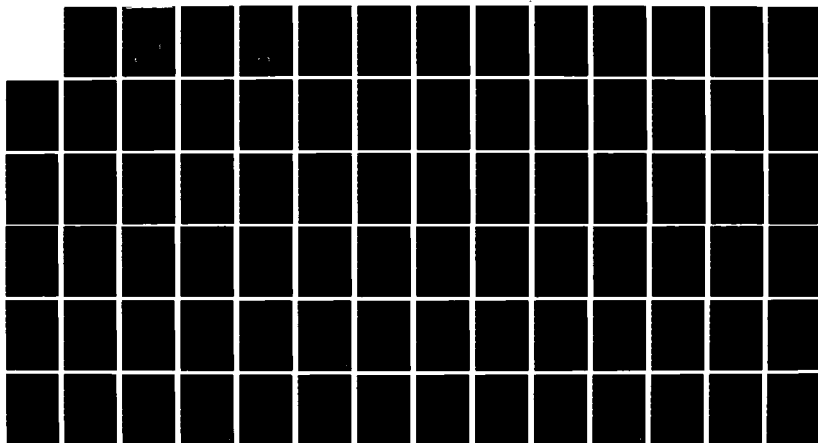
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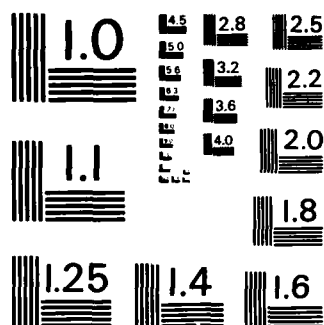
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GENERAL PERTURBATION SATELLITE THEORY

BY RUSSELL H. LYDDANE
EBELING ASSOCIATES, INC.

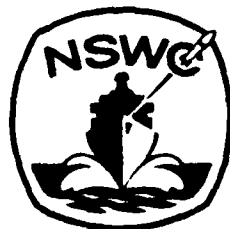
PREPARED FOR
SPACE AND SURFACE SYSTEMS DIVISION
STRATEGIC SYSTEMS DEPARTMENT

SEPTEMBER 1984

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FOREWORD

The work described in this report was performed by Dr. Russell H. Lyddane of Ebeling Associates, Inc., Scotia, New York. Given are the mathematical derivations and the complete FORTRAN Code.

The Lyddane program is estimated to operate at least two orders of magnitude faster than standard orbit integration techniques. It is eminently suitable for the computation of satellite orbits with moderate accuracy over long periods of time.

Released by:

Thomas A. Clare

THOMAS A. CLARE, Head
Strategic Systems Department

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INTRODUCTION

The General Perturbation Satellite (GPS) computer program was designed to conduct the integration of the equations of motion of an earth satellite under the influence of (1) an arbitrary zonal-harmonic gravitational field of the earth and (2) the gravitational attractions of the sun and moon. The zonal harmonics (as the program stands) go up to the twentieth, but additional terms can be added by changing a few parameters at the start of the program. There is no provision for tesseral harmonics. Sun and moon coordinates as functions of time are read from a data file that will not be described here, since its construction is outside the program.

The program employs Poincaré-like canonical variables and transforms them by general perturbation theory (von Zeipel) to remove first-order short-period terms (higher-order short-period terms are neglected). The equations in the mean variables thus produced are integrated numerically, with a large step size (set currently at 1/4 day). Since long-period terms receive no special treatment, there is nothing special about the critical angle; and since the variables are Poincaré-like, there are no difficulties at low eccentricities or inclinations.

A listing of the GPS program, in FORTRAN IV, is shown in Appendix A, with line numbers and with concordances for the various subdivisions. This is referenced as appropriate to relate derivations to the final program. The relationships among the subroutines and common variables are shown in Appendix B.

COORDINATES, ELEMENTS, AND CANONICAL VARIABLES

The basic Cartesian coordinate system employed is the nonrotating system with center at the earth's center, Z-axis along the north polar axis, epoch 1950.0. This is the system in which solar and lunar coordinates are also given. The coordinates of the satellite are x , y , z and the velocity components v_x , v_y , v_z .

The (osculating) elliptic elements are the usual set:

a = semi-major axis

e = eccentricity

i = inclination

M = mean anomaly

ω = argument of pericenter

Ω = argument of node

The auxiliary variables follow:

f = true anomaly

E = eccentric anomaly

$\tilde{\omega} = \omega + \Omega$ argument of longitude

$\alpha = E + \tilde{\omega}$ eccentric longitude

$u = f + \tilde{\omega}$ true longitude

The Delaunay canonical variables, used only as convenient intermediate variables, are also as usual:

$$L = \sqrt{\mu a} \qquad \ell = M$$

$$G = L \sqrt{1 - e^2} \qquad g = \omega$$

$$H = G \cos i \qquad h = \Omega$$

where μ is the gravitational constant. Usually ℓ , g , and h will be employed instead of M , ω , and Ω .

The canonical variables used in the program follow:

$$\begin{aligned} L & & \lambda &= \ell + \tilde{\omega} \\ \xi &= \sqrt{2(L-G)} \cos \tilde{\omega} & \eta &= -\sqrt{2(L-G)} \sin \tilde{\omega} \\ \sigma &= \sqrt{2(G-H)} \cos h & \tau &= -\sqrt{2(G-H)} \sin h \end{aligned}$$

Of course, λ is the mean longitude. Proof that this is indeed a canonical transformation is easily derived (Reference 1).

Several more intermediate quantities are convenient to use with these:

$$\xi = G/L = \sqrt{1-e^2} = 1 - (\xi^2 + \eta^2)/2L$$

$$e_c = e \cos \tilde{\omega} = \sqrt{\frac{1+\xi}{2L}} \xi$$

$$e_s = e \sin \tilde{\omega} = \sqrt{\frac{1+\xi}{2L}} (-\eta)$$

$$\rho = H/G = \cos i = 1 - (\sigma^2 + \tau^2)/2 \xi L$$

$$I_c = \sin i \cos h = \sigma \sqrt{(1+\rho)/2L\xi}$$

$$I_s = -\sin i \sin h = \tau \sqrt{(1+\rho)/2L\xi}$$

Details of the transformations between these sets of coordinates are given in Appendix C.

¹H. Goldstein, "Classical Mechanics"

MEAN ELEMENTS: THE VON ZEIPPEL TRANSFORMATION

The perturbing potential in the Hamiltonian is dominated by the contribution of the second zonal harmonic, whose coefficient is the small perturbing parameter (~ 0.001). All other terms in the perturbing potential are of the order of the square of this parameter. We expand the Hamiltonian in powers of the parameter:

$$F \approx F_0 + F_1 + F_2 \quad F_0 = \mu^2/2L^2$$

and introduce a determining function S expanded in the same way:

$$S = S_0 + S_1 + S_2 \quad \text{where } S_0 = L'\lambda + \xi'\eta + \sigma'\tau$$

and the primed quantities are the mean elements.

We will choose S so that the first-order short-period terms are transformed out. Higher-order short-period terms will be neglected, except for taking into account their effect on the initial value of L' , as described below.

$$\text{Since } F(L, \xi, \sigma, \lambda, \eta, \tau) = F^*(L', \xi', \sigma', -, \eta, \tau).$$

$$F_0 \left(\frac{\partial S}{\partial L} \right) + F_1 \left(\frac{\partial S}{\partial \lambda}, \frac{\partial S}{\partial \eta}, \frac{\partial S}{\partial \tau}, \lambda, \eta, \tau \right) + F_2 =$$

$$F_0^*(L') + F_1^* \left(L', \xi', \sigma', -, \frac{\partial S}{\partial \xi'}, \frac{\partial S}{\partial \sigma'} \right) + F_2^*$$

Expanding this in the usual Taylor series,

$$F_0^*(L') + \frac{\partial F_0^*}{\partial L}(L') \left(\frac{\partial S_1}{\partial \lambda} + \frac{\partial S_2}{\partial \lambda} \right) + \frac{1}{2} \frac{\partial^2 F_0^*}{\partial L^2}(L') \left(\frac{\partial S_1}{\partial \lambda} \right)^2 + F_1(L', \xi', \sigma', \lambda, \eta, \tau)$$

$$+ \frac{\partial F_1}{\partial L}(L', \xi', \sigma', \lambda, \eta, \tau) \frac{\partial S_1}{\partial \lambda} + \frac{\partial F_1}{\partial \xi} \frac{\partial S_1}{\partial \eta} + \frac{\partial F_1}{\partial \sigma} \frac{\partial S_1}{\partial \tau} + F_2 =$$

$$F_0^*(L') + F_1^*(L', \xi', \sigma', -, \eta, \tau) + \frac{\partial F_1^*}{\partial \eta} \frac{\partial S_1}{\partial \xi} + \frac{\partial F_1^*}{\partial \tau} \frac{\partial S_1}{\partial \sigma} + F_2^*.$$

This can be separated into orders:

$$0: F_0^* = F_0(L') = \mu^2/2L'^2$$

$$\frac{\partial F_0}{\partial L}(L') = -\mu^2/L'^3 \quad \frac{\partial^2 F_0}{\partial L^2} = 3\mu^2/L'^4$$

$$1: -(\mu^2/L'^3) \frac{\partial S_1}{\partial \lambda} + F_1 = F_1^*$$

Define $F_1 = F_{1s} + F_{1p}$; that is, F_{1s} is the secular and F_{1p} the periodic part. Then,

$$F_1^* = F_{1s}$$

$$\frac{\partial S_1}{\partial \lambda} = \frac{L'^3}{\mu^2} F_{1p}$$

The solution is exactly that of Brouwer (Reference 2); we shall return to it.

$$2: -\frac{\mu^2}{L'^3} \frac{\partial S_2}{\partial \lambda} + \frac{3}{2} \frac{\mu^2}{L'^4} \left(\frac{\partial S_1}{\partial \lambda} \right)^2 + \frac{\partial F_{1s}}{\partial L} \frac{\partial S_1}{\partial \lambda} + \frac{\partial F_{1s}}{\partial \xi} \frac{\partial S_1}{\partial \eta} + \frac{\partial F_{1s}}{\partial \sigma} \frac{\partial S_1}{\partial \tau} \\ + \frac{\partial}{\partial L} \left(\frac{\mu^2}{L'^3} \frac{\partial S_1}{\partial \lambda} \right) \frac{\partial S_1}{\partial \lambda} + \frac{\mu^2}{L'^3} \frac{\partial^2 S_1}{\partial \xi \partial \lambda} \frac{\partial S_1}{\partial \eta} + \frac{\mu^2}{L'^3} \frac{\partial^2 S_1}{\partial \sigma \partial \lambda} \frac{\partial S_1}{\partial \tau} + F_2 =$$

$$\frac{\partial F_{1s}}{\partial \eta} \frac{\partial S_1}{\partial \xi} + \frac{\partial F_{1s}}{\partial \tau} \frac{\partial S_1}{\partial \sigma} + F_2^*$$

F_2^* is obtained by averaging this equation over λ to remove short-period terms. It consists of the secular part of the summed contributions of the zonal harmonics, omitting the second, plus the solar and lunar contributions, plus the second-order contribution from the second zonal harmonic, which is the same as given in Reference 2, translated into the canonical variables. F_2 is the instantaneous contribution of the perturbing potential, except for the second harmonic.

² Dirk Brouwer, A. J. 64, 378 (1959)

$$\begin{aligned} \text{Since } L &= \frac{\partial S}{\partial \lambda} = L' + \frac{\partial S_1}{\partial \lambda} (L', \xi', \sigma', \lambda, \eta, \tau) + \frac{\partial S_2}{\partial \lambda} \\ &= L' + \frac{\partial S_1}{\partial \lambda} (L, \xi, \sigma, \lambda, \eta, \tau) - \frac{\partial^2 S_1}{\partial L \partial \lambda} \frac{\partial S_1}{\partial \lambda} - \frac{\partial^2 S_1}{\partial \xi \partial \lambda} \frac{\partial S_1}{\partial \eta} - \frac{\partial^2 S_1}{\partial \sigma \partial \lambda} \frac{\partial S_1}{\partial \tau} + \frac{\partial S_2}{\partial \lambda} \end{aligned}$$

we can solve the second-order part of the Hamiltonian above:

$$\begin{aligned} \frac{\partial S_2}{\partial \lambda} - \frac{\partial^2 S_1}{\partial L \partial \lambda} \frac{\partial S_1}{\partial \lambda} - \frac{\partial^2 S_1}{\partial \xi \partial \lambda} \frac{\partial S_1}{\partial \eta} - \frac{\partial^2 S_1}{\partial \sigma \partial \lambda} \frac{\partial S_1}{\partial \tau} &= - \frac{3}{2L} \left(\frac{\partial S_1}{\partial \lambda} \right)^2 \\ + \frac{L^3}{\mu^2} \left(F_2 - F_2^* + \frac{\partial F_{1s}}{\partial L} \frac{\partial S_1}{\partial \lambda} + \frac{\partial F_{1s}}{\partial \xi} \frac{\partial S_1}{\partial \eta} + \frac{\partial F_{1s}}{\partial \sigma} \frac{\partial S_1}{\partial \tau} \right. \\ &\quad \left. - \frac{\partial F_{1s}}{\partial \eta} \frac{\partial S_1}{\partial \xi} - \frac{\partial F_{1s}}{\partial \tau} \frac{\partial S_1}{\partial \sigma} \right) \end{aligned}$$

Substitute in the equation for L :

$$\begin{aligned} L &= L' + \frac{\partial S_1}{\partial \lambda} \left(1 - \frac{3}{2L} \frac{\partial S_1}{\partial \lambda} + \frac{L^3}{\mu^2} \frac{\partial F_{1s}}{\partial L} \right) + \frac{L^3}{\mu^2} \left(F_2 - F_2^* + \frac{\partial F_{1s}}{\partial \xi} \frac{\partial S_1}{\partial \eta} \right. \\ &\quad \left. + \frac{\partial F_{1s}}{\partial \sigma} \frac{\partial S_1}{\partial \tau} - \frac{\partial F_{1s}}{\partial \eta} \frac{\partial S_1}{\partial \xi} - \frac{\partial F_{1s}}{\partial \tau} \frac{\partial S_1}{\partial \sigma} \right) \end{aligned}$$

Now F_{1s} can be readily written [from Brouwer (Reference 2, Equation 13)] in our variables:

$$F_{1s} = (\mu^4 R_c^2 J_2 / 4 L^6 \xi^2) (-1 + 3 \rho^2)$$

From Reference 2, Equation 15:

$$\begin{aligned} S_1 &= (6 \mu^2 R_c^2 J_2 / (2L \xi)^3) \left(\frac{1}{3} (-1 + \rho^2) U_Q \right. \\ &\quad \left. + ((1 + \rho) / 4 L \xi) ((\sigma^2 - \tau^2) S_{up} + 2\sigma\tau C_{up}) \right) \end{aligned}$$

where

$$U_{\ell} = f - \ell + e \sin f$$

$$S_{up} = \sin 2u + e \sin (u + \tilde{\omega}) + (e/3) \sin (3u - \tilde{\omega})$$

$$C_{up} = \cos 2u + e \cos (u + \tilde{\omega}) + (e/3) \cos (3u - \tilde{\omega})$$

Appendix D contains the tedious details of the calculation of the partial derivatives of S_1 to determine the van Zeipel transformation.

$$L' = L - \frac{\partial S_1}{\partial \lambda} \quad \lambda' = \lambda + \frac{\partial S_1}{\partial L}$$

$$\xi' = \xi - \frac{\partial S_1}{\partial \eta} \quad \eta' = \eta + \frac{\partial S_1}{\partial \xi}$$

$$\sigma' = \sigma - \frac{\partial S_1}{\partial \tau} \quad \tau' = \tau + \frac{\partial S_1}{\partial \sigma}$$

When and only when the first entry is made to the system, it is necessary to compute L' (the initial value) to the second order, in order that $n_0 = \mu^2/L'^3$ does not contain a second-order error (Reference 3). For this case, evaluation of the derivatives of F_{1s} in the equation above leads to

$$\begin{aligned} L = & L' + \frac{\partial S_1}{\partial \lambda} \left(1 - \frac{3}{2L} \frac{\partial S_1}{\partial \lambda} + \gamma'_2 (\xi (1 - 3\rho^2) + 1 + 2\rho - 5\rho^2) \right) \\ & + \frac{L^3}{\mu^2} (F_2 - F_2^*) + \gamma'_2 (1 + 2\rho - 5\rho^2) \left(\eta \frac{\partial S_1}{\partial \xi} - \xi \frac{\partial S_1}{\partial \eta} \right) \\ & + \gamma'_2 \cdot 2\rho \left(\tau \frac{\partial S_1}{\partial \sigma} - \sigma \frac{\partial S_1}{\partial \tau} \right) \end{aligned}$$

Note that this equation is required only for the initial transformation from osculating to mean variables and that in it the derivatives of S are to be evaluated at the point $(L, \xi, \sigma, \lambda, \eta, \tau)$. Any other of the transformations from mean to osculating variables and back again involves only the first derivatives of S_1 , and these may be evaluated indifferently in terms of mean or osculating variables, since the difference is second-order short-period and hence to be neglected.

³R. H. Lyddane and C. J. Cohen, A. J. 67, 176 (1962)

Details of the calculation of the second-order contribution of L' are found in Appendix E.

THE HAMILTONIAN IN TERMS OF MEAN VARIABLES

From this point on, all quantities will be in terms of mean variables and the primes on the variables will be omitted for simplicity of writing. The next task is to average the Hamiltonian over the short period and to put the results into a form suitable for calculation.

ZONAL HARMONICS

We set aside for later consideration the secular second-order contribution of the second zonal harmonic and write F_1^* plus the zonal-harmonic portion of F_2^* as

$$\Delta F_Z = -(\mu/a) \sum_{n=2}^{\infty} J_n (R_e/a)^n \overline{(a/r)^{n+1}} P_n(\cos \Theta)$$

This comes from the standard expansion of an arbitrary axially-symmetric gravitational field into spherical harmonics, which is valid as long as $r > R_e$. The bar indicates the average over λ (removal of short-period terms), and Θ is the colatitude of the satellite, that is, the angle between the unit vector to the satellite $\hat{r}$ and the unit vector along the z -axis $\hat{r}_z$:

$$\cos \Theta = \hat{r} \cdot \hat{r}_z$$

The sum is terminated with the highest harmonic to be considered.

The addition theorem for the spherical harmonics,

$$P_n(\cos \Theta) = \sum_{m=-n}^n (2 - \delta_{m0}) \frac{(n-m)!}{(n+m)!} P_n^m(\cos \theta_1) P_n^m(\cos \theta_2) \cos m(\varphi_1 - \varphi_2),$$

where $\cos \Theta = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\varphi_1 - \varphi_2)$, may be conveniently applied by inserting the representations of $\hat{r}$ and $\hat{r}_z$ in the $\hat{L}_1 \hat{L}_2 \hat{h}$ frame, namely,

$$\hat{r} = \theta_1 = \pi/2 \quad \varphi_1 = u$$

```

(0119) C
(0120) SUBROUTINE ENTER(KEY)
(0121) C
(0122) C KEY=1: TRANSFORM FROM COORDINATES TO OSCULATING ELEMENTS
(0123) C KEY=2: TRANSFORM FROM ELLIPTIC ELEMENTS TO OSCULATING ELEMENTS
(0124) C
(0125) DOUBLE PRECISION X,Y,Z,VX,VY,VZ,XI,ETA,SIGMA,TAU,L,LAMBDA,ZETA,
(0126) + L2Z,RHO,C1,C2,ECLON,U,COSU,SINU,ECOSF,ESINF,GAM2P,R,Z9,Z8,IC,IS,
(0127) + HM,EC,ES,A,ECC,INC,ELL,G,H,DRAD,MU
(0128) COMMON /COREL/X,Y,Z,VX,VY,VZ,A,ECC,INC,ELL,G,H/INTEL/ETA,XI,
(0129) + TAU,SIGMA,LAMBDA,L,ZETA,L2Z,RHO,C1,C2,ECLON,U,COSU,SINU,ECOSF,
(0130) + ESINF,EC,ES,GAM2P/COEF/DRAD,MU
(0131) C
(0132) IF(KEY.EQ.1) GOTO 100
(0133) C
(0134) L=DSQRT(MU*A)
(0135) Z8=(G+H)/DRAD
(0136) LAMBDA=ELL/DRAD+Z8
(0137) ZETA=DSQRT(1-ECC*ECC)
(0138) Z9=DSQRT(2*L*(1-ZETA))
(0139) XI=Z9*DCOS(Z8)
(0140) ETA=-Z9*DSIN(Z8)
(0141) Z9=2*DSIN(INC/DRAD/2)*DSQRT(L*ZETA)
(0142) SIGMA=Z9*DCOS(H/DRAD)
(0143) TAU=-Z9*DSIN(H/DRAD)
(0144) CALL INTERM
(0145) GOTO 200
(0146) C
(0147) 100 IS=-(Y*VZ-Z*VY)
(0148) IC=-(Z*VX-X*VZ)
(0149) RHO=X*VY-Y*VX
(0150) HM=DSQRT(IS**2+IC**2+RHO**2)
(0151) IS=IS/HM
(0152) IC=IC/HM
(0153) RHO=RHO/HM
(0154) R=DSQRT(X**2+Y**2+Z**2)
(0155) Z9=(X*IS+Y*IC)/(1+RHO)-Z
(0156) U=DATAN2(Y-IC*Z9,X-IS*Z9)
(0157) COSU=DCOS(U)
(0158) SINU=DSIN(U)
(0159) ECOSF=HM*HM/(MU*R)-1
(0160) ESINF=(X*VX+Y*VY+Z*VZ)*HM/(MU*R)
(0161) ZETA=DSQRT(1-ECOSF**2-ESINF**2)
(0162) EC=ECOSF*COSU+ESINF*SINU
(0163) ES=ECOSF*SINU-ESINF*COSU
(0164) Z9=ESINF/(1+ZETA)
(0165) ECLON=DATAN2(SINU+ES-EC*Z9,COSU+EC+ES*Z9)
(0166) LAMBDA=ECLON-EC*DSIN(ECLON)+ES*DCOS(ECLON)
(0167) L=HM/ZETA
(0168) C1=DSQRT((1+ZETA)/(2*L))
(0169) XI=EC/C1
(0170) ETA=-ES/C1
(0171) L2Z=2*L*ZETA

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NEQ	I //	017740	0005S	0027I	0052	0063	0071	0098	0101
NTERM	I /COEF/	000030	0004S	0005S	0027I				
OSTCOR	R EXTERNAL	000000	0084	0110					
RE	D /COEF/	000014	0001S	0005S	0025I	0035			
READ	R EXTERNAL	000000	0048	0061	0096				
S	D	001002	0001S	0064M	0065	0067	0102M	0104	
SQRT	R EXTERNAL	000000	0076						
STEP	D	000002	0001S	0027I	0044				
TIMO	D	000006	0001S	0027I					
VAR	D //	000000	0001S	0005S	0051M	0099M			
VEL	D //	000630	0001S	0005S	0053	0064	0099	0102	
VX	D /COREL/	000014	0001S	0005S	0029I	0088	0115		
VY	D /COREL/	000020	0001S	0005S	0029I	0088	0115		
VZ	D /COREL/	000024	0001S	0005S	0029I	0088	0115		
X	D /COREL/	000000	0001S	0005S	0029I	0088	0115		
Y	D /COREL/	000004	0001S	0005S	0029I	0088	0115		
Z	D /COREL/	000010	0001S	0005S	0029I	0088	0115		
Z8	R	001006	0057M	0077					
Z9	R	001010	0037M	0057	0059M	0065M	0076M	0077	
10		000045	0038	0039	0040	0041D			
-1010		000454	0079D	0088	0115				
-1012		000464	0080D						
-1020		000525	0089D						
-110		000201	0052	0053D					
-120		000224	0057D	0077	0078				
-130		000334	0058	0063	0066	0068D			
-150		000374	0069	0071	0072	0074D			
-20		000123	0043	0045D					
-200		000530	0094D	0108	0116				
-210		000556	0098	0099D					
-220		000670	0103	0105D					
-250		000676	0101	0106D					
-30		000160	0050	0051D					

```

(0107)      KS=KS+1
(0108)      IF(MOD(KS,48).NE.3) GOTO 200
(0109)      CALL ELTRAN(KR,2)
(0110)      CALL OSTCOR
(0111) C    WRITE(1,1012)KS
(0112) C    WRITE(1,1010)A,ECC,INC,ELL,G,H
(0113) C    WRITE(1,1010)(O(I),I=1,6)
(0114) C    WRITE(1,1010)(VAR(I,KR),I=1,6)
(0115)      WRITE(1,1010)X,Y,Z,VX,VY,VZ
(0116)      IF(KS.LT.1445) GOTO 200
(0117)      CALL EXIT
(0118)      END

```

A	D /COREL/	000030	0001S	0005S	0029I				
ADAM	D /COEF/	000153	0001S	0005S	0007I	0044M	0045		
ADSUM	D	000755	0001S	0042M	0045M	0099			
DEL	D //	001460	0001S	0005S	0041M	0053M	0064	0067M	0074M
			0099	0102	0104M				
DERIV	R EXTERNAL	000000	0049	0100					
DRAD	D /COEF/	000000	0001S	0005S	0025I				
ECC	D /COREL/	000034	0001S	0005S	0029I				
ELL	D /COREL/	000044	0001S	0005S	0029I				
ELTRAN	R EXTERNAL	000000	0047	0083	0109				
ENTER	R EXTERNAL	000000	0046						
EXIT	R EXTERNAL	000000	0117						
EXTRAP	R EXTERNAL	000000	0062	0097					
G	D /COREL/	000050	0001S	0005S	0029I				
GAM2N	D /COEF/	000010	0001S	0005S	0035M				
H	D /COREL/	000054	0001S	0005S	0029I				
HAR	D /COEF/	000033	0001S	0005S	0016I	0035			
I	I	000765	0038M	0041	0043M	0044	0045	0050M	0051
INC	D /COREL/	000040	0001S	0005S	0029I				
INIT	I	000012	0029I	0046A					
J	I	000770	0039M	0041					
K	I	000771	0040M	0041	0069M	0070			
K9	I	000772	0082M	0083A					
KC	I	000773	0066M	0067	0072M	0073	0103M	0104	
KC1	I	000774	0073M	0074					
%CYC	I	000775	0036M	0061	0075M	0078			
KMAX	I //	017741	0005S	0027I	0043	0058	0069	0070	0072
			0073	0093	0103				
KR	I	000776	0058M	0060	0061A	0062A	0064	0066	0067
			0070M	0074	0095M	0096A	0097A	0099	0100A
			0102	0104	0109A				
KR1	I	000777	0060M	0062A	0094M	0097A			
KS	I	001000	0093M	0094	0095	0107M	0108	0116	
MM	D /COEF/	000020	0001S	0005S	0025I				
MOD	I EXTERNAL	000000	0094	0095	0108				
MS	D /COEF/	000024	0001S	0005S	0025I				
MU	D /COEF/	000004	0001S	0005S	0025I	0035			
N	I	001001	0052M	0053	0063M	0064	0067	0071M	0074
			0098M	0099	0101M	0102	0104		

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NEQ	I //	017740	0005S	0027I	0052	0063	0071	0098	0101
NTERM	I /COEF/	000030	0004S	0005S	0027I				
OSTCOR	R EXTERNAL	000000	0084	0110					
RE	D /COEF/	000014	0001S	0005S	0025I	0035			
READ	R EXTERNAL	000000	0048	0061	0096				
S	D	001002	0001S	0064M	0065	0067	0102M	0104	
SQRT	R EXTERNAL	000000	0076						
STEP	D	000002	0001S	0027I	0044				
TIMO	D	000006	0001S	0027I					
VAR	D //	000000	0001S	0005S	0051M	0099M			
VEL	D //	000630	0001S	0005S	0053	0064	0099	0102	
VX	D /COREL/	000014	0001S	0005S	0029I	0088	0115		
VY	D /COREL/	000020	0001S	0005S	0029I	0088	0115		
VZ	D /COREL/	000024	0001S	0005S	0029I	0088	0115		
X	D /COREL/	000000	0001S	0005S	0029I	0088	0115		
Y	D /COREL/	000004	0001S	0005S	0029I	0088	0115		
Z	D /COREL/	000010	0001S	0005S	0029I	0088	0115		
Z8	R	001006	0057M	0077					
Z9	R	001010	0037M	0057	0059M	0065M	0076M	0077	
10		000045	0038	0039	0040	0041D			
-1010		000454	0079D	0088	0115				
-1012		000464	0080D						
-1020		000525	0089D						
-110		000201	0052	0053D					
-120		000224	0057D	0077	0078				
-130		000334	0058	0063	0066	0068D			
-150		000374	0069	0071	0072	0074D			
-20		000123	0043	0045D					
-200		000530	0094D	0108	0116				
-210		000556	0098	0099D					
-220		000670	0103	0105D					
-250		000676	0101	0106D					
-30		000160	0050	0051D					

```

(0107)      KS=KS+1
(0108)      IF(MOD(KS,48).NE.3) GOTO 200
(0109)      CALL ELTRAN(KR,2)
(0110)      CALL OSTCOR
(0111) C    WRITE(1,1012)KS
(0112) C    WRITE(1,1010)A,ECC,INC,ELL,G,H
(0113) C    WRITE(1,1010)(O(I),I=1,6)
(0114) C    WRITE(1,1010)(VAR(I,KR),I=1,6)
(0115)      WRITE(1,1010)X,Y,Z,VX,VY,VZ
(0116)      IF(KS.LT.1445) GOTO 200
(0117)      CALL EXIT
(0118)      END

```

A	D /COREL/	000030	0001S	0005S	0029I				
ADAM	D /COEF/	000153	0001S	0005S	0007I	0044M	0045		
ADSUM	D	000755	0001S	0042M	0045M	0099			
DEL	D //	001460	0001S	0005S	0041M	0053M	0064	0067M	0074M
			0099	0102	0104M				
DERIV	R EXTERNAL	000000	0049	0100					
DRAD	D /COEF/	000000	0001S	0005S	0025I				
ECC	D /COREL/	000034	0001S	0005S	0029I				
ELL	D /COREL/	000044	0001S	0005S	0029I				
ELTRAN	R EXTERNAL	000000	0047	0083	0109				
ENTER	R EXTERNAL	000000	0046						
EXIT	R EXTERNAL	000000	0117						
EXTRAP	R EXTERNAL	000000	0062	0097					
G	D /COREL/	000050	0001S	0005S	0029I				
GAM2N	D /COEF/	000010	0001S	0005S	0035M				
H	D /COREL/	000054	0001S	0005S	0029I				
HAR	D /COEF/	000033	0001S	0005S	0016I	0035			
I	I	000765	0038M	0041	0043M	0044	0045	0050M	0051
INC	D /COREL/	000040	0001S	0005S	0029I				
INIT	I	000012	0029I	0046A					
J	I	000770	0039M	0041					
K	I	000771	0040M	0041	0069M	0070			
K9	I	000772	0082M	0083A					
KC	I	000773	0066M	0067	0072M	0073	0103M	0104	
KC1	I	000774	0073M	0074					
KCYC	I	000775	0036M	0061	0075M	0078			
KMAX	I //	017741	0005S	0027I	0043	0058	0069	0070	0072
			0073	0093	0103				
KR	I	000776	0058M	0060	0061A	0062A	0064	0066	0067
			0070M	0074	0095M	0096A	0097A	0099	0100A
			0102	0104	0109A				
KR1	I	000777	0060M	0062A	0094M	0097A			
KS	I	001000	0093M	0094	0095	0107M	0108	0116	
MM	D /COEF/	000020	0001S	0005S	0025I				
MOD	I EXTERNAL	000000	0094	0095	0108				
MS	D /COEF/	000024	0001S	0005S	0025I				
MU	D /COEF/	000004	0001S	0005S	0025I	0035			
N	I	001001	0052M	0053	0063M	0064	0067	0071M	0074
			0098M	0099	0101M	0102	0104		

```

(0054) C
(0055) C      CYCLE ON STARTING SOLUTION
(0056) C
(0057) 120    Z8=Z9
(0058)        DO 130 KR=2,KMAX
(0059)        Z9=0
(0060)        KR1=KR-1
(0061)        IF(KCYC.EQ.0)CALL READ(KR)
(0062)        CALL EXTRAP(KR1,KR)
(0063)        DO 130 N=1,NEQ
(0064)        S=VEL(N,KR)-DEL(1,N,KR)
(0065)        Z9=Z9+S*S
(0066)        DO 130 KC=1,KR
(0067)        DEL(KC,N,KR)=DEL(KC,N,KR)+S
(0068) 130    CONTINUE
(0069)        DO 150 K=2,KMAX
(0070)        KR=KMAX+1-K
(0071)        DO 150 N=1,NEQ
(0072)        DO 150 KC=2,KMAX
(0073)        KC1=KMAX+2-KC
(0074) 150    DEL(KC1,N,KR)=DEL(KC1,N,KR+1)-DEL(KC1+1,N,KR+1)
(0075)        KCYC=KCYC+1
(0076)        Z9=SQRT(Z9)
(0077)        IF (Z9.LT.Z8)GOTO 120
(0078)        IF(KCYC.LT.6)GOTO 120
(0079) 1010   FORMAT(1HX,6D18.10)
(0080) 1012   FORMAT(3I5)
(0081) C      DO 1020 K9=1,KMAX
(0082)        K9=2
(0083)        CALL ELTRAN(K9,2)
(0084)        CALL OSTCOR
(0085) C      WRITE (1,1010)A,ECC,INC,ELL,G,H
(0086) C      WRITE(1,1010)(O(K),K=1,6)
(0087) C      WRITE(1,1010)(VAR(K,K9),K=1,6)
(0088)        WRITE(1,1010)X,Y,Z,VX,VY,VZ
(0089) 1020   CONTINUE
(0090) C
(0091) C      END OF CYCLE: CONTINUE THE SOLUTION
(0092) C
(0093)        KS=KMAX+1
(0094) 200    KR1=MOD(KS-2,17)+1
(0095)        KR=MOD(KS-1,17)+1
(0096)        CALL READ(KR)
(0097)        CALL EXTRAP(KR1,KR)
(0098)        DO 210 N=1,NEQ
(0099) 210    VAR(N,KR)=VAR(N,KR)+ADSUM*(VEL(N,KR)-DEL(1,N,KR))
(0100)        CALL DERIV(KR,1)
(0101)        DO 250 N=1,NEQ
(0102)        S=VEL(N,KR)-DEL(1,N,KR)
(0103)        DO 220 KC=1,KMAX
(0104)        DEL(KC,N,KR)=DEL(KC,N,KR)+S
(0105) 220    CONTINUE
(0106) 250    CONTINUE

```

```

(0001)      DOUBLE PRECISION VAR(6,17),VEL(6,17),ADAM(17),DEL(18,6,17),
(0002)      + STEP,S,TIM,TIMO,ADSUM,DRAD,MU,RE,MM,MS,GAM2N,X,Y,Z,VX,VY,VZ,
(0003)      + A,ECC,INC,ELL,G,H,HAR(20),O(6)
(0004)      DIMENSION NTERM(3)
(0005)      COMMON //VAR,VEL,DEL,NEQ,KMAX/COEF/DRAD,MU,GAM2N,RE,MM,MS,NTERM,
(0006)      + HAR,ADAM/COREL/X,Y,Z,VX,VY,VZ,A,ECC,INC,ELL,G,H,O
(0007)      DATA ADAM(1),ADAM(2),ADAM(3),ADAM(4),ADAM(5),ADAM(6),ADAM(7),
(0008)      + ADAM(8),ADAM(9),ADAM(10),ADAM(11),ADAM(12),ADAM(13),ADAM(14),
(0009)      + ADAM(15),ADAM(16),ADAM(17)/
(0010)      + -0.1000000000000000D 01,0.5000000000000000D 00,0.833333333333333D-01,
(0011)      + 0.416666666666667D-01,0.263888888888889D-01,0.187500000000000D-01,
(0012)      + 0.14269179894180D-01,0.11367394179894D-01,0.93565365961199D-02,
(0013)      + 0.78925540123457D-02,0.67858499846347D-02,0.59240564123377D-02,
(0014)      + 0.52366932579503D-02,0.46774984070423D-02,0.42149522390055D-02,
(0015)      + 0.38268995532120D-02,0.34973498453500D-02/
(0016)      DATA HAR(2),HAR(3),HAR(4),HAR(5),HAR(6),HAR(7),HAR(8),HAR(9),
(0017)      + HAR(10),HAR(11),HAR(12),HAR(13),HAR(14),HAR(15),HAR(16),HAR(17),
(0018)      + HAR(18),HAR(19),HAR(20)/
(0019)      + 0.108263 4 D-02, -0.2536 D-05,
(0020)      + -0.1664 D-05, -0.2195 D-06,
(0021)      + 0.6355 D-06, -0.3720 D-06,
(0022)      + -0.3508 D-06, -0.8733 D-07,
(0023)      + -0.5730 D-07, 0.1686 D-06,
(0024)      + -0.3809 D-06, 0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0/
(0025)      DATA MU,RE,MM,MS,DRAD/.398601 178977 8 D06,.637814 5 D04,.122999
(0026)      + 7171 D-01,.332945 561925 44 D06,.572957 795130 823 D02/
(0027)      DATA NEQ,KMAX,STEP,TIMO,NTERM(1),NTERM(2),NTERM(3)/5,9,.216D5,
(0028)      + 0.D0,12,5,2/
(0029)      DATA INIT,X,Y,Z,VX,VY,VZ,A,ECC,INC,ELL,G,H/2,0,0,0,0,0,0,
(0030)      + 0.748503 712201 72 D04, 0.8255 D-02, 0.634300 470727 88 D02,
(0031)      + 0.1033005 D03, 0.19952 D03, 0.1249632 D03/
(0032)      C
(0033)      C      INITIALIZE
(0034)      C
(0035)      GAM2N=12*HAR(2)*(MU*RE)**2
(0036)      KCYC=0
(0037)      Z9=1.E20
(0038)      DO 10 I=1,17
(0039)      DO 10 J=1,6
(0040)      DO 10 K=1,18
(0041) 10      DEL(K,J,I)=0
(0042)      ADSUM=0
(0043)      DO 20 I=1,KMAX
(0044)      ADAM(I)=-STEP*ADAM(I)
(0045) 20      ADSUM=ADSUM+ADAM(I)
(0046)      CALL ENTER(INIT)
(0047)      CALL ELTRAN(1,1)
(0048)      CALL READ(1)
(0049)      CALL DERIV(1,2)
(0050)      DO 30 I=1,17
(0051) 30      VAR(6,I)=VAR(6,1)
(0052)      DO 110 N=1,NEQ
(0053) 110      DEL(1,N,1)=VEL(N,1)

```



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APPENDIX A
GPS PROGRAM LISTING

where the only zero-order term is μ^2/L^3 , a constant. There are, thus, five simultaneous first-order differential equations to be integrated step by step (L being a constant of the motion), after the partial derivatives of F_1 have been evaluated at each step.

The program (in the subroutine DERIV) defines the array

$$DF(I) = \left(\frac{\partial F_1}{\partial \alpha}, \frac{\partial F_1}{\partial \beta}, \frac{\partial F_1}{\partial \gamma}, \frac{\partial F_1}{\partial \xi}, \frac{\partial F_1}{\partial a} \right)$$

whose elements are defined piecemeal in the previous section since $\partial/\partial \xi = (-1/\xi^2)(\partial/\partial x)$. The right-hand sides of the equations of motion are contained in the array

$$DE(I) = \left(\frac{\partial F_1}{\partial \eta}, \frac{\partial F_1}{\partial \xi}, \frac{\partial F_1}{\partial \tau}, \frac{\partial F_1}{\partial \sigma}, \frac{\partial F_1}{\partial L} \right)$$

The connection between these two is the Jacobian matrix

$$\frac{J(\alpha, \beta, \gamma, \xi, a)}{J(\eta, \xi, \tau, \sigma, L)}$$

The components of the matrix are evaluated in Appendix G. The program equates the matrix to the array $C(I, J)$, where for example $C(3, 4) = \partial \gamma / \partial \sigma$. Then clearly,

$$DE(I) = \sum_J DF(J) C(J, I)$$

is the set of time derivatives of the variables, and this is the output of DERIV.

The equations of motion are integrated by a standard Adams numerical integration routine. Although provision is made for an order as high as 17, experimentation indicated that an order of 9 gave the best compromise between roundoff error and truncation error, so, the program is currently set to 9 (KMAX Value). The starting procedure is found in the main program (lines 57-78). At least 6 cycles are performed, and more if the rms change in the derivatives per cycle continues to decline. After the starting values are stabilized, the running procedure applies a predictor-corrector pair once per time step. The extrapolation forward of the backward difference table and the application of the predictor are to be found in the subroutine EXTRAP; the corrector is applied in the main program, lines 94-108.

$$\begin{array}{lll}
\frac{\partial}{\partial \beta} \Delta F_M & C_m & -mS_{m-1} \\
\frac{\partial}{\partial \gamma} \Delta F_M & Q_n^m & Q_n^{m+1} \\
\frac{\partial}{\partial x} \Delta F_M & A_{n+1}^m & \frac{-n+m}{x} (A_{n+1}^m + A_{n+1}^{m+1}) \\
\frac{\partial}{\partial a} \Delta F_M & \left(\frac{a}{r_M} \right)^n & \frac{n}{a} \left(\frac{a}{r_M} \right)^n
\end{array}$$

THE SOLAR ATTRACTION

The contribution of the sun is exactly the same as the moon's except that μ_S and r_S are substituted for μ_M and r_M and x'_S, y'_S, z'_S are substituted for x'_M, y'_M, z'_M .

THE EQUATIONS OF MOTION IN THE MEAN VARIABLES

The Hamiltonian in the mean variables may be written

$$F = \mu^2/2L^2 + F_1$$

$$F_1 = \Delta F_Z + \Delta F_{2s} + \Delta F_M + \Delta F_S$$

Since the variables are canonical, the Hamiltonian equations of motion are

$$\begin{array}{ll}
\dot{\eta} = -\frac{\partial F_1}{\partial \xi} & \dot{\xi} = \frac{\partial F_1}{\partial \eta} \\
\dot{\tau} = -\frac{\partial F_1}{\partial \sigma} & \dot{\sigma} = \frac{\partial F_1}{\partial \tau} \\
\dot{\lambda} = \mu^2/L^3 - \frac{\partial F_1}{\partial L} & \dot{L} = 0
\end{array}$$

THE LUNAR ATTRACTION

The moon's contribution to the Hamiltonian can also be expanded into a spherical harmonic series and averaged over the short period:

$$F_M = \left(\frac{\mu_M}{r_M} \right) \sum_{n=2}^{\infty} \left(\frac{a}{r_M} \right)^n \overline{\left(\frac{r}{a} \right)^n P_n(\cos \Theta)}$$

Here $\cos \Theta = \hat{r} \cdot \hat{r}_M$, i.e., Θ is the angle between the radius vector of the satellite and the vector from the origin to the moon. The distance to the moon is r_M , and the components of the unit vector $\hat{r}_M$ are x'_M, y'_M, z'_M . The expansion is valid as long as $r < r_M$ and the series is terminated when the terms become negligibly small.

The development of this contribution is exactly like that for the zonal harmonics, with two exceptions: α, β , and γ contain x'_M, y'_M, z'_M in place of x'_Z, y'_Z, z'_Z and the coefficients in x , instead of $B_n^m(x)$, are defined by

$$\overline{\left(\frac{r}{a} \right)^n \cos mf} = e^m A_{n+2}^m(x)$$

The upshot is

$$\Delta F_M = \frac{\mu_M}{r_M} \sum_{m=0}^N (2 - \delta_{m0}) C_m(\alpha, \beta) \sum_n \left(\frac{a}{r_M} \right)^n D_n^m Q_n^m(\gamma) A_{n+2}^m(x)$$

The sum over m being from $\max(m, 2)$ by steps of 2 to N .

The recursion relationships for $D_n^m, Q_n^m(\gamma)$, and $C_m(\alpha, \beta)$ are exactly the same as for the zonal case, although of course the arguments are different. The recursions and the derivatives of the recursive functions are given in Appendix F. The derivatives of ΔF_M can be written as follows:

In ΔF_M , obtain:

$$\frac{\partial}{\partial \alpha} \Delta F_M \text{ by replacing } C_m \text{ with } m C_{m-1}$$

$$\frac{\partial}{\partial a} \Delta F_z = \frac{\mu}{a} \left(\frac{R_e}{a} \right)^n - \frac{n+1}{a} \frac{\mu}{a} \left(\frac{R_e}{a} \right)^n$$

THE SECULAR SECOND-ORDER CONTRIBUTION OF THE SECOND HARMONIC

Brouwer's Equation 29 (Reference 2) can readily be put in terms of the variables being used here:

$$\begin{aligned} \Delta F_{2s} = & (\mu^2/L^3) \gamma_2'^2 L(\xi/24) \left\{ 5 \xi^2 (1 - 18 \rho^2/5 + \rho^4) \right. \\ & \left. + 4 \xi (1-6 \rho^2 + 9\rho^4) - 5 (1-2 \rho^2 - 7 \rho^4) - 2(1-15 \rho^2)(\alpha^2 - \beta^2) \right\} \end{aligned}$$

It follows that, if $M = (\mu^2/L^3) \gamma_2'^2 L \xi/24$ and $\gamma = \rho$,

$$\frac{\partial}{\partial \alpha} \Delta F_{2s} = M(-4\alpha)(1-15 \rho^2)$$

$$\frac{\partial}{\partial \beta} \Delta F_{2s} = M(4\beta)(1-15 \rho^2)$$

$$\begin{aligned} \frac{\partial}{\partial \gamma} \Delta F_{2s} = & M(4\rho) \left\{ \xi^2 (-9 + 5\rho^2) + 12 \xi (-1 + 3 \rho^2) \right. \\ & \left. + 5 (1 + 7 \rho^2) + 15 (\alpha^2 - \beta^2) \right\} \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial x} \Delta F_{2s} = & M(-\xi^2) \left\{ 10 \xi (1-18 \rho^2/5 + \rho^4) + 4 (1-6 \rho^2 + 9 \rho^4) \right\} \\ & + 7 \xi \cdot \Delta F_{2s} \end{aligned}$$

$$\frac{\partial}{\partial a} \Delta F_{2s} = - (5/a) \Delta F_{2s}$$

Then, $\alpha = e_c \psi_1 + e_s \psi_2$

$$\beta = e_s \psi_1 - e_c \psi_2$$

and $C_m(\alpha, \beta) = \text{Re}[(\alpha + i\beta)^m] = e^m (1 - \gamma^2)^{m/2} \cos m(\tilde{\omega} - \varphi_2)$

Introduce for convenience one more set of polynomials by

$$D_n^m = \frac{(n-m)!}{(n+m)!} Q_n^m(0)$$

and introduce the new variable $x = 1/\xi$.

Then,

$$\Delta F_\gamma = - \sum_{m=0}^N (2 - \delta_{m0}) C_m(\alpha, \beta) \sum_n J_n(\mu/a) (R_c/a)^n D_n^m Q_n^m(\gamma) B_n^m(x)$$

The sum over n starts at 2 or m , whichever is greater, and goes in steps of 2 to N .

This formulation has the advantages that the Hamiltonian depends only on five intermediate variables $[\alpha, \beta, \gamma, \xi$ (or x), and $a]$ through three functions (Q_n^m , B_n^m , and C_m) and one set of constants (D_n^m), all of which can be calculated recursively; and the partial derivatives are reasonably simple.

The recursion relationships and the derivatives of the functions are developed in Appendix F. The derivatives of ΔF_γ can, with the aid of those results, be written as the following schema:

In ΔF_γ , obtain:

$\frac{\partial}{\partial \alpha} \Delta F_\gamma$	by replacing C_m	with mC_{m-1}
$\frac{\partial}{\partial \beta} \Delta F_\gamma$	C_m	$-m S_{m-1}$
$\frac{\partial}{\partial \gamma} \Delta F_\gamma$	Q_n^m	Q_n^{m+1}
$\frac{\partial}{\partial x} \Delta F_\gamma$	B_n^m	$\frac{n+m}{x} (B_n^m + B_n^{m+1})$

$$\hat{r}_z: \sin \theta_2 \cos \varphi_2 = \psi_1 = \hat{L}_1 \cdot \hat{r}_z = x'_z + \frac{I_s}{1+\rho} (\gamma + z'_z)$$

$$\sin \theta_2 \sin \varphi_2 = \psi_2 = \hat{L}_2 \cdot \hat{r}_z = y'_z + \frac{I_c}{1+\rho} (\gamma + z'_z)$$

$$\cos \theta_2 = \gamma = \hat{h} \cdot \hat{r}_z = -I_s x'_z - I_c y'_z + \rho z'_z$$

$$\text{where the unit vector } \hat{r}_z = \begin{pmatrix} x'_z \\ y'_z \\ z'_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

(There is an obvious simplification if the actual values for x'_z , y'_z , z'_z are substituted in the foregoing, but this form is used for parallelism with the lunar and solar contributions to the Hamiltonian.)

$$P_n(\cos \Theta) = \sum_{m=0}^n (2 - \delta_{m0}) \frac{(n-m)!}{(n+m)!} P_n^m(0) P_n^m(\gamma) \cos m(u - \varphi_2)$$

If $Q_n^m(\gamma) = d^m/d\gamma^m P_n(\gamma)$, then $P_n^m(\gamma) = (1 - \gamma^2)^{m/2} Q_n^m(\gamma)$ and

$$\overline{(a/r)^{n+1} P_n(\cos \Theta)} = \sum_m (2 - \delta_{m0}) \frac{(n-m)!}{(n+m)!} Q_n^m(0) Q_n^m(\gamma) \overline{(a/r)^{n+1} \cos mf.}$$

$$(1 - \gamma^2)^{m/2} \cos m(\tilde{\omega} - \varphi_2)$$

We define B_n^m by $\overline{(a/r)^{n+1} \cos mf} = e^m B_n^m$ (knowing that the Cayley coefficients are proportional to e^m) and consider

$$[e \sqrt{1-\gamma^2} \exp i(\tilde{\omega} - \varphi_2)]^m = [(e_c + i e_s)(\psi_1 - i \psi_2)]^m$$

$$= [e_c \psi_1 + e_s \psi_2 + i(e_s \psi_1 - e_c \psi_2)]^m$$

Define polynomials C_m and S_m by

$$(\alpha + i \beta)^m = C_m(\alpha, \beta) + i S_m(\alpha, \beta)$$

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(0172)      C2=DSQRT((1+RHO)/L2Z)
(0173)      SIGMA=IC/C2
(0174)      TAU=IS/C2
(0175)  C
(0176)  200  CALL XFR(0,3)
(0177)      RETURN
(0178)      END

```

A	D /COREL/	000030	0125S	0128S	0134				
C1	D /INTEL/	000044	0125S	0128S	0168M	0169	0170		
C2	D /INTEL/	000050	0125S	0128S	0172M	0173	0174		
COSU	D /INTEL/	000064	0125S	0128S	0157M	0162	0163	0165	
DATAN2	D EXTERNAL	000000	0156	0165					
DCOS	D EXTERNAL	000000	0139	0142	0157	0166			
DRAD	D /COEF/	000000	0125S	0128S	0135	0136	0141	0142	0143
DSIN	D EXTERNAL	000000	0140	0141	0143	0158	0166		
DSQRT	D EXTERNAL	000000	0134	0137	0138	0141	0150	0154	0161
			0168	0172					
EC	D /INTEL/	000104	0125S	0128S	0162M	0165	0166	0169	
ECC	D /COREL/	000034	0125S	0128S	0137				
ECLON	D /INTEL/	000054	0125S	0128S	0165M	0166A			
ECOSF	D /INTEL/	000074	0125S	0128S	0159M	0161	0162	0163	
ELL	D /COREL/	000044	0125S	0128S	0136				
ES	D /INTEL/	000110	0125S	0128S	0163M	0165	0166	0170	
ESINF	D /INTEL/	000100	0125S	0128S	0160M	0161	0162	0163	0164
ETA	D /INTEL/	000000	0125S	0128S	0140M	0170M			
G	D /COREL/	000050	0125S	0128S	0135				
H	D /COREL/	000054	0125S	0128S	0135	0142	0143		
HM	D	001007	0125S	0150M	0151	0152	0153	0159	0160
			0167						
IC	D	001013	0125S	0148M	0150	0152M	0155	0156	0173
INC	D /COREL/	000040	0125S	0128S	0141				
INTERM	I EXTERNAL	000000	0144						
IS	D	001017	0125S	0147M	0150	0151M	0155	0156	0174
KEY	I ARGUMENT	000003	0120S	0132					
L	D /INTEL/	000024	0125S	0128S	0134M	0138	0141	0167M	0168
			0171						
L2Z	D /INTEL/	000034	0125S	0128S	0171M	0172			
LAMBDA	D /INTEL/	000020	0125S	0128S	0136M	0166M			
MU	D /COEF/	000004	0125S	0128S	0134	0159	0160		
R	D	001023	0125S	0154M	0159	0160			
RHO	D /INTEL/	000040	0125S	0128S	0149M	0150	0153M	0155	0172
SIGMA	D /INTEL/	000014	0125S	0128S	0142M	0173M			
SINU	D /INTEL/	000070	0125S	0128S	0158M	0162	0163	0165	
TAU	D /INTEL/	000010	0125S	0128S	0143M	0174M			
U	D /INTEL/	000060	0125S	0128S	0156M	0157A	0158A		
VX	D /COREL/	000014	0125S	0128S	0148	0149	0160		
VY	D /COREL/	000020	0125S	0128S	0147	0149	0160		
VZ	D /COREL/	000024	0125S	0128S	0147	0148	0160		
X	D /COREL/	000000	0125S	0128S	0148	0149	0154	0155	0156
			0160						

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XFR	R EXTERNAL	000000	0176						
XI	D /INTEL/	000004	0125S	0128S	0139M	0169M			
Y	D /COREL/	000004	0125S	0128S	0147	0149	0154	0155	0156
			0160						
Z	D /COREL/	000010	0125S	0128S	0147	0148	0154	0155	0160
Z8	D	001027	0125S	0135M	0136	0139A	0140A		
Z9	D	001033	0125S	0138M	0139	0140	0141M	0142	0143
			0155M	0156	0164M	0165			
ZETA	D /INTEL/	000030	0125S	0128S	0137M	0138	0141	0161M	0164
			0167	0168	0171				
100		000176	0132	0147D					
200		000766	0145	0176D					

```

(0179) C
(0180) SUBROUTINE OSTCOR
(0181) C
(0182) C TRANSFORM FROM OSCULATING ELEMENTS TO COORDINATES/ELLIPTIC ELEMENTS
(0183) C
(0184) DOUBLE PRECISION X,Y,Z,VX,VY,VZ,XI,ETA,SIGMA,TAU,L,LAMBDA,ZETA,
(0185) + L2Z,RHO,C1,C2,ECLON,U,COSU,SINU,ECOSF,ESINF,R,Z9,EC,ES,
(0186) + A,ECC,INC,ELL,G,H,DRAD,MU
(0187) COMMON /COREL/X,Y,Z,VX,VY,VZ,A,ECC,INC,ELL,G,H/INTEL/ETA,XI,
(0188) + TAU,SIGMA,LAMBDA,L,ZETA,L2Z,RHO,C1,C2,ECLON,U,COSU,SINU,ECOSF,
(0189) + ESINF,EC,ES/COEF/DRAD,MU
(0190) C
(0191) CALL XFR(0,1)
(0192) CALL INTERM
(0193) C
(0194) A=L*L/MU
(0195) ECC=DSQRT(1-ZETA*ZETA)
(0196) Z9=DSQRT(1-RHO*RHO)
(0197) INC=DRAD*DATAN2(Z9,RHO)
(0198) IF(SIGMA.NE.0.OR.TAU.NE.0) H=DRAD*DATAN2(-TAU,SIGMA)
(0199) G=DRAD*DATAN2(-ETA,XI)-H
(0200) ELL=DRAD*LAMBDA-G-H
(0201) ELL=DMOD(ELL,3.6D2)
(0202) IF(ELL.LT.0.D0)ELL=ELL+3.6D2
(0203) IF(ELL.GT.1.8D2)ELL=ELL-3.6D2
(0204) C
(0205) R=(1-EC*DCOS(ECLON)-ES*DSIN(ECLON))*L*L/MU
(0206) X=R*(COSU*(1-TAU*TAU/L2Z)-SINU*SIGMA*TAU/L2Z)
(0207) Y=R*(-COSU*SIGMA*TAU/L2Z+SINU*(1-SIGMA*SIGMA/L2Z))
(0208) Z9=DSQRT((1+RHO)/L2Z)
(0209) Z=R*Z9*(SINU*SIGMA+COSU*TAU)
(0210) VX=((ECLON-LAMBDA)*X+ZETA*(-C2*SIGMA*Z-RHO*Y))*L/(R*R)
(0211) VY=((ECLON-LAMBDA)*Y+ZETA*(RHO*X+C2*TAU*Z))*L/(R*R)
(0212) VZ=((ECLON-LAMBDA)*Z+ZETA*(-C2*TAU*Y+C2*SIGMA*X))*L/(R*R)
(0213) RETURN
(0214) END

```

A	D /COREL/	000030	0184S	0187S	0194M			
C2	D /INTEL/	000050	0184S	0187S	0210	0211	0212	
COSU	D /INTEL/	000064	0184S	0187S	0206	0207	0209	
DATAN2	D EXTERNAL	000000	0197	0198	0199			
DCOS	D EXTERNAL	000000	0205					
DMOD	D EXTERNAL	000000	0201					
DRAD	D /COEF/	000000	0184S	0187S	0197	0198	0199	0200
DSIN	D EXTERNAL	000000	0205					
DSQRT	D EXTERNAL	000000	0195	0196	0208			
EC	D /INTEL/	000104	0184S	0187S	0205			
ECC	D /COREL/	000034	0184S	0187S	0195M			
ECLON	D /INTEL/	000054	0184S	0187S	0205A	0210	0211	0212
ELL	D /COREL/	000044	0184S	0187S	0200M	0201M	0202M	0203M
ES	D /INTEL/	000110	0184S	0187S	0205			
ETA	D /INTEL/	000000	0184S	0187S	0199			
G	D /COREL/	000050	0184S	0187S	0199M	0200		

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H	D /COREL/	000054	0184S	0187S	0198M	0199	0200		
INC	D /COREL/	000040	0184S	0187S	0197M				
INTERM	I EXTERNAL	000000	0192						
L	D /INTEL/	000024	0184S	0187S	0194	0205	0210	0211	0212
L2Z	D /INTEL/	000034	0184S	0187S	0206	0207	0208		
LAMBDA	D /INTEL/	000020	0184S	0187S	0200	0210	0211	0212	
MU	D /COEF/	000004	0184S	0187S	0194	0205			
R	D	000535	0184S	0205M	0206	0207	0209	0210	0211
			0212						
RHO	D /INTEL/	000040	0184S	0187S	0196	0197A	0208	0210	0211
SIGMA	D /INTEL/	000014	0184S	0187S	0198A	0206	0207	0209	0210
			0212						
SINU	D /INTEL/	000070	0184S	0187S	0206	0207	0209		
TAU	D /INTEL/	000010	0184S	0187S	0198	0206	0207	0209	0211
			0212						
VX	D /COREL/	000014	0184S	0187S	0210M				
VY	D /COREL/	000020	0184S	0187S	0211M				
VZ	D /COREL/	000024	0184S	0187S	0212M				
X	D /COREL/	000000	0184S	0187S	0206M	0210	0211	0212	
XFR	R EXTERNAL	000000	0191						
XI	D /INTEL/	000004	0184S	0187S	0199A				
Y	D /COREL/	000004	0184S	0187S	0207M	0210	0211	0212	
Z	D /COREL/	000010	0184S	0187S	0209M	0210	0211	0212	
Z9	D	000541	0184S	0196M	0197A	0208M	0209		
ZETA	D /INTEL/	000030	0184S	0187S	0195	0210	0211	0212	

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(0215) C
(0216) SUBROUTINE ELTRAN(KROW,KEY)
(0217) C
(0218) C KEY=1: TRANSFORM FROM OSCULATING TO MEAN ELEMENTS
(0219) C KEY=2: TRANSFORM FROM MEAN TO OSCULATING ELEMENTS
(0220) C
(0221) DOUBLE PRECISION XI,ETA,SIGMA,TAU,L,LAMBDA,ZETA,
(0222) + L2Z,RHO,C1,C2,ECLON,U,COSU,SINU,ECOSF,ESINF,GAM2P,C(6),D(6),
(0223) + E(6),O(6),ZETA2,RHO3,Z9,Z8,Z7,Z6,Z5,Z4,Z,C2U,S2U,SU3,CU3,U',
(0224) + SUP,CUP,SUPS2,SGH,EC,ES,VAR(6,17),DRAD,MU
(0225) COMMON /COREL/C,E,O//VAR/INTEL/ETA,XI,TAU,SIGMA,LAMBDA,L,
(0226) + ZETA,L2Z,RHO,C1,C2,ECLON,U,COSU,SINU,ECOSF,ESINF,EC,ES,GAM2P
(0227) + /COEF/DRAD,MU
(0228) C
(0229) CALL XFR(KROW,KEY)
(0230) 20 CALL INTERM
(0231) ZETA2=ZETA*ZETA
(0232) Z=1+ECOSF
(0233) RHO3=(-1+3*RHO*RHO)/3
(0234) Z9=SIGMA*SIGMA-TAU*TAU
(0235) Z8=2*SIGMA*TAU
(0236) Z7=DSIN(2*U)
(0237) Z6=DCOS(2*U)
(0238) Z5=SINU+DSIN(3*U)/3
(0239) Z4=COSU+DCOS(3*U)/3
(0240) C2U=Z6*Z9-Z7*Z8
(0241) S2U=Z7*Z9+Z6*Z8
(0242) SU3=Z5*Z9+Z4*Z8
(0243) CU3=(2*COSU-Z4)*Z9+(-2*SINU+Z5)*Z8
(0244) UL=U-LAMBDA+ESINF
(0245) SUP=Z7+EC*Z5+ES*(2*COSU-Z4)
(0246) CUP=Z6+EC*Z4+ES*(Z5-2*SINU)
(0247) SUPS2=SUP*Z9+CUP*Z8
(0248) SGH=(1+RHO*(2-5*RHO))*UL-(3+5*RHO)*SUPS2/(2*L2Z)
(0249) Z9=Z*(1+Z)*(RHO3+(1+RHO)/L2Z*C2U)
(0250) D(5)=-GAM2P*(SGH-(ESINF*Z9+ZETA2*(RHO3*ESINF+(1+RHO)/(2*L2Z)*
(0251) + (SUPS2-S2U)))/(1+ZETA))
(0252) Z8=(Z/ZETA)**3
(0253) D(6)=GAM2P*(RHO3*(Z8-1)*L2Z+(1+RHO)*Z8*C2U)/2
(0254) Z9=(RHO3+Z*(1+RHO)*C2U/L2Z)/(2*ZETA2)
(0255) Z8=(1+Z)*Z9+RHO3*(Z/ZETA)**2/2
(0256) Z7=ESINF*(1+2*ZETA)*Z8+(1+RHO)*(SUPS2-S2U)/(8*L)
(0257) Z7=SGH+RHO3+Z7/(1+ZETA)
(0258) Z6=(1+ZETA+ZETA2)*Z9
(0259) Z5=(1+RHO)*C1/4
(0260) Z9=2*L*C1*Z8
(0261) D(1)=-GAM2P*(-XI*Z7+Z9*SINU-Z6*ETA+Z5*SU3)
(0262) D(2)=GAM2P*(-ETA*Z7+Z9*COSU+Z6*XI-Z5*CU3)
(0263) Z9=2*RHO*UL+SUPS2/(2*L2Z)
(0264) D(3)=-GAM2P*(-SIGMA*Z9+(1+RHO)/2*(SIGMA*SUP+TAU*CUP))
(0265) D(4)=GAM2P*(-TAU*Z9+(1+RHO)/2*(-TAU*SUP+SIGMA*CUP))
(0266) C
(0267) IF(KEY.EQ.1)GOTO 100

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```

(0268) C
(0269)      DO 50 I=1,6
(0270) 50    O(I)=VAR(I,KROW)+D(I)
(0271)      RETURN
(0272) C
(0273) 100   Z8=1+RHO*(2-5*RHO)
(0274)      D(6)=D(6)*(1-3/(2*L)*D(6)+GAM2P*(-3*ZETA*RHO3+Z8))+
(0275)      + GAM2P*(-Z8*(ETA*D(1)+XI*D(2))-2*RHO*(TAU*D(3)+SIGMA*D(4)))
(0276)      DO 150 I=1,6
(0277) 150   VAR(I,KROW)=O(I)-D(I)
(0278)      RETURN
(0279)      END

```

C1	D /INTEL/	000044	0221S	0225S	0259	0260			
C2U	D	001376	0221S	0240M	0249	0253	0254		
COSU	D /INTEL/	000064	0221S	0225S	0239	0243	0245	0262	
CU3	D	001402	0221S	0243M	0262				
CUP	D	001406	0221S	0246M	0247	0264	0265		
D	D	000006	0221S	0250M	0253M	0261M	0262M	0264M	0265M
			0270	0274M	0277				
DCOS	D EXTERNAL	000000	0237	0239					
DSIN	D EXTERNAL	000000	0236	0238					
EC	D /INTEL/	000104	0221S	0225S	0245	0246			
ECOSF	D /INTEL/	000074	0221S	0225S	0232				
ES	D /INTEL/	000110	0221S	0225S	0245	0246			
ESINF	D /INTEL/	000100	0221S	0225S	0244	0250	0256		
ETA	D /INTEL/	000000	0221S	0225S	0261	0262	0274		
GAM2P	D /INTEL/	000114	0221S	0225S	0250	0253	0261	0262	0264
			0265	0274					
I	I	001446	0269M	0270	0276M	0277			
INTERM	I EXTERNAL	000000	0230						
KEY	I ARGUMENT	000004	0216S	0229A	0267				
KROW	I ARGUMENT	000003	0216S	0229A	0270	0277			
L	D /INTEL/	000024	0221S	0225S	0256	0260	0274		
L2Z	D /INTEL/	000034	0221S	0225S	0248	0249	0250	0253	0254
			0263						
LAMBDA	D /INTEL/	000020	0221S	0225S	0244				
O	D /COREL/	000060	0221S	0225S	0270M	0277			
RHO	D /INTEL/	000040	0221S	0225S	0233	0248	0249	0250	0253
			0254	0256	0259	0263	0264	0265	0273
			0274						
RHO3	D	001451	0221S	0233M	0249	0250	0253	0254	0255
			0257	0274					
S2U	D	001455	0221S	0241M	0250	0256			
SGH	D	001461	0221S	0248M	0250	0257			
SIGMA	D /INTEL/	000014	0221S	0225S	0234	0235	0264	0265	0274
SINU	D /INTEL/	000070	0221S	0225S	0238	0243	0246	0261	
SU3	D	001465	0221S	0242M	0261				
SUP	D	001471	0221S	0245M	0247	0264	0265		
SUPS2	D	001475	0221S	0247M	0248	0250	0256	0263	
TAU	D /INTEL/	000010	0221S	0225S	0234	0235	0264	0265	0274
U	D /INTEL/	000060	0221S	0225S	0236	0237	0238	0239	0244

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UL	D	001501	0221S	0244M	0248	0263			
VAR	D //	000000	0221S	0225S	0270	0277M			
XFR	R EXTERNAL	000000	0229						
XI	D /INTEL/	000004	0221S	0225S	0261	0262	0274		
Z	D	001505	0221S	0232M	0249	0252	0254	0255	
Z4	D	001511	0221S	0239M	0242	0243	0245	0246	
Z5	D	001515	0221S	0238M	0242	0243	0245	0246	0259M
			0261	0262					
Z6	D	001521	0221S	0237M	0240	0241	0246	0258M	0261
			0262						
Z7	D	001525	0221S	0236M	0240	0241	0245	0256M	0257M
			0261	0262					
Z8	D	001531	0221S	0235M	0240	0241	0242	0243	0247
			0252M	0253	0255M	0256	0260	0273M	0274
Z9	D	001535	0221S	0234M	0240	0241	0242	0243	0247
			0249M	0250	0254M	0255	0258	0260M	0261
			0262	0263M	0264	0265			
ZETA	D /INTEL/	000030	0221S	0225S	0231	0250	0252	0255	0256
			0257	0258	0274				
ZETA2	D	001541	0221S	0231M	0250	0254	0258		
100		001217	0267	0273D					
150		001347	0276	0277D					
20		000042	0230D						
50		001167	0269	0270D					

```

(0280) C
(0281) SUBROUTINE INTERM
(0282) C
(0283) C CALCULATE INTERMEDIATE QUANTITIES IN ELEMENT TRANSFORMATIONS
(0284) C
(0285) DOUBLE PRECISION XI,ETA,SIGMA,TAU,L,LAMBDA,ZETA,L2Z,RHO,C1,C2,
(0286) + ECLON,U,COSU,SINU,ECOSF,ESINF,EC,ES,DRAD,MU,GAM2N,GAM2P,Z8,Z9
(0287) COMMON /INTEL/ETA,XI,TAU,SIGMA,LAMBDA,L,ZETA,L2Z,RHO,C1,C2,
(0288) + ECLON,U,COSU,SINU,ECOSF,ESINF,EC,ES,GAM2P/COEF/DRAD,MU,GAM2N
(0289) C
(0290) ZETA=1-(XI*XI+ETA*ETA)/(2*L)
(0291) L2Z=2*L*ZETA
(0292) GAM2P=GAM2N/L2Z**4
(0293) RHO=1-(SIGMA*SIGMA+TAU*TAU)/L2Z
(0294) C1=DSQRT((1+ZETA)/(2*L))
(0295) C2=DSQRT((1+RHO)/L2Z)
(0296) EC=C1*XI
(0297) ES=-C1*ETA
(0298) ECLON=LAMBDA
(0299) 100 Z9=ECLON
(0300) Z8=1-EC*DCOS(Z9)-ES*DSIN(Z9)
(0301) ECLON=Z9-(Z9-LAMBDA-EC*DSIN(Z9)+ES*DCOS(Z9))/Z8
(0302) IF(DABS(ECLON-Z9).GT.2.D-14*(DABS(Z9)+1))GOTO 100
(0303) Z9=(ECLON-LAMBDA)
(0304) Z8=Z8+ZETA
(0305) U=ECLON+2*DATAN2(Z9,Z8)
(0306) SINU=DSIN(U)
(0307) COSU=DCOS(U)
(0308) ESINF=EC*SINU-ES*COSU
(0309) ECOSF=EC*COSU+ES*SINU
(0310) RETURN
(0311) END

```

C1	D /INTEL/	000044	0285S	0287S	0294M	0296	0297		
C2	D /INTEL/	000050	0285S	0287S	0295M				
COSU	D /INTEL/	000064	0285S	0287S	0307M	0308	0309		
DABS	D EXTERNAL	000000	0302						
DATAN2	D EXTERNAL	000000	0305						
DCOS	D EXTERNAL	0000C3	0300	0301	0307				
DSIN	D EXTERNAL	000000	0300	0301	0306				
DSQRT	D EXTERNAL	000000	0294	0295					
EC	D /INTEL/	000104	0285S	0287S	0296M	0300	0301	0308	0309
ECLON	D /INTEL/	000054	0285S	0287S	0298M	0299	0301M	0302	0303
			0305						
ECOSF	D /INTEL/	000074	0285S	0287S	0309M				
ES	D /INTEL/	000110	0285S	0287S	0297M	0300	0301	0308	0309
ESINF	D /INTEL/	000100	0285S	0287S	0308M				
ETA	D /INTEL/	000000	0285S	0287S	0290	0297			
GAM2N	D /COEF/	000010	0285S	0287S	0292				
GAM2P	D /INTEL/	000114	0285S	0287S	0292M				
L	D /INTEL/	000024	0285S	0287S	0290	0291	0294		
L2Z	D /INTEL/	000034	0285S	0287S	0291M	0292	0293	0295	

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LAMBDA	D /INTEL/	000020	0285S	0287S	0298	0301	0303		
RHO	D /INTEL/	000040	0285S	0287S	0293M	0295			
SIGMA	D /INTEL/	000014	0285S	0287S	0293				
SINU	D /INTEL/	000070	0285S	0287S	0306M	0308	0309		
TAU	D /INTEL/	000010	0285S	0287S	0293				
U	D /INTEL/	000060	0285S	0287S	0305M	0306A	0307A		
XI	D /INTEL/	000004	0285S	0287S	0290	0296			
Z8	D	000344	0285S	0300M	0301	0304M	0305A		
Z9	D	000350	0285S	0299M	0300A	0301A	0302A	0303M	0305A
ZETA	D /INTEL/	000030	0285S	0287S	0290M	0291	0294	0304	
<u> </u> 100		000144	0299D	0302					

```

(0312) C
(0313) SUBROUTINE XFR(KROW,KEY)
(0314) C
(0315) C TRANSFER OSCULATING ELEMENTS O TO WORKING ELEMENTS W (KEY=1)
(0316) C OR MEAN ELEMENTS VAR TO WORKING ELEMENTS (KEY=2) OR THE REVERSE
(0317) C (KEY=3 OR 4).
(0318) C
(0319) DOUBLE PRECISION C(6),E(6),O(6),VAR(6,17),W(6)
(0320) COMMON /INTEL/W/COREL/C,E,O//VAR
(0321) C
(0322) GOTO (10,20,30,40),KEY
(0323) 10 DO 15 I=1,6
(0324) 15 W(I)=O(I)
(0325) RETURN
(0326) 20 DO 25 I=1,6
(0327) 25 W(I)=VAR(I,KROW)
(0328) K9=KROW
(0329) RETURN
(0330) 30 DO 35 I=1,6
(0331) 35 O(I)=W(I)
(0332) RETURN
(0333) 40 DO 45 I=1,6
(0334) 45 VAR(I,KROW)=W(I)
(0335) RETURN
(0336) END

```

I	I	000134	0323M	0324	0326M	0327	0330M	0331	0333M
			0334						
K9	I	000136	0328M						
KEY	I ARGUMENT	000004	0313S	0322					
KROW	I ARGUMENT	000003	0313S	0327	0328	0334			
O	D /COREL/	000060	0319S	0320S	0324	0331M			
VAR	D //	000000	0319S	0320S	0327	0334M			
W	D /INTEL/	000000	0319S	0320S	0324M	0327M	0331	0334	
10		000014	0322	0323D					
15		000016	0323	0324D					
20		000034	0322	0326D					
25		000036	0326	0327D					
30		000065	0322	0330D					
35		000067	0330	0331D					
40		000105	0322	0333D					
45		000107	0333	0334D					

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```

(0337) C
(0338)      SUBROUTINE READ(KROW)
(0339) C
(0340) C      READ SUN AND MOON COORDINATES FROM DISK FILE
(0341) C
(0342)      DOUBLE PRECISION TSL,SL(17,8),ZZ
(0343)      COMMON/LUNSOL/TSL,SL
(0344)      READ(5,100)TSL,(SL(KROW,I),I=5,7)
(0345)      READ(5,100)(SL(KROW,I),I=1,3)
(0346)      READ(5,100)ZZ
(0347)      READ(5,100)ZZ
(0348) 100    FORMAT(4D22.14)
(0349)      DO 200 I=1,2
(0350)      N=4*(I-1)
(0351)      SL(KROW,N+4)=DSQRT(SL(KROW,N+1)**2+SL(KROW,N+2)**2+SL(KROW,
(0352)      + N+3)**2)
(0353)      DO 150 J=1,3
(0354) 150    SL(KROW,N+J)=SL(KROW,N+J)/SL(KROW,N+4)
(0355) 200    CONTINUE
(0356)      RETURN
(0357)      END

```

DSQRT	D EXTERNAL	000000	0351					
I	I	000231	0344M	0345M	0349M	0350		
J	I	000234	0353M	0354				
KROW	I ARGUMENT	000003	0338S	0344	0345	0351	0354	
N	I	000235	0350M	0351	0354			
SL	D /LUNSOL/	000004	0342S	0343S	0344M	0345M	0351M	0354M
TSL	D /LUNSOL/	000000	0342S	0343S	0344M			
ZZ	D	000236	0342S	0346M	0347M			
100		000077	0344	0345	0346	0347	0348D	
150		000155	0353	0354D				
200		000211	0349	0355D				

```

(0358) C
(0359) SUBROUTINE QUE(M,Q0,Y,NT,Q,J)
(0360) C
(0361) C      Q(JS,N)= Q M/N      :      Q(JL,N)= Q M+1/N
(0362) C
(0363) DOUBLE PRECISION Q0(2,20),Y,Q
(0364) IF(M.GT.1) Q0(J,M-1)=0
(0365) IF(M.GT.NT)RETURN
(0366) IF(M.EQ.0)GOTO 100
(0367) Q0(J,M)=Q
(0368) IF(M.EQ.NT)RETURN
(0369) Q0(J,M+1)=(2*M+1)*Y*Q
(0370) N=M+2
(0371) GOTO 200
(0372) 100  Q0(J,1)=Y
(0373)      Q0(J,2)=1.5*Y*Y-.5
(0374)      N=3
(0375) 150  Q0(J,N)=((2*N-1)*Y*Q0(J,N-1)-(N+M-1)*Q0(J,N-2))/(N-M)
(0376)      N=N+1
(0377) 200  IF(N.LE.NT)GOTO 150
(0378)      Q=(2*M+1)*Q
(0379)      RETURN
(0380)      END

```

J	I ARGUMENT	000010	0359S	0364	0367	0369	0372	0373	0375
M	I ARGUMENT	000003	0359S	0364	0365	0366	0367	0368	0369
			0370	0375	0378				
N	I	000202	0370M	0374M	0375	0376M	0377		
NT	I ARGUMENT	000006	0359S	0365	0368	0377			
Q	D ARGUMENT	000007	0359S	0363S	0367	0369	0378M		
Q0	D ARGUMENT	000004	0359S	0363S	0364M	0367M	0369M	0372M	0373M
			0375M						
Y	D ARGUMENT	000005	0359S	0363S	0369	0372	0373	0375	
100		000070	0366	0372D					
150		000112	0375D	0377					
200		000156	0371	0377D					

```

(0381) C
(0382) SUBROUTINE AYBEE(M,BO,X,NT,B,J,IDEC)
(0383) C
(0384) DOUBLE PRECISION BO(2,20),B,X,X2,P0,P1
(0385) X2=X*X
(0386) IF(IDEC.EQ.1)GOTO 200
(0387) C
(0388) C      BO(JS,N)= A M/N+2      :      BO(JL,N)= A M+1/N+2
(0389) C
(0390) P0=B
(0391) P1=(2*M+1)*B/(M+1)
(0392) B=-P1
(0393) N=M+1
(0394) IF(M.GT.2) BO(J,M-2)=P0
(0395) IF(M.GT.1) BO(J,M-1)=P1
(0396) GOTO 150
(0397) 100 BO(J,N)=((2*N+1)*P1-(N+M)*(N-M)*P0/(X2*N))/(N+1)
(0398) P0=P1
(0399) P1=BO(J,N)
(0400) N=N+1
(0401) 150 IF(N.GT.NT)RETURN
(0402) GOTO 100
(0403) C
(0404) C      BO(JS,N)= B M/N      :      BO(JL,N)= B M+1/N
(0405) C
(0406) 200 IF(M.EQ.0) GOTO 210
(0407) BO(J,M)=0
(0408) IF(M.GT.1) BO(J,M-1)=0
(0409) IF(M.GE.NT)RETURN
(0410) 210 BO(J,M+1)=B
(0411) IF(M.EQ.NT-1)RETURN
(0412) BO(J,M+2)=(M+1)*X2*B
(0413) N=M+3
(0414) GOTO 250
(0415) 220 BO(J,N)=(N-1)*X2/((N-M-1)*(N+M-1))*((2*N-3)*BO(J,N-1)-(N-2)
(0416) + *BO(J,N-2))
(0417) N=N+1
(0418) 250 IF(N.LE.NT)GOTO 220
(0419) B=B*X2/2
(0420) RETURN
(0421) END

```

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B	D ARGUMENT	000007	0382S	0384S	0390	0391	0392M	0410	0412
			0419M						
BO	D ARGUMENT	000004	0382S	0384S	0394M	0395M	0397M	0399	0407M
			0408M	0410M	0412M	0415M			
IDEC	I ARGUMENT	000011	0382S	0386					
J	I ARGUMENT	000010	0382S	0394	0395	0397	0399	0407	0408
			0410	0412	0415				
M	I ARGUMENT	000003	0382S	0391	0393	0394	0395	0397	0406
			0407	0408	0409	0410	0411	0412	0413
			0415						
N	I	000371	0393M	0397	0399	0400M	0401	0413M	0415
			0417M	0418					
NT	I ARGUMENT	000006	0382S	0401	0409	0411	0418		
PO	D	000372	0384S	0390M	0394	0397	0398M		
P1	D	000376	0384S	0391M	0392	0395	0397	0398	0399M
X	D ARGUMENT	000005	0382S	0384S	0385				
X2	D	000402	0384S	0385M	0397	0412	0415	0419	
100		000106	0397D	0402					
150		000172	0396	0401D					
200		000177	0386	0406D					
210		000227	0406	0410D					
220		000263	0415D	0418					
250		000344	0414	0418D					

TCOR

By a call to XFR, moves the osculating elements into the working area. Call INTERM to get the intermediate quantities appropriate to those elements. Calculate both elliptic elements and Cartesian coordinates.

TRAN (KROW, KEY)

KROW points to the step value in VAR to which the mean elements apply.

KEY = 1

Call XFR and INTERM to move the osculating elements into position. Calculate the short-period terms that are the differences between mean and osculating elements. Apply these and store the mean elements obtained in the proper place in VAR.

NOTE: In this process, only *part* of the second-order short-period correction to L is made, the remainder being done more conveniently in DERIV.

KEY = 2

Execute the reverse process, from mean elements to osculating elements.

TERM

Using the working elements as a base, calculate the intermediate quantities (e.g., ZETA, TSINE, FC). (Here also Kepler's equation is solved).

R

Moves osculating elements or mean elements into working area or reverse.

AD

Reads the file LUNSOL to acquire lunar solar coordinates and calculates the unit vector and the distance to each body.

GPS

1. The program starts with DATA statements defining the numerical constants and the initial parameters of the problem, which include the initial Cartesian coordinates or elliptic elements for the satellite in the last of the DATA statements (line 29 of the listings).
2. After initialization and calculation of the starting values of the mean elements and their time derivatives, the program in line 57 begins the cycling to set up the starting solution.
3. When this is complete, lines 81 through 89 allow printing information about the starting solution. Note that all of these lines can be commented out or removed without affecting the operation of the program. See also the note below in paragraph 5.
4. Lines 93 through 107 then apply one predictor and one corrector step to move the solution forward one time step; line 116 governs the repetitions of this.
5. Lines 108 through 115 allow printing information about the continuing solution, again these lines can be changed or removed.

NOTE: The form of the write statements in this program, as well as that of the READ statements in subroutine READ (lines 344-347), might need to be altered for running the program on a different computer. PRIME Fortran IV (in which the program is written) recognizes logical unit 1 as the terminal and logical unit 5 as the first file channel. The proper file for reading is opened at the operating system level by OPEN LUNSOL 1 1, which opens the disk file LUNSOL on file channel 1 for reading only.

ENTER (KEY)

1. KEY = 1

Transforms Cartesian coordinates (position and velocity) into osculating elements and, by a call to INTERM, prepares the intermediate quantities useful in further transformations. Calls XFR to move the working set of osculating elements to safe storage.

2. KEY = 2

Transforms elliptic elements into osculating elements (calculating the intermediate quantities in the process) and, by calling XFR, moves the working elements into safe storage as the osculating elements.



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APPENDIX B
PROGRAM AND SUBROUTINES

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```

(0656) C
(0657) SUBROUTINE EXTRAP(KROW1,KROW)
(0658) C
(0659) DOUBLE PRECISION DEL(18,6,17),ADAM(17),VAR(6,17),VEL(6,17),
(0660) + S,XX(6),HH(20)
(0661) DIMENSION NT(3)
(0662) COMMON //VAR,VEL,DEL,NEQ,KMAX/COEF/XX,NT,HH,ADAM
(0663) DO 110 N=1,NEQ
(0664) S=0
(0665) DO 100 KC=1,KMAX
(0666) KC1=KMAX+1-KC
(0667) DEL(KC1,N,KROW)=DEL(KC1,N,KROW1)+DEL(KC1+1,N,KROW)
(0668) 100 S=S+ADAM(KC1)*DEL(KC1,N,KROW)
(0669) 110 VAR(N,KROW)=VAR(N,KROW1)+S
(0670) CALL DERIV(KROW,1)
(0671) RETURN
(0672) END

```

ADAM	D /COEF/	000153	0659S	0662S	0668		
DEL	D //	001460	0659S	0662S	0667M	0668	
DERIV	R EXTERNAL	000000	0670				
KC	I	000146	0665M	0666			
KC1	I	000147	0666M	0667	0668		
KMAX	I //	017741	0662S	0665	0666		
KROW	I ARGUMENT	000004	0657S	0667	0668	0669	0670A
KROW1	I ARGUMENT	000003	0657S	0667	0669		
N	I	000150	0663M	0667	0668	0669	
NEQ	I //	017740	0662S	0663			
S	D	000151	0659S	0664M	0668M	0669	
VAR	D //	000000	0659S	0662S	0669M		
100		000053	0665	0668D			
110		000106	0663	0669D			

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100	002411	0484	0510D		
110	002537	0517	0518	0519D	
120	002662	0533	0535D		
130	002702	0529	0537D		
140	002752	0542	0543D		
150	002771	0536	0544D		
160	003037	0551	0553D		
180	003115	0557	0562D		
200	003175	0561	0569D		
300	003246	0573	0576D		
400	003452	0577	0588	0590D	
410	003525	0594	0596D		
500	003571	0555	0576	0598D	
510	003577	0597	0599D		
530	004165	0599	0617D		
540	004171	0617	0618	0619D	
550	004350	0628	0630D		
560	004401	0623	0632D		
570	004547	0641	0643D		
600	004572	0443	0620	0631	0644D
650	004637	0645	0648D		
70	001170	0438	0440D		
80	001434	0465	0466	0467D	
90	002006	0489	0490	0491D	

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QUE	R EXTERNAL	000000	0558	0570	0627	0640				
R	D	005027	0428S	0622M	0624	0625	0632	0635		
RE	D /COEF/	000014	0428S	0434S	0531	0624				
RHO	D /INTEL/	000040	0428S	0434S	0451	0452	0458	0459	0468	
			0483	0503	0504	0509	0603	0612	0636	
RHO2	D	005033	0428S	0603M	0605	0606	0608	0609	0612	
SIGMA	D /INTEL/	000014	0428S	0434S	0449	0450	0454	0455	0460	
			0462	0471	0485	0494	0498	0507	0621	
			0637							
SINB	D	005037	0428S	0621M	0627A	0636	0637			
SINU	D /INTEL/	000070	0428S	0434S	0621	0637				
SL	D /LUNSOL/	000004	0428S	0434S	0485	0486	0487	0488	0491	
			0493	0494	0495	0496	0498	0499	0500	
			0501	0507	0508	0537	0632	0637		
SM	D	005043	0428S	0550M	0566	0568M	0597			
SM1	D	005047	0428S	0566M	0567	0568	0592			
TAU	D /INTEL/	000010	0428S	0434S	0449	0450	0453	0456	0461	
			0463	0472	0485	0493	0499	0508	0621	
			0637							
V	D	005053	0428S	0609M	0614	0615	0616			
V1	D	005057	0428S	0449M	0451	0481	0486	0493	0494	
			0495	0496	0497					
V2	D	005063	0428S	0450M	0458	0487	0487	0498	0499	
			0500	0501	0502					
VAR	D //	000000	0428S	0434S	0647M					
VEL	D //	000630	0428S	0434S	0648M	0649M	0650M	0651M	0652M	
			0653M							
X	D	005067	0428S	0527M	0548	0559A	0571A	0585		
XFR	R EXTERNAL	000000	0441							
XI	D /INTEL/	000004	0428S	0434S	0449	0450	0453	0456	0460	
			0462	0469	0475	0486	0487	0493	0496	
			0498	0500	0505	0511	0514			
Z5	D	005073	0428S	0608M	0609	0610	0611			
Z6	D	005077	0428S	0607M	0609	0612				
Z7	D	005103	0428S	0485M	0486	0487	0488	0492M	0493	
			0494	0495	0496	0497	0498	0499	0500	
			0501	0502	0503	0538M	0539M	0540	0626M	
			0609	0614	0625M	0629M	0630	0633M	0634M	
			0635M	0642M	0643					
Z8	D	005107	0428S	0451M	0452	0455	0456	0458M	0459	
			0462	0463	0504M	0505	0506	0532M	0534M	
			0535	0540M	0541	0582M	0583	0586	0589	
			0605M	0609	0614	0624M	0625	0629	0632M	
			0635	0642						
Z9	D	005113	0428S	0452M	0453	0454	0457	0459M	0460	
			0461	0464	0468M	0469	0470	0473	0474M	
			0475	0476	0479	0503M	0504	0507	0508	
			0509	0510M	0511	0512	0513	0514	0515	
			0516	0531M	0532	0534	0537M	0540	0541	
			0543	0581M	0582	0585	0604M	0609	0610	
			0611	0612	0614	0636M	0637M	0640A		
ZETA	D /INTEL/	000030	0428S	0434S	0474	0479	0510	0513	0516	
			0527	0605	0609	0612	0614	0622		

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ECOSF	D /INTEL/	000074	0428S	0434S	0622				
ETA	D /INTEL/	000000	0428S	0434S	0449	0450	0454	0455	0461
			0463	0470	0476	0486	0487	0494	0495
			0499	0501	0506	0512	0515		
FO	D	000546	0428S	0552M	0583M	0584M	0585M	0586M	0591
			0592	0593M	0595	0596M			
GAM2P	D /INTEL/	000114	0428S	0434S	0604				
GAMMA	D	004775	0428S	0483M	0488M	0558A	0570A		
H	D	000576	0428S	0535M	0541M	0543M	0581	0584	
HO	D /COEF/	000033	0428S	0434S	0535	0630			
I	I	005001	0438M	0439	0440	0466M	0467	0490M	0491
			0518M	0519	0551M	0552	0553	0594M	0595
			0596	0617M	0619				
INTERM	I EXTERNAL	000000	0442						
ISUM	I	005005	0443M	0448	0484	0526	0529	0539	0548
			0559A	0571A	0579	0589	0599	0623	0634
J	I	005006	0465M	0467	0489M	0491	0517M	0519	0618M
			0619						
JL	I	005007	0569M	0570A	0571A	0584	0585		
JS	I	005010	0545M	0562M	0569	0580	0581	0582	0584
			0585						
KEY	I ARGUMENT	000004	0423S	0588	0616	0620	0645		
KR	I ARGUMENT	000003	0423S	0441A	0485	0486	0487	0488	0491
			0493	0494	0495	0496	0498	0499	0500
			0501	0507	0508	0537	0632	0637	0647
			0648	0649	0650	0651	0652	0653	
KS	I	005011	0448M	0485	0486	0487	0488	0491	0493
			0494	0495	0496	0498	0499	0500	0501
			0507	0508	0537	0632	0637		
L	D /INTEL/	000024	0428S	0434S	0474	0480	0510	0528	0604
			0615	0646M	0647	0652			
L2Z	D /INTEL/	000034	0428S	0434S	0449	0450	0468	0471	0472
			0481	0482	0492	0604			
M	I	005013	0556M	0557	0563	0570	0571	0572	0573
			0574	0575	0585	0587	0591	0592	
MM	D /COEF/	000020	0428S	0434S	0538	0633			
MS	D /COEF/	000024	0428S	0434S	0539	0634			
MU	D /COEF/	000004	0428S	0434S	0480	0528	0532	0540	0604
			0615	0625	0635	0646	0652		
MUP	I	005014	0555M	0556					
N	I	005015	0533M	0535	0542M	0543	0577M	0578	0579
			0581	0582	0584	0585	0587	0589	0628M
			0630	0641M	0643				
NO	I	005016	0572M	0574M	0576	0577	0597		
N5	I	005017	0578M	0579M	0585	0586			
NMU	I	005020	0554M	0555					
NN	I	005021	0580M						
NTER	I	005022	0526M	0533	0542	0554	0558A	0559A	0570A
			0571A	0576	0577	0627A	0628	0640A	0641
NTERM	I /COEF/	000030	0433S	0434S	0526				
Q	D	000716	0428S	0558A	0570A	0581	0584	0627A	0630
			0640A	0643					
Q1	D	005023	0428S	0544M	0558A	0570A	0626M	0627A	0639M
			0640A						

```

(0634)      IF (ISUM.EQ.3) Z7=MS
(0635)      Z7=Z7*MU*Z8*Z8/R
(0636)      Z9=C2*SINB/(1+RHO)
(0637)      Z9=SL(KR,KS+1)*(COSU-Z9*TAU)+SL(KR,KS+2)*(SINU-Z9*SIGMA)+
(0638)      + SL(KR,KS+3)*SINB
(0639)      Q1=1
(0640)      CALL QUE(0,Q,Z9,NTER,Q1,1)
(0641)      DO 570 N=2,NTER
(0642)      Z7=Z7*Z8
(0643) 570  DELF2=DELF2-Z7*Q(1,N)
(0644) 600  CONTINUE
(0645)      IF (KEY.EQ.1) GOTO 650
(0646)      L=L+L**3/(MU*MU)*DELF2
(0647)      VAR(6,KR)=L
(0648) 650  VEL(1,KR)=-DE(2)
(0649)      VEL(2,KR)=DE(1)
(0650)      VEL(3,KR)=-DE(4)
(0651)      VEL(4,KR)=DE(3)
(0652)      VEL(5,KR)=MU*MU/L**3-DE(5)
(0653)      VEL(6,KR)=0
(0654)      RETURN
(0655)      END

```

A	D	004715	0428S	0528M	0531	0532	0537	0540	0586
			0622						
ALPHA	D	004721	0428S	0481M	0486M	0511	0512	0513	0520M
			0567	0568	0607	0610			
AYBEE	R EXTERNAL	000000	0559	0571					
B	D	000006	0428S	0559A	0571A	0582	0584	0585	
B1	D	004725	0428S	0547M	0548M	0559A	0571A		
BETA	D	004731	0428S	0482M	0487M	0514	0515	0516	0521M
			0567	0568	0607	0611			
C	D	000246	0428S	0439M	0453M	0454M	0455M	0456M	0457M
			0460M	0461M	0462M	0463M	0464M	0467M	0469M
			0470M	0471M	0472M	0473M	0475M	0476M	0477M
			0478M	0479M	0480M	0491M	0493M	0494M	0495M
			0496M	0497M	0498M	0499M	0500M	0501M	0502M
			0505M	0506M	0507M	0508M	0509M	0511M	0512M
			0513M	0514M	0515M	0516M	0519M	0619	
C1	D /INTEL/	000044	0428S	0434S	0519	0520	0521		
C2	D /INTEL/	000050	0428S	0434S	0467	0481	0482	0488	0505
			0506	0507	0508	0509	0621	0636	
CM	D	004735	0428S	0549M	0565	0567M	0589	0595	0597
CM1	D	004741	0428S	0565M	0567	0568	0591		
COSU	D /INTEL/	000064	0428S	0434S	0621	0637			
D	D	004745	0428S	0560M	0564M	0575M	0581	0584	0587M
D1	D	004765	0428S	0546M	0560	0563M	0564		
DE	D	000466	0428S	0440M	0619M	0648	0649	0650	0651
			0652						
DELF2	D	004771	0428S	0530M	0589M	0616M	0630M	0643M	0646
DF	D	000516	0428S	0553M	0591M	0592M	0595M	0610M	0611M
			0612M	0614M	0615M	0619			

```

(0581)      Z9=D*H(N)*Q(JS,N)
(0582)      Z8=Z9*B(JS,N)
(0583)      F0(1)=F0(1)+Z8
(0584)      F0(3)=F0(3)+D*H(N)*Q(JL,N)*B(JS,N)
(0585)      F0(4)=F0(4)+(M-N5-1)/X*Z9*(B(JS,N)+B(JL,N))
(0586)      F0(5)=F0(5)+N5/A*Z8
(0587)      D=-D*(N-M+1)/(N+M+2)
(0588)      IF (KEY.EQ.1) GOTO 400
(0589)      IF (N.GT.2.OR.ISUM.GT.1) DELF2=DELF2+Z8*CM
(0590) 400  CONTINUE
(0591)      DF(1)=DF(1)+F0(1)*M*CM1
(0592)      DF(2)=DF(2)-F0(1)*M*SM1
(0593)      F0(1)=0
(0594)      DO 410 I=3,5
(0595)      DF(I)=DF(I)+F0(I)*CM
(0596) 410  F0(I)=0
(0597)      IF(CM*CM+SM*SM.LT.1.0D-8.AND.NO.GT.2)GOTO 510
(0598) 500  CONTINUE
(0599) 510  IF (ISUM.NE.1)GOTO 530
(0600)      C
(0601)      C      SECOND ORDER CONTRIBUTION OF SECOND HARMONIC
(0602)      C
(0603)      RHO2=RHO*RHO
(0604)      Z9=(MU*GAM2P)**2*L2Z/(48*L**3)
(0605)      Z8=5*ZETA*(1+RHO2*(-3.6+RHO2))
(0606)      Z7=4*(1+RHO2*(-6+9*RHO2))
(0607)      Z6=ALPHA*ALPHA-BETA*BETA
(0608)      Z5=2-30*RHO2
(0609)      V=Z9*((Z8+Z7)*ZETA+5*(-1+RHO2*(2+7*RHO2))-Z6*Z5)
(0610)      DF(1)=DF(1)-2*Z9*Z5*ALPHA
(0611)      DF(2)=DF(2)+2*79*Z5*BETA
(0612)      DF(3)=DF(3)+Z9*4*RHO*(5*(1+7*RHO2)+ZETA*(12*(-1+3*RHO2)+ZETA*
(0613)      + (-9+5*RHO2))+15*Z6)
(0614)      DF(4)=DF(4)-ZETA*(ZETA*Z9*(2*Z8+Z7)-7*V)
(0615)      DF(5)=DF(5)-5*MU*V/(L*L)
(0616)      IF (KEY.NE.1) DELF2=DELF2+V
(0617) 530  DO 540 I=1,5
(0618)      DO 540 J=1,5
(0619) 540  DE(I)=DE(I)+DF(J)*C(J,I)
(0620)      IF (KEY.EQ.1) GOTO 600
(0621)      SINB=C2*(SIGMA*SINU+TAU*COSU)
(0622)      R=A*ZETA*ZETA/(ECOSF+1)
(0623)      IF (ISUM.NE.1) GOTO 560
(0624)      Z8=RE/R
(0625)      Z7=-MU/R*Z8*Z8
(0626)      Q1=1
(0627)      CALL QUE(0,Q,SINB,NTER,Q1,1)
(0628)      DO 550 N=3,NTER
(0629)      Z7=Z7*Z8
(0630) 550  DELF2=DELF2-Z7*H0(N)*Q(1,N)
(0631)      GOTO 600
(0632) 560  Z8=R/SL(KR,KS+4)
(0633)      Z7=MM

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(0528)      A=L* $\mu$ /MU
(0529)      IF (ISUM.NE.1)GOTO 130
(0530)      DELF2=0
(0531)      Z9=RE/A
(0532)      Z8=MU/A*Z9
(0533)      DO 120 N=2, NTER
(0534)      Z8=Z8*Z9
(0535) 120   H(N)=-H0(N)*Z8
(0536)      GOTO 150
(0537) 130   Z9=A/SL(KR,KS+4)
(0538)      Z7=MM
(0539)      IF (ISUM.EQ.3)Z7=MS
(0540)      Z8=MU*Z7/A*Z9
(0541)      H(1)=Z8*Z9
(0542)      DO 140 N=2, NTER
(0543) 140   H(N)=H(N-1)*Z9
(0544) 150   Q1=1
(0545)      JS=1
(0546)      D1=2
(0547)      B1=1
(0548)      IF (ISUM.EQ.1)B1=X
(0549)      CM=1
(0550)      SM=0
(0551)      DO 160 I=1,6
(0552)      FO(I)=0
(0553) 160   DF(I)=0
(0554)      NMU=NTER+1
(0555)      DO 500 MUP=1, NMU
(0556)      M=MUP-1
(0557)      IF (M.GT.0)GOTO 180
(0558)      CALL QUE(0,Q,GAMMA,NTER,Q1,1)
(0559)      CALL AYBEE(0,B,X,NTER,B1,1,ISUM)
(0560)      D=D1/2
(0561)      GOTO 200
(0562) 180   JS=3-JS
(0563)      D1=D1/(2*M)
(0564)      D=D1
(0565)      CM1=CM
(0566)      SM1=SM
(0567)      CM=CM1*ALPHA-SM1*BETA
(0568)      SM=SM1*ALPHA+CM1*BETA
(0569) 200   JL=3-JS
(0570)      CALL QUE(M+1,Q,GAMMA,NTER,Q1,JL)
(0571)      CALL AYBEE(M+1,B,X,NTER,B1,JL,ISUM)
(0572)      NO=M
(0573)      IF (M.GE.2) GOTO 300
(0574)      NO=M+2
(0575)      D=-D/(2*M+2)
(0576) 300   IF (NO.GT.NTER)GOTO 500
(0577)      DO 400 N=NO, NTER, 2
(0578)      N5=N
(0579)      IF (ISUM.EQ.1)N5=-N-1
(0580)      NN=JS

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(0475)      C(4,2)=-XI*Z9
(0476)      C(4,1)=-ETA*Z9
(0477)      C(4,4)=0
(0478)      C(4,3)=0
(0479)      C(4,5)=(1-ZETA)*Z9
(0480)      C(5,5)=2*L/MU
(0481)      ALPHA=C2*V1*L2Z
(0482)      BETA=C2*V2*L2Z
(0483)      GAMMA=RHO
(0484)      IF (ISUM.EQ.1)GOTO 100
(0485)      Z7=SL(KR,KS+1)*TAU+SL(KR,KS+2)*SIGMA
(0486)      ALPHA=SL(KR,KS+3)*ALPHA+SL(KR,KS+1)*XI-SL(KR,KS+2)*ETA-V1*Z7
(0487)      BETA=SL(KR,KS+3)*BETA-SL(KR,KS+1)*ETA-SL(KR,KS+2)*XI-V2*Z7
(0488)      GAMMA=SL(KR,KS+3)*GAMMA-C2*Z7
(0489)      DO 90 J=1,5
(0490)      DO 90 I=1,3
(0491) 90    C(I,J)=SL(KR,KS+3)*C(I,J)
(0492)      Z7=Z7/L2Z
(0493)      C(1,2)=C(1,2)+SL(KR,KS+1)-Z7*(TAU+2*V1*X1)
(0494)      C(1,1)=C(1,1)-SL(KR,KS+2)+Z7*(SIGMA-2*V1*ETA)
(0495)      C(1,4)=C(1,4)-SL(KR,KS+2)*V1+ETA*Z7
(0496)      C(1,3)=C(1,3)-SL(KR,KS+1)*V1-X1*Z7
(0497)      C(1,5)=C(1,5)+2*V1*Z7
(0498)      C(2,2)=C(2,2)-SL(KR,KS+2)+Z7*(SIGMA-2*V2*X1)
(0499)      C(2,1)=C(2,1)-SL(KR,KS+1)+Z7*(TAU-2*V2*ETA)
(0500)      C(2,4)=C(2,4)-SL(KR,KS+2)*V2+X1*Z7
(0501)      C(2,3)=C(2,3)-SL(KR,KS+1)*V2+ETA*Z7
(0502)      C(2,5)=C(2,5)+2*V2*Z7
(0503)      Z9=Z7/(1+RHO)
(0504)      Z8=2*RHO*Z9
(0505)      C(3,2)=C(3,2)-C2*X1*Z8
(0506)      C(3,1)=C(3,1)-C2*ETA*Z8
(0507)      C(3,4)=C(3,4)-C2*(SL(KR,KS+2)-SIGMA*Z9)
(0508)      C(3,3)=C(3,3)-C2*(SL(KR,KS+1)-TAU*Z9)
(0509)      C(3,5)=C(3,5)+C2*2*RHO*Z9
(0510) 100   Z9=1/(2*L*(1+ZETA))
(0511)      C(1,2)=C(1,2)-ALPHA*X1*Z9
(0512)      C(1,1)=C(1,1)-ALPHA*ETA*Z9
(0513)      C(1,5)=C(1,5)-2*ALPHA*ZETA*Z9
(0514)      C(2,2)=C(2,2)-BETA*X1*Z9
(0515)      C(2,1)=C(2,1)-BETA*ETA*Z9
(0516)      C(2,5)=C(2,5)-2*BETA*ZETA*Z9
(0517)      DO 110 J=1,5
(0518)      DO 110 I=1,2
(0519) 110   C(I,J)=C1*C(I,J)
(0520)      ALPHA=C1*ALPHA
(0521)      BETA=C1*BETA
(0522)  C
(0523)  C      RECURSIVE CALCULATION OF DERIVATIVES OF PERTURBING
(0524)  C      POTENTIAL WITH RESPECT TO XI,ETA,SIGMA,TAU,L
(0525)  C
(0526)      NTER=NTERM(ISUM)
(0527)      X=1/ZETA

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(0422) C
(0423) SUBROUTINE DERIV(KR,KEY)
(0424) C
(0425) C KEY=1: DO NOT CALCULATE SECOND ORDER CORRECTIONS TO L
(0426) C KEY=2: DO
(0427) C
(0428) DOUBLE PRECISION VAR(6,17),VEL(6,17),XI,ETA,SIGMA,TAU,L,LAMBDA,
(0429) + ZETA,L2Z,RHO,C1,C2,V1,V2,Z5,Z6,Z7,Z8,Z9,ALPHA,BETA,GAMMA,C(6,6),
(0430) + DRAD,MU,GAM2N,RE,MM,MS,X,A,B1,D,D1,Q1,CM1,SM1,CM,SM,R,B(2,20),
(0431) + Q(2,20),FO(6),DF(6),DE(6),SL(17,8),H(20),HO(20),TT,DELF2,SINB,
(0432) + ECLON,U,SINU,COSU,ECOSF,ESINF,EC,ES,GAM2P,RHO2,V
(0433) DIMENSION NTERM(3)
(0434) COMMON/INTEL/ETA,XI,TAU,SIGMA,LAMBDA,L,ZETA,L2Z,RHO,C1,C2,
(0435) + ECLON,U,COSU,SINU,ECOSF,ESINF,EC,ES,GAM2P
(0436) + //VAR,VEL/COEF/DRAD,MU,GAM2N,RE,MM,MS,NTERM,HO/LUNSOL/TT,SL
(0437) C
(0438) DO 70 I=1,5
(0439) C(5,I)=0
(0440) 70 DE(I)=0
(0441) CALL XFR(KR,2)
(0442) CALL INTERM
(0443) DO 600 ISUM=1,3 /* 1=ZONALS, 2=MOON, 3=SUN
(0444) C
(0445) C JACOBIAN OF ALPHA,BETA,GAMMA,X,A WITH RESPECT TO
(0446) C ETA,XI,TAU,SIGMA,L
(0447) C
(0448) KS=4*(ISUM-2)
(0449) V1=(XI*TAU-ETA*SIGMA)/L2Z
(0450) V2=(-XI*SIGMA-ETA*TAU)/L2Z
(0451) Z8=V1/(1+RHO)
(0452) Z9=2*RHO*Z8
(0453) C(1,2)=TAU+Z9*XI
(0454) C(1,1)=-SIGMA+Z9*ETA
(0455) C(1,4)=-ETA-SIGMA*Z8
(0456) C(1,3)=XI-TAU*Z8
(0457) C(1,5)=-Z9
(0458) Z8=V2/(1+RHO)
(0459) Z9=2*RHO*Z8
(0460) C(2,2)=-SIGMA+Z9*XI
(0461) C(2,1)=-TAU+Z9*ETA
(0462) C(2,4)=-XI-SIGMA*Z8
(0463) C(2,3)=-ETA-TAU*Z8
(0464) C(2,5)=-Z9
(0465) DO 80 J=1,5
(0466) DO 80 I=1,2
(0467) 80 C(I,J)=C2*C(I,J)
(0468) Z9=2*(1-RHO)/L2Z
(0469) C(3,2)=-XI*Z9
(0470) C(3,1)=-ETA*Z9
(0471) C(3,4)=-2*SIGMA/L2Z
(0472) C(3,3)=-2*TAU/L2Z
(0473) C(3,5)=Z9
(0474) Z9=-1/(L*ZETA*ZETA)

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B	D ARGUMENT	000007	0382S 0419M	0384S	0390	0391	0392M	0410	0412
BO	D ARGUMENT	000004	0382S 0408M	0384S 0410M	0394M 0412M	0395M 0415M	0397M	0399	0407M
IDEC	I ARGUMENT	000011	0382S	0386					
J	I ARGUMENT	000010	0382S 0410	0394 0412	0395 0415	0397	0399	0407	0408
M	I ARGUMENT	000003	0382S 0407 0415	0391 0408	0393 0409	0394 0410	0395 0411	0397 0412	0406 0413
N	I	000371	0393M 0417M	0397 0418	0399	0400M	0401	0413M	0415
NT	I ARGUMENT	000006	0382S	0401	0409	0411	0418		
PO	D	000372	0384S	0390M	0394	0397	0398M		
P1	D	000376	0384S	0391M	0392	0395	0397	0398	0399M
X	D ARGUMENT	000005	0382S	0384S	0385				
X2	D	000402	0384S	0385M	0397	0412	0415	0419	
100		000106	0397D	0402					
150		000172	0396	0401D					
200		000177	0386	0406D					
210		000227	0406	0410D					
220		000263	0415D	0418					
250		000344	0414	0418D					

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(0422) C
(0423) SUBROUTINE DERIV(KR,KEY)
(0424) C
(0425) C KEY=1: DO NOT CALCULATE SECOND ORDER CORRECTIONS TO L
(0426) C KEY=2: DO
(0427) C
(0428) DOUBLE PRECISION VAR(6,17),VEL(6,17),XI,ETA,SIGMA,TAU,L,LAMBDA,
(0429) + ZETA,L2Z,RHO,C1,C2,V1,V2,Z5,Z6,Z7,Z8,Z9,ALPHA,BETA,GAMMA,C(6,6),
(0430) + DRAD,MU,GAM2N,RE,MM,MS,X,A,B1,D,D1,Q1,CM1,SM1,CM,SM,R,B(2,20),
(0431) + Q(2,20),FO(6),DF(6),DE(6),SL(17,8),H(20),HO(20),TT,DELF2,SINB,
(0432) + ECLON,U,SINU,COSU,ECOSF,ESINF,EC,ES,GAM2P,RHO2,V
(0433) DIMENSION NTERM(3)
(0434) COMMON/INTEL/ETA,XI,TAU,SIGMA,LAMBDA,L,ZETA,L2Z,RHO,C1,C2,
(0435) + ECLON,U,COSU,SINU,ECOSF,ESINF,EC,ES,GAM2P
(0436) + //VAR,VEL/COEF/DRAD,MU,GAM2N,RE,MM,MS,NTERM,HO/LUNSOL/TT,SL
(0437) C
(0438) DO 70 I=1,5
(0439) C(5,I)=0
(0440) 70 DE(I)=0
(0441) CALL XFR(KR,2)
(0442) CALL INTERM
(0443) DO 600 ISUM=1,3 /* 1=ZONALS, 2=MOON, 3=SUN
(0444) C
(0445) C JACOBIAN OF ALPHA,BETA,GAMMA,X,A WITH RESPECT TO
(0446) C ETA,XI,TAU,SIGMA,L
(0447) C
(0448) KS=4*(ISUM-2)
(0449) V1=(XI*TAU-ETA*SIGMA)/L2Z
(0450) V2=(-XI*SIGMA-ETA*TAU)/L2Z
(0451) Z8=V1/(1+RHO)
(0452) Z9=2*RHO*Z8
(0453) C(1,2)=TAU+Z9*XI
(0454) C(1,1)=-SIGMA+Z9*ETA
(0455) C(1,4)=-ETA-SIGMA*Z8
(0456) C(1,3)=XI-TAU*Z8
(0457) C(1,5)=-Z9
(0458) Z8=V2/(1+RHO)
(0459) Z9=2*RHO*Z8
(0460) C(2,2)=-SIGMA+Z9*XI
(0461) C(2,1)=-TAU+Z9*ETA
(0462) C(2,4)=-XI-SIGMA*Z8
(0463) C(2,3)=-ETA-TAU*Z8
(0464) C(2,5)=-Z9
(0465) DO 80 J=1,5
(0466) DO 80 I=1,2
(0467) 80 C(I,J)=C2*C(I,J)
(0468) Z9=2*(1-RHO)/L2Z
(0469) C(3,2)=-XI*Z9
(0470) C(3,1)=-ETA*Z9
(0471) C(3,4)=-2*SIGMA/L2Z
(0472) C(3,3)=-2*TAU/L2Z
(0473) C(3,5)=Z9
(0474) Z9=-1/(L*ZETA*ZETA)

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QUE

Calculates the quantities Q required for the harmonics.

AYBEE

Calculates the quantities A and B required for the harmonics.

DERIV (KEY)

Loads the mean elements with XFR and INTERM; and in separate calculations for zonals, moon, and sun:

1. Finds the Jacobian matrix of the variables ALPHA, BETA, GAMMA, X, A (in terms of which the derivatives of the Hamiltonian are naturally determined) with respect to the mean elements.
2. Calculates the derivatives of the perturbing potential as well as the potential value itself. The second order of the second harmonic is included.
3. If KEY=2 (at entry to the program only), finds the instantaneous value of the perturbing potential and uses it to calculate the remaining part of the second-order-short-period correction for L. If KEY=1, this step is omitted.
4. Use the Jacobian and the derivatives to accumulate the derivatives of the mean elements required for the next integration step, storing these in VEL.

EXTRAP (KROW1, DROW)

Extends the difference table DEL from the preceding row KROW1 to the current row KROW, calculates the step in VAR, and calls DERIV for the new row.

SUBROUTINE TAXONOMY

<u>NAME</u>	<u>CALLS</u>	<u>IS CALLED BY</u>	<u>USES COMMON</u>
GPS (MAIN)	ENTER OSTCOR ELTRAN READ DERIV EXTRAP		(BLANK) COEF COREL
ENTER	INTERM XFR	GPS	COEF COREL INTEL
OSTCOR	INTERM XFR	GPS	COEF COREL
ELTRAN	INTERM XFR	GPS	(BLANK) COEF COREL INTEL
INTERM		ENTER OSTCOR ELTRAN	COEF INTEL
XFR		ENTER OSTCOR ELTRAN	(BLANK) COREL INTEL
READ		GPS	LUNSOL
QUE		DERIV	
AYBEE		DERIV	
DERIV	INTERM XFR QUE AYBEE	GPS EXTRAP	COEF INTEL LUNSOL
EXTRAP	DERIV	GPS	(BLANK) COEF

VARIABLES AND COMMON BLOCKS

(BLANK) Integration quantities

VAR (6,17)

The six integration variables (mean elements) for 17 consecutive time steps

VEL (6,17)

The derivatives in a similar array

DEL (18,6,17)

The differences used in the numerical integration

NEQ

Number of equations

KMAX

Order of numerical integration procedure

COEF Numerical coefficients

DRAD

Degrees/radians

MU

Gravitational const

GAM2N

 $12 * J_2 * (MU * RE)^{**2}$

RE

Earth radius

MM

Lunar mass

MS

Solar mass

NTERM (3)

Highest order harmonic: zonal, lunar, solar

HAR(20)

Coefficients of zonal harmonics

ADAM (17)

Adams coefficients

COREL Coordinates and elements

X,Y,Z,VX,VY,VZ (or C(6))

Coordinates

A,ECC,INC,ELL,G,H (or E(6))

Elliptic elements

O(6)

Osculating elements

INTEL Intermediate quantities in element transformations

ETA,XI,TAU,SIGMA,LAMDA,L (or W(6))

Working elements (osculating on mean)

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[14 variables, such as ZETA, RHO, U,
ECOSF— see listing]

Intermediate variables

LUNSOL Lunar and solar coordinates

TSL

Time

SL (17, 8)

For 17 time steps, the three
components of the unit vector
to the moon and the lunar
distance; followed by the
corresponding 4 quantities
for the sum

The variables not found in common declarations are few in number and usually nonce variables. An exception is the variable KS that counts the steps in the integration. The variable TIM (which is defined but not used) is the time; its definition is

$$TIM = STEP * (KS - 2) + TIMO \text{ (seconds)}$$

External procedures not in the subroutine list (system subroutines or functions):

EXIT	DSQRT
MOD	DABS
SQRT	DATAN2
DCOS	DMOD
DSIN	DSIGN

All of the DOUBLE PRECISION declarations and functions are to produce a 64-bit value (48-bit mantissa). On a machine with sufficient word length, all the DOUBLE PRECISION declarations could be replaced with REAL and all the double functions with their single-precision counterparts.

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APPENDIX C

COORDINATE TRANSFORMATIONS

TRANSFORMATION FROM CARTESIAN COORDINATES TO OSCULATING CANONICAL VARIABLES

[This is merely summarized here, since most of it is to be found in any text on orbital mechanics and the rest is obvious from the definition of the canonical variables. In the program, it is found in subroutine ENTER (lines 147-178)]. Vectors are defined by an underline ; unit vectors by a caret $\hat{}$.

$$\text{Angular momentum } \underline{h} = \underline{r} \times \underline{v}$$

$$\text{magnitude } h = \sqrt{h_x^2 + h_y^2 + h_z^2}$$

$$\text{Unit vector } \hat{h} = \begin{pmatrix} \sin i \sin h \\ -\sin i \cos h \\ \cos i \end{pmatrix}$$

$$\text{Hence, } \hat{h} = \begin{pmatrix} -I_s \\ -I_c \\ \rho \end{pmatrix}$$

$$\text{Unit vector } \hat{L}_1 = \begin{pmatrix} 1 - I_s^2/(1+\rho) \\ -I_s I_c/(1+\rho) \\ I_s \end{pmatrix}$$

$$\text{Unit vector } \hat{L}_2 = \begin{pmatrix} -I_s I_c/(1+\rho) \\ 1 - I_c^2/(1+\rho) \\ I_c \end{pmatrix}$$

$\hat{L}_1$, $\hat{L}_2$, and $\hat{h}$ are mutually orthogonal and define a Cartesian system rotated from the XYZ system by the angle i about the line of nodes. Therefore, $\hat{L}_1$ and $\hat{L}_2$ define the orbital plane, the radius vector makes the angle u with $\hat{L}_1$, so that

$$\cos u = \hat{r} \cdot \hat{L}_1$$

$$\sin u = \hat{r} \cdot \hat{L}_2$$



[The formulation here differs from the customary one in terms of the basis defined by $\hat{N}$ (the vector along the line of nodes), $\hat{h}$, and $\hat{h} \times \hat{N}$. When the inclination is small, $\hat{N}$ and $\hat{h} \times \hat{N}$ are ill-defined, a disadvantage not suffered by $\hat{L}_1$ and $\hat{L}_2$. The change entails also using the longitudes (u, α, λ) instead of the anomalies (f, E, ℓ)].

From here on the algorithm is straightforward:

$$\left. \begin{aligned} h_x &= yv_z - zv_y \\ h_y &= zv_x - xv_z \\ h_z &= xv_y - yv_x \end{aligned} \right\}$$

$$h = \sqrt{h_x^2 + h_y^2 + h_z^2}$$

$$I_S = -h_x/h$$

$$I_C = -h_y/h$$

$$\rho = h_z/h$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\cos u = (x + zI_S/(1+\rho))/r$$

$$\sin u = (y + zI_C/(1+\rho))/r$$

$$e \cos f = (h^2/\mu r) - 1$$

$$e \sin f = (h/\mu r)(xv_x + yv_y + zv_z)$$

$$e^2 = (e \sin f)^2 + (e \cos f)^2$$

$$\xi = \sqrt{1-e^2}$$

$$e \cos \tilde{\omega} = e \cos f \cos u + e \sin f \sin u$$

$$e \sin \tilde{\omega} = e \cos f \sin u - e \sin f \cos u$$

$$\frac{h^2}{\mu r} \cos \alpha = \cos u + e \cos \tilde{\omega} + e \sin \tilde{\omega} \cdot e \sin f / (1 + \xi)$$

$$\frac{h^2}{\mu r} \sin \alpha = \sin u + e \sin \tilde{\omega} - e \cos \tilde{\omega} \cdot e \sin f / (1 + \xi)$$

$$\alpha = \tan^{-1} \frac{(h^2/\mu r) \sin \alpha}{(h^2/\mu r) \cos \alpha}$$

$$L = h/\xi$$

$$\lambda = \alpha - e \cos \tilde{\omega} \sin \alpha + e \sin \tilde{\omega} \cos \alpha$$

$$\xi = \sqrt{2L/(1+\xi)} e \cos \tilde{\omega}$$

$$\eta = -\sqrt{2L/(1+\xi)} e \sin \tilde{\omega}$$

$$\sigma = \sqrt{2L\xi/(1+\rho)} I_c$$

$$\tau = \sqrt{2L\xi/(1+\rho)} I_s$$

TRANSFORMATION FROM ELLIPTIC ELEMENTS TO OSCULATING CANONICAL VARIABLES

[This is the first section of the ENTER subroutine (lines 134-145) invoked when the KEY argument is 2.]

$$L = \sqrt{\mu a}$$

$$\lambda = \ell + g + h$$

$$\xi = \sqrt{1 - e^2}$$

$$\xi = \sqrt{2L(1 - \zeta)} \cos (g + h)$$

$$\eta = - \sqrt{2L(1 - \zeta)} \sin (g + h)$$

$$\sigma = 2 \sqrt{L\zeta} \sin i/2 \cos h$$

$$\tau = - 2 \sqrt{L\zeta} \sin i/2 \sin h$$

TRANSFORMATION FROM OSCULATING CANONICAL VARIABLES TO ELLIPTIC ELEMENTS

[This is in subroutine OSTCOR (lines 194-203).] See the last section in this Appendix for the calculation of the intermediate quantities used below.

$$a = L^2/\mu$$

$$e = \sqrt{1 - \zeta^2}$$

$$i = \cos^{-1} \rho$$

$$h = \tan^{-1} (-\tau/\sigma)$$

$$g + h = \tan^{-1} (-\eta/\xi)$$

$$\ell = \lambda - g - h$$

TRANSFORMATION FROM OSCULATING CANONICAL VARIABLES TO CARTESIAN COORDINATES

[This is subroutine OSTCOR (lines 205-212).] Again the intermediate quantities described in the next section are used.

$$r = (1 - e \cos \tilde{\omega} \cos \alpha - e \sin \tilde{\omega} \sin \alpha)L^2/\mu$$

$$x = r(1 - \tau^2/2L\zeta) \cos u - (\sigma\tau/2L\zeta) \sin u$$

$$y = r(-\sigma\tau/2L\zeta) \cos u + (1 - \sigma^2/2L\zeta) \sin u$$

$$z = r (\sigma \sin u + \tau \cos u) \sqrt{(1+\rho)/2L\xi}$$

$$v_x = ((\alpha - \lambda) x + \xi (-\sigma z \sqrt{(1+\rho)/2L\xi} - \rho y))L/r^2$$

$$v_y = ((\alpha - \lambda) y + \xi (\rho x + \tau z \sqrt{(1+\rho)/2L\xi}))L/r^2$$

$$v_z = ((\alpha - \lambda) z + \xi (-\tau y + \sigma x) \sqrt{(1+\rho)/2L\xi})L/r^2$$

CALCULATION OF INTERMEDIATE QUANTITIES FROM CANONICAL VARIABLES

(This is the subroutine INTERM.)

$$\xi = 1 - (\xi^2 + \eta^2)/2L$$

$$\rho = 1 - (\sigma^2 + \tau^2)2L\xi$$

$$e \cos \tilde{\omega} = \xi \sqrt{(1+\xi)/2L}$$

$$e \sin \tilde{\omega} = -\eta \sqrt{(1+\xi)/2L}$$

Solve for α , by Newton-Raphson, the Kepler equation:

$$\lambda = \alpha - e \cos \tilde{\omega} \sin \alpha + e \sin \tilde{\omega} \cos \alpha$$

$$u = \alpha + 2 \tan^{-1} ((\alpha - \lambda)/(1 + \xi - e \cos \tilde{\omega} \cos \alpha - e \sin \tilde{\omega} \sin \alpha))$$

$$e \sin f = e \cos \tilde{\omega} \sin u - e \sin \tilde{\omega} \cos u$$

$$e \cos f = e \cos \tilde{\omega} \cos u + e \sin \tilde{\omega} \sin u$$

$$c_1 = \sqrt{(1+\xi)/2L}$$

$$c_2 = \sqrt{(1+\rho)2L\xi}$$

$$\gamma'_2 = 12 J_2 (\mu R_e)^2 / (2L\xi)^4$$

APPENDIX D

**DIFFERENTIAL RELATIONSHIPS AND THE TRANSFORMATION
BETWEEN OSCULATING AND MEAN CANONICAL VARIABLES**

DIFFERENTIAL RELATIONSHIPS AND THE TRANSFORMATION BETWEEN OSCULATING AND MEAN CANONICAL VARIABLES

The first-order determining function, whose partial derivatives give the transformation between mean and osculating variables, is given by

$$S_1 = 6 (\mu R_e)^2 J_2 / (2L\xi)^3 \left\{ (-1 + 3\rho^2) U_\psi + \right. \\ \left. + ((1 + \rho)/4L\xi)(\sigma^2 - \tau^2) S_{up} + 2\sigma\tau C_{up} \right\}$$

$$U_\psi = f - \ell + e \sin f$$

$$S_{up} = \sin 2u + e \sin (u + \tilde{\omega}) + (e/3) \sin (3u - \tilde{\omega})$$

$$C_{up} = \cos 2u + e \cos (u + \tilde{\omega}) + (e/3) \cos (3u - \tilde{\omega})$$

The advantage of this dissection is that none of the last three quantities defined depend upon σ or τ . Quantities such as e , f , and ℓ are to be considered as expressed in terms of the canonical variables being used. Hence, it is necessary to derive a miscellany of differential relationships that will be needed in the process.

$$\text{Since } \xi^2 = 1 - e^2 \text{ and } 1 - \xi = (\xi^2 + \eta^2)/2L,$$

$$de = -\xi d\xi / e = \frac{\xi e}{1+\xi} \left(\frac{2(\xi d\xi + \eta d\eta)}{\xi^2 + \eta^2} - \frac{dL}{L} \right)$$

$$\text{Also, } d\tilde{\omega} = d(\tan^{-1} (-\eta/\xi)) = \frac{\eta d\xi - \xi d\eta}{\xi^2 + \eta^2}$$

From the usual Keplerian orbit relationships:

$$\ell = E - e \sin E$$

$$r/a \sin f = \xi \sin E$$

$$r/a \cos f = \cos E - e$$



Differentiation, substitution, and rearrangement yields

$$df = (Z^2/\xi^3) d\ell + ((Z + 1) \sin f)/\xi^2 de$$

$$\text{where } Z = \xi^2 a/r = 1 + e \cos f.$$

It will also be convenient to use

$$e_c = \sqrt{(1 + \xi)/2L} \xi = c_1 \xi$$

$$e_s = -c_1 \eta$$

from which by differentiation:

$$de_c = c_1 d\xi + e_c/(2L(1 + \xi)) (-2\xi dL - \xi d\xi - \eta d\eta)$$

$$de_s = -c_1 d\eta + e_s/(2L(1 + \xi)) (-2\xi dL - \xi d\xi - \eta d\eta)$$

If the foregoing are applied,

$$dU_k = df(1 + e \cos f) - d\ell + \sin f de$$

$$= (Z^3/\xi^3 - 1) (d\lambda - d\tilde{\omega}) + (Z^2 + Z + \xi^2) \sin f de/\xi^2$$

$$= \frac{1}{\xi^2 + \eta^2} [(Z^3/\xi^3 - 1)(-\eta d\xi + \xi d\eta) + 2 \frac{(Z^2 + Z + \xi^2)}{\xi(1 + \xi)} e \sin f (\xi d\xi + \eta d\eta)]$$

$$+ (Z^3/\xi^3 - 1)d\lambda - \frac{2(Z^2 + Z + \xi^2)}{L\xi(1 + \xi)} e \sin f dL$$

The coefficients of $d\xi$ and $d\eta$ are not convenient for computing, since at low eccentricities they become the ratios of small quantities. It is expedient to transform them using the easily derived relations:

$$-\eta \cos f + \xi \sin f = \sqrt{2L/(1 + \xi)} e \sin u$$

$$\eta \sin f + \xi \cos f = \sqrt{2L/(1 + \xi)} e \cos u$$

arriving finally at

$$\begin{aligned}
 dU_{\ell} = & \frac{1 + Z + Z^2}{2L\zeta^3} [2L(1 + \zeta) \sin u \, d\xi + \cos u \, d\eta) \\
 & - \frac{2\zeta+1}{1+\zeta} e \sin f (\xi d\xi + \eta d\eta)] + \frac{1 + \zeta + \zeta^2}{2L\zeta^3} (-\eta \, d\xi + \xi \, d\eta) \\
 & - (\xi \, d\xi + \eta \, d\eta)/L\zeta - \frac{Z^2 + Z + \zeta^2}{L\zeta (1 + \zeta)} e \sin f \, dL + (Z^3/\zeta^3 - 1)d\lambda
 \end{aligned}$$

Next C_{up} may be written $= \cos 2u + e_c(\cos u + (1/3) \cos 3u)$

$$+ e_s (-\sin u + (1/3) \sin 3u)$$

so that $dC_{up} = - (2 \sin 2u + e_c (\sin u + \sin 3u) + e_s (\cos u - \cos 3u) (df + d\omega)$

$$+ (\cos u + (1/3) \cos 3u) de_c + (-\sin u + (1/3) \sin 3u) de_s$$

Note that $2 \sin 2u + e_c (\sin u + \sin 3u) + e_s (\cos u - \cos 3u) = 2Z \sin 2u$;

substitute, rearrange, and make the same transformation as for U_{ℓ} . The result is

$$\begin{aligned}
 dC_{up} = & - (Z \sin 2u/L\zeta^3) [(Z + 1) \frac{1 + \zeta}{c_1} (\sin u \, d\xi + \cos u \, d\eta) \\
 & - \frac{1 + 2\zeta}{1 + \zeta} e \sin f (\xi \, d\xi + \eta \, d\eta) + (1 + \zeta + \zeta^2) (-\eta \, d\xi + \xi \, d\eta)] \\
 & + c_1 [\cos u + (1/3) \cos 3u] d\xi + (\sin u - (1/3) \sin 3u) d\eta] \\
 & - \frac{C_{up} - \cos 2u}{2L(1 + \zeta)} (\xi \, d\xi + \eta \, d\eta) + \frac{dL}{L\zeta (1 + \zeta)} [2Z (Z + 1) \sin 2u, e \sin f \\
 & - \zeta^2 (C_{up} - \cos 2u)] - 2Z^3 \sin 2u \, d\lambda/\zeta^3
 \end{aligned}$$

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$$\frac{\partial \beta}{\partial L} = c_1 (x'\tau + y'\sigma) 2v_2/(2L\xi) + c_1 c_2 z' (-2v_2 \rho)/(1 + \rho) - 2\beta\xi/(2L(1 + \xi))$$

$$\frac{\partial \gamma}{\partial \xi} = (2\xi/(2L\xi))(-c_2 \rho (x'\tau + y'\sigma)/(1 + \rho) - z'(1 - \rho))$$

$$\frac{\partial \gamma}{\partial \eta} = (2\eta/(2L\xi))(-c_2 \rho (x'\tau + y'\sigma)/(1 + \rho) - z'(1 - \rho))$$

$$\frac{\partial \gamma}{\partial \sigma} = c_2 \sigma (x'\tau + y'\sigma)/(2L\xi (1 + \rho)) - c_2 y' - 2z'\sigma/(2L\xi)$$

$$\frac{\partial \gamma}{\partial \tau} = c_2 \tau (x'\tau + y'\sigma)/(2L\xi (1 + \rho)) - c_2 x' - 2z'\tau/(2L\xi)$$

$$\frac{\partial \gamma}{\partial L} = 2c_2 \rho (x'\tau + y'\sigma)/(2L\xi (1 + \rho)) + 2z'(1 - \rho)/(2L\xi)$$

$$\frac{\partial \xi}{\partial \xi} = -\xi/L$$

$$\frac{\partial \xi}{\partial \eta} = -\eta/L$$

$$\frac{\partial \xi}{\partial L} = (1 - \xi)/L$$

$$\frac{\partial a}{\partial L} = 2L/\mu$$

The other partial derivatives in the matrix are zero.

$$\xi = 1 - (\xi^2 + \eta^2)/2L, \quad a = L^2/\mu$$

we may write

$$\alpha_x = c_1 (\xi - v_1 \tau) \quad \alpha_y = c_1 (-\eta - v_1 \sigma) \quad \alpha_z = c_2 v_1 (2L\xi)$$

$$\beta_x = c_1 (-\eta - v_2 \tau) \quad \beta_y = c_1 (-\xi - v_2 \sigma) \quad \beta_z = c_2 v_2 (2L\xi)$$

$$\gamma_x = -c_2 \tau \quad \gamma_y = -c_2 \sigma \quad \gamma_z = \rho$$

and easily but tediously differentiate these to find

$$\frac{\partial \alpha}{\partial \xi} = c_1 x' - c_1 (\tau + 2v_1 \xi)(x'\tau + y'\sigma)/(2L\xi) + c_1 c_2 z' (\tau + 2v_1 \xi \rho/(1 + \rho)) - \alpha\xi/(2L(1 + \xi))$$

$$\frac{\partial \alpha}{\partial \eta} = -c_1 y' + c_1 (\sigma - 2v_1 \eta)(x'\tau + y'\sigma)/(2L\xi) + c_1 c_2 z' (-\sigma + 2v_1 \eta \rho/(1 + \rho)) - \alpha\eta/(2L(1 + \xi))$$

$$\frac{\partial \alpha}{\partial \sigma} = c_1 (x'\tau + y'\sigma) \eta/(2L\xi) - c_1 y'v_1 + c_1 c_2 z' (-\eta - \sigma v_1/(1 + \rho))$$

$$\frac{\partial \alpha}{\partial \tau} = -c_1 (x'\tau + y'\sigma) \xi/(2L\xi) - c_1 x'v_1 + c_1 c_2 z' (\xi - \tau v_1/(1 + \rho))$$

$$\frac{\partial \alpha}{\partial L} = c_1 (x'\tau + y'\sigma) 2v_1/(2L\xi) + c_1 c_2 z' (-2v_1 \rho)/(1 + \rho) - 2\sigma\xi/(2L(1 + \xi))$$

$$\frac{\partial \beta}{\partial \xi} = -c_1 y' + c_1 (\sigma - 2v_2 \xi)(x'\tau + y'\sigma)/(2L\xi) + c_1 c_2 z' (-\sigma + 2v_2 \xi \rho/(1 + \rho)) - \beta\xi/(2L(1 + \xi))$$

$$\frac{\partial \beta}{\partial \eta} = -c_1 x' + c_1 (\tau - 2v_2 \eta)(x'\tau + y'\sigma)/(2L\xi) + c_1 c_2 z' (-\tau + 2v_2 \eta \rho/(1 + \rho)) - \beta\eta/(2L(1 + \xi))$$

$$\frac{\partial \beta}{\partial \sigma} = c_1 (x'\tau + y'\sigma) \xi/(2L\xi) - c_1 y'v_2 + c_1 c_2 z' (-\xi - v_2 \sigma/(1 + \rho))$$

$$\frac{\partial \beta}{\partial \tau} = c_1 (x'\tau + y'\sigma) \eta/(2L\xi) - c_1 x'v_2 + c_1 c_2 z' (-\eta - v_2 \tau/(1 + \rho))$$

CALCULATION OF THE JACOBIAN MATRIX

To complete the differentiation of the Hamiltonian with respect to the mean variables, we need the Jacobian matrix

$$\frac{J(\alpha, \beta, \gamma, \xi, a)}{J(\eta, \xi, \tau, \sigma, L)}$$

for the three cases: zonal harmonics, lunar attraction, and solar attraction. It is clear from THE HAMILTONIAN IN TERMS OF MEAN VARIABLES section that we can subsume the three into one by writing

$$\alpha = \alpha_x x' + \alpha_y y' + \alpha_z z'$$

$$\beta = \beta_x x' + \beta_y y' + \beta_z z'$$

$$\gamma = \gamma_x x' + \gamma_y y' + \gamma_z z'$$

where the unit vector

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \text{ is } \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ for the zonals, } \begin{pmatrix} x'_M \\ y'_M \\ z'_M \end{pmatrix} \text{ for the moon, and } \begin{pmatrix} x'_S \\ y'_S \\ z'_S \end{pmatrix} \text{ for the sun}$$

(and constant in all three cases), and $\xi, a, \alpha_x, \alpha_y, \alpha_z, \beta_x, \beta_y, \beta_z, \gamma_x, \gamma_y,$ and γ_z are the same for all three cases.

If we define

$$v_1 = (\xi\tau - \eta\sigma)/2L\xi$$

$$v_2 = -(\xi\sigma + \eta\tau)/2L\xi$$

and remember that $c_1 = \sqrt{(1+\xi)/2L},$

$$c_2 = \sqrt{(1+\rho)/2L\xi}$$

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APPENDIX G

CALCULATION OF THE JACOBIAN MATRIX

Thus,

$$C_m(\alpha, \beta) = \alpha C_{m-1}(\alpha, \beta) - \beta S_{m-1}(\alpha, \beta)$$

$$S_m(\alpha, \beta) = \alpha S_{m-1}(\alpha, \beta) + \beta C_{m-1}(\alpha, \beta)$$

and the starting values are

$$C_0(\alpha, \beta) = 1 \quad S_0(\alpha, \beta) = 0$$

Differentiation from the definition is

$$\frac{\partial C_m}{\partial \alpha}(\alpha, \beta) = m C_{m-1}(\alpha, \beta)$$

$$\frac{\partial C_m}{\partial \beta}(\alpha, \beta) = -m S_{m-1}(\alpha, \beta)$$

Derivation of the results below is parallel to that in the section above.

$$A_{n+1}^m(x) = (-)^m \frac{(n-m)!}{n!} \frac{P_n^m(x)}{x^n e^m}$$

Upon substitution for P_n^m (see the previous section),

$$A_{n+1}^m(x) = (-)^m \frac{(n-m)!}{n!} x^{-n+m} Q_n^m(x)$$

The recursion relation is

$$n(n+1) A_{n+2}^m(x) = n(2n+1) A_{n+1}^m(x) - \frac{(n+m)(n-m)}{x^2} A_n^m(x)$$

with starting values

$$A_m^m(x) = 0 \quad A_{m+1}^m(x) = (-)^m \frac{Q_m^m(x)}{m!}$$

and starting recursion

$$A_{m+1}^m(x) = -\frac{2m-1}{m} A_m^{m-1}(x); \quad A_1^0(x) = 1$$

Differentiation, from the function definition, is

$$\frac{d}{dx} A_{n+1}^m(x) = -\frac{n-m}{x} (A_{n+1}^m(x) + A_{n+1}^{m+1}(x))$$

RECURSION ON $C_m(\alpha, \beta)$ AND $S_m(\alpha, \beta)$

These coefficients are defined by

$$(\alpha + i\beta)^m = C_m(\alpha, \beta) + i S_m(\alpha, \beta)$$

so that

$$C_m(\alpha, \beta) + i S_m(\alpha, \beta) = (\alpha + i\beta)(C_{m-1}(\alpha, \beta) + i S_{m-1}(\alpha, \beta))$$

RECURSION ON $B_n^m(x)$

These are almost the coefficients defined by Cefola and Broucke (Reference F-1, Equations 68-80), but multiplied by x^{2n+1} , which simplifies the calculations required.

$$B_{n+1}^m(x) = \frac{n!}{(n+m)!} \frac{x^{n+1} P_n^m(x)}{e^m} \quad (\text{eq. 76})$$

(Note that this P_n^m has a difference: it is as defined in Reference F-2:

$$P_n^m(x) = (x^2 - 1)^{m/2} Q_n^m(x), \text{ not } (1 - x^2)^{m/2}.$$

$$B_{n+1}^m(x) = \frac{n!}{(n+m)!} x^{n+m+1} Q_n^m(x)$$

Now the recursion follows from that for Q_n^m above.

$$(n-m+1)(n+m+1) B_{n+2}^m(x) = (n+1)x^2 [(2n+1) B_{n+1}^m(x) - n B_n^m(x)]$$

with starting values $B_m^m(x) = 0$, $B_{m+1}^m(x) = x^{2m+1}/2^m$,

and the starting recursion $B_{m+1}^m(x) = (x^2/2) B_m^{m-1}(x)$, $B_1^0(x) = x$

Differentiation of the function, from its definition, is

$$\frac{d}{dx} B_{n+1}^m(x) = \frac{n+m+1}{x} (B_{n+1}^m(x) + B_{n+1}^{m+1}(x))$$

RECURSION ON $A_n^m(x)$

These are the coefficients described by Cefola and Broucke (Reference F-1, Equations 54-67), but with an additional factor of $(-1)^m$.

F-1 Paul J. Cefola and Roger Broucke, AIAA Paper No. 75-9 (no date)

F-2 Milton Abramowitz and Irene A. Stegun, Eds., "Handbook of Mathematical Functions" (Dover).

RECURSION ON $Q_n^m(\gamma)$

The function is defined by $Q_n^m(\gamma) = \frac{d^m}{d\gamma^m} P_n(\gamma)$ with P_n the Legendre polynomial of the first kind. The recursion relationship is easily obtained by differentiating the recursion relationship for the Legendre polynomial:

$$(n+2-m) Q_{n+2}^m(\gamma) = (2n+3) \gamma Q_{n+1}^m(\gamma) - (n+m+1) Q_n^m(\gamma)$$

with starting values $Q_m^{m+1}(\gamma) = 0$

$$Q_m^m(\gamma) = (2m-1)!!$$

$$Q_{m+1}^m(\gamma) = \gamma (2m+1)!!$$

where $(2m+1)!! = 1.3.5.7 \dots (2m+1)$. The starting values may themselves be determined recursively:

$$Q_m^m(\gamma) = (2m-1) Q_{m-1}^{m-1}(\gamma)$$

$$Q_{m+1}^m(\gamma) = (2m+1) \gamma Q_m^m(\gamma)$$

$$Q_0^0(\gamma) = 1$$

Differentiation, from the function definition, is

$$\frac{d}{d\gamma} Q_n^m(\gamma) = Q_n^{m+1}(\gamma)$$

RECURSION OF D_n

D_n^m is defined by $\frac{(n-m)!}{(n+m)!} Q_n^m(0)$ and its recursive relationships

follow from those of Q_n^m : $D_n^{m+2} = -\frac{n-m+1}{n+m+2} D_n^m$

Starting values are $D_m^m = \frac{1}{2^m m!}$, $D_{m+1}^m = 0$

and the former allows the recursion

$$D_m^m = \frac{1}{2m} D_{m-1}^{m-1}, \text{ with } D_0^0 = 1$$

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APPENDIX F

RECURSIVE FUNCTIONS AND THEIR DERIVATIVES

THE SECOND-ORDER SHORT-PERIOD CORRECTION TO L

The last equation in the MEAN ELEMENTS: THE VON ZEIPPEL TRANSFORMATION shows the transformation from L to L' with the second-order correction. The portions of this involving S_2 are calculated in the subroutine ELTRAN, lines 273-278. The term remaining, $(L^3/\mu^2)(F_2 - F_2^*)$, is more conveniently calculated in the subroutine DERIV, lines 621-643, where F_2^* is already available. The calculation of F_2 , the instantaneous second-order portion of the Hamiltonian, is as follows:

- (1) The zonal harmonics contribution to F_2 .

$$(\Delta F)_{2Z} = - \sum_{n=3} (\mu/r) J_n (R_e/r)^n P_n(\sin \bar{\beta})$$

where $r = L^2 \xi^2 / \mu Z$ and

$$\sin \bar{\beta} = \sin i \sin(f + g) = c_2 (\sigma \sin u + \tau \cos u)$$

- (2) The lunar contribution to F_2 .

$$(\Delta F)_{2M} = \sum_{n=2} (\mu/r_M) (r/r_M)^n P_n(\sin \bar{\beta}')$$

where r is as above and

$$\sin \bar{\beta}' = x'_M (\cos u - c_2 \tau \sin \bar{\beta} / (1 + \rho))$$

$$+ y'_M (\sin u - c_2 \sigma \sin \bar{\beta} / (1 + \rho)) + z'_M \sin \bar{\beta}$$

The distance to the moon is r_M , and the components of the unit vector to the moon x'_M , y'_M , z'_M , in the coordinate frame $\hat{L}_1 \hat{L}_2 \hat{h}$, and $c_2 = \sqrt{(1 + \rho/L\xi)}$.

- (3) The solar contribution to F_2 is exactly the same, except that the solar coordinates are substituted for lunar coordinates.

Since F_2 is second-order, it makes no difference whether it is calculated in terms of mean or osculating variables.

For a derivation of the formulas above see THE HAMILTONIAN IN TERMS OF MEAN VARIABLES section.

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APPENDIX E

THE SECOND-ORDER SHORT-PERIOD CORRECTION TO L

$$\begin{aligned}
dS_1 = \gamma'_2 \bigg\{ & L\xi \left(-\frac{1}{3} + \rho^2 \right) dU_\xi + ((1 + \rho)/4) (\sigma^2 - \tau^2) dS_{up} + 2 \sigma \tau dC_{up} \\
& + [(1 + 2\rho - 5\rho^2) U_\xi + \\
& ((3 + 5\rho)/4L\xi) ((\sigma^2 - \tau^2) S_{up} + 2 \sigma \tau C_{up})] (dL - \xi d\xi - \eta d\eta) \\
& - (2\rho U_\xi + (1/4L\xi) ((\sigma^2 - \tau^2) S_{up} + 2\sigma\tau C_{up})) (\sigma d\sigma + \tau d\tau) \\
& + ((1 + \rho)/2) ((\sigma d\sigma - \tau d\tau) S_{up} + (\sigma d\tau + \tau d\sigma) C_{up}) \bigg\}
\end{aligned}$$

The transformation between mean and osculating coordinates is taken from these equations and is contained in the subroutine ELTRAN, lines 215-271.

In exactly similar fashion,

$$\begin{aligned}
 dS_{up} &= (Z \cos 2u/L\xi^3) [(Z+1) \frac{(1+\xi)}{c_1} (\sin u d\xi + \cos u d\eta) \\
 &\quad - \frac{1+2\xi}{1+\xi} e \sin f(\xi d\xi + \eta d\eta) + (1+\xi+\xi^2)(-\eta d\xi + \xi d\eta)] \\
 &\quad + c_1 [\sin u + (1/3) \sin 3u] d\xi - [\cos u + (1/3) \cos 3u] d\eta \\
 &\quad - \frac{S_{up} - \sin^2 u}{2L(1+\xi)} (\xi d\xi + \eta d\eta) + \frac{dL}{L\xi(1+\xi)} [-2Z(Z+1) \cos 2u e \sin f \\
 &\quad - \xi^2 (S_{up} - \sin^2 u)] + 2Z^3 \cos 2u d\lambda/\xi^3
 \end{aligned}$$

There is more differentiation to be done. We may write

$$\begin{aligned}
 S_1 &= \gamma'_2 L\xi [(-\frac{1}{3} + \rho^2) U_\xi + \frac{1+\rho}{4L} (\sigma^2 - \tau^2) S_{up} + 2\sigma\tau C_{up}] \\
 &= \gamma'_2 (L\xi)^4 \left[\left(-\frac{1}{3} \frac{1}{(L\xi)^3} + \frac{(\rho L\xi)^2}{(L\xi)^5} \right) U_\xi + \frac{1}{4} \left(\frac{1}{(L\xi)^4} + \frac{\rho L\xi}{(L\xi)^5} \right) \right. \\
 &\quad \left. ((\sigma^2 - \tau^2) S_{up} + 2\sigma\tau C_{up}) \right]
 \end{aligned}$$

and observe that $\gamma'_2 (L\xi)^4$ is a constant and that

$$L\xi = L - (\xi^2 + \eta^2)/2$$

$$\rho L\xi = L\xi - (\sigma^2 + \tau^2)/2$$

so that differentiating S_1 with respect to $L\xi$ and $\rho L\xi$, and $L\xi$ and $\rho L\xi$ with respect to the variables yields the required derivatives.

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