AD-A158 615 ALTERNATIVE APPROACHES TO THE STANDARD BASE SUPPLY SYSTEM ECONOMIC ORDER (U) AIR FORCE LOGISTICS MANAGEMENT CENTER GUNTER AFS AL D J BLAZER ET AL. 1/1

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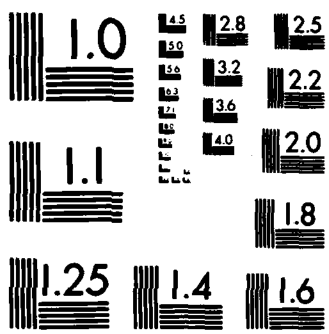

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ALTERNATIVE APPROACHES TO THE STANDARD
BASE SUPPLY SYSTEM ECONOMIC ORDER QUANTITY
DEPTH MODEL

BY

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ABSTRACT

The current Air Force inventory policy for Economic Order Quantity (EOQ) items in the Standard Base Supply System (SBSS) minimizes total variable costs to determine how much to order and when to order. There are many inventory experts in both the Air Force and academic communities who believe minimization of variable costs is not the correct way to determine Air Force inventory policy. This report documents the comparison of ten alternative inventory systems to the two Standard Base Supply System models. Using a 40-item selected sample and many larger samples, we compared the performance of these inventory systems in terms of the number of backorders, the number of orders, unit fill rates and total variable costs. We found an aggregate model that minimizes backorders subject to a limitation on investment funds and the number of requisitions was more efficient and reduced backorders up to 95% as compared to the current SBSS. The aggregate model outperformed all other models as well. In addition, the aggregate model is well suited for computing EOQ stock fund requirements since each base operates under a stock fund dollar constraint.

EXECUTIVE SUMMARY

The purpose of this study was to compare ten alternative inventory systems to the two Standard Base Supply System models. The current Standard Base Supply System determines inventory policy by minimizing total variable costs. There are problems inherent in a cost minimization approach. First, variable costs are difficult to accurately measure. Secondly, the variable costs are applied equally to all items, regardless of their obsolescence or essentiality. In fact, many essential aircraft parts have low demand and high cost which, using a variable cost approach, causes us to stock fewer of these items. Finally, there are other criteria we can use to set inventory policy that seem better suited to Air Force needs. In this study we tested a model which uses another criteria.

Using a 40-item sample, we compared the performance of ten alternative inventory models to the two current Standard Base Supply System methods. We then selected the best performing models from the 40-item sample and compared their performance with two large samples taken from actual Air Force base data. We determined that an aggregate model which minimized the total number of backorders, subject to constraints on both funding and requisitioning workload, performed more efficiently and effectively than any of the other models.

We next tested to see if the aggregate model was feasible for Air Force wide use. We ran the aggregate model on all consumable items at England AFB. The aggregate model had 95% fewer backorders than the Standard Base Supply System at the same cost. We found the model was feasible in terms of Air Force computing and software capability. The aggregate model is currently being developed for prototype testing as part of the Civil Engineering Materiel Acquisition System (CEMAS). Pending successful completion of the CEMAS prototype testing, we recommend the aggregate model be used as part of the Standard Base Supply System.

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CHAPTER 1

THE PROBLEM

PROBLEM STATEMENT

There are many scientific inventory management models that can be used to determine the amount of stock for each consumable item at the retail level. The current Air Force retail level system determines the depth of stock based on the Wilson Economic Order Quantity (EOQ) model. The purpose of this study was to compare the effectiveness and efficiency of the two current Air Force retail level models to ten other inventory systems.

BACKGROUND

The current Standard Base Supply System employs the Wilson Economic Order Quantity (EOQ) model to determine the amount of stock to order. The EOQ is designed to minimize the sum of two variable costs: holding and ordering. As a result of the Retail Inventory Management and Stockage Policy (RIMSTOP) Working Group, DOD units were to set inventory policy based on minimizing total variable costs. However, the DOD model in DODI 4140.45 included a backorder penalty cost as well as the holding and ordering costs.

Many inventory experts, in both academic and practical environments, have serious criticisms of the total variable cost approach. They contend the measurement of the cost variables necessary to compute the classical EOQ are suspect. Even if the variable costs are estimated correctly, the costs are usually applied equally to each item in the inventory system. For example, the holding cost for two items with the same unit price is the same regardless of their obsolescence rate, pilferability or shelf life. In the current SBSS depth model, backorder penalty cost is not considered. We do not stock more of an item that causes a lot of backorders or grounds a weapon system. In fact, many of these items that ground weapon systems are relatively high in cost and low in demand, thus we actually stock less of these items. Another criticism of the Air Force's use of minimizing total variable costs is the use of a 10% cost of capital. Some believe there should not be a cost of capital, especially for Systems Support Division items because these items are already bought and are in the Air Force inventory. THE POINT IS MINIMIZING TOTAL VARIABLE COSTS IS NOT THE BEST APPROACH FOR DETERMINING INVENTORY POLICY FOR AIR FORCE CONSUMABLE ITEMS.

The current SBSS stockage formulas also make some assumptions that do not strictly apply to Air Force demand patterns. For example, the current SBSS models assume demand usage at a constant rate, whereas Air Force demand patterns tend to be "lumpy" and erratic. The current system assumes a variance-to-mean ratio of three and that leadtime is known and constant. We have previously documented the inaccuracies of these assumptions and have recommended changes to the current stockage policy [1,2]. These changes have been approved for Air Force-wide implementation and are included in this study.

Minimizing total variable costs is not the proper criteria to use to set stockage policy. Minimizing backorders or maximizing mission capable weapon systems may be better criteria for stockage policy. In addition, the base level depth model provides managers little or no flexibility in that it ignores resource constraints. For example, if there is a limit to stock fund dollars, the current system provides no capability to assure the best effectiveness with the limited dollars.

CHAPTER 2

ANALYSIS

OVERVIEW: We document our analysis in three sections. In the first section we explain the experimental design, which involves developing data samples and documenting the alternative inventory systems. In the second section we compare the performance of alternative models. In the final section we discuss implementation issues.

EXPERIMENTAL DESIGN

We conducted our analysis in two phases. The first phase was to test alternative inventory systems on a 40-item sample of items that was representative of Air Force consumable items. In the second phase we selected the models that perform the best on the small sample and compared these models using random samples of items selected from England and Upper Heyford Air Force Bases. In this section we document the selection of the 40-item samples and the 10 alternative inventory systems that we compared to the two SBSS.

We selected a sample of 40 items to meet two criteria. First, the 40 items were representative of Air Force consumable items. In Table 2-1, we compare the aggregate characteristics for the 40 item sample to the aggregate characteristics of Air Force consumable (Economic Order Quantity) items. The second criteria in the 40-item sample selection was to take a wide spectrum of items; items that displayed as many different demand characteristics as were found in a real-life setting. We did this to determine if one inventory system's performance was superior for certain types of items. Appendix A lists the data characteristics of the 40-item sample. The sample has both high and low order frequency, demand totals, costs, and variances. Thus the sample contains a wide spectrum of data characteristics.

AGGREGATE CHARACTERISTICS

	AVERAGE DEMAND DURING LEADTIME	AVERAGE ACTUAL VARIANCE OF DEMAND DURING LEADTIME
40-ITEM SAMPLE	12.3	23.77
TYPICAL USAF BASE	11.4	25.00

Table 2-1

The sample was used to compare twelve alternative inventory systems. Table 2-2 lists the twelve alternative inventory systems. Appendix B provides a more detailed discussion of the twelve inventory systems.

ALTERNATIVE INVENTORY SYSTEMS

TITLE	ABBREVIATION
1. Standard Base Supply System (C Factor = 1)	SBSS1
2. Standard Base Supply System (C Factor = 2)	SBSS2
3. Standard Base Supply System with Revised Safety Level using Constant Poisson Demand Variance (C Factor = 1)	RSL1
4. Standard Base Supply System with Revised Safety Level using Constant Poisson Demand Variance (C Factor = 2)	RSL2
5. Standard Base Supply System with Revised Safety Level using Actual Variance (C Factor = 1)	RSL3
6. Standard Base Supply System with Revised Safety Level using Actual Variance (C Factor = 2)	RSL4
7. Ninety Days EOQ (C Factor = 1)	EOQ901
8. Ninety Days EOQ (C Factor = 2)	EOQ902
9. Aggregate Model with an Investment Constraint	AGG1
10. Aggregate Model with an Investment and Workload Constraint	AGG2
11. RIMSTOP model	RIM
12. Power Approximation Model	POWER

Table 2-2

The first two alternative inventory systems reflect the current Standard Base Supply System's method to determine inventory policy. The C factor is a multiplier of the safety level and is used to set the percent of time materiel should be on hand to support a customer during a replenishment order (see Table 2-3). Note the fill rate in Table 2-3 is during the time a replenishment order has been submitted until its receipt. The overall fill rate is higher than the reorder cycle fill rate.

EXPECTED FILL RATE DURING THE REORDER CYCLE

C FACTOR	FILL RATE (PERCENT)
1	84
2	97

Table 2-3

The next alternative inventory systems are a set of four that provide the same policy as the Standard Base Supply System, except the safety level uses an actual measure of the variance of demand. The first two methods (RSL1 and RSL2) use a variance of demand from the constant-Poisson demand distribution. This revised safety level is recommended for Air Force-wide implementation in the AFLMC Demand Forecasting Report [1] when the sample variance cannot be computed (i.e., not enough demands). The other two inventory systems (RSL3 and RSL4) in Table 2-2 are also documented in [1,2] and have been approved for world-wide implementation. The revised safety level method uses the same EOQ as the SBSS, however the safety level uses historical data to estimate the variance of demand. In [1] we showed significant improvements to stockage effectiveness will occur as a result of implementation of this system. The C factors are again multipliers of the safety level with the target fill rates as shown in Table 2-3.

The ninety day inventory systems use the same inventory policy as the Standard Base Supply System except the order quantity (the amount of stock to order when the reorder point is reached) is 90 days of stock. The reorder point was the same as the revised safety level methods using the actual demand variance. We use a C factor of 1 and 2.

The next two inventory systems are called aggregate models [5]. The aggregate models minimize the number of backorders for the entire system subject to certain constraints. The first aggregate model uses one constraint; it limits the amount of inventory investment. The second aggregate model uses two constraints, inventory investment and the number of requisitions. This model will assure the best inventory policy within an investment threshold and within a specified number of requisitions that can be accommodated with the existing number of manpower resources.

The Retail Inventory Management and Stockage Policy (RIMSTOP) model is documented in [3,6]. The RIMSTOP model is an optimization model; it minimizes total variable costs. It is a fairly complicated model and we describe it in Appendix B.

The final inventory model is the Power Approximation method [4]. The Power model provides approximately optimal inventory policy for a periodic review inventory system. A periodic review system checks the stock level (x) periodically and when stock is below the reorder point (s), S-x stock is ordered, where the quantity S is the order up to level. Periodic review models are subject to stock outs during the review period, therefore these models compute higher stock levels than continuous review models [11].

PERFORMANCE OF ALTERNATIVE MODELS

In this section we provide the performance results of the twelve alternative inventory systems using the 40-item sample. In order to determine inventory performance, we assumed a demand distribution for each of the 40 items. To build a demand distribution we need two parameters--the number of customer orders and the size of the order. We extracted these parameters from the past year's history for each item. The number of customer orders is shown in Appendix A. The lot size of the order is the average size of a customer order in the previous year, which is:

$$\text{Order Lot Size} = \frac{\text{Cumulative Recurring Demands}}{\text{Number of Orders}} .$$

With these demand distributions, we compared the performance of the twelve inventory systems. Table 2-4 displays the results in terms of 5 criteria (see Appendix C):

(1) Number of orders: This is the number of stock replenishment requisitions required on average in one year.

(2) Backorders (units per year): This is the average number of units per year that were not available at the time of a customer request.

(3) Average on hand inventory: This is the average dollar value amount of stock on-hand during the year.

(4) Total Variable Costs: This is the sum of the cost to order, cost to hold, and backorder cost for a year. The order cost, holding cost and backorder cost factors are those currently being used by the Standard Base Supply System.

(5) Overall Unit Fill Rate: This is the percent of time that customer demand is filled upon request. Partial fills are counted as the percent of the demand filled. For example, if the customer orders 10 units and only 5 units are available, the unit fill rate is 50 percent.

PERFORMANCE OF ALTERNATIVE INVENTORY SYSTEMS
(40 ITEM SAMPLE)

SYSTEM	TOTAL ORDERS	BACKORDERS (UNITS/YEAR)	AVERAGE ON HAND INVENTORY	TOTAL VARIABLE COST	OVERALL UNIT FILL RATE
1. SBSS1	116	322	$3744	$1912	96.8
2. SBSS2	116	143	$5239	$2087	98.6
3. RSL1	116	263	$4019	$1853	97.3
4. RSL2	116	56	$5692	$2070	99.4
5. RSL3	116	113	$4742	$1968	98.9
6. RSL4	116	21	$7133	$2427	99.8
7. EOQ901	139	83	$6325	$2482	99.2
8. EOQ902	139	16	$8716	$2942	99.8
9. AGG1	215	105	$3754	$2251	99.0
10. AGG2	117	210	$3723	$1909	97.9
11. RIM	116	292	$3747	$1855	97.1
12. POWER	117	17	$8588	$2794	99.8

Table 2-4

Note from Table 2-4 the SBSS inventory systems experience a high incidence of backorders which was mainly due to the system's inaccurate measure of demand variance [1]. Thus the SBSS inventory system was not as effective as the other systems. The ninety-day stock and the Power Approximation systems greatly reduced the number of backorders but at a large increase in stock. These systems were less efficient than the other systems. Therefore the choice of the "best" system resided with the Revised Safety Level (RSL) methods or one of the aggregate systems. The Revised Safety Level with a C factor of one (RSL1) and the RIMSTOP model required more inventory and had more backorders but at a lower total variable cost than the other methods. Regardless of DOD policy, from Table 2-4 it is not clear that the Air Force should set inventory policy based on minimizing total variable cost for the reasons stated earlier. However, we included these methods in the second phase of our analysis.

In Table 2-5, we show the results of phase two of our analysis. We compare the models that performed the best in the 40-item sample in two random samples of items from England and Upper Heyford Air Force Bases. We include the SBSS models as the baseline.

PERFORMANCE OF INVENTORY SYSTEMS

482 Items - England AFB

SYSTEM	ORDERS	BACKORDERS	AVERAGE ON HAND INVENTORY	TOTAL VARIABLE COST	UNIT FILL RATE
SBSS1	943	2562	$ 58,445	$22,640	95.6
SBSS2	943	1585	$ 85,215	$27,968	97.3
RSL1	943	1819	$ 58,044	$22,167	96.9
RSL2	943	473	$ 73,127	$24,277	99.2
RSL3	943	1193	$ 61,381	$22,645	98.0
RSL4	943	440	$ 87,146	$27,875	99.2
AGG1	1540	131	$ 59,175	$25,772	99.8
AGG2	864	172	$ 60,336	$23,515	99.7
RIM	941	1766	$ 52,520	$21,829	96.9

833 - Upper Heyford AFB

SYSTEM	ORDERS	BACKORDERS	AVERAGE ON HAND INVENTORY	TOTAL VARIABLE COST	UNIT FILL RATE
SBSS1	2078	3929	$143,667	$52,779	96.9
SBSS2	2078	1536	$222,526	$69,396	98.8
RSL1	2078	3195	$127,792	$49,052	97.5
RSL2	2078	635	$177,499	$57,192	99.5
RSL3	2078	1928	$136,057	$50,231	98.5
RSL4	2078	377	$195,100	$61,634	99.7
AGG1	3215	140	$146,530	$61,687	99.9
AGG2	2061	175	$144,874	$56,655	99.9
RIM	2067	3407	$119,582	$49,292	97.3

Table 2-5

As Table 2-5 shows, the aggregate models outperformed all the models in terms of the fewest backorders. For the results shown in Table 2-5, we set the aggregate models' on-hand inventory constraint to the SBSS on hand inventory amount. We next constrained the aggregate models to the on-hand inventory achieved by the RIMSTOP model (which was the smallest of all the models), and show the performance in Table 2-6.

AGGREGATE MODEL PERFORMANCE
USING THE RIMSTOP INVESTMENT CONSTRAINT

482 Items - England AFB

SYSTEM	ORDERS	BACKORDERS	AVERAGE ON HAND INVENTORY	TOTAL VARIABLE COST	UNIT FILL RATE
RIMSTOP	941	1766	$ 52,520	$21,829	96.9
AGG1	1722	449	$ 50,188	$25,300	99.1
AGG2	879	879	$ 52,444	$22,889	98.5

833 - Upper Heyford AFB

SYSTEM	ORDERS	BACKORDERS	AVERAGE ON HAND INVENTORY	TOTAL VARIABLE COST	UNIT FILL RATE
RIMSTOP	2067	3407	$119,582	$49,292	97.3
AGG1	2990	389	$122,030	$56,662	99.4
AGG2	1876	838	$118,379	$53,247	99.3

Table 2-6

The aggregate models clearly dominated the other models. For the same inventory investment, the aggregate models produced fewer backorders than any other model. Since they optimized on backorders, that was as expected. They did not provide as low a total variable cost as the RIMSTOP model, because the RIMSTOP model is an optimization model that minimizes total variable costs. However, it seemed apparent that backorders and inventory investment were more important criteria than variable costs. The aggregate models were much superior to the current SBSS system in both effectiveness and efficiency. Note from Table 2-5 the revised safety level methods using an actual measure of the variance of demand were also significantly better than the current system and recall they are scheduled to be implemented into the SBSS. However, the aggregate models were clearly more effective than the revised safety level methods in terms of both backorders and inventory investment. Finally, the aggregate model with both an inventory and workload constraint seemed to best fit the needs of the base level supply system. It kept the number of yearly requisitions at or below the current SBSS level; whereas the aggregate model with no workload constraint almost doubled the required yearly orders.

The performance of these inventory systems in a real-life setting will not match the performance shown in Table 2-5 and Table 2-6. The models were run with perfect information; the mean and variance of demand were known exactly and the actual form of the distribution was known. Additionally leadtimes were known and constant. In a real-life environment without these conditions the unit fill rate will not be as high as 99%. However, Table 2-5 and Table 2-6 are used to show the <u>relative</u> performance of the models. That is, the aggregate model will perform much better than the SBSS in a real-life environment but will not reach a 99% fill rate.

IMPLEMENTATION ISSUES

Prior to recommending a change in the Air Force retail level inventory system, we want to insure the "two constraint" aggregate system will work in a "real-life" environment. We tested to see if the system can accommodate all the consumable items at a base and it can operate without the need of operations research analysts and computer programmers. To determine if the aggregate model can handle a larger group of items, we tested the model with all the consumable items at England AFB. We ran the aggregate model with the workload and investment constraints and compared it to the SBSS method. The results are shown in Table 2-7.

AGGREGATE MODEL PERFORMANCE AT
ENGLAND AFB (7478 ITEMS)

SYSTEM	ORDERS	BACKORDERS	$ ON HAND INVENTORY	TOTAL VARIABLE COST	UNIT FILL RATE
SBSS1	16,943	64,796	$1,139,009	$446,466	89.1
AGG2	12,399	3,524	$1,159,205	$445,600	99.4

Table 2-7

The benefits of the aggregate system were readily apparent. For the same amount of inventory investment, we can reduce backorders by 95%. To determine the operational impact of implementing the aggregate model, we selected all items from England AFB that have a stockage priority code of 1 or A. Stockage priority code 1 or A means that item had recently experienced a MICAP reportable demand or high priority awaiting parts request. A total of 1336 items were selected. Table 2-8 displays the results for these 1336 essential items.

AGGREGATE MODEL PERFORMANCE FOR ESSENTIAL ITEMS
(ENGLAND AFB - 1336 ITEMS)

SYSTEM	EXPECTED UNITS SHORT (YEAR)
SBSS1	8125
RSL1	4715
AGG2	381

Table 2-8

We can reduce unit backorders for operationally essential items by 95% over the current system and 92% over the revised safety level method. The revised safety level method reduced the MICAP rate by 4% over the SBSS [1]. Table 2-8 shows that the RSLI method caused a reduction of 3410 (8125-4715) units backordered. The aggregate model reduced unit backorders by another 4334 (4715-381). Therefore we assume the MICAP rate would be reduced at least another 4% over the revised safety level methods.

In addition, computing time for the aggregate model took only 174 computer processing seconds on the IBM 4331, which has less computing power than the Phase IV hardware. In terms of benefits gained, new computer hardware, and software capability, the aggregate system is feasible for Air Force-wide use.

The second implementation issue was to insure the model can be used by base-level supply personnel. The aggregate model requires the input of two constraint values: inventory investment and workload. The inventory investment constraint is a limitation on average inventory. We programmed the model to determine the average amount of inventory for the SBSS model and then used that inventory figure as a constraint for the aggregate model. In other words, we minimized backorders using the same amount of inventory as the baseline system. Of course other inventory investment constraints can be used. We also developed the logic to convert a limit on the amount of stock fund purchasing monies to an inventory investment constraint. Thus, the inventory investment constraint was automated and the user need not make any input or the user can enter a purchasing funds dollar limitation. Similarly, we can provide the logic to determine a workload constraint; either as the amount of requisitions used by the SBSS or based on the number of manpower resources available at the base.

The aggregate model uses the same demand data inputs as collected and used by the current SBSS. Thus no additional data collection is necessary. Demand level updates for the aggregate model must be done concurrently for all the items in the system. Therefore, demand leveling would be done periodically (say quarterly) for all the items rather than after a demand as can occur with the current system. However, Phase IV will provide sufficient computer processing power to update the levels quarterly or more often if necessary.

Yet another issue is whether the software can be maintained by the Data Systems Design Office (DSDO) Supply directorate. The model is rather sophisticated and requires some knowledge of calculus. The Logistics Management Center with its operations research staff would be able to provide technical assistance.

Another implementation issue is whether the model is too complex for base level personnel. Prior to recommending the aggregate model, it would be advantageous to prototype the model and examine its operation in a real-life retail level inventory setting. Fortunately we will have that opportunity. The aggregate model has been programmed and will be tested as part of the Civil Engineering Materiel Acquisition System (CEMAS). The final decision to implement the aggregate model for all base level consumable items must await complete conversion to the Phase IV system. By that time, CEMAS will have been fully tested for Civil Engineering support. The decision to use the aggregate model can be made after reviewing the Civil Engineers' experience with CEMAS.

In summary, we have shown that the aggregate inventory model is more efficient and effective than the Standard Base Supply System and the aggregate model is feasible in terms of both hardware and software performance. Civil Engineer Materiel Acquisition System will provide a prototype test of the benefits and operability of the aggregate model in a real-life base level setting.

CHAPTER 3

CONCLUSIONS/RECOMMENDATIONS

CONCLUSIONS

a. An inventory model that minimizes total variable cost does not best suit the operational needs of the Air Force.

b. Air Force inventory variable costs are very difficult to accurately measure.

c. Applying the same variable cost measurement to each consumable item, regardless of the item's characteristics, decreases the performance of an inventory system.

d. An aggregate inventory system that minimizes backorders subject to inventory investment and workload constraints better suits the operational needs of the Air Force.

e. An aggregate inventory system is superior in effectiveness and efficiency to both the current SBSS and the SBSS with its scheduled improvements. We estimate a 95% decrease in the number of units backordered for stocked EOQ items and an additional 4% decrease in the number of MICAP occurrences over the revised safety level method.

f. An aggregate inventory system can be implemented on Phase IV hardware and used by base level supply personnel.

RECOMMENDATIONS: Observe the aggregate model's performance as part of the Civil Engineering Materiel Acquisition System (CEMAS). If the system performs as well as expected, implement the system for all Air Force retail level consumable items. (OPR: HQ USAF/LEYS)

APPENDIX A

40-ITEM DATA SAMPLE

APPENDIX A

40 ITEM DATA SAMPLE

The 40 item sample used to test our eight alternative inventory models was extracted from actual demand data from England AFB. We constructed a sample that represents typical Air Force consumable items in terms of the demand rate and variance of demand. In addition, the sample contains a wide spectrum of data characteristics for demand frequency, units demanded, cost, and demand variance. Table A-1 lists the data characteristics for the entire 40-item sample. It should be noted that the lot size represents the average number of units ordered per customer.

40-ITEM DATA SAMPLE CHARACTERISTICS

ITEM	UNIT COST ($)	YEARLY CUSTOMERS	DAILY CUSTOMER ARRIVAL RATE (CUSTOMERS/ 365)	YEARLY UNITS DEMANDED	LOT SIZE (UNITS/ CUSTOMER)	CONSTANT POISSON LEADTIME VARIANCE OF DEMAND
1	4.71	83	.227	626	7.54	400.99
2	2.19	5	.014	521	104.20	4610.78
3	27.02	1	.003	1	1.00	0.08
4	2.34	15	.041	321	21.40	583.43
5	0.33	20	.055	173	8.65	127.09
6	0.20	5	.014	27	5.40	12.38
7	0.53	16	.044	94	5.87	46.86
8	42.69	15	.041	234	15.60	310.03
9	15.00	4	.011	4	1.00	0.34
10	2.60	4	.011	7	1.75	1.04
11	19.80	3	.008	3	1.00	0.25
12	2.25	37	.101	69	1.86	10.90
13	3.50	78	.214	160	2.05	27.86
14	0.67	11	.030	143	13.00	157.89
15	0.92	32	.088	148	4.63	58.19
16	0.95	1	.003	2	2.00	0.34
17	0.60	1	.003	1	1.00	0.08
18	0.44	1	.003	1	1.00	0.08
19	2.58	22	.060	35	1.59	4.73
20	0.39	5	.014	26	5.20	11.48
21	0.10	3	.008	3	1.00	0.25
22	0.69	1	.003	2	2.00	0.34
23	2.20	34	.093	306	9.00	233.90
24	0.35	10	.027	41	4.10	14.27
25	2.85	22	.060	245	11.14	231.80
26	0.11	8	.022	29	3.63	8.94
27	81.20	1	.003	1	1.00	0.08
28	1.83	4	.011	8	2.00	1.36
29	28.10	54	.148	126	2.33	24.93
30	0.90	122	.334	631	5.17	277.07
31	0.73	47	.129	1853	39.43	6205.42
32	0.55	18	.049	95	5.28	42.60
33	41.50	2	.005	4	2.00	0.68
34	2.40	23	.063	89	3.87	29.25
35	1.44	158	.433	1827	11.56	1793.76
36	1.56	159	.436	1946	12.24	2022.99
37	7.60	14	.038	42	3.00	10.70
38	1.15	8	.022	35	4.38	13.02
39	24.71	3	.008	3	1.00	.25
40	3.38	9	.025	23	2.56	5.00

Table A-1

Note the wide variety of item characteristics; for example, unit prices range from 10 cents to $81.20, customer arrivals vary from 1 to 159, units demanded range from 1 to 1946, and lotsizes take on values from 1 to more than 100 units per order. Also, the leadtime demand variance ranges from near zero for some items to extremely high values for several of the larger demand items. Thus we satisfied our goal to provide a sample of items with a wide spectrum of demand characteristics.

Based on these data characteristics as well as literature sources [7,9] we used the constant-Poisson probability distribution to describe the leadtime customer arrival and demand patterns as input to our alternative models. The constant-Poisson distribution assumes customers arrive according to a Poisson parameter, λ, and each orders a constant lotsize of L units. Table A-2 shows the form of the distribution and its mean and variance.

CONSTANT-POISSON PROBABILITY DISTRIBUTION

Probability Function	Mean	Variance
$Pr(X=x) = \frac{(\lambda t)^{x/L} e^{-\lambda t}}{(x/L)!}$; $x = 0, 1L, 2L, \ldots$	$\lambda t L$	$\lambda t L^2$

where L = Lotsize (in units)
λ = Customer arrival rate (per day)
t = Replenishment cycle leadtime (in days)
X = Units ordered during leadtime

Table A-2

The constant-Poisson distribution realistically describes demand for items that are ordered in consistent lotsizes and describes demand reasonably well for items where the variance of units ordered is small [7]. Table A-1 contains computed ($\lambda t L^2$) demand variance for the 40-item sample.

APPENDIX B

ALTERNATIVE INVENTORY SYSTEMS

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We compared the performance of twelve alternative inventory systems in this study. For purposes of this discussion similar models are grouped into six general categories. The discussion will cover the Standard Base Supply system (SBSS), the SBSS with Revised Safety Level, the Ninety Day inventory system, the Aggregate Models, the RIMSTOP model, and the Power Aproximation Model.

STANDARD BASE SUPPLY SYSTEM (SBSS) MODEL

The SBSS method, outlined in AFM 67-1, Volume II, Part Two computes a reorder quantity (how much to order) and a reorder point (when to order) based on economic considerations (total variable costs). The reorder quantity, Q, is given by the Wilson EOQ formula and the reorder point, R, is the mean demand during leadtime plus some number of standard deviations of leadtime demand which is referred to as a safety level. The equations used to compute reorder quantity and reorder point are:

$$Q = \sqrt{\frac{2LDA}{IP}} \quad , \tag{B-1}$$

where; L = Customer lotsize (units/customer)
D = Expected number of customers annually
A = Order Cost ($)
I = Inventory holding cost fraction
P = Unit price of item ($/unit).

$$R = OSTQ + SLQ \tag{B-2}$$

where; OSTQ = λLt
λ = Customer arrival rate (customers/day)
L = Customer lotsize (units/customer)
t = Replenishment leadtime (days)
and SLQ = Maximum ($C\sqrt{3 \text{ X } OSTQ}$, $C15\lambda L$)
C = Safety Level Factor (C=1 unless directed by HQ USAF).

An estimate for one standard deviation of leadtime demand is given by $\sqrt{3 \text{ X } OSTQ}$. If this estimate is less than the average unit demand rate for 15 days, the safety level is set equal to 15 days worth of inventory ($15\lambda L$). The C-factor is a multiplier of the safety level and is used to set the number of standard deviations of leadtime demand that will be on hand to support a customer during a reorder cycle. We ran the SBSS model for two cases--one with a C-factor of 1 and one with a C-factor of 2 which is supposed to provide an 84% and 97% service level respectively.

SBSS MODEL WITH REVISED SAFETY LEVEL

This model is actually four models that provide the same policy as the SBSS model except the safety level is computed using an actual measure of the variance of leadtime demand. This revised safety level was recommended for Air Force-wide implementation in the AFLMC Demand Forecasting Report[1].

For the first two revised safety level models, we used the variance of lead time demand provided by the assumed constant-Poisson probability distribution (Table A-2) and hence one standard deviation of leadtime demand is

$$\text{STANDARD DEVIATION} = \sigma = \sqrt{L^2 \lambda t} \quad . \tag{B-3}$$

As mentioned in Appendix A, this is a good estimate for demand variance especially when the variance of units per demand is small.

This model's reorder quantity, Q, was computed as it was in the SBSS model and the reorder point, R, is given by

$$R = \text{OSTQ} + C\sqrt{L^2 \lambda t} \ . \tag{B-4}$$

Again, we ran this SBSS revised safety level model for two cases; one with the C factor equal to 1 and one with the C factor equal to 2.

The next two revised safety level models are the same as above except the actual historical demand variance is used to compute the safety level. We compute the demand variance via

$$\begin{array}{l}\text{Variance of} \\ \text{Demand}\end{array} = \frac{\sum \text{Demand}^2 - \dfrac{(\sum \text{Demand})^2}{n}}{n} \tag{B-5}$$

where Demand = number of units ordered by a customer, and

n = number of days since date of first demand.

Thus the reorder point is

$$R = \text{OSTQ} + C\sqrt{\text{Variance of Demand X OST}} \quad . \tag{B-6}$$

We use a C factor of 1 and 2.

NINETY DAY INVENTORY SYSTEM

The ninety day inventory system is the same as the Standard Base Supply with the revised safety level except that the order quantity is 90 days of stock. The order quantity (Q) is shown by;

$$Q = 90\lambda L. \tag{B-7}$$

The reorder point is the same as (B-4) and again we ran 2 models, one with C equal to 1 and one with C equal to 2.

AGGREGATE MODEL

The next two models are different from the other ten in that stockage policy is determined considering the entire inventory system rather than on a single-item basis. The inventory policy is determined by optimally solving, via Lagrangian methods, a mathematical model. The two models minimize expected backorders per year subject to constraints. The first model has only one constraint; a limit on the amount of inventory investment. The second model has the inventory investment constraint and also a limitation on the workload or number of requisitions per year. We describe the model with two constraints below. The model with one constraint is similar [5].

The model minimizes expected backorders per year:

$$\text{minimize} \sum_i \frac{L_i{}^2 D_i}{Q_i} \sum_{x=R_i+1}^{\infty} (x - R_i)\, f(x), \tag{B-8}$$

where L_i, D_i are customer lotsize and expected number of customers annually for item i, and

Q_i = order quantity in units for item i,

R_i = reorder point in customers for item i, and

$f(x)$ = probability of x customer arrivals during a reorder leadtime.

The model is subject to the investment constraint;

$$\sum_i p_i \left(\frac{Q_i}{2} + R_i L_i - \lambda_i t_i L_i\right) \leq I \tag{B-9}$$

where p_i, λ_i and t_i are the price, customer arrival rate and leadtime for item i and I is the inventory investment ceiling in dollars, and a workload constraint;

$$\sum_i \frac{L_i D_i}{Q_i} \leq w \tag{B-10}$$

where w is the maximum number of yearly requisitions.

We can now formulate the Lagrangian function, $\mathcal{L}$,

$$\mathcal{L} = \sum_i \frac{L_i^2 D_i}{Q_i} \sum_{x=R_i+1}^{\infty} (x-R_i)\ f(x) + \ell_I [\sum_i p_i \left(\frac{Q_i}{2} + R_i L_i - \lambda_i t_i L_i\right) - I]$$

$$+ \ell_w [\sum_i \frac{L_i D_i}{Q_i} - w]. \qquad \text{(B-11)}$$

where;

ℓ_I = investment constraint lagrangian multiplier, and

ℓ_w = workload constraint lagrangian multiplier.

It is proved in [5] that the functions are convex; therefore the solutions to the first order conditions are optimal. Thus we differentiate the lagrangian function with respect to the variables of interest (Q, R, ℓ_I, and ℓ_w) and solve the simultaneous equations. It follows that,

$$Q_i = \sqrt{\frac{2 L_i D_i (\ell_w + L_i E_i)}{\ell_I\ p_i}} \qquad \text{(B-12)}$$

$$\sum_0^{R_i} f(x) = 1 - \frac{\ell_I p_i Q_i}{L_i D_i} \qquad \text{(B-13)}$$

$$\ell_I = \frac{\sum_i L_i D_i B_i}{2(I - \sum_i S_i)} \qquad \text{(B-14)}$$

$$\ell_w = \frac{1}{w} [\ell_I \sum_i \frac{p_i Q_i}{2} - \sum_i \frac{L_i^2 D_i E_i}{Q_i}] \qquad \text{(B-15)}$$

where;

B_i is the probability of stocking out, and

S_i is the dollar value of safety stock;

$$B_i = \sum_{x=R_i+1}^{\infty} f(x)$$

$$S_i = L_i p_i\ (R_i - \lambda_i t_i)\ .$$

The model must be solved iteratively starting with small values for the order quantity (Q_i) and reorder point (R_i) which correspond to an assumption of zero safety stock. After computing the initial values, a check is made to see if the constraints are violated. If the constraints are not violated, the order quantity and reorder point are increased systematically to achieve the largest decrease in backorders. The constraints are then rechecked and, if the inventory investment and workload totals are still below their ceilings, the process is repeated. The final solution is found when any additional units of stock cause the constraints to be violated. The process is shown below.

STEP 1. Compute initial values for all items;

$$\ell_I = \frac{\sum .5L_iD_i}{2I}$$

$$Q_i = \frac{.5D_i}{\ell_I P_i}$$

$$\ell_w = \frac{1}{w}\left[\ell_I \frac{\sum_i P_iQ_i}{2} - \sum_i \frac{L_iD_i(.398942\sigma_i)}{Q_i}\right]$$

$$\sum_0^{R_i} = 1 - \frac{\ell_I P_i Q_i}{L_i D_i}$$

STEP 2. Compute the total inventory investment and workload and compare them to the constraints. If the values are less than or equal to the constraints go to step 3. Else go to Step 4.

STEP 3. Recompute ℓ_I and ℓ_w using (B-14) and (B-15):

Recompute the new Q_i and R_i using (B-12) and (B-13):

Go to Step 2.

STEP 4. Print out the values for Q_i and R_i found in the previous iteration; the iteration that did not violate the constraints. This is the optimal inventory policy. Stop.

RIMSTOP

The inventory model entitled RIMSTOP is described in [3,6]. Its solution minimizes the total variable cost equation (per item) given below

$$TVC = \frac{LDA}{Q} + Ip\left(\frac{Q}{2} + RL - \lambda tL\right) + \pi\frac{L^2D}{Q}\sum_{x=R+1}^{\infty}(x-R)f(x) \quad , \qquad \text{(B-16)}$$

where $\frac{LDA}{Q}$ = total annual order cost,

L = customer lotsize,
D = expected number of customers annually,
A = order cost,
Q = order quantity (units),

$Ip(\frac{Q}{2} + RL - \lambda tL)$ = total annual holding cost,

I = holding cost fraction,
p = unit price,
R = reorder point (customers),
λ = customer arrival rate,
t = replenishment leadtime (days),

and $\pi\frac{L^2D}{Q} \sum_{x=R+1}^{\infty} (x-R)f(x)$ = total annual backorder cost, where

π = backorder penalty cost, given by

$$\pi = \frac{IpQ}{.16LD} \tag{B-17}$$

$\sum_{x=R+1}^{\infty} (x-R)f(x)$ = expected number of customer backorders during leadtime (E) and it follows [5] that

$$E = \lambda t - R + \sum_{x=0}^{R} (R-x)f(x). \tag{B-18}$$

The total annual cost equations are described in detail in Appendix C.

In order to find the solution to the total variable cost equation (B-16) we take partial derivatives with respect to the variables of interest, Q and R, equate them to zero, and obtain

$$Q = \sqrt{\frac{2LDA + \pi LE}{Ip}} \tag{B-19}$$

and $$\sum_{0}^{R} f(x) = \frac{\pi LD - IpQ}{\pi LD} = 1 - \frac{IpQ}{\pi LD} \quad . \tag{B-20}$$

Equations (B-19) and (B-20) are solved iteratively, starting with an assumption of zero backorders. Hence, initially the reorder quantity simply becomes the Wilson-EOQ,

$$Q = \sqrt{\frac{2LDA}{Ip}} \quad . \tag{B-21}$$

This Q is substituted into the backorder penalty cost equation (B-17), which is then used to calculate the reorder point using (B-20). Expected backorders during leadtime (B-18) are then computed and used to compute a "new" reorder quantity (Q) using (B-19). We compare the "new Q" to the "old Q" and continue the process until the Q's converge to a specified significance. That iteration's Q and R becomes the item's reorder quantity and reorder point, respectively.

POWER APPROXIMATION

The Power Approximation Model is a periodic review inventory model which provides approximately optimal stockage policy. A periodic review inventory model assumes inventory is checked periodically rather than continually and when the existing level is below the reorder point, s, sufficient stock is ordered to bring the level up to a point, S. Since the chance of a stockout can occur during the review period as well as the replenishment cycle, more stock is required [11] than for a continuous review system.

The Power Approximation model is documented in [4]. We present the algorithm below.

Compute:

$$F = (1.463) \left(\frac{DtL}{365}\right)^{.364} \left(\frac{A}{Ip}\right)^{.498} [(T + 1)\, \sigma^2]^{.0691} \qquad \text{(B-22)}$$

where
- D = expected number of customers annually (customers),
- L = customer lot size (units/customer),
- t = reorder cycle leadtime (days) per period,
- T = number of leadtime periods (t/30),
- A = order cost (we use the Air Force order cost of 4.54),
- I = inventory holding cost fraction (the current Air Force fraction is .26),
- p = unit price of an item ($/unit), and
- σ^2 = variance of demand during a leadtime.

Compute:

$$s_1 = (T + 1) \left(\frac{DtL}{365}\right) + \left[(T + 1)\left(\frac{DtL}{365}\right)\right]^{.416} \left[\frac{\sigma^2}{\left(\frac{DtL}{365}\right)}\right]^{.603} \mu(z) \qquad \text{(B-23)}$$

and $S_1 = s_1 + F$

where $\mu(z) = \dfrac{.182}{z} + 1.142 - 3.466z$ and

$$z = \left[\frac{\left(\frac{DtL}{365}\right)^{.364} \left(\frac{A}{Ip}\right)^{.498}}{\left(1 + \frac{\pi}{Ip}\right) [(T + 1)\sigma^2]^{.431}}\right]^{1/2}$$

π = backorder penalty cost.

If $F/\left(\frac{DtL}{365}\right) > 1.5$, then $s = s_1$ and $S = S_1$. Otherwise

$$S_2 = (T+1)\,\frac{DtL}{365} + C[(T + 1)\,\sigma^2]^{1/2} \qquad \text{(B-24)}$$

where C = C factor.

Then

$$s = \min\,(s_1, S_2)$$

$$S = S_1 \quad . \qquad \text{(B-25)}$$

Thus the Power Approximation yields an (s,S) policy which states as the on-hand stock level (x), reaches s, order S-x units. The policy is analogous to the SBSS policy where

Reorder point (R) = s

and Reorder quantity (Q) = S - s.

APPENDIX C

MATHEMATICAL FORMULATION OF PERFORMANCE CRITERIA

APPENDIX C

MATHEMATICAL FORMULATION OF PERFORMANCE CRITERIA

In this appendix, we have documented the mathematics to derive the five criteria used to compare the alternative inventory systems. The five criteria are:

(1) Number of orders,
(2) Backorders in units per year,
(3) Average on hand inventory ($),
(4) Total variable costs, and
(5) Overall unit fill rate.

We first list the notation and then derive the mathematics for each criteria.

We used the constant-Poisson probability distribution to describe the customer arrival and demand patterns during leadtime for our inventory models. This distribution was used to formulate our performance measures and involves the following notation:

- Q = order quantity (units),
- R = reorder point quantity in customers (lots),
- L = customer lot size (units/customer),
- RL = reorder point quantity in units,
- D = expected number of customers annually (customers),
- A = order cost (we use the Air Force order cost of 4.54),
- I = inventory holding cost fraction (the current Air Force fraction is .26),
- π = backorder penalty cost ($/unit),
- p = unit price of an item ($/unit),
- E = expected customer backorders during leadtime (customers),
- λ = customer arrival rate (customers/day),
- t = reorder cycle leadtime (days), and
- $f(x)$ = Poisson probability distribution with parameter λt, where $f(x)$ is the probability of having x customers during the leadtime t.

The discussion of performance criteria is in terms of individual items. However, overall aggregate performance can be found with the summation or averaging of results for all items in the data set.

(1) Number of Orders: The number of orders or requisitions per year is the yearly demand in units divided by the order quantity

$$\text{Annual Orders} = \frac{LD}{Q} . \tag{C-1}$$

(2) Backorders: The expected number of units backordered annually for each item is simply its number of annual orders times the expected units backordered per reorder cycle

$$\text{Annual Backorders} = \left(\frac{LD}{Q}\right) \cdot (LE) = \frac{L^2 D}{Q} E . \tag{C-2}$$

The expected backordered customers per reorder cycle, E, is given by the sum of all possible orders greater than the reorder point multiplied by the probability of receiving an order of that given size

$$E = \sum_{x=R+1}^{\infty} (x-R)f(x) . \qquad \text{(C-3)}$$

This summation can be simplified [5] to

$$E = \lambda t - R + \sum_{x=0}^{R} (R-x)f(x) . \qquad \text{(C-4)}$$

(3) Average on hand inventory ($): Each item's dollar value of average on-hand inventory is found by multiplying its unit price by average on hand inventory.

$$\text{Dollar Value of Average On-Hand Inventory} = p(\frac{Q}{2} + RL - \lambda tL). \qquad \text{(C-5)}$$

(4) Total Variable Costs: Total variable cost (TVC) per year can be expressed as

$$TVC = OC + HC + BC \qquad \text{(C-6)}$$

where OC = total annual order costs,
HC = total annual holding costs, and
BC = total annual backorder costs.

The order cost is easily calculated by multiplying the annual orders by the ordering cost (A)

$$OC = \frac{LD}{Q} A . \qquad \text{(C-7)}$$

Likewise, the holding cost is the holding cost fraction times the dollar value of average on-hand inventory

$$HC = Ip(\frac{Q}{2} + RL - \lambda tL) . \qquad \text{(C-8)}$$

Since the probability function of customer demand during leadtime is known (constant-Poisson) we calculate the backorder cost as the product of the penalty cost (π), and (C-2)

$$BC = \pi \frac{L^2 D}{Q} E . \qquad \text{(C-9)}$$

It is shown that the penalty cost can be set to achieve a desired reorder cycle service level [6]. The Air Force uses 84% as its desired reorder cycle fill rate therefore the penalty cost is set so:

$$\sum_{0}^{R} f(x) = \frac{\pi LD - IpQ}{\pi LD} = .84 \ .$$

Solving for π:

$$\pi = \frac{IpQ}{.16LD} \qquad \text{(C-10)}$$

it necessarily follows that backorder cost (C-9) becomes

$$BC = \frac{IpLE}{.16} \ . \qquad \text{(C-11)}$$

(5) Overall Unit Fill Rate: An inventory system's unit fill rate is defined as the probability that a unit will be available when it is requested or

$$\text{Unit fill rate} = \frac{\text{Units Available}}{\text{Total Units Requested}} \ . \qquad \text{(C-12)}$$

The units not available during stock replenishment are the expected backorders from the formula shown as (C-2). Since the sum of the units available and the units not available is the total units requested,the unit fill rate can be computed via:

$$\text{Unit fill rate} = 1 - \frac{\text{Units backordered}}{\text{Total Units Requested}} \qquad \text{(C-13)}$$

where;

$$\text{Units backordered} = \frac{L^2 D_E}{Q},$$

and Total Units Requested = $365\lambda L$.

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