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MULTIAXIAL FATIGUE AND FRACTURE— A LITERATURE REVIEW

by

P. W. BEAVER

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SUMMARY

Problems often arise when attempting to determine the fatigue behaviour of structural components from laboratory data. A major reason for this lack of correlation is that most engineering components operate in stress environments significantly more complicated than uniaxial tension, the stress state in which most research studies are conducted.

A review of the literature has shown that virtually all fatigue and fracture properties of metals and components are affected by multidirectional loading. In particular, variations in stress state compared with uniaxial tension produce the following effects:

(a) *decreases in the fatigue limit by up to approximately 50%;*

(b) *increases or decreases in low-cycle fatigue life by factors of up to 20, depending on whether the stress in the second direction is tensile or compressive and is static or cyclic;*

(c) *out-of-phase loading also reduces the low-cycle fatigue life by a factor of up to 4 compared with in-phase loading; and*

(d) *accelerations and retardations of fatigue crack growth rates by factors of 3 to 4 depending on the nature of the transverse stress.*

It is also evident that multiaxial criteria used for design purposes can be non-conservative, especially under out-of-phase loading, and consequently can lead to unsafe estimates of the fatigue life of a component.

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1. INTRODUCTION

During the past few decades a considerable amount of data concerning both the fatigue and fracture behaviour of commercial materials has been generated. However, problems often arise when attempting to predict the fatigue behaviour of mechanical components and structural elements from laboratory data and the failure of engineering structures and components by fatigue still remains as a major problem.

Several factors are responsible for the difficulties encountered in correlating laboratory fatigue data with the behaviour of structural components. Many engineering components operate in stress environments significantly more complicated than uniaxial tension, the stress state in which most research studies are conducted. For example, biaxial stress states exist in the leading edges of gas turbine blades, in pressure vessels and in the skins of aircraft wings, whilst components, such as crank-shafts or propeller shafts are subjected to simultaneous, and usually out-of-phase, bending and torsion. In addition, most research investigations are performed using specimens of relatively simple geometry. In contrast, geometric discontinuities and notches are present in engineering components, such as joints in aircraft wings or cooling holes in gas turbine blades, which introduce localized regions experiencing both high strains and complex stress states. Therefore, an area of research which still requires considerable attention is the fatigue behaviour of materials under complex stress states, especially in the low-cycle, high-strain region, and in the presence of geometric discontinuities.

At present the amount of multiaxial fatigue research of metals has been very limited compared with that undertaken for uniaxial fatigue. This situation has arisen for the following reasons:

(1) There has been a lack of reliable multiaxial or biaxial testing machines as the design, development and experimental work associated with building such equipment tends to be complicated, time consuming and expensive compared with that for uniaxial testing systems [1, 2].

(2) Intricate specimen designs are required, hence the specimens also are expensive [3].

(3) No accurate analytical methods for determining the stress distributions in the various test specimens are available [4].

(4) There is no substantial theoretical framework for analysing the data [5].

(5) The experimental work is time consuming.

(6) A large number of test results are required to develop and prove or disprove empirical criteria [3].

Most multiaxial fatigue research conducted on metals has been under biaxial loading as fatigue cracks generally originate at a free surface where biaxial stress states exist. Compared with that for metals, multiaxial fatigue data for fibre reinforced composites are even more scarce, as several unusual additional problems are associated with multiaxial fatigue testing of these materials [6].

Several review papers surveying particular aspects of the multiaxial fatigue of metals are presently available [3, 7–14]. A comprehensive review up to 1974 of the phenomenological aspects of multiaxial fatigue in the low-cycle range ($< 10^5$ cycles) was undertaken by Krempl [7].

McDiarmid [ ] and Parsons and Pascoe [9] have reviewed multiaxial high-cycle fatigue whilst Brown and Miller [10] have reviewed both low-cycle and high-cycle multiaxial fatigue. More recently Brown and Miller [11–13] have reviewed several aspects of multiaxial fatigue, such as the effects of stress state on the cyclic deformation behaviour and mode of crack growth in metals, together with the evolution of the various multiaxial low-cycle fatigue criteria [3]. Garud [14] has reviewed multiaxial fatigue with an emphasis on the criteria for assessing the fatigue strength of metals in multiaxial stress states at room temperature. Additional references are given at the end of this review concerning other aspects of multiaxial fatigue, such as cumulative fatigue damage, notch effects, cyclic deformation behaviour and creep-fatigue interactions.

To the best of the author's knowledge there is no review paper presently available regarding the multiaxial fatigue behaviour of fibre reinforced composites. This topic will be reviewed in a later paper.

At present there is considerable lack of agreement in the literature regarding the apparent effects of multiaxial stress states on fatigue properties and especially on fatigue crack growth rates. For example, experimental results have shown that applying a tensile stress parallel to a crack and perpendicular to the principal stress can increase, decrease, or have no effect on the fatigue crack growth rate. Several factors are responsible for the inconsistency in the literature and their effects can be sub-divided into the following groups [15]:

(1) Material effects resulting from differences in the type of material or alloy tested and its mechanical working and heat treatment process.

(2) Environmental effects, such as the interaction between a growing crack and its chemical environment.

(3) Geometrical effects arising from the use of a variety of test methods and/or specimen and crack geometries.

(4) Load state effects such as whether the transverse loads parallel to a crack are static or cyclic, tensile or compressive and in- or out-of-phase.

Other contributing factors, which have been mentioned previously, are (1) the lack of methods for accurately analysing the stress distributions in the test specimens, and (2) the absence of a satisfactory theoretical basis for interpreting the test data.

The objective of this report is to systematically review published data concerning the effects of multiaxial stress on fatigue and fracture properties, and thereby clarify many of the conflicting results in the literature. Emphasis is placed on the effects of biaxial loads on cyclic properties, deformation mechanisms and crack growth behaviour, as the various multiaxial fatigue criteria have been adequately reviewed elsewhere [3, 7–10, 14].

2. DEFINITIONS AND NOMENCLATURE

2.1 Definitions

The stress states associated with the various biaxial testing techniques, which will be described in the next section, have been defined using a variety of parameters. For example, the biaxial stress states in cruciform type specimens, such as shown in Figure 1(*a*), may be expressed in terms of either,

(1) the ratio of the NOMINAL stresses remote from the test area $S_x/S_y = \lambda$; or

(2) the ratio of the LOCAL stresses within the test area $\sigma_x/\sigma_y = k$.

Both biaxial stress ratios have values of between -1 for pure shear and $+1$ for equibiaxial tension with a value of 0 for uniaxial tension. Similarly, the multiaxial strain states encountered in strain controlled, low-cycle fatigue tests may be expressed in terms of the ratio of the principal strains in the test section, $\epsilon_x/\epsilon_y = \phi$. This parameter also has values between -1 (pure shear) and $+1$ (equibiaxial tension). However, the biaxial strain ratio has a value of $-\nu$ in uniaxial tension, where ν is Poisson's ratio. In comparison, a parameter frequently used for combined tension–torsion tests is the ratio of the torsional to axial strain range $\Delta\gamma/\Delta\epsilon = \psi$. The range of values for this parameter is between 0 (uniaxial tension) and ∞ (pure shear).

No single parameter, including those above, can completely specify the loading system as the following factors must also be considered:

(a) The phase difference between the two loading directions as the ratio of σ_x/σ_y will vary with time if a phase difference exists.

(b) The mean loads of the cyclic loading in each of the two directions.

(c) Whether the loading in one of the directions is static tensile or compressive.

Therefore, two or more parameters are required to completely specify the loading system.

An additional factor which must be considered in specifying the stress state for biaxial specimens containing cracks is the direction of the maximum principal stress with respect to the plane of the crack. In general, the biaxial stress ratio σ_x/σ_y is used to define the stress state in cracked specimens. However, values of greater than 1 are frequently reported in the literature. By convention the loading axis perpendicular to the crack is defined as the y-direction and that parallel to the crack as the x-direction. Consequently, when the biaxial stress ratio is less than 1, the maximum principal stress is perpendicular to the plane of the crack. In comparison, the maximum principal stress is parallel to the crack when the biaxial stress ratio is greater than 1. Therefore, these two situations represent different crack/loading configurations.

Many authors have used the same symbol to represent different parameters. For example, some authors have used the symbol λ to represent the biaxial stress ratio σ_x/σ_y, whereas others have used this symbol to represent the ratio of torsional to axial strain $\Delta\gamma/\Delta\epsilon$. The various symbols used to represent such parameters in this report are as shown in the following section.

Most of the biaxial fatigue test examined in this report have been used completely reversed, in-phase loading cycles. Therefore, unless stated otherwise, the cyclic loads in the x- and y-directions can be assumed to be in-phase and acting about a zero mean stress.

2.2 Nomenclature

P_x, P_y	NOMINAL loads remote from the test area
S_x, S_y	NOMINAL stresses remote from the test area
σ_x, σ_y	LOCAL stresses within the test area
ϵ_1, ϵ_2	principal surface strains
ΔP	axial load range
$\Delta\tau$	torsional stress range
$\Delta\sigma$	axial local stress range
$\Delta\gamma$	torsional strain range

$\Delta\epsilon$	axial strain range
λ	NOMINAL biaxial stress ratio (S_x/S_y)
k	LOCAL biaxial stress ratio ([illegible])
η	ratio of torsional to axial stress range ($\Delta\tau/\Delta\sigma$)
ψ	ratio of torsional to axial strain range ($\Delta\gamma/\Delta\epsilon$)
ϕ	biaxial strain ratio ($\Delta\epsilon_2/\Delta\epsilon_1$)
t	fatigue strength in torsion at 10^7 cycles
b	fatigue strength in bending at 10^7cycles
S_t	torsional stress
S_b	bending stress
S_{max}, S_{min}	maximum and minimum NOMINAL stress in a cycle
R	stress ratio $= S_{min}/S_{max}$
$\sigma_1, \sigma_2, \sigma_3$	principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$)
$\epsilon_1, \epsilon_2, \epsilon_3$	principal strains ($\epsilon_1 > \epsilon_2 > \epsilon_3$)
τ_{oct}	octahedral shear stress
γ_{oct}	octahedral shear strain
τ_{max}	maximum shear stress
γ_{max}	maximum shear strain
ϵ_n	strain normal to the plane of maximum shear
Γ	definition of the plot of $\frac{\gamma_{max}}{2}\left(=\frac{\epsilon_1-\epsilon_3}{2}\right)$ vs. $\epsilon_n\left(=\frac{\epsilon_1+\epsilon_3}{2}\right)$
g, h and j	life dependent constants
K	stress intensity factor
K_c	critical stress intensity factor
J	the J-integral
G	strain energy release rate
COD	crack opening displacement
E	modulus of elasticity in tension
ν	Poisson's ratio

n'	cyclic hardening exponent
k'	cyclic strength co-efficient
$\Delta\epsilon_e/2$	elastic strain amplitude
$\Delta\epsilon_p/2$	plastic strain amplitude
$\Delta\epsilon_T/2$	total strain amplitude
N_f	number of cycles to failure
α, A	Manson–Coffin exponent and intercept respectively
β, B	Basquin exponent and intercept respectively
X, Y	constants
θ	phase angle between the torsional and axial strain
a	half crack length
da/dN	crack growth rate
r_p	plastic zone size
K_t	theoretical stress concentration factor
K_f	fatigue strength reduction factor

Subscripts

x, y	cartesian co-ordinates
1, 2, 3	principal values
I, II, III	mode of crack surface displacement
c	critical
oct	octahedral

*Conversion Units**

1 in. = 25·4 mm

1 mil = 25·4 μm

1 lb_f = 4·448 N

* NOTE: All diagrams are reproduced from the original source and contain the units in use at that time.

1 kip = $4 \cdot 448 \times 10^3$ N

1 psi = $6 \cdot 895$ Nm^{-2}

1 ton_f $in.^{-2}$ = $15 \cdot 4$ MNm^{-2}

1 kpsi$\sqrt{in.}$ = $1 \cdot 099$ MNm^{-2}

3. MULTIAXIAL FATIGUE TESTING TECHNIQUES

A wide variety of techniques have been used to fatigue test laboratory specimens under multiaxial stress states. These techniques include:

(1) *reversed bending* of wide cantilever specimens. Varying the width-to-thickness ratio produces biaxial stress ratios of $\nu \geqslant \lambda \geqslant 0$, where ν is Poisson's ratio;

(2) *anticlastic bending* of square and rhombic-shaped plates. The range of biaxial stress ratios is $0 \geqslant \lambda \geqslant -1$ depending on the ratio of the lengths of the major and minor axes.

(3) *bulge testing* where round or oval-shaped plates are clamped around the periphery and subjected to fluctuating pressure on one or both sides. An equibiaxial stress state ($\lambda = +1$) is produced in the central area of the round plates whereas the oval-shaped plates produce biaxial stress ratios of $+1 > \lambda > \nu$;

(4) *cruciform specimen technique* where in-plane orthogonal loads are applied to cross-shaped plate specimens. The complete range of biaxial stress ratios, i.e., $1 \geqslant \lambda \geqslant -1$ can be obtained with this technique; and

(5) *thin-walled tubes* subjected to axial tension–compression plus (*a*) torsion loading where the range of stress states available is limited to $0 \geqslant \lambda \geqslant -1$, or (*b*) internal-external pressure which increases the stress states available to $1 \geqslant \lambda \geqslant -1$.

Evans [16], Krempl [7], Lohr [1, 2] and Brown [13, 17] have reviewed the above techniques and discussed the respective advantages and disadvantages. These reviews show that the two methods most frequently used are:

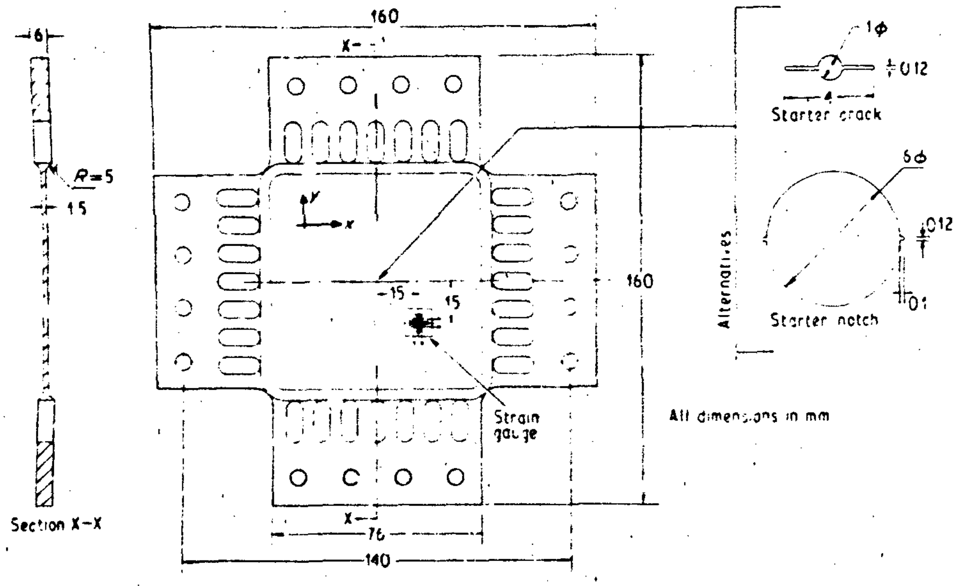
(1) the cruciform specimen technique; and

(2) combined tension-torsion of thin-walled tubular specimens.

Examples of the types of specimens used are shown in Figures 1(*a*) and 1(*b*). The cruciform specimen technique has several advantages which make it suitable for biaxial fatigue studies, namely:

(1) A full range of biaxial stress and strain ratios can be obtained using a single specimen geometry.

(2) The test section area is easily observed during a test.



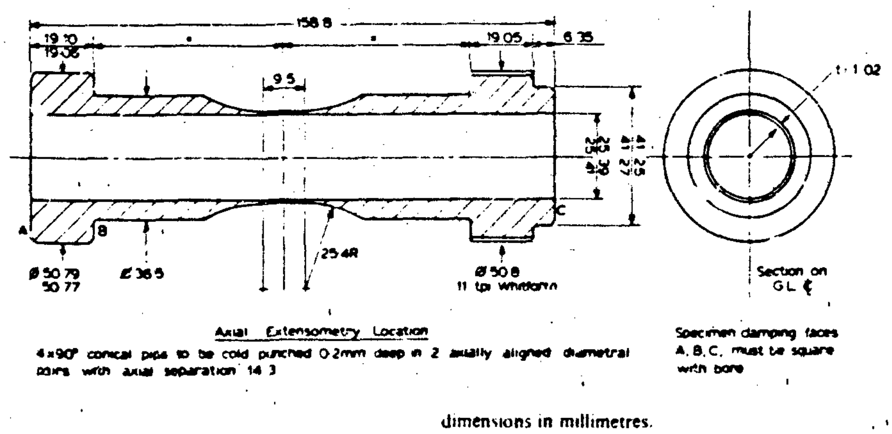


FIG. 1. EXAMPLES OF BIAXIAL FATIGUE SPECIMENS

a) A CRUCIFORM SHAPED SPECIMEN [68]

b) A THIN-WALLED TUBULAR SPECIMEN [2]

(3) The specimen shape is suitable for microscopy.

(4) The mean load level and amplitude on each axis can be controlled independently.

(5) It is the most versatile biaxial fatigue system available for elevated temperature work.

(6) It is most suitable for fatigue crack growth studies.

(7) Low-cycle fatigue studies are possible.

(8) The test system is suitable for component testing.

However, there are several disadvantages associated with this technique: these are:

(1) The thickness of the test section must be reduced to ensure that crack initiation and failure occur within that region. This means that the stresses in this region may not be accurately estimated from the load cell readings as an unknown proportion of each load is carried by the thick outer section [16, 17]. Local stresses and strains must be determined using finite element analyses or strain gauge measurements.

(2) The best profile for strain uniformity in the test section is a flat-bottomed profile. However, this profile is susceptible to buckling under compressive loads. This problem can be overcome by using a continually radiused test section although this would result in a decrease in the volume of uniform strain distribution. Therefore, the specimen design must be a compromise, preventing buckling yet producing an adequate volume of uniform strain.

(3) Once yielding has occurred the stress distribution in the test section cannot be calculated using resistance gauges.

(4) The load frame and actuator displacements limit the range of specimen sizes.

Similarly, the advantages and disadvantages of the tension-torsion technique using thin-walled specimens can be summarised as follows :

Advantages

(1) The loads in the test section can be easily determined as the applied loads are fully supported in this region. Similarly, the strain can be easily measured using a biaxial extensometer.

(2) Accurate calculation of plastic strain is not difficult.

(3) The stresses and strains can be continuously monitored thus simplifying the generation of hysteresis loops.

(4) High-strain low-cycle fatigue data are readily obtainable.

(5) The system can be used at elevated temperatures.

(6) This technique is suitable for understanding the fundamental behaviour of materials in complex stress states as both stresses and strains (elastic and plastic) are, in principle, measurable quantities and the ratio of shear tensile stress can be controlled. This latter point is important as shear predominates in the formation and fracture processes under cyclic stresses.

Disadvantages

(1) A limited range of biaxial stress and strain ratios is available.

(2) Strain gradients through the wall of the specimen can extend the fatigue life. This effect can be overcome by reducing the tube-wall thicknesses, but buckling may limit the extent to which this can be done and thus again, the specimen design must be based on an acceptable compromise.

Increasing the range of stress ratios possible by using pressurisation instead of torsion introduces additional problems, namely.

(a) Through-thickness cracks would limit pressurisation being used for biaxial fatigue crack growth studies.

(b) The environmental interactions of the liquid used for pressurisation may influence the initiation and growth of cracks.

(c) The "hydrowedge effect" may be encountered, i.e. where high pressure liquid enters cracks and wedges them open. This effect provides an additional Mode I opening stress and alters the crack closure characteristics.

(d) High temperature work may not be possible because of the limited temperature range available with such liquids as mineral oils.

Effects (b) and (c) may be minimised by used a rubber sleeve to eliminate contact between the oil and the specimen surface [17]. However, suitable sleeving material may not be available for high temperature work.

4. THE EFFECTS OF MULTIAXIAL STRESS ON FATIGUE AND TENSILE STRENGTH

The effects of combined stress on high-cycle fatigue properties, such as fatigue strength or fatigue limit have been studied since the turn of the century. Consequently, much information is available for laboratory specimens subjected to nominally elastic bulk stresses and with fatigue lives in excess of 10^5 cycles. Brown and Miller [3, 10], and Garud [14] have reviewed the most important papers published on this subject. However, since infinite life is no longer a widely used criterion for fatigue design (especially in structural components) a brief discussion only will be given. Furthermore, most of the work on the effects of multiaxiality on fatigue strength was published over 20 years ago.

Most of the experimental data were obtained from combined bending and torsion tests. For example, Gough *et al* [18, 20] determined the fatigue limits at 10^7 cycles (zero mean stress) for numerous ferrous alloys under alternating bending, alternating torsion and five different combinations of in-phase alternating bending and torsion. Three typical sets of results are shown in Figure 2. In addition, as shown in Table 1, Forrest [21] compiled similar data from a number of investigators. This compilation shows that, 1) the torsional fatigue strength (t) is less than the bending fatigue strength (b) and, 2) the ratio of the fatigue strength in torsion to that in bending (t/b) varios markedly from one material to another. Forrest also claimed that the *t/b* ratio bears no relationship to the tensile strength of the metal.

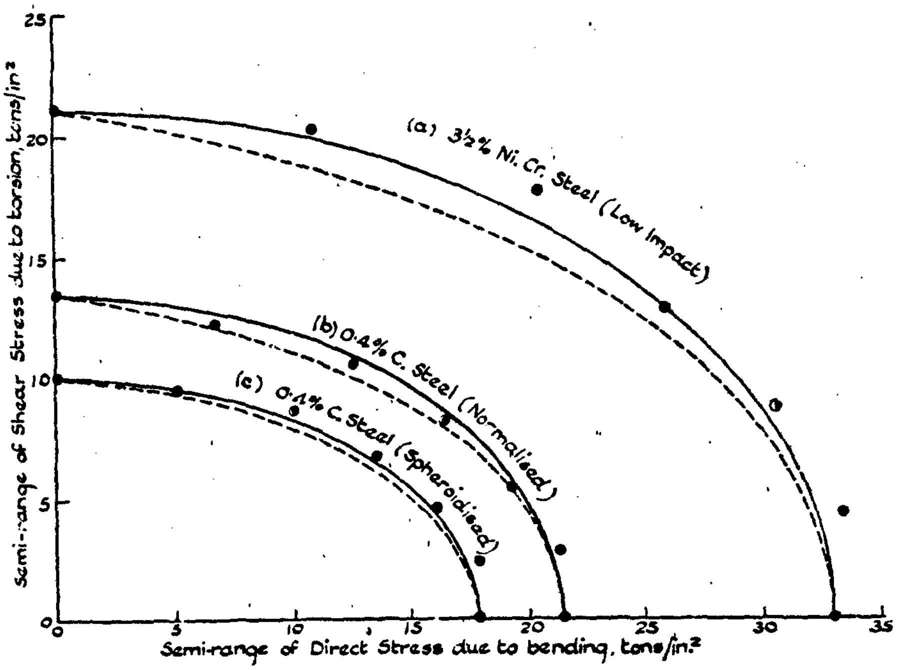


FIG. 2. FATIGUE LIMITS OF THREE STEELS UNDER COMBINED BENDING AND TORSIONAL ALTERNATING STRESSES [22]

The large variation in the ratio of t/b indicated that none of the traditional failure criteria could be relied upon for predicting the fatigue strengths under other combined stress conditions. This fact (which will be discussed in the next section) was realised by Gough who derived two empirical equations for this purpose, based on the experimental values of t/b. Ductile metals could be represented by an equation of the form,

TABLE 1

Ratio of Fatigue Strength in Torsion (t) to that in Bending (b) for Various Materials as Compiled by Forrest [21]

Material	Range of Ratio t/b	No. of Results Considered	Average Value of t/b
Wrought steels	0·52–0·69	31	0·60
Wrought aluminium alloys	0·43–0·74	13	0·55
Wrought copper and copper alloys	0·41–0·67	7	0·56
Wrought magnesium alloys	0·49–0·60	2	0·54
Titanium	0·37–0·57	3	0·48
Cast iron	0·79–1·01	9	0·90
Cast aluminium and magnesium alloys	0·71–0·91	5	0·85

$$\left(\frac{S_t}{t}\right)^2 + \left(\frac{S_b}{b}\right)^2 = 1, \qquad (1)$$

and brittle metals by an equation of the form,

$$\left(\frac{S_t}{t}\right)^2 + \left(\frac{S_b}{b}\right)^2\left(\frac{b}{t} - 1\right) + \left(\frac{S_b}{b}\right)\left(2 - \frac{b}{t}\right) = 1 \qquad (2)$$

where S_t and S_b are the torsional and bending stresses, and t and b are the torsional and bending fatigue strengths respectively at 10^7 cycles. Equation (1) is known as the Ellipse Quadrant relationship as the data, when plotted, form part of an ellipse. Similarly, equation (2) is known as the Ellipse Arc relationship. More accurate predictions of fatigue strength under combined stress states and for a wider range of materials were possible with these equations.

The effects of both triaxial and biaxial stress states on the static and high-cycle fatigue strengths of a Cr-V and a low C-steel (using test specimens of the same geometry) have been established in a recent paper by Habetineck [22]. He found that the ultimate tensile strengths of these steels increased by up to 18% as the number of loading axes increased from one to three, Table 2. In contrast, the fatigue strengths of the triaxial specimens were higher than in uniaxial tension by up to 48% compared with equibiaxial or equitriaxial tension loading, Table 3. The fatigue strengths of the latter two stress states were approximately the same. Habetineck concluded that the fatigue strength of the specimens which he used was determined by the strength of the surface layer. This was in a state of biaxial stress in both biaxially and triaxially loaded specimens. Similarly, other workers [23–25] have concluded that the intermediate principal stress has no effect on the fatigue strengths of various metals.

TABLE 2

The Effect of Stress State on Ultimate Tensile Strength

The values given in this table are the average of three tests [22]

Material	Stress State	Ultimate Tensile Strength (PMa)
Chromium steel (CSN 15230)		
(a) Heat treated and tempered	uniaxial	980·6
	equibiaxial	1049·3
	equitriaxial	1157·1
(b) Soft annealed	uniaxial	648·2
	equibiaxial	650·2
	equitriaxial	674·6
Low carbon steel (SCN 12010) Normalized	uniaxial	519·7
	equibiaxial	539·4
	equitriaxial	558·9

TABLE 3

The Effect of State of Stress on Fatigue Limit

Tests were conducted using:

(1) A Symmetric Cycle Around a Zero Mean Load ($R = -1$) and

(2) A Zero to Tension Cycle ($R = 0$) [22].

Material	Stress State	Fatigue Limit* (MPa) (at 10^7 cycles)	
		$R = -1$	$R = 0$
Chromium steel (CSN 15230)			
(a) Heat treated and tempered	uniaxial	75·5	144·1
	equibiaxial	49·0	90·2
	equitriaxial	42·1	85·3
(b) Sof annealed	uniaxial	70·6	137·2
	equibiaxial	46·0	85·3
	equitriaxial	40·2	71·6
Low carbon steel (SCN 12010) Normalized	uniaxial	63·7	112·7
	equibiaxial	40·2	70·6
	equitriaxial	39·2	69·6

*NOTE : Here, the fatigue limit if defined using the maximum stress S_{max} and not the stress amplitude S_a.

5. MULTIAXIAL FATIGUE CRITERIA

The ability to predict the fatigue behaviour of structural components is an important factor in the safe design and reliable operation of many engineering systems. Since most components in service are subjected to complex stress states, it is necessary to consider multiaxial fatigue behaviour when predicting the safe life or structural integrity of any component. However, these predictions are usually based on simple laboratory tests under uniaxial loading and utilizing multiaxial fatigue criteria. The purpose of these criteria is to reduce complex multiaxial stress states to an equivalent uniaxial stress state.

At present more than 20 multiaxial fatigue criteria exist [26]. The theoretical and experimental bases for many of these criteria have been reviewed recently by Brown and Miller [10], Krempl [7] and Garud [14]. These reviews show that the early multiaxial fatigue criteria were based on either static yield theories for ductile materials, or fracture criteria for brittle or notched materials. For example, two yield theories which have been used with some success for fatigue strength correlations in the high-cycle fatigue regime are :

(1) Tresca's maximum shear stress criterion,

$$\tau_{max} = \text{constant} = (\sigma_1 - \sigma_3)/2 \tag{3}$$

and (2) Von Mises octahedral shear stress criterion, which is equivalent to a critical distortional strain energy criterion,

$$\tau_{oct} = \text{constant} = \tfrac{1}{3}\sqrt{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}. \tag{4}$$

An example of a fracture criterion is Rankine's critical tensile stress,

$$\sigma_1 = \text{constant}, \tag{5}$$

which was initially proposed for the fracture of brittle materials. These stress based criteria are more suitable for high-cycle fatigue as the stresses can be easily calculated using elastic analyses. In contrast, strain based criteria, such as the octahedral shear strain theory,

$$\gamma_{oct} = \text{constant} = \tfrac{2}{3}\sqrt{[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2]} \tag{6}$$

are more suitable in the low-cycle fatigue (LCF) regime, particularly since strain controlled cycling is often encountered in laboratory and practical LCF situations.

Unfortunately, predictions using these classical theories have sometimes proven unsatisfactory [7, 10, 12-14]. For example, Brown and Miller [10] concluded that the octahedral shear strain theory can be non-conservative when used to predict LCF life. Furthermore, these authors concluded that it is not possible, by using the classical theories, to correlate all multiaxial fatigue data as : (1) they cannot separate out the two possible cracking systems, i.e. one along the surface (Case A) and one away from the surface (Case B) as in Figure 3, and (2) they cannot account for the effects of the orientation of the three-dimensional strain field with respect to the surface [11-13].

The lack of correlation between the classical theories and experimental data has resulted in more complex theories being developed. For example, Brown and Miller [10] proposed a new theory based on a physical interpretation of the mechanisms of plastic deformation and fatigue crack growth. They suggested that fatigue life is controlled by the maximum shear strain,

$$\frac{\gamma_{max}}{2} = \left(\frac{\epsilon_1 - \epsilon_3}{2}\right), \tag{7}$$

and the strain normal to the plane of maximum shear,

$$\epsilon_n = \left(\frac{\epsilon_1 + \epsilon_3}{2}\right). \tag{8}$$

These two strains govern the direction of crack growth, the fatigue life, i.e. rate of crack growth, and help to differentiate between the two possible cracking systems, as described in the following paragraph.

The theory of Brown and Miller can be represented graphically by contours of constant fatigue life called "Γ plots" which can be expressed mathematically as,

$$\left(\frac{\epsilon_1 - \epsilon_3}{2}\right) = f\left(\frac{\epsilon_1 + \epsilon_3}{2}\right). \tag{9}$$

These Γ plots consist of two separate curves for any given fatigue life, depending on whether the cracking system is Case A or Case B as shown, for example, in Figure 4. Specific forms of equation (9) were subsequently developed by Brown and Miller [3, 13, 27]. The equation which describes Case A cracking is of the form :

$$\left(\frac{\gamma_{max}}{2g}\right)^j + \left(\frac{\epsilon_n}{h}\right)^j = 1 \tag{10}$$

where the values of the constants g, h and j are life dependent, although useful approximations for j are 2 for ductile materials and 1 for brittle materials. These approximations also correspond to the ductility effect observed in high-cycle fatigue by Gough, since the ellipse quadrant and ellipse arc, equations (1) and (2), may be transformed into strain to give the quadratic and linear forms of equation (10) respectively [27]. For Case B cracking, a Tresca maximum shear stress type criterion was proposed [28] :

$$\frac{\gamma_{max}}{2} = \text{constant}. \tag{11}$$

Compared with the classifical theories these equations give better correlations with fatigue life and crack growth rates under multiaxial conditions, and Brown [13] advocates that they should be used in design procedures. However, these new criteria are more complex and require additional test information, and therefore may be more difficult to implement. Current design procedures are often based on the octahedral shear stress or strain theory in spite of the lack of agreement with experimental results [29, 30].

6. THE EFFECTS OF BIAXIAL STRESS ON FRACTURE PROPERTIES OF CRACKED COMPONENTS

6.1 Laboratory Specimens

Fracture mechanics concepts can be used for predicting the fatigue life of cracked structural components under complex loading conditions. Consequently, several theoretical and experimental investigations into the effects of biaxial stresses on fracture properties have been undertaken [31-44]. However, some of these investigations have produced conflicting results for the following reasons :

(1) An implication of linear elastic fracture mechanics (LEFM) is that fracture properties are not affected by stresses acting parallel to a crack. For example, Liebowitz and co-workers [31-33] have performed linear elastic finite element analyses (FEA's) on finite centre-cracked specimens subjected to symmetrical biaxial stresses under Mode I loading. Their calculations showed that the stress intensity factor K_I, the strain energy release rate G and the J-integral are not affected by a stress applied parallel to a crack. Similarly, Miller and Kfouri [34] and Liu and co-workers [35, 36] analysed a centre-cracked specimen in the elastic range. They found that the stress intensity factor and the crack opening displacement (COD) are independent of the biaxial stress ratio λ.

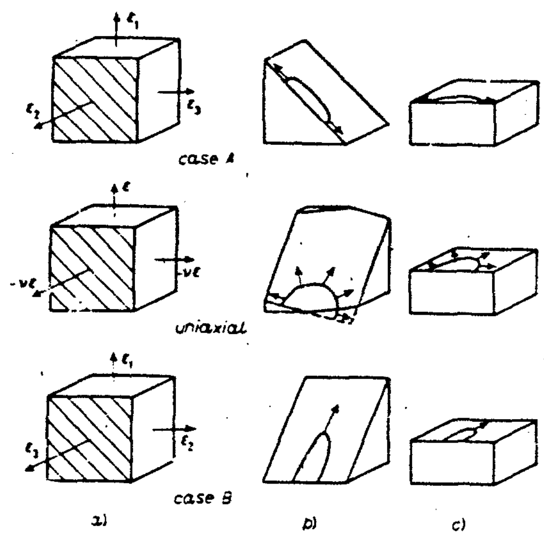


FIG.3. FATIGUE CRACK GROWTH SYSTEMS OBSERVED IN MULTIAXIAL STRAIN FIELDS SHOWING THE MAJOR CRACK GROWTH DIRECTION AND CHARACTERISTIC CRACK SHAPE [13].

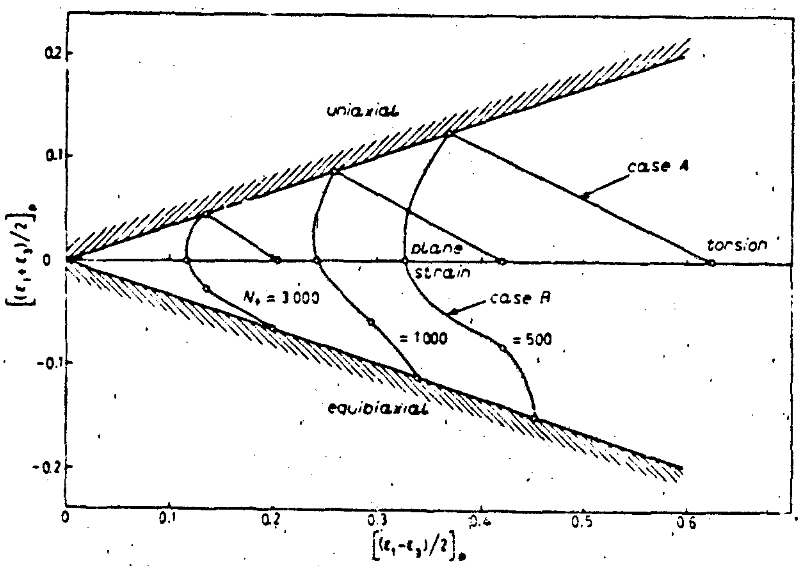


FIG. 4. A TYPICAL Γ PLOT [10]

(2) The effects of biaxial stress on fatigue and fracture properties are very dependent on specimen geometry, and the location and orientation of the crack with respect to the loadings axes.
(3) Crack tip plasticity, which is stress state dependent, can drastically modify the local stresses and displacements and thereby influence the critical load at fracture. This effect cannot be predicted using LEFM.

Even though some results are inconsistent, the limited experimental data show that biaxial loads affect fracture properties, such as the critical load at fracture, fracture toughness K, J-integral, strain energy release rate G, the direction of crack extension and the rate of crack growth.

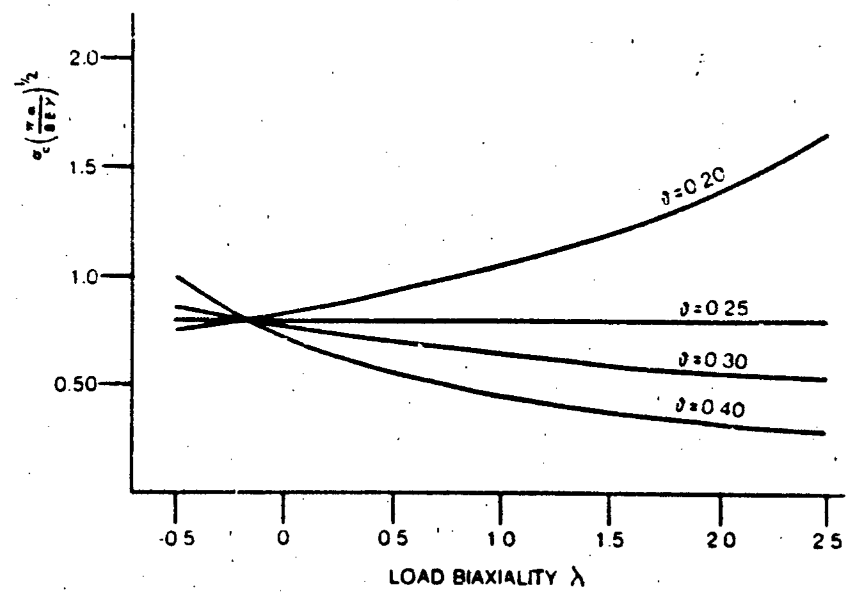
Various finite element analyses have been undertaken to determine the dependence of fracture properties on the biaxial stress ratio. These analyses can be divided into two general groups (1) linear elastic FEA's and (2) non-linear elastic-plastic FEA's. Liebowitz, Lee and Eftis [31] have shown using a linear elastic FEA, that the elastic stresses and displacements in the crack tip region cannot be adequately represented by the so-called "singular solution". In fracture mechanics analyses these local stresses and displacements are approximated by the first term of a series expansion of the functions representing them. Eftis, Subramonian and Liebowitz [37, 38] showed that this approximation, which appears to be reasonable on face value, is unacceptable because the effects of loads applied parallel to the crack appear entirely in the second term of the series. Inclusion of the non-singular stress terms enables proper evaluation of the maximum shear stresses, crack tip strain energy density and crack extension direction. All of these parameters show a biaxial load dependence even in the elastic range [31, 32, 39].

Liebowitz and co-workers [31-33] also considered a non-linear elastic-plastic analysis of a finite-width centre-cracked specimen subjected to uniform Mode I biaxial loading. The found that the strain energy release rate G, the J-integral, the stress intensity factor K, and the strain intensity factor (after Hilton and Hutchinson [40]) are all dependent on the biaxial stress ratio λ. The major part of this biaxial load dependency comes from the inelastic material behaviour at the crack tip.

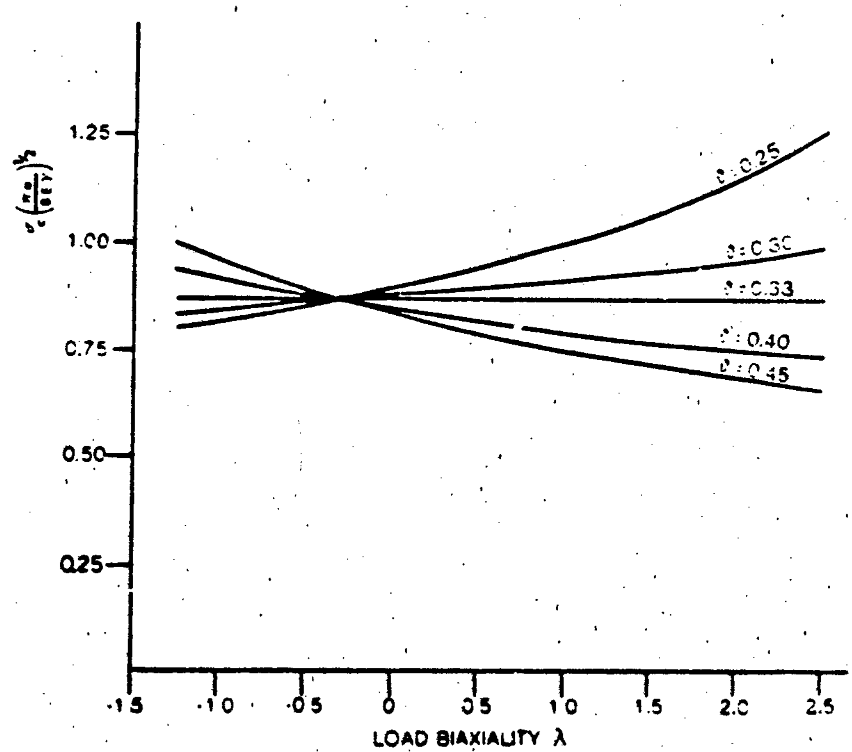
Miller and Kfouri [34] and Hilton [41] used elastic-plastic analyses to study biaxial load effects for normal stresses beyond the small scale yielding range. Results of the analysis by Miller and Kfouri showed that changing the biaxial stress ratio λ produced different crack tip stress fields and crack face displacements. The equibiaxial loading mode produced the greatest separation stress and the smallest crack face displacement. In contrast, the shear loading mode produced the maximum crack face displacement and the smallest crack tip separation stress. Hilton showed that the addition of a transverse tensile load increased the fracture initiation stress and decreased the plastic strain intensity factor. Compression loading parallel to the crack had the opposite effect. Furthermore, Hilton concluded that the effects of biaxial loads were increased at higher plastic strains. Similarly, Smith and Pascoe [42] concluded that the increased ductility associated with lower strength alloys tends to emphasise the effects of biaxial loads on fracture properties compared with medium and high strength alloys.

Recently, Jones and Eftis [39] undertook a general fracture mechanics analysis of a finite centre-cracked plate. Their calculations showed that the fracture load varies with the Poisson's ratio (ν) and the biaxial stress ratio (λ) under both plane stress and plane strain conditions; Figure 5(a) and (b). The fracture load (and hence K_c value) increase with increasing λ for all values of Poisson's ratio less than $\frac{1}{3}$ and decrease with increasing λ values when ν is greater than $\frac{1}{3}$. The effect of λ on fracture load, while present, is rather small for Poisson's ratios between 0·3 and 0·4. Most structural metals have Poisson's ratios within this range.

Jones and Eftis supported their analytical predictions by performing biaxial tests on materials used extensively in aircraft structures. Tests on aluminium alloy 7075 T6 (ν = 0·3) specimens showed that the critical plane stress fracture load increased 15 to 20% as the λ values increased from 0 to 1·8, Figures 6(a) and (b). Since the crack length and specimen dimensions were identical in all tests the critical stress and K_c values would increase proportionately. The difference indicated in Figures 6(a) and (b) is attributed to the anisotropy of the microstructure introduced during the rolling process. The opposite trend was observed for tests on Plexiglass (PMMA), with a Poisson's ratio between 0·40 and 0·45, where the fracture load decreased with increasing value of λ, Figure 7(a) and (b).

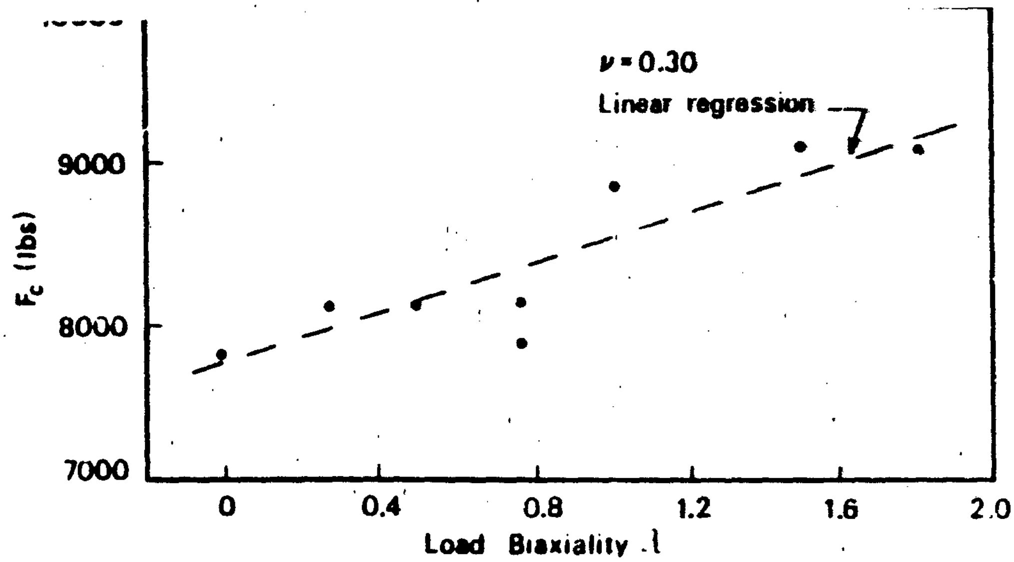


a) PLANE STRESS



b) PLANE STRAIN

FIG. 5. CRITICAL STRESS VARIATION WITH BIAXIAL STRESS RATIO λ AND POISSON'S RATIO ν [39]



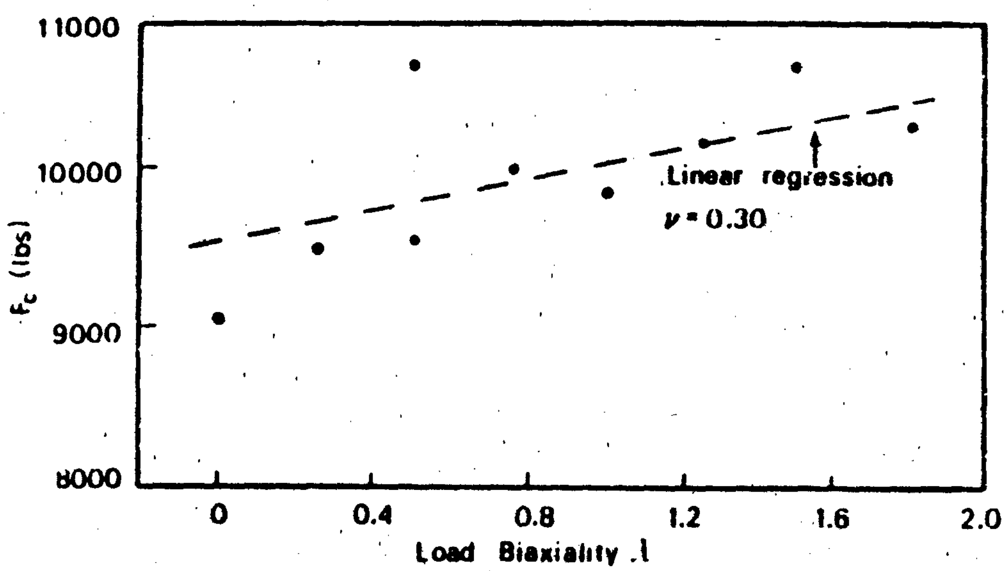
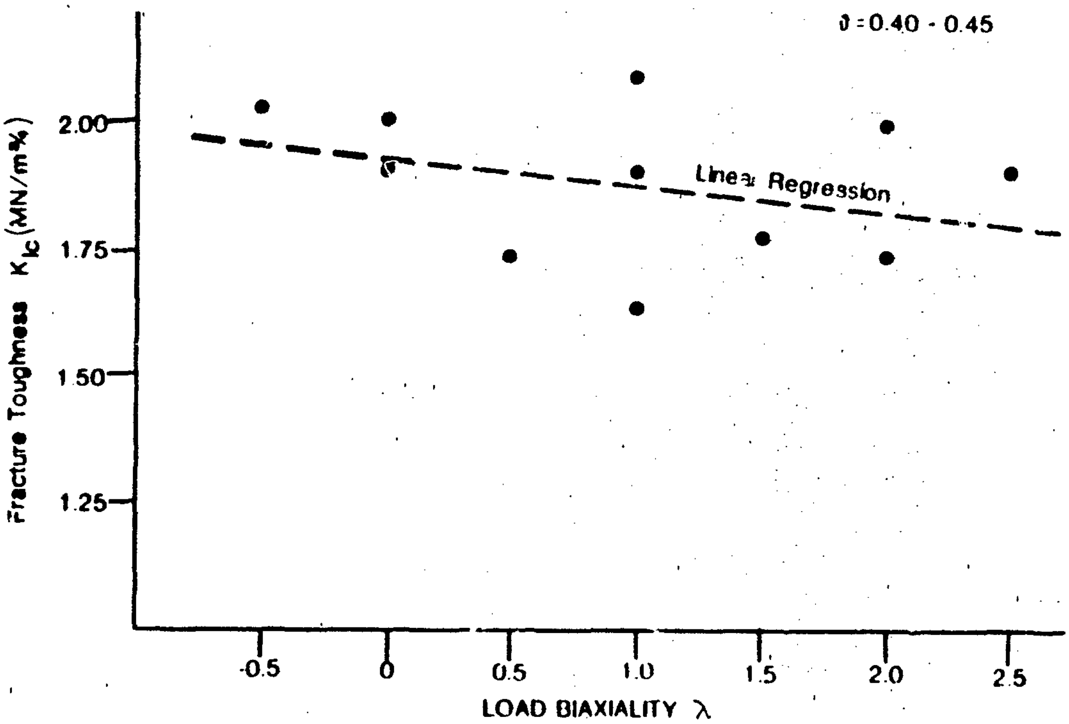


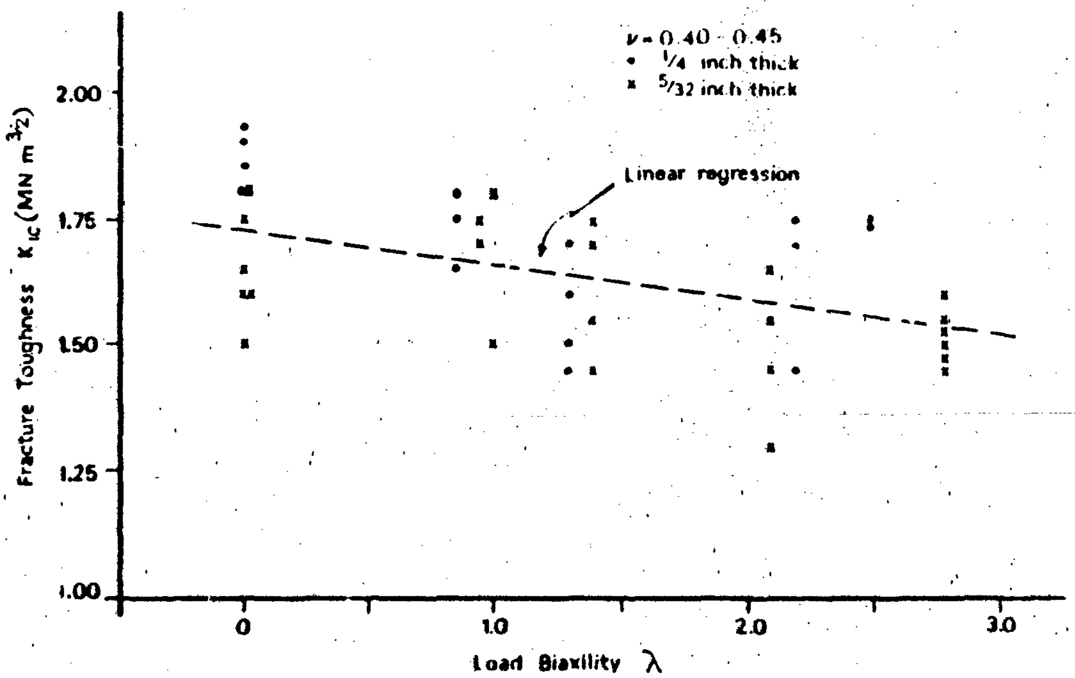
FIG. 6. FRACTURE LOAD VARIATIONS WITH BIAXIAL STRESS RATIO λ FOR ALLOY 7075-T6 SHEETS [39]

a) CRACK PARALLEL TO THE ROLLING DIRECTION

b) CRACK PERPENDICULAR TO THE ROLLING DIRECTION



a) UNIVERSITY OF WASHINGTON TESTS



b) IMPERIAL COLLEGE OF TECHNOLOGY TESTS

FIG. 7. FRACTURE TOUGHNESS VARIATIONS WITH BIAXIAL STRESS RATIO λ FOR POLYMETHYLMETHACRYLATE (PMMA) SHEET [39]

Experimental studies by other workers have indicated a variable influence of biaxiality on fracture strength. Kibler and Roberts [43] found that a transverse tensile stress increased the fracture toughness of Plexiglass and the aluminium alloy 6061 by up to 25%, compared with uniaxial tension. These tests were carried out under plane stress conditions. Kibler and Roberts expected little or no effect for tests carried out under plane strain conditions. In contrast Leevers, Radon and Culver [5] reported a slight decrease in the critical fracture load of PMMA, as the biaxial load ratio increases. Tests on PMMA by Ueda and co-workers [44, 45] produced a similar trend when the transverse load was parallel to the crack. This effect diminished as the angle between a slanted crack and the transverse load increased.

The results of Jones and Eftis, Leevers, Radon and Culver, and Ueda and co-workers for PMMA are in agreement, whereas the results of Kibler and Roberts do not follow the same trends. This inconsistency can be explained as follows.

(1) The specimen and loading arrangements used by Kibler and Roberts leave some doubt as to whether the stress intensity factor remains independent of the transverse stress [46]. The proper design of a specimen should not permit the interaction between the stresses in the two orthogonal directions [32].

(2) For an uncracked specimen of the type used by Kibler and Roberts the stresses along the transverse x-axis were not constant [43].

(3) The fracture tests made by Kibler and Roberts at different biaxial load ratios were not conducted at the same loading rate. In addition, the rate of loading was much slower compared with, for example, that used by Leevers, Radon and Culver. Therefore, differences in results would arise from time rate effects associated with materials such as PMMA [32].

6.2 Structural Components

Information regarding the effect of biaxial load ratio on the fracture and fatigue properties of structural components is very scarce. Swift [47, 48] has studied the effects of biaxial loads on the toughness and crack growth rates of stiffened panels, such as commonly used in aircraft and ship structures. The geometry of these panels is shown in Figure 8. Tests on panels with all stiffeners intact showed that biaxial loading increased the critical stress intensity factor K_c, Table 4. In addition, using a semianalytical method, Swift showed that :

(1) The calculated stress intensity factor increased with increasing biaxial load ratio for panels with a two bay crack and the central stiffener intact, Figure 8(*a*) (this is in agreement with his experimental results) ; and
(2) The opposite effect is observed when the central stiffener is broken, Figure 8(*b*). The effect of biaxial loads was less pronounced for these panels than for those with the central stiffener intact.

Maynor and co-workers [49] tested steel cylindrical pressure vessels with surface cracks and compared the critical stress intensity factors K_{Ic} with those obtained using uniaxial tensile specimens. Their results showed that the average K_{Ic} values for the biaxially stressed pressure vessels and the uniaxial specimens were similar. This result was unexpected as the average stress at which the pressure vessels yielded was less than that of the tensile specimens. Consequently, they compared their K_{Ic} values for the pressure vessels with those obtained by Blat [50] for tensile specimens of similar strength. From this comparison it was concluded that the K_{Ic} value in biaxial tension was two-thirds the K_{Ic} value in uniaxial tension. Unfortunately, this comparison is questionable as pressure and geometrical effects were not considered fully.

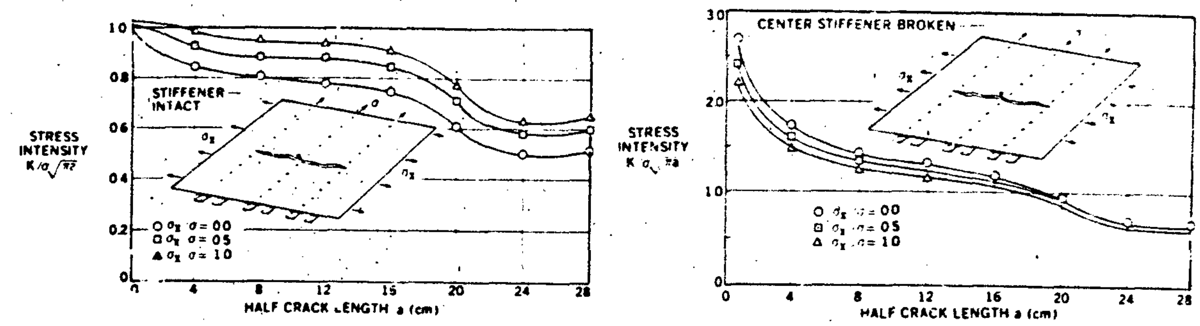


FIG. 8. EFFECTS OF BIAXIAL STRESS RATIO ON STRESS INTENSITY IN STIFFENED PANELS [47]

a) STIFFENERS INTACT

b) STIFFENERS BROKEN

TABLE 4

Test Results for Longitudinal Crack Tests on Curved Panels of 7075–T73 Aluminium Alloy with all Stiffeners Intact [48]

Test Number	Panel Con-figuration	Pressure at Fast Fracture (psi)	Hoop Stress Normal to Crack (psi)	Axial Stress Parallel to Crack (psi)	Crack Length at Fast Fracture (in)	Critical Stress Intensity Factor (psi√in.)
1	one-bay crack	12·80	14,912	0	7·24	66,460
2		13·36	15,622	2,500	8·05	72,000
3	two-bay crack with centre frame intact	10·80	12,600	0	12·10	65,114
4		11·55	15,380	28,495	12·83	80,573

For example, tests on curved panels indicate a loss of strength of up to 30% because of bulging from internal pressure [48]. Maynor and co-workers modified their equations for calculating K_{Ic} for the pressure vessels so as to allow for this effect. However, Swift [48] concluded that the bulging effect in pressure vessels cannot be simulated exactly by flat panel testing. Therefore, it is difficult to determine whether the difference between the biaxial K_{Ic} values of Maynor and co-workers and the uniaxial K_{Ic} values of Blat is the result of stress state, or pressure and geometrical effects.

Erdogan and Ratawani [51] examined the fatigue and fracture properties of cylindrical and spherical shells containing a through crack and subjected to internal pressure or torsion. In addition, rectangular plates and cruciform specimens were tested. Results of rupture tests for the various specimens, made from aluminium alloy 6061-T6, are shown in Figure 9(*a*) and (*b*). These Figures show that the nominal critical stress intensity factor K_{cn} (where K was obtained using the rupture load and the initial crack length) increases with increasing stress component parallel to the crack. In addition, Erdogan and Ratawani found that curvature may have a considerable effect on the stress intensity factors in shells with cracks. Their theoretical analysis showed that curvature increases the local stresses at the crack tip and thereby reduces both the critical strain intensity factor and rupture strength of the component. Therefore, the difference in critical stress intensity values between the pressure vessels of Maynor and co-workers and the flat uniaxial specimens of Blat appears to be a result of geometrical and not stress state effects.

7. MULTIAXIAL LOW-CYCLE FATIGUE

Many high performance components, such as gas turbine blades or nuclear pressure vessels can occasionally experience large mechanical or thermal transients during operation, i.e. during start-up and shut-down procedures. These transients can cause significant damage accumulation after only a few hundred or a few thousand of these cycles. Even under normal operating conditions the material in the vicinity of structural discontinuities, such as notches or welds, can experience local cyclic plasticity. These discontinuities, where fatigue cracks usually nucleate, are invariably at the free surface which is in a state of biaxial stress. Therefore, the prediction of low-cycle high-strain fatigue failure under biaxial stress conditions is an important design consideration for many structural components.

Compared with high-cycle fatigue, correlation between uniaxial and biaxial low-cycle fatigue is complicated by the effects of plasticity which is non-linear and history path dependent. Consequently, criteria are necessary for both the cyclic stress-strain behaviour and fatigue life data in low-cycle fatigue.

7.1 Cyclic Stress–Strain Data

Limited experimental data are available on the cyclic stress-strain behaviour of materials under biaxial stress conditions [9, 52–57]. In addition, there are only a few published papers where a single test specimen geometry has been used for the full range of stress states [9, 55–57].

Parsons and Pascoe [9] performed strain controlled tests on cruciform specimens of two alloy steels under states of biaxial stress. These tests showed that the resistance to cyclic deformation is a function of the material, the applied strain and the strain ratio ($\phi = \Delta\epsilon_2/\Delta\epsilon_1$). The annealed austenitic steel AISI 304 cyclically strain hardened at all strain amplitudes, whereas the quenched and tempered steel QT35 cyclically strain softened. Maximum hardening for AISI 304 and maximum softening for QT35 occurred under equibiaxial conditions ($\phi = +1$) and the minimum occurred under shear conditions ($\phi = -1$). In addition, the load range (ΔP) applied to the specimen arms to maintain a given total strain range was dependent on the strain ratio. For example, the values of ΔP required to maintain any given strain range (at one fifth of the fatigue life) increased by up to a factor of 3 as ϕ changed from -1 to $+1$.

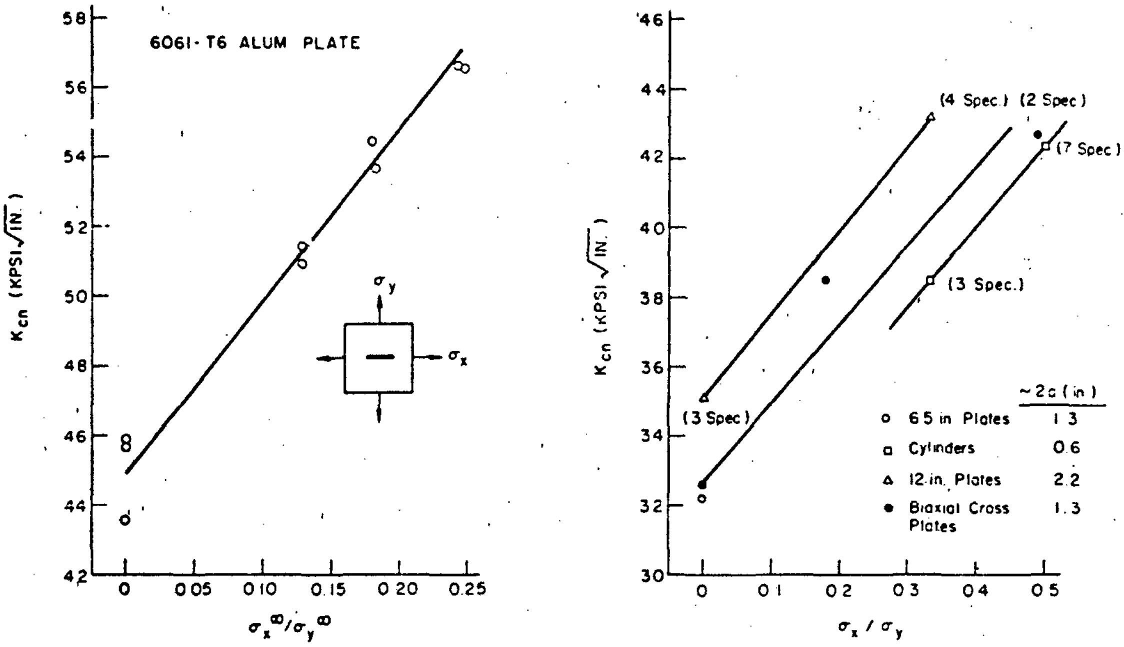


FIG.9. EFFECT OF BIAXIAL STRESS RATIO ON THE RUPTURE STRENGTH K_{cn}
of a/ 0.6mm THICK 6061-T6 ALUMINIUM PLATES
and b/ 6061-T4 ALUMINIUM PLATE AND SHELLS [51]

Havard and Topper [55, 56] tested thin-walled tubes of normalised AISI 1018 mild steel under in-phase axial loading and internal/external pressure. The data from fully reversed load tests for several different stress states were used to establish the cyclic behaviour of the mild steel. Each cyclic stress-strain curve could be represented by the sum of a power law term for the plastic strain and a linear (generalised Hooke's law) term for the elastic strain. The equation which provided the best fit for each set of data was of the form :

$$\Delta\epsilon_1 = \frac{\Delta\sigma_1(1 - \nu\eta)}{E} + \frac{k'.(\Delta\sigma_1)^{1/n'}}{50} \qquad (12)$$

The values of the cyclic strength co-efficient k' and the cyclic hardening exponent n' are both dependent on the stress state. For example, Table 5 shows that the range of values of k' and n' obtained for the various biaxial stress ratios were 0·083 to 0·589 and 0·271 to 0·572 respectively. Table 5 further shows that increasing the biaxial stress ratio tends to increase the cyclic hardening exponent n'. Therefore, the resistance to cyclic deformation also increases as the biaxial stress ratio increases.

TABLE 5

Values of the Cyclic Strength Coefficient k' and Hardening Exponent n' from Equation (12) for Various Biaxial Stress States [55]

Biaxial Stress Ratio (circumferential to axial stress ratio) η	Direction of Control	Parameters from Cyclic Stress-Strain Curve	
		k'	n'
Uniaxial (0) (rolling direction)	Axial	0·166	0·314
Uniaxial (0) (transverse direction)	Axial	0·155	0·294
Torsional (∞)	Shear	0·589	0·271
Biaxial (−2·9)	Circumferential	0·214	0·307
Biaxial (0·50)	Circumferential	0·108	0·310
Biaxial (0·86)	Circumferential	0·083	0·368
Biaxial (−0·28)	Axial	0·219	0·307
Biaxial (0·26)	Axial	0·174	0·572

In general two criteria have been used to correlate multiaxial cyclic properties : (1) a Tresca maximum shear stress-shear strain correlation, and (2) a Von Mises octahedral shear stress-shear strain correlation. Brown and Miller [57] determined the biaxial cyclic stress-strain curves for a 1% Cr–Mo–V steel and an AISI 316 stainless steel at various temperatures and strain-rates. In addition, these authors undertook a statistical evaluation of the Tresca and Von Mises correlations using most of the experimental data available. They found that cyclic stress-strain curves under multiaxial stresses are best represented by the Tresca maximum shear stress-shear strain criterion, as shown for example in Figure 10.

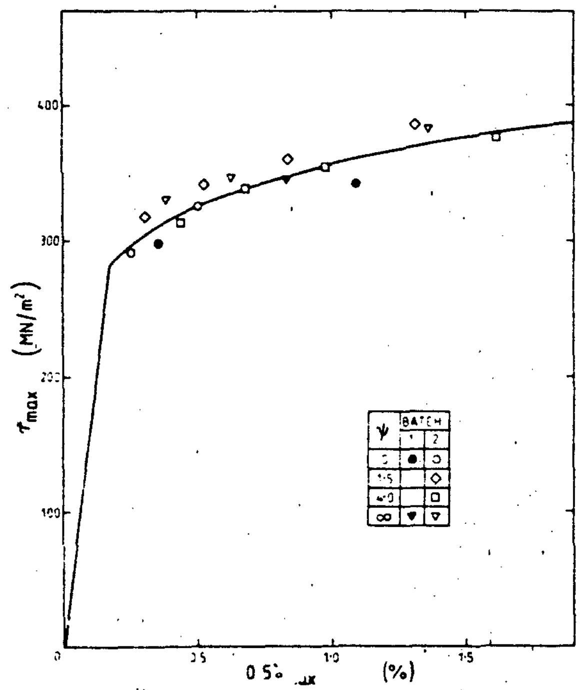


FIG. 10. CYCLIC STRESS-STRAIN CURVE FOR 1% Cr-Mo-V STEEL AT 20°C. ψ=RATIO OF TORSIONAL TO AXIAL STRAIN [57]

7.2 Fatigue Life Relationships

Well known relationships exist for relating low-cycle fatigue life in the uniaxial stress state to stable values of stress or strain amplitudes. Manson [58] and Coffin [59] used a simple power law to relate fatigue life to the plastic strain range, whilst Basquin [60] used a similar law for the stress range. These equations are :

$$\frac{\Delta\epsilon_p}{2} \cdot N_f^{\alpha} = X \tag{13}$$

$$\frac{\Delta\sigma}{2} \cdot N_f^{\beta} = Y \tag{14}$$

In addition, the stress amplitude in the elastic range can be expressed as,

$$\frac{\Delta\sigma}{2} = \frac{\Delta\epsilon_e}{2} \cdot E \tag{15}$$

Therefore, the total strain range may be represented by the sum of two power law functions of fatigue life, namely :

$$\frac{\Delta\epsilon_T}{2} = \frac{\Delta\epsilon_p}{2} + \frac{\Delta\epsilon_e}{2} = A\,N_f^{-\alpha} + B\,N_f^{-\beta}. \tag{16}$$

Fatigue life data under biaxial stress conditions also can be represented in terms of these equations [55, 56, 61–63]. For example, Ellison and Andrews [61] presented high-strain biaxial fatigue data for aluminium alloy RR58 at several biaxial strain ratios and for fatigue lives between 100 and 3000 cycles. These authors used thin-walled tubes subjected to axial loading combined with internal/external pressure under strain control. Plots of the maximum principal plastic strain range ($\Delta\epsilon_{1p}$) versus fatigue life (N_f) showed that the Manson–Coffin exponent increased from 0·753 to 1·436 as the biaxial strain ratio ϕ increased from −1 to +1, see Figure 11 and Table 6. In addition, increasing the strain ratio from −1 to +1 decreased the fatigue life by a factor of 2 at high plastic strain amplitudes ($\Delta\epsilon_{1p} \simeq 0{\cdot}5\%$) and by a factor of 5 at lower amplitudes ($\Delta\epsilon_{1p} \simeq 0{\cdot}05\%$).

TABLE 6

Effect of Biaxial Strain Ratio ϕ on the Manson–Coffin Exponent and Intercept Constant for Aluminium Alloy RR58 [61]

Strain Ratio ϕ / Parameter	−1	−½	0	+½	+1
α	0·753	0·761	0·938	1·218	1·436
A	0·2[illegible]	0·235	0·486	1·91	5·45

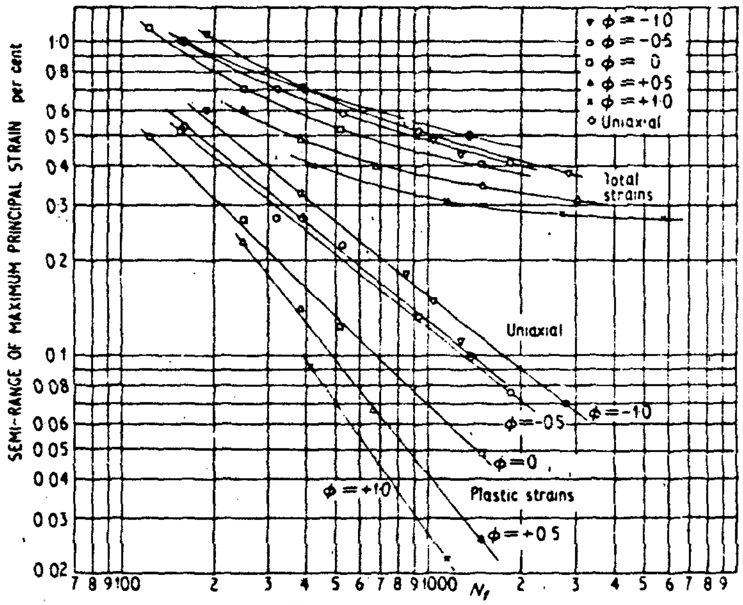


FIG. 11. BIAXIAL STRAIN-LIFE DATA FOR ALUMINIUM ALLOY RR58 AT DIFFERENT STRAIN RATIOS ϕ AND AT ROOM TEMPERATURE [61]

Similar results were obtained for a ferritic 1% Cr–Mo–V steel by Lohr and Ellison [1, 2] using a modified version of the equipment of Ellison and Andrews. In addition, a few tests at different strain ratios were conducted using the aluminium alloy RR58 of Ellison and Andrews. Agreement between the total strain versus fatigue life curves was good. The tests on the 1% Cr–Mo–V steel showed that the constant A in the Manson-Coffin equation, and both the constant B and the exponent β in the Basquin equation all decrease with increasing biaxial strain ratio ϕ, Table 7. The exponent α in the Manson-Coffin equation was constant. Lohr and Ellison also observed a decrease in the fatigue lives by a factor of 2 to 3 for both the 1% Cr–Mo–V steel and the aluminium alloy RR58 as the biaxial strain ratio changed from −1 to +1. Again this is in agreement with the trends observed by other workers [62, 63].

TABLE 7

Effect of Biaxial Strain Ratio ϕ on the Manson-Coffin and Basquin Exponents and Intercept Constants for a 1% Cr-Mo-V Steel [2]

Strain Ratio ϕ / Parameter	−1	Uniaxial	0	+1
Manson-Coffin				
A	1·28	0·84	0·66	0·58
α	0·9	0·9	0·9	0·9
Basquin				
B	0·0092	0·0074	0·0072	0·0037
β	0·16	0·12	0·13	0·009

The fatigue life behavoiur of a ferritic steel QT35 and an austenitic steel AISI 304, using flat cruciform specimens, is slightly different from the above results. Parsons and Pascoe [9] found that the intercept and exponent constants in the above equations were different in the high strain (10^2 cycles) and low strain (10^5 cycles) regimes. The change in slope of these curves occurred in the vicinity of 10^4 cycles. (Most of the previous authors had restricted their tests to less than approximately 10^4 cycles). On subsequent examination of the literature Parsons and Pascoe concluded that this behaviour was typical of hardened alloys of steel and aluminium, and of certain austenitic steels. In general, increasing the strain ratio from −1 to +1 decreased the intercept and exponent constants in the total strain versus fatigue life equation. Consequently, increasing the strain ratio from −1 to +1 decreased the fatigue lives in the high and low-strain regimes by factors of 10 and 20 respectively.

8. THE EFFECTS OF BIAXIAL STRESS ON FATIGUE CRACK GROWTH RATES

8.1 Laboratory Specimens

The most popular and widely used analytical tool for predicting the safe life of aircraft and many other flawed structural components, based on laboratory data, is linear elastic fracture mechanics. Fatigue crack growth rates da/dN can be correlated with the change in the stress intensity factor ΔK. These data are usually obtained under uniaxial loading conditions.

A fracture mechanics approach for the analysis of fatigue crack growth in biaxial stress fields has not yet been successfully developed. Linear elastic fracture mechanics predicts that components of stress parallel to a crack should have no effect on the fatigue crack growth rate [64]. In contrast, experimental results have shown that the growth rate of fatigue cracks can be enhanced [65, 66], retarded [15, 39, 43, 46, 67–72] or remain unchanged [4, 35, 36, 73–77] by the application of a static or cyclic tensile stress parallel to the crack. Similarly, applying a static or cyclic compressive stress in the transverse direction has been reported to increase [15, 68, 69, 72], decrease [4, 74, 75, 78, 79] or have no effect [35, 36, 76, 77] on fatigue crack growth rates. Furthermore, the effects of biaxial load are enhanced by (1) cyclic transverse stresses of constant amplitude compared with static transverse stresses, [15, 71] and, (2) cyclic transverse stresses of variable amplitude compared with those of constant amplitude [35, 80].

One factor which is responsible for the inconsistency in the literature is the method of evaluating the stresses applied to cruciform type specimens. In most cases the practice has been to define the biaxial stress ratio λ in terms of the NOMINAL stresses applied parallel (S_x) and perpendicular (S_y) to the crack but remote from it [72, 81]. Usually the biaxial stress ratio $\lambda = S_x/S_y$ reduces to the ratio of the loads P_x/P_y in the loading arms of the cruciform specimens. Results from these tests generally indicate that biaxial stresses have a signifiant effect on fatigue crack growth rates, as shown for example in Figure 12 [68]. In contrast, when tests are carried out using the LOCAL stresses, e.g. at the centre of an uncracked plate, the effects of the biaxial stress ratio on crack growth rates are small [4, 35, 36, 74–77]. (The stress distribution across the test section of cruciform specimens can be evaluated using either strain gauges or finite element analysis. Both methods are in excellent agreement [35]). This difference between NOMINAL and LOCAL stress state effects can be rationalised as follows: The NOMINAL load P_y required to maintain a constant LOCAL stress σ_y significantly increases as the LOCAL biaxial stress ratio k changes from -1 to $+1$ [81]. Therefore, the increase in crack growth rate associated with the increase in P_y between, say, uniaxial tension ($k = 0$) and equibiaxial tension ($k = +1$) nullifies the decrease in growth rate from the change in stress state. A similar counter-balancing effect occurs as the LOCAL stress state changes from uniaxial tension ($k = 0$) to pure shear ($k = -1$).

Another factor may also be responsible for the above difference, that is, in any give test the NOMINAL and LOCAL biaxial stress ratios may not necessarily be the same. For example, various workers have shown that the local biaxial stress ratio $k = \sigma_x/\sigma_y$ can vary from $-0{\cdot}26$ to $-0{\cdot}35$ for cruciform specimens loaded in uniaxial tension, i.e. when $\lambda = S_x/S_y = 0$ [74–76]. Therefore, the local stress state at the crack tip can be biaxial with a compressive static transverse stress although the nominal stress state may be uniaxial tension. Similarly, Kitagawa, Yuuki and Tohgo [71] found that a static tensile load parallel to a crack produced a local transverse compressive stress. They also found that the local stress intensity ratio, defined as $R_K = K_{min}/K_{max}$, in the y direction, varies with the biaxial load condition and crack length even if the stress ratio in the y loading arm, $R_y = S_{y\,min}/S_{y\,max}$, is kept constant.

Experimental results have shown that in uniaxial tension the application of a mean stress can increase the fatigue crack growth rates and hence shorten fatigue life [82]. Changing the mean stress value in the x direction also influences fatigue crack growth rates in biaxial tension [71, 83, 84]. For example, Hoshide, Tanaka and Yamada [84] found that when the local stress ratio R_L was -1, the fatigue crack growth rate increased as the local biaxial stress ratio (k) decreased from 1 to -1. One the other hand, when $R = 0$ the growth rate was the lowest for a k value of -1. Therefore, different crack growth rates may be produced depending on whether a nominal or local stress parameter is used to control the test.

The growth of fatigue cracks in multiaxial stress states is affected by the orientation of the crack with respect to both the surface of the component and the applied stress field. In uniaxial tension, fatigue cracks usually nucleate at the surface and initially grow along planes close to the planes of maximum shear stress, denoted as Stage I growth by Forsyth [85]. After the crack has propagated through one or more grains in depth the crack changes direction and grows along planes perpendicular to the maximum principal stress direction, denoted as Stage II crack growth [85]. The stages of crack growth in multiaxial stress fields are the same as those just described for uniaxial tension. Parsons and Pascoe [86] also observed that LCF crack growth in biaxial strain states occurred on or close to the planes associated with Stage I and Stage II cracking. However, Stage I crack growth was dominant under shear ($\phi = -1$) and plane surface

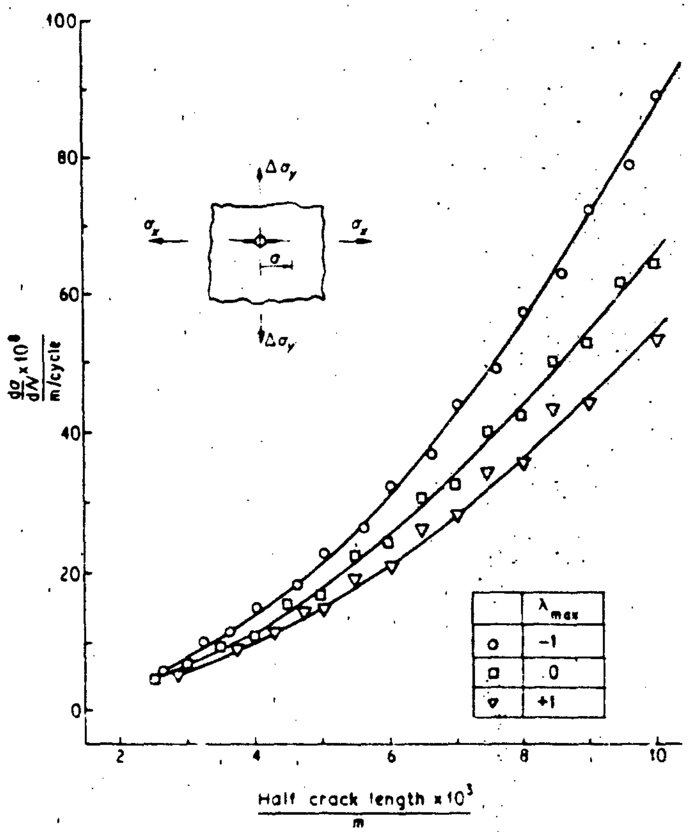


FIG. 12. EFFECTS OF BIAXIAL STRAIN RATIO ON FATIGUE CRACK PROPAGATION RATES FOR UNNOTCHED CRUCIFORM SPECIMENS OF ALUMINIUM ALLOY RR58; $\Delta\sigma_y$ = 70 MPa AND σ_{ymax} = 105 MPa [68]

strain ($\phi = 0$) loading. Similarly, Brown and Miller [87] found a transition from Stage I to Stage II crack growth for biaxial strain ratios (ϕ) of approximately 1·5. At elevated temperatures this transition occurred for biaxial strain ratios (ϕ) of approximately 4. Brown and Miller concluded that the transition from Stage I to Stage II crack growth in ductile metals is dependent on the applied stress or strain state, and not on the crack length.

Brown and Miller [10] also observed that two types of cracks can develop for each stage of crack growth. Cracks can grow along the surface of a component, as occurs in tension-torsion type loading, or just under but parallel to the surface as occurs in rolling contact fatigue. Alternatively, cracks can grow away from the surface in the through-thickness direction, as occurs in the biaxial testing of pressurised tubes, plate bending (anticlastic) tests, or biaxial tension tests using cruciform specimens. Brown and Miller [10] have denoted surface cracks as Type A cracks and through-thickness cracks as Type B cracks, Figure 3. Type B cracking is the most dangerous case as the crack growth rate is much faster and the fatigue strength lower than compared with Type A cracks [10, 13, 69]. Consequently, the fatigue lives of components having type A and Type B cracks may be quite different even though the equivalent stresses and strains may be the same [10, 11, 69].

The orientation of the crack plane with loading direction also may be classified as either Mode I, II or III depending on the loading configuration, Figure 13. For example, a Stage I crack may grow under Mode I loading in uniaxial tension. However, a Stage I crack under torsional loading conditions can grow along the surface under Mode II or in a radial direction under Mode III loading. In service, most components are subjected to mixed loading conditions. For example, tubine shafts are simultaneously subjected to a constant torque, which provides the driving force, and an alternating bending stress, which arises from the weight of the rotor or any slight imbalances [78, 79]. Consequently, cracks growing perpendicular to the surface of the shaft may be subjected to a combination of Mode I plus either Mode II or Mode III loading.

Hourlier and Pineau [78, 79] investigated the fatigue crack growth rates in four materials under cyclic Mode I plus steady Mode II or Mode III loading conditions. They found that the simultaneous application of a cyclic Mode I plus a steady Mode III loading produced two main effects compared with Mode I loading, namely :

(1) A large decrease in the fatigue crack growth rates of up to two orders of magnitude.

(2) A significant change in the appearance of the fracture surface.

The major effects of applying a steady Mode II to a cyclic Mode I loading were to change the direction and reduce the rate of crack growth compared with Mode I crack growth.

The special test environments sometimes used for testing biaxial specimens can influence fatigue crack growth characteristics. For example, the fluid used in tests of thin-walled tubes subject to axial loading plus internal external pressure can interact with a growing crack [13, 16, 88]. In addition, the crack may be subjected to the "hydrowedge" effect where high pressure oil enters the crack and thereby provides an additional Mode I opening stress [13, 88].

In summarising this section, at present there are conflicting results in the literature concerning the effects of biaxial stresses on fatigue crack growth characteristics. Conflicting results have arisen between the various workers because of differences associated with the material used, geometrical shapes of specimens, test control conditions, and test environment. However, one method of overcoming these problems is to conduct a series of "changeover tests" as initially proposed by Leevers, Radon and Culver [70]. Changeover tests consist of fatigue loading a specimen using a fixed biaxial stress (strain) ratio until the crack growth behaviour has stabilised. The biaxial stress (strain) ratio is then suddenly changed to another value and the effects on the crack growth curve monitored. This procedure can be repeated several times during a test. Consequently, the effects of stress state on fatigue crack growth behaviour can be identified as the one specimen, geometry and environment are used for each stress state. In addition, the local stress and the stress intensity factor at each changeover are the same even though the actual value may not be known, and therefore changes in fatigue crack growth rates after a changeover are legitimate and not transient effects.

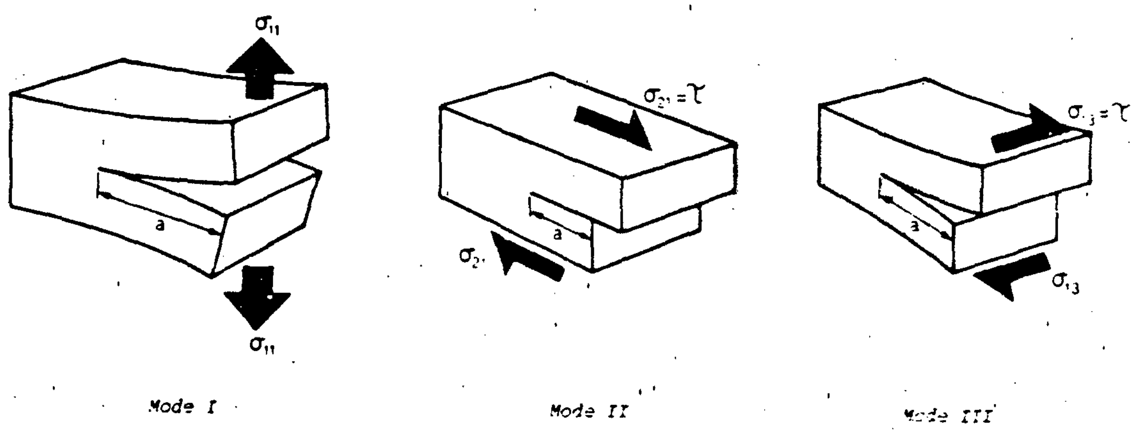


FIG. 13. BASIC MODES OF CRACK SURFACE DISPLACEMENT

Garrett and co-workers [15, 72, 81] performed such changeover using centre-cracked cruciform specimens of mild steel. The results of some of these tests can be summarised as follows :

(1) An instantaneous change in stress state from uniaxial to equibiaxial tension (σ_x cyclic) at the same nominal applied stress normal to the crack reduces the fatigue crack growth rate by a factor of 2, Figure 14(*a*). In comparison the fatigue crack growth rate is accelerated by a factor of 4 when the stress state changes from equibiaxial tension to uniaxial tension, Figure 14(*b*). Leevers, Radon and Culver observed the same trends for PMMA as shown in Figure 15. The changeover from uniaxial to biaxial tension ($P_x = 2\,P_y$ and is cyclic) decreased the fatigue crack growth rate by a factor of 2·5 whilst the opposite changeover increased the growth rate by a factor of approximately 4.

(2) Compared with uniaxial tension, the fatigue crack growth rate is increased by a factor of 3 when a cyclic compressive stress is applied parallel to a propagating crack, Figure 14(*c*).

(3) The effects of biaxial loading were the same for steel specimens in the annealed and cold worked states.

8.2 Full-Scale Stiffened Panels Containing Cracks

Many structural elements consist of flat plates or shells reinforced with stiffeners, such as the fuselage panels in aircraft structures. The design of such structures according to damage tolerance principles requires a knowledge of fatigue crack growth characteristics, especially under complex loading.

Research into the effects of biaxial stresses on the fatigue crack growth rates of stiffened panels is very limited. Swift [47, 48] has examined the effects of both biaxial loads and curvature on the fatigue crack growth rates in this type of structure. In this work the crack growth behaviour in a curved panel subjected to biaxial loads combined with internal pressure was compared with the behaviour of a flat panel in uniaxial tension with the tensile axis parallel to the stiffeners. The nominal axial stresses and the stress ratios (R) were the same in each case except that the curved panel was also subjected to a cyclic internal pressure. The combined effects of curvature and biaxial loading caused an increase in the fatigue crack growth rates of 50 to 100% depending on the crack length, as shown in Figure 16.

Ansee and Morrow [89] have investigated fatigue crack growth rates in large flat panels containing stiffeners under uniaxial and biaxial loading conditions. These panels were completely representative of the fuselage panels used in the Concorde aircraft in all respects except for curvature. (In contrast to Swift, these authors investigated fatigue crack growth rates in flat panels only. However, as noted by Swift, considerable care should be exercised when using the results obtained from tests on flat panels to predict the allowable stresses in curved panels). The test assembly for the stiffened panels is shown schematically in Figure 17 where the actual test panel is represented by ABCD. The cracks in each panel were symmetrical about a stiffener which was also completely cracked. Results from these tests showed that when compared with uniaxial tension, tensile biaxial stresses (σ_x cyclic) reduced the fatigue crack growth rate by a factor of 1·5 for a nominal biaxial stress ratio $\lambda = 0{\cdot}5$, and by a factor of 2·2 when $\lambda = 1{\cdot}0$, Figure 18. Ansee and Morrow concluded that tensile loads parallel to a crack can be safely neglected when determining the safe life of this type of structure. However, they did not examine the effects of compressive stresses parallel to a crack.

The experimental results of Ansee and Morrow for panels where both the skin and the central stiffener are broken are in agreement with the theoretical analysis of Swift [47]. However, Swift predicted the reverse trend if the stiffener was intact, that is biaxial loads would increase the fatigue crack growth rates compared with uniaxial loading. The finite element analysis of Ratwani and Wilhem [90, 91] also predicted this reverse effect. Therefore, these theoretical

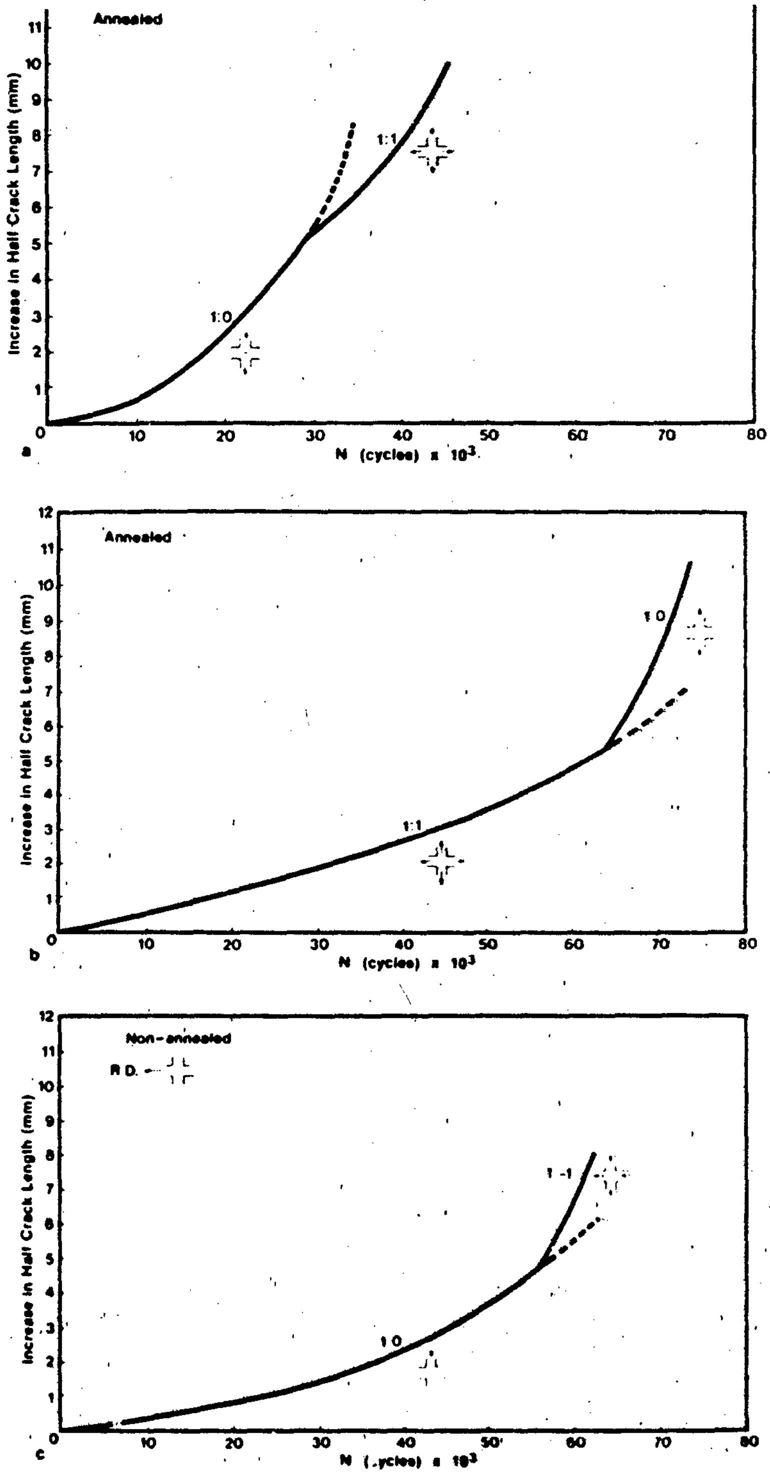
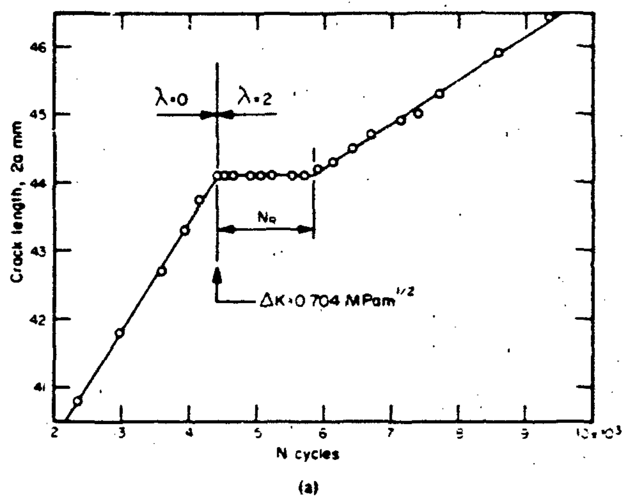


FIG. 14. BIAXIAL "CHANGEOVER" TESTS FOR MILD STEEL AT CONSTANT LOAD P_y [72]

a) UNIAXIAL TO EQUIBIAXIAL TENSION (σ_x = CYCLIC) DECREASES THE FATIGUE CRACK GROWTH RATE BY A FACTOR OF 2

b) EQUIBIAXIAL TENSION(σ_x = CYCLIC) TO UNIAXIAL INCREASES THE FATIGUE CRACK GROWTH RATE BY A FACTOR OF 4.

c) UNIAXIAL TO EQUIBIAXIAL TENSION/COMPRESSION (PURE SHEAR) INCREASES THE FATIGUE CRACK GROWTH RATE BY A FACTOR OF 3



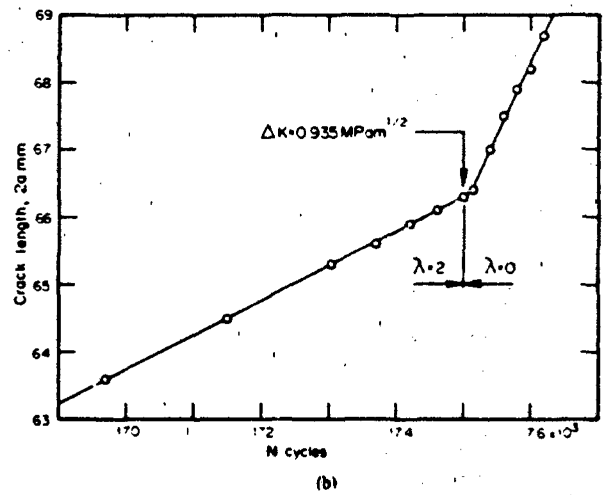


FIG. 15. BIAXIAL "CHANGEOVER" TESTS FOR PMMA AT CONSTANT LOAD RANGE ΔP_y [46]

a) UNIAXIAL TO BIAXIAL ($P_x = 2.P_y$) TENSION DECREASE THE FATIGUE CRACK GROWTH RATE BY A FACTOR OF 2.5

b) BIAXIAL ($P_x = 2.P_y$) TENSION TO UNIAXIAL TENSION INCREASES THE FATIGUE CRACK GROWTH RATE BY A FACTOR OF 4

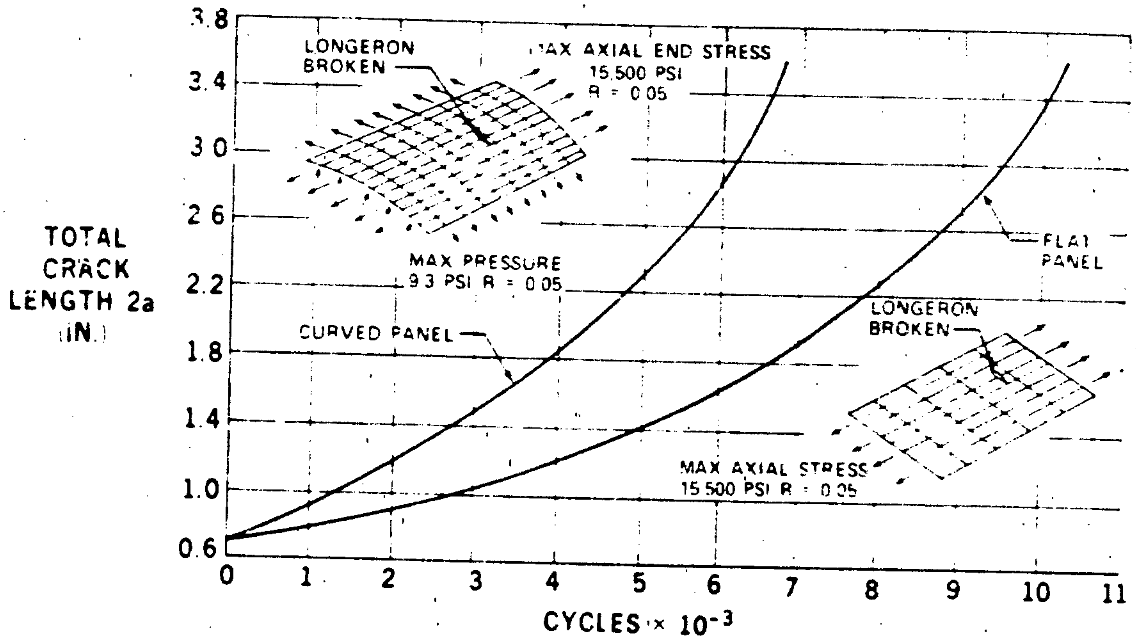


FIG. 16. COMPARISON OF FATIGUE CRACK GROWTH RATE CURVES OF FLAT AND CURVED PANELS OF 2024-T3 SKIN MATERIAL [48]

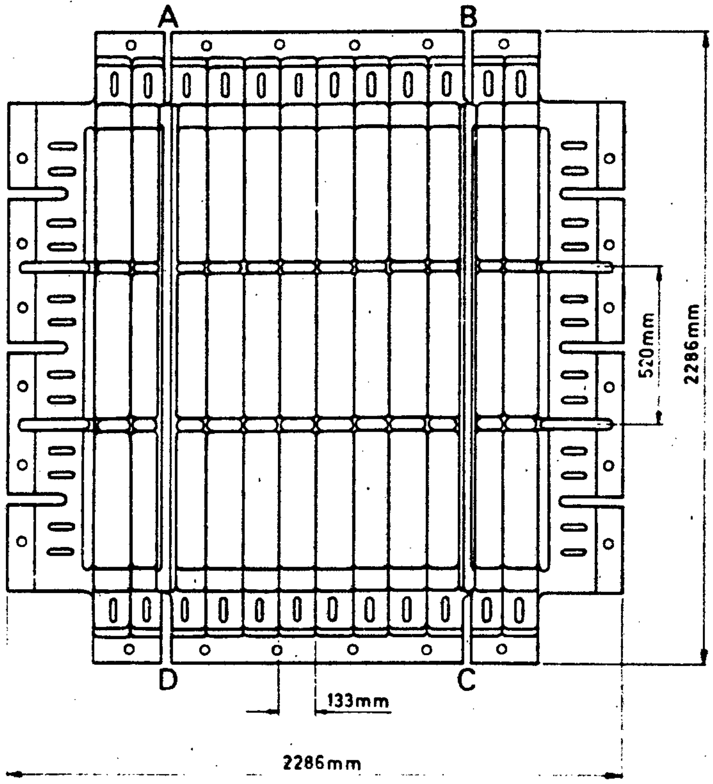


FIG.17. TEST ASSEMBLEY OF ANSEE AND MORROW [89]. ABCD REPRESENTS THE ACTUAL TEST PANEL.

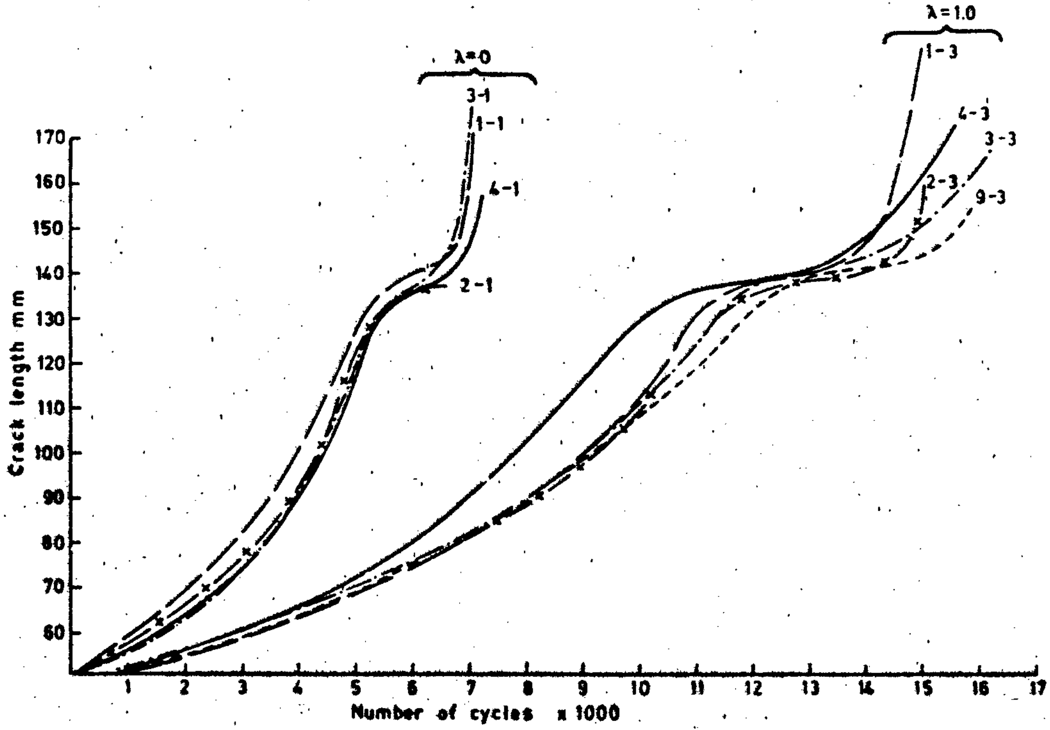


FIG.18. COMPARISON BETWEEN THE FATIGUE CRACK GROWTH CURVES IN UNIAXIAL TENSION (λ=0) AND EQUIBIAXIAL TENSION (λ=1) [89].

analyses suggest that neglecting the effects of tensile loads parallel to a crack could lead to an unsafe life estimate for panels with the stiffeners intact and small critical crack lengths in the panel.

As a result of the predictions by Swift, and Ratwani and Wilhem, Morrow [92] conducted a further series of tests on panels with intact stiffeners. This series was conducted using the changeover test described in the previous section. The tests were started in biaxial tension and then changed to uniaxial tension after a certain period of crack growth. The loading condition was then alternated between biaxial and uniaxial tension. Results from these tests show that, unlike the predictions, biaxial tension reduces the fatigue crack growth rate both in panels with stiffeners intact compared with that in uniaxial tension, Figure 19, and in panels with broken stiffeners. Equibiaxial tension $\lambda = 1$ reduced the fatigue crack growth rate by a factor of 1·7 in the former case and by a factor of 2·3 in the latter case compared with uniaxial tension. Morrow suggested that crack closure and plastic zone size effects as possible reasons for the differences between his experimental results and the theoretical predictions. Both of these effects appear to have been ignored by Swift, and Ratwani and Wilhem. Furthermore, Morrow concluded that a purely elastic analysis of biaxial loading effects in stiffened panels is inadequate.

In summary, the results of Ansee and Morrow, and Morrow suggest that the effects of tensile loads parallel to a crack in a stiffened panel, with either cracked or intact stiffeners, can be safely neglected in safe life estimates. However, the effects of compressive transverse loads on fatigue crack growth rates in stiffened panels have not yet been determined. Non-conservative estimates of the safe life of these structures would occur if the effects of transverse compressive stresses are similar to those observed in laboratory tests.

9. THE EFFECTS OF OUT-OF-PHASE BIAXIAL LOADING ON FATIGUE PROPERTIES

All of the biaxial fatigue research reviewed so far has been conducted under proportional (in-phase) loading, that is, the ratio of the applied stresses or strains remains constant throughout the loading cycle with the peaks and the troughs occurring at the same time. In service, many engineering components are subjected to cyclic biaxial loading conditions where the waveforms of mutually perpendicular stresses or strains are out-of-phase with each other. For example, aircraft structures are vulnerable to this type of loading which can reduce the fatigue life of the structure [93–94].

Very little experimental work has been undertaken to establish the effects of out-of-phase biaxial loading on fatigue properties of materials and components [11, 35, 71, 77, 95–99]. This situation has arisen due to the difficulty in determining representative stresses and strains as the magnitude and direction of the principal stresses and strains continually change during each cycle. Rotation of the principal stresses and strains means that the biaxial stress (strain) ratio is not constant but also varies in a cyclic manner during each test. For example, Liu, Dittmer and Holloway [77] found that the biaxial stress ratio in their tests could vary by a factor of 10 during a complete cycle for out-of-phase loading of cruciform type specimens. The problem is further enhanced in low-cycle fatigue as the principal stresses and strains rotate at varying speeds in every cycle and therefore are not coincident [95].

The rotation of the principal stress and strain directions during out-of-phase biaxial testing affects the fatigue properties of materials. For example, Grubisic and Simbürger [96] examined the effects of out-of-phase loading on the biaxial fatigue strength of carbon steel using thin-walled cylindrical specimens. These specimens were tested under biaxial loading conditions by either :

(1) Axial tension plus internal pressure with a phase difference between the normal stresses σ_x and σ_y ;

(2) Tension plus torsion with out-of-phase normal and shear stresses σ_x and τ_{xy} respectively ; or

(3) Combined bending and torsion.

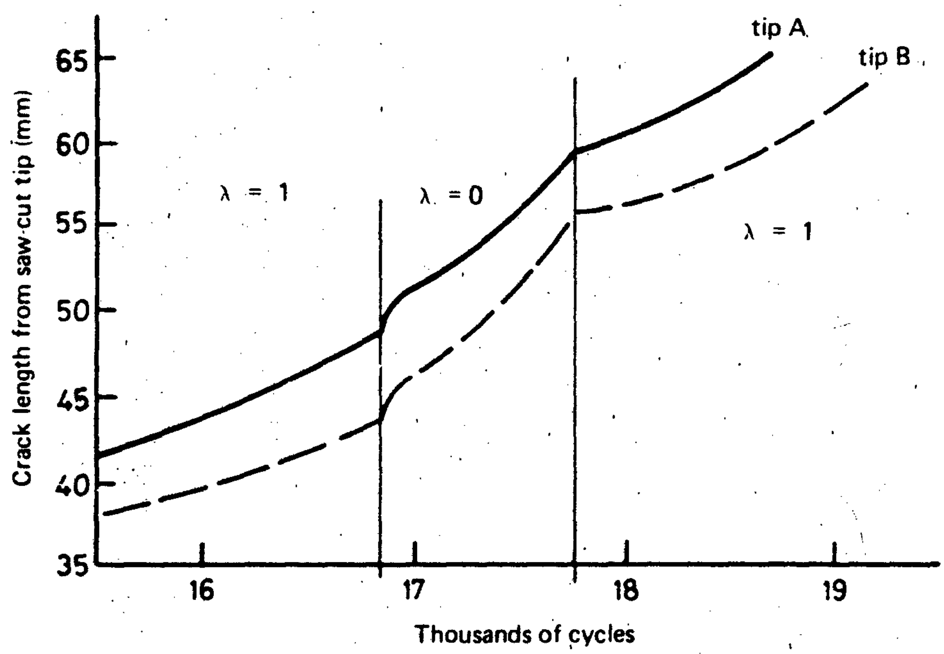


FIG. 19. THE EFFECTS OF "CHANGEOVER" TESTS ON THE FATIGUE CRACK GROWTH CURVES IN STIFFENED PANELS WITH STIFFENERS INTACT [92]

Results from these tests showed that a phase difference between the normal stresses, and especially for phase angles greater than 90°, decreased the fatigue strength. Similarly, the fatigue strength was lower for out-of-phase compared with in-phase testing for both tension-torsion and bending-torsion loading.

Grubisic and Simbürger also observed that yielding initiated at a lower stress for in-phase loading as a result of the higher peak strains encountered compared with out-of-phase loading. Consequently, if the stress amplitides were the same, in-phase loading could produce plastic deformation whereas out-of-phase loading might not.

Earlier work by Nishihara and Kawamoto [97] using combined tension and torsion showed that the fatigue limit for ductile materials increased for out-of-phase compared with in-phase cyclic loading. This result is the opposite to that observed by Grubisic and Simbürger. Little [98] re-analysed the data of Nishihara and Kawamoto and showed that the apparent increase in fatigue strength is very misleading. Little expressed the out-of-phase data in terms of the true shear stress amplitudes whereas they were originally expressed in terms of the maximum in-phase shear stress amplitudes. The analysis of the data by Little showed that the fatigue limit actually decreases as the phase difference increases. The decrease is of the order of 25% for a shear stress to normal stress ratio of 0·5 and a phase difference of 90°. This is a special case as every plane in the surface material is a plane of maximum range of shear stress.

Kanazawa, Miller and Brown [95] also conducted biaxial tests using combined cyclic tensile and torsional loads with various phase differences. They found that the fatigue life of a 1% Cr-Mo-V steel is reduced by out-of-phase loading conditions, with phase angles of 90° reducing the fatigue lives by a factor of between 2 and 4. In addition, these authors observed that the highest fatigue limits were for in-phase loading which agrees with the results of Grubisic and Simbürger. Usually, data from in-phase tests and either the Tresca or octahedral shear strain criterion are used for design purposes. However, Kanazawa, Miller and Brown showed that both of these criteria are non-conservative under out-of-phase conditions and therefore can lead to non-conservative estimates of safety factors. For example, if the Tresca criterion is used to establish a safe life based on fatigue life in torsion, then the safe life for a biaxial ratio stress of 1·5 and a phase difference of 90° will be over estimated by a factor of 10.

The cyclic stress-strain response of metals in biaxial stress states is affected by out-of-phase loading. Kanazawa, Miller and Brown [99] showed in other work using the 1% Cr Mo V steel that out-of-phase loading produces additional work hardening. This was attributed to the increasing complexity of the deformation behaviour resulting from the rotation of the principal stress directions compared with in-phase loading. In addition, rotation of the principal axes distorts the hysteresis loops at higher stresses where gross plastic deformation is taking place. The hysteresis loops, as shown in Figure 20, obtained from tension-torsion tests with phase angles other than 0° or 180°, are difficult to analyse. The distortion of the hysteresis loops means that the traditional definitions of elastic and plastic strain ranges used in fatigue life assessment may be inappropriate [11]. Despite this difficulty Kanazawa, Miller and Brown produced a cyclic stress-strain curve from their results (Figure 21) by using the maximum shear strain and the corresponding shear stress during each cycle. These authors also used a correction factor to compensate for the additional work hardening produced by the rotation of the principal axes.

The fatigue crack growth rate for metals under biaxial stress conditions also is affected by out-of-phase loading as different cracking systems can operate at different times in each cycle [11]. Kitagawa, Yuuki and Tohgo [71] found that the fatigue crack growth rates in cruciform specimens of a structural steel increased as the phase angle changed from 0° to 180° for equi-biaxial conditions, as shown in Figure 22. Similarly, Zamrik and Frishmuth [93] observed a definite phase effect on the crack growth direction and mode of failure using thin walled tubular specimens. In contrast, Liu and Dittmer [35, 77] concluded that the direction and rate of fatigue crack growth were the same for cruciform specimens under out-of-phase and in-phase loading conditions. These conflicting results once again highlight the problems in determining representative stresses and strains under out-of-phase loading conditions.

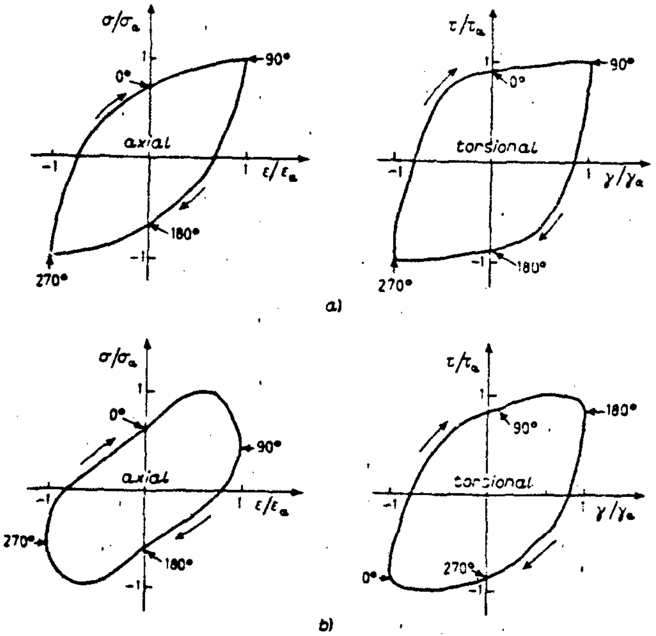


FIG. 20. HYSTERESIS LOOP FOR IN- AND OUT-OF-PHASE AXIAL AND TORSIONAL STRAINS FOR A 1% Cr-MO-V STEEL AT 20°C.
(a) $\psi = 4$, $\Theta = 0°$, $\epsilon_a = 0.5\%$,
(b) $\psi = 4$, $\Theta = 90°$, $\epsilon_a = 0.5\%$, [99].

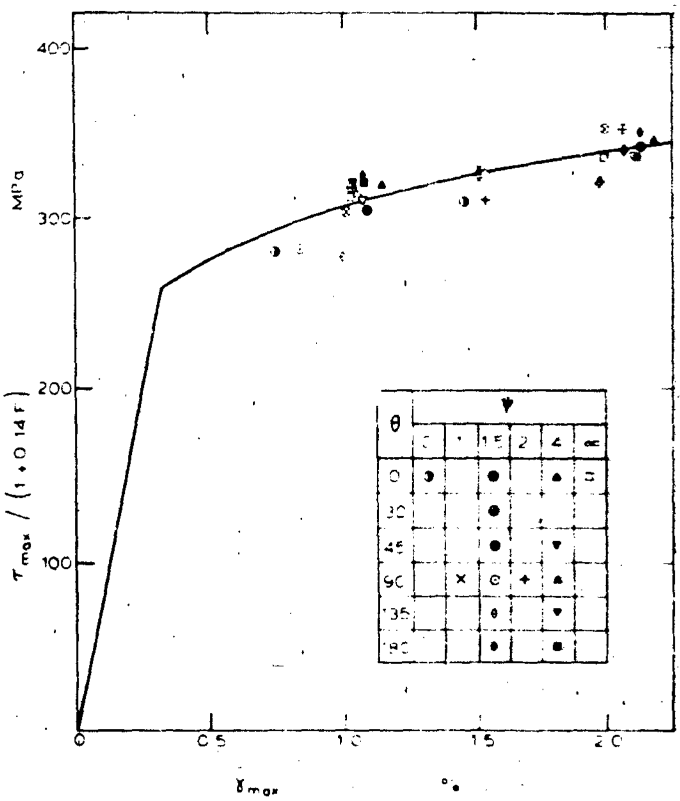


FIG. 21. BIAXIAL CYCLIC STRESS-STRAIN CURVE FOR OUT-OF-PHASE LOADING [99]

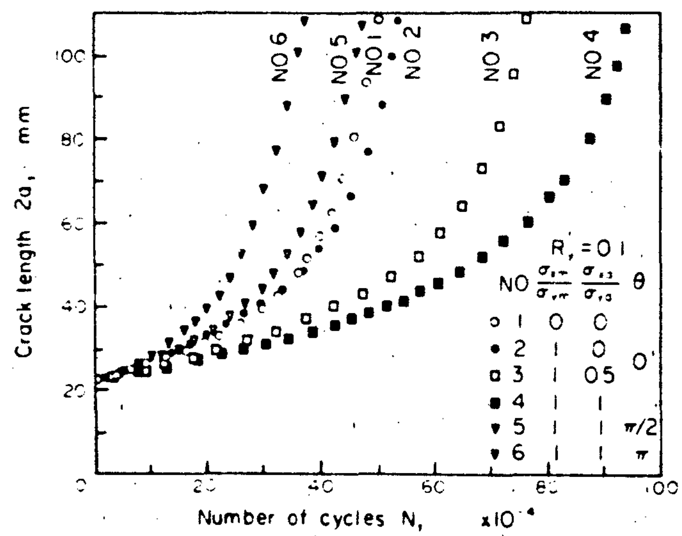


FIG. 22. CRACK LENGTH VERSUS NUMBER OF LOAD CYCLES FOR VARIOUS BIAXIAL FATIGUE TESTS [71]

10. THE EFFECTS OF BIAXIAL STRESS ON CRACK TIP PLASTICITY

The analysis of fatigue crack growth behaviour in biaxial stress fields using linear elastic fracture mechanics has proven inadequate as plasticity and anisotropy effects are not considered. In real materials fatigue crack growth rates are dependent on the amount of local plastic deformation at the crack tip. Consequently, fatigue crack growth rates can be related to the plastic zone size r_p, or alternatively to a critical crack opening displacement COD at the crack tip. For example, Hoshide, Tanaka, Yamada and Taira [4, 84, 100] showed a unique correlation between the rate of crack growth da/dN and the crack tip opening displacement as shown, for example, in Figure 23. These authors concluded that the range of COD is a parameter which is uniquely related to the fatigue crack growth rate under both elastic-plastic and general yield conditions. In addition, they [4, 69] concluded that of the three fracture mechanics parameters examined, i.e. the stress intensity factor range ΔK, the effective stress intensity factor range ΔK_{eff} (which accounts for crack closure effects), and the COD range, the latter was the best for correlating crack growth rates da/dN in all stress states. Figure 23 shows that the COD range produces a unique relationship with da/dN for all stress states compared with the relationship between da/dN and ΔK shown in Figure 24.

The plastic deformation response of a material is dependent on the stress state [26] (as will be described in the last section), hence r_p and COD may also be expected to be stress state dependent. At present, to the best of the author's knowledge, no experimental data on the effects of biaxial loading on plastic zone size have been reported in the literature. However, several elastic-plastic analyses have been undertaken to establish the dependence of both r_p and COD on the biaxial stress ratio. The elastic-plastic analyses of Miller and Kfouri [34], Hilton [41] and Tanaka *et al.* [4] have suggested that the plastic zone size decreases as the NOMINAL biaxial stress ratio λ increases from -1 to $+1$ as shown, for example, in Figure 25. Similarly, Liu and Dittmer [35, 36] found that the plastic zone sizes for tension-compression ratios ($\lambda = -$ve) are significantly larger than those for uniaxial tension ($\lambda = 0$) or tension-tension ($\lambda = +$ve) biaxial loading conditions. The plastic zone size increases as the tension-compression ratio becomes more negative. However, these authors found that the plastic zone sizes for biaxial ratios of 0·5 to 1·0 are approximately the same and only slightly smaller than those for uniaxial tension.

The elastic-plastic analyses of Adams [101] and Liebowitz, Lee and Eftis [31] produced similar results to the above, that is, the plastic zone size decreased as the LOCAL biaxial stress ratio k increased from negative values to a value of approximately 1. The plastic zone size reached a minimum value at a biaxial stress ratio of approximately 1 and then increased very rapidly as k became greater than 1 as shown, for example, in Figure 26(*a*). Adams also found a similar dependence of COD on the biaxial stress ratio, as shown in Figure 26(*b*). The biaxial ratio at which r_p and COD reach minimum values, and the percentage reduction in each case, depend on the ratio of the applied yield stress normal to the crack. When the applied yield stress ratio is 0·9 the plastic zone size is decreased by up to 65% and the crack opening displacement by 33% compared with the uniaxial tension values. Adams concluded that the fatigue crack growth rate also will decrease and reach a minimum value as the biaxial ratio increases because of the reduced values of either r_p or COD. However, the minima in the curves of r_p and COD versus biaxial stress ratio at biaxial ratios of approximately 1 may result from a change in the crack loading configuration. As mentioned in Section 2.1, the crack loading configuration for biaxial stress ratios greater than 1 is not the same as that for ratios less than 1.

Miller and Kfouri [34] performed both an elastic and an elastic-plastic analysis to establish the dependence of the crack opening displacement on the biaxial stress ratio. Results from the elastic analysis suggested that the crack opening displacement was independent of the biaxial ratio. However, the elastic-plastic analysis suggested that the COD value was significantly larger for pure shear ($\lambda = -1$) than for pure tension ($\lambda = 0$). In addition, the value of COD for pure tension was slightly larger than that for equibiaxial tension ($\lambda = +1$). The results of this elastic-plastic analysis are in agreement with those of Hoshide and co-workers [84] (as shown in Figure 27) and those of Adams. This latter analysis predicted that for an applied yield stress ratio of between 0·5 and 0·6, as used by Miller and Kfouri, the COD value would decrease with increasing k until a minimum was reached at a value of k of approximately 1.

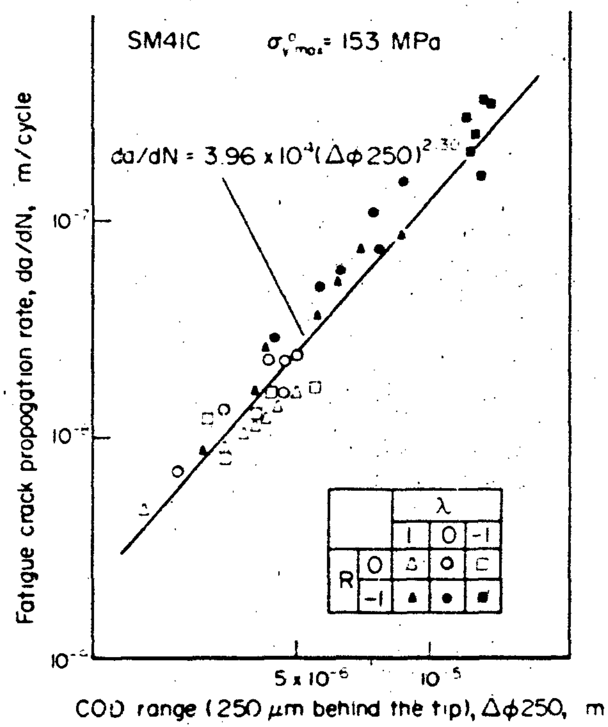


FIG. 23. RELATION BETWEEN CRACK GROWTH RATE AND CRACK OPENING DISPLACEMENT RANGE [84]

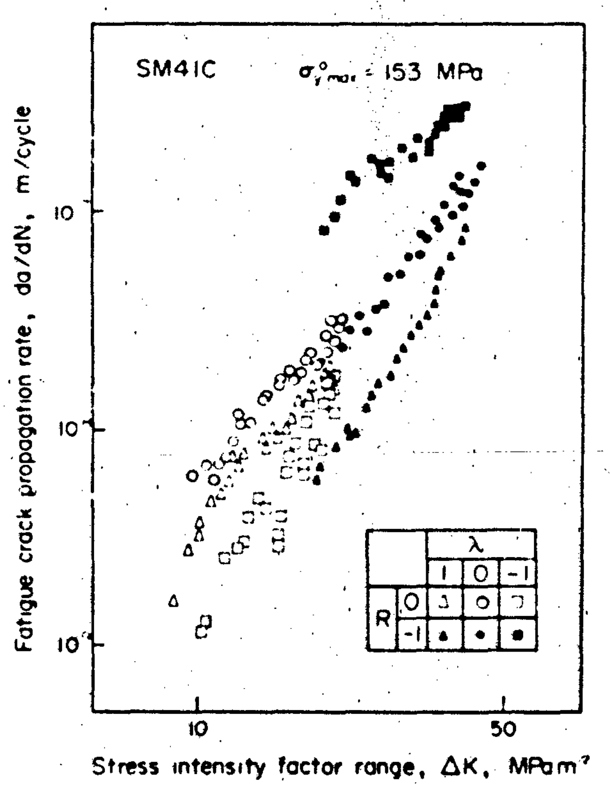


FIG. 24. RELATION BETWEEN CRACK GROWTH RATE AND STRESS INTENSITY RANGE [84]

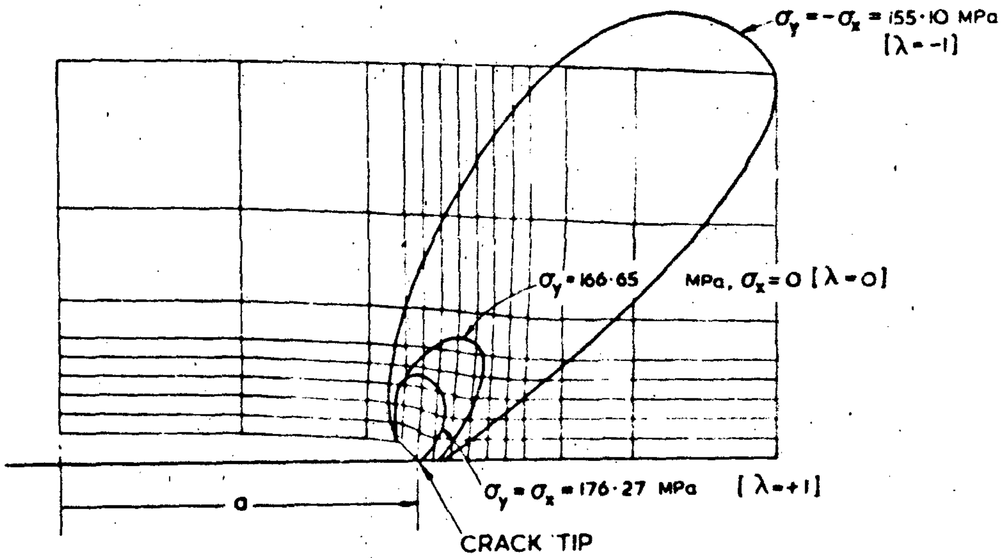


FIG. 25. EFFECT OF STRESS STATE ON CRACK TIP PLASTIC ZONE SIZE [34]

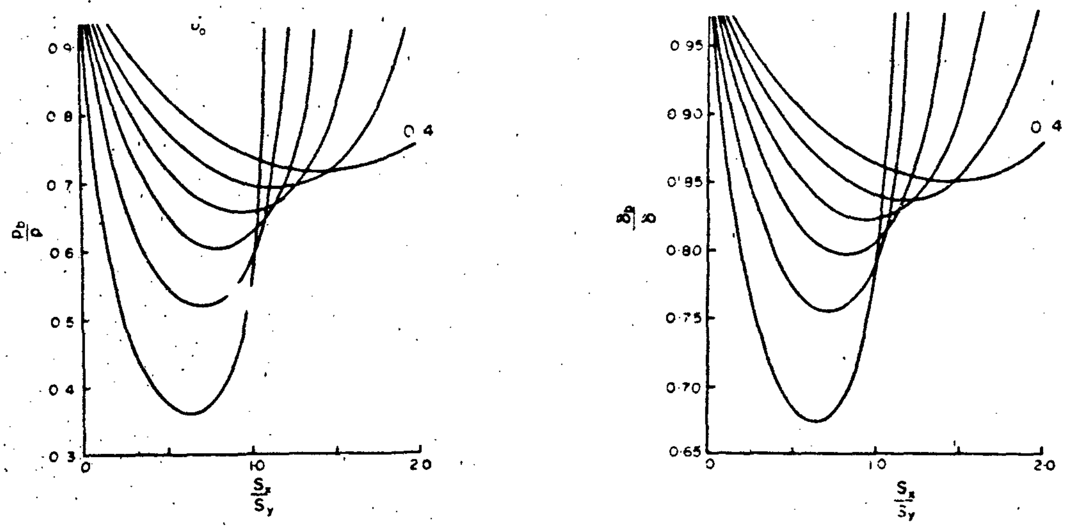


FIG. 26 (a) VARIATION IN PLASTIC ZONE SIZE WITH INCREASING BIAXIAL STRESS RATIO [101]

(b) VARIATION IN CRACK OPENING DISPLACEMENT WITH INCREASING BIAXIAL STRESS RATIO [101]

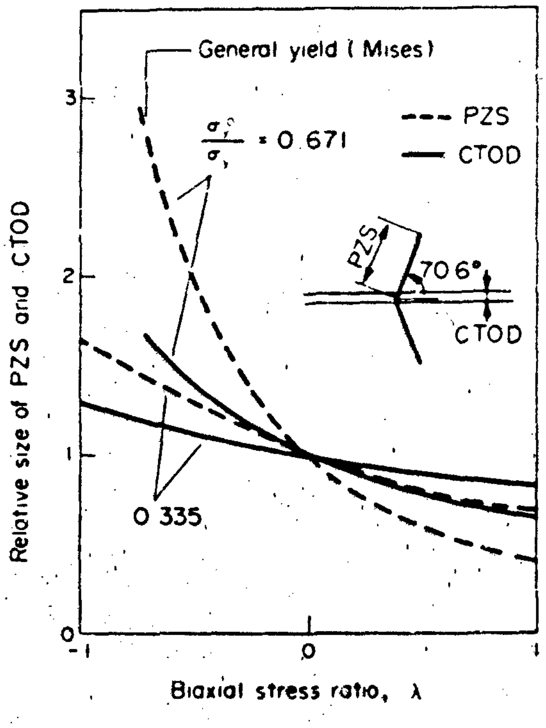


FIG. 27. EFFECT OF BIAXIAL STRESS RATIO ON PLASTIC ZONE SIZE AND CRACK TIP OPENING DISPLACEMENT RELATIVE TO THE VALUES IN UNIAXIAL TENSION, $\lambda = 0$. (σ_Y = UNIAXIAL YIELD STRENGTH AND σ_Y' IS THE STRESS IN THE CENTRE OF THE CRUCIFORM SPECIMEN IN THE Y DIRECTION [84]

The effects of biaxial loads on COD have been established experimentally by a few workers. Abou-Sayed *et al.* [102] found that the average COD value per unit load depends almost linearly on the biaxial stress ratio, as shown in Figure 28. The rate of decrease in COD with increasing biaxial ratio is greater for long cracks compared with short cracks. Hoshide, Tanaka and Yamada [84] also found that the COD values decreased with increasing biaxial stress ratios when cycling around zero mean stress values ($R = -1$). Consequently, the fatigue crack growth rates also decreased with increasing values of λ, as shown in Figure 24. However, the COD was independent of the biaxial stress ratio when positive mean loads were applied to each cycle and the minimum loads were zero ($R = 0$).

Fatigue crack growth rates can be influenced by crack closure effects as initially suggested by Elber [102], even under cyclic tensile loading conditions. Elber proposed that the residual deformation, associated with the plastic zones left in the wake of a moving fatigue crack, could close the crack tip and thereby reduce the effective portion of the loading cycle. Since crack closure is dependent on the plastic zone size, this phenomenon should also be dependent on the biaxial stress ratio. Ogura and co-workers [104] undertook a finite element analysis to predict the fatigue crack closure behaviour for varous biaxial stress ratios. They found that the effective fraction of the applied stress range decreased, and hence the amount of crack closure increased, as the biaxial stress ratio increased from -1 to $+1$ when cycling about zero mean stresses ($R = -1$). However, the biaxial stress ratio had no significant effect on crack closure when cycling between zero and a tensile stress ($R = 0$). The experimental results of Hoshide, Tanaka and Yamada [84] are qualitatively in agreement with these analytical results. Those authors found that the effective stress intensity factor range decreased by approximately 35% and consequently, the fatigue crack growth rate decreased by a factor of approximately 4, as λ increased from -1 to $+1$ when $R = -1$. The effect of biaxial stress ratio on crack closure was not significant when the sytess ratio was $R = 0$.

11. THE EFFECTS OF BIAXIAL STRESS IN THE PRESENCE OF A NOTCH

The initiation and subsequent growth of fatigue cracks in engineering components is usually from geometric discontinuities such as a notches or holes. These discontinuities act as stress and strain concentrators* and thereby introduce localised, highly strained regions experiencing complex stress conditions. Consequently, the fatigue strength and fatigue life of components can be seriously reduced in the presence of a geometric discontinuity.

The usual method for predicting the fatigue strength or fatigue life of a notched component is to describe the notch severity in terms of a concentration factor, such as the theoretical (elastic) stress concentration factor K_t or the fatigue strength reduction factor K_f. These correction factors can be most useful when designing a component for an infinite life, however, they are less useful in the finite life regime [105]. For example, the theoretical stress concentration factor K_t is, by definition, independent of mean stress level, and does not consider the "size effect" i.e., large components fail sooner than small components. In addition, the experimentally determined strength reduction factors K_f are usually smaller than (and often inconsistently related to) the corresponding theoretical K_t values [106], and are life dependent.

Leis and Topper [106] suggested that three main factors are responsible for the discrepancies between the theoretical K_t and experimental K_f values : 1) the type of failure criteria used, 2) the inelastic local stress-strain response not being accurately accounted for, and 3) the effects of local multiaxial stresses. Newman [107], and Hopper and Miller [68] have shown analytically that the severity of the notch effect depends on both the method used to represent the notch correction factor and on the stress state, as shown in Figures 29 and 30. It should be noted that Newman's results, Figure 29, were presented in terms of a stress intensity factor based on the

* It should be noted that surface roughness and metallurgical defects, such as porosity, inclusions and heat affected zones also can act as stress concentrators.

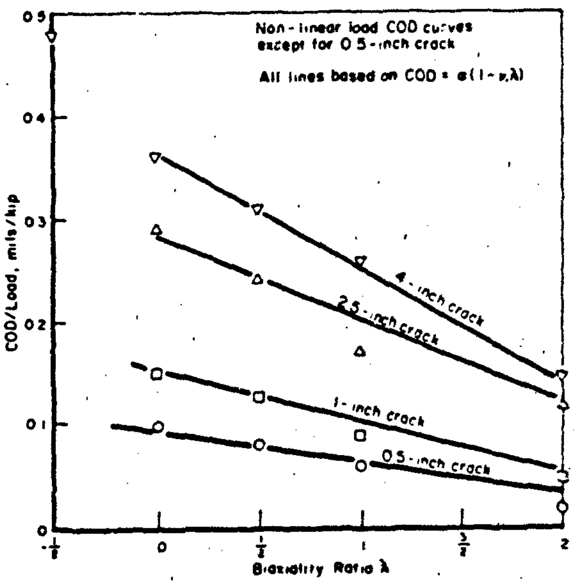


FIG. 28. EFFECT OF BIAXIAL STRESS RATIO ON CRACK OPENING DISPLACEMENT [102]

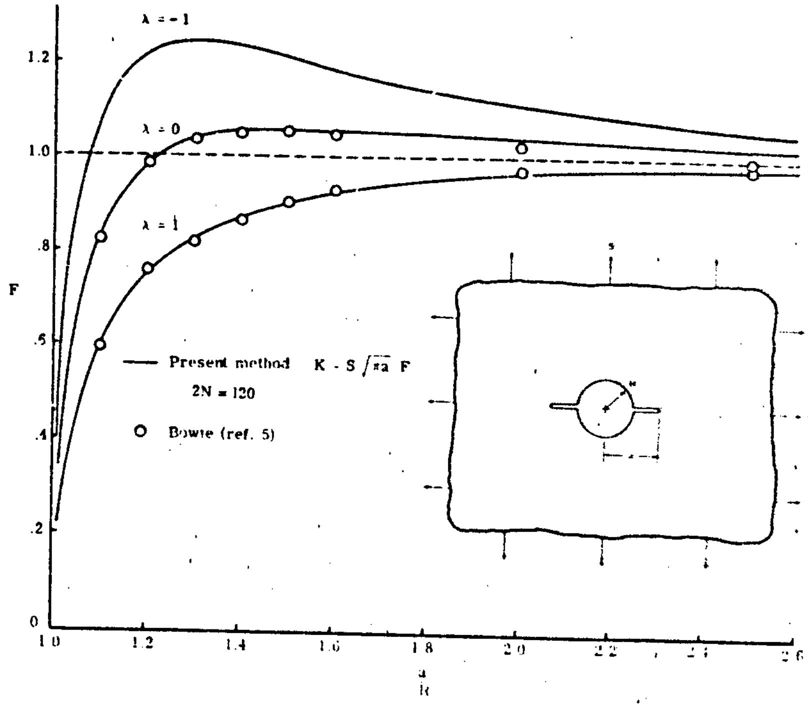


FIG.29. STRESS INTENSITY CORRECTION FACTORS FOR CRACK EMANATING FROM A CIRCULAR HOLE IN AN INFINITE PLATE SUBJECTED TO BIAXIAL STRESS [107]

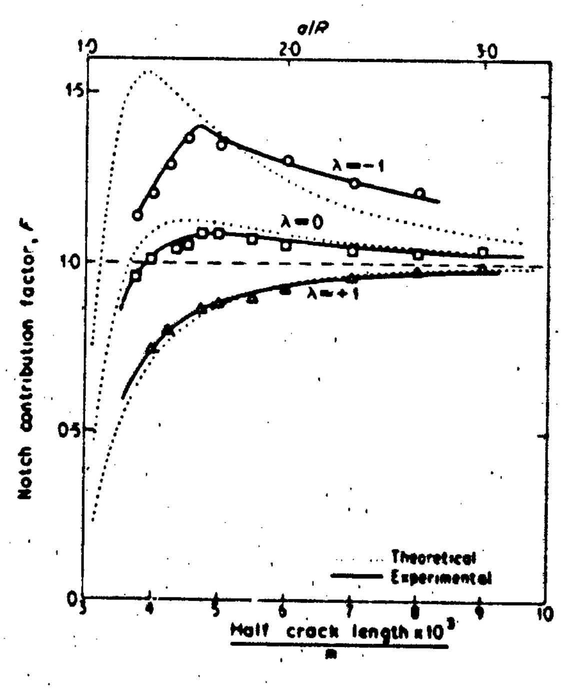


FIG.30. EXPERIMENTAL AND THEORETICAL NOTCH CONTRIBUTION FACTORS [68]

complex variable method of Muskhelishvili and a modified boundary collocation method—not on LEFM. In contrast, Hopper and Miller's approach (Fig. 30) was to define a notch contribution factor as the non-dimensional ratio of the equivalent crack length (which represents a crack growing in a notched field) to the crack length in an unnotched field when both cracks have the same growth rates. The results of both analyses are in agreement. For example, shear loading ($\lambda = -1$) gives the highest stress intensity correction factors and equibiaxial loading ($\lambda = +1$) gives the lowest. Therefore, the fatigue crack rate is expected to increase for a crack emanating from a notch when $\lambda = -1$ and decrease when $\lambda = +1$, when compared with the growth rates in unnotched specimens. This trend is in agreement with the experimental results of Hopper and Miller, Figures 12 and 31.

Hopper and Miller also calculated the notch contribution factors from their experimental results. As shown in Figure 30, the closest agreement between the theoretical and the experimental notch contribution factors is for equibiaxial loading ($\lambda = +1$) whereas the greatest difference is for the shear loading case ($\lambda = -1$). Hopper and Miller suggested that the change in plastic zone size with stress state was responsible for this trend. The plastic zone size at a crack tip is largest under shear loading conditions, Figure 26. Therefore, the elastic analysis used to determine the notch contribution factor would be less accurate for this stress state compared with equibiaxial tension. This is in agreement with the second factor suggested by Leis and Topper to explain the discrepancies between the theoretical K_t and the experimental K_f values.

12. MICROSTRUCTURAL ASPECTS OF MULTIAXIAL FATIGUE

The review so far has concentrated on the engineering aspects of multiaxial fatigue as most research has been conducted in this area. Very few papers have been published on the microstructural aspects of fatigue in complex stress states. For example, Habetinek and co-workers [26, 108] are the only researchers known to have studied the effects of stress state on the deformation mechanisms and microstructure of metals during fatigue. They observed the development of the dislocation substructure and the mode of failure of a low-C steel under uniaxial, biaxial and triaxial stress states.

Habetinek and co-workers found that increasing the number of loading axes increased the resistance to dislocation mobility, and thereby reduced the degree of stress relaxation and retarded the development of the dislocation substructure. Under uniaxial loading conditions a cellular dislocation substructure was forming after 20% of the fatigue failure life. However, under biaxial loading conditions, a similar substructure was not observed until 90% of the fatigue life had elapsed. No such substructure was observed in any of the fatigue specimens tested under triaxial conditions. The increased resistance to dislocation motion and substructure formation which occurred as the stress state changed from uniaxial to triaxial was associated with a decrease in fatigue strength. In addition, the mode of failure was dependent on the stress state. Striations were observed on the fracture surface under uniaxial loading conditions indicating a relatively continuous mode of crack growth. However, no striations were observed under triaxial conditions as the fracture surface consisted of smooth, brittle-type facets. A mixed mode of failure occurred under biaxial loading conditions with both small areas of striations and brittle facets being present.

Tanaka and co-workers [4, 75] observed a similar change in mode of failure in cruciform shaped specimens of a 0·04% C steel. The fracture surfaces resulting from the use of positive biaxial stress ratios were relatively smooth, with secondary cracking and brittle-like facets being frequently observed. In comparison, for negative biaxial stress ratios, the fracture surfaces were much roughter with secondary cracking and brittle-like facets being virtually absent. These authors suggested that this change in mode of failure was a result of an increased hydrostatic stress component near the crack tip at larger biaxial stress ratios.

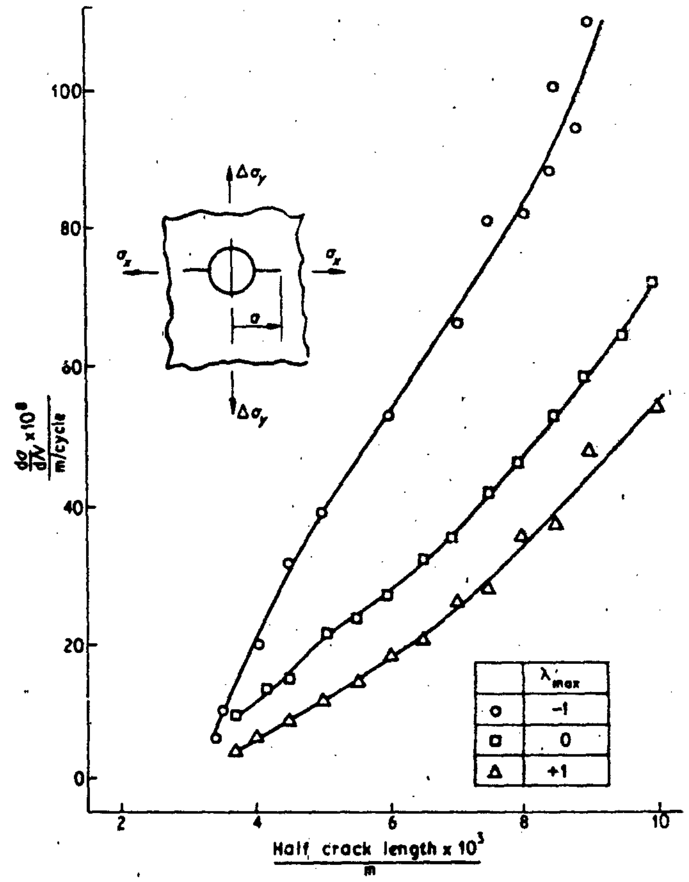


FIG. 31. FATIGUE CRACK GROWTH RATES FOR NOTCHED SPECIMENS OF RR58
$\Delta\sigma_y$ = 70MPa, σ_{ymax} = 105MPa [68]

The nature of the surface deformation and the direction and mode of crack growth also are dependent on stress or strain state [86, 87]. Parsons and Pascoe [86] found that the pattern of surface deformation, i.e. slip lines, varies characteristically with the strain state. In general, there exists a preferred orientation for the slip lines in each strain state which coincides with the direction of subsequent surface crack growth. The change in the mode of crack growth with strain state observed by Brown and Miller [87] has been discussed in Section 8.

Finally, the orientation of the crack path with respect to the loading axes is stress state dependent [35, 39, 76, 109, 110]. For low biaxial stress ratios ($\lambda < 1$) the crack path tends to be straight and perpendicular to the maximum principal stress (σ_y). However, for high biaxial stress ratios ($\lambda > 1$) and therefore a different crack loading configuration, the crack tends to follow an S-shaped path centred on the initial straight notch, as shown for example in Figure 32. The curvature of the path increases with increasing biaxial stress ratio for values greater than 1.

13. SUMMARY AND CONCLUSIONS

A review of the literature has shown that there are significant differences in the fatigue and fracture properties of materials and components when loaded under uniaxial and multiaxial stress conditions. The effects of stress state on the various parameters used to describe these properties can be summarised as follows :

(1) The ultimate tensile strength of steel increases by up to 18% and the fatigue limit decreases by up to 48% as the stress state changes from uniaxial to biaxial to triaxial.

(2) The resistance of a metal to cyclic deformation is dependent both on the biaxial stress ratio and the type of metal. Increasing the biaxial stress ratio from −1 (shear loading) to +1 (equibiaxial loading) increases the resistance to cyclic deformation.

(3) The fatigue life of metals is dependent on the biaxial strain ratio. Increasing this ratio from −1 to +1 can decrease fatigue lives in the high and low-strain regimes by factors of up to 10 and 20 respectively.

(4) Although there are conflicting results in the literature concerning the effects of biaxial stresses on fatigue crack growth rates, changeover tests have shown conclusively that :

  (a) An instantaneous change in the stress state from uniaxial ~~to equibiaxial~~ at the same nominal stress applied normal to the crack decreases the fatigue crack growth rate by a factor of 2. By comparison, the growth rate is increased by a factor of 4 when the stress state is suddenly changed from equibiaxial to uniaxial. A similar trend is observed on stiffened panels with the stiffeners either cracked or intact.

  (b) An instantaneous change in stress state from uniaxial tension to pure shear, by suddenly applying a cyclic compressive stress parallel to the crack, increases the fatigue crack growth rate by a factor of 3.

(5) Experiments have shown that when cycling about zero mean stress the crack opening displacement and the degree of crack closure decrease with increasing biaxial stress ratio. Both properties are independent of biaxial ratio when the R-value is from zero to a positive value.

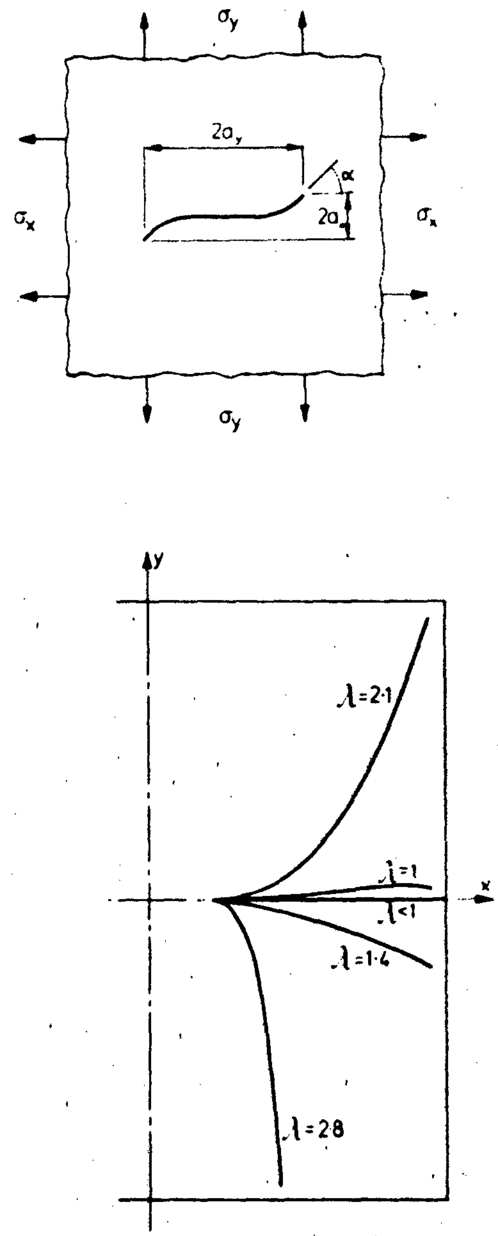


FIG. 32. CRACK TRAJECTORIES AT VARIOUS BIAXIAL STRESS RATIOS λ [110]

(6) Fatigue properties are affected by out-of-phase biaxial stresses and the associated rotation of the principal stresses as follows :

(*a*) At any given biaxial ratio fatigue strength is decreased as the phase angle between the normal stresses is increased.

(*b*) Yield is initiated at a lower stress for in-phase compared with out-of-phase loading.

(*c*) The cyclic stress strain response of a material is affected by the phase angle – hysteresis loops change shape and rotation of the principal stresses produces additional cyclic hardening.

(*d*) LCF life is reduced by up to a factor of 4 compared with out-of-phase cycling – a phase angle of 90° giving the lowest life.

(*e*) Both the Tresca and octahedral shear strain criteria, commonly used for design purposes, are non-conservative under out-of-phase conditions.

(7) (*a*) Linear elastic fracture mechanics analyses which use the so-called "singular solution" to represent the local stresses and strains at the crack tip, predict that the various fracture parameters are independent of the biaxial stress ratio. However, when the second term of the series expansion is included, all of the fracture parameters show a biaxial load dependence, even in the elastic range.
(*b*) Non-linear elastic-plastic analyses also show that the fracture parameters are dependent on stress state. The major part of this dependence comes from the inelastic material response at the crack tip.

(8) The critical fracture load (and critical stress intensity for fracture) are dependent on both the biaxial stress ratio and Poisson's ratio. Fracture loads increase with increasing biaxial ratio for Poisson's ratios of less than $\frac{1}{3}$ and the reverse occurs when Poisson's ratios are greater than $\frac{1}{3}$.

(9) The critical stress intensity factors for stiffened panels, and cylindrical and spherical shells containing cracks, increases with increasing biaxial stress ratio.

(10) The stress intensity correction factors used to predict the fatigue lives of notched components are dependent on the biaxial stress ratio. Compared with uniaxial tension, a biaxial ratio of −1 increases the correction factors whereas a ratio of +1 decreases the correction factors.

(11) The effects of a notch on fatigue crack growth rates are greater for biaxial ratios of −1 and +1 compared with a ratio of zero.

(12) The stress state affects the deformation mechanisms and the deformed microstructure of metals during cyclic loading. The resistance to dislocation movement and the formation of a recovered dislocation sub-structure are increased, and the degree of stress relaxation is reduced, as the stress state changes from uniaxial to biaxial to triaxial.

(13) The orientation of the crack path and the failure mode are dependent on the degree of multiaxiality.

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16. Abstract

A review of the literature has shown that virtually all fatigue and fracture properties of metals and components are affected by multiaxial loading. In particular, variations in stress state compared with uniaxial tension produce the following effects:

(a) *Decreases in the fatigue limit by up to approximately 50%.*

(b) *Increases or decreases in low-cycle fatigue life by factors of up to 20, depending on whether the stress in the second direction is tensile or compressive, and is static or cyclic.*

(c) *Out-of-phase loading also reduces the low-cycle fatigue life by a factor of 4 compared with in-phase loading.*

This page is to be used to record information which is required by the Establishment for its own use but which will not be added to the DISTIS data base unless specifically requested.

16. Abstract (Contd)

(*d*) *Accelerations or retardations of fatigue crack growth rates by factors of 3 to 4 depending on the nature of the transverse stress.*

It is also evident that multiaxial criteria used for design purposes can be non-conservative, especially under out-of-phase loading, and consequently can lead to unsafe estimates of the fatigue life of a component.

17. Imprint

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