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PITCH RATE VERSUS G COMMAND AS THE LONGITUDINAL FLIGHT CONTROL SYSTEM DESIGN STRATEGY FOR A STATICALLY UNSTABLE FIGHTER TYPE AIRCRAFT WITH TWO CONTROL SURFACES

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PITCH RATE VERSUS G COMMAND AS THE LONGITUDINAL FLIGHT
CONTROL SYSTEM DESIGN STRATEGY FOR A STATICALLY UNSTABLE
FIGHTER TYPE AIRCRAFT WITH TWO CONTROL SURFACES

Thomas P. Webb[*]

Abstract

Pitch rate command and normal G command longitudinal flight control systems were designed using linear optimal control theory for a statically unstable, two control surface, fighter-type aircraft at both a power approach and an up-and-away flight condition. The closed-loop systems were then evaluated in man-in-the-loop simulations with pilots attempting random altitude tracking and pitch tracking tasks. The evalutation results indicated that in the power approach flight condition, normal G command was more suitable for altitude tracking and pitch rate command was preferred for pitch tracking. Results for the up-and-away flight condition were inconclusive.

I. Introduction

The recent development of reliable aircraft fly-by-wire flight controls has allowed some significant changes in aircraft design methodology. Airframe designers have been able to take advantage of the benefits of relaxed static stability. Aircraft have been made to fly in unconventional ways (control configured vehicles). Along with these new capabilities have come the challenges associated with the enormous increase in the complexity of aircraft flight control systems. The use of multiple sensors and multiple control surfaces obviously elevates the control design task from single input-single output (SISO) to multi input-multi output (MIMO). In addition, the higher control surface rates necessary to control unstable aircraft mandate that actuator and sensor dynamics and structural modes be accounted for in the

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control design process. Previously, these higher order dynamics could be neglected in most cases.

The complexity of the control problem has made the linear optimal control design techniques from "modern" control theory very attractive because they are well equipped to handle MIMO systems. These techniques are not a cure-all, however. Plant uncertainties, limited sensors, noise contamination, cycle rates for digital control systems, etc. all present problems that keep the control design task interesting. (In many cases it is necessary for the controls designer to augment "modern" control strategies with techniques from "classical" SISO theory.)

Another consideration is that in manned aircraft, the flight control system must interface with the pilot. The question arises as to how the optimal control design techniques can be tailored to the man-in-the-loop control problem. Although there are certain proven guidelines for the desired dynamics of human controlled aircraft (namely Military Specification MIL-F-8785C, reference 3) there is still a great deal of latitude left in applying these techniques, particularly if there is more than one control surface available. The purpose of this experiment was to compare two of the many options available in answering this last question.

Flight control systems were designed using linear optimal control for a simplified longitudinal model of a statically unstable fighter type aircraft with two control surfaces. Two design schemes were used. In one scheme the closed-loop system output was optimized assuming that the pilot's single input was a pitch rate (q) command. In the other scheme, the input was assumed to be a normal G (n_z) command. These design procedures were accomplished at two flight conditions: a power approach case and an up-and-away case. This produced four different closed-loop systems. The closed-loop dynamics were then

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simulated on an analog computer. Pilots flew each configuration in an altitude
tracking and a pitch tracking task. Pilot ratings and error histories were
obtained and compared to determine which, if either, design procedure would be
more appropriate for this type of aircraft.

II. Theory

A. Open-Loop Models

The aircraft open-loop mathematical models for both the power approach
(PA) and up-and-away (UA) flight conditions were obtained from reference 2.
They are two degree-of-freedom, short period approximations of the linearized
longitudinal equations of motion for a representation of the Grumman X-29A
airframe. These approximations assume rigid body and no actuator dynamics. The
equations take the standard first order form:

$$\dot{\overline{x}} = [A] \, \overline{x} \; + \; [B] \, \overline{u} \qquad\qquad (1)$$

where: $\overline{x}$, the state vector, is $[\alpha, q]^T$

$\overline{u}$, the control vector, is $[\delta_c, \delta_s]^T$ for PA

and $[\delta_c, \delta_f]^T$ for UA

A and B are matrices
α is angle of attack in radians
q is pitch rate in radians per second
δ is control deflection from equilibrium in
        degrees, positive for trailing edge
        down
the subscripts c, s, and f denote canard,
        strake, and flap, respectively

The A and B matrices for both flight conditions are listed in Appendix A.

The PA flight condition is for Mach .2 at sea level. The two control
surfaces used are the canard and strake. The two open-loop poles are 1.19 and
-1.77. The aperiodic short period with one pole in the right half plane is
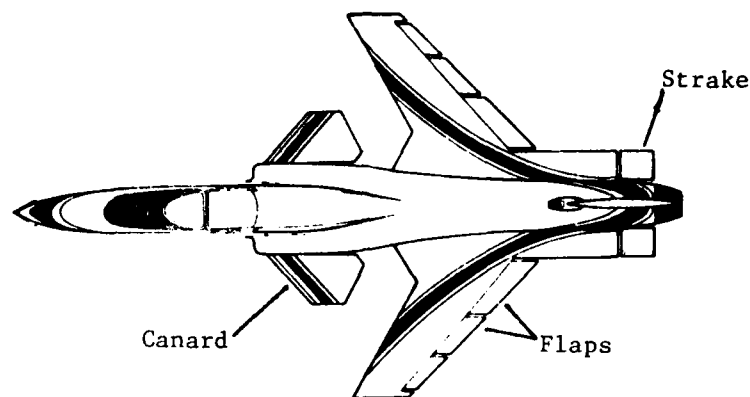characteristic of statically unstable aircraft.

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Figure 1. Aircraft Control Surface Locations

The UA flight condition is for Mach .59 at 5000 feet altitude. The two
control surfaces used are the canard and flaps. The two open-loop poles are
3.45 and -5.29. Again, note the unstable aperiodic mode. As expected, the
dynamics are quite a bit faster than for the PA condition as evidenced by the
larger magnitude of the poles. The unstable mode has a time to double amplitude
of .2 seconds.

B. Flight Control System Structure

The structure for both the pitch rate (q) command and the G command
closed-loop configurations is the standard continuous tracker problem as
depicted in figure 2. The only difference in the problem formulation is that in
the q command system, the pilot's stick input (δ_p) represents a commanded pitch
rate while in the G command, δ_p represents a commanded change in normal load
factor (n_z). Full state feedback with perfect sensors is assumed. F_2 is the 2
by 1 feedforward matrix containing the stick gains. F_1 is the 2 by 2 matrix
containing the feedback gains. With the structure defined, the problem

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remaining is to determine F_1 and F_2.
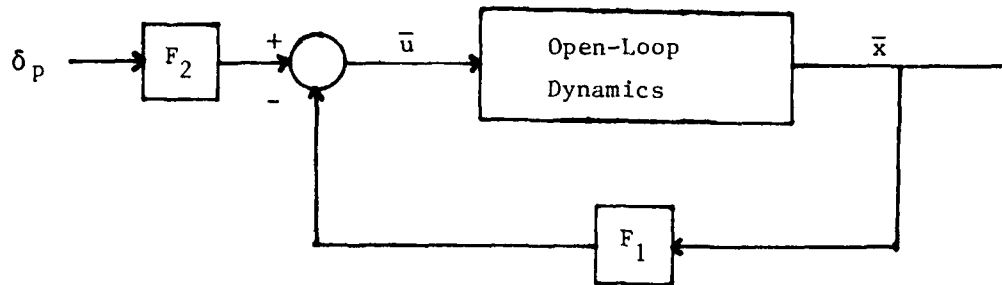
Figure 2. Closed-Loop System

C. Formulation of the Optimization Problem

The gain matrices F_1 and F_2 were chosen by transforming the tracking problem of figure 2 into the well known time-invariant stochastic linear optimal regulator problem (reference 1, page 255). In order to do this, some assumptions had to be made about how the input δ_p would vary with time. As is normally done in cases like this, $\delta_p(t)$ was assumed to be random and modeled as the output of a linear differential system driven by white noise. A first order system was chosen with a break frequency of 10 rad/sec giving the scalar equation:

$$\dot{\delta}_p = -10\,\delta_p + w(t) \qquad (2)$$

where w(t) is the white noise

δ_p can be thought of as white noise passed through a low-pass filter. The intensity of w(t) has no effect on the values of the optimum gain matrices. Equation 2 was used in all cases to model the pilot's stick input.

The problem was converted into regulator form by augmenting the state equation, 1, with equation 2:

$$\begin{bmatrix} \dot{\overline{x}} \\ \dot{\delta}_p \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & -10 \end{bmatrix} \begin{bmatrix} \overline{x} \\ \delta_p \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \overline{u} + \begin{bmatrix} 0 \\ w(t) \end{bmatrix} \qquad (3)$$

or, defining the augmented state, $\widetilde{x} = [\overline{x}^T, \delta_p]^T$:

$$\dot{\widetilde{x}} = \begin{bmatrix} A & 0 \\ 0 & -10 \end{bmatrix} \widetilde{x} + \begin{bmatrix} B \\ 0 \end{bmatrix} \overline{u} + \begin{bmatrix} 0 \\ w(t) \end{bmatrix} \qquad (4)$$

Referring back to figure 2, it can be seen that for the closed-loop system, $\overline{u}$ is a linear combination of the augmented state, $\widetilde{x}$. The block diagram can be redrawn in regulator form (figure 3). The augmented feedback matrix, $\widetilde{F}$, is $[F_1, -F_2]$.
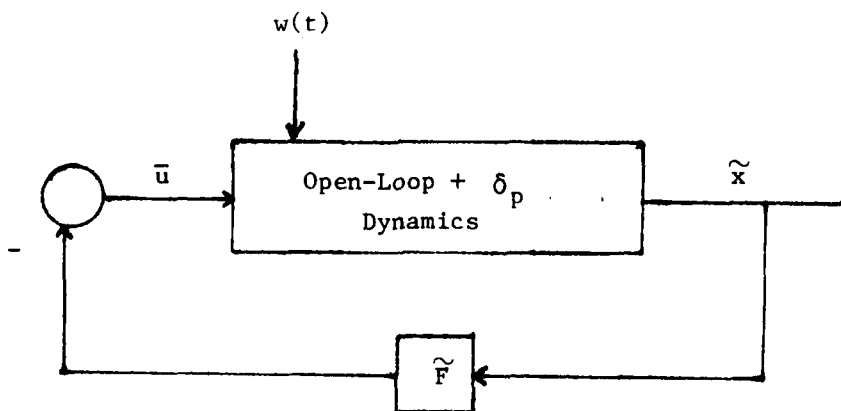
Figure 3. Aircraft States Combined With Modeled Pilot Input Into Regulator Format

The solution of the linear optimal regulator problem will find $\widetilde{F}$ such that the following performance index (J) is minimized:

$$J = \int_0^\infty (\widetilde{x}^T Q \widetilde{x} + \rho \overline{u}^T R \overline{u}) \, dt \qquad (5)$$

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where: Q is a positive-definite symmetric matrix that
determines what function of the states
is minimized
R is a positive-definite symmetric matrix that
determines what function of the
controls is minimized
ρ is a scalar that is used to weight the
relative importance of the controls
versus the states in J

This problem is frequently called the Linear Quadratic Regulator (LQR) problem
in the literature because of the quadratic nature of the two terms in J.

The only difference between the q command and G command design schemes is
the Q matrix. For the q command system, recall that δ_p represents a commanded
pitch rate. It is therefore desirable to minimize the integral of $(\delta_p - q)^2$
over time. The smaller this integral is, the closer q tracks δ_p. The Q matrix
used for the q command design in both PA and UA flight conditions was:

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

This results in:

$$\tilde{x}^T Q \tilde{x} = q^2 - 2\delta_p q + \delta_p^2 = (\delta_p - q)^2$$

For the G command system the Q matrix was a little more involved due to the fact
that n_z must be formed as a combination of q and α. This is shown in Appendix
B. The Q matrices used for the PA and UA conditions are listed there.

The same R matrix was used in all cases:

$$R = \begin{bmatrix} .5 & 0 \\ 0 & 1 \end{bmatrix}$$

.5 was chosen as the canard deflection weight to penalize it less than the other
control surface (flap or strake) since the canard has more travel.

D. Gain Matrix Selecti

Values of the gain matrix $\widetilde{F}$ were selected for each of the four flight
control systems: q command (PA and UA) and G command (PA and UA). For each
system, a series of six optimal gain matrices was computed for values of ρ (the
control weighting scalar) from 100 down to .001. The gains were calculated
using a computer program maintained by the Department of Astronautics at USAFA
(reference 4) which finds the gains using the well-established algebraic matrix
Riccati equation (reference 1, page 237). The particular optimal gain matrix to
be used for each system was selected from the series based on the poles of the
closed-loop system as seen by the pilot (see figure 2). The gains with the
"best" set of short period poles according to reference 3 were used.

The closed-loop poles for each computer run are listed in Appendix C along
with the actual gains and transfer functions for the systems selected. The
poles for the systems selected are repeated in table 1 below.

	Poles	ω_n (rad/s)	ζ
Power Approach (PA)			
MIL-F-8785C requirements		.86 – 3.0	.35 – 1.3
q command	−1.14, −1.80	1.43	1.03
G command	−1.40, −1.57	1.48	1.00
Up and Away (UA)			
MIL-F-8785C requirements		2.6 – 9.7	.35 – 1.3
q command	−3.03, −6.12	4.31	1.06
G command	−4.89 ±j 1.98	5.27	.93

Table 1.   Closed-Loop Poles

Notice that the short period mode mil. spec. requirements were met in all cases.
Also, note that both design schemes yielded similiar poles. The transfer

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function numerators, as might be expected, show a little more variety (see Appendix C).

III. Apparatus and Procedure

Each of the four closed-loop systems was simulated on an analog computer. The systems were evaluated by six pilots who flew each configuration in both a pitch tracking and an altitude tracking task. Only longitudinal dynamics were simulated -- lateral-directional motion was not present.

A. Simulation

The simulations were conducted on an Electronic Associates, Inc. TR-20 analog computer. The closed-loop systems were obtained from figure 2 using the gains and open-loop dynamics appropriate for each condition. The resulting systems are described by:

$$\dot{\overline{x}} = [A - BF_1]\overline{x} + [BF_2]\,\delta_p \qquad (6)$$

For the tracking tasks, it was necessary to generate aircraft pitch attitude change (θ) and altitude change (**h**). Pitch attitude was obtained by simply integrating pitch rate, q. Altitude change was obtained by integrating vertical velocity which was approximated by $U_1(\theta - \alpha)$ where U_1 is the trim airspeed. The quantity $\theta - \alpha$, of course, is the flight path angle.

The single stick input from the pilot was provided through a modified Kraft model airplane radio control joystick. Information was displayed to the pilot on an oscilloscope. Only pitch attitude and the error signal (pitch or altitude) were presented.

B. Tasks

The same second order random process was used for both the pitch and the altitude tracking tasks. The process is represented by:

$$\ddot{c} = -.73 + \quad w(t) \qquad (7)$$

where: c is the commanded value
(either θ or h)
w(t) is noise

This is equivalent to filtering the noise through a second order filter with break frequency of 1 rad/sec and damping ratio of .35. The command was generated on the TR-20 using a homemade device for the noise source. The output of the device was evidently far from white, having much less power in the low frequencies of interest than in the higher frequencies. The filter output had to be amplified one thousand times to obtain the required amplitude. A typical example of the generated command is shown in figure 4 scaled for both altitude and pitch attitude.
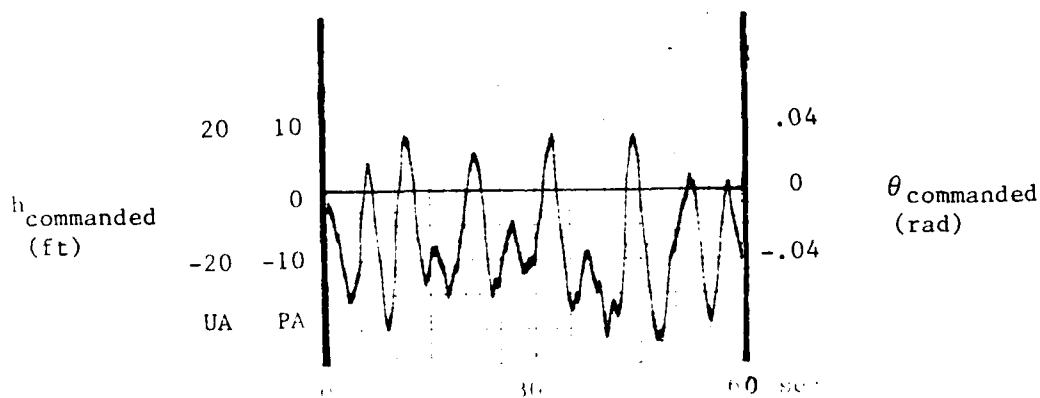
Figure 4. Typical Command Signal for Tracking Task

In the simulations, the h signal was scaled so that the magnitude of the h command was double the number of feet for UA configurations as for PA

configurations.

A tracking error signal was presented to the pilot on the oscilloscope. For the altitude tracking task, $h_{commanded} - h$ was shown . When the aircraft was on commanded altitude, the command trace was centered. If the command increased (or the altitude decreased) the command trace on the oscilloscope would increase or go up an amount proportional to the error, indicating an increase in altitude was required. The display worked much like an instrument landing system (ILS) glide slope indicator and the task appeared similiar to terrain following. The aircraft pitch attitude was also presented for reference using the second trace on the oscilloscope. As might be expected, it was impossible to fly this task without pitch reference.

For the pitch tracking task, $\theta_{commanded} - \theta$ was displayed on the command trace. This task was more like a nose pointing exercise or following a pitch steering bar. The basic pitch attitude trace was not necessary for this task and most pilots requested it be turned off. In both tasks, a low amplitude, high frequency noise on the command trace (due to the high amplification of the command signal) made it easy for the pilots to discern between the pitch attitude trace and the command trace.

C. Evaluation Procedure

Six experienced Air Force pilots, including one test pilot, rated all four closed-loop systems in both the altitude and the pitch tracking tasks. A brief background for each pilot can be found in Appendix E.

Each pilot made all eight simulation runs at one sitting in the following order:

PA	q	command	altitude tracking
PA	G	"	altitude "
PA	G	"	pitch "
PA	q	"	pitch "

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UA	q command	altitude tracking
UA	G	altitude "
UA	G	pitch "
UA	q	pitch "

The subjects gave each confi ration a rating for both tasks using the Cooper-Harper rating scale included in Appendix D. They were also encouraged to write subjective comments about each flight control system configuration. A one minute record of the pilot's performance (tracking error) was made for each run using a strip chart recorder.

The tasks and display were explained to each subject prior to the runs. Subjects were allowed as much time as they wanted to familiarize themselves with each task and set of dynamics before the strip chart data was taken. They were told what the flight conditions were for the PA and UA cases. The q command and G command flight control systems for each condition were simply referred to as "A" and "B", respectively.

The oscilloscope was set so that the pitch attitude sensitivity was .02 rad (1.145 deg)/cm. This was also the sensitivity of the pitch command trace. The sensitivity of the altitude command trace was set at 10 ft/cm. This imformation was briefed to the pilots.

IV. Results

All of the simulation results are contained in Appendix E. Tables 2 and 3 are tabulations of the pilot ratings and total tracking error for each of the runs. The total tracking error is the absolute value of the error integrated over time. This number was obtained by counting squares under the strip chart output in Appendix E. Tables 2 and 3 give the average error and Cooper-Harper rating for each simulation. The "better rating" entry for a simulation is the number of pilots that rated that particular flight control system the better of

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the two for the given task. (The two numbers do not necessarily add to six since some pilots gave the same rating to both aircraft.) The "lower error" similarly shows the number of pilots who had the lower total tracking error in that flight control system for the given task.

PILOT		ALTITUDE TRACKING		PITCH TRACKING	
		q Command	G Command	q Command	G Command
PH	Error[1]	580	485	.36	.5
	Rating	4.5	4	4	5
CS	Error	828	765	.98	.95
	Rating	6	10	3	5
CL	Error	330	783	.44	.51
	Rating	6	6	7.5	7
NB	Error	363	658	.38	.51
	Rating	4	3	2	3
JA	Error	668	445	.43	.48
	Rating	3	3	3	3
SW	Error	895	498	.73	.76
	Rating	9	4	3	6
Ave.	Error	611	606	.55	.62
	Rating	5.4	5.0	3.8	4.8
Better Rating		1	3	4	1
Lower Error		2	4	5	1

[1] ft-sec for Alt. tracking
rad-sec for pitch tracking

Table 2. Simulation Results for Power Approach (PA)

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PILOT		ALTITUDE TRACKING		PITCH TRACKING	
		q Command	G Command	q Command	G Command
PH	Error[1]	605	455	.29	.24
	Rating	6	6	5.5	3.5
CS	Error	1210	720	.4	.73
	Rating	9	5	2	5
CL	Error	720	1075	.69	.48
	Rating	6.5	7	6.5	7
NB	Error	765	640	.34	.39
	Rating	3	3	3	2
JA	Error	935	1315	.4	.41
	Rating	3	4	3	3
SW	Error	1450	995	.66	.57
	Rating	7	5	2	3
Ave.	Error	948	867	.46	.47
	Rating	5.8	5.0	3.7	3.9
Better Rating		2	2	3	2
Lower Error		2	4	3	3

[1] ft-sec for alt. tracking
rad-sec for pitch tracking

Table 3.   Simulation Results for Up-and-Away (UA)

V. Analysis and Discussion

The results as presented in tables 2 and 3 are somewhat disappointing as no clear-cut winner for either task jumps out of the data. There is quite a bit of variation from pilot to pilot in both tracking error and Cooper-Harper rating. The most reliable data is probably the "better rating" and "lower error" rows in the tables as these are direct comparisons between the two systems in each task.

From intuition, it would seem that the G command system should work better

for altitude tracking and the q command system should be better suited to pitch tracking. Recall that the pilot's stick input in the G command system represents a commanded normal acceleration while the input in the q command system is a commanded pitch rate.

The results for the PA simulations (table 2) appear to support this observation. In the altitude tracking task, four of the six pilots had lower errors with the G command systems. Three of the pilots gave the G command system a better rating, two rated both systems the same, and only one, CS, rated the q command system better. Note that CS gave the G command system a "10". This could possibly be due to his momentarily misinterpreting the display during the one minute data run. Most of the pilots did this during practice at least once. The "better rating/lower error" results for the pitch tracking task, on the other hand, clearly give the advantage to the q command system.

The UA results (table 3) appear to give no consensus whatsoever. The average ratings and errors are too close to call. The "better rating/lower error" results are split about evenly. At this point, it cannot be determined whether the use of flaps instead of strakes for the second control surface in the design scheme was a factor.

Some of the data scatter could have been caused by problems in the tracking task signal. The poor noise source used did drift and cause some variations in the frequency spectrum and amplitude of the signal from run to run as evidenced in the strip chart traces in Appendix E. Also, the commands seemed about right for pitch tracking, but changed too rapidly for a reasonable altitude tracking task.

Four factors related to the pilot subjects would have probably improved the results of the experiment by making the ratings more consistent. First, not enough time was allowed for the pilots to read and understand the rating

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procedure. In this experiment the subjects just hurriedly read the information in Appendix D. A greater effort should have been made to ensure that they understood the significance of the major divisions in the ratings. Second, the desired and/or minimum level of performance should have been specified to give the subjects a more common basis of comparison (for instance "commanded altitude must be maintained within plus or minus 50 feet"). This can be related to real aircraft constraints such as the requirement to discontinue an instrument approach upon full scale glide slope indicator deflection (altitude tracking) or gun site settling parameters (pitch tracking). Third, presenting two or three levels of difficulty of the same task would have promoted a finer or more accurate evaluation. For example, the altitude tracking task could have started with a simple step change in altitude, progressed to a slowly varying altitude command, and then ended with the rather rapidly changing task used in this experiment. Finally, too many simulation runs and configurations were rushed past the subjects at one sitting. This could have led to some confusion and perhaps even fatigue on their part. Doing only one flight condition (PA or UA) at a given sitting would have made for less confusion and possibly allowed a second pass through the four runs to promote consistency.

VI. Conclusions and Recommendations

Of the two optimal longitudinal flight control system design schemes (q command and G command) applied to a two control surface statically unstable aircraft, the results of this experiment indicate that for the power approach flight condition, the q command system is more desirable for pitch tracking maneuvers and the G command system is more desirable for altitude tracking maneuvers. For the up-and-away flight condition simulated, the results were inconclusive.

The recommendations for continuing work along the lines of this experiment are based on the discussion in the previous section and are listed below:

1. Improve the noise source used to drive the tracking tasks.

2. Use a "slower" task for altitude tracking.

3. Ensure the pilot subjects understand the Cooper-Harper rating scale.

4. Specify the levels of performance desired or required in the tasks.

5. Present several levels of difficulty in the tasks.

6. Only conduct simulations from one flight condition at a given sitting.

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APPENDIX A – Aircraft Open–Loop Linear Models

These longitudinal aircraft models are two degree-of-freedom (short period)
approximations of the linear X-29A rigid body equations of motion from reference
2. The power approach (PA) equations are linearized about a steady-state flight
condition of M = .2 at sea level. The up and away (UA) steady-state flight
condition is M = .59 at 5000 feet altitude.

$$\dot{\overline{x}} = [A]\overline{x} + [B]\overline{u}$$

$$\text{where:} \quad \overline{x} = [\,\alpha\,,\,q\,]^T$$

$$\overline{u} = [\,\delta_{\text{canard}}\,,\,\delta_{\text{strake or canard}}\,]^T$$

PA: $\quad A = \begin{bmatrix} -.3716 & .9878 \\ 2.143 & -.2085 \end{bmatrix}$, $\quad B = \begin{bmatrix} -.000869 & -.0003469 \\ .02162 & -.008776 \end{bmatrix}$

UA: $\quad A = \begin{bmatrix} -1.262 & .9391 \\ 19.12 & -.5675 \end{bmatrix}$, $\quad B = \begin{bmatrix} -.001294 & -.005529 \\ .1734 & -.09192 \end{bmatrix}$

To find Q such that $\widetilde{x}^T Q \widetilde{x} = (\delta_p - n_z)^2$, n_z must be approximated as a combination of the states. The approximation used is:

$$n_z = \frac{U_1 \; q - U_1 \dot{\alpha}}{g} = \frac{U_1 \; q - U_1 \; (a_{11}\alpha + a_{12} \; q)}{g}$$

$$\equiv [d_1 \; , \; d_2, 0] \widetilde{x}$$

where: U_1 is the trim velocity

g is acceleration of gravity

a is the appropriate component of the A matrix from Appendix A

d is the appropriate constant defined above

$\widetilde{x}$ is $[\alpha \; , \; q, \; \delta_p]^T$

Inspection will show that if the above approximation is used:

$$(\delta_p - n_z)^2 = \widetilde{x}^T \begin{bmatrix} 0 & d_1 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ d_1 & d_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \widetilde{x}$$

The Q matrix is the product of the three inside matrices.

$$Q = \begin{bmatrix} d_1^2 & d_1 d_2 & -d_1 \\ d_1 d_2 & d_2^2 & -d_2 \\ -d_1 & -d_2 & 1 \end{bmatrix}$$

For the power approach configuration:

$$Q = \begin{bmatrix} 6.6 & .217 & -2.57 \\ .217 & .00714 & -.0845 \\ -2.57 & -.0845 & 1 \end{bmatrix}$$

For the up-and-away flight condition:

$$Q = \begin{bmatrix} 643 & 5.55 & -25.4 \\ 5.55 & .048 & -.219 \\ -25.4 & -.219 & 1 \end{bmatrix}$$

APPENDIX C – Optimization Results

ρ	Poles		ζ	ω_n	
Power Approach, q Command					
100	−1.20,	−1.75	1.01	1.45	
10	−1.17,	−1.75	1.02	1.43	
1	−1.17,	−1.75	1.02	1.43	
.1	−1.16,	−1.75	1.02	1.43	
.01	−1.14,	−1.80	1.03	1.43	*
.001	−.974,	−2.12	1.08	1.44	
Power Approach, G Command					
100	−1.13,	−1.75	1.02	1.40	
10	−1.17,	−1.75	1.02	1.43	
1	−1.17,	−1.75	1.02	1.43	
.1	−1.18,	−1.74	1.02	1.43	
.01	−1.40,	−1.57	1.00	1.48	*
.001	−1.66 ± j .729		.915	1.81	
Up-and-Away, q Command					
100	−3.57,	−5.28	1.02	4.34	
10	−3.44,	−5.28	1.02	4.26	
1	−3.44,	−5.29	1.02	4.27	
.1	−3.39,	−5.38	1.03	4.27	
.01	−3.03,	−6.12	1.06	4.31	*
.001	−2.09,	−10.2	1.33	4.62	
Up-and-Away, G Command					
100	−3.45,	−5.28	1.02	4.27	
10	−3.49,	−5.25	1.02	4.28	
1	−3.93,	−4.94	1.01	4.40	
.1	−4.89 ± j 1.98		.927	5.27	*
.01	−6.71 ± j 4.89		.808	8.3	
.001	−11.04 ± j 9.44		.760	14.52	

*   Selected Configuration

Gain Matrices Selected and Transfer Functions:

PA, q Command: $F_1 = \begin{bmatrix} 144.3 & 104.5 \\ -32.74 & -23.68 \end{bmatrix}$ $\quad F_2 = \begin{bmatrix} .3393 \\ -.06819 \end{bmatrix}$

$$\alpha/\delta_s(s) = \frac{-.00028\ (s - 27.68)}{s^2 + 2.933s + 2.042}$$

$$q/\delta_s(s) = \frac{.00794\ (s + .301)}{s^2 + 2.933s + 2.042}$$

PA, G Command: $F_1 = \begin{bmatrix} 150.3 & 106.1 \\ -34.29 & -24.12 \end{bmatrix}$ $\quad F_2 = \begin{bmatrix} .07581 \\ -.03266 \end{bmatrix}$

$$\alpha/\delta_s(s) = \frac{-.00006\ (s - 31.83)}{s^2 + 2.967s + 2.195}$$

$$q/\delta_s(s) = \frac{.00193\ (s + .295)}{s^2 + 2.967s + 2.195}$$

UA, q Command: $F_1 = \begin{bmatrix} 134.0 & 35.45 \\ -45.78 & -11.87 \end{bmatrix}$ $\quad F_2 = \begin{bmatrix} 1.883 \\ -.4745 \end{bmatrix}$

$$\alpha/\delta_s(s) = \frac{.00019\ (s + 1896)}{s^2 + 9.147s + 18.54}$$

$$q/\delta_s(s) = \frac{.3702\ (s + 1.338)}{s^2 + 9.147s + 18.54}$$

UA, G Command: $F_1 = \begin{bmatrix} 172.9 & 38.03 \\ -63.81 & -13.31 \end{bmatrix}$ $\quad F_2 = \begin{bmatrix} .3619 \\ -.2236 \end{bmatrix}$

$$\alpha/\delta_s(s) = \frac{.00076\ (s + 114.2)}{s^2 + 9.777s + 27.79}$$

$$q/\delta_s(s) = \frac{.08332\ (s + 1.237)}{s^2 + 9.777s + 27.29}$$

Cooper-Harper Pilot Opinion Rating Scale

AIRCRAFT CHARACTERISTICS	DEMANDS ON THE PILOT IN SELECTED TASK OR REQUIRED OPERATION	PILOT RATING	PILOT QUALITIES LEVELS
Excellent Highly desirable	Pilot compensation not a factor for desired performance	1	1
Good Negligible deficiencies	Pilot compensation not a factor for desired performance	2	
Fair-some mildly unpleasant deficiencies	Minimal pilot compensation required for desired performance	3	
Minor but annoying deficiencies	Desired performance requires moderate pilot compensation	4	2
Moderately objectionable deficiencies	Adequate performance requires considerable pilot compensation	5	
Very objectionable but tolerable deficiencies	Adequate performance requires extensive pilot compensation	6	
Major deficiencies	Adequate performance not attainable with maximum tolerable pilot compensation. Controllability not in question.	7	3
Major deficiencies	Considerable pilot compensation is required for control	8	
Major deficiencies	Intense pilot compensation is required to retain control	9	
Major deficiencies	Control will be lost during some portion of required operation	10	

ADEQUACY FOR SELECTED TASK OR REQUIRED OPERATION

Is it satisfactory without improvement?

No → Deficiencies warrant improvement

Yes

Is adequate performance obtainable with a tolerable pilot workload

No → Deficiencies require improvement

Yes

Is it controllable

No → Improvement mandatory
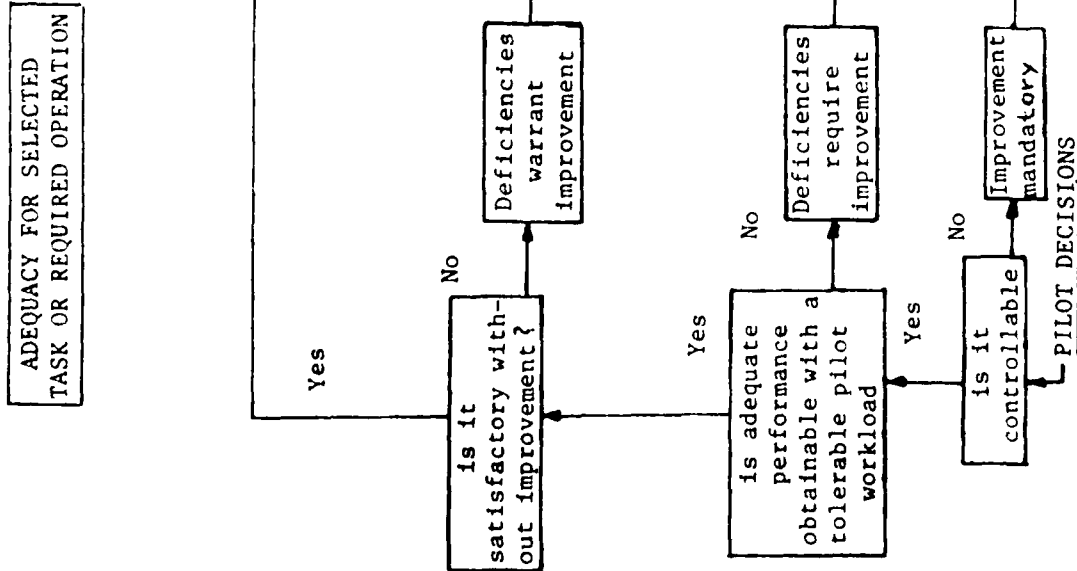
PILOT DECISIONS

Run #	Flight Condition	Stick Command	Flight Control System	Tracking Task
1	PA	q	A	altitude
2	PA	G	B	altitude
3	PA	G	B	pitch
4	PA	q	A	pitch
5	UA	q	A	altitude
6	UA	G	B	altitude
7	UA	G	B	pitch
3	UA	q	A	pitch

Strip Chart Key

330 | 783
1 | 2

.51 | .44
3 | 4

720 | 1075 | .48 | .69
5 | 6 | 7 | 8
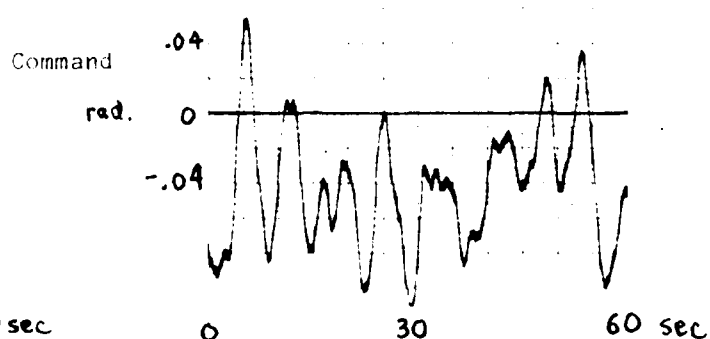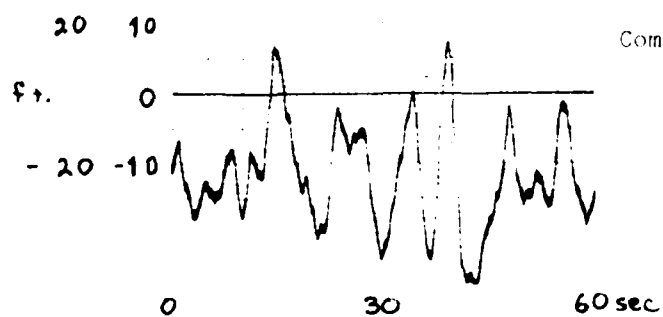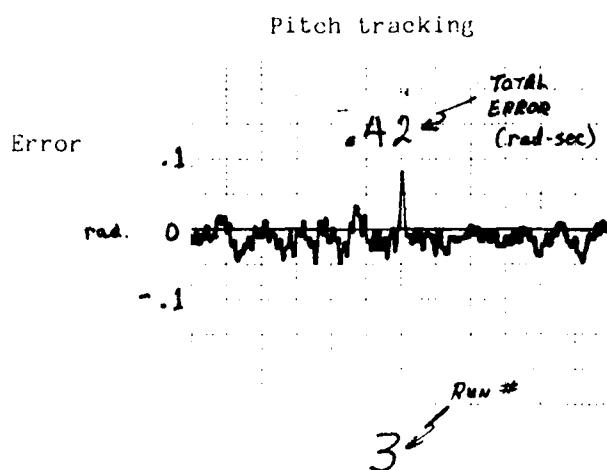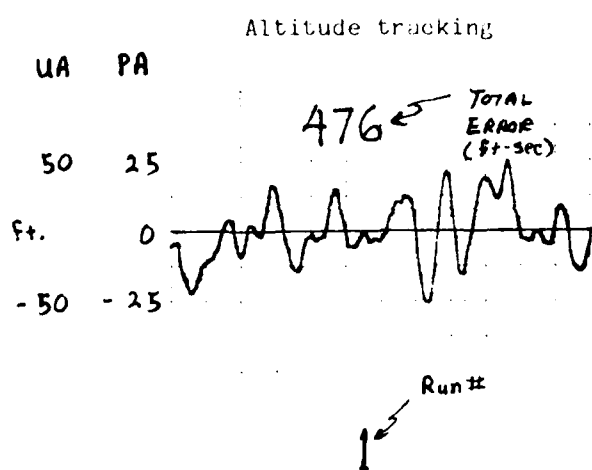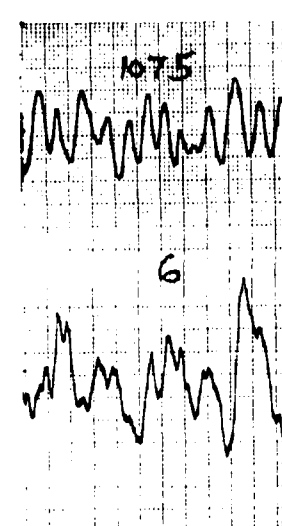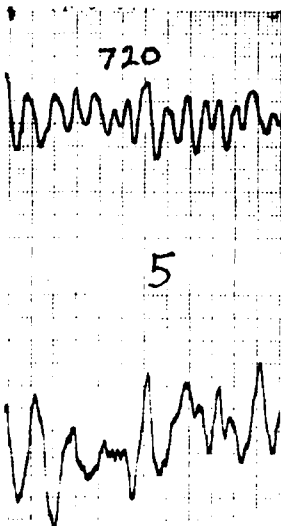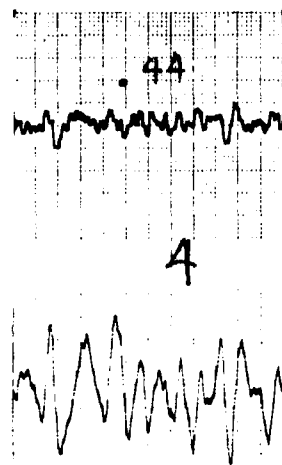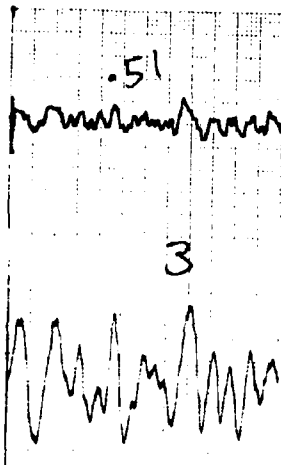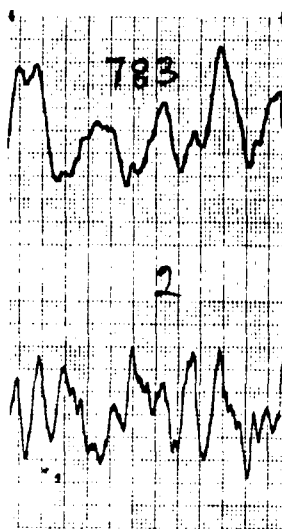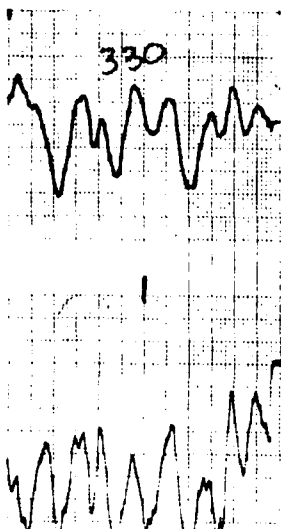
PILOT ID C.

Air Force pilot? Yes , AF Test Pilot

Experience: 2500 hrs total time
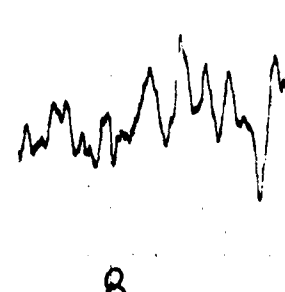
Type aircraft F-4   A-7
               C-2   A-10
               F-105

TEST DATA

Flight Condition #1 (Power Approach)

Cooper-Harper Rating

FCS Config.	Altitude Tracking	Pitch Tracking	Comments
A	6 *	75	
B	6	7 *	

Flight Condition #2 (Up and Away)

A	65	6.	
B	7.0	7.5	

828

1

765

2

.95

3

.98

4

1210

5

720

6

.73

7

.4

8
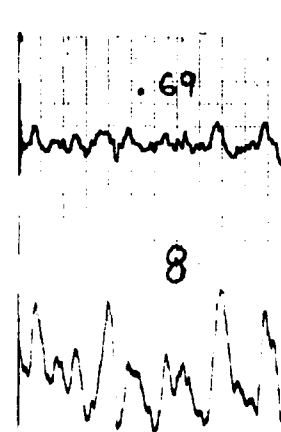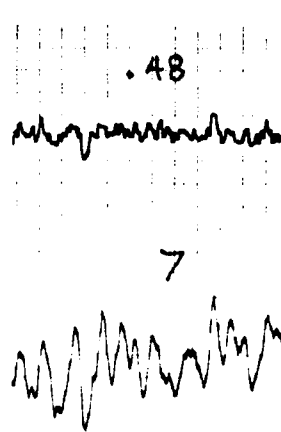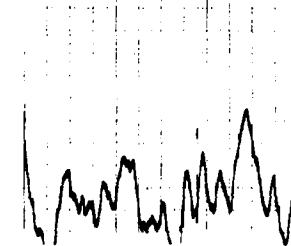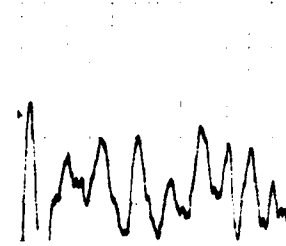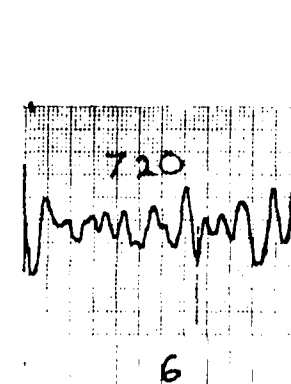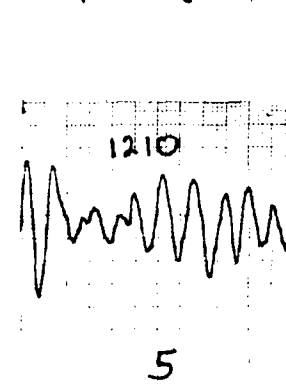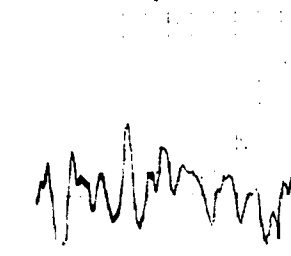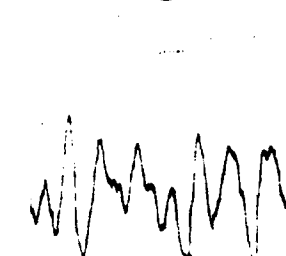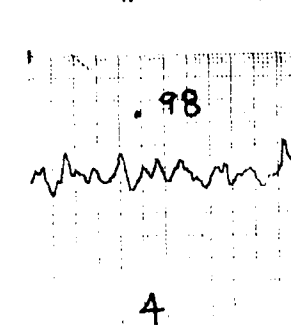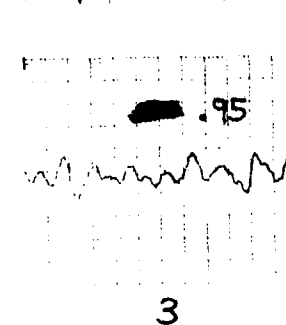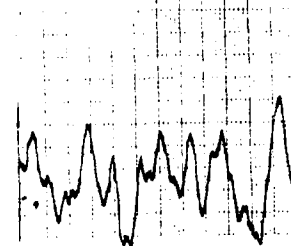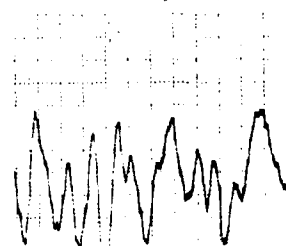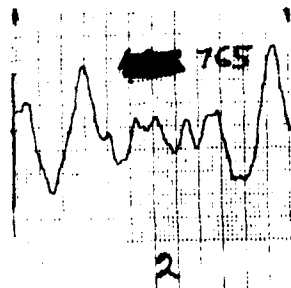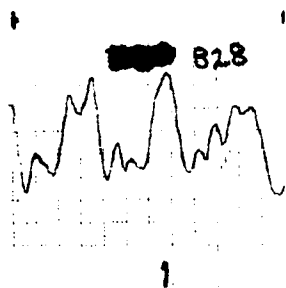
PILOT ID CS

Air Force pilot? YES

Experience: 1600 hrs total time

Type aircraft T-37

TEST DATA

Flight Condition #1 (Power Approach)

FCS Config.	Cooper-Harper Rating		Comments
	Altitude Tracking	Pitch Tracking	
A	6	3	
B	10	5	seemed far slow, tried no to articulate the "depth" command in end the number entire

Flight Condition #2 (Up and Away)

FCS Config.			Comments
A	9	2	the quicker was nice to pitch but seemed to make altitude harder. Tended to overcorrect
B	5	5	took very large stick or inputs when to make large errors, but error could easily when required corrections were smaller.

1 — 363

2 — 658

3 — .51

4 — .38
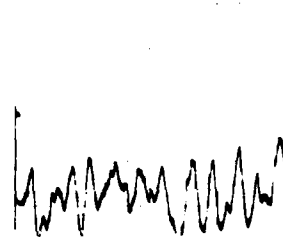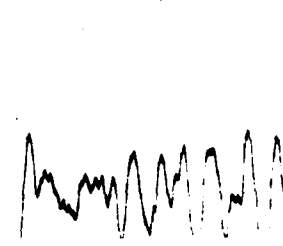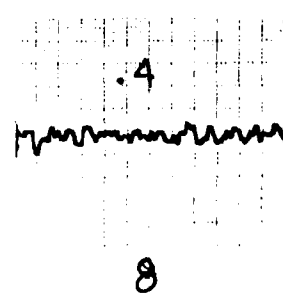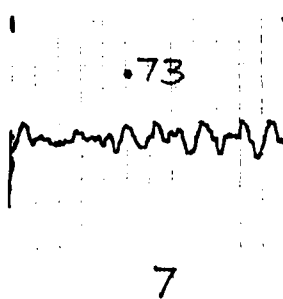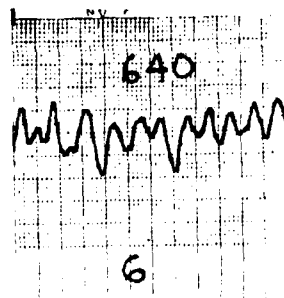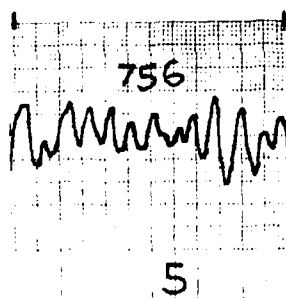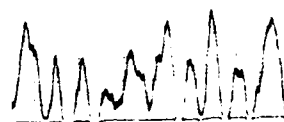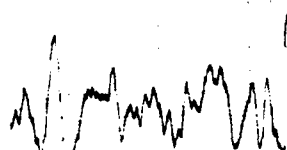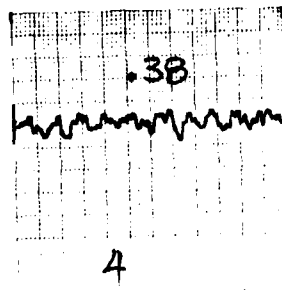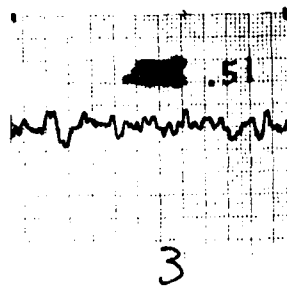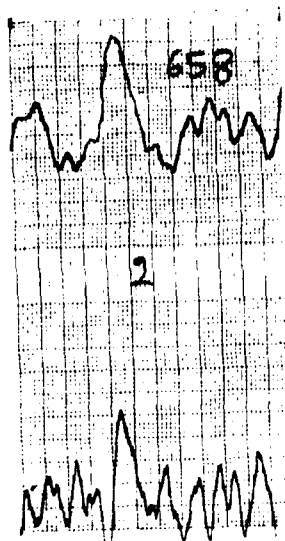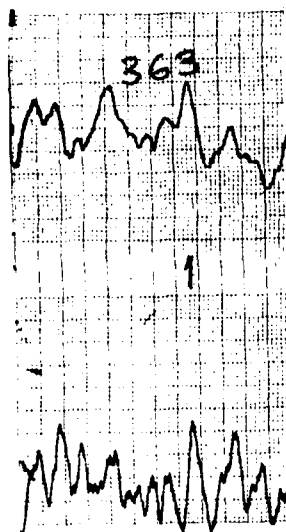
PILOT ID _H B_

Air Force pilot? _YES_
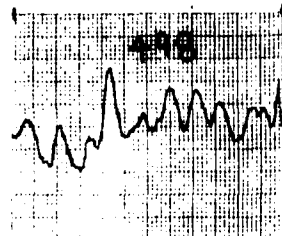
Experience: _2500_ hrs total time

Type aircraft _T-38_

_F-41_

TEST DATA
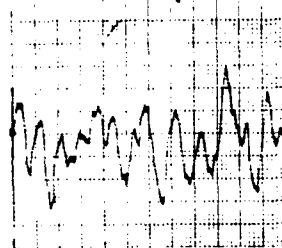
Flight Condition #1 (Power Approach)

FCS Config.	Cooper-Harper Rating Altitude Tracking	Pitch Tracking	Comments
A	4	2	Better feeling for pitch, but less control in Alt tracking
B	3	3	Altitude tracking easier than A but worse in pitch

Flight Condition #2 (Up and Away)

FCS Config.	Altitude Tracking	Pitch Tracking	Comments
A	3	3	"Pitch very crisp, no pitch overshoot
B	3	2	

5 — 756

6 — 640

7 — .39

8 — .34

668

445

1

2

.48

.43

3

4

PILOT ID   1?

    Air Force pilot?   /

    Experience: ??? hrs total time

           Type aircraft   E

TEST DATA

Flight Condition #1   (Power Approach)
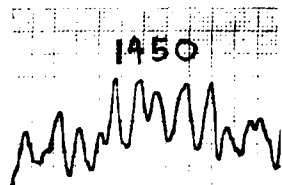
FCS Config.	Cooper-Harper Rating		Comments
	Altitude Tracking	Pitch Tracking	
A	3	3	
B	3	3	

Flight Condition #2   (Up and Away)

A	3	3	
B	4	3	very

935

1315

.41

5

6

7

8

.4

58 0 — 1

405 — 2

.5 — 3

.36 — 4

605 — 5

455 — 6

.24 — 7

.29 — 8

PILOT ID _PH_

Air F. r pilot? _Yes_

Experience: _1800_ hrs total time

Type aircraft T-37
F 4
F-111

TEST DATA

Flight Condition #1 (Power Approach)

FCS Config.	Cooper-Harper Rating		Comments
	Altitude Tracking	Pitch Tracking	
A	4	4	_(illegible)_
B	_(illegible)_	5	_(illegible)_

Flight Condition #2 (Up and Away)

A	_(illegible)_	< 1½	_(illegible)_
B	4	5½	_(illegible)_

895

1

498

2

.76

3

.73

4

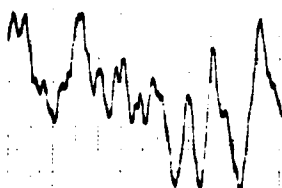

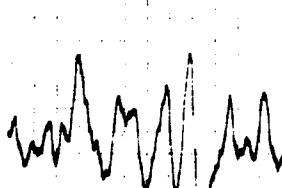1450

5

995

6

.57

7

.66

8

PILOT ID ___SW___

Air Force pilot? _Yes_

Experience: __1920__ hrs total time

Type aircraft KC-135
T-A
T-37
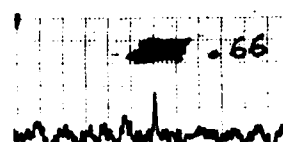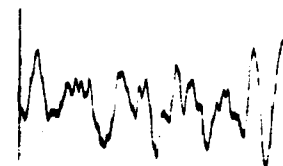
TEST DATA

Flight Condition #1   (Power Approach)

FCS Config.	Cooper-Harper Rating		Comment:
	Altitude Tracking	Pitch Tracking	
A	1	3	
B		6	

Flight Condition #2   (Up and Away)

FCS Config.	Altitude Tracking	Pitch Tracking	Comment:
A	1	1	
B	5	3	

END

FILMED

10-85

DTIC