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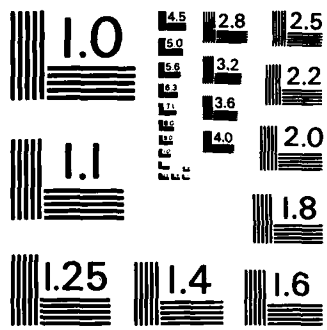

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Babinet's Principle for an Anisotropic Resistive Surface Using Different Approaches

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Extension of Babinet's principle in electromagnetic theory to an inhomogeneous anisotropic (with non-zero off-diagonal terms) resistive sheet is made. Different methods are employed for this purpose so that a better understanding can be gained. Some basic properties of electromagnetic waves, such as odd and even symmetry, are also reviewed for clarity of presentation.

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CONTENTS

BABINET'S PRINCIPLE FOR AN ANISOTROPIC RESISTIVE SURFACE USING DIFFERENT APPROACHES

INTRODUCTION

A rigorous vector formulation of Babinet's principle for electromagnetic waves is very useful in simplifying many microwave diffraction problems. Copson [1] formulated a rigorous statement of Babinet's principle for diffraction of electromagnetic waves by an aperture in a perfectly conducting screen and its complementary surface (a conducting disc). Recently, Elliott [2] presented a review of Copson's work in a manner easily understandable to engineers and scientists. In addition to providing a rigorous statement of Babinet's principle, this analysis displays some important symmetry (odd and even) properties of electromagnetic fields with respect to the thin diffracting screen. Introducing such concept of symmetrical (i.e., with even symmetry) and unsymmetrical (i.e., with odd symmetry) excitations of electromagnetic fields, Collin [3] rederived the Babinet's principle. In this derivation it is not necessary to know the explicit expressions of scattered fields in terms of surface integrals containing equivalent induced surface current (electric or magnetic). Such expressions can be found in references (1) and (2). In an analogous manner, advancing the properties of image fields, Jones [4] also presented the relationships associated with Babinet's principle. Although these various approaches are interrelated, they can provide better insight into understanding the principle involved and may help in devising different experiments for the same purpose. A comprehensive list of references on this subject can be found in [4].

Manuscript approved May 24, 1985.

The preceding discussion on the various methods of derivation of Babinet's principle is associated with diffraction or scattering of electromagnetic waves by a conducting screen with apertures and the corresponding complementary surface consisting of conducting disks having the same size and shape of respective apertures. However, over the years interest has also been shown in extending this Babinet's principle to absorbing and resistive surfaces [5,6,7]. This has important applications in reducing reflections of electromagnetic waves. Neugebaur [5] considered an absorbing surface, whereas Lang [6] studied a resistive surface. Lang took the complementary surfaces as a perfectly conducting screen with a resistive insert and a resistive screen with a perfectly conducting insert. In his attempt to extend Babinet's principle to resistive surfaces, Lang followed the method used by Jones [4], but the derivation has been criticized [8] for the assumptions made involving the normal components of the field. It appears that Baum and Singaraju [9] have also considered resistive sheets in an unpublished work. Applying the same idea [9] in mind Senior [7] extended Babinet's principle to diagonally anisotropic and inhomogeneous resistive sheets with the appropriate complementary problems. In this derivation Senior assumed the usual symmetry properties [1,2] of electromagnetic fields.

The present work also gives a derivation of Babinet's principle and the appropriate field relations for a resistive surface which is anisotropic and may be a function of position of the coordinates of the surface. Unlike the resistive sheet (or surface) considered by Senior, the anisotropic resistive sheet treated here has non-zero off-diagonal terms. A relation between this anisotropic (or dyadic) resistivity and that for its complementary surface is also derived. The method used parallels Senior's. In addition, we have extended Collin's technique to include the effect of such anisotropic resistive sheets on Babinet's principle. Since Jones' approach is equally valid, although Lang made an error as mentioned earlier, this procedure is also used to generalize the result for anisotropic and inhomogeneous resistive sheets. Even though these various methods are interrelated, they offer an opportunity to look into this problem from different perspectives and thus provide a better insight which may help in setting up experiments in different ways. Since all of these methods use some well known basic properties which are important in diffraction or scattering of electromagnetic waves, (such as odd and even symmetry), they are reviewed for the clarity of presentation.

SOME BASIC PROPERTIES OF THE ELECTROMAGNETIC FIELD

Consider the following time-harmonic Maxwell's equation in a source free homogeneous medium.

$$\nabla \times \vec{E} = i\omega\mu\vec{H} \tag{1a}$$

$$\nabla \times \vec{H} = -i\omega\epsilon\vec{E} \, . \tag{1b}$$

The assumed time dependance $\exp(-i\omega t)$ is suppressed for convenience. If $\vec{E}$ and $\vec{H}$ satisfy the Maxwell's equations (1a) and (1b), so also the set $\vec{E}_c$, $\vec{H}_c$ which are related to $\vec{E}$, $\vec{H}$ in the following manner.

$$\vec{E}_c = \mp\sqrt{\mu/\epsilon}\ \vec{H} = \mp Z\vec{H} \tag{2a}$$

$$\vec{H}_c = \pm\sqrt{\epsilon/\mu}\ \vec{E} = \pm Y\vec{E} \, . \tag{2b}$$

where $\sqrt{\mu/\epsilon} = Z = 1/Y$ is the intrinsic impedance of the homogeneous medium. In the derivation of Babinet's principle the concept of complementary problem and its associated field is essential. If the field $\vec{E}$, $\vec{H}$ refers to the original problem, then the field $\vec{E}_c$, $\vec{H}_c$ satisfying (2a) and

(2b) represents the corresponding complementary problem. The subscript c is then introduced to designate the complementary field. The sign convention in (2a) and (2b) depends on the problem at hand. This will be discussed in appropriate places. The relations (2a) and (2b) are also known as duality properties.

It follows from the source free Maxwell's equations that $\vec{E}$ and $\vec{H}$ satisfy the following divergence equations.

$$\nabla\cdot\vec{E} = 0 \text{ and } \nabla\cdot\vec{H} = 0 \quad . \tag{3}$$

In a rectangular or a general cylindrical coordinate system, if z is assumed to be the preferred direction of propagation, then it will be found convenient to express, $\vec{E}$, $\vec{H}$ and ∇ in the following manner.

$$\begin{aligned} \vec{E} &= \vec{E}_t + \hat{z}_0\, E_z \\ \vec{H} &= \vec{H}_t + \hat{z}_0\, H_z \\ \nabla &= \nabla_t + \hat{z}_0 \frac{\partial}{\partial z} \quad . \end{aligned} \tag{4}$$

where $\hat{z}_0$ is a unit vector in the z direction. The subscript t refers to the components transverse to z. E_z and H_z are the respective z components. Then the relations in (3) may be expressed as

$$\nabla_t\cdot\vec{E}_t + \frac{\partial}{\partial z} E_z = 0 \tag{5a}$$

$$\nabla_t\cdot\vec{H}_t + \frac{\partial}{\partial z} H_z = 0 \quad . \tag{5b}$$

Futhermore, it follows from Maxwell's equations that

$$i\omega\mu\vec{H}_t = -\hat{z}_0 \times \nabla_t E_z + \hat{z}_0 \times \frac{\partial}{\partial_z} \vec{E}_t \tag{6a}$$

$$-i\omega\epsilon\, \vec{E}_t = -\hat{z}_0 \times \nabla_t H_z + \hat{z}_0 \times \frac{\partial}{\partial_z} \vec{H}_t \,. \tag{6b}$$

It is evident from (5a) that if E_z is an even (odd) function of z, then $\vec{E}_t$ is an odd (even) function of z. In view of (6a), one then finds that when E_z is an even (odd) function of z, so also $\vec{H}_t$. Similarly, from (5b) and (6b) one can conclude that when H_z is an even (odd) function of z, so also $\vec{E}_t$, however, $\vec{H}_t$ is an odd (even) function of z. Therefore, when E_z and $\vec{H}_t$ are odd (even) function of z, the fields H_z and $\vec{E}_t$ are even (odd) function of z. Excitation which creates odd (even) $\vec{E}_t$ and H_z (together with even (odd) $\vec{H}_t$ and E_z) is also known as odd (even) or unsymmetrical (symmetrical) excitation. For example, the incident and the specularly reflected fields from a very thin perfectly conducting infinite plane (which may contain apertures) parallel to x-y plane at z = 0, are related to each other by odd symmetrical property with respect to z, i.e.,

$$\vec{E}_t^{\,i}(\vec{\rho},z) = -\vec{E}_t^{\,r}(\vec{\rho},-z),\ E_z^{i}(\vec{\rho},z) = E_z^{r}(\vec{\rho},-z) \tag{7a}$$

$$\vec{H}_t^{\,i}(\vec{\rho},z) = \vec{H}_t^{\,r}\vec{\rho},-z),\ H_z^{i}(\vec{\rho},z) = -H_z^{r}(\vec{\rho},-z) \tag{7b}$$

where the superscripts i and r refer to the incident and the specularly reflected fields respectively. The vector $\vec{\rho}$ represents coordinates transverse to z. On the otherhand, the diffracted or scattered fields in the two regions z>0 and z<0 (due to the presence of an aperture on the conducting screen) are related via even symmetry, i.e.,

$$\vec{E}_{2t}^{\,s}(\vec{\rho},z) = \vec{E}_{1t}^{\,s}(\vec{\rho},-z),\ E_{2z}^{s}(\vec{\rho},z) = -E_{1z}^{s}(\vec{\rho},-z) \tag{8a}$$

$$\vec{H}_{2t}^{\,s}(\vec{\rho},z) = -\vec{H}_{1t}^{\,s}(\vec{\rho},-z),\ H_{2z}^{s}(\vec{\rho},z) = H_{1z}^{s}(\vec{\rho},-z)\ . \tag{8b}$$

The superscript s refers to the scattered fields (which also include the diffracted fields). The subscripts 1 and 2 designate fields in the regions $z<0$ and $z>0$ respectively. More about odd and even symmetrical fields will be discussed later on.

BABINET'S PRINCIPLE USING UNSYMMETRICAL AND SYMMETRICAL EXCITATIONS

Let us consider an electrically thin resistive sheet occupying the entire plane $z = 0$ described by a rectangular coordinate (x,y,z) or any other cylindrical coordinate systems $(\vec{\rho},z)$. The normalized resistivity $\bar{\bar{R}}$ is a dyadic with non-zero elements in general. The elements of the resistivity tensor (or dyadic) may be functions of coordinates of the surface. The form of $\bar{\bar{R}}$ is given by

$$\bar{\bar{R}} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \hat{x}_0\hat{x}_0 R_{11} + \hat{x}_0\hat{y}_0 R_{12} + \hat{y}_0\hat{x}_0 R_{21} + \hat{y}_0\hat{y}_0 R_{22} \tag{9}$$

where $\hat{x}_0$, $\hat{y}_0$ and $\hat{z}_0$ are unit vectors in the x, y, and z directions, respectively.

The procedure adopted here will generalize the technique used by Collin [3] introducing unsymmetrical (odd) and symmetrical (even) excitations. We consider first unsymmetrical excitation.

Let $\vec{E}_1^i$ be an incident field originated from the region $z<0$, and $\vec{E}_1^r$ be the corresponding specularly reflected field. The region $z>0$ is also excited by another incident field $\vec{E}_2^i$, such that

$$\vec{E}_2^i(\vec{\rho},z) = \vec{E}_1^r(\vec{\rho},z) \text{ and } \vec{E}_2^r(\vec{\rho},z) = \vec{E}_1^i(\vec{\rho},z) \tag{10}$$

where $\vec{E}_2^r$ is the specularly reflected field created by the incident field $\vec{E}_2^i$ in the region $z>0$. Then the complete unsymmetrical excitation can be represented in the following manner.

$$\vec{E}_{10} = \vec{E}_1^{\,i} + \vec{E}_1^{\,r} \qquad (11a)$$

$$\vec{H}_{10} = \vec{H}_1^{\,i} + \vec{H}_1^{\,r} \qquad (11b)$$

$z<0$

and

$$\vec{E}_{20} = \vec{E}_2^{\,i} + \vec{E}_2^{\,r} \qquad (12a)$$

$$\vec{H}_{20} = \vec{H}_2^{\,i} + \vec{H}_2^{\,r} \,. \qquad (12b)$$

$z>0$

The subscript 0 indicates the fields belonging to odd (or unsymmetrical) excitation. The subscripts 1 and 2 designate regions $z<0$ and $z>0$, respectively. In addition to the relations (10), the fields in (11a) to (12b) have the following properties (dropping the transverse coordinates $\vec{\rho}$ for expediency).

$$\vec{H}_2^{\,i}(z) = \vec{H}_1^{\,r}(z),\ \vec{H}_2^{\,r}(z) = \vec{H}_1^{\,i}(z) \qquad (14a)$$

$$\vec{E}_{2t}^{\,i}(z) = -\vec{E}_{1t}^{\,i}(-z),\ E_{2z}^{i}(z) = E_{1z}^{i}(-z) \qquad (14b)$$

$$\vec{H}_{2t}^{\,i}(z) = \vec{H}_{1t}^{\,i}(-z),\ H_{2z}^{i}(z) = -H_{1z}^{i}(-z) \qquad (14c)$$

$$\vec{E}_{1t}^{\,r}(z) = -\vec{E}_{1t}^{\,i}(-z) = \vec{E}_{2t}^{\,i}(z) = -\vec{E}_{2t}^{\,r}(-z) \qquad (14d)$$

$$E_{1z}^{r}(z) = E_{1z}^{i}(-z) = E_{2z}^{i}(z) = E_{2z}^{r}(-z) \qquad (14e)$$

$$\vec{H}_{1t}^{\,r}(z) = \vec{H}_{1t}^{\,i}(-z),\ \vec{H}_{2t}^{\,i}(z) = \vec{H}_{2t}^{\,r}(-z) \qquad (14f)$$

$$H_{1z}^{r}(z) = -H_{1z}^{i}(-z) = H_{2z}^{i}(z) = -H_{2z}^{r}(-z) \,. \qquad (14g)$$

Since this odd excitation consists only of incident and specularly reflected fields in each side of the surface, the relations in (14) show that the surface behaves like a perfect conductor.

If we call the fields given by (11a) to (12b) as the unsymmetrical fields for the original problem, then the corresponding unsymmetrical fields associated with the complementary problem can be generated from the original fields by using the transformations (2a) and (2b) in the following manner. The lower signs in (2a) and (2b) should be taken for waves propagating in the positive z direction; whereas the upper signs belong to waves propagating in the negative z direction. Note that this choice is not arbitrary. It is necessary for the proper continuity of the complementary fileds at $z = 0$, i.e., the tangential components of $\vec{E}_{10c}$ and $\vec{E}_{20c}$ at $z = 0$ are continuous, where $\vec{E}_{10c}$ and $\vec{E}_{20c}$ are the total unsymmetrical (odd) complementary electric fields in regions 1 and 2 respectively. Thus, the unsymmetrical complementary fields can be expressed in the following manner:

$$\left.\begin{aligned} \vec{E}_{10c} &= -Z[\vec{H}_1^i - \vec{H}_1^r] && \text{(15a)} \\ \vec{H}_{10c} &= Y[\vec{E}_1^i - \vec{E}_1^r] && \text{(15b)} \end{aligned}\right\} \quad z<0$$

$$\left.\begin{aligned} \vec{E}_{20c} &= Z[\vec{H}_2^i - \vec{H}_2^r] && \text{(16a)} \\ \vec{H}_{20c} &= -Y[\vec{E}_2^i - \vec{E}_2^r] \ . && \text{(16b)} \end{aligned}\right\} \quad z>0$$

Superscript c refers to fields for the complementary problem.

We shall now construct the field components for symmetrical (even) excitation. In this case the total field in each region ($z<0$ and $z>0$) consists of the incident field, the corresponding specularly reflected field and the scattered field which obeys the even symmetry. Thus, the even symmetric fields are given by introduced previously in sections (3) and (4). For this so called magnetically conductive surface, the tangential magnetic field is continuous, but there is a discontinuity in the tangential electric field. This surface supports a magnetic current. The boundary conditions on the surface are given by the following expressions [7].

$$\hat{z}_0 \times \vec{H}_2(+0) = \hat{z}_0 \times \vec{H}_1(-0) = \hat{z}_0 \times \vec{H}_t(0) \tag{57a}$$

$$Y\overline{\overline{R}}^{*}\cdot[\vec{E}_2(+0) \times \hat{z}_0 - \vec{E}_1(-0) \times \hat{z}_0] = \vec{H}_t(+0) = \vec{H}_t(-0) \ . \tag{57b}$$

Note that in this case the situations $\overline{\overline{R}}^{*}=0$ and $\overline{\overline{R}}^{*} = \infty$ represent respectively a "perfect ferrite" with infinite permeability and the absence of this surface. Here also $\overline{\overline{R}}^{*}$ has non-zero off-diagonal terms.

For a combination resistive-conductive sheet, the fields on both sides of the sheet are obtained by combining (56a) and (57b) in the following way [7].

$$2Z\overline{\overline{R}}\cdot\{\hat{z}_0 \times [\vec{H}_{2t}(+o) - \vec{H}_{1t}(-0)]\} = \vec{E}_{2t}(+o) + \vec{E}_{1t}(-0) \tag{58a}$$

$$2Y\overline{\overline{R}}^{*}\cdot\{[\vec{E}_{2t}(+0) - \vec{E}_{1t}(-0)] \times \hat{z}_0\} = \vec{H}_{2t}(+0) + \vec{H}_{1t}(-0) \ . \tag{58b}$$

If we now require that the fields on each side of the sheet must satisfy impedance boundary conditions separately (see relations (60)), then the following relation between $\overline{\overline{R}}$ and $\overline{\overline{R}}^{*}$ can be established.

$$\overline{\overline{R}}^{*} = \overline{\overline{R}}^{T}\cdot[4 \det \overline{\overline{R}}^{*}] = \overline{\overline{R}}^{T}/[4\det \overline{\overline{R}}] \tag{59}$$

which is very similar to (40). The condition (59) then implies the following relations.

$$\vec{H}_{1t}(-0) = -2Y\overline{\overline{R}}^{*}\cdot[\vec{E}_{1t}(-0) \times \hat{z}_0] \tag{60i}$$

$$\vec{H}_{2t}(+0) = 2Y\overline{\overline{R}}^{*}\cdot[\vec{E}_{2t}(+0) \times \hat{z}_0] \tag{60ii}$$

with respect to the coordinate z, it can be shown [1,2] that for the computation of the scattered fields (including diffracted field) one needs only one type of source currents (i.e., either electric current $\vec{J}_s$ or the magnetic current $\vec{M}_s$). Then the scattered field can be derived [1,2,7] by using the Hertz potential. In this way the scattered fields are expressed by the surface integrals (44) and (45). More about this will be discussed later.

In this section we generalize Senior's results [7] by including the off-diagonal terms of the normalized dyadic (or tensor) resistivity $\bar{\bar{R}}$, the form of which is shown in (9). Here also we shall outline the method without much repetition of the results already given elsewhere in this paper.

For a resistive sheet (whether it is isotropic, anisotropic or inhomogeneous) the tangential electric field is continuous, whereas there is a discontinuity in the tangential magnetic field, indicating that the surface supports electric current. Therefore, the electromagnetic field satisfies the following resistive boundary conditions on the surface.

$$\hat{z}_0 \times \vec{E}_2(+0) = \hat{z}_0 \times \vec{E}_1(-0) = \hat{z}_0 \times \vec{E}_t(0) \tag{56a}$$

$$Z\bar{\bar{R}}\cdot[\hat{z}_0 \times \vec{H}_2(+0) - \hat{z}_0 \times \vec{H}_1(-0)] = \hat{z}_0 \times \vec{E}_t(0) \times \hat{z}_0 = \vec{E}_t(0) \ . \tag{56b}$$

As before, $(\vec{E}_1, \vec{H}_1)$ and $(\vec{E}_2, \vec{H}_2)$ are the fields in the region $z<0$ and $z>0$, respectively. The dyadic $\bar{\bar{R}}$ is the normalized resistivity as shown in (9). In particular, the situations $\bar{\bar{R}} = 0$ and $\bar{\bar{R}} = \infty$ represent prefectly conducting surface and the absence of any material surface respectively. The dual of this surface is a "magnetically conductive" surface with conductivity $Y\bar{\bar{R}}^*$ mhos per square meter. Note that $\bar{\bar{R}}^*$ is not the complex conjugate of $\bar{\bar{R}}$. This surface may be considered as the second kind of complementary surface. The other complementary surface, represented by its resistivity $Z\bar{\bar{R}}_c$ was

Furthermore, letting $z \to 0$ the conditions (51a) and (51b) can be used to derive the behavior of the various components of the field at the resistive surface. In this way it can be shown that the field satisfies the conditions already presented in Eqs. (32a) to (33b).

Noting that the expressions (52a) to (52d) provide the fields for the original scattering or diffractions problem, the fields for the complementary problem can be constructed in the following manner

$$\left.\begin{aligned} \vec{E}_{1c} &= Z[\vec{H}_I - \vec{H}_1] \qquad (55a) \\ \vec{H}_{1c} &= Y[\vec{E}_1 - \vec{E}_I^i] \qquad (55b) \end{aligned}\right\} \quad z<0$$

where $\vec{E}_I^i = \vec{E}_1^r$, and $\vec{H}_I^i = \vec{H}_1^r$

$$\left.\begin{aligned} \vec{E}_{2c} &= Z[\vec{H}_2 - \vec{H}_1^i] \qquad (55c) \\ \vec{H}_{2c} &= Y[\vec{E}_1^i - \vec{E}_2]\ . \qquad (55d) \end{aligned}\right\} \quad z>0$$

It can easily be shown that these equations are equivalent to those given by (28a) to (29b). The resistive boundary condition and the corresponding complementary boundary condition are the same as given by (34) and (37) respectively. Similarly, other properties and relations of the various fields can be found in the previous section.

BABINET'S PRINCIPLE USING SCATTERING THEORY

It has already been mentioned that the scattered field is created by the induced surface currents on the resistive infinite screen. These surface currents are related to the total tangential fields on the surface. By using the odd and even symmetry of the various components of the scattered fields

$$\vec{E}(Z) + \vec{E}_I(z) = \vec{E}_1^{\,i}(z) + \vec{E}_I^{\,i}(z) \tag{51a}$$

$$\vec{H}(z) + \vec{H}_I(z) = \vec{H}_1^{\,i}(z) + \vec{H}_I^{\,i}(z) \ . \tag{51b}$$

Let us now write

$$\left.\begin{aligned} &\vec{E}_1(z) = \vec{E}_1^{\,i}(z) + \vec{E}_1^{\,r}(z) + \vec{E}_1^{\,s}(z) \qquad && (52a) \\ &\vec{H}_1(z) = \vec{H}_1^{\,i}(z) + \vec{H}_1^{\,r}(z) + \vec{H}_1^{\,s}(z) \qquad && (52b) \end{aligned}\right\} \quad z<0$$

$$\left.\begin{aligned} &\vec{E}_2(z) = \vec{E}_2^{\,s}(z) \qquad && (52c) \\ &\vec{H}_2(z) = \vec{H}_2^{\,s}(z) \qquad && (52d) \end{aligned}\right\} \quad z>0$$

$$\left.\begin{aligned} &\vec{E}_{I1}(z) = \vec{E}_{I1}^{\,s}(z) \qquad && (53a) \\ &\vec{H}_{I1}(z) = \vec{H}_{I1}^{\,s}(z) \qquad && (53b) \end{aligned}\right\} \quad z<0$$

$$\left.\begin{aligned} &\vec{E}_{I2}(z) = \vec{E}_2^{\,i}(z) + \vec{E}_2^{\,r}(z) + \vec{E}_{I2}^{\,s}(z) \qquad && (53c) \\ &\vec{H}_{I2}(z) = \vec{H}_2^{\,i}(z) + \vec{H}_2^{\,r}(z) + \vec{H}_{I2}^{\,s}(z) \ . \qquad && (53d) \end{aligned}\right\} \quad z>0$$

When both the incident field $(\vec{E}_1^{\,i}, \vec{H}_1^{\,i})$ and its image $(\vec{E}_I^{\,i}, \vec{H}_I^{\,i})$ are present simultaneously, then the conditions in (51a) and (51b) imply that the resultant scattered field vanishes everywhere. In otherwords one finds

$$\left.\begin{aligned} &\vec{E}_1^{\,s}(z) + \vec{E}_{I1}^{\,s}(z) = 0 \qquad && (54a) \\ &\vec{H}_1^{\,s}(z) + \vec{H}_{I1}^{\,s}(z) = 0 \qquad && (54b) \end{aligned}\right\} \quad z<0$$

$$\left.\begin{aligned} &\vec{E}_2^{\,s}(z) + \vec{E}_{I2}^{\,s}(z) = 0 \qquad && (54c) \\ &\vec{H}_2^{\,s}(z) + \vec{H}_{I2}^{\,s}(z) = 0 \ . \qquad && (54d) \end{aligned}\right\} \quad z>0$$

Therefore, when these fields are present simultaneously, they constitute the unsymmetrical excitation represented by (11a) to (12b).

Consider now the total field $(\vec{E}_I, \vec{H}_I)$ created by the incident image field $(\vec{E}_I^i, \vec{H}_I^i)$. The field $(\vec{E}_I, \vec{H}_I)$ may be expressed as

$$\vec{E}_I = \begin{cases} \vec{E}_{I1}(z), & z<0 \quad (49a) \\ \vec{E}_{I2}(z), & z>0 \quad (49b) \end{cases}$$

$$\vec{H}_I = \begin{cases} \vec{H}_{I1}(z), & z<0 \quad (49c) \\ \vec{H}_{I2}(z), & z>0 \; . \quad (49d) \end{cases}$$

Since the field $(\vec{E}_I, \vec{H}_I)$ is the image of $(\vec{E}, \vec{H})$ given by (47a) to (47d), the combination represent unsymmetrical solutions of Maxwell's equation, i.e.,

$$\vec{E}_I(z) = \vec{E}_{It}(z) + \hat{z}_0 E_{Iz}(z) = -\vec{E}_t(-z) + \hat{z}_0 E_z(-z) \quad (50a)$$

$$\vec{H}_I(z) = \vec{H}_{It}(z) + \hat{z}_0 H_{Iz}(z) = \vec{H}_t(-z) - \hat{z}_0 H_z(-z) \; . \quad (50b)$$

If the incident field $(\vec{E}_1^i, \vec{H}_1^i)$ and its image $(\vec{E}_I^i, \vec{H}_I^i)$ exist simultaneously, the gross total field is obtained by adding the two separate total fields $(\vec{E}, \vec{H})$ and $(\vec{E}_I, \vec{H}_I)$. Since the combined field due to the original incident field $(\vec{E}_1^i, \vec{H}_1^i)$ and its image $(\vec{E}_I^i, \vec{H}_I^i)$ satisfies the boundary condition on the resistive sheet (and the edge condition if there is any), the resistive sheet could be removed. This implies that the resultant field created by the original incident field $(\vec{E}_1^i, \vec{H}_1^i)$ and its image $(\vec{E}_I^i, \vec{H}_I^i)$ is exactly the same as if the screen were not there. This statement is mathematically equivalent to the following relations.

BABINET'S PRINCIPLE USING IMAGE THEORY

In principle, the technique to be used here is similar to that presented by Jones [4]. The screen is an anisotropic resistive sheet as descirbed in the previous Section 3. Since many of the results which can be derived by using this method, are the same given in the preceding section, they will not be repeated. Therefore, we shall simply outline the procedure and whenever possible refer to the results obtained previously.

Let $(\vec{E}_1^i, \vec{H}_1^i)$ be the field incident from the region z<0. This incident field then creates a total field $(\vec{E}, \vec{H})$ which may be expressed as

$$\vec{E} = \begin{cases} \vec{E}_1(z), & z<0 \quad (47a) \\ \vec{E}_2(z), & z>0 \quad (47b) \end{cases}$$

$$\vec{H} = \begin{cases} \vec{H}_1(z), & z<0 \quad (47c) \\ \vec{H}_2(z), & z>0 \;. \quad (47c) \end{cases}$$

Assume further that the field $(\vec{E}_I^i, \vec{H}_I^i)$, the image (i.e., mirror image) of the field $(\vec{E}_1^i, \vec{H}_1^i)$, is also present. The subscript I indicates image. Since the field $(\vec{E}_1^i, \vec{H}_1^i)$ is incident from z<0, its image field $(\vec{E}_I^i, \vec{H}_I^i)$ may be regarded as the field incident from z>0. Let us then write

$$\vec{E}_I^i = \vec{E}_2^i, \; \vec{H}_I^i = \vec{H}_2^i \tag{48}$$

which are fields incident from z>0.

Let $(\vec{E}_1^r, \vec{H}_1^r)$ be the specularly reflected field in the region z<0, created by the incident field $(\vec{E}_1^i, \vec{H}_1^i)$. Similarly, the field $(\vec{E}_2^r, \vec{H}_2^r)$ is the specularly reflected field caused by the image incident field $(\vec{E}_I^i = \vec{E}_2^i, \vec{H}_I^i = \vec{H}_2^i)$. Then these fields satisfy the relations (10) and (14a) to (14g).

due to the incident fields, are unknown. Formal expressions of these scattered fields in terms of the induced surface current, which is also unknown, can be derived in the following manner.

$$\text{Let } \vec{M}_s(\vec{\rho}) = \vec{E}_t(\vec{\rho},0) \times z_0 \ . \tag{43}$$

$\vec{M}_s(\vec{\rho})$ is equivalent to a surface magnetic current and $\vec{E}_t(\vec{\rho},0)$ is the same tangential electric field at $z = 0$ defined by (32a). Then it can be shown [1,2,7] that

$$\vec{E}_1^s(\vec{\rho},z) = -2\iint \vec{M}_s(\vec{\rho}') \times \nabla g(\vec{r},\vec{r}')dx'dy' = -\vec{E}_2^s(\vec{\rho},z) \tag{44}$$

$$\vec{H}_1^s(\vec{\rho},z) = -2ikY\iint [\vec{M}_s g + (\vec{M}_s \cdot \nabla)\nabla g/k^2]dx'dy' = -\vec{H}_2^s(\vec{\rho},z) \ . \tag{45}$$

Intergration in (44) and (45) is over the entire surface at $z = 0$.

$$g(\vec{r},\vec{r}') = \frac{e^{ik|\vec{r}-\vec{r}'|}}{4\pi\,|\vec{r}-\vec{r}'|}, \quad r = \sqrt{x^2+y^2+z^2}, \quad r' = \sqrt{x'^2+y'^2}, \quad k = \omega\sqrt{\mu\varepsilon} \tag{46}$$

where (x,y,z) refers to observation point and (x',y') denotes a point on the resistive surface at $z = 0$.

Note that the integral representations (44) and (45) of the scattered fields satisfy the symmetry conditions shown in (8a) and (8b). It may be pointed out that the mathematical statement of Babinet's principle given by (28a) to (31b) differs from that provided by Collin [3]. The reason may lie in the manner in which the symmetrical excitation is derived by Collin.

The symmetrical excitation represented by Eqs. (17a) to (18b) displays explicitly the presence of the specularly reflected fields, $\vec{E}_1^r$, $\vec{E}_2^r$, $\vec{H}_1^r$, and $\vec{H}_2^r$. However, the corresponding expressions given by Collin do not show them. It appears that Collin absorbed these quantities in his definition of various scattered fields.

where $\vec{E}_{ct}(0)$ is defined by (36a) and $\bar{\bar{R}}_c$ is given by

$$\bar{\bar{R}}_c = \begin{bmatrix} R_{11c} & R_{12c} \\ R_{21c} & R_{22c} \end{bmatrix} = \hat{x}_0\hat{x}_0 R_{11c} + \hat{x}_0\hat{y}_0 R_{12c} + \hat{y}_0\hat{x}_0 R_{21c} + \hat{y}_0\hat{y}_0 R_{22c} \quad . \tag{38}$$

The boundary condition (37) may also be re-expressed as

$$\begin{aligned} \vec{H}_{2ct}(+0) - \vec{H}_{1ct}(-0) &= -2Y\vec{E}_t(0) = -2Y\vec{E}^s_{1t}(-0) \\ &= Y[(\bar{\bar{R}}_c)^{-1} \cdot \vec{E}_{ct}(0)] \times \hat{z}_0 \quad . \end{aligned} \tag{39}$$

Combining now (35) and (39), the following relation between $\bar{\bar{R}}$ and $\bar{\bar{R}}_c$ can be established.

$$\bar{\bar{R}}_c = \bar{\bar{R}}^T/[4 \det \bar{\bar{R}}] = \bar{\bar{R}}^T \cdot [4 \det \bar{\bar{R}}_c] \tag{40}$$

where $\bar{\bar{R}}^T$ is the transpose of $\bar{\bar{R}}$ and

$$\det \bar{\bar{R}} = R_{11}R_{22} - R_{12}R_{21} = \text{determinant of } \bar{\bar{R}} \tag{41a}$$

$$\det \bar{\bar{R}}_c = R_{11c}R_{22c} - R_{12c}R_{21c} = \text{determinant of } \bar{\bar{R}}_c \quad . \tag{41b}$$

The relation (40) may also be expressed explicitly in the following form.

$$R_{11}/R_{11c} = R_{21}/R_{12c} = R_{22}/R_{22c} = R_{12}/R_{21c} = 4(\det \bar{\bar{R}}) \tag{42a}$$

$$4(\det\bar{\bar{R}}) = 1/[4\det \bar{\bar{R}}_c] \quad . \tag{42b}$$

In equations (24a) to (24b) the incident and the corresponding specularly reflected fields are assumed to be known. However, the scattered fields, which are the fields radiated by the currents induced in the surface where $\bar{\bar{R}}$ is given by (9) and $\vec{E}_t(0)$ is defined by (32a). It will be found convenient to re-express (34) in the following way using (33a).

$$\left.\begin{aligned} \vec{H}_{2t}(+0) - \vec{H}_{1t}(-0) &= Y[(\bar{\bar{R}})^{-1}\cdot\vec{E}_t(0)] \times \hat{z}_0 \\ &= 2[\vec{H}^{i}_{1t}(0) - \vec{H}_{1t}(-0)] \ . \end{aligned}\right\} \qquad (35)$$

From Eqs. (26a) to (27b) and the relations (32a) to (32e), behavior of the complementary fields at the surface z = 0 can be expressed as follows.

$$\left.\begin{aligned} \vec{E}_{1ct}(-0) &= Z[\vec{H}_{2t}(+0) - \vec{H}^{i}_{1t}(0)] = \vec{E}_{2ct}(+0) = \vec{E}_{ct}(0) \\ &= \frac{Z}{2}[\vec{H}_{2t}(+0) - \vec{H}_{1t}(-0)] \end{aligned}\right\} \qquad (36a)$$

$$E_{1cz}(-0) = -Z[H^{i}_{1z}(0) + H_{1z}(-0)] \qquad (36b)$$

$$E_{2cz}(+0) = Z[H_{2z}(+0) - H^{i}_{1z}(0)] \qquad (36c)$$

$$\vec{H}_{1ct}(-0) = Y[\vec{E}_{1t}(-0) + \vec{E}^{i}_{1t}(0)] \qquad (36d)$$

$$H_{1cz}(-0) = Y[E_{1z}(-0) - E^{i}_{1z}(0)] \qquad (36e)$$

$$\vec{H}_{2ct}(+0) = Y[\vec{E}^{i}_{1t}(0) - \vec{E}_{2t}(+0)] \qquad (36f)$$

$$H_{2cz}(+0) = Y[E^{i}_{1z}(0) - E_{2z}(+0)] \ . \qquad (36g)$$

The above relations show that $\vec{E}_{ct}$ and H_{cz} are continuous across the surface at z = 0, whereas $\vec{H}_{ct}$ and E_{cz} are discontinuous.

Let $Z\bar{\bar{R}}_c$ be the resistivity of the complementary surface which may be viewed as an anisotropic conductive sheet. Then the complementary fields satisfy the following boundary condition.

$$Z\bar{\bar{R}}_c\cdot[\hat{z}_0 \times \{\vec{H}_{2ct}(+0) - \vec{H}_{1ct}(-0)\}] = \vec{E}_{ct}(0) \qquad (37)$$

Alternatively, Babinet's principle is sometimes expressed as

$$\left.\begin{aligned} \vec{E}_1^{s} &= Z\vec{H}_{1c}^{s} = Z\vec{H}_{1c} - \vec{E}_1^{i} && (30a) \\ \vec{H}_1^{s} &= -Y\vec{E}_{1c}^{s} = -Y\vec{E}_{1c} - \vec{H}_1^{i} && (30b) \end{aligned}\right\} \quad z<0$$

$$\left.\begin{aligned} \vec{E}_2^{s} &= \vec{E}_2 = -Z\vec{H}_{2c}^{s} = -Z\vec{H}_{2c} + \vec{E}_1^{i} && (31a) \\ \vec{H}_2^{s} &= \vec{H}_2 = Y\vec{E}_{2c}^{s} = Y\vec{E}_{2c} + \vec{H}_1^{i} \ . && (31b) \end{aligned}\right\} \quad z<0$$

Using the symmetry (i.e., even) property of the tangential scattered electric fields, the following behavior of the fields given by (24a) to (25b) at the surface $z = 0$ can be established.

$$\vec{E}_{1t}(z = -0) = \vec{E}_{1t}^{s}(-0) = \vec{E}_{2t}^{s}(+0) = \vec{E}_{2t}(+0) = \vec{E}_t(0) \tag{32a}$$

$$E_{1z}(z = -0) = 2E_{1z}^{i}(0) + E_{1z}^{s}(-0) \tag{32b}$$

$$\vec{H}_{1t}(z = -0) = 2\vec{H}_{1t}^{i}(0) + \vec{H}_{1t}^{s}(-0) \tag{32c}$$

$$H_{1z}(z = -0) = H_{1z}^{s}(0) = H_{2z}^{s}(+0) = H_{2z}(+0) \tag{32d}$$

$$\vec{H}_{2t}(z = +0) = \vec{H}_{2t}^{s}(+0) = -\vec{H}_{1t}^{s}(-0) \ . \tag{32e}$$

Alternatively, one may write

$$\vec{H}_{2t}(+0) + \vec{H}_{1t}(-0) = 2\vec{H}_{1t}^{i}(0) \tag{33a}$$

$$E_{2z}(+0) + E_{1z}(-0) = 2E_{1z}^{i}(0) \ . \tag{33b}$$

The resistive boundary condition may now be expressed as

$$\bar{\bar{Z}}R\cdot[\hat{z}_0 \times \{\vec{H}_{2t}(+0) - \vec{H}_{1t}(-0)\}] = \vec{E}_t(0) \tag{34}$$

Note that the relations (24a) to (25b) provide the formal representations of the fields associated with the scattering or diffraction of the field ($\vec{E}_1^i$, $\vec{H}_1^i$) incident from the region z<0 on an anisotropic and inhomogeneous resistive thin surface at z = 0. Since we have not imposed the boundary conditions explicitly at the surface z = 0, the expressions (24a) to (25b) are also valid for an infinite conducting plane (at z = 0) with apertures. Although no boundary conditions are applied at z = 0 explicitly, the superposition of the symmetrical and unsymmetrical fields implies the continuity of the tangential components of $\vec{E}_1$ and $\vec{E}_2$ at z = 0. If we designate the fields represented by (24a) to (25b) as the fields associated with the original diffraction or scattering problem, then the fields given by (26a) to (27b) represent the corresponding complementary problem. Furthermore, if the field ($\vec{E}_1^i$, $\vec{H}_1^i$) represent the source of the original problem, then the field ($-Z\vec{H}_1^i$, $Y\vec{E}_1^i$) is the source of the complementary problem. The latter source field can be obtained by rotating the former (i.e., $\vec{E}_1^i$, $\vec{H}_1^i$) through 90° in the plane parallel to the screen.

The formal relationship concerning Babinet's principle can be obtained by connecting (24a), (24b) with (26a), (26b) and (25a), (25b) with (27a), (27b). Thus, we have the following relations expressing Babinet's principle.

$$\left.\begin{aligned} \vec{E}_1 - Z\vec{H}_{1c} &= \vec{E}_1^r \qquad (28a) \\ \vec{H}_1 + Y\vec{E}_{1c} &= \vec{H}_1^r \qquad (28b) \end{aligned}\right\} \quad z<0$$

$$\left.\begin{aligned} \vec{E}_2 + Z\vec{H}_{2c} &= \vec{E}_2^r = \vec{E}_1^i \qquad (29a) \\ \vec{H}_2 - Y\vec{E}_{2c} &= \vec{H}_2^r = \vec{H}_1^i \; . \qquad (29b) \end{aligned}\right\} \quad z>0$$

$$\vec{H}_1 = 1/2\ [\vec{H}_{10} + \vec{H}_{1e}] \tag{23c}$$

$$\vec{H}_1^s = 1/2\ \hat{\vec{H}}_2^s \tag{23d}$$

$$\vec{E}_{1c} = 1/2\ [\vec{E}_{10c} + \vec{E}_{1ec}] \tag{23e}$$

$$\vec{E}_{1c}^{\,s} = 1/2\ \hat{\vec{E}}_{1c}^{\,s} \tag{23f}$$

$$\vec{H}_{1c} = 1/2\ [\vec{H}_{10c} + \vec{H}_{1ec}] \tag{23g}$$

$$\vec{H}_{1c}^{\,s} = 1/2\ \hat{\vec{H}}_{1c}^{\,s}\ . \tag{23h}$$

Similiar expressions with subscript 2 represent fields for the region $z>0$. We now add (11a), (17a) and then dividing by 2 obtain the following expression where use has been made of (23a) and (23b).

$$\vec{E}_1 = \vec{E}_1^i + \vec{E}_1^r + \vec{E}_1^s\ . \qquad (24a) \qquad z<0$$

In a similiar way the following expressions can be obtained.

$$\vec{H}_1 = \vec{H}_1^i + \vec{H}_1^r + \vec{H}_1^s \qquad (24b) \qquad z<0$$

$$\left.\begin{array}{ll} \vec{E}_2 = \vec{E}_2^s & \qquad (25a) \\ \vec{H}_2 = \vec{H}_2^s & \qquad (25b) \end{array}\right\} \quad z>0$$

$$\left.\begin{array}{ll} \vec{E}_{1c} = -Z[\vec{H}_1^i + \vec{H}_1^s] & \qquad (26a) \\ \vec{H}_{1c} = Y[\vec{E}_1 + \vec{E}_1^s] & \qquad (26b) \end{array}\right\} \quad z<0$$

$$\left.\begin{array}{ll} \vec{E}_{2c} = -Z[\vec{H}_2^r - \vec{H}_1^s] & \qquad (27a) \\ \vec{H}_{2c} = Y[\vec{E}_2^r - \vec{E}_2^s]\ . & \qquad (27b) \end{array}\right\} \quad z>0$$

used for unsymmetrical complementary excitation. In the present situation the lower signs in (2a) and (2b) are applied to fields (17a) and (17b) for the region z<0; whereas the upper signs in (2a) and (2b) are applied to (18a) and (18b) for fields in the region z>0. Such choice of signs is again necessary to preserve the proper symmetry and behavior at the boundary z = 0 for the complementary fields. In this way the symmetrical fields for the complementary problem can be represented in the following manner.

$$\left.\begin{aligned}\vec{E}_{1ec} &= -Z[\vec{H}_1^i + \vec{H}_1^r + \hat{\vec{H}}_1^s] \qquad (21a)\\ \vec{H}_{1ec} &= Y[\vec{E}_1^i + \vec{E}_1^r + \hat{\vec{E}}_1^s] \qquad (21b)\end{aligned}\right\} \; z<0$$

$$\left.\begin{aligned}\vec{E}_{2ec} &= -Z[\vec{H}_2^i + \vec{H}_2^r - \hat{\vec{H}}_2^s] \qquad (22a)\\ \vec{H}_{2ec} &= Y[\vec{E}_2^i + \vec{E}_2^r - \hat{\vec{E}}_2^s] \;. \qquad (22b)\end{aligned}\right\} \; z>0$$

Now the formal expressions of the total fields for the original diffraction problem (associated with a resistive anisotropic screen at z=0, or an aperture in a perfectly conducting screen which is a special case of the former) are given by the superposition of the fields (11a) to (12b) and (17a) to (18b) in an appropriate manner. Similarly, the formal expressions of the total fields for the corresponding complementary problem can be obtained by the superposition of the fields (15a) to (16b) and (22a) to (22b). Let us introduce the fields $\vec{E}_1$, $\vec{H}_1$, $\vec{E}_1^s$, $\vec{H}_1^s$, $\vec{E}_2$, $\vec{H}_2$, $\vec{E}_2^s$, $\vec{H}_2^s$, $\vec{E}_{1c}$, $\vec{H}_{1c}$, $\vec{E}_{1c}^s$, $\vec{H}_{1c}^s$, $\vec{E}_{2c}$, $\vec{H}_{2c}$, $\vec{E}_{2c}^s$ and $\vec{H}_{2c}^s$ by defining them in the following way.

$$\vec{E}_1 = 1/2\;[\vec{E}_{10} + \vec{E}_{1e}] \qquad (23a)$$

$$\vec{E}_1^s = 1/2\;\hat{\vec{E}}_1^s \qquad (23b)$$

$$\vec{E}_{1e} = \vec{E}_1^{\,i} + \vec{E}_1^{\,r} + \hat{\vec{E}}_1^{\,s} \qquad (17a)$$

$$\vec{H}_{1e} = \vec{H}_1^{\,i} + \vec{H}_1^{\,r} + \hat{\vec{H}}_1^{\,s} \qquad (17b)$$

$z<0$

$$\vec{E}_{2e} = -[\vec{E}_2^{\,i} + \vec{E}_2^{\,r}] + \hat{\vec{E}}_2^{\,s} \qquad (18a)$$

$$\vec{H}_{2e} = -[\vec{H}_2^{\,i} + \vec{H}_2^{\,r}] + \hat{\vec{H}}_2^{\,s} \,. \qquad (18b)$$

$z>0$

The subscript e refers to the even symmetrical fields. Note that $(\vec{E}_1^{\,i}, \vec{H}_1^{\,i})$ is the same field incident from $z<0$ for the unsymmetrical as well as symmetrical excitations. However, when the field $(\vec{E}_2^{\,i}, \vec{H}_2^{\,i})$ is incident from $z>0$ for the unsymmetrical excitation, the corresponding incident field is $(-\vec{E}_2^{\,i}, -\vec{H}_2^{\,i})$ from $z>0$ for the symmetrical excitation. Therefore, the total symmetrical fields have the following properties.

$$\vec{E}_{2et}(-z) = \vec{E}_{1et}(z),\ E_{2ez}(-z), = -\,E_{1ez}(z) \qquad (19a)$$

$$\vec{H}_{2et}(-z) = -\vec{H}_{1et}(z),\ H_{2ez}(-z) = H_{1ez}(z) \,. \qquad (19b)$$

The even symmetry of (19a) and (19b) implies also the following even symmetry of the scattered field.

$$\hat{\vec{E}}_{2t}^{\,s}(-z) = \hat{\vec{E}}_{1t}^{\,s}(z),\ \hat{E}_{2z}^{\,s}(-z) = -\hat{E}_{1z}^{\,s}(z) \qquad (20a)$$

$$\hat{\vec{H}}_{2t}^{\,s}(-z) = -\hat{\vec{H}}_{1t}^{\,s}(z),\ \hat{H}_{2z}^{\,s}(-z) = \hat{H}_{1z}^{\,s}(z) \,. \qquad (20b)$$

Note that Eqs. (8a) and (8b) define the even symmetrical solution of Maxwell's equations.

Corresponding to these symmetrical fields given by (17a) to (18b), we need to construct the fields for the complementary problem. This time the transformation (2a) and (2b) will be applied in a way different from that

$$\vec{E}_{1t}(-0) = -\ 2Z\overline{\overline{R}}\cdot[\hat{z}_0 \times \vec{H}_{2t}(-0)] \qquad (60iii)$$

$$\vec{E}_{2t}(+0) = \ 2Z\overline{\overline{R}}\cdot[\hat{z}_0 \times \vec{H}_{2t}(+0)]\ . \qquad (60iv)$$

The relations (60i) to (60iv) are equivalent to the Leontovich boundary conditions at a sheet with relative surface impedance [7]

$$\overline{\overline{\eta}} = 2\overline{\overline{R}} \qquad (61)$$

on each side.

Let us define [2] electric and magnetic Hertz potentials, $\vec{\Pi}(r)$ and $\vec{\Pi}^*(r)$ respectively, by the following relations.

When the source is an electric current, we have

$$(\nabla^2 + k^2)\vec{\Pi} = \vec{J}/(i\omega\varepsilon) \qquad (62i)$$

$$\vec{E}^s = k^2\ \vec{\Pi} + \nabla\nabla\cdot\vec{\Pi} \qquad (62ii)$$

$$\vec{H}^s = -i\omega\varepsilon\nabla\ \times\vec{\Pi} \qquad (62iii)$$

where $k^2 = \omega^2\mu\varepsilon$.

In an unbounded medium

$$\vec{\Pi}(\vec{r}) = (1/i\omega\varepsilon) \iiint \vec{J}(\vec{r}')g(\vec{r},\vec{r}')d^3r' \qquad (63)$$

where $g(\vec{r},\vec{r}') = e^{ik|\vec{r}-\vec{r}'|}/4\pi|\vec{r}-\vec{r}'|$.

Similarly,

$$(\nabla^2 + k^2)\ \vec{\Pi}^* = \vec{M}/(i\omega\mu) \qquad (64i)$$

$$\vec{E}^s = i\omega\mu\nabla \times \vec{\Pi}^* \qquad (64ii)$$

$$\vec{H}^s = k^2\vec{\Pi}^* + \nabla\nabla\cdot\vec{\Pi}^* \tag{64iii}$$

$$\vec{\Pi}^*(\vec{r}) = (-1/i\omega\varepsilon) \iiint \vec{M}(\vec{r}')g(\vec{r},\vec{r}')d^3r' \ . \tag{64iv}$$

Then for a resistive sheet the scattered fields can be derived from (64ii) to (64iv), replacing the volume integrals by surface integral with $\vec{M}(\vec{r}') = 2\vec{M}(\vec{r}') = 2\vec{E}_t(+0) \times \hat{z}_0$ [see Eq. (43)]. The factor of 2 arises due to the symmetry properties of the scattered field [1,2]. The scattered fields are then given by (44) and (45) and the total field can be expressed as in (24a) to (25b). Using the relations (24a), (25b), (45) and (56b), one can derive the following integral equation for the magnetic surface current $\vec{M}_t$.

$$(1/2)Y\left(\frac{\bar{\bar{R}}^T\cdot\vec{M}_t}{(\det \bar{\bar{R}})}\right) = (1/2)[\vec{H}_{2t}(+0) - \vec{H}_{1t}(-0)] = \vec{H}_{2t}(+0) - \vec{H}^i_{1t}(0)$$

$$= \vec{H}_{1t}(0) - \vec{H}_{1t}(-0) = -\vec{H}^i_{1t}(0) + 2iYk \iint_{z\to 0} \left[\vec{M}g + \frac{(\vec{M}\cdot\nabla)\nabla g}{k^2}\right]_t dx'dy' \tag{65}$$

The subscript t in the surface integral indicates the transverse component of the vector quantity. Note that $\vec{M}$ in (65) is the same as $\vec{M}_s$ in (45).

For the magnetically conductive sheet occupying the entire plane z=0 and characterized by the boundary conditions (57a) and (57b), we assume that the same field ($\vec{E}_1^{\,i}$, $\vec{H}_1^{\,i}$) is also incident from z<0. Let us also assume the total field ($\vec{E}_c'$, $\vec{H}_c'$) in this case satisfies the conditions (57a) and (57b). Then the total magnetic surface current is defined by

$$\vec{M} = \vec{J}^* = [\vec{E}'_{2tc}(+0) - \vec{E}'_{1tc}(-0)] \times z_0 \ . \tag{67}$$

Then in this "second kind" of complementary or dual problem the scattered field can be expressed by

$$\vec{E}_c^{s\prime} = \iint \vec{J}^*(\vec{r}^{\prime}) \times \nabla g(\vec{r},\vec{r}^{\prime})dx^{\prime}dy^{\prime} \tag{68a}$$

$$\vec{H}_c^{s\prime} = ikY \iint \left[\vec{J}^*g + \frac{(\vec{J}^* \cdot \nabla)\nabla g}{k^2} \right] dx^{\prime}dy^{\prime} \tag{68b}$$

which are valid for both $z<0$ and $z>0$. Therefore, the total field in this situation is given by the following relations for all z.

$$\vec{E}_c^{\prime} = \vec{E}_1^{\,i} + \vec{E}_c^{s\prime} \tag{69a}$$

$$\vec{H}_c^{\prime} = \vec{H}_1^{\,i} + \vec{H}_c^{s\prime} . \tag{69b}$$

An integral equation for determining this surface magnetic current $\vec{J}^*$ is given by

$$\frac{Y}{4} \frac{\bar{\bar{R}}^T \cdot \vec{J}^*}{(\det \bar{\bar{R}})} = \vec{H}_{1t}^{\,i}(0) + iYk \iint_{z\to 0} \left[\vec{J}^*g + \frac{(\vec{J}^* \cdot \nabla)\nabla g}{k^2} \right]_t dx^{\prime}dy^{\prime} \quad . \tag{70}$$

If we now set $2\vec{M} = -\vec{J}^*$ in (65) and compare it with (70), once again the relation (59) is obtained. When the condition (59) is satisfied, the two sets of fields given by (24a) to (25b) and (69a), (69b) are related in the following manner providing statement of Babinet's principle.

$$\left.\begin{aligned} \vec{E}_c^{\prime} &= \vec{E}_1 - \vec{E}_1^{\,r} && (71a) \\ \vec{H}_c^{\prime} &= \vec{H}_1 - \vec{H}_1^{\,r} && (72b) \end{aligned}\right\} \quad z<0$$

$$\left.\begin{aligned} \vec{E}_c^{\prime} &= \vec{E}_1^{\,i} - \vec{E}_2 && (71c) \\ \vec{H}_c^{\prime} &= \vec{H}_1^{\,i} - \vec{H}_2 . && (71d) \end{aligned}\right\} \quad z>0$$

Note that these relations (71a) to (71d) are different from those given by (28a) to (29b).

Let us now consider the other type of complementary problem which was treated in previous sections. In this case, the scattering surface has an electrical resistivity $Z\bar{\bar{R}}_c$ and located at $z = 0$. The form of $\bar{\bar{R}}_c$ is shown in (38). The incident field $(\vec{E}_c{}^i,\vec{H}_c{}^i)$ originated in $z<0$ is related to the original incident field $(\vec{E}_1{}^i,\vec{H}_1{}^i)$ via

$$\vec{E}_c{}^i = -Z\vec{H}_1^i,\ \vec{H}_c^i = Y\vec{E}_1^i \ . \tag{72}$$

This means that the incident field $(\vec{E}_c{}^i,\vec{H}_c{}^i)$ can be generated by rotating the former $(\vec{E}_1{}^i,\ \vec{H}_1{}^i)$ through 90° in the plane parallel to the resistive sheet. In this situation the total fields $(\vec{E}_{1c},\ \vec{H}_{1c})$ and $(\vec{E}_{2c},\ \vec{H}_{2c})$ satisfy the relations (56a) and (56b) adhering the subscript c to the field components and the normalized resistivity. In other words, the impedance boundary condition is given by (37). The total induced current on the sheet is given by

$$\vec{J}_c = \hat{z}_0 \times [\vec{H}_{2ct}(+0) - \vec{H}_{1ct}(-0)] \ . \tag{73}$$

Then the scattered fields $(\vec{E}_{1c}^s,\ \vec{H}_{1c}^s)$ and $(\vec{E}_{2c}^s,\ \vec{H}_{2c}^s)$ can be calculated by using (62ii) to (63iv)

$$\vec{H}_{1c}^s = -\iint \vec{J}_c(\vec{r}') \times \nabla g(\vec{r},\vec{r}')dx'dy' = \vec{H}_{2c}^s \tag{74a}$$

$$\vec{E}_{1c}^s = ikZ \iint [\vec{J}_c g + (\vec{J}_c\cdot\nabla)\nabla g/k^2]dx'dy' = \vec{E}_{2c}^s \ . \tag{74b}$$

Therefore, the total field is given by

$$\vec{E}_{1c} = \vec{E}_{1c}^i + \vec{E}_{1c}^s = \vec{E}_{2c} \tag{75}$$

$$\vec{H}_{1c} = \vec{H}_{1c}^i + \vec{H}_{1c}^s = \vec{H}_{2c} \ . \tag{76}$$

From (37), (74a) and (76), the following integral equation for $\vec{J}_c$ can be formulated.

$$\bar{\bar{R}}_c \cdot \vec{J}_c = - \vec{H}^i_{1ct}(0) + ik \iint_{z \to 0} [\vec{J}_c g + (\vec{J}_c \cdot \nabla)\nabla g/k^2]_t dx' dy' = Y\vec{E}_{ct}(0) \quad . \tag{77}$$

If we let

$$\vec{J}_c = 2Y\vec{M} \tag{78}$$

in (77) and then compare it with (65), we obtain again the relation (40). The fields (75), (76) and those generated by $\vec{M}$ lying on the resistive sheet $Z\bar{\bar{R}}$, are related through Babinet's principle stated in (28a) to (29b).

Note that the resistivity $\bar{\bar{R}}_c$ has the same relationship to $\bar{\bar{R}}$ as does the magnetic conductivity $\bar{\bar{R}}^*$ with $\bar{\bar{R}}$.

CONCLUSION

Babinet's principle relates the diffracted or scattered fields associated with one diffracting surface to those associated with the complementary surface (or screen). It does not, however, state how the fields can be computed in principle. The methods of field computation are provided by the appropriate diffraction theory pertinent to the problem concerned. For example, Collin [3] and Jones [4] presented derivations of Babinet's principle without requiring how to compute the fields. On the other hand, Copson [1] and Elliott [2] provided the derivation of Babinet's principle as well as the formal methods of calculating the fields. In doing so, they had to establish the necessary even and odd symmetrical properties of the various components of the electromagnetic fields. Senior [7], in his derivation of Babinet's principle assumed the required symmetry of the fields and presented the formal integral expressions of the diffracted or scattered fields together with the boundary conditions appropriate to a diagonally anisotropic resistive screen.

In this paper, section 3 provides the necessary properties (odd and even symmetry), of the electromagnetic fields, derivation of Babinet's principle, the boundary conditions appropriate to a general anisotropic resistive surface, which may be inhomogeneous and also the integral representation of the diffracted fields. In a sense, section 3 is self-contained and offers a generalization of Collin's method. In section 4, the same problem is addressed with somewhat different viewpoint generalizing Jones' method. It also rectifies Lang's results. Finally, in section 5 we have extended Senior's results to a general anisotropic resistive surface. The resistive surfaces have applications in reducing undesirable electromagnetic reflections. The theory is also valid for a general impedance screen provided a complementary surface of such an impedance screen can be conceived.

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