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## THESIS

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OPERATIONS RESEARCH TECHNIQUES  
FOR HUMAN FACTORS ENGINEERS

by

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June 1985

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| Human performance                                                                                                                                                                                                                                                                                                                                                                                                                        | Linear programming                         | Markov chains                                                      |
| Operations research                                                                                                                                                                                                                                                                                                                                                                                                                      | Decision analysis                          | Simulation                                                         |
| Operations analysis                                                                                                                                                                                                                                                                                                                                                                                                                      | Distribution models                        | Queueing                                                           |
| Man-machine systems                                                                                                                                                                                                                                                                                                                                                                                                                      | Stochastic models                          | Modeling                                                           |
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# ABSTRACT

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## TABLE OF CONTENTS

|      |                                                                 |    |
|------|-----------------------------------------------------------------|----|
| I.   | INTRODUCTION . . . . .                                          | 9  |
|      | A. BACKGROUND . . . . .                                         | 10 |
|      | B. OBJECTIVES AND PROCEDURE . . . . .                           | 12 |
| II.  | DEFINITIONS . . . . .                                           | 13 |
|      | A. HUMAN FACTORS ENGINEERING . . . . .                          | 13 |
|      | B. SYSTEMS ANALYSIS AND OPERATIONS ANALYSIS . . . . .           | 16 |
|      | C. OPERATIONS RESEARCH . . . . .                                | 20 |
|      | D. OPERATIONS RESEARCH TECHNIQUES . . . . .                     | 24 |
|      | E. COMBINING HUMAN FACTORS AND OPERATIONS<br>RESEARCH . . . . . | 27 |
| III. | NAVY MISSIONS: AIR COMBAT MANEUVERING . . . . .                 | 31 |
| IV.  | LITERATURE SEARCH . . . . .                                     | 33 |
| V.   | MODELS AND MODELING . . . . .                                   | 36 |
|      | A. THE NATURE OF MODELS . . . . .                               | 36 |
|      | B. THE PROCESS OF MODELING . . . . .                            | 42 |
|      | C. FINDING AND EVALUATING SOLUTIONS . . . . .                   | 46 |
| VI.  | MODELS FOR DESCRIBING AND PREDICTING . . . . .                  | 49 |
|      | A. REGRESSION ANALYSIS . . . . .                                | 51 |
|      | B. FACTOR ANALYSIS . . . . .                                    | 54 |
|      | C. DISCRIMINANT ANALYSIS . . . . .                              | 57 |
|      | D. CANONICAL CORRELATION . . . . .                              | 58 |
|      | E. MULTIDIMENSIONAL SCALING . . . . .                           | 60 |
|      | F. MANUAL CONTROL MODELS . . . . .                              | 62 |
|      | G. OPTIMAL CONTROL MODELS . . . . .                             | 64 |
|      | H. TIME SERIES MODELS . . . . .                                 | 67 |
|      | I. FINITE MARKOV CHAINS . . . . .                               | 69 |

|       |                                                           |     |
|-------|-----------------------------------------------------------|-----|
| J.    | POISSON PROCESSES . . . . .                               | 79  |
| K.    | QUEUEING PROCESSES . . . . .                              | 81  |
| L.    | RELIABILITY MODELS . . . . .                              | 95  |
| M.    | SIMULATION MODELS . . . . .                               | 97  |
| VII.  | MODELS FOR OPTIMIZING . . . . .                           | 117 |
| A.    | LINEAR PROGRAMMING MODELS . . . . .                       | 118 |
| B.    | NONLINEAR PROGRAMMING MODELS . . . . .                    | 133 |
| C.    | NETWORK MODELS . . . . .                                  | 135 |
| D.    | DISTRIBUTION MODELS . . . . .                             | 146 |
| VIII. | MODELS FOR DECISIONS . . . . .                            | 159 |
| A.    | DECISION THEORY MODELS AND DECISION<br>ANALYSIS . . . . . | 160 |
| B.    | SIGNAL DETECTION THEORY MODELS . . . . .                  | 179 |
| IX.   | SUMMARY . . . . .                                         | 183 |
|       | APPENDIX A: GLOSSARY . . . . .                            | 185 |
|       | LIST OF REFERENCES . . . . .                              | 188 |
|       | INITIAL DISTRIBUTION LIST . . . . .                       | 191 |

## LIST OF TABLES

|    |                                                                    |     |
|----|--------------------------------------------------------------------|-----|
| 1. | HUMAN FACTORS OBJECTIVES AND CATEGORIES . . . . .                  | 17  |
| 2. | OPERATIONS RESEARCH MODELS AND TECHNIQUES . . . . .                | 25  |
| 3. | MATHEMATICAL TOOLS FOR OPERATIONS RESEARCH<br>TECHNIQUES . . . . . | 28  |
| 4. | DESCRIPTION OF OPERATIONS RESEARCH<br>MODEL/TECHNIQUE . . . . .    | 29  |
| 5. | NAVY MISSIONS . . . . .                                            | 31  |
| 6. | SAMPLE SIMULATION RUNS: FIVE CYCLES . . . . .                      | 114 |
| 7. | PERCENT OF TIME INFORMATION SOURCES MUST BE<br>MONITORED . . . . . | 153 |
| 8. | RELATIVE DIFFICULTY OF MONITORING INFORMATION<br>SOURCES . . . . . | 154 |

## LIST OF FIGURES

|     |                                                                 |     |
|-----|-----------------------------------------------------------------|-----|
| 2.1 | Simple Model of a Man-Machine System . . . . .                  | 18  |
| 5.1 | Modeling Process, as Described by Olkin . . . . .               | 43  |
| 6.1 | Transition States During a Simulated Engagement . .             | 75  |
| 6.2 | One-Step Transition Diagram for an ACM<br>Engagement . . . . .  | 76  |
| 6.3 | Exponential Distributions for Arrivals and<br>Service . . . . . | 89  |
| 6.4 | Illustration of Single Server Queueing System . . .             | 90  |
| 6.5 | Flow Diagram for Simulation Model . . . . .                     | 110 |
| 7.1 | Graphical Solution to ACM Ammunition Problem . .                | 130 |
| 7.2 | Network Graph Illustration for ACM Example . . .                | 142 |
| 7.3 | Initial ACM Distribution Tableau, First<br>Solution . . . . .   | 154 |
| 7.4 | Second Solution to ACM Distribution Problem . . .               | 156 |
| 8.1 | Payoff Matrix for the Example Problem . . . . .                 | 176 |
| 8.2 | Decision Tree for the Example Problem . . . . .                 | 177 |

## I. INTRODUCTION

This thesis is intended for human factors engineers who seek additional techniques to use in evaluating the adequacy of new systems. The techniques discussed here are commonly used in the field of operations research. They are readily applicable to many human factors problems, but are not widely known in that discipline. Most of these techniques result in quantitative answers--numbers or equations which represent how much money or time will be saved, or how many errors can be expected, etc., if this particular system or change is implemented.

Human factors engineers certainly recognize the need for numerical answers to engineering problems, based on measures of effectiveness. The design and systems engineers who oversee major programs are justifiably skeptical of qualitative judgements--especially ones like, "It is my informed opinion that the operator will make fewer errors if we raise the widget 2 inches." Since it costs money to change a design and raise a widget, at least three questions need to be asked. How much does an error cost? By how much will errors be reduced? In the long run, will it be cost effective (in money or lives), if we raise the widget?

Numerical answers may be available, in specific cases, as a result of directly-applicable experimentation (either previously carried out or done especially to answer a current question). Usually, however, they are not. When empirical data are not available, the human factors engineer must rely either on intuition or on extrapolation of what is known, through the use of some model, to make evaluations. The operations research techniques presented here are intended to assist in the latter process.

## A. BACKGROUND

For some time, human factors engineers have been intrigued by a possible liaison of human factors with operations research, notes DeGreene [Ref. 1]. However, he continues, human factors and operations research have largely gone their separate ways. The former perhaps have made somewhat more use of operations research techniques than the latter have incorporated psychological knowledge from human factors research. Still, much of the mathematics and many of the concepts used in operations research remain outside the general knowledge pool of human factors specialists.

DeGreene describes the gap between the two disciplines as follows:

Operations research tends to be formal and quantitative, and applied at the subsystem level, or lower, and toward suboptimizations, in the systems sense, rather than towards optimizations. To a great extent, there is an emphasis on theory over applications.

Yet in many ways, operations research represents a natural extension of the methods long practiced in psychological and human factors research. In both cases, problem "fields" and methodologies (e.g., sampling, surveying, and simulation) are similar, and quantitative procedures have a basis in probability theory.

Operations research tends to require more mathematics than most psychologists and human factors specialists possess or need. We know of no texts, articles, or university courses entitled "Operations Research Techniques for the Human Factors Specialist". Such courses would require a respectable understanding of human behavior, as well as mathematical manipulations.

DeGreene does acknowledge a number of human factors problems that actually have utilized operations research techniques. These include panel layout; work space design; visual sampling and display design; information system data-file, data-bank, and data retrieval designs; organizational data flows; and manpower determinations and allocations.

However, he notes that a large number of potentially useful techniques are not widely used. He lists queueing theory, linear programming, and game theory as especially fruitful areas for human factors applications.

This study is intended as a slender bridge across the gap DeGreene has described. It may be useful to operations researchers who happen to work with man-machine systems. However, the primary intended audience consists of human factors engineers. Specific operations research techniques are identified and described here, then correlated with human factors-related problems. Thus, human factors engineers may be able to pick up some of these procedures, as applicable to their individual areas of interest, without having to study the whole of operations research.

According to Cogan [Ref. 2], good operations research and good human factors have several points in common. Both are concerned with how to implement innovations. And both require a general imagination, elastic feats of the mind, and adventurousness. Effective work in both fields must be based on an intimate knowledge of the system being studied. Discovery, invention, or creativity puts this knowledge in a new light. A formal epistemological or mathematical review then provides a firm foundation for innovation.

While human factors principles are applied (or can be applied) throughout the whole range of human activities, one of the most promising areas for the melding of that discipline with operations research is in military systems. The enormous expense of these systems, coupled with the severe consequences of human error, suggest that all applicable techniques that might result in system improvements should be utilized. Therefore, military applications are emphasized in the examples given below.

To aid in understanding the techniques presented here, it is useful to provide a common thread throughout the

explanations and examples. Actually, two "threads" are used. The first is the employment of a constant format in describing all of the techniques (as discussed in the next chapter). The second is the application of each technique to the same (or close to the same) scenario or mission. The Navy's mission area of antiair warfare (AAW) has been selected for this purpose. Insofar as practical, that tactical category known as air combat maneuvering (ACM)--direct air battles between two or more fighter aircraft--is used to illustrate how the various operations research techniques may be applied.

## B. OBJECTIVES AND PROCEDURE

The objectives of this study are:

1. To identify and classify a number of operations research models and techniques which are applicable to certain human factors problems.
2. To describe several of these models and techniques in enough detail to enable the human factors engineer to determine if they are useful for his or her particular problems.
3. To illustrate the use of some of these techniques in a military-related human factors setting.
4. To provide references for additional information on each listed technique, to enable further study if the human factors engineer is interested.

In short, this is intended as a "how to" manual, not a "why so" textbook. Readers who desire or need a theoretical basis for the techniques discussed here are referred to the more standard mathematics and operations research texts cited in each section.

## II. DEFINITIONS

Before beginning discussion of operations research techniques applicable to human factors engineering problems, the following sections briefly describe what is meant (in this study) by the following terms:

1. Human factors and human factors engineering
2. Systems engineering, systems analysis, and operations analysis
3. Operations research
4. Operations research techniques

The procedure used in this study to combine human factors engineering problems with operations research techniques also is discussed.

### A. HUMAN FACTORS ENGINEERING

Human factors engineering is the application of information about human behavior in the design of equipment, facilities, and environments, in order to meet man-machine system objectives. It is not synonymous with, but rather is a subset of, the more general field of human factors. The latter also includes research into human capabilities and the enhancement of capabilities through training, as well as application of research findings to design problems.

Closely related to human factors engineering is the field of engineering psychology. The difference is in focus. Human Factors engineering adopts the perspective of the engineer, and is concerned with the entire human body and its performance. Engineering psychology, on the other hand, adopts the perspective of the psychologist, and so focuses on the brain, the mind, and behavior.

According to McCormick and Sanders [Ref. 3], the primary focus of the general field of human factors is on human beings carrying out functions to meet an objective (or, as Bailey [Ref. 4] puts it, "Somebody doing something some-place"--that is, a person converting inputs to specified outputs, via work activities). The combination of person, activity, and surroundings--including equipment needed to perform the activity--can be considered a man-machine system, and thus is amenable to the techniques of systems analysis.

The approach used in human factors engineering is the systematic application of relevant information about human abilities, characteristics, behavior, and motivation, in the execution of functions or activities as described above. In other words, engineering techniques and knowledge about the human being are applied to the man-machine system, to bring about some desired output from given inputs. In doing so, as Jones points out [Ref. 5], the psychologist is well aware that the man is, at best, statistical in nature (while the typical systems engineer is only vaguely aware that human behavior is variable, nonlinear, and time varying).

It is important to note that the human factors engineer attempts to apply research-based information (as opposed to logic or common sense) to systems problems, as is emphasized by Chapanis [Ref. 6]. He notes:

Because psychologists work so intimately with the tangled skein of relationships which constitute human behavior, they are not much inclined to trust their common sense, intuitions, or logical powers of analysis when it comes to matters of this kind. Most good human engineers, I find, are always a little uneasy when they have to make decisions unsupported by empirical findings. To such people, one good experiment is worth a hundred guesses, because they know how often guesses turn out to be wrong....

As a result, the two kinds of engineers--human and systems--do not often understand each other. The one is reluctant to play his hunches, and argues constantly for empirical evidence; the other is impatient at the long-haired scientific attitude which demands validation, and

argues that an "informed guess" is better than none. In the long run, however, the validity of any model must face the stern test of empirical validation. In this respect, the human engineer's scepticism can contribute to the work of the operations analyst.

It also is important to note that knowledge about human performance is unavailable, to cover every possible situation and under all conditions. What we know--through human factors research--is how a typical or anticipated user probably will perform in some previously-studied situations.

Thus, when a new situation arises (through development of a new system) we usually have only two ways to "find out" how well we may expect the man-machine system to do its intended job. We can perform research and measure performance under the precise conditions of interest. Or we can take the closest, best data presently at hand and extend its usefulness (make predictions) through some form of analysis.

The field of human factors engineering, and the measurement and analysis techniques used by that discipline to attack various problems, often are divided into established categories. It is useful for our purposes to classify these categories under the objectives for which they are intended--objectives which then can be related to the operations research techniques to be discussed here. These objectives may be stated as:

1. Describing individual human differences: permanent differences, such as those that are inherent or due to experience, or transitory differences, such as those due to physical or emotional state, motivation, etc.
2. Describing a man-machine system.
3. Designing (or modifying) a system for optimum performance.
4. Evaluating human performance within the system, to judge whether given criteria are met for such things

as perceiving inputs, performing mental activities (mediation), communicating, and making responses (motor processes).

Table 1 illustrates how various "standard" human factors categories fit within these four general objectives. As noted, these objectives sometimes have been met by measuring attributes or performance, through tests and experiments. At other times, techniques of prediction have been used, via analysis and modeling.

Those human factors engineering methods and procedures listed in Table 1 that have been extensively exploited in the past are not covered in this study. These include function, task, timeline, workload, link, and environment analysis, as well as the design, conduct, and analysis of experiments and tests. The intent here is to break new ground, not to review the entire spectrum of techniques. Similar human factors objectives often can be met, in somewhat different ways (and sometimes with better results), using the less familiar operations research techniques discussed below.

## B. SYSTEMS ANALYSIS AND OPERATIONS ANALYSIS

Analysis, of course, is the separation of a whole into its component parts. Analysis can be looked upon as a detailed examination of anything complex, in order to understand the nature and to determine the essential features of that complex object or concept.

A system can be considered an assemblage of constituents (people, hardware, and software) that interact to fulfill a common purpose, transcending the individual purposes of the components [Ref. 7]. Thus a system consists of several parts (each with attributes) plus the relationships among them. Often the inputs to and outputs from this collection

**TABLE 1**  
**HUMAN FACTORS OBJECTIVES AND CATEGORIES**

| HUMAN FACTORS OBJECTIVE                    | HUMAN FACTORS CATEGORY                                                                                                                                                                                                                                                                                    |
|--------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. <u>Describe Individual Differences:</u> | <u>Inherent</u><br>Anthropometric measures<br>Physiological measures<br>Intelligence tests<br>Psychological tests<br><br><u>Experience</u><br>Knowledge level tests<br>Skill level tests<br><br><u>Transitory</u><br>Physical states<br>Emotional states<br>Motivation measures<br>Satisfaction measures  |
| 2. <u>Describe Systems</u>                 | Functional analyses<br>Task analyses<br>Workload analyses<br>Link analyses<br>Environment analyses                                                                                                                                                                                                        |
| 3. <u>Design Systems</u>                   | Function allocation<br>Equipment design<br>Environment design<br>Job design<br>Personnel selection<br>Training design                                                                                                                                                                                     |
| 4. <u>Evaluation of Human Performance:</u> | <u>Perception measurement</u><br>Search, identify,<br>monitor, recognize<br><br><u>Mediation modeling</u><br>Information processing<br>Decision making<br><br><u>Communications measurement</u><br>Verbal<br>Nonverbal<br><br><u>Motor Processes measurement</u><br>Simple/discrete<br>Complex/continuous |

of objects also are considered part of the system. Since, as John Muir pointed out, "Everything in the universe seems to be hitched to everything else," systems (and systems problems) often are large and complex. Figure 2.1 is a very simple model of a man-machine system, using this definition.

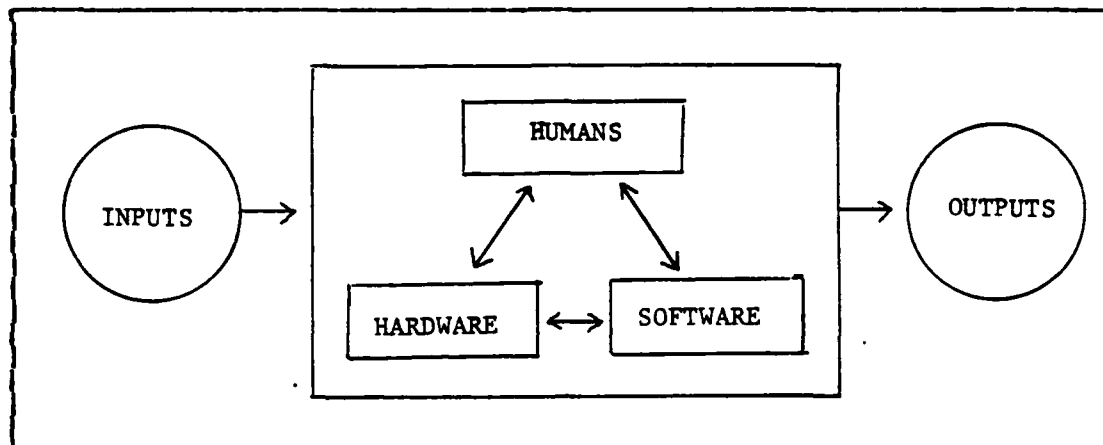


Figure 2.1 Simple Model of a Man-Machine System

Systems engineering is the application of scientific and engineering knowledge to the planning, design, evaluation, and construction of man-machine systems and system components, according to Chapanis [Ref. 8]. The process primarily is concerned with the construction of new hardware and software systems. Since the knowledge to be applied must include information about human behavior, human factors engineering may be considered a subcategory of systems engineering.

Systems analysis is the process of taking unmanageably large problems of system design or control (especially problems that are ill-defined) and "cutting" them into small problems--known as suboptimization. Solutions to these small problems can be sought, then combined in some manner to yield solutions for the large ones [Ref. 9].

When does a problem exist? According to Daellenbach and others [Ref. 10], for a problem to exist,

1. There must be a decision maker who has a goal to be achieved.
2. At least two alternative courses of action are available.
3. There must be some doubt about which is the best course of action.
4. The problem must be treatable within a relevant environment.

Operations analysis is the term usually applied to analyses of the operation of an existing system (as opposed to the design or development of a new system) [Ref. 11]. According to Raiffa [Ref. 12], problems tackled by operations research are more limited in character than those of systems analysis, and have better defined structure and goals. There is no hard and fast demarcation line between the two, however.

The basic role of operations analysis is to provide carefully reasoned, technical, and predictive advice to the system's users, according to DeGreene [Ref. 13]. He lists as the sequential steps used for all operations analyses:

1. Recognition that a problem exists and that the solution may be amenable to operations analysis techniques.
2. Definition of that problem in an appropriate form, including definition of objectives, requirements, and constraints.
3. Definition of the system itself, beginning with gross approximations and working toward minute preciseness; the result should be a conceptual model on which quantitative analysis may be performed.
4. Definition of performance criteria for the system as a whole, for the various levels of organization, and for the combination of its constituents.

5. Definition of alternative configurations, and evaluation of trade-offs (using operations research techniques).
6. Presentation of alternatives and trade-offs to the user.
7. Performance of ongoing, iterative engineering and human factors analyses during system development.
8. Analysis of operational systems, to gather performance data.

Although operations analysis may have a qualitative beginning, quantification is required as the system develops. The ensuing quantitative analyses then include:

1. Determination of the functional relationships of performance parameters, using mathematical models to describe subsystems and systems.
2. Optimization of the system, using predetermined criteria.
3. Determination of the variations in system performance associated with changes in constraints, external requirements, etc.

Note that "quantitative" refers to the degree or level of measurement of some quality or attribute. Thus it includes ordinal relationships such as "more versus less" and "better versus worse", in addition to interval and ratio measurements. It also includes probabilities, as well as discrete numbers.

### C. OPERATIONS RESEARCH

According to DeGreene [Ref. 14], operations research is the application of quantitative, mostly probabilistic techniques (largely at the subsystem level) to the management and control of specific complex systems. He contrasts this with systems analysis or operations analysis, which applies

a similar body of techniques to systems to obtain general (as opposed to system-specific) data--which then can be used for general predictions of system and human performance reliability.

As noted in the above section, the techniques developed under the umbrella of operations research are used during the process of operations analysis-- specifically, during definition of alternative configurations and evaluation of trade-offs. Thus, in this sense, operations research can be considered a subcategory of operations analysis.

Operations research techniques usually are applied to problems of conducting or coordinating operations or activities within an organization, according to Hillier and Lieberman [Ref. 15]. The nature of the organization is immaterial; breadth of applications is wide. The discipline is concerned with optimal decision making in, and modeling of, deterministic and probabilistic systems that originate from real life.

Not all operations research problems involve systems engineering or human factors engineering. However, when predictions about the most efficient operation of a not-yet-constructed man-machine system are needed, the knowledge provided by human factors engineering becomes vital [Ref. 16].

The approach of operations research basically involves use of the scientific method. Daellenbach and others provide five major steps or phases for a successful operations research project [Ref. 17]. These include:

1. Defining and formulating the problem.
2. Constructing a mathematical model to represent the operation studied.
3. Deriving a solution to the model.
4. Testing the model with empirical or other practical data, evaluating whether the solution yields

acceptable values for the measures of effectiveness, and, if not, making appropriate changes or refinements.

5. Implementing, maintaining, and using the solution for predictions

The above steps assume that the problem is well-structured, a condition necessary if we are to approach it with the usual operations research procedures. Daellenbach and others [Ref. 18] list six characteristics of a well-structured problem:

1. Any knowledge relevant to the problem can be represented in an acceptable model.
2. An acceptable model will encompass all feasible solutions.
3. Definite criteria are available for judging the feasibility and optimality of any solution.
4. A programmable method (that is, one which can be laid out in logical steps) exists for finding the optimal solution.
5. The solution method does not require more computation than is economically practical.
6. All information required by the acceptable model is available or can be obtained economically.

Wagner [Ref. 19] notes that the distinguishing characteristics of operations research include the following:

1. A primary focus on decision making: the analysis must have direct and unambiguous implications for action.
2. An appraisal resting on criteria of economic effectiveness: a recommended solution must take into account the cost and return tradeoffs, based on some measure of effectiveness, so that a balance has been struck.

3. Reliance on a formal mathematical model: data manipulation procedures should be so explicit that they can be described to another analyst, who could then derive the same results.
4. Dependence on an electronic computer: this characteristic is not necessarily desirable, but is a reflection of the complexity and size of most problems tackled under the banner of operations research.

The concept of measure of effectiveness (MOE) deserves elaboration. Under the "systems point of view", final criteria of overall system performance are used to evaluate individual design decisions [Ref. 20]. The measure of performance or effectiveness used most often in operations research is cost in dollars.

Some operations research techniques are intended for problems where only a single objective (e.g., cost) is to be met, and only one measure of effectiveness is used. Other techniques can handle several objectives at once. Still others are used within a framework of continuous (rather than discrete) variables and objectives.

A conflict between operations research and human factors engineering must be considered here. For the human factors engineer, measures of effectiveness usually are based on some human performance outcome, described by one or more observable attributes (such as speed or accuracy), that is associated with each of the operator's alternative courses of action. These attributes are used to measure how effectively each outcome will meet the decision maker's objectives. Pre-set criteria or standards must be available in order to measure the "goodness" of each outcome (for example, a criterion that data will be entered on a keypad with an error rate of five or fewer incorrect entries per 100 keystrokes).

The problem, as Chapanis points out [Ref. 21], is that most human factors research is carried out under carefully-controlled, generalizable conditions so that results will be widely applicable. This creates an extremely serious shortcoming in most human factors data: since they were not obtained under realistic conditions (which are not so generalizable), they cannot be entered directly into the operations researcher's cost equations. It does little good to set a measure of effectiveness of error rate unless we know how much an error costs--in dollars, lives, time, etc.

Chapanis recommends that, whenever human factors results are to be used in operations research models, these results be expressed in systems-relevant measures. These include measures such as a pilot's delay-time expressed in the amount of fuel consumed by the aircraft during the delay, and pilot error rates expressed as the probability of mid-air collision as a function of these errors.

#### D. OPERATIONS RESEARCH TECHNIQUES

An operations research technique can be considered to be a verbal, physical, or mathematical procedure (usually mathematical), defining or performed on a model, that either

1. elucidates a specific question about a system, condition, or event (using a descriptive model); or
2. that gives a quantitative answer to such a question (using either a descriptive or prescriptive model).

The basic categories of models (descriptive, prescriptive, etc.) and the relationships between models and techniques are discussed further in Chapter V.

Table 2 lists a number of operations research models and techniques, applicable to human factors engineering problems, which are discussed in this study. These have been categorized under three "purposes":

TABLE 2  
OPERATIONS RESEARCH MODELS AND TECHNIQUES

| PURPOSE                                                         | MODEL                                     | TECHNIQUE                |
|-----------------------------------------------------------------|-------------------------------------------|--------------------------|
| 1. <u>Describing Systems, and Description-Based Predictions</u> |                                           |                          |
|                                                                 | Deterministic models                      |                          |
|                                                                 |                                           | Regression analysis      |
|                                                                 |                                           | Factor analysis          |
|                                                                 |                                           | Discriminant analysis    |
|                                                                 |                                           | Canonical correlation    |
|                                                                 |                                           | Multidimensional scaling |
|                                                                 |                                           | Manual control           |
|                                                                 |                                           | Optimal control          |
|                                                                 |                                           | Time series models       |
|                                                                 | Stochastic models                         |                          |
|                                                                 |                                           | Markov chains            |
|                                                                 |                                           | Poisson processes        |
|                                                                 |                                           | Queueing processes       |
|                                                                 |                                           | Reliability models       |
|                                                                 | Simulation models                         |                          |
| 2. <u>Maximizing/Minimizing and Meeting Constraints</u>         |                                           |                          |
|                                                                 | Linear programming models                 |                          |
|                                                                 | Nonlinear programming models              |                          |
|                                                                 | Network models                            |                          |
|                                                                 | Distribution models                       |                          |
| 3. <u>Making Choices and Decisions</u>                          |                                           |                          |
|                                                                 | Decision theory models, decision analysis |                          |
|                                                                 | Signal detection theory models            |                          |

1. Describing systems, and description-based predictions. These techniques involve finding or developing some mathematical formulation that represents existing knowledge about the system well enough so it can be extrapolated to predict the performance which is expected under somewhat different conditions. These describing functions are divided here into deterministic, stochastic, and simulation models--to make it easier to see differences and relationships. Meanings of these terms are discussed in Chapter V or in the introduction to Chapter VI (and also in the Glossary).
2. Maximizing, minimizing, and meeting constraints. These techniques find ways to satisfy a number of criteria simultaneously (or sometimes serially), within bounds set by nature or human organizations, to obtain the best possible solution to a problem. Linear and nonlinear programming, network, and distribution models are included in this category.
3. Making choices and decisions. Given a list of alternatives, these techniques are used to perform analyses and evaluations in order to determine which alternative will best meet some given criterion or aspiration level. This category includes techniques derived from decision theory and from signal detection theory.

Any scheme for separating models into some set of categories is of course based on someone's judgement. It is also pointless, unless there is some value to be gained by this categorization. The value here lies in the fact that models and techniques must be used, must have a purpose. Otherwise, they are merely intellectual exercises. The purposes or categories used here suggest what the models included in this study are good for. They should aid the

human factors engineer as he screens the various techniques to see which (if any) are applicable to whatever questions he needs answered.

Other scientific disciplines will be required for implementing such techniques--primarily those of mathematics. Therefore, we will define a mathematical tool as a procedure which does not, in and of itself, answer a specific question, but which is necessary in order to use an operations research technique. That is, it can be considered an instrument, or a means to an end.

Table 3 lists some of the most common mathematical tools used with operations research techniques. Several of these already are an integral part of the psychologist's or human factors engineer's bag of tools (probability, statistics, experimental design, etc.). These tools for measuring and estimating, for making decisions about hypotheses, and for planning are equally important for many operations research techniques.

Others of these mathematical tools may not be so familiar. It is important to note that not all of these tools are necessary for any single operations research technique. As each technique is discussed, we will note which specific tools are needed, so the user can determine whether he already has the requisite skills or whether he can obtain them easily enough to make the technique practical for his use.

## **E. COMBINING HUMAN FACTORS AND OPERATIONS RESEARCH**

The basic procedure used in this study is to identify and describe specific operations research models and their accompanying techniques, and to apply some of these models to given human factors-related problems. Table 4 illustrates the format which is used throughout the report. For each model or technique, the following are provided:

**TABLE 3**  
**MATHEMATICAL TOOLS FOR OPERATIONS RESEARCH TECHNIQUES**

Arithmetic  
 Algebra, simple  
 Algebra, linear or matrix  
 Algebra, Boolean  
  
 Geometry, plane  
 Geometry, spherical  
 Geometry, analytic  
  
 Trigonometry  
  
 Calculus, single variable  
 Calculus, multiple variable  
  
 Logic and set theory  
 Fuzzy set theory  
  
 Probability theory  
  
 Statistics, descriptive  
 Statistics, inferential  
  
 Experimental design  
  
 Graphs and plots  
  
 Computer programming  
 Computer packages

1. The kinds of questions or problems for which it is especially useful--that is, what it is good for.
2. The kinds of mathematical skills required for proper use.
3. In general, the kinds of human factors engineering applications we see as particularly appropriate.
4. Examples of the use of the model for human factors-related problems, as reported in the literature (where available).
5. References and texts for more information, if desired.

**TABLE 4**  
**DESCRIPTION OF OPERATIONS RESEARCH MODEL/TECHNIQUE**

**OPERATIONS RESEARCH MODEL/TECHNIQUE:**

- 1. PURPOSE OF MODEL/TECHNIQUE:**
- 2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:**
- 3. HUMAN FACTORS APPLICATIONS:**
- 4. DESCRIPTION:\***
  - a) Model:
  - b) Assumptions:
  - c) Strengths:
  - d) Weaknesses:
  - e) Procedures:
  - f) Other calculations that may be made:
- 5. ACM EXAMPLE:\***
  - a) Situation:
  - b) Procedures:
- 6. USED IN LITERATURE:**
- 7. REFERENCES AND TEXTS:**

**\*For selected models only**

For several of these techniques (primarily those that are among the most important for the operations research field), additional elaboration on the model or technique is provided:

6. The assumptions underlying the use of this model or technique.
7. What its strengths and weaknesses are.
8. General procedures for using the technique.
9. A worked-out example of application of this technique to a human factors problem.

Descriptions necessarily are brief; no attempt is made to be mathematically rigorous or to cover all of the rich complexity of many of these techniques. Such a compendium would require many volumes. It also would defeat the purpose of this study, which is to familiarize human factors engineers with a set of practical tools, and help them decide which of these tools may be applicable to their special problems.

### III. NAVY MISSIONS: AIR COMBAT MANEUVERING

The U.S. Navy is tasked with a number of critical missions, all related in some way to defense of our forces at sea. Table 5 lists these mission areas.

TABLE 5  
NAVY MISSIONS

Sea control  
Power projection  
Fundamental missions  
    Antiair warfare  
    Antisubmarine warfare  
    Antisurface ship warfare  
    Mine warfare  
    Air-to-ground warfare  
Supporting missions  
    Mobility  
    Command, Control, Communications  
    Intelligence  
    Electronic warfare  
    Logistics

For simplicity and consistency, a single mission area, that of antiair warfare, has been selected here for illustration of how operations research techniques may be applied to a variety of human factors engineering problems. Within that broad area fall both surface-to-air and air-to-air combat. The latter of these will be given primary emphasis in this study.

Air-to-air combat most often is referred to as "air combat maneuvering" or ACM. It involves in-air battles between two or more adversary aircraft. The three-dimensional nature of such battles in space makes them especially rich material for modeling. Navy fighter aircraft carry either a single pilot or a pilot and a weapons system officer (often called a radar officer (RO) or a radar intercept officer (RIO)). Depending on the type of plane, on-board sensors include various radars, infrared systems, television systems, and laser detectors (plus occasionally rifle scopes purchased at a local sporting goods store). Electronic countermeasures also can add complexity to sample scenarios, as can various rules of engagement (such as a requirement for visual identification of the adversary before missile engagement).

The Navy's fighter aircraft include the F-4 Phantom, F-14 Tomcat, and F-18 Hornet. The first two are two-seat and the third a single-seat aircraft. U.S. Navy air-to-air weapons consist of:

1. Aircraft guns, for close-in engagements (often called dogfights)
2. Sidewinder (AIM-9) heat-seeking missiles, for short-range engagements
3. Sparrow (AIM-7) radar-guided missiles, for short- and intermediate-range engagements
4. Phoenix (AIM-54) radar-guided missiles, for long-range engagement of enemy aircraft (F-14 aircraft only)

For this study, data, procedures, and tactics from various ACM-related activities will be used in modeling, analysing, and making decisions about this type of mission, using a variety of operations research techniques.

#### IV. LITERATURE SEARCH

A major attempt was made to review existing literature involving some aspect of the combination of human factors and operations research. Some 1500 citations were obtained, using keywords related to both of those two disciplines. Review of abstracts of these publications revealed a significant point: authors and abstractors use extremely broad definitions of these two fields. Numerous citations involved neither of the two, as they are defined for this study.

An early review, conducted by Raben in 1960 [Ref. 22], apparently yielded a similar result. She reviewed 1000 references, and included approximately 500 of these in her report. The definition of operations research she uses is very broad:

1. Moving scientific research into the everyday world of business, government, and industry.
2. Providing decision makers with an efficient basis for making decisions regarding the operations under their control.
3. Going after the immediate problems in complex organizations.

Operations research was still a young field at that time (the original text on the subject, Morse and Kimball's Methods of Operations Research, was only 10 years old). Basically, four operations research techniques are included in Raben's study:

1. Communicaton and information theory
2. Game or decision theory (which includes a very brief reference to linear programming)
3. Computers and simulations

4. Queueing theory (including work measurement techniques)

For choosing an operations research technique, Raben quotes Hoag [Ref. 23] as proposing that one examine the problem at hand and ask:

1. What are the relevant alternatives?
2. What test of preferences should be applied in choosing among alternatives?
3. How do we go about the process of weighing objectives against costs?

Of the 500 reports cited by Raben, less than 20 appear to meet our present criteria of human factors engineering and operations research in combination. The rest are about evenly divided between relatively "pure" psychology or human factors books, reports, or studies (i.e., E.J. McCormick's Human Engineering), and relatively "pure" mathematics or operations research books, reports, or studies (i.e., R.L. Ackoff's Principles of Operations Research).

For this present study, citations were obtained from the Defense Technical Information Center (DTIC), and the DIALOG Information Services PSYCINFO database, along with the Naval Postgraduate School thesis and reports database retrieval system. In addition, a collection of about 500 citations of operations research/human factors reports, compiled by students for human factors courses in the Operations Research Department, were reviewed for applicability to this project.

Of these varied citations, 55 definitely pertain to both operations research and human factors, and are listed in this thesis either in the list of references or under the technique or model to which they apply. An additional 234 citations have some applicability; the most pertinent of these are included here. Another 159 were noted and reviewed, to some extent, but were found of little interest

for this study. All-in-all, more than 450 reports were reviewed, either in the form of the original publications or from an abstract.

## V. MODELS AND MODELING

Before beginning discussion of some of the specific types of models used in operations research, it is useful to consider the topic of models in general. What is a model? How do we develop or select an appropriate model? And what do we do with a model, after we have one?

### A. THE NATURE OF MODELS

Model building is considered by Wagner (and many others) to be the essence of operations research [Ref. 24]. By formulating, manipulating, and analysing models, it is possible

1. to put the complexities and uncertainties attending a decision-making problem into a logical framework amenable to comprehensive analysis,
2. to clarify decision alternatives and their anticipated effects,
3. to indicate the data that are relevant for analysing the alternatives, and
4. to lead to informative conclusions.

In short, the model is a vehicle for arriving at a well-structured view of reality.

Even more important, the model is what is used with operations research techniques. These procedures are not intended for operations on real-world objects, but rather on some abstraction of reality--on some representation (shadowy or concrete) of such objects.

But what is a model? According to Kantowitz and Sorkin [Ref. 25], models are abstract representations of systems or subsystems. Models attempt to describe, explain, predict,

or control the behavior of whatever they represent. Models can be verbal, physical, mathematical, or a combination of these. Examples include:

1. Verbal/symbolic: a description in words and geometric symbols, such as a sequential flow diagram (as in Figure 2.1) or an operational sequence diagram.
2. Physical: a model airplane in a wind tunnel, or a set of electronic components wired together to represent a tornado--or even a military exercise.
3. Mathematical: almost any function, equation, or inequality:  $e=mc^2$  is a model of the energy-mass relationship;  $\text{speed} \leq 55 \text{ mph}$  is a model representing a standard constraint placed on highway travel; computer simulations usually are based on math models.

Rouse likens a model to an analogy [Ref. 26]. One of the most powerful problem solving methods in science, this involves viewing a new problem as if it were an old problem for which one may know the answer, or at least possess considerable insight. The set of analysis tools already proven for the old problem then is available for attacking the new one.

Continuing in this vein, Rouse suggests nine analogies of human behavior he considers useful in the modeling process [Ref. 27]. These are:

1. Electrochemical network: treating the human as a simple net of neurons which interact according to basic physical laws.
2. Information processor: using models normally used for storage and retrieval of symbols, or during employment of information theory (as in a communications channel).
3. Pattern recognizer: use of product inspection and process monitoring modeling techniques.

4. Ideal observer: models used in estimation theory or in signal detection theory.
5. Servomechanism: vehicle control, tracking, and process control analogies and models.
6. Time-shared computer: resource allocation models.
7. Logical problem solver: use of set theory operations, including logical implications and procedures.
8. Planner: rule-based (or production) systems and models, or time/frequency domain series models.
9. Reflector-daydreamer: the upper limit on modeling the human, requiring techniques that are not fully established.

These individual analogies or simple models may be combined into what Rouse calls composite analogies: structures or frameworks integrating two or more of the basic analogies into a cohesive, purposeful entity. Such a structure then may be used to describe behavior.

When an analogy within a particular area of research gathers a sufficient number of adherents, it is often termed a paradigm. Rouse awards paradigm status to the analogy of the human as a servomechanism--an error-nulling or self-correcting device.

What characterizes a good model? According to Daellenbach and others [Ref. 28], there are five important qualities:

1. Simplicity. Only those aspects of the system that have significant effects on performance should be included.
2. Robustness. It should be difficult to cause the model to give bad answers, particularly answers that are outside the previous range of experience.
3. Ease of manipulation and use. Extensive training or experience should not be required.

4. Adaptability. It should be easy to change input parameters and obtain updated solutions.
5. Completeness. All important aspects of the system should be included in the model.
6. Ease of communication. The user should be able to change inputs easily and obtain answers quickly.

There are a number of ways models can be classified into categories. Two of the most useful divide them into either deterministic or probabilistic classes, and into either descriptive or prescriptive classes.

In brief, a deterministic or mechanistic model considers nature to be a fully predictable machine, and implies that one will use it "to find out exactly". This is known as decision making under certainty. Such a model yields a number (or a sequence of numbers) as its end product--with certainty. For example, given a specific input, the output will be a number or range of values (dollars, kilograms, etc.) which will result from a given manufacturing process, under controlled conditions.

A probabilistic model deals with problems of decision making under uncertainty or risk, typically assuming that the probabilities of the alternative states of nature are known. A stochastic model is a probabilistic model which has time as one of its factors. These types of models work with a collection of assumed possible outcomes, and usually yield a probability or set of probabilities for these outcomes as its end product, according to Larson [Ref. 29]. Weather forecasting models are probabilistic. So are many other forecasting models, which provide the likelihood of some occurrence, based on a given set of conditions.

A descriptive model is just what it sounds like: it attempts to describe the system being modeled. This description then can be used for prediction or decision making, if desired. Most stochastic models are descriptive in nature.

A prescriptive model prescribes what action should be taken with a system to obtain a desired outcome. Linear and nonlinear programming models are examples of prescriptive models. Given a set of variables,  $X$ , under our control, and a set of variable outcomes,  $Y$ , not under our control except that they depend on  $X$ , then our criterion function or objective function is  $f(x,y)$ , a function of both of these variables. This criterion function often is used in setting up a measure of effectiveness (MOE). The possible values of  $X$  result in various values for  $Y$  and for the function  $f$ --some "better" or more effective than others, for the system of interest. The goal is to select a value of  $X$  which will yield a solution that is "good enough" (or, to use the decision theory term, is "satisficing").

Rouse suggests that models can serve four purposes [Ref. 30]. These include:

1. Providing insight into the system and its interrelationships, for which the modeling process in and of itself is beneficial (regardless of the ability to make further use of the model).
2. Giving succinct representations and explanations of data, allowing clearer comparisons among tasks and experiments.
3. Assisting in design of experiments, after an approximate model suggests what parameters may affect performance most strongly.
4. Yielding quantitative predictions about the system that was modeled.

Rouse also makes a distinction between human-system behavior models and performance models [Ref. 31]. The former are more general and more difficult to develop, since a variety of patterns of behavior might result in the same performance. For many engineering applications, performance predictions are all that are necessary. While the

"stronger" behavioral models more completely describe the human as the task of interest is performed, they also may result in such generality that the engineer cannot use their answers.

Models may be special purpose (peculiar to a specific problem) or general purpose (adaptable to any system which satisfies the underlying assumptions). Although general purpose models are sometimes simply referred to as techniques [Ref. 32], for purposes of this study, the two terms will be kept separate. Here, the model will be used to represent the system, while the technique will be used to shed light on the system or to answer questions about the system, via operations performed on the model.

Many problems may be solved through the use of several techniques (perhaps using several different models), each offering certain advantages. In order to choose the technique that best fits the problem at hand, it is necessary to be familiar with the features of each which make it especially useful under various conditions. We will attempt, in this study, to point out these features.

As a final point here, the limitations of models and of their corresponding techniques must be considered [Ref. 33]. The ability to develop practical, useful models of human-machine interaction is limited by:

1. Measurement and computational difficulties, since many human processes cannot be directly observed, and no two individuals may perform alike.
2. The fact that the human often behaves in a non-optimal manner, settling for "good enough"--a very difficult situation to model. This may be a function of the artificiality of the laboratory environment. It also may be due to the limitations of working (short term) memory and the division of attention--for example, conservatism in decision making is

nonoptimal, yet is consistent with known human limitations.

3. Criticality of the environment in which a task is to be performed, making it difficult to generalize from one set of data to another.
4. The virtual impossibility of modeling the totality of human behavior, including the above-mentioned reflecting and daydreaming activities.
5. The difficulty of truly understanding most models by anyone other than the model's developer.

#### B. THE PROCESS OF MODELING

Sinclair and Drury suggest four questions to ask before beginning any modeling process [Ref. 34].

1. Is mathematical modeling likely to be applicable and useful? Two areas representing safe ground for modeling are:
  - a) Where it is reasonable to assume the man in the system is acting as a "logical machine".
  - b) Where physiological or biodynamic processes of the human body are being modeled, with no "willful" control by the operator.
2. At what level are you working? The man-machine system as a whole can be modeled, or only individual components within a single man-machine system can be considered.
3. Which is the limiting subsystem? Is it the man's anatomy and physiology, his perceptual capability, or his decision making ability? Choosing the limiting subsystem will to a large extent force the selection of the type of model needed.
4. Does an appropriate model already exist? The authors include a fairly comprehensive list of models (14 in

all) which have been successfully used in human factors work. These are categorized as biomechanics, man-in-loop, decision theory, visual search, and metabolic. Only if there is no satisfactory existing model should one consider building a new one "from scratch".

Olkin [Ref. 35] considers models to be abstract and simplified descriptions of given phenomena. To build a model, certain basic aspects of the phenomenon are isolated as being of primary interest. An analogy is drawn between these aspects and some logical structure--concerning which we already have detailed information (see Figure 5.1). Models most often are based on mathematical structures of various kinds.

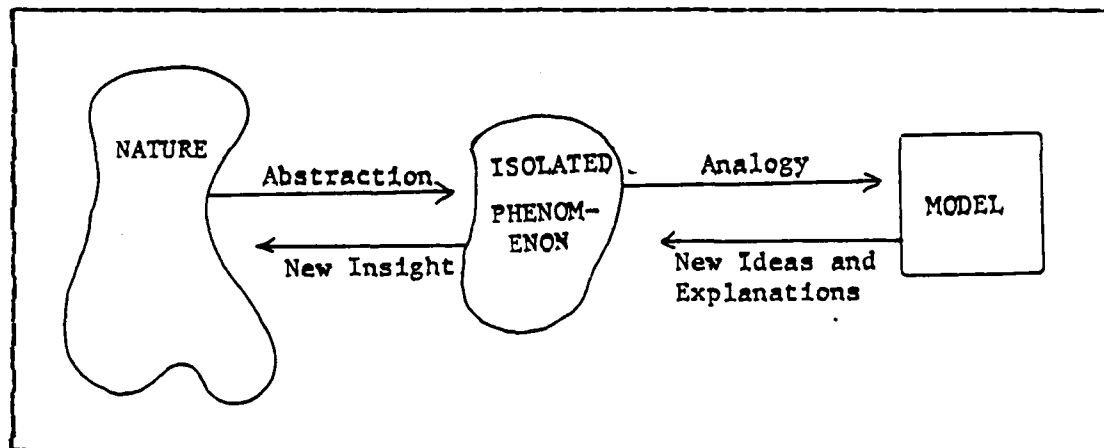


Figure 5.1 Modeling Process, as Described by Olkin

A model need not be complex or completely precise to be useful. Criteria for choosing a model are practical, not metaphysical, Olkin emphasizes. Does the model provide a simple, yet comprehensive explanation of the known phenomenon? At the same time, does it have strong potentiality for providing insight into the natural world? If so, it warrants consideration.

To be of use, a model must be elaborate enough to represent reality, but also sufficiently simple to remain tractable, Daellenbach and others emphasize [Ref. 36]. Simplicity in a model can be achieved only by making suitable approximations. These authors list six useful ways to do so:

1. Omitting variables. To determine whether a variable has a significant effect on the measure of effectiveness, statistical tests and techniques such as correlation, regression analysis, and analysis of variance and covariance are used. Variables which contribute only insignificantly to description of the system should be removed.
2. Aggregating variables. Activities and items which are similar can be lumped into a single variable, as can those which individually have low values.
3. Changing the nature of variables. Sometimes variables may be treated as constants, for simplicity's sake--such as when an average value is substituted for a random variable, or when conducting a parametric analysis. Discrete variables may be treated as continuous, and vice versa, when it is useful.
4. Approximating the relationship between variables. Linear and quadratic functional relationships are easier to deal with than are cubic or other nonlinear functions, and the simpler relationship may be entirely adequate for modeling purposes.
5. Omitting constraints. Limitations which make modeling difficult may be ignored, initially. If the solution is found to violate one or more of these constraints, they subsequently may be introduced.
6. Disaggregating the entity modeled. One single model that covers the entire system may be highly complex and difficult to find a solution for; such a problem

may be broken into smaller and partially self-contained submodels, as an approximation.

There is no "recipe" for making models, notes Morris [Ref. 37]. The teaching of modeling is not the same as the teaching of models, he states. Modeling tends to be an intuitive or artistic skill, largely the result of imitation and practice. Facility in modeling appears to be associated with a feeling of being at ease with mathematics, an appreciation of the various purposes models may serve, and familiarity with the characteristics of models.

Morris provides seven suggestions for the novice model-builder:

1. Factor the system problem into simpler problems, which can be modeled individually, then recombined into a system model.
2. Establish a clear statement of the deductive objectives: the purpose of the model and what the results are to be used for.
3. Seek analogies between the problem at hand and some previously well-developed logical structure; is the problem one in linear programming, in queueing, or in inventory?
4. Consider a specific numerical instance of the problem; retreating from generality and complexity helps to make clear the assumptions which characterize the example, and frequently allows "solution" by inspection.
5. Establish some symbols: write down in symbolic terms (letters and numbers and arithmetic operator characters) some of the obvious things which can be seen in the numerical example.
6. Write down the obvious: conservation laws, input-output relations, ideas expressed in the assumptions, or the consequences of trivially simple policies.

7. If a tractable model is obtained, enrich it.  
Otherwise, simplify.

Given the model, regardless of its type or degree of complexity, it now should be possible to apply various operations research techniques in order to answer some of the questions of interest (or at least to understand the problems better).

### C. FINDING AND EVALUATING SOLUTIONS

Having settled upon an appropriate model for a given human factors engineering problem, the next step is to find a solution. Operations researchers speak of solving the model or of finding its solution. Specific ways of finding solutions using the operations research techniques covered in this study are included in each technique's section. However, the more generalized concept of finding optimal or acceptable solutions should be discussed first.

The desired solution sometimes may be discovered simply by breaking a problem down into its component parts, laying these out in some logical pattern, and inspecting them closely. This is solution by analysis.

More often, numeric methods are required. The most powerful numeric methods are based on an algorithm [Ref. 38]. An algorithm may be defined as a set of logical and mathematical operations performed in a specific sequence. Sometimes this is done by hand--paper and pencil and a hand calculator. More often (in operations research, at least) a computer is used.

To use an algorithm, usually an "initial solution" is needed. This may be obtained by some arithmetic or logical technique, or simply by guessing, based on whatever data are available about the system of interest. The algorithm is applied to the initial solution, in order to derive a new

(and, ideally, better) solution. The sequence of operations leading to the new solution is called an iteration. The new solution is substituted as the starting point, and the process repeated. This continues until certain conditions (called stopping rules) are satisfied. At this point, the optimal solution has been reached with the desired degree of accuracy--or else no feasible and bounded solution exists, as the problem is presently set up.

Daellenbach and others list the properties of a practical, useable algorithm:

1. Each successive solution must be an improvement over the preceding one.
2. Successive solutions must converge to the optimum solution.
3. Convergence must occur in a finite number of iterations.
4. The computational requirements of each iteration must be sufficiently small to remain economically feasible.

Given a possible solution, sensitivity analysis usually is performed on it. How the optimal solution would change if input data are changed (as they might change in the real world) is systematically evaluated. This is especially useful in determining just how accurate the input data for the model must be. It also establishes the range within which input values may vary, given this model, and still result in a near-optimum solution. And if some of the variables represent resources which are scarce, sensitivity analysis enables one to place a value on these resources [Ref. 39].

Before any solution is implemented into a real-world system, it must be validated or tested. This is necessary to determine that the solution will remain feasible when introduced into the actual (versus the model) situation, and

that the benefits will be sufficient to warrant the required changes.

Cross validation usually is done by checking the proposed solution with new data (not that used to derive the model and optimal solution). These data values must be representative of future behavior of the system, and the testing should be extensive enough to allow for evaluation of the variability of the outcomes with time.

## VI. MODELS FOR DESCRIBING AND PREDICTING

A vastly heterogeneous set of mathematical procedures is included in this chapter. They are linked together by their common purpose: describing some system in mathematical terms, so that predictions may be made about the system. Under some conditions, each technique provides more predictive ability than could be obtained simply by using the average value of past performance as a predictor of future performance. This is a good criterion for a useful model [Ref. 40].

Researchers collect varying kinds of data, in a variety of ways, depending on the situation of interest. According to Nie and others [Ref. 41], the most common situation is one in which only a relatively small number of variables are to be analysed. For any one piece of analysis, it usually is possible to arrive at one dependent variable that is to be explained and at a limited number of other variables with which to explain it. Multiple regression is the procedure of choice in such an instance.

As the number of variables in the data set becomes larger, multiple regression becomes an unwieldy technique. Some form of data reduction is needed in order to make sense of it all. Factor analysis is the most common technique used for this purpose, combining several variables together to yield a single new variable (representing some larger concept).

Closely related to the above two techniques is that of discriminant analysis. This is the technique of preference if we desire to separate two or more groups of individuals on the basis of some discriminating factor. As with the above two (and also the following two), this technique

relies on a deterministic model, and requires that the measurement level of the variables be at least interval in nature, that relationships be linear, and that data values be known constants.

A fourth related technique is canonical correlation. To use this procedure, the experimenter divides his variables into two sets, each of which can be given theoretical meaning as a set (such as a behavioral set and an attitudinal set). Then a linear combination is derived from each set in such a way that the correlatiuon between the two linear combinations is maximized. The goal is to account for a maximum amount of relationship between two sets of variables (rather than accounting for the maximum total variance). A redundancy index is used for this purpose.

Multidimensional scaling is the fifth technique covered here. This procedure is intended for analysis of opinions about proximity from populations of interest, rather than for analysing and modeling experimental data. The end result is a graphical representation which describes a system and perhaps may be used in making predictions.

Control models are already in wide use in human factors analysis, at least in theoretical descriptions of the human as a controller. Thus these are touched on only briefly here. Time series models also are described only briefly here. They require computer packages to be of use (Box-Jenkins in SPSS, for example), and are best understood in the context of the particular computer and software that are available.

The next three techniques fall in the category of stochastic models. Markov chains, Poisson processes, and queueing processes are important general operations research models. Two of the three are discussed in depth, so that human factors engineers may observe the usefulness of such procedures for their own modeling problems.

Reliability models also usually are stochastic in nature, since they rely heavily on various probability distributions to predict failure rates. Little used by human factors engineers, they deserve consideration where human errors are an important consideration (and the available data suggests that a known distribution is an adequate approximation for error rate).

The final modeling technique covered in this chapter falls in the broad category known as simulation models. These versatile models can be either deterministic or stochastic. They are growing in popularity with human factors personnel, and are discussed here in depth.

#### A. REGRESSION ANALYSIS

1. PURPOSE OF MODEL/TECHNIQUE: Determining the mathematical relationship between a dependent or criterion variable and one or more independent or predictor variables, in order to describe that relationship and to make predictions based on whatever data values are available.

For example, the researcher may wish to predict the effect of age and of IQ (independent variables) on ability to operate a new tactical computer system. The criterion is the time required to solve a standard problem using that computer. One hundred subjects of known ages and IQs are tested on that problem, and their times for solution recorded. Multiple regression techniques then are used to develop an equation of the form:

$$Y = A + BX(1) + CX(2),$$

where Y represents the estimated or predicted time value that will result from this equation. A is the

Y-axis intercept, B is the regression coefficient related to age,  $X(1)$ , and C is the regression coefficient related to IQ,  $X(2)$ . Once A, B, and C have been determined from the collected data through multiple regression techniques, various values of age and IQ may be substituted for  $X(1)$  and  $X(2)$  in the equation, to come up with predicted time values, Y.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra
- b) Descriptive statistics
- c) Inferential statistics
- d) Graphs and plots
- e) Computer packages

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, in terms of expected responses or characteristics resulting from given inputs or from other personal characteristics.
- b) Describing systems, when a functional relationship is desired between one or more independent variables and some other variable which presumably is dependent on these.
- c) Designing systems, when predictions or inferences are needed in order to determine whether a system with certain given characteristics will result in desired (or necessary) human performance.
- d) Evaluating human performance, where performance can be measured in numerical terms and seems to be a direct result of certain conditions (which also can be measured numerically) imposed on or inherent in the human population of interest.

4. USED IN LITERATURE: It should be noted that the following list contains only a very small sample of the use of regression analysis for human factors research. This technique, which is an important one in operations research, has long been a mainstay of psychology, also.

- a) Bateman, R.P. "An Heuristic Approach to Work Analysis", Proceedings of the 23rd Annual Meeting of the Human Factors Society, Boston, MA, 1979, pp. 554-557.

Regression analysis is used to develop an equation showing the relationship of tracking performance to certain variables associated with multifunction keyboards.

- b) Chawla, S., and others. "Human Factors Considerations for a Combined Brake-Accelerator Pedal", Ergonomics, Vol. 14, No. 2, 1971, pp. 279-292.

Linear regression is used to relate accelerator and brake reaction time with various pedal designs.

- c) Kvalseth, T.O. "A Generalized Model of Temporal Motor Control Subject to Movement Constraints", Ergonomics, Vol 20, No. 1, 1977, pp. 41-50.

First and second order linear regression models are formulated which relate mean arm movement time to Fitts's index of difficulty variable and to a lateral movement constraint variable, for a number of kinds of constraints.

- d) Wardle, M.G. "A Psychophysical Approach to Estimating Endurance in Performing Physically Demanding Work", Human Factors, Vol. 20, No. 6, 1977, pp. 745-747.

Regression equations are developed which provide point estimates of the maximum working time to be expected at various levels of strenuous workloads.

- e) Williges, R.C., and Williges, B.H. "Modeling the Human Operator in Computer-Based Data Entry", Human Factors, Vol. 24, No. 3, 1982, pp. 285-299.

Human-computer interfaces were evaluated via operator satisfaction ratings, work-sampling techniques, and imbedded performance measurement. Polynomial regression analysis was used to generate functional relationships among these four metrics and four independent variables representing system delay, display rate, keyboard echo rate, and keyboard buffer rate.

5. REFERENCES AND TEXTS:

- a) Larson, H.J. Introduction to Probability Theory and Statistical Inference, Third Edition. New York: John Wiley and Sons, 1982.  
Provides theoretical and mathematical foundations for regression, for the mathematically inclined.
- b) Mendenhall, William, Scheaffer, R.L., and Wackerly, D.D. Mathematical Statistics With Applications, Second Edition. Boston: Duxbury Press, 1981.  
Clearly written, and nicely laid out for reference. Heavy reliance on matrix algebra. Again, for the mathematically sophisticated.
- c) Nie, N.H., and others. SPSS: Statistical Package for the Social Sciences, Second Edition. New York: McGraw Hill Book Company, 1975.  
An excellent introduction to the technique, as well as to the SPSS software programs to perform it. Note that the later SPSS-X Manual does not have the useful introductory information.
- d) Wright, R.L.D. Understanding Statistics: An Informal introduction for the Behavioral Sciences. New York: Harcourt Brace Jovanovich, Inc., 1976.  
Easy, non-threatening introduction to the subject of regression and to statistics in general. Only simple linear regression is included.
- e) Wonnacott, T.H., and Wonnacott, R.J. Introductory Statistics, Third Edition. New York: John Wiley and Sons, 1977.  
An excellent introduction to statistics in general, and to regression in particular, for those with limited mathematical experience. Simple and multiple linear regression and nonlinear regression are discussed.
- f) Younger, M.S. Handbook for Linear Regression. North Scituate, Mass.: Duxbury Press, 1979.  
Clear, complete, and easy to read text and reference book. Includes the use of computer programs for regression, such as BMD, SPSS, and SAS.

B. FACTOR ANALYSIS

1. PURPOSE OF MODEL/TECHNIQUE: Obtaining a parsimonious description of observed data. This is done by reducing an apparently large number of variables (many of which are correlated with others) to a

smaller number of source variables representing the same concepts, but under broader categories.

For example, perhaps empirical data values have been obtained for a number of children on their age, height, weight, reading ability, and grade in school. Age, height, and weight are correlated, as are age, reading ability, and grade in school. Factor analysis may be used to reduce the five data values for each child to two--probably representing a physical maturity variable and an intellectual maturity variable.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra, simple, linear
- b) Calculus, single variable, multiple variable
- c) Logic and set theory
- d) Descriptive statistics
- e) Inferential statistics
- f) Graphs and plots
- g) Computer packages

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, where data values have been obtained for several human variables, and it is desired to reduce these dimensions to a smaller, more meaningful set (factors).
- b) Describing systems, when numerous kinds of variable values are available for a system, and there is a need for reduced dimensionality.
- c) Designing systems, when the variables for the proposed system are needed in a succinct and orthogonal form.

- d) Evaluating human performance, if a few, relatively uncorrelated descriptive variables are desired for use in the measurements.

4. USED IN LITERATURE:

- a) Burke, E.J. "A Factor Analytic Investigation of Tests of Physical Working Capacity", Ergonomics, Vol. 22, No. 1, 1979, pp. 11-18.  
Sixteen tests of physical working capacity, submitted to factor analysis, are reduced to three factors which account for 71 percent of the total variance.
- b) Haslegrave, C.M. "Anthropometric Profile of British Car Drivers", Ergonomics, Vol. 23, No. 5, 1980, pp. 437-467.  
Factor analysis is used to explore the relationships among 17 dimensions used in design of cars. Three factors are extracted, then used in construction of a set of body indices for use in designing of anthropometric dummies.

5. REFERENCES AND TEXTS:

- a) Harmon, H.H. Modern Factor Analysis. Chicago: The University of Chicago Press, 1967.  
A comprehensive and detailed text for those who want to know the theoretical and mathematical formulations for this procedure.
- b) Morrison, D.F. Multivariate Statistical Methods. New York: McGraw-Hill Book Company, 1967.  
A mathematical explanation, with heavy reliance on matrix algebra.
- c) Nie, N.H., and others. SPSS: Statistical Package for the Social Sciences, Second Edition. New York: McGraw Hill Book Company, 1975.  
An excellent introduction to the technique, as well as to the SPSS software programs to perform it. Note that the later SPSS-X Manual does not have the useful introductory information.
- d) Rulon, P.J., and others. Multivariate Statistics for Personnel Classification. New York: John Wiley and Sons, 1967.  
A fairly complete explanation, including mathematical derivations, relying on graphical and matrix techniques.

### C. DISCRIMINANT ANALYSIS

1. PURPOSE OF MODEL/TECHNIQUE: To find and make use of characteristic variables which can distinguish between two or more groups. This is done via a collection of discriminating variables that measure levels of the characteristics on which the groups are expected to differ. These discriminating variables are weighted and combined so as to force the groups (and individuals in them) to be as statistically distinct as possible. Both linear and nonlinear combinations are possible.

For example, it may be desired to discriminate between persons who will work effectively with computer systems and those who will not. A set of discriminating questions could be devised and tested, for this purpose. These might query the individuals' attitudes towards working alone, self-correction of errors, sedentary occupations, etc.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra, simple, linear
- b) Logic and set theory
- c) Descriptive statistics
- d) Inferential statistics
- e) Graphs and plots
- f) Computer packages

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, by selecting group characteristics which can be used to define those differences.
- b) Describing systems, making use of the known characteristics of groups into which they might fall, and with which analogies might be useful.

- c) Designing systems, taking advantage of knowledge of those characteristics that user groups prefer.
- d) Evaluating human performance, by determining whether various individuals fall into various performance categories (inexperienced, average, superior, etc.).

#### 4. USED IN LITERATURE:

- a) Miller, R.E., Optimal Assignment of Air Force Pilots, Final Report. Government Reports Announcements, February 1974. (AD-781 035/1GA)  
A multiple discriminant analysis is performed, using ten test scores and training grades to classify a new pilot as optimally assignable to a transport, fighter, or reconnaissance aircraft or mission.

#### 5. REFERENCES AND TEXTS:

- a) Nie, N.H., and others. SPSS: Statistical Package for the Social Sciences, Second Edition. New York: McGraw Hill Book Company, 1975.  
An excellent introduction to the technique, as well as to the SPSS software programs to perform it. Note that the later SPSS-X Manual does not have the useful introductory information.
- b) Rulon, P.J., and others. Multivariate Statistics for Personnel Classification. New York: John Wiley and Sons, 1967.  
A good, clear explanation, including mathematical derivations, relying on graphical, calculus, and matrix techniques.

### D. CANONICAL CORRELATION

- 1. PURPOSE OF MODEL/TECHNIQUE: Given two sets of variables, each of which has a theoretical meaning, a linear combination is derived from each set so that correlation between the two linear combinations is maximized. Several such pairs (canonical variates) of linear combinations may be derived from one data set. A redundancy index is used to account for a maximum amount of relationship between the two sets

of variables. This technique can be considered an extension of multiple regression analysis to the case of multiple criteria.

For example, individual attitudes toward reading and toward arithmetic may be scaled by a group of individuals (from zero for dislike to 10 for extreme enjoyment). The same individuals may be tested on reading speed and on computational speed. Data values for these two sets of variables (two variables per set, one set representing attitude and the other behavior) then can be compared, using canonical correlation. The resulting correlation coefficients are used in a mathematical function to describe the relationships between the two attitudes and two behaviors.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra
- b) Descriptive statistics
- c) Inferential statistics
- d) Graphs and plots
- e) Computer packages

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, where differences can be measured and grouped into two sets of variables whose correlations are useful.
- b) Describing systems, when system parameters can be treated as described above for individual differences.
- c) Designing systems, when it is important to relate one set of parameters with another in order to maximize the system's usefulness.

- d) Evaluating human performance, when performance variables can be measured and correlated with other variables in order to determine that performance will be good enough for the job to be done.
4. USED IN LITERATURE: No examples of the use of canonical correlation were found, relating both to human factors and to operations research.
5. REFERENCES AND TEXTS:
- a) Morrison, D.F. Multivariate Statistical Methods. New York: McGraw-Hill Book Company, 1967.  
A brief explanation in mathematical terms, with heavy reliance on matrix algebra.
  - b) Nie, N.H., and others. SPSS: Statistical Package for the Social Sciences, Second Edition. New York: McGraw Hill Book Company, 1975.  
An excellent introduction to the technique, as well as to the SPSS software programs to perform it. Note that the later SPSS-X Manual does not have the useful introductory information.
  - c) Stewart, Douglas, and Love, William. "A General Canonical Correlation Index", Psychological Bulletin, vol. 70, No. 3, 1968, pp. 160-163.  
A redundancy index is described, to handle the previous canonical correlation problem of not providing a measure of redundancy in one set of variables, with respect to a second set. The index represents the amount of predicted variance in a set of variables.

## E. MULTIDIMENSIONAL SCALING

1. PURPOSE OF MODEL/TECHNIQUE: Determining the underlying structure of a set of points in n-dimensional space (for n-dimensional scaling), after individuals have assigned a set of proximities to the points describing how close (in distance or similarity) they believe each point is to every other point. The resulting multidimensional scaling solution provides the minimum number of dimensions underlying the

structure, and gives the order of the instances along each dimension.

This technique may be used in determining the dimensions of a given task, thus leading to development of as many unidimensional scales as the task has dimensions. If each task component is rated in criticality, then weights can be assigned to each dimension.

For example, 100 aviators may be tested with a symbol set proposed for use on a cockpit map display, to determine whether they feel they can discriminate rapidly between each pair. The result is a "confusion matrix", showing the percent of time each symbol will be confused with another. This matrix of values is entered into a multidimensional scaling computer program for analysis. The result is a two-dimensional plot which places easily confused symbols next to one another, and those seldom or never confused at greater distances from one another. The researcher then provides an interpretation to the results, based on his expertise and on this configuration of points.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra, simple, linear
- b) Logic and set theory
- c) Descriptive statistics
- d) Inferential statistics
- e) Graphs and plots
- f) Computer packages

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, where opinions about similarities and dissimilarities between individuals are desired.

- b) Describing systems, where similarities or proximities between various components can be judged by individuals and then subjected to analysis for use in predictions.
- c) Designing systems, where it is necessary to determine that two or more components are or are not similar or close, in order to make choices and decisions.
- d) Evaluating human performance, when performance can be judged in terms of proximities or similarities to various criteria.

4. USED IN LITERATURE:

- a) Harris, D.H. "Human Dimensions of Water Resources Planning", Human Factors, Vol. 19, No. 3, 1977, pp. 241-251.  
Computer-based multidimensional scaling techniques are used to determine the underlying dimensional structure of 42 factors related to water resources planning and decisions. A value reflecting social importance is developed for each of these and for the five basic dimensions emerging from the multidimensional analysis.

5. REFERENCES AND TEXTS:

- a) Kruskal, J.B., and Wish, Myron. Multidimensional Scaling. Beverly Hills: Sage Publications, 1978.  
Complete and readable explanation of the theory and procedures. Computer programs for the technique also are discussed.
- b) SAS Institute, Inc. SAS User's Guide. SAS Institute, Inc., 1979.  
Instructions for the Alternating Least Squares Algorithm (ALSCAL) program, using the SAS computer package.

F. MANUAL CONTROL MODELS

- 1. PURPOSE OF MODEL/TECHNIQUE: Describing human behavior in terms of a servomechanism (error-nulling

device), in order to predict the effect of varying conditions on the operator's performance while controlling a system. The most important performance criterion is minimization of deviation of the state of the controlled process from a desired state. A simple tracking task, where an observer must keep a pipper on a target, is a good example of a use of this model.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra
- b) Single variable calculus
- c) Descriptive statistics

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, when an individual receives information through his senses about some world state and uses that information to control some situation manually.
- b) Describing systems, when a system's present state is used as input in order to provide direct outputs back to the system via some manipulation, with the goal of minimizing system error.
- c) Evaluating human performance, when performance at simple control of some system is being measured and compared with a criterion.

4. USED IN LITERATURE:

- a) Bekey, G.A., Burnham, G.O., and Seo, J. "Control Theoretic Models of Human Drivers in Car Following", Human Factors, Vol. 10, No. 1, 1977, pp. 399-413.  
Three mathematical models are used to describe control behaviors of human drivers. First is classical (manual) control, with assumptions about the driver's stimulus-response characteristics and control strategy algorithms. Second is based on optimal control theory, assuming a performance index and a driver's control strategy intended to minimize this index. Third is a set of heuristic models.

- b) Lau, C.G.Y. A Review of Human Operator Models in Manual Control Systems. Pacific Missile Test Center, Pt. Mugu, CA, February 1977 (PMTC-TP-76-40, AD B016 783L). Includes both quasi-linear describing function models and optimal control models. An extensive bibliography of manual control models is included.
- c) Levison, W. "A Methodology for Quantifying the Effects of Aging on Perceptual-Motor Capability", Human Factors, Vol. 23, No. 1, 1981, pp. 87-96. The application of experimental and analytical techniques of manual control is made, for quantification of aging effects in a model of human perception and control.
- d) Pew, R.W. et al. Critical Review and Analysis of Performance Models Applicable to Man-Machine System Evaluation. Bolt, Berenak, and Newman, Inc., Cambridge, MA, March 1977 (BBN No. 3446, AFOSR-TR-77-0520, AD-A038 597). Simple, quasilinear, and optimal control models are among the numerous models reviewed as part of this 300-page comprehensive report.

## 5. REFERENCES AND TEXTS:

- a) Rouse, W.B. Systems Engineering Models of Human-Machine Interaction. New York: North Holland, 1980. Theoretical presentation, with some application information; greatest emphasis is on discrete time optimal control, but manual control also is covered.
- b) Sheridan, T.B., and Farrell, W.R. Man-Machine Systems: Information, Control, and Decision Models of Human Performance. Cambridge, Mass.: The MIT Press, 1974, 1981. Detailed theoretical and mathematical presentation on manual control, with lesser discussion of optimal control.

## G. OPTIMAL CONTROL MODELS

1. PURPOSE OF MODEL/TECHNIQUE: Describing human behavior in terms of an optimum regulator (well-motivated, highly-trained individual, subject to known personal limitations). The controller is affected by and affects state variables of a system. Given a process to control, constraints on the accuracy with which he may observe state variables, and

limited energy or time for control tasks, the optimal controller seeks to maximize a cost function or performance criterion within his own constraints. The criterion usually is stated as a linear quadratic function of error, control effort, and time.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra
- b) Calculus, single variable, multiple variable
- c) Descriptive statistics

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, when the operator is considered sophisticated enough to recognize his own dynamics and the dynamics of the controlled process, his own variability, and the criterion to be met.
- b) Describing systems, when the situation is similar to that discussed above for the individual.
- c) Evaluating human performance, when actual performance can be compared to optimal behavior as described by this model.

4. USED IN LITERATURE:

- a) Barron, Sheldon. "A Model for Human Control and Monitoring Based on Input Control Theory", Journal of Cybernetics and Information Science, Vol. 1, No. 1, 1976, pp. 3-18.  
The state variable optimal control model of the human operator is reviewed and described in detail. Examples of its use in prediction and analysis are presented, along with advantages and limitations of the model.
- b) Barron, Sheldon, and Levison, W.H. "Display Analysis with the Optimal Control Model of the Human Operator", Human Factors, Vol. 19, No. 5, 1977, pp. 437-457.  
The optimal control model is applied in determining what information is needed on a display so the operator can meet his performance objective. The techniques are then applied to analysis of advanced display and control systems.

- c) Harvey, T.R. Application of an Optimal Control Pilot Model to Air-to-Air Combat. Master's thesis, School of Engineering, Air Force Institute of Technology, Wright Patterson AFB, OH, 1974.  
Two-dimensional kinematics of air-to-air combat tracking with a lead computing optical sight system were simulated on an analog computer, using a fixed-base simulator. Tracking error data values were collected from three pilots, using the simulator.
- d) Hess, R.A. "Prediction of Pilot Opinion Ratings Using an Optimum Pilot Model", Human Factors, Vol. 19, No. 5, 1977, pp. 459-475.  
Multiloop piloting tasks can be modeled via the optimal control formulation with relative ease. Numerical pilot opinion ratings concerning particular vehicles and tasks are related to the numerical value of the model's index of performance, using data from piloted simulations.
- e) Ince, Fuat. Application of Modern Control Theory to the Design of Man-Machine Systems. University of Illinois, August 1973 (NPS U 158758).  
Results from optimal control theory and the optimal control model of the human operator are used in design of control and display dynamics, and in predicting tracking performance.
- f) Seifert, D.J. Combined Discrete Network Continuous Control Modeling of Man-Machine Systems. Aerospace Medical Research Lab, Wright-Patterson AFB, OH, March 1979 (AMRL-TR-79-34, AD-A071 574).  
Open-loop optimal control models are combined with network models to describe the human operator as supervisor of a system. Tasks include information retrieval and cognitive processing, as well as flight control.

## 5. REFERENCES AND TEXTS:

- a) Rouse, W.B. Systems Engineering Models of Human-Machine Interaction. New York: North Holland, 1980.  
Theoretical presentation, with some application information; greatest emphasis is on discrete time optimal control, but manual control also is covered.
- b) Sheridan, T.B., and Farrell, W.R. Man-Machine Systems: Information, Control, and Decision Models of Human Performance. Cambridge, Mass.: The MIT Press, 1974, 1981.  
Detailed theoretical and mathematical presentation on manual control, with lesser discussion of optimal control models.

## H. TIME SERIES MODELS

1. PURPOSE OF MODEL/TECHNIQUE: To assess the magnitude and statistical significance of any changes in behavior or performance, as a result of an interruption (change of conditions) during a series of measurements over time.

For example, the accuracy with which an observer tracks a symbol on a CRT display can be measured over a period of days. The interruption is considered to occur when the observer is given a different type of display on which the same task will be done in the future. Time series analysis is used to determine whether any real change in performance (not due to simple learning) is evident, after the interruption.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra
- b) Probability theory
- c) Descriptive statistics
- d) Graphs and plots
- e) Computer packages

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, where the differences can be attributed to different responses to an interruption in a time series of measurements.
- b) Describing systems, where system performance over time is the factor of interest, especially as affected by some kind of change in conditions.
- c) Evaluating human performance, when performance changes may be expected as a result of known changed conditions, and a series of measurements (before and after) will be made.

#### 4. USED IN LITERATURE:

- a) Carter, R.C. "Time Series Models of Human Factors Dynamics," Human Factors, Vol. 26, No. 1, 1984, pp. 83-95.  
The Box and Jenkins multivariate time series model is used for analysis of human factors data representing U.S. Navy enlistments, career progression of technicians, spatial and verbal time cycles, and simple and choice reaction times.
- b) Krause, P.B. "The Impact of High Intensity Street Lighting on Nighttime Business Burglary," Human Factors, Vol. 19, No. 3, 1977, pp. 235-239.  
An interrupted time series design is used in an experiment which demonstrates the effect of night lighting on crime. Hazards that can arise if the serial dependence of successive observations is ignored also is illustrated.
- c) Shinnars, S.M. "Modelling of Human Operator Performance Utilizing Time Series Analysis", IEEE Transactions on Systems, Man, and Cybernetics, Vol. 4, No. 5, 1974, pp. 446-458.  
The time series approach is a useful method for modelling any set of discrete observables corrupted with noise, be it human or some other deterministic/stochastic process. Actual input-output data are used, in this time series model. The technique first identifies the model, then estimates the parameters of the identified model, based on the data. Finally, model improvement is made, by checking the fitted model with data.

#### 5. REFERENCES AND TEXTS:

- a) Cook, T.D. and Campbell, D.T. Quasi-Experimentation: Design and Analysis Issues for Field Settings. Boston: Houghton Mifflin Company, 1979.  
Complete and detailed discussion of the background and use of time series models in general and the Autoregressive Integrated Moving Average (ARIMA or Box-Jenkins) model in particular.
- b) SPSS, Inc. SPSS-X User's Guide. New York: McGraw-Hill Book Company, 1983.  
Detailed instructions for carrying out the Box-Jenkins procedure for time series data. SPSS Update 7-9 (1981) also includes the Box-Jenkins technique.

## I. FINITE MARKOV CHAINS

1. PURPOSE OF MODEL/TECHNIQUE: Describing possible "states" of a system, and predicting the probability the system will be in one of these states at some time in the future.
2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:
  - a) Linear or matrix algebra
  - b) Probability theory
  - c) Descriptive statistics
  - d) Computer programming (the APL programming languaging is especially useful for operations on matrices)
3. HUMAN FACTORS APPLICATIONS:
  - a) Describing individual differences; for example, illustrating the probability that someone will transition from one to another of four states (asleep, bored, alert, frantic) in the next time period.
  - b) Describing systems; for example, noting in concise form (matrix) the probability that a system will move from one state (OFF) to any one of three others (WARMUP, MANUAL, AUTOMATIC) in its next transition.
  - c) Evaluating human/system performance; for example, determining the probability that an individual or system will be in a state-of-interest, at some given time in the future.
4. DESCRIPTION:
  - a) Model: A system or process is described as a function of four things:
    - i) A finite set of objects (usually one).

- ii) A finite collection of possible discrete states the objects can be in or assume.
- iii) The initial probability: probability for each state that the process begins there.
- iv) The transition probability: probability the object will stay in that state or enter another state in the next time period.

b) Assumptions:

- i) Markov property. The likelihood of entering any state in the future depends only on the present state the object is in, not on any past states (also called the memoryless property); that is,

$$P[X(n+1)=j | X(n)=i, X(n-1)=i(n-1), \dots, X(0)=i(0)]$$

$$= P[X(n+1)=j | X(n)=i] = P(ij)$$

for all states  $i(0), i(1), \dots, i(n-1), i, j$ ;  
and all  $n \geq 0$ .

- ii) Stationarity. Transition probabilities do not change with time (are "stationary").
- iii) Certainty. All states and all transition probabilities used in the model are known constants, obtained through some empirical data collection process; the system is completely characterized by the set of states, initial probabilities, and transition probabilities.

c) Strengths:

- i) The procedure is simple to follow, appeals to logic, and is easy to defend.

d) Weaknesses:

- i) Values for the transition probabilities are critical; small errors in estimates for these can give significantly incorrect answers.
- ii) The independence of the probability of the next state from that of all past states, except the present one (Markov property), often does not mirror reality.
- iii) The requirement that states of existence be discrete can be difficult to meet, for many continuously varying situations.

e) Procedures:

- i) Define an exhaustive and mutually exclusive set of states the system can assume over time (preferably no more than a dozen).
- ii) Taking one state at a time, assign (by whatever means) a probability to the event that the process begins in that state, plus another probability that it either stays there or enters each of the other possible states from that state. These latter probabilities must sum to 1, for each beginning state. That is, if there are four possible states (including the one the system presently is in) and the chances are equally likely that it will be in any one of the four during the next time period, each event is given the probability 0.25).
- iii) As the first stage in the modeling process, put this information into matrix form, as

illustrated below in 5.c. This is the transition matrix,  $P$  (also called the one-step transition matrix). This transition matrix plus the set of initial probabilities now (under the model's assumptions) completely characterizes the system or process--this is the complete model.

iv) To determine the probability the system will be in a given state two steps or stages onward (two time periods ahead), square the transition matrix (use matrix multiplication to multiply it by itself) to obtain the two-step transition matrix,  $P(ij)^2$ . Cubing the matrix yields the three-step transition matrix,  $P(ij)^3$  (the probability the system will be in a given state three time periods ahead), etc.

v) The long-run probabilities (also known as steady-state, stationary, or equilibrium probabilities),  $\pi$ , are obtained by repeatedly multiplying the transition matrix by itself until limiting (essentially constant) values for the probabilities are reached (if they exist). These represent the long-run proportion of time the system will be in each of its possible states. These values also may be obtained by solving a set of linear steady state equations of the form

$$\begin{aligned}\pi(1) &= \pi(1)p(11) + \pi(2)p(21) + \dots + \pi(n)p(n1), \\ \pi(2) &= \pi(1)p(12) + \pi(2)p(22) + \dots + \pi(n)p(n2), \\ \pi(j) &= \pi(1)p(1j) + \pi(2)p(2j) + \dots + \pi(n)p(nj), \\ \pi(n) &= \pi(1)p(1n) + \pi(2)p(2n) + \dots + \pi(n)p(nn).\end{aligned}$$

Values are found for each of the limiting probabilities,  $\pi(i)$ , using a subset of  $(n-1)$  of these equations, along with the relationship that the sum of all the  $\pi(i)$ ,  $i=1, 2, \dots, n$ , must equal 1 (normalization equation).

f) Other calculations that may be made, using the transition matrix, include the following. See references below, for details of these calculations.

- i) Probability of first passage time: the likelihood the system will enter a specific state,  $t(i)$ , at a future time, given that it started in some specified state,  $t(j)$ .
- ii) Expected first passage time: the mean value of the time it takes the system to move from state  $t(i)$  to state  $t(j)$ .
- iii) Expected recurrence time: the average value of the time it takes the system to return to state  $t(i)$ , when it has been there once.
- iv) Absorption probabilities: the probability that a system will enter one of its possible states and never be able to leave that state (be "absorbed" into that state).

5. ACM EXAMPLE (adapted from Oberle; see below):

- a) Situation: A one-on-one engagement between a fighter and adversary. The relative positions and orientations of the two combatants are divided into five tactically meaningful states (although such transitions between states in actuality would be continuous, they are discretized here for use

with the Markov property), as follows (see Figure 6.1):

- i) Offensive weapon (OW): the fighter has a "rule of thumb" weapon opportunity, giving him an almost-sure kill opportunity.
  - ii) Offensive (O): the fighter is acting in the role of pursuer and has a tactically significant advantage in position.
  - iii) Neutral (N): both combatants are maneuvering head-to-head in an attempt to achieve a position of tactical advantage.
  - iv) Defensive (D): the adversary is acting in the role of pursuer and the fighter is reacting to the adversary's positional advantage.
  - v) Fatal defensive (FD): the adversary has a "rule of thumb" weapon opportunity.
- b) Procedures:
- i) Direct (one-step) transitions can occur only between adjacent states. At each 5-second time division during an engagement, the fighter is classified as being in one of the five states. Oberle provides the simulated data shown in Figure 6.1 for a hypothetical 115-second exercise (artificially divided here into 23 5-second time segments).
  - ii) Although Oberle does not do so, the same data can also be given in the format of a more standard Markov chain model, as is illustrated as in Figure 6.2. States are shown as circles, transitions as arrows, and transition probabilities as fractions.

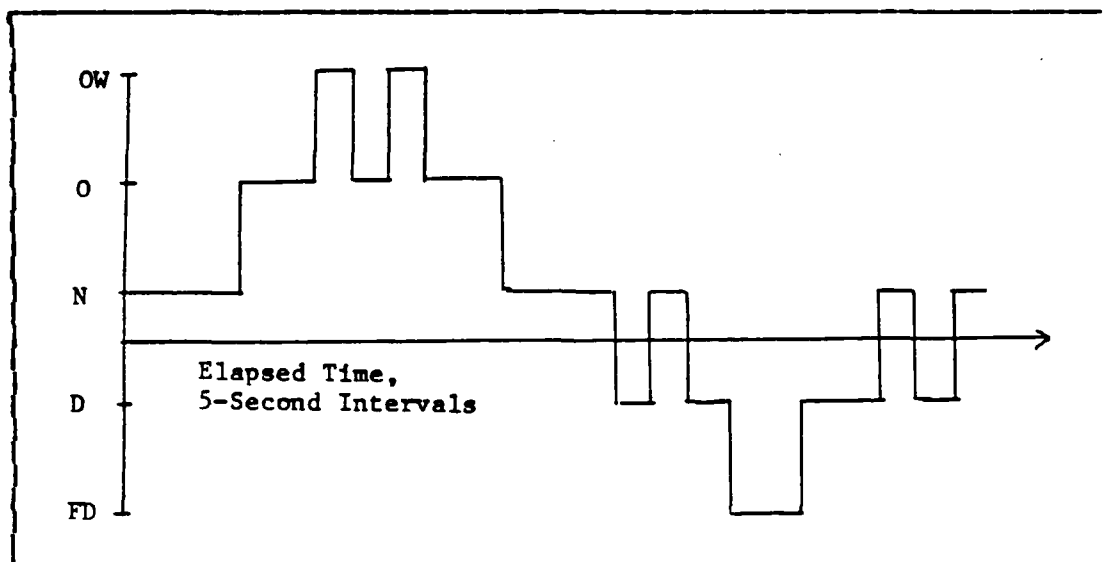


Figure 6.1 Transition States During a Simulated Engagement

- iii) One-step transition matrix: Oberle does not put his simulated data into transition matrix format, nor does he make any further calculations in this report. However, given this data, it is easy to carry out additional operations usually performed with Markov chains.

For purposes of this example, it is hypothesized that the initial probability of being in a neutral position at time 0 is 0.5; the probability the fighter will still be in that neutral position at the end of the first time segment is 0.56 (calculated from the data in Figure 6.1 or 6.2), that he will be in an offensive position is 0.11, and that he will be in a defensive position is 0.33.

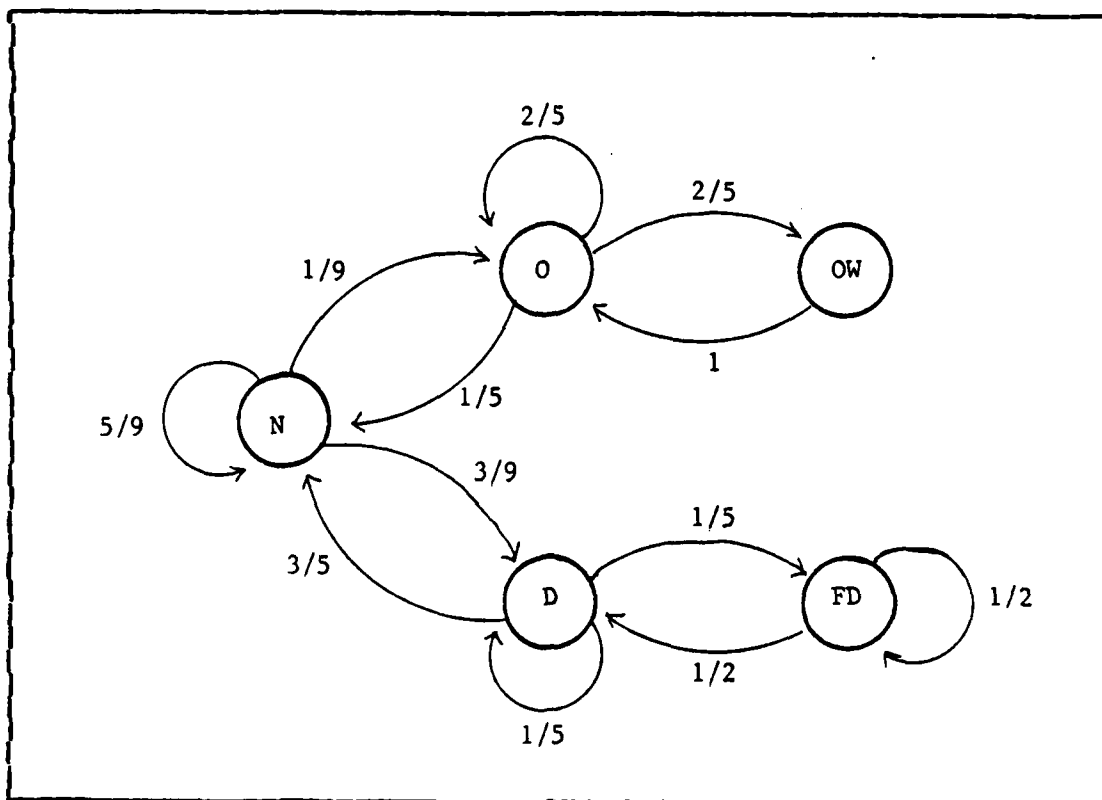


Figure 6.2 One-Step Transition Diagram for an ACM Engagement

Similarly, it is hypothesized that the initial probability of being in an offensive position at the start of the engagement is 0.25, and that he will still be in an offensive position at the end of the first time segment is 0.40. This process is repeated, with results as shown in the one-step transition matrix:

| Initial<br>prob. |     |     | OW | O   | N   | D   | FD  |     |
|------------------|-----|-----|----|-----|-----|-----|-----|-----|
| OW               | 0   | P = | OW | 0   | 1   | 0   | 0   | 0   |
| O                | .25 |     | O  | .40 | .40 | .20 | 0   | 0   |
| N                | .50 |     | N  | 0   | .11 | .56 | .33 | 0   |
| D                | .25 |     | D  | 0   | 0   | .60 | .20 | .20 |
| FD               | 0   |     | FD | 0   | 0   | 0   | .50 | .50 |

- iv) The total probability that the fighter will be in an offensive position at the end of the first time segment is:

$$(0.25)(0.40) + (0.50)(0.11) = 0.16.$$

- v) Two-step transition matrix:

|                  |    | OW  | O   | N   | D   | FD  |
|------------------|----|-----|-----|-----|-----|-----|
| P <sup>2</sup> = | OW | .40 | .40 | .20 | 0   | 0   |
|                  | O  | .16 | .58 | .19 | .07 | 0   |
|                  | N  | .04 | .11 | .53 | .25 | .07 |
|                  | D  | 0   | .06 | .46 | .34 | .14 |
|                  | FD | 0   | 0   | .30 | .35 | .35 |

- vi) Long-run probability values:

The long-run proportion of time the fighter aircraft can be expected to spend in each of the five states is:

|            | OW   | O    | N    | D    | FD   |
|------------|------|------|------|------|------|
| $\pi(i) =$ | .087 | .217 | .394 | .217 | .087 |

## 6. USED IN LITERATURE:

- a) Bell, E.L. Optimal Bayesian Estimation of the State of a Probabilistically Mapped Memory-Conditional Markov Process with Applications to Manual Morse Decoding. Doctor of Engineering thesis, Naval Postgraduate School, September 1977 (NPS T 180049).  
First and second order Markov chain models are used to describe the decoding of hand-keyed Morse code signals. A Bayesian solution process is then used to find an optimal estimate of the state of the Morse process.
- b) El Shanawani, A.A. Availability of Maintained Systems. Master's thesis, School of Engineering, Air Force Institute of Technology, Wright-Patterson AFB, March 1983 (AFIT/GOR/MA/82D-7, AD-A127 365).  
A survey and classification of the literature relevant to availability, with emphasis on probability density functions of failure times and repair times. Models include those based on Markov processes.
- c) Newman, R.A., and Tiffany, P.B. "Discrimination of Density and Clustering on Four Versions of a Stochastic Display", Proceedings of the 21st Meeting of the Human Factors Society, San Francisco, CA, 1977, pp. 113-117.  
A two-dimensional Markov process is used to control the variables, in a study of the interaction of two texture variables (density and cluster) with two display parameters (positive/negative image and adjacency/separation of images).
- d) Oberle, R.A. Air Combat Evaluation: the Reduced Dimension Measures. D2S Associates, Escondido, CA, May 31, 1983 (RES 83-6-2).  
See the above example.
- e) Snow, R.E. "Eye Movement and Cognitive-Process Research in Europe: Some Examples from Switzerland," European Science News, Vol. 38, No. 6, 1984, pp. 291-294.  
Eye movements and fixations, associated with problem solving during stimulus search and processing, are modelled as Markov chains to predict different eye fixation paths and lengths.
- f) Thomas, M.U. "A Human Response Model of a Combined Manual and Decision Task," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-3, NO. 5, 1973, pp. 478-484.  
A combined manual and decision task is described in terms of a Markov chain model combined with a discrete probability function.
- g) Thomas, M.U. Some Models of Human Error for Man-Machine System Evaluation. Department of

System Design, Wisconsin University, December 1978  
(TR-79-5, AD-A072 838).

A general semi-Markov formulation is used to describe transitions among error states. Interdecision times are treated as a renewal process.

## 7. REFERENCES AND TEXTS:

- a) Bronson, Richard. Schaum's Outline, Theory and Problems of Operations Research. New York: McGraw-Hill Book Company, 1982.

A brief explanation, with a number of worked out examples.

- b) Olkin, Ingram, Glaser, L.J., and Derman, Cyrus. Probability Models and Applications. New York: Macmillan Publishing Co., Inc., 1978.

A readable introduction to Markov processes and chains.

- c) Ross, S.M. Introduction to Probability Models. New York: Academic Press, 1980.

Extremely succinct explanation, in strictly mathematical terms; a high level of mathematical sophistication is advised.

- d) Taylor, H.M., and Karlin, Samuel. An Introduction to Stochastic Modeling. Orlando: Academic Press, Inc., 1984.

Highly mathematical explanation; not for beginners.

## J. POISSON PROCESSES

1. PURPOSE OF MODEL/TECHNIQUE: Determining such quantities as the number of arrivals into a system (or tasks that must be performed) over a period of time, the expected "population" size at a given time, and the probability of any given population size at a specified time. Arrivals must be according to an exponential distribution with a known rate parameter. Then the number of arrivals by a given time has a Poisson distribution dependent on the rate parameter and elapsed time.

For example, a person may be tasked with learning the Chinese language. The length of time required to

learn any given character is a random variable. If data collection indicates that the distribution of this random variable is approximately exponential and a rate parameter can be calculated, the learning process can be described as a Poisson process. Then it is possible to predict such quantities as how many characters will be learned in an hour and the probability that the student will know 100 characters at the end of five hours.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra
- b) Probability theory
- c) Descriptive statistics
- d) Graphs and plots

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, where the differences are a function of some characteristic that can be considered in terms of arrivals, with exponential distribution.
- b) Describing systems in which events occur with time, expressible as an exponential random variable.
- c) Evaluating human performance under conditions which meet the Poisson and exponential distribution requirements of this model. This is especially useful in accident and error predictions, as is discussed in the section on reliability models.

4. USED IN LITERATURE:

- a) Haight, F.A. "A Mathematical Model of Driver Alertness", Ergonomics, Vol. 15, No. 4, 1972, pp. 367-378.  
A non-homogeneous Poisson process is used to develop a model of driver decision making, based on observations of driver behaviors.

5. REFERENCES AND TEXTS:

- a) Barlow, R.E., and Proschan, Frank. Statistical Theory of Reliability and Life Testing: Probability Models. Silver Springs, Md., To Begin With Press, 1975, 1981.  
Comprehensive discussion of the use of Poisson processes in reliability determination.
- b) Bronson, Richard. Schaum's Outline, Theory and Problems of Operations Research. New York: McGraw-Hill Book Company, 1982.  
A very brief explanation.
- c) Cox, D.R. Renewal Theory. London: Methuen and Company, Ltd., 1962.  
A brief mathematical look at Poisson processes as they relate to reliability and renewal.
- d) Ross, S.M. Introduction to Probability Models. New York: Academic Press, 1980.  
Complete and thorough explanation; a high level of mathematical sophistication is advised.
- e) Taylor, H.M., and Karlin, Samuel. An Introduction to Stochastic Modeling. Orlando: Academic Press, Inc., 1984.  
Good, brief explanation, but not for beginners.

K. QUEUEING PROCESSES

- 1. PURPOSE OF MODEL/TECHNIQUE: Determining the length of time a customer (person, object, or task) must wait "in a line" (in queue) to get attention, the time needed to provide service or perform the task, the number of customers in the system and in queue at one time, etc.
- 2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:
  - a) Algebra
  - b) Probability theory
  - c) Descriptive statistics
  - d) Graphs and plots

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences in task performance.
- b) Describing systems, where these can be viewed as consisting of "customers" and "services".
- c) Evaluating human performance, such as how long it probably will take to perform a series of tasks.

4. DESCRIPTION:

- a) Model: A system or process is described as a function of seven things:
  - i) Population size of the arriving customers or input source, either finite or infinite (usually assumed to be infinite, since calculations are easier).
  - ii) State of the system, that is, the number of customers actually in the queueing system at a given time of interest, either waiting or being served.
  - iii) Arrival patterns, usually specified by the interarrival time, the time between successive customers into the system. More complex models may specify single versus batch arrivals, and whether customers may balk (refuse to enter the system because lines are too long), renege (a customer in a queue gets tired of waiting and leaves before being served), or jockey (move from his original line into another that is shorter). Simple models assume single arrivals, and no balking, renegeing, or jockeying.

- iv) Service patterns, usually specified by the service time required to serve one customer. This can be deterministic (a constant, known value), or a random variable with some known probability distribution. More complex models specify whether a customer requires a series of servers or is served completely by one server (the usual simplifying assumption).
- v) Number of parallel servers at the facility who provide the needed services for customers in the queue. All such servers are assumed to be interchangeable and equal.
- vi) Queue discipline or service discipline, which specifies the order in which customers are served. Usual orders are first-come, first-served or first-in, first-out (FIFO), as with customers at a supermarket checkout stand; last-in, first-out (LIFO), as with items in a suitcase; random order (RO); or priority order (PO), where certain customers get preferential treatment. Most commonly a service order is not specified, but the model is referred to as a general discipline (GD) model.
- vii) Kendall's notation, which is simply a standardized shorthand for specifying the above parameters:

$$(X/Y/Z):(U/V/W).$$

X is the interarrival time distribution and Y the service time distribution. These usually are denoted as M (for Markovian or

exponential),  $D$  (for deterministic),  $E(k)$  (for Erlang), or  $G$  (for general).

$Z$  is the quantity of parallel servers.

$U$  represents the service discipline.

$V$  and  $W$  indicate the system capacity and size of the task population, respectively--both often infinite in size ( $\infty$ ).

b) Assumptions: The first four assumptions are common for most queueing problems. The last seven assumptions will be made for this study, in order to keep the model at its most basic, easy-to-follow level:  $(M/M/1):(GD/\infty/\infty)$ .

i) Stationarity. Arrival time and service time probabilities do not change with time (are "stationary").

ii) Certainty. The population size, system capacity, and number of servers used in the model are known constants--have been empirically determined in some manner.

iii) Homogeneity and equivalence. Customers, servers, and service all are homogeneous. It makes no difference who serves whom.

iv) Non-negativity. All variables are non-negative (exist in quantities greater-than-or-equal-to zero).

v) Exponentially-distributed interarrival times and service times (also known as Markovian or Poisson processes). It is a property of this probability distribution that the arrival time of the next customer is independent of when the last one arrived, and

that the expected time for completion of service is independent of how long the customer already has been in service (memoryless property or Markovian property).

- vi) Single events. Time increments under consideration are small enough that the probability is approximately zero that two or more events will occur in one time increment (two arrivals, one arrival and one service, or two services). There is, however, a positive probability of either one arrival or one service during any time increment.
- vii) General queue discipline (GD), with a single server.
- viii) Infinite population size and system capacity, at least as an approximation.
- ix) Simple arrival patterns. Customers are not allowed to balk, renege, or jockey.
- x) Underutilization of servers. Server occupancy or utilization is not perfect. If servers are always busy (100%), waiting lines slowly will become infinitely long. A useful rule of thumb, according to Rouse (see References and Texts), is that servers are occupied 70% of the time, for an efficient system.
- xi) Steady state conditions. The system has been in operation long enough to have reached equilibrium or steady state behavior. That is, we are not considering a

new queue just forming when a store has just opened for the day.

c) Strengths:

- i) The procedure is applicable to a wide variety of problems which can be viewed as having "waiting line"-type characteristics.
- ii) Qualitative and approximate quantitative answers to a number of questions of interest about a given queueing system can be obtained via this relatively simple model.
- iii) Using advanced mathematics and a computer, large and complex problems can be solved via sophisticated queueing model techniques.

d) Weaknesses:

- i) For the model to remain simple and easily tractable, both interarrival times and service times must follow an exponential distribution--a condition not always easy to justify in the real world.
- ii) The model requires that customers (tasks) be handled serially; yet, in many situations, simultaneous attention often is necessary to accomplish a job.
- iii) Many queueing problems are analytically intractable, and require both approximation and simulation to obtain even rough answers.
- iv) The non-equilibrium situation is especially difficult to deal with, limiting the usefulness of the model to on-going, mature processes.

e) Procedures: The example that follows illustrates the process in more detail.

i) Determine that interarrival times and service times are approximately exponential random variables, from available data (see Figure 6.3 in the example). Note the values of the mean time between arrivals,  $\lambda$ , and the mean service time,  $\mu$ .

ii) Also determine that other parameters of the situation may approximately be modeled as an  $(M/M/1):(GD/\infty/\infty)$  system. That is, there should be a single server, the service discipline should be general in nature, and the customer population and number of customers allowed in the system should be very large, if not actually infinite.

iii) Decide what the state of the system,  $n$ , is likely to be at the time the modeling process begins. What quantity of customers, either waiting or being served, are already in the system?

iv) The quantities that will be calculated are:

Server occupancy or server utilization,  
 $\rho = \lambda / \mu$ .

Probability the system will be in state  $n$ ,  
 $P(n)$ ;

if  $n = 2$ ,  $P(2) = \rho^2 (1 - \rho)$ .

Average number of customers in the system,  
 $L = \rho / (1 - \rho)$ .

Average number of customers in queue,  
 $L_q = \rho^2 / (1 - \rho)$ .

Average time a customer spends in the system,

$$W = 1/(\mu - \lambda).$$

Average time a customer spends in queue,

$$Wq = \rho / (\mu - \lambda).$$

Probability a customer spends more than  $t$  units of time in the system,

$$W(t) = \exp (-t/W).$$

Probability a customer spends more than  $t$  units of time in queue,

$$Wq(t) = \rho \exp (-t/W).$$

- v) Three other equations may be used, if desired, for calculations:

$$W = Wq + 1/\mu.$$

$$L = \lambda W.$$

$$Lq = \lambda Wq.$$

- f) Other calculations that may be made: Queueing models considerably more complex than the one illustrated here have been used to obtain answers to questions similar to those described above (and illustrated below), when a simple model is not applicable. See the various authors cited under References and Texts for details.

5. MILITARY EXAMPLE (hypothetical)

a) Situation:

- i) A fighter aircraft is ingressing toward a fixed target, crossing hostile territory. Various enemy ground-based radar systems illuminate the aircraft from time to time,

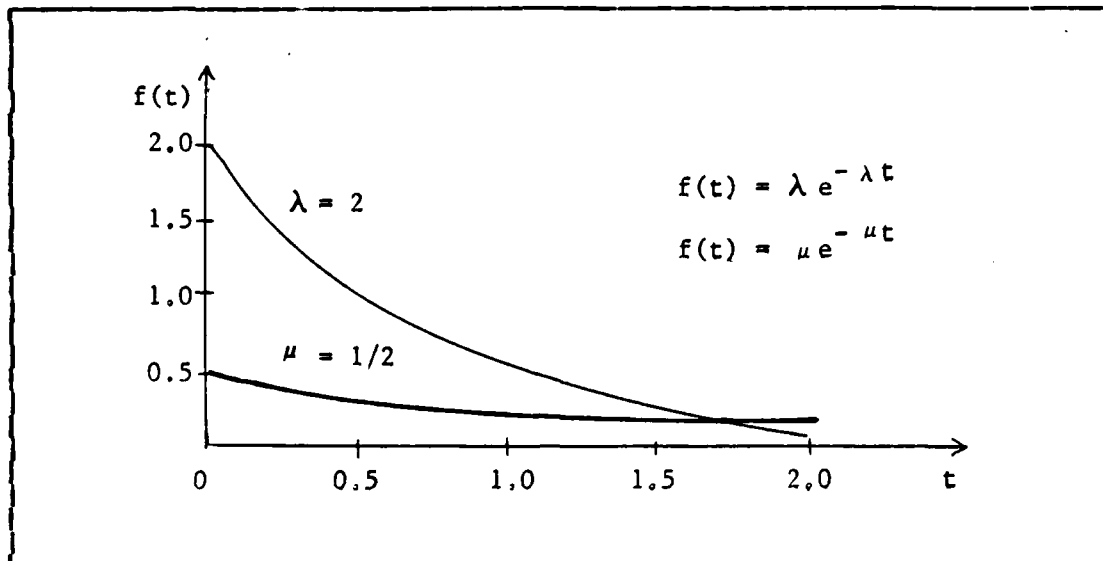


Figure 6.3 Exponential Distributions for Arrivals and Service

with time distribution approximately that of an exponential random variable with rate  $\lambda = 1/2$  per minute (2 minutes average time between threats). Figure 6.3 illustrates this distribution.

- ii) Warning that he is being illuminated by a threat radar is provided to the pilot on his radar warning display (RWD). He must take some kind of defensive or deceptive action when this occurs: drop chaff or flares, jam the radar, or make jinking maneuvers with his aircraft. The time required to take an appropriate action also approximately is an exponential random variable (Figure 6.3), with rate  $\mu = 2$  per minute (0.5 minute or 30 seconds average time to take an action).

- iii) At time zero (the start of the scenario being modeled), the aircraft will be considered to be in a steady state condition, since it has been behind enemy lines for five minutes. One radar system presently is illuminating the aircraft, and no defensive maneuvers are underway.

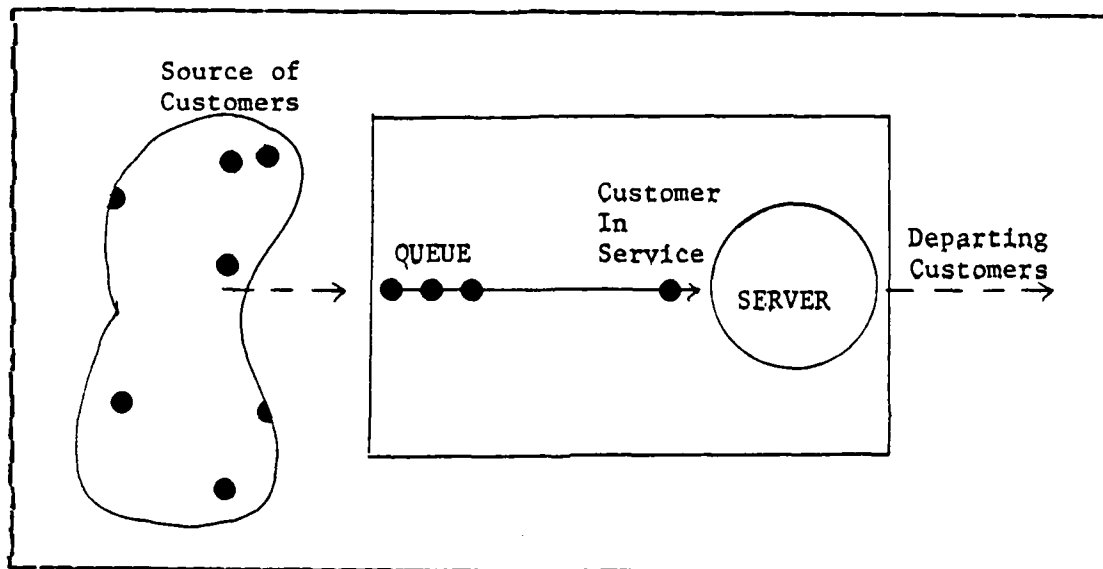


Figure 6.4 Illustration of Single Server Queueing System

b) Procedures:

- i) This model can be specified as  $(M/M/1):(GD/\infty/\infty)$ . Both the interarrival times and service times are exponentially distributed (Markovian). There is a single server (the aircraft), an infinite "customer" population (the enemy radars), and an unspecified (presumably infinite) number of customers allowed in the system.

No particular service discipline is specified, so it will be considered "general".

ii) A sketch of this very simple system is shown in Figure 6.4

iii) To perform the desired calculations, we first note that:

Arrival rate  $\lambda = 1/2$  per minute.

Service rate  $\mu = 2$  per minute.

Number of servers  $c = 1$ .

State of the system  $n = 1$ .

iv) The "server occupancy",  $\rho$ , is a rough measure of the pilot's workload. In this steady state condition,

$$\rho = \lambda / \mu = 1/2 \text{ divided by } 2 = 0.25.$$

Thus we see that, on the average, the pilot is spending 25% of his time responding to threats.

v) The probability of the system being in the state where  $n = 1$  at any given time (that is, where exactly one threat is present) is given by

$$P(1) = \rho^1 (1 - \rho) = (0.25)^1 (0.75) = 0.188.$$

That is, there is less than a 20% chance of this state occurring at any specified time.

vi) The average number of threats illuminating the aircraft is given by:

$$L = \rho / (1 - \rho) = (0.25) / (0.75) = 0.333.$$

The average number of threats "in queue", or illuminating the aircraft and not yet countered, is:

$$L_q = \rho^2 / (1 - \rho) = (0.25)^2 / (0.75) = 0.083.$$

- vii) The average time a threat will illuminate the aircraft is given by:

$$W = 1 / (\mu - \lambda) = 1 / (2 - 1/2) = 2/3 = 0.667 \text{ minutes.}$$

The average time a threat will illuminate the aircraft before it is countered is:

$$W_q = \rho / (\mu - \lambda) = (0.25) / (1.5) = 0.167 \text{ minutes.}$$

- viii) The probability that a threat will illuminate the aircraft for longer than one minute is:

$$W(t) = \exp(-t/W)$$

$$W(1) = \exp(-1/W) = \exp(-1/0.667) = 0.223.$$

The probability that a threat will illuminate the aircraft for longer than a minute before it is countered is:

$$W_q(t) = \rho \exp(-t/W)$$

$$W_q(1) = (0.25) \exp(-1/0.667) = (0.25)(0.233) = 0.056.$$

- ix) The other three equations may be used to check our work:

$$W = Wq + (1/\mu) = 0.167 + 0.5 = 0.667.$$

$$L = \lambda W = (0.5) (0.667) = 0.333.$$

$$Lq = \lambda Wq = (0.5) (0.167) = 0.083.$$

## 6. USED IN LITERATURE:

- a) Carbonell, J.R. "A Queueing Model of Many-Instrument Visual Sampling", IEEE Transactions on Human Factors in Electronics, HFE-4, No. 4, 1966, pp. 157-164.

Carbonell, J.R., Ward, J.L., and Sanders, J.W. "A Queueing Model of Visual Sampling: Experimental Validation", IEEE Transactions on Man-Machine Systems, MMS-9, No. 3, 1968, pp. 82-87.

The instrument scanning behavior of pilots was studied, and a queueing model developed to predict the fraction of time devoted to each instrument. The model later was compared with performance of three pilots flying simulated airport approaches, and compared well with observed performance.

- b) Eltermann, L.J. Computer Simulation Design, Development and Validation. Mitre Corp., Bedford, MA, June 1982 (MTR-8416, AD-B067 977L).  
SIMSCRIPT is used for a discrete-event queueing simulation of a communications system control system, as part of system support analyses of man-machine resources within a computer system.

- c) Groves, A.W., and Kaercher, R.L. A Simulation to Analyse Pilot Workload in an Electro-Optical, Night, Low-Level Environment. Air Force Institute of Technology, Wright-Patterson AFB, OH, March 1981 (AFIT/GST/OS/81M-5, AD-A101 138).

A time-sequenced network of required tasks, with priority servicing by a single server, is used to model 30 minutes of visual navigation and terrain following, incorporating 20 tasks.

- d) Schmidt, D.K. "A Queueing Analysis of the Air Traffic Controller's Workload", IEEE Transactions on Systems, Man, and Cybernetics, SMC-8, No. 6, 1978, pp. 492-493.

A queueing theory formulation was used to analyse the workload of air traffic controllers. The model was then used to predict average delay and server occupancy as a function of demand.

- e) Taguchi, K, and Murotsu, Y. "Simulation Studies of Evacuation of Passengers and Crews on Board," Ergonomics, Vol. 20, No. 3, 1977, p 329.

Queueing models are applied to the flows of passengers from doorways and exits, to determine selection of passages and widths of passageways, based on delays of the flows and evacuation times.

- f) Wichansky, A.M. "Human Factors Aspects of Queueing: A Critical Review", Human Factors, Vol. 18, No. 2, 1976, pp. 161-172.  
A general look at queueing theory and its use in describing customer behavior and human waiting behavior.

7. REFERENCES AND TEXTS:

- a) Bronson, Richard. Schaum's Outline, Theory and Problems of Operations Research. New York: McGraw-Hill Book Company, 1982.  
Excellent, clear introduction to queueing systems and (M/M/1) queues, plus separate explanation of more complex models and how to use them.
- b) Daellenbach, H.G., and others. Introduction to Operations Research Techniques, Second Edition. Boston: Allyn & Bacon, Inc., 1983.  
Simple models are glossed over; heavier reliance on mathematical derivations than is usual in this text.
- c) Hillier, F.S., and Lieberman, G.L. Introduction to Operations Research. San Francisco: Holden-Day, Inc., 1980.  
Heavy emphasis on exponential distribution, but no simple explanation of how to use the simple models to find answers. Detailed discussion of mathematical derivations and manipulations.
- d) Ross, S.M. Introduction to Probability Models. New York: Academic Press, 1980.  
Succinct explanation, in mathematical terms, with no numerical examples provided.
- e) Rouse, W.B. Systems Engineering Models of Human-Machine Interaction. New York: North Holland, 1980.  
Considers queues of tasks, and queueing theory is used to predict human performance at completing tasks. However, simple models are glossed over, and great detail is given to one rather complex model of flight management.
- f) Wagner, H.M. Principles of Operations Research. New Jersey: Prentice-Hall, Inc., 1975.  
Excellent, clear introduction to the subject, and to the exponential distribution family. Clear explanation of more complex models, also.

## I. RELIABILITY MODELS

1. PURPOSE OF MODEL/TECHNIQUE: Estimating expected time of failure, probability of failure in a given time period, etc., for a given system with known failure rate (or survival rate) distribution.

For example, data collected on frequency of misinterpreting information on a given CRT display format may indicate that time between errors is a random variable that closely fits an exponential distribution, with rate  $\lambda = 2$  per hour. Given this, we can use properties of that distribution to make calculations. On the average, the mean time to failure will be  $1/\lambda = 1/2$  hour. The probability that no failure will occur in the first 15 minutes is the exponential survival function,  $\exp(-\lambda t) = \exp(-2 \times 0.25) = 0.61$ . Probability of no failure in  $1/2$  hour is 0.37, in 1 hour is 0.14, etc.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra
- b) Single variable calculus ,
- c) Probability theory
- d) Descriptive statistics
- e) Graphs and plots

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, as related to accuracy and errors.
- b) Describing systems, and predicting how long we expect them to perform.
- c) Evaluating human performance over a long period of time, based on measured failures/errors during a shorter data collection period which can be used to develop a model.

4. USED IN LITERATURE:

- a) Askren, W.B., and Regulinski, T.L. "Quantifying Human Performance for Reliability Analysis of Systems," Human Factors, Vol. 11, No. 4, 1969, pp. 343-350.

A general mathematical model of the probability of errorless human performance was derived, and equated to human reliability for time-continuous tasks. Weibull, gamma, and log-normal density functions were determined to be relevant descriptors of the data.

- b) Meister, David. Comparative Analysis of Human Reliability Models. Bunker-Ramo Corp., November 1971. (L0074-107, NPS U 147484).

A total of 22 models were analysed to evaluate their ability to predict performance of humans in operating and maintaining military systems. Simulation models were found more powerful than analytic models. Output usually consisted of probability of successful task/system performance and completion time.

- c) Meister, David. "Methods of Predicting Human Reliability in Man-Machine Systems", Human Factors, Vol. 6, 1964, pp. 621-646.

A simple multiplicative probability model for human error prediction is reviewed and evaluated. Performance reliabilities for task elements are progressively combined through the use of the series product rule, to yield reliability estimates for tasks, mission phases, and the overall system. Altman's Data Store is used to obtain the elemental reliability values.

- d) Naval Sea Systems Command. Human Reliability Prediction System User's Manual. Washington, D.C., December 1977 (AD-A058 668).

Both human and equipment mean-time-before-failure and mean-time-to-repair values are used in predicting and demonstrating system effectiveness and for predicting human reliability, in a weapons system environment.

- e) Pew, R.W. et al. Critical Review and Analysis of Performance Models Applicable to Man-Machine System Evaluation. Bolt, Berenak, and Newman, Inc., Cambridge, MA, March 1977 (BBN No. 3446, AFOSR-TR-77-0520, AD-A038 597).

Altman's Data Store and other data bank-type models of human reliability are surveyed and evaluated, as part of this 300-page comprehensive report.

- f) Pollard, D., and Cooper, M.B. "An Extended Comparison of Telephone Keying and Dialing Performance", Ergonomics, Vol. 21, No. 12, 1979, p. 107.

The reliability of office workers performing dialing and keying tasks was investigated. An

exponential curve was found to be a good fit for keying error data.

- g) Siegel, A.I., Wolf, J.J., and Lautman, M.R. "A Family of Models for Measuring Human Reliability", Proceedings of the 1975 Annual Reliability and Maintainability Symposium, IEEE, Washington, DC, 1975, pp. 110-115.  
A set of stochastic, digital simulation models for human performance in man-machine systems is described. One of these will yield predictions of integrated system reliability, considering both equipment and human performance.

## 5. REFERENCES AND TEXTS:

- a) Barlow, R.E., and Proschan, Frank. Statistical Theory of Reliability and Life Testing: Probability Models. Silver Springs, Md., To Begin With Press, 1975, 1981.  
Probabilistic underpinnings of reliability determination; very complete; mathematically sophisticated.
- b) Cox, D.R. Renewal Theory. London: Methuen and Company, Ltd., 1962.  
A brief, succinct mathematical text on the renewal aspect of reliability.

## M. SIMULATION MODELS

1. PURPOSE OF MODEL/TECHNIQUE: Building an experimental model of a system when uncertainties, dynamic or complicated interactions, and interdependence among variables makes the development of an analytical model difficult or impossible. The simulation model then can be used with a computer to evaluate and compare specific alternatives, and to make predictions about the system. Simulation can be considered the laboratory or experimental arm of operations research.
2. MATHEMATICAL TOOLS REQUIRED OR USEFUL: While any of the following may be needed for a given simulation model, only logic and set theory, descriptive statistics, experimental design, computer programming, and

computer software packages will almost always be required.

- a) Algebra, simple, linear, Boolean
- b) Geometry, plane, spherical, analytic
- c) Trigonometry
- d) Calculus, single variable, multiple variable
- e) Logic and set theory
- f) Fuzzy set theory
- g) Probability theory
- h) Statistics, descriptive, inferential
- i) Experimental design
- j) Graphs and plots
- k) Computer programming (in languages such as SIMSCRIPT, GPSS, etc.)
- l) Computer packages

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, where the individual is considered in more detail or complexity than can be handled through simple analytical models; examples include the Computerized Accomodated Percentage Evaluation model (see Bittner), and Computerized Biomechanical Man-Machine Model (see McDaniel), both anthropometric descriptions of humans.
- b) Describing systems, where the systems are dynamic in nature or where variables are known to interact; examples include describing a man-machine system during an air intercept mission (see Meldrum).
- c) Designing systems such as control panels (see Bonney and Williams), workspaces (see McDaniels), task allocations (see Parks and Springer), and individual tasks (see Wortman and others).

- d) Evaluating human performance such as that observed in complex crewstations (see Strieb and Wherry).

4. DESCRIPTION:

a) Model: A system or process is described as a function of ten things:

- i) The system itself, which has dynamic phenomena--inputs, components, behaviors, and outputs--that are being studied.
- ii) Entities or elements of the system--components whose behaviors are traced through the system or time period of interest. Classes of entities can be concrete or abstract, and include people, machines, various objects, signals, bits of data, and tasks.
- iii) Attributes of the entities--size, quantity, requirements, responses--that characterize their behaviors in the system. Attribute values can be numerical or can be word descriptions (responses can be verbal, hand-written, keyed, etc.).
- iv) Membership relationships of entities, such as shared attributes which cause them to belong to sets or files (temporarily or permanently). Files also may have attributes, such as capacity or a finite useful life.
- v) Activities related to the entities: dynamic operations which entities can perform or which can be performed upon them.

- vi) States of the entities and of the system as a whole: the configuration at a given point in time which has been defined by file or entity attributes and ongoing activities. The initial state is a special case which is defined by the experimenter at the start of the simulation.
- vii) Events, which describe any change in the state of a system and which result in its dynamic behavior. Events can be exogenous, the result of some occurrence outside the system, or endogenous, resulting from activities of the system's own entities. If all events result either from constant exogenous inputs or from deterministic endogenous consequences, the simulation is called deterministic that is, the same set of inputs always will result in exactly the same simulation outputs. If events result from inputs that are subject to random phenomena, this is considered to be a stochastic simulation or a Monte Carlo simulation. Randomness in the initial values of entity attributes, in changes in attribute values, or in the timing of events can be provided through inputting of random numbers representing the probability distribution most appropriate for the system under study. As a result of this randomness, simulation outputs will differ from run to run, reflecting that specified probability distribution.

- viii) Time representation, as the system progresses through the events of interest. Fixed-time increments (also known as time-step incrementation) can be used if events occur on a fairly regular basis, so that there are not long periods of inactivity; in this case, time elapses period by period (second by second, or day by day). Variable-time incrementing (also called event-step incrementation) is used if many time periods will contain no activities. This kind of program progresses according to an event list, which governs the progress of the program in much the same manner as seconds or days would--except that the length of periods is not a constant. This latter type of programming requires more skill than does the simpler time-step incrementing.
- ix) Decision rules or operating rules, which provide logical links between entities, activities, events, and resulting states of the system. To use computer jargon, if certain entity and activity requirements are met, then a specified change of state will occur, else the system's state will remain unchanged.
- x) A flow diagram or algorithm. This is a useful tool which depicts the orderly and logical flow of events as a series of boxes and arrows, covering the sequence or time period of interest. The above-noted system parameters are described in that diagram (see Figure 6.5).

b) Assumptions: Simulation models vary widely, depending on the system being modeled. As a result, assumptions must be made on a case-by-case basis (and should be clearly stated for each model). However, the following three assumptions probably apply to most such models.

i) Algorithm validity. The flow diagram describing the system, used for developing the computer program, is an adequate representation of the real system for obtaining useful results.

ii) Known constants. All constant parameters used in the model are known values, obtained through some empirical data collection process or through logical deductions.

iii) Known probability distributions. Randomly-distributed events can be adequately characterized by known discrete or continuous probability distributions; values for these distributions can be obtained for use in computer runs by means of mathematical transforms of values obtained from a random number generator. For many simulations, the assumption is made that the individual observations of the variables (variable values obtained for use in the simulation) are independent (uncorrelated) and are drawn from a single normal (Gaussian) distribution with constant mean and standard deviation. A second popular distribution is the exponential distribution, for which the same assumptions of a

constant mean and independent values are made.

c) Strengths:

- i) Simulation techniques can be used for problems which cannot be handled through analytical modeling techniques.
- ii) Simulations provide a means of experimenting with proposed systems before they actually are developed and implemented.
- iii) Simulation models do not require as great a degree of abstraction, simplification, and approximation as do analytical models; simulation models may be fairly true representations of the real world.
- iv) The procedure of preparing an algorithmic flow diagram is a very useful tool in designing a model which represents a system adequately; the orderly thought process required can aid the experimenter in picking up flaws in his logic.

d) Weaknesses:

- i) Simulation cannot be used to find the "best" solution for a system problem. Rather, it is an aid to analysis which can be used to compare various alternatives--but does not find a better one if the experimenter has not already thought of it. Optimization is done via trial and error.
- ii) Simulation models must include a great deal of detail in order to be successful representations of a system. Thus model building

effort is much greater than for an analytical model.

- iii) The answers resulting from stochastic simulation must be considered estimates, and are subject to statistical error. A large number of simulation runs are necessary in order to achieve statistical significance (similar to other forms of experimentation).
- iv) Simulation models, being complicated, frequently can eat up computer time at an enormous rate. Large numbers of runs can be required to validate that the model behaves like the real system, to estimate model responses to various parameter settings, and to determine relationships among these parameters. The process is expensive.
- v) Although many simulation studies concern investigation of systems that operate continually in a steady-state condition, simulation models cannot operate continually; they must start and stop. The performance of the simulated system cannot be representative of the real one until it essentially has reached a steady-state condition, through many runs. This makes it especially difficult to use these models to predict steady-state behavior.
- vi) Selection of values for starting conditions (initial states) is important in determining how soon a simulated state similar to the real system's steady state is achieved. Yet estimating such values with an adequate

degree of accuracy can be impossible--indeed, may be the purpose for which the simulation is intended!

vii) The assumption of statistically independent random observations from a given probability distribution often is not correct, when modeling the real world. For example, the waiting time of one customer in a queue is definitely dependent on the waiting time of the person ahead of him in line.

e) Procedures: Not all of the procedures listed here will be appropriate for any one simulation model; the user must pick and choose according to what the system actually is like.

i) Define the system of interest, setting limits on just what portions will be modeled and to what degree simplification and approximation will be allowed.

ii) Specify the classes of entities to be included, and enumerate the entities themselves.

iii) Assign attributes to the entities, including only those attributes which are appropriate for this system and this degree of system representation. Give a range of allowed values for each attribute (numerical or otherwise descriptive).

iv) Determine the relationships among the entities--what similarities do they have, and how does a change in one affect another? Identify appropriate sets or files into

which entities will fall during the simulation.

- v) For each entity, define what activities it will be allowed to perform, and what operations can be performed on it.
- vi) Define the allowable states for each entity, and specify the initial value which will be used for the state of each entity.
- vii) Determine the events which will occur during the simulation process, including both those that are exogenous and those that are endogenous. Ascertain which events are most appropriately represented as deterministic inputs and which are better modeled as stochastic inputs. For stochastic values, decide what probability distribution will be used to generate those values. Find the correct mathematical transform formula to convert randomly-generated numbers into values of that probability distribution (for inverse transformations, see Daellenbach and others, p. 469; Hillier and Lieberman, p. 650; or Wagner, p. 930).
- viii) Decide whether fixed-time or variable-time incrementation will be used.
- ix) Prepare the set of decision rules which will be used in the program. These should be based on the entities, attributes, relationships, activities, and states of the system.
- x) Design the experiment which is to be run via simulation. This includes both the

selection of the constants, independent variables, and dependent variables, and also determination of the statistical procedures which will be used to evaluate experimental results. Use of statistical analysis computer packages (SAS, SPSS, etc.) can be very helpful.

- xi) Draw an algorithmic flow diagram describing the system and the process it will go through during the simulation runs.
- xii) Write a computer program for the algorithm (or have it written by someone who does that for a living). Use of one of the specially-designed simulation languages is highly recommended (SIMSCRIPT, GASP, SIMULA, GPSS, etc.). For some simulation problems, canned software packages may already be available (for human factors use: SAINT, HOS, CAPABLE, COMBIMAN, CAFES, etc.; see literature references at the end of this section).
- xiii) Using the values of the independent variables previously decided upon in the computer program, run the simulation to validate its ability to represent the system being modeled. Modify the program, if necessary. Once the model is validated, continue the runs until the desired degree of precision is reached for the resulting dependent variables (for a stochastic simulation).
- xiv) Use the pre-selected statistical evaluation techniques on data obtained from the

simulation runs to determine if results are statistically significant, etc. Sensitivity analysis also is especially important here, to determine which variables are the critical ones.

f) Other calculations that may be made: The variety of purposes for which simulation models are being used in the human factors field is illustrated below in the literature references. Other uses certainly will be found, as human factors engineers become familiar with simulation techniques.

5. MILITARY EXAMPLE (adapted from examples in Hillier and Lieberman, and in Wagner; see References and Texts)

a) Situation:

i) Essentially the same situation will be modeled here as was described in Chapter 6, Section H., Queueing Processes. However, interarrival times and service times will be assumed to follow a uniform (square) probability distribution rather than an exponential one. Thus, queueing model calculations cannot be made in the usual way.

ii) A fighter aircraft is ingressing toward a fixed target, crossing hostile territory. Various enemy ground-based radar systems illuminate the aircraft from time to time, with time distribution approximately that of a continuous uniform random variable with range 6 to 24 seconds (average time 15 seconds).

- iii) Warning that he is being illuminated by a threat radar is provided to the pilot on his radar warning display (RWD). He must take some kind of defensive or deceptive action when this occurs: drop chaff or flares, jam the radar, or make jinking maneuvers with his aircraft. The time required to take an appropriate action also approximately is a continuous uniform random variable, with range 1 to 19 seconds (average time 10 seconds).
- iv) At time zero (the start of the scenario being modeled), the aircraft will be considered to be in a steady state condition, since it has been behind enemy lines for five minutes. No radar systems presently are illuminating the aircraft, and no defensive countermeasures maneuvers are underway.

b) Procedures:

- i) The system of interest will be defined as a single-server queueing system.
- ii) There are two classes of entities: radar illuminations (threat warnings), which are the "customers", and aircraft pilots (the "servers"). There is only one entity in the class of pilots: the single pilot of our aircraft of interest. The class of threat warning entities is infinite in size, with entities considered all equivalent, and identified only by the sequence of their arrivals.

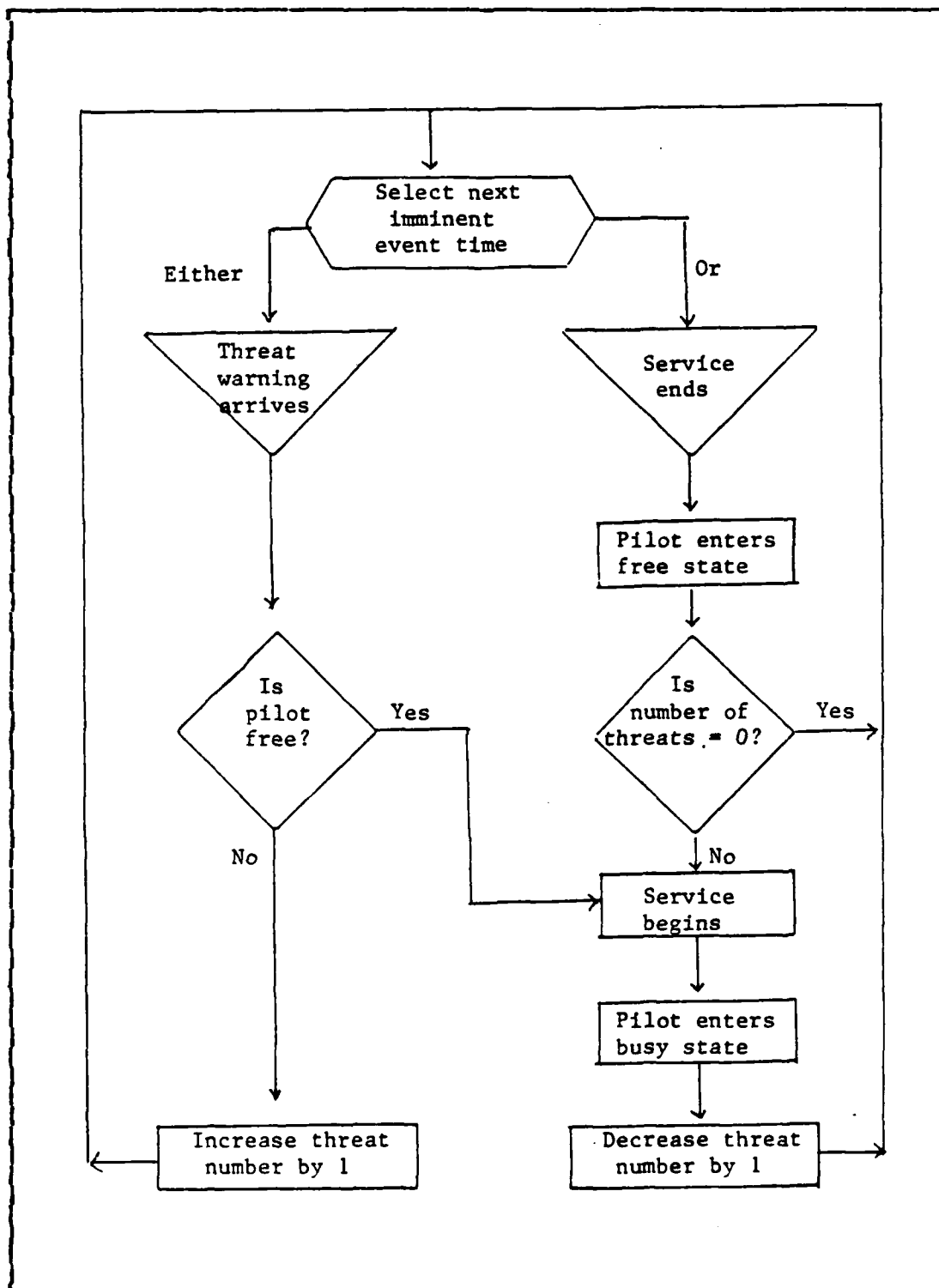


Figure 6.5 Flow Diagram for Simulation Model

- iii) Threat warning entities have only one attribute of interest for this study: their interarrival times. As previously noted, these are considered to be uniformly distributed, with range 6 to 25 seconds. The pilot also has only one attribute of interest: his countermeasure response times. These also are uniformly distributed, within a range of 1 to 19 seconds.
- iv) Relationships between the entities that are of interest to this study are: (a) a threat warning must precede a pilot response, and (b) a threat warning will not go away until the pilot has responded to it.
- v) Activities of the two entities will be limited to the following: (a) a radar illumination will result in a threat warning to the pilot, and (b) the pilot will take a defensive countermeasure action which will result in disappearance of the threat warning (and presumably suppression of the threat).
- vi) Allowable states of the radar illuminations are (a) absent, or (b) present. Allowable states for the pilot are (a) free, or (b) busy.
- vii) Only two kinds of events will occur: (a) arrival of radar illumination threat warnings, as stochastic inputs, and (b) completion of a defensive maneuver by the pilot, also as a stochastic input.

- viii) Time incrementing for this example will be done using the event-step procedure for variable time increments.
- ix) Two decision rules may prove useful in programming this situation: (a) If the pilot is free when a threat warning arrives, then the threat is countered, else the threat joins the queue; (b) If a pilot countermeasure is completed when at least one threat is in queue, then the next threat will be countered, else the pilot is free.
- x) The experiment to be performed for this simulation is to determine the percent of time the pilot will not be able to complete his mission, given these circumstances. It will be assumed that "mission completion" will be equivalent to "no more than two threats in the system at any one time" (an extremely simple definition, but one that can be tested and measured). A "mission" will be considered to be one complete cycle, or that period of time between queues: a new mission will be started (for testing purposes) each time the number of threats in the system drops to zero.

The statistic to be calculated is:

$$\text{Expected \% of mission failures} = 100 \times \frac{\text{no. of cycles containing 3 threats}}{\text{total no. of cycles.}}$$

- xi) An algorithmic flow diagram of the system of interest is shown in Figure 6.5.

- xii) One-digit random numbers can be used to generate the random observations from the two uniform distributions. If random number 0 is drawn, this will represent an interarrival time of 6 seconds, or a service time of 1 second; 1 will give an interarrival time of 8, or a service time of 3; 2 will give an interarrival time of 10, or a service time of 5; and so on, incrementing interarrival times by twos, up to random number 9 representing 24 seconds, and incrementing service times by twos, up to random number 9 representing 19 seconds.
- xiii) Although a computer program could be written to perform the required simulation (and certainly should be, if enough runs are to be made for statistical significance), the required procedure can be illustrated here simply as is shown in Table 6.
- xiv) This table follows the simulation through five cycles (number of threats drops to zero five times). In only one cycle does the number of threats in the system climb to three--criterion for mission failure.

The resulting statistic for mission failure is:

$$\begin{aligned} \text{Expected \% of mission failures} &= \\ 100 \times 1/5 &= 20\%. \end{aligned}$$

#### 6. USED IN LITERATURE:

- a) Bittner, A.C., Jr. Computerized Accomdated  
Percentage Evaluation: Review and Prospectus.

**TABLE 6**  
**SAMPLE SIMULATION RUNS: FIVE CYCLES**

| <u>Random number</u> | <u>Arrival time increment</u> | <u>Next arrival</u> | <u>Service time increment</u> | <u>Next service completion</u> | <u>Event time</u> | <u>Number of threats</u> |
|----------------------|-------------------------------|---------------------|-------------------------------|--------------------------------|-------------------|--------------------------|
| <u>Cycle 1</u>       |                               |                     |                               |                                |                   |                          |
| 9                    | 24                            | 24                  | 13                            | 37                             | 0                 | 0                        |
| 2,6                  | 10                            | 34                  |                               |                                | 24                | 1                        |
| 4                    | 14                            | 48                  |                               |                                | 34                | 2                        |
| 6                    |                               |                     | 13                            | 50                             | 37                | 1                        |
| 4                    | 14                            | 62                  |                               |                                | 48                | 2                        |
| 1                    |                               |                     | 3                             | 5                              | 50                | 1                        |
| <u>Cycle 2</u>       |                               |                     |                               |                                |                   |                          |
| 1,1                  | 8                             | 70                  | 3                             | 65                             | 53                | 0                        |
| <u>Cycle 3</u>       |                               |                     |                               |                                |                   |                          |
| 3,9                  | 12                            | 82                  | 19                            | 89                             | 62                | 1                        |
| 1                    | 8                             | 90                  |                               |                                | 65                | 0                        |
| 4                    |                               |                     | 9                             | 98                             | 70                | 1                        |
| 1                    | 8                             | 98                  |                               |                                | 82                | 2                        |
| 1,5                  | 8                             | 106                 | 11                            | 109                            | 89                | 1                        |
| 6                    | 18                            | 124                 |                               |                                | 90                | 2                        |
| 2                    |                               |                     | 5                             | 114                            | 98                | 2                        |
| 1                    |                               |                     | 3                             | 117                            | 106               | 3                        |
| <u>Cycle 4</u>       |                               |                     |                               |                                |                   |                          |
| 5,6                  | 16                            | 140                 | 13                            | 137                            | 109               | 2                        |
| <u>Cycle 5</u>       |                               |                     |                               |                                |                   |                          |
| 9,3                  | 24                            | 164                 | 7                             | 147                            | 114               | 1                        |
| ...                  |                               |                     |                               |                                | 117               | 0                        |
|                      |                               |                     |                               |                                | 124               | 1                        |
|                      |                               |                     |                               |                                | 137               | 0                        |
|                      |                               |                     |                               |                                | 140               | 1                        |
|                      |                               |                     |                               |                                | 147               | 0                        |

Pacific Missile Test Center, Pt. Mugu, CA, December 1976. (PMTC-TP-76-46, AD-A035 205).

About a dozen research efforts are reviewed which employ Monte Carlo simulation for computerized accommodated percentage evaluation (CAPE). These models are used to determine what proportion of the population will be able to use a given system, based on anthropometry.

- b) Bonney, M.C., and Williams, R.W. "CAPABLE: A Computer Program to Lay Out Controls and Panels", Ergonomics, Vol. 20, No. 3, 1979, pp. 297-316. A computer program called Controls and Panel Arrangement by Logical Evaluation (CAPABLE) is described, and results of its use are discussed.

- c) Hudson, E.M. "Adaptive Techniques on Multiparameter Problems", Human Factors, Vol. 11, No. 6, 1969, pp. 561-568.  
A simulation technique is used for conducting multiparameter experiments so that the number of data points investigated is a minimum. The method is based on observations that human responses to psychophysiological inputs are lawful rather than random, and so can be predicted from mathematical equations. Data collected from a few points in the experimental matrix are fitted with a low-order polynomial, using a computer program to evaluate the coefficients. Various values predicted from this equation are compared with other data values, and improvements are made in the fit as needed.
- d) McDaniel, J.W. Computerized Biomechanical Man-Model. Aerospace Medical Research Lab, Wright-Patterson AFB, OH, July 1976 (AMRL-TR-78-30, AD-A032 402).  
COMBIMAN is a computerized interactive graphics technique for workplace design. The simulation allows manipulation of a three-dimensional male form of variable anthropometry, and the designing of a workspace around him, using a lightpen.
- e) Meldrum, W.G. A Digital Simulation with Human Interaction of One vs. Many Air-to-Air Intercept. Master's thesis, Naval Postgraduate School, March 1973 (NPS T 154600).  
A single- vs. multiple-aircraft intercept mission is modeled using digital simulation, incorporating computer graphics and dynamic human interaction. MOE is probability of kill at each position of possible weapon release.
- f) Parks, D.L., and Springer, W.E. "Human Factors Engineering Analytic Process Definition and Criterion Development for CAFES", Ergonomics Abstracts, Vol. 10, No. 1, 1978, p. 94.  
The Computer Aided Function-Allocation Evaluation System (CAFES) is evaluated for ability to support human factors engineering in systems development.
- g) Shubik, Martin, and Brewer, G.D. Models, Simulations, and Games--a Survey. Rand Corporation, May 1972 (R-1060-ARPA/RC, NPS U 151521).  
Approximately 450 active military models, simulations, and games were identified, from which 132 were chosen for study. Four types were identified: analytic models, machine simulation, man-machine simulation, and free-form gaming. Purpose, usefulness, and expense of each model was analyzed. An inverse relationship between size and usefulness was observed.
- h) Siegel, A.I., and Wolf, J.J. Digital Behavioral Simulation: State-of-the-Art and Implications. Applied Psychological Services, Inc., Wayne, PA, June 1981 (AD-A128 641).

A review, analysis, and appraisal, along with presentation of examples of current models. Problems in model design, cost-benefit tradeoffs, and future trends in behavioral modeling also are discussed, along with recommendations for development and maintenance of current Army models.

- i) Streib, M.I., and Wherry, R.J., Jr. An Introduction to Human Operator Simulator. Analytics Inc., Willow Grove, PA, December 1979 (TR-1400.62-D, AD-A097 520).

HOS is a digital computer program used in evaluation of performance in complex crewstations. The activities of an operator (perception, physical movement, decision making, etc.) are simulated dynamically. The system predicts how long each activity will take.

See also The Human Operator Simulator, Vol. 9, HOS Study Guide, by M.I. Streib, F.A. Glenn, and R.J. Wherry, Jr. (September 1978, TR-1320-Vol-9, AD-A094 353).

- j) Wortman, D.B., and others. The SAINT User's Manual. Pritsker and Associates, Inc., West Lafayette, IN, June 1978 (AD-A058 724).

SAINT (Systems Analysis of Integrated Networks of Tasks) is a network modeling and simulation technique used in design and analysis of complex man-machine systems. Systems can consist of discrete tasks, continuous state variables, and interactions between them.

See also Simulation Using SAINT: A User-Oriented Instruction Manual, by the same authors (July 1978, AD-A058 671).

## 7. REFERENCES AND TEXTS:

- a) Daellenbach, H.G., and others. Introduction to Operations Research Techniques, Second Edition. Boston: Allyn & Bacon, Inc., 1983.

Good introduction to the subject. Excellent examples of flow diagrams, and of setting up data for simulation runs. Good discussion of simulation programming languages.

- b) Hillier, F.S., and Lieberman, G.L. Introduction to Operations Research. San Francisco: Holden-Day, Inc., 1980.

Good examples. Good discussion of variance reducing techniques (Monte Carlo techniques).

- c) Wagner, H.M. Principles of Operations Research. New Jersey: Prentice-Hall, Inc., 1975.

Excellent, clear introduction to the subject, with emphasis on the procedures used in building a simulation model.

## VII. MODELS FOR OPTIMIZING

The techniques covered here are representative of those used to obtain the best possible solution to a problem when a number of constraints also must be met. These constraints may be laid on us by physical laws (we cannot exceed the speed of light, for example), man-made laws (55-mile-per-hour speed limits), or simple economics (we have only so much money to spend on gasoline).

In this section, we will consider linear programming, nonlinear programming, network analysis, and distribution models. These four operations research models (and corresponding techniques) are used to find satisfactory solutions to problems involving the allocation, use, or distribution of scarce resources. The scarce resources usually are money, time, equipment, or people--all available in less-than-infinite quantities. The optimum solution for such a problem may be one that maximizes some measure of benefit or utility (such as profit or survivability), or minimizes some measure of cost [Ref. 42].

Linear programming, the first of these four techniques, is a geometric or algebraic procedure for optimum allocation of some resource between two or more alternatives, in light of certain goals and in light of certain constraints or conditions [Ref. 43]. Emphasis is on optimum allocation or mix, and on linear (straight line) relationships among variables. The term "programming" does not refer to computer programming (although that usually is involved, for real-life problems); rather, it is a synonym for "planning"--for an orderly, step-wise approach to a problem.

Nonlinear programming also is used for obtaining an optimum allocation of resources, but does not require that

relationships be linear. Only rarely can graphical or algebraic procedures be used "by hand" to solve real-life problems which have nonlinear constraints. Computer software packages are widely used for this purpose.

Network flow models are useful for determining the best path along which to transport resources, in order to meet needs for these at various locations or times. In addition to their use for transportation of physical goods, network analysis techniques are used for project planning and control--the flow of a project through the steps needed for its completion. Two well-known network techniques for the latter problem are the Program Evaluation and Review Technique (PERT), and the Critical Path Method (CPM).

Finally, distribution models are used when a commodity is available at a number of sources and is needed at a number of destinations. The goal is to identify the least-cost transportation plan, from sources to destinations, while meeting the requirements of the users at the destinations and remaining within the amounts of the commodity available for distribution.

#### A. LINEAR PROGRAMMING MODELS

1. PURPOSE OF MODEL/TECHNIQUE: Determining the best way to allocate scarce resources among the demands of competing activities so that either the level of service (productivity) is maximized or the cost is minimized--while operating within a set of constraints.
2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:
  - a) Algebra, simple, linear
  - b) Descriptive statistics
  - c) Graphs and plots
  - d) Computer programming
  - e) Computer packages

3. HUMAN FACTORS APPLICATIONS:

- a) Designing systems, where relationships among system variables can be described in terms of a set of linear equations, and an optimum allocation of a scarce resource is needed.

4. DESCRIPTION:

- a) Model: A system or process is described as a function of seven things:

i) A set of decision variables (factors over which the allocator has control) that represent the amounts of each scarce resource (people, dollars, weapons) to be allocated among those who want them.

ii) An objective function or mathematical statement which relates the decision variables to each other via a linear equation; for example:

$$a(x1) + b(x2) = z,$$

representing the proportions, a and b, of dollars, x, going to activities 1 and 2, to yield a total of z dollars. This function may be minimized (e.g., total costs, z, be as small as possible) or maximized (e.g., productivity, z, be as great as possible)--depending on whether the decision variable represents costs or benefits.

iii) A set of maximum-setting constraints (supply constraints), which say that various decision variables cannot exceed certain values (that the amounts are limited--by law,

physics, economics, the nature of measurements, etc.). These must be expressed as linear equations or as linear inequalities.

- iv) A set of minimum-setting constraints (demand constraints), which say that at least a certain amount of some decision variables must be available--again expressed as linear equations or inequalities. This includes non-negativity constraints, which say that the amount of each decision variable must be greater-than-or-equal-to zero.
  - v) At least one feasible solution which simultaneously satisfies all constraints. If there are more than one, the set of feasible solutions is called a feasible region.
  - vi) At least one optimal solution, which is a feasible solution that yields the most favorable value for the objective function (sometimes there will be an infinite number of these, if the line representing the objective function happens to be parallel to the constraint line that is setting the limits).
  - vii) A stopping rule (needed for the algebraic simplex method) which specifies a way to recognize an optimal solution and to discontinue the iterations that have been seeking that optimum.
- b) Assumptions:
- i) Divisibility. All variables can assume any real value--fractional or integer (integer

linear programming is a subcategory which does not require this assumption; see Daellenbach and others, p. 519; Hillier and Lieberman, p. 714; Wagner, p. 469).

- ii) Non-negativity. All variables are non-negative (exist in quantities greater-than-or-equal-to zero). If it should happen that an activity can occur at negative as well as positive levels (e.g., we have the option of either buying or selling one of the items we consider to be decision variables), two separate decision variables are introduced:  $x^+$  for non-negative levels and  $x^-$  for non-positive levels. Their difference,

$$x = (x^+) - (x^-)$$

represents the actual level of the activity.

- iii) Linearity. All relationships among variables are linear or can be represented linearly through transformations. That is, the contribution of each variable is strictly proportional to its value, constant over the entire range of values that variable can assume. Also, the contributions of the variables are additive: the total equals the sum of the individual contributions, regardless of the values of the variables (i.e., there are no interactions). Even if this assumption is not met exactly, linear programming remains a convenient and powerful approximation, if relationships are close to linear within the range of solution values [Ref. 44].

- iv) Known constants. All parameters of the model are known constants. That is, the relationships expressed in the describing equations (the coefficients of the decision variables in the objective function and in the constraints) have already been determined. This assumption also is violated at times, since linear programming models are used to select some future course of action. This requires that the parameters be based on predictions of future conditions--introducing some uncertainty.
  - v) Convexity. The set of constraints must form a feasible region which is a convex polyhedron. This guarantees that any locally maximum solution is also globally maximum, and that no two constraints are mutually exclusive.
- c) Strengths:
- i) Commercial computer programs are available that are capable of solving huge problems with thousands of variables and constraints, using variations of the simplex method. This is possible since the number of iterations needed to find a solution increases only linearly with the number of constraints.
  - ii) Smaller problems (two or three variables and a half-dozen constraints) can be solved graphically (as is illustrated below). Axes on the graph represent the decision variables, and the constraints are shown as lines setting bounds for these variables.

d) Weaknesses:

- i) It is necessary to set up both the objective function and all constraints as linear equations or inequalities. Thus relationships among the variables must be expressible in the same general terms and be approximately linear.
- ii) Even with the use of efficient computer programs, solving large systems of equations simultaneously is time-consuming and expensive.
- iii) While the algebraic simplex method can be used "by hand" for problems of up to half-a-dozen variables and constraints, computations are arduous and prone to error.

e) General Procedures:

- i) Define clearly the resources that are to be allocated (the decision variables); determine what units these will be expressed in (dollars, man-months, kilograms, years, miles, etc.--as is appropriate to the problem). Assign a different symbol to represent each decision variable ( $x_1$ ,  $x_2$ ,  $x_3$ , etc., for dollars going to activity 1, activity 2, activity 3, etc.). Determine whether maximizing or minimizing will be done.
- ii) Determine the mathematical relationships among the decision variables, and express these in the form of a linear equation (objective function). Linear regression may

be useful for setting up this equation, if empirical data are available but the linear relationships among them are not obvious. The resulting equation should be of the form:

$$(a_1)(x_1) + (a_2)(x_2) + \dots = z, \quad (7.1)$$

where  $a_1$  and  $a_2$  are the coefficients relating the variables,  $x_1$  and  $x_2$ , and  $z$  is the total value to be maximized or minimized.

- iii) Enumerate the constraints which must be met. Formulate these into linear equations or inequalities, using the same symbols and units for decision variables as appear in the objective function. These constraints usually will be in the form of inequalities, such that the sum of some of the variables cannot be greater than some specific number (or less than a given value, in other cases).
- f) Procedures for a Graphical Solution. (no more than three decision variables; two preferred; see example below).
  - i) Label the coordinate axes of a standard Cartesian coordinate system to represent the decision variables.
  - ii) Plot all constraints (including non-negativity constraints) onto these axes to define the feasible region.

- iii) Lay out the objective function as a series of contour lines which represent the constant slope of that equation, at several values of  $z$ , as it intersects the axes at various values of the decision variables
  - iv) For a maximizing problem, that point farthest to the "northeast" where a contour lies within the feasible region represents the best (biggest) possible combination of decision variables, and the optimum solution for the objective function within the constraints.
  - v) For a minimizing problem, the optimum point will be found in the "southwest" corner of the feasible region, in that non-negative quadrant.
- g) Procedures for an Algebraic Solution (using a computer software package; see references below, for details of how these computations are done).
- i) Convert all inequalities to equations by introducing slack (for  $\leq$  inequalities) or surplus (for  $\geq$  inequalities) variables. These represent the amount by which the sum of the decision variables could be increased (slack) or decreased (surplus) and still lie within the feasible region. For example, if a constraint says that

$$5(x_1) + 2(x_2) \leq 30, \quad (7.2)$$

we can introduce a new variable, (s1), and say that

$$5(x1) + 2(x2) + (s1) = 30. \quad (7.3)$$

- ii) Represent the entire linear programming problem in the form of a table, in detached coefficient form (see example below). Variables are laid out across the top of the table to form columns (x1, x2, x3, s1, etc.). The far right-hand column contains the right-hand side of each constraint equation. Each row represents one of the constraints. The coefficients for each variable in each constraint then form the body of the table (or matrix). For convenience in entering data, the objective function also is laid out in this form, either at the top or the bottom of the constraint matrix.
- iii) Follow the instructions that came with the computer package, for data entry and for running the program.
- iv) The computer program will provide an optimum solution for the problem (or say why it cannot do so), yielding the recommended amounts of each scarce resource to be allocated to each activity.

5. ACM EXAMPLE (Hypothetical)

a) Situation:

- i) A one-on-one engagement is planned between a fighter and a simulated adversary, in a

practice dogfight. The fighter is testing the concept of carrying two types of pod-mounted guns, each using a different ammunition. The pilot's performance in being able to switch between the two, as needed, will be measured.

- ii) One type of ammunition (UR) uses spent-uranium rounds. It weighs 250 lb per 1000 rounds, compared with 200 lb per 1000 rounds for standard rounds (SR). The aircraft can carry a maximum of 1500 lb of ammunition.
- iii) Since this is a practice engagement, it is necessary to keep the cost of ammunition below \$20,000, while enabling the fighter to be as "lethal" as possible in the dogfight. The UR ammunition costs more than the SR (\$7000 per 1000 rounds, versus \$4000)--but is considered twice as lethal (a fact to use, if we wish to maximize "lethality" value).
- iv) The gun using the UR ammo is less efficient, firing rounds at a rate of 75 rounds-per-second (13 sec per 1000 rounds), to the SR gun's rate of 100 rounds-per-second (10 sec for 1000 rounds). For this engagement, a total of at least 30 sec of gun employment time is desired.
- v) In order to ensure a fair test, at least 1000 rounds of each of UR and SR must be carried.

b) Procedures:

- i) Decision variables are UR and SR, representing the amounts of the two kinds of ammunition to be carried on one engagement, in 1000-round units.
- ii) The objective function, to be maximized, is the lethality of fighter performance. Since UR contributes twice as much as SR to lethality, the equation is

$$2UR + SR = z. \quad (7.4)$$

Our goal is to find the values of UR and SR which will yield the maximum value for z, while meeting the constraints below.

- iii) Our constraints, placed in inequality form, are costs:

$$7UR + 4SR \leq 20 \quad (\text{in } \$1000 \text{ units}), \quad (7.5)$$

weight:

$$0.25UR + 0.2SR \leq 1.5 \quad (\text{in } 1000\text{-lb units}), \quad (7.6)$$

time:

$$13UR + 10SR \geq 30 \quad (\text{in seconds}), \quad (7.7)$$

quantities:

$$UR \geq 1 \quad (\text{in } 1000\text{-round units}), \quad (7.8)$$

$$SR \geq 1 \quad (\text{in 1000-round units}) \quad (7.9)$$

(note that this also satisfies non-negativity requirements).

iv) This same constraint information can be represented in tabular form:

| <u>Per 1000 Rounds</u> | <u>UR</u> | <u>SR</u> | <u>Total</u> |
|------------------------|-----------|-----------|--------------|
| \$1000 cost            | 7         | 4         | $\leq 20$    |
| 1000 lb weight         | 0.25      | 0.20      | $\leq 1.5$   |
| Firing time, sec       | 13        | 10        | $\geq 30$    |
| Quantities, 1000s      | 1         | 1         | $\geq 2$     |

c) Graphical Solution. Figure 7.1 illustrates how the constraints are mapped onto a two-dimensional representation of the decision variables, UR and SR (the axes) and the objective function, z (dashed lines). The point where the largest possible z-contour still lies within the feasible region (cross-hatched) is at (2.2, 1), and represents the optimum values for UR and SR, respectively, for this linear programming problem. "Lethality value" of 5.4 is the largest we can get, within the constraints. Note that the weight constraint is not a determining factor in the solution--maximum weight allowance is generous enough that it does not limit the amounts of the decision variables, in this instance, and the line

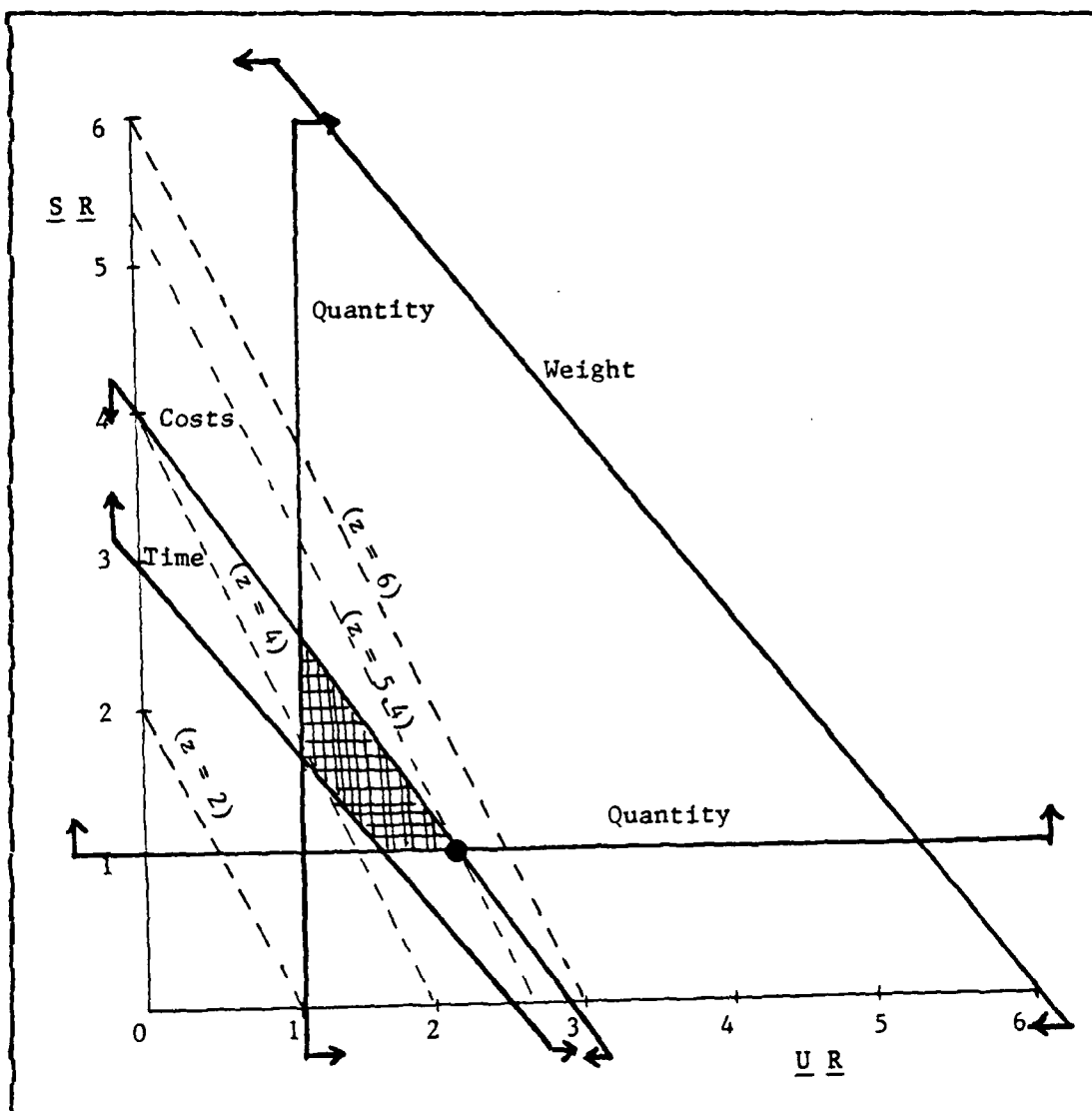


Figure 7.1 Graphical Solution to ACM Ammunition Problem

representing it does not help define the feasible region.

- d) Checking the Graphical Solution. Substituting the values of 2.2 for  $U R$  and 1 for  $S R$  in each of the constraints, we can show that these are indeed met:

$$(2)(2.2) + (1)(1) = 5.4 \text{ "lethality units"}$$

$$(7)(2.2) + (4)(1) = 19.4 \leq \$20K$$

$$(0.25)(2.2) + (0.2)(1) = 0.75 \leq 1.5 \text{ K lb}$$

$$(13)(2.2) + (10)(1) = 38.6 \geq 30 \text{ sec}$$

$$(2.2) \geq 1 \qquad (1) \geq 1 \quad 1000 \text{ rounds}$$

e) Simplex solution.

i) Introduce slack and surplus variables:

$$\begin{array}{rclcl} 7UR + 4SR + s1 & & & & = 20 \\ 0.25UR + 0.2SR + s2 & & & & = 1.5 \\ 13UR + 10SR - s3 & & & & = 30 \\ UR & & & - s4 & = 1 \\ & SR & & - s5 & = 1 \end{array}$$

ii) Prepare a detached coefficient table:

| <u>Constraints</u> | <u>UR</u> | <u>SR</u> | <u>s1</u> | <u>s2</u> | <u>s3</u> | <u>s4</u> | <u>s5</u> | <u>RHS</u> |
|--------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|------------|
| costs              | 7         | 4         | 1         | 0         | 0         | 0         | 0         | 20         |
| weight             | .25       | .2        | 0         | 1         | 0         | 0         | 0         | 1.5        |
| time               | 13        | 10        | 0         | 0         | -1        | 0         | 0         | 30         |
| quantity           | 1         | 0         | 0         | 0         | 0         | -1        | 0         | 1          |
| quantity           | 0         | 1         | 0         | 0         | 0         | 0         | -1        | 1          |
| object. funct.     | 2         | 1         | 0         | 0         | 0         | 0         | 0         | max. z     |

iii) Enter the above data values into whatever linear programming software package you have available on your computer, and follow instructions for obtaining a solution.

#### 6. USED IN LITERATURE:

- a) Ayoub, M.A., Ayoub, M.M., and Walvekar, A.G. "A Biomechanical Model for the Upper Extremity Using Optimization Techniques", Human Factors, Vol. 16, No. 6, 1974, pp. 585-594.  
Three approaches are used for solving an optimization model for arm articulation joints: linear and geometric programming, dynamic programming, and simulation.
- b) Benjamin, R. "Resources Deployment", Ergonomics, Vol. 15, No. 2, 1972, pp. 192-208.  
A basic optimization technique is used to allocate skilled workers according to job requirements. The technique should be useful for small scale problems.
- c) Bland, R.G. "The Allocation of Resources by Linear Programming", Scientific American, Vol. 244, No. 6, June 1981, pp. 126-144.  
The simplex method is discussed in terms of a "polytope" (three-dimensional solid). Several assignment problems are considered in depth. An excellent tutorial.
- d) Freund, L.E., and Sadosky, T.L. "Linear Programming Applied To Optimization of Instrument Panel and Workplace Layout", Human Factors, Vol. 9, No. 4, 1967, pp. 295-300.  
Small linear programming problems are solved by hand via the Hungarian method (see Daellenbach and others, p. 175), and product method (described). The simplex method is used with a computer program, for a slightly larger problem in instrument layout.
- e) Reid, R.A., and Sheets, E.E. "Applying Linear Programming to Logistics Planning", Defense Management Journal, Vol. 20, No. 2, 1984, pp. 26-30.  
Use of "canned" linear programming packages for desktop microcomputers is described in detail. A fine tutorial.

#### 7. REFERENCES AND TEXTS:

- a) Bazaraa, M.S., and Jarvis, J.J. Linear Programming and Network Flows. New York: JOHN Wiley and Sons, 1972.  
Highly technical; requires much comfort with mathematics to follow.

- b) Daellenbach, H.G., and others. Introduction to Operations Research Techniques, Second Edition. Boston: Allyn & Bacon, Inc., 1983.  
An excellent introductory text, for both linear and nonlinear programming; easy to read.
- c) Hillier, F.S., and Lieberman, G.L. Introduction to Operations Research. San Francisco: Holden-Day, Inc., 1980.  
A readable explanation, for both linear and nonlinear models.
- d) Nagel, S.S., and Neef, Marian. Operations Research Techniques. Beverly Hills: Sage Publications, 1976.  
Provides a very clear, brief example of how linear programming can be used.
- e) Wagner, H.M. Principles of Operations Research. New Jersey: Prentice-Hall, Inc., 1975.  
A large number of examples are provided--clever, but not always easy to follow. Both linear and nonlinear cases are included.

## B. NONLINEAR PROGRAMMING MODELS

1. PURPOSE OF MODEL/TECHNIQUE: Determining the best way to allocate scarce resources among the demands of competing activities so that either the level of service (productivity) is maximized or the cost is minimized--while operating within a set of constraints.

There is no "universal" NLP algorithm or technique. Algorithms are tailored to specific program classes. Computer software packages vary widely, both in applications and in requirements for use. Thus is it quite difficult to generalize about this technique. Potential users are advised to determine whether a nonlinear programming package is available to them; if so, they should study documentation on that particular software package.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra, simple, linear
- b) Calculus, single variable, multiple variable
- c) Logic and set theory
- d) Descriptive statistics
- e) Graphs and plots
- f) Computer programming
- g) Computer packages

3. HUMAN FACTORS APPLICATIONS:

- a) Designing systems, where relationships among system variables can be described mathematically in a form acceptable to whatever nonlinear programming computer package is available, and an optimum allocation of a scarce resource is needed.

4. USED IN LITERATURE:

- a) Faerber, R.H., Jr. Optimal Multimodel Parameter Identification in the State Space Model of the Human Operator. Air Force Institute of Technology, Wright-Patterson AFB, OH, December 1974 (GE/EE/74-42, AD-A008 707).  
Bounded random search techniques are used to identify parameters of interest, which are input into a clustering algorithm which identifies the human's (modeled) j-dimensional hypersurface. Newton-Raphson or gradient search techniques then are used to determine local and global maxima for the performance parameters.
- b) Kou, R.S., Glass, B.C., and Vikmanis, M.W. Reduced Order Observer Model for Antiaircraft Artillery (AAA) Tracker Response. Systems Research Labs, Inc., Dayton, OH, August 1979 (SRL-6872-7, AD-A080 632).  
Luenberger reduced-order observer theory, least squares curve fitting, and the Gauss-Newton gradient algorithm are used in an iterative simulation of human tracking error.

5. REFERENCES AND TEXTS:

- a) Daellenbach, H.G., and others. Introduction to Operations Research Techniques. Second Edition. Boston: Allyn & Bacon, Inc., 1983.  
An excellent introductory text; easy to read.
- b) Hillier, F.S., and Lieberman, G.L. Introduction to Operations Research. San Francisco: Holden-Day, Inc., 1980.  
A readable explanation.

- c) Nagel, S.S., and Neef, Marian. Operations Research Techniques. Beverly Hills: Sage Publications, 1976.  
Provides a very clear, brief example of how nonlinear programing can be used.
- d) Wagner, H.M. Principles of Operations Research. New Jersey: Prentice-Hall, Inc., 1975.  
A large number of examples are provided--clever, but not always easy to follow.

### C. NETWORK MODELS

1. PURPOSE OF MODEL/TECHNIQUE: Determining the best possible path through a series of events or locations, in order to maximize flow (or minimize cost or time) between the start of a process (or a source of goods) and a specified endpoint.
2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:
  - a) Logic and set theory
  - b) Probability theory (for PERT)
  - c) Descriptive statistics
  - d) Graphs and plots
  - e) Computer programming
  - f) Computer packages
3. HUMAN FACTORS APPLICATIONS:
  - a) Describing systems, where one or more paths through a system of events can be determined.
  - b) Designing systems, so that the best possible path is determined.
4. DESCRIPTION:
  - a) Model: A network system or process is described as a function of eight things:

- i) Nodes (vertices): points in time or space which represent events, tasks, or locations (usually shown as circles on a network graph).
- ii) Links (lines, arcs, edges, or branches): connections between any two nodes, associated with a flow from one to the next.
- iii) Direction of the flow, as it moves between nodes (shown by an arrow head). All links can have a flow in either direction (although flow capacity may have a value of zero in one direction); net flow is the difference between the two opposing flows.
- iv) Capacity (distance, cost) of the flow along a link between two nodes; a numerical value which is used in maximizing or minimizing the quantity of interest, over the entire network, by choosing the best links. A flow direction that is not permitted is given a capacity limit of zero. Positive excess capacity is whatever capacity is unused, in a given link.
- v) Source: a node which has all those links that are connected to it directed away from it.
- vi) Sink: a node which has all those links that are connected to it directed toward it.
- vii) Path: a set of connected links such that any node is passed through at most once. Excess capacity of the path is the minimum of the excess capacities of all links in

that path. A feasible path is a path that has positive excess capacity, as it goes from the source to a given node.

viii) Tree: A network having one more node than links; i.e., the path through the network is unique for each pair of nodes.

b) Assumptions:

- i) Divisibility. Flow capacity can assume any real value--fractional or integer. This assumption often is violated in the case of discrete units, if they are sufficiently numerous to be "essentially" continuous.
- ii) Conservation of flow. No flow is lost within the network.
- iii) Optimality. The solution is optimal if there is no path from source to sink which has positive excess capacity in every link.
- iv) Non-negativity. All variables exist in quantities greater-than-or-equal-to-zero.
- v) Linearity. All relationships among variables are linear (contributions proportional to values, constant over the possible range of values, and additive).
- vi) Known constants. All parameters of the model are known constants--have been empirically determined in some manner.
- vii) Homogeneity and equivalence. The product or commodity is homogeneous, regardless of its source or destination. All sources are equivalent (as are all destinations), except

for flow capacities along the links. Otherwise, we do not care from which source any destination gets its product.

c) Strengths:

- i) The procedure is simple to follow, appeals to logic, and is easy to defend.
- ii) The algorithm is easy to program for computer use, as is illustrated in the references below (Daellenbach and others, Hillier and Lieberman).

d) Weaknesses:

- i) Only problems with less than a dozen nodes and links can be done using the graphical method shown here.
- ii) The capacity values for the links are critical, and must be known with fair accuracy and precision if a useful answer is to be obtained.

e) Procedures: The process described here is known as the labeling technique. It is used to keep track of a feasible path (if one exists) from the source to each node, and to record excess capacities of the feasible paths to each node [Ref. 45].

- i) Identify nodes and links for the problem of interest, and assign capacity values, using a network graph to lay out the problem (as is illustrated in Figure 7.2, in the example below). It is convenient to identify nodes with alphabet letters.
- ii) Starting at the source, find any path from source to sink that can accommodate a

positive flow of material (or whatever the flow consists of). Only one path through the network should be kept track of at a time; it is not necessary to consider all feasible paths for each iteration. The smallest capacity value of any link in that path will determine the total flow for that path.

- iii) Write down the amount of the excess capacity that will be required for the total flow along that path, for the link from the first node to the second. Also write down the letter-designator of the previous node in the path (A, in this case). These are the labels for that second node (B), and are noted next to it in vector form: (excess capacity value, previous node letter). Do not label a node if the flow equals zero; even though a link exists there, no feasible path exists.
- iv) Taking the nodes in alphabetic order (convention), continue labeling, taking in turn each node in that path, as above. Continue until the sink is reached. At the sink, note the maximum amount (m) that can be transported along this path.
- v) Subtract the value m from the excess capacity (in the source-to-sink direction) for each link along that path. Add m to the reverse-flow (sink-to-source) capacity for each link. This process yields the "updated excess capacity" value for each link, once

that first path has been considered--the amount that still can be carried along that link, if another feasible path can be found.

- vi) Return to the source, and choose another path to the sink. Using the updated excess capacity values, repeat the above process.
  - vii) Add the amount of flow resulting from this new path to that obtained from following the first path. This is the updated total flow,  $m_1$ , which is also subtracted from each link's capacity, to obtain a new "updated excess capacity" value.
  - viii) Continue this path-definition process, from source to sink, until all feasible paths have been traced. At this point, an optimum solution for the maximal flow from source to sink has been obtained.
- f) Other calculations that may be made: See references below, for details of these calculations.
- i) Determination of the shortest route through a network, from source to sink.
  - ii) Minimization of the total length of connections among all nodes ("minimal spanning tree problem"), needed (for example) for transporting goods which are used at a number of locations along a network of roads.
  - iii) Project planning and control, for which events are scheduled along a timeline so that scheduled project completion date is met, at minimal cost.

5. ACM EXAMPLE (Hypothetical)

a) Situation:

- i) A total of 25 fighter aircraft aboard a carrier must be moved to the catapult area and launched, for a combat air patrol (CAP) mission.
- ii) There are three routes along which the fighters may be transported. One of these is a direct route from parking area to catapult, on the carrier deck. The other two routes involve moving aircraft from below deck, via elevators, to the deck.
- iii) Based on accessibility and conditions of the aircraft in their present locations and on the personnel available to move them, ten aircraft (maximum) can be moved to one elevator area (node B) and seven to the other (node C) within the allotted time. Five aircraft may be transported between the two elevator loading areas, in either direction (or both directions, if needed), within that timeframe. From elevator B, four aircraft can be gotten to the catapult within the time limits, and ten may be transported from elevator area C.
- iv) The maximum quantity of aircraft possible must be gotten from storage to catapult, within the time available.

b) Procedures:

- i) Prepare a network graph, as is illustrated in Figure 7.2, to describe the problem and the initial information that is available.

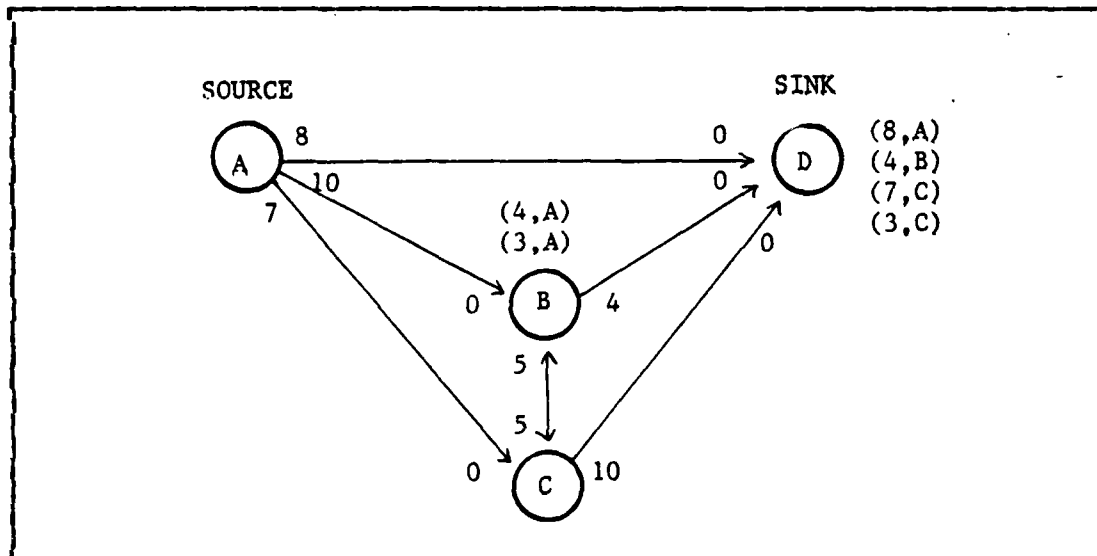


Figure 7.2 Network Graph Illustration for ACM Example

- ii) Taking first the direct route (A,D), label node D as (8,A). The total flow resulting from path (A,D) is 8.
- iii) Subtract the value of 8 from the (A,D) link capacity value, leaving a remainder of 0. Add 8 to the reverse flow value (originally zero), yielding a value of 8.
- iv) Returning to the source, A, trace out path (A,B,D). The label for node B is (4,A), and for node D, at this iteration, is (4,B).
- v) Flow resulting from this path has a value of 4, which is added to the value of 8 from the first path traced, to get a current total flow of 12.
- vi) Retracing the path just followed, 4 is subtracted from link (B,D)'s capacity,

leaving a flow capacity of 0 (and added in the (D,B) direction, to yield 4). For the (A,B) link, subtracting 4 from 10 leaves an unused capacity of 6 for this link (and a reverse direction value of 4).

vii) At this point, either the path (A,B,C,D) or the path (A,C,D) may be traced. For simplicity, (A,C,D) will be used next. Labels at this iteration become (7,A) for node C and (7,C) for node D. Total flow is increased by 7, yielding a value of 19 at this point.

viii) The final path (A,B,C,D) now is traced. Labels for this iteration are (3,A) for node B, (3,B) for node C, and (3,C) for node D. It should be noted that the remaining excess flow capacity of 3 at link (C,D) has limited this entire path to a maximum of that value.

ix) The final total flow value is 22, when 3 aircraft are added from this final path iteration. Within the time constraints, this is the maximum number of aircraft which can be gotten to the catapult.

x) The reverse-flow values which were calculated during the problem-solving process were not needed, for this example. In other cases, however, it will be found that increased flow values will be obtained sometimes by what at first appears to be "backtracking".

6. USED IN LITERATURE:

- a) Callahan, L.G., Jr., and Lovell, James. Graceful Degradation of Air Defense Capabilities. Georgia Institute of Technology, June 1982. (AD No. B075 562).  
Includes a brief review of approaches to weapons systems modeling, a review of network methodologies, and a model of a battalion-level air defense network.
- b) Fakan, J.C. Application of Modern Network Theory to Analysis of Manned Systems. National Aeronautics and Space Administration, October 1970. (TN-D-6034, NPS U 135276).  
Network theory is used for describing man's functional roles in a human subsystem. Human parameters include heart rate, as a measure of work output. A FORTRAN program is provided which outputs time of expected task degradation, based on input of human performance characteristics, etc.
- c) Lewis, Leslie, and Copeland, Melinda. Human Performance Requirements in C<sup>3</sup>I Systems and Their Implications in System Design. TRW Defense and Space Systems, Redondo Beach, CA, March 1983. (AD NO. P000 890).  
The user interface with C<sup>3</sup>I systems is modeled, defining cognitive processes as quantitative, testable units. Techniques include the use of networks to translate these processes into system requirements.
- d) Pew, R.W., and others. Critical Review and Analysis of Performance Models Applicable to Man-Machine System Evaluation. Bolt, Berenak, and Newman, Inc., Cambridge, MA, March 1977 (BBN No. 3446, AFOSR-IR-77-0520, AD-A038 597).  
Five network-based techniques for predicting human performance are surveyed and evaluated, as part of this 300-page comprehensive report. These include the SAINT, PERT, and THERP models.
- e) Pulat, B.M. A Computer Aided Workstation Assessor for Crew Operations--WOSTAS. North Carolina State University, May 1982. (AD NO. A116 045).  
A network-based model which is part of the Multi-Man-Machine Work Area Design and Evaluation System (MAWADES). WOSTAS groups activities or tasks of a crew so that all job stations have a fairly equal amount of work, in terms of time to perform tasks.
- f) Randolph, P.H., and Ringeisen, R.D. "A Network Learning Model with GERT Analysis", Journal of Mathematical Psychology, Vol. 11, No. 1, 1974, pp. 59-70.  
Graphical Evaluation and Review Technique (GERT) is used to analyse the teaching and learning process, when that process is represented as a stochastic network. A topology equation for a closed network is used to obtain parameters for the teaching-learning process.

- g) Smillie, R.T., and Ayoub, M.A. "Job Performance Aids: Evaluation of Design Alternatives Via Network Simulation", Ergonomics Vol. 23, No. 4, 1980, pp. 319-339.

Network simulation is used as an alternative to laboratory experimentation to evaluate different combinations of job performance aid formats, combined with the effects of stress.

- h) Wortman, D.B., and others. The SAINT User's Manual. Pritsker and Associates, Inc., West Lafayette, IN, June 1978 (AD-A058 724).

SAINT (Systems Analysis of Integrated Networks of Tasks) is a network modeling and simulation technique used in design and analysis of complex man-machine systems. Systems can consist of discrete tasks, continuous state variables, and interactions between them.

See also Simulation Using SAINT: A User-Oriented Instruction Manual, by the same authors (July 1978, AD-A058 671).

## 7. REFERENCES AND TEXTS:

- a) Bazaraa, M.S., and Jarvis, J.J. Linear Programming and Network Flows. New York: John Wiley and Sons, 1972.

Highly technical; requires much comfort with mathematics to follow.

- b) Bronson, Richard. Schaum's Outline, Theory and Problems of Operations Research. New York: McGraw-Hill Book Company, 1982.

A brief but clear discussion of the maximal-flow, minimum-span, and shortest route problems.

- c) Daellenbach, H.G., and others. Introduction to Operations Research Techniques. Second Edition. Boston: Allyn & Bacon, Inc., 1983.

An excellent introductory text; easy to read; good sections on CPM and PERT techniques.

- d) Hillier, F.S., and Lieberman, G.L. Introduction to Operations Research. San Francisco: Holden-Day, Inc., 1980.

An excellent, readable explanation, with clear applications of the technique to four classes of problems, including CPM and PERT.

- e) Wagner, H.M. Principles of Operations Research. New Jersey: Prentice-Hall, Inc., 1975.

Several examples are provided--clever, but not always easy to follow.

#### D. DISTRIBUTION MODELS

1. PURPOSE OF MODEL/TECHNIQUE: Finding the least-cost distribution schedule for transporting a commodity between a number of sources and a number of destinations, to meet demands from current inventory.
2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:
  - a) Matrix or linear algebra
  - b) Logic and set theory
  - c) Probability theory
  - d) Descriptive statistics
  - e) Graphs and plots
  - f) Computer programming
  - g) Computer packages
3. HUMAN FACTORS APPLICATIONS:
  - a) Describing systems, where they can be considered as a set of starting places and end points, connected by paths.
  - b) Designing systems, in order to find the most efficient path between points.
4. DESCRIPTION:
  - a) Model: A system or process is described as a function of four things:
    - i) Sources, each of which has available a given quantity of units of a specified, homogeneous commodity or product.
    - ii) Destinations, each of which requires a given quantity of units of that same commodity or product.
    - iii) Cost of transporting one unit of product from one of these sources to one of these destinations.

- iv) Variables, that is, the to-be-determined number of units to be shipped between a given source and a given destination. Basic variables are variables which are assigned numerical (non-zero) values in the current solution. Nonbasic variables are "unsigned variables"; that is, they have a value of zero (no goods are shipped from that source to that destination, for this solution).
- b) Assumptions:
- i) Integral units. The product occurs in integer units only; that is, a unit cannot be further broken down or fractionalized.
  - ii) Non-negativity. All variables exist in quantities greater-than-or-equal-to-zero.
  - iii) Linearity. All relationships among variables are linear (contributions proportional to values, constant over the possible range of values, and additive).
  - iv) Known constants. All parameters of the model are known constants--have been empirically determined in some manner.
  - v) Conservation of flow. No product is lost within the transportation network.
  - vi) Equal supply and demand. Total supply and total demand are equal. If this is not true in actuality, either a fictitious source or a fictitious destination is created to provide or absorb the extra product.

vii) Homogeneity and equivalence. The product or commodity is homogeneous, regardless of its source or destination. All sources are equivalent (as are all destinations), except for cost of distribution. Otherwise, we do not care from which source any destination gets its product.

c) Strengths:

- i) The procedure is simple to follow and appeals to logic, so is easy to defend.
- ii) The algorithm is easy to program for computer use (see Daellenbach and others, pp. 157-168).

d) Weaknesses:

- i) The procedure of optimization becomes arduous, if there are more than a handful of sources and destinations. In this case, the aid of a computer is mandatory.
- ii) Degeneracy is a frequent occurrence in the distribution problem. This results in the "stepping stone" algorithm going from iteration to iteration without any improvement in the distribution. See Daellenbach and others, p. 165, or Bronson, p. 72, for a treatment of this problem.

e) Procedures: These procedures are illustrated below in the example.

- i) Set up a matrix or tableau (see Figure 7.3) showing complete data for the problem: sources and availability (supply) of products (rows), destinations and requirements

(demand) for products (columns), and distribution costs (noted in the upper left corner in each cell).

- ii) Find an initial solution, via the "Northwest Corner" rule. Beginning with the northwest (top left) corner cell in the tableau, allocate from the amount available at Source 1 as many units as possible to Destination 1 (up to the total amount available or the total required). Write this number in the cell. Thereafter, continue by moving one cell to the right (if some product remains), allocating units of product to the next Destination. If no supply remains in Source 1, move down the matrix one cell. Now the product from Source 2 will be allocated to the Destinations, until it is all used up. The procedure is continued until the "south east" (lower right) corner of the matrix is reached. This yields an initial feasible solution.
- iii) Test this solution for optimality, which is a function of the cost of this particular solution for the given problem. To do so, create testing variables,  $u$  (associated with the "supply" rows; see Figure 7.3), and  $v$  (associated with the "demand" columns). For each cell, the sum ( $u + v$ ) must equal the cost value,  $c$ , for that cell. Arbitrarily choose some  $u$  variable associated with one supply row, and set it equal to zero. Now we can set up sufficient ( $u + v = c$ ) equations to solve for  $u$  and  $v$  for each column

and row, starting with the basic variable cells. Note that some values of  $u$  and  $v$  may be negative at this point. Next, subtract both  $u$  and  $v$  from  $c$ , for each nonbasic variable cell in the matrix, to find the value  $(c - u - v)$  for cells that presently have no allocation of product from that source to that destination. Place this number in the lower right-hand corner of this nonbasic variable cell. If at least one of these  $(c - u - v)$  values is negative, the current solution is not optimal. A better solution will be found by increasing the allocation (presently zero) in the cell having the most negative value for  $(c - u - v)$ . Place a "+" sign in that cell, to signify that increase is desired.

- iv) Improve the solution. Identify a loop in the matrix, so that the loop contains the cell with the "+" and at least three other cells all of which contain values for basic variables. The sequence of cells in a loop must be such that each pair of consecutive cells lies either in the same row or the same column (no diagonals), but no three consecutive cells do. No cell can appear in a loop more than once, and the loop must be closed, with beginning and end lying in the same cell. Increase the allocation to the "+" cell as much as possible, while adjusting other cell allocations in the loop so that supply, demand, and nonnegativity constraints are not violated. This results

in a new solution to the distribution problem. Prepare a new tableau showing the solution (see Figure 7.4).

- v) Once again check for optimality, as in step (iii). If the solution still is not optimal, repeat step (iv). Continue this process until a solution is obtained for which no value of  $(c - u - v)$  is negative. This solution will be optimal.
- f) Other calculations that may be made: See references below, for details of these calculations.
  - i) Transshipment problems, with "warehouses" available to facilitate shipments in two stages, rather than directly from source to destination.
  - ii) Assignment problems, where a given number of candidates must be assigned uniquely to a specified number of jobs (one-to-one) in such a way that all jobs are completed in the minimum total time. The Hungarian method is the most efficient technique for this problem (see Daellenbach and others, p. 175, or Bronson, p. 85).
  - iii) Traveling salesman problems, where one individual must leave a base location and visit a number of other locations, one time each, then return to the starting location. Objective is to minimize the distance or cost of travel (see Bronson, p. 85, or Daellenbach and others, p. 662).

5. ACM EXAMPLE (Hypothetical)

a) Situation:

- i) A new cockpit is being designed for a two-man fighter aircraft. The crew station will contain several CRT-type displays, capable of providing a wide variety of information to the crew from a number of sources. These displays will be placed so that each is available for monitoring by either crew member during a typical air-to-air mission. However, it is critical that each display be monitored, and that neither crew member be overloaded with monitoring tasks, which also include radio communications and visual out-the-window inspections of the area.
- ii) Task analyses indicate that the pilot must spend 60% of his time in flight control tasks, leaving 40% for monitoring the information displays, etc. The radar officer (RO) will be busy with navigation and weapon delivery tasks 40% of the time, leaving 60% for monitoring-type tasks.
- iii) Five sources of information must be monitored: a radar warning receiver display (RWR), a tactical information display (TID), and the air-to-air radar scope (RDR), plus radio communications (COM) and frequent out-the window (OTW) checks of the surroundings. Table 7 shows the percent of time each must be monitored, based on analysis of the mission (totaling 90% of available time). Since it is desired to account for

100% of the crew's "spare" time, a "dummy" column is included for the remaining 10% of the time--perhaps representing time to stretch, scratch, etc.

**TABLE 7**  
**PERCENT OF TIME INFORMATION SOURCES MUST BE MONITORED**

|                       | INFORMATION SOURCE |            |            |            |            |              |
|-----------------------|--------------------|------------|------------|------------|------------|--------------|
|                       | <u>RWR</u>         | <u>OTW</u> | <u>TID</u> | <u>COM</u> | <u>RDR</u> | <u>DUMMY</u> |
| MONITORING<br>TIME, % | 15                 | 30         | 10         | 10         | 25         | 10           |

- iv) Monitoring each of these information sources is not equally easy for both crew members, due to locations, to interference with primary tasks, and to difficulty of interpreting the information. "Costs" or difficulty values have been assigned to these monitoring tasks, as is shown in Table 8.

b) Procedures:

- i) Set up an initial tableau for the problem, using all the information that has been provided. The tableau is shown in Figure 7.3.

**TABLE 8**  
**RELATIVE DIFFICULTY OF MONITORING INFORMATION SOURCES**

| CREW MEMBER   | INFORMATION SOURCES |            |            |            |            |              |
|---------------|---------------------|------------|------------|------------|------------|--------------|
|               | <u>RWR</u>          | <u>OTW</u> | <u>TID</u> | <u>COM</u> | <u>RDR</u> | <u>DUMMY</u> |
| Pilot         | 5                   | 3          | 9          | 1          | 9          | 0            |
| Radar Officer | 5                   | 7          | 6          | 1          | 7          | 0            |

|        | RWR     | OTW     | TID     | COM     | RDR     | Dummy   | Supply | u  |
|--------|---------|---------|---------|---------|---------|---------|--------|----|
| Pilot  | 5<br>15 | 3<br>25 | 9<br>7  | 1<br>5  | 9<br>2  | 0<br>4  | 40     | -4 |
| RO     | 5<br>+  | 7<br>5  | 6<br>10 | 1<br>10 | 7<br>25 | 0<br>10 | 60     | 0  |
| Demand | 15      | 30      | 10      | 10      | 25      | 10      | 100    |    |
| v      | 9       | 7       | 6       | 1       | 7       | 0       |        |    |

**Figure 7.3 Initial ACM Distribution Tableau, First Solution**

- ii) The initial solution, shown in Figure 7.3, is found using the Northwest Corner rule. The pilot would spend 15% of his time monitoring the RWR display and 25% looking out

the window. The RO would spend 5%, 10%, 10%, and 25%, respectively, with the window monitoring, TID, radio communications, and radar display tasks, and 10% on the "dummy" task (unassigned time).

- iii) This solution is tested for optimality by finding, first, the values of  $u$  and  $v$ , via solution of the equations,  $(u + v = c)$ : For ease of computation,  $u_2$  is assigned the value of zero. Then, from the cost for cell (2,2),  $v_2$  is found to be 7 ( $0 + v_2 = 7$ , or  $v_2 = 7$ ). Similarly,  $v_3 = 6$ ,  $v_4 = 1$ , etc. Now from the  $v_2$  value of 7 and the cell (1,2) cost of 3, we can determine that  $u_1$  must be -4 ( $u_1 + 7 = 3$ , or  $u_1 = -4$ ). Finally, we determine the value of  $v_1$  from ( $u_1 + v_1 = 5$ ):  $-4 + v_1 = 5$ , or  $v_1 = 9$ . Now we examine the nonbasic variable cells to find the values for  $(c - u - v)$ . For cell (1,3),  $(9 - (-4) - 6 = 7)$ , as is noted in the lower right-hand corner. For cell (1,4),  $(1 - (-4) = 5)$ , etc. Continuing the process, we discover that the value for cell (2,1) is  $(5 - 0 - 9 = -4)$ . Thus we find that this solution is not optimal. This value of (-4) is the most negative (only negative, in this instance), so a "+" is placed in cell (2,1).
- iv) A loop is now constructed (as is shown by the heavy lines in Figure 7.3), containing the cell with the "+" (cell (2,1)) along with the three nearest cells with basic variables (adjacent cells, in this instance). The most by which cell (2,1) can

be increased is 5, in order to remain within the "demand" constraints of 15 for the RWR. Thus, 5% of the RO's time will be taken away from monitoring OTW so that he can spend 5% of his time on the RWR display. The resulting new solution is shown in Figure 7.4.

|        | RWR       | OTW       | TID       | COM       | RDR       | Dummy     | Supply | u |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|--------|---|
| Pilot  | 5<br>(10) | 3<br>(30) | 9<br>3    | 1<br>0    | 9<br>2    | 0<br>0    | 40     | 0 |
| RO     | 5<br>(5)  | 7<br>4    | 6<br>(10) | 1<br>(10) | 7<br>(25) | 0<br>(10) | 60     | 0 |
| Demand | 15        | 30        | 10        | 10        | 25        | 10        | 100    |   |
| v      | 5         | 3         | 6         | 1         | 7         | 0         |        |   |

Figure 7.4 Second Solution to ACM Distribution Problem

- v) The optimality of this solution now is tested, as above. This time none of the  $(c - u - v)$  values is found to be negative (see Figure 7.4). The solution is optimal, with the pilot spending 10% of his time monitoring the RWR display and 30% looking out the window. The RO has no out-the-window tasks, but instead spreads his time over all the other displays and has 10% "free" time to do things not called out in the model.

c) Caveat: This example points up the importance of assumptions. For simplification, we have assumed here (assumption (vii)) that it makes little difference whether each of the tasks is performed by the pilot or by the RO, as long as the assigned costs are considered. Thus, under the formulation here, the pilot ends up doing all the out-the-window monitoring, and gets none of the "surplus" (dummy) time. Under the Northwest Corner initialization process, a completely different allocation of tasks would result were the RO assignments listed in the first row of the matrix and the pilot assignments in the second. Whether the resulting allocation under this set-up would be equally good is debatable.

6. USED IN LITERATURE: No examples of use of distribution models were found in the human factors literature.

7. REFERENCES AND TEXTS:

- a) Bazaraa, M.S., and Jarvis, J.J. Linear Programming and Network Flows. New York: John Wiley and Sons, 1972.  
Highly technical; requires much comfort with mathematics to follow.
- b) Bronson, Richard. Schaum's Outline, Theory and Problems of Operations Research. New York: McGraw-Hill Book Company, 1982.  
A brief but clear discussion of the transportation problem and degeneracy.
- c) Daellenbach, H.G., and others. Introduction to Operations Research Techniques, Second Edition. Boston: Allyn & Bacon, Inc., 1983.  
An excellent introductory text; easy to read.
- d) Hillier, F.S., and Lieberman, G.L. Introduction to Operations Research. San Francisco: Holden-Day, Inc., 1980.

A good, general explanation of the transportation problem and related algorithms.

- e) Wagner, H.M. Principles of Operations Research.  
New Jersey: Prentice-Hall, Inc., 1975.  
Several examples are provided--clever, but not  
always easy to follow.

### VIII. MODELS FOR DECISIONS

A decision usually is considered to be a choice among alternatives. If there is only one solution or course of action worthy of consideration, there is no decision, as such, to be made. Instead, optimization techniques may be used to make the best of the situation (as described in Chapter VII). Or descriptive models may be developed to describe the situation better, and make predictions about the results of taking that course of action (see Chapter VI).

The various alternatives may be discrete, separate entities ("Shall I hire John Smith or Mary Jones?"); or they may be continuous functions (or nearly so) within a given range ("How certain should I be that an aircraft is unfriendly, on a target recognition continuum scale, before I shoot it down?").

One factor common to decision models is the need for at least one measure of effectiveness (MOE) and for some criterion or standard for making a choice. The decision model will not provide these; they come from the decision maker himself, outside the modeling process. A MOE is needed if we are to measure the "acceptableness" of a given alternative. A criterion is required to tell us exactly how good an alternative must be, on that "acceptableness scale", in order to be "good enough" (in terms of money, time, pleasure, etc.). This concept is called "satisficing" (as opposed to the process of "optimizing", or finding the optimum solution).

If no numerical MOES and cut-off criteria are available, it still will be possible to rank alternatives. However, subjective techniques then will be needed to choose the most value-effective possibility.

In the first section of this chapter we consider in detail the models and procedures used in what variously is described (with differing emphasis) as decision theory, game theory, or utility theory. Decision theory is the broadest of the three, and may be considered to include the others. Decision analysis is the basic technique used with these decision theory models.

Game theory emphasizes decisions where two or more individuals are in conflict over their opposing goals. According to Raiffa [Ref. 46], classical game theory attempts to offer advice to each of the conflicting individuals (a jointly prescriptive approach). More recent theoretical studies have considered conflict situations from a one-sided prescriptive point of view, with the goal of helping one (and only one) party win.

Utility theory emphasizes the expected usefulness or value of the various outcomes. Its major feature is development of a utility function (usually linear) that transforms payoffs (say in dollars) into a utility scale, based on some useful value of each payoff (perhaps whether that many dollars will be enough to pay the rent). The resulting scale is then used in the decision analysis procedure.

Signal detection theory models are briefly covered in the second section. These models already are used extensively in human factors analyses, so are not discussed in great detail here. They can be valuable tools in evaluating the various outcomes of choosing different alternatives along a continuum of values. Interested persons may refer to the listed references for more details.

#### A. DECISION THEORY MODELS AND DECISION ANALYSIS

1. PURPOSE OF MODEL/TECHNIQUE: Choosing one of several well-defined alternatives that will meet an

aspiration level, or predetermined criteria or standards of adequacy.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra
- b) Boolean algebra
- c) Logic and set theory
- d) Fuzzy set theory
- e) Probability theory
- f) Descriptive statistics
- g) Graphs and plots

3. HUMAN FACTORS APPLICATIONS:

- a) Describing systems, where a system includes alternative outcome states, with varying probabilities of occurrence.
- b) Designing systems, where a choice among alternative systems or subsystems must be made.
- c) Evaluating human performance, when utility values can be assigned to various levels of performance, along with criteria for acceptable performance.

4. DESCRIPTION:

- a) Model: A system or process is described as a function of 11 things:
  - i) A problem which requires that at least one decision (choice) be made.
  - ii) A time horizon within which that decision is required.
  - iii) The sequence of decisions which are required, in a multiple-stage problem.
  - iv) A well-defined set of alternative actions (decision variables) from which a choice

must be made, for solution of the problem. These are under the decision maker's control.

- v) A set of events that possibly may occur (future states of nature, or chance points). If possible, these should be mutually exclusive and collectively exhaustive. These are not under the control of the decision maker.
- vi) The probabilities of occurrence for these events.
- vii) The set of payoffs or outcomes (outcome variables) which accompany these alternative actions and events.
- viii) The structural relationships between alternatives/events and their corresponding outcomes, expressed as a mathematical function, if possible. The parameters of these relationships are included in this function.
- ix) The utility values or "expected worth" of each of the outcomes. This may be the same as the outcomes themselves. If more than one factor is included in assessing the value or worth of a given outcome, these are considered multi-attribute utilities. Associated with the utilities are measures of effectiveness (MOEs), used to evaluate the outcomes. This may be as simple as "more is better". If more than one MOE will be used, the importance of each should be weighted. Then the MOEs may be aggregated into a single criterion function (usually a

linear combination), which can be used for evaluations.

- x) At least one criterion or aspiration level, which is used to determine that an alternative and its associated outcome will result in a satisfactory solution.
- xi) A payoff matrix and/or a decision tree. These lay out the above information in logical form, so that the analysis can be performed. A payoff matrix usually is adequate for a single-stage decision problem. A decision tree is required if a series of alternatives and events must be evaluated in order to reach a final outcome.

b) Assumptions:

- i) Steady state conditions. The system is in equilibrium; we are considering a problem that is not in a state of flux.
- ii) Relationship validity. The choices open to the decision maker may be adequately described in terms of payoff values or utilities and their associated probabilities. The payoff matrix/decision tree used to describe the system is an adequate representation of the system, for purposes of obtaining useful results.
- iii) Certainty, risk, and uncertainty. It is possible to place the decision being made in one of the following categories:
  - under certainty: we assume that one given state will occur, and all others have zero probability.

- under risk: we can estimate the probability of occurrence for each possible future state, and two or more of these probabilities are positive.
  - under uncertainty: we are unable to estimate the probabilities of various future states (although we can list those states).
- iv) Known constants. All constant parameters and utilities used in the model are known values, obtained through some empirical data collection process (objective values) or through logical deductions (subjective or Bayesian values).
- v) Known probabilities and probability distributions. Randomly distributed events can be characterized by known discrete or continuous probability distributions. These probabilities may be strictly objective, obtained through observations and measurements (the limit of long-term relative frequencies). They also may be subjective (Bayesian), based on a priori probabilities assigned by experts.
- vi) Stationarity. Probabilities of events and outcomes do not change with time, within any one stage of a decision analysis process.
- vii) Independent alternatives and events. All alternatives and all events are mutually exclusive. Thus, joint probabilities may be obtained by multiplying individual probabilities together.

viii) Logical consistency or coherence. Rational beliefs and actions all are logically consistent with one another, involving no mutual contradictions. Note that coherence is not sufficient to guarantee rationality, but it is necessary.

ix) Ordering. It is possible for the decision maker to express preference or indifference between any pair of payoffs. That is, he can rank payoffs in order of value to him, or he can express no preference at all among them.

x) Linearity of multi-attribute utilities. If several MOEs are considered in developing a utility value, relationships among them may be expressed as a linear combination (contributions are proportional to values, constant over the possible range of values, and additive).

c) Strengths:

i) The technique of decision analysis is an excellent way to provide greater insight into a decision problem, and especially to open it up for discussion and conflict resolution. It encourages scrutiny of the problem as a whole, and forces the decision maker to determine quantitative relationships among the various parts of his problem. New sources for gathering and organizing information may be suggested by the process, and new alternative actions may be uncovered.

- ii) The structures of payoff matrices and decision trees provide a convenient basis for communicating and justifying an analysis.
  - iii) The process can aid in identifying who the decision maker actually should be, for a given problem, once alternatives and outcomes are laid out. The person most affected by (and affecting) the system then more easily can be identified.
  - iv) The decision maker's preferences for various outcomes can be separated from his judgments about probabilities, using this technique.
- d) Weaknesses:
- i) Emphasis on the construction of a disciplined structure (payoff matrix or decision tree) may divert attention from the value of creative inputs to problem solving. This analytical pattern of thinking does not take advantage of other styles of thought, such as intuitive, lateral, and imaginative.
  - ii) It is easy to oversimplify a problem during its decomposition into manageable pieces. This is especially true in utility function assessment, where simple, contrived questions may be used to elicit relative values in some usable form--when the values actually are much more complex than the process would indicate.
  - iii) Analysing the wrong problem is a real hazard. The decision analyst seldom is the

decision maker. The analyst must take great care to learn precisely the nature of the problem being faced. Otherwise, he may seize on some facet of the situation that interests him (and that he can handle)--but that is of no real concern to the client.

- iv) Independence of variables is rare, in the real world. It may be necessary to partition uncertain quantities into categories that then may be nearly independent. Or mathematical transformations sometimes may be used (for example, using differences between values rather than the values themselves), which more nearly meet the requirements of independence.
  - v) Utility functions are not always linear, in real life. A given risk when a person is at one state in a system ("I'm broke anyway") will be viewed differently than when at another state ("I'm already comfortably off"). Also, differences between utility values usually express merely the rank of an outcome, not the actual proportional strengths of preference.
  - vi) Utilities are not comparable from person to person. A utility function is a personal statement of an individual's risk attitude, and cannot be aggregated with the utility function of another individual without the use of normalization techniques.
- e) Procedures: Not all of the procedures listed here will be applicable (in this exact form) to all

decision problems. The user must select those that are appropriate to his situation, and revise the steps as necessary. This general procedure is illustrated below in the example.

- i) Define the problem: what is the immediate decision to be made? Determine that choices among alternatives are required, and that some measure of effectiveness and criterion of a satisfactory solution can be found. Check that the required assumptions can be (approximately) met. Strip away irrelevant factors from the situation and system. Determine who the actual decision maker should be.
- ii) Set the time horizon which will be considered for this study. Will we begin with the situation right now, and look at the next two days? Or might we begin with hypothetical states five years hence, and consider the period of the following 20 years?
- iii) Determine whether this is a single-stage or multiple-stage decision. After the problem has been laid out, will we make one decision and be done? Or will a series of decisions be made, each relying on the preceding decision? If multiple-stage, lay out the sequence in which choices will be made.
- iv) List the alternative courses of action open to the decision maker. Be as comprehensive as possible; less useful options can be eliminated as we go along. Remember that "do nothing" and "delay the decision" also

are alternative courses of action. Insofar as possible, the alternatives should be mutually exclusive.

- v) Lay out the events that possibly may occur, in their expected sequence. These events will determine the "state of nature" of the system by their occurrence. Decide if this will be a decision under certainty, under risk, or under uncertainty.
- vi) If this will be a decision under risk, assign probabilities to the occurrence of each of the above events. These probabilities should be based on available data, or on some logical process of determination.
- vii) List all possible outcomes that can result from the above-noted alternatives and the possible events. If possible, state these in terms of payoffs--though not necessarily in money alone. Remember that payoffs can be negative as well as positive.
- viii) If possible, express the relationships between alternatives/events and their resulting outcomes in the form of a mathematical equation or other function. This will be easiest for decisions under certainty. A logical flow diagram can be used if numbers cannot be assigned to the various parameters, showing relationships in time.
- ix) Determine the utility value for each listed outcome, based on the decision maker's value

system. If a number of attributes of each outcome must be considered in valuing it (cost, weight, color, size), this must be considered a multi-attribute utility problem. It will be necessary to add extra sub-steps here to combine these into a single, useful utility function--a process beyond the scope of this presentation (see References and Texts below for books that cover this situation). It is preferable to choose one significant, numerical result that easily can be determined and ranked (such as profit or time saved). This utility value may be considered the measure of effectiveness, or it may be some function of the MOE (which then also must be defined here).

- x) Set the aspiration level, or criterion for satisfaction, success, or usefulness, based on the MOE, outcome, and/or utility values. Early determination of this criterion, before the actual analysis begins, lessens the chance of biasing criterion point selection by knowledge of "what is possible".
- xi) Prepare a payoff matrix incorporating the above information (see Figure 8.1 for an example). The various events that may occur are listed at the top of the matrix, along with their respective probabilities of occurrence. Down the left side are listed the alternatives from which the decision maker may choose. The body of the matrix contains the payoffs which result from each

pairing of alternative and event. The completed matrix organizes the information needed to make a decision into a convenient form for beginning the analysis. If this is a multiple-stage problem and parameters differ for the various stages, a separate payoff matrix may be needed for each stage.

- xii) Check the payoff matrix for dominance. If one alternative is as good as or better than another under all states resulting from the events, the dominated alternative should be eliminated from the analysis.
- xiii) For a multiple-stage problem, draw a decision tree similar to that shown in Figure 8.2. In such a diagram, time moves from left to right. A square box indicates a decision point, where the decision maker chooses one of his alternatives. A circle denotes a chance point, where an event outside the decision maker's control occurs (or its pre-existence comes to light). Branches and twigs represent the alternative paths leading to the various outcomes--with the outcomes themselves at the ends of the twigs. Probabilities are noted along the branches and twigs, wherever they apply.
- xiv) The completed decision tree now is used to determine a strategy which will achieve the aspiration level or criterion set earlier. If more than one alternative path results in a satisfactory outcome, the first one encountered may be selected or all

strategies may be evaluated and the one yielding the "best" outcome may be chosen.

5. ACM EXAMPLE (hypothetical)

a) Situation:

- i) A fighter aircraft is on a combat air patrol (CAP) mission, protecting a carrier worth \$2 billion. Replacement cost for the aircraft itself and for similar friendly aircraft is \$25 million. It carries long-range and short-range air-to-air missiles (each costing about \$1 million), and also carries an internal gun (negligible cost per encounter).
- ii) The fighter is equipped with an automatic target recognition system which can tell the pilot whether an observed aircraft is friendly or hostile with 90% probability, when in range. He also knows that 70% of the aircraft in the area are friendly and that 30 % are hostile, from pre-briefed information.
- iii) The pilot has observed an aircraft beyond the range of his target recognition system. He has four alternatives:
  - assume it is a friendly aircraft and continue his patrol pattern,
  - assume it is an enemy and fire a long-range missile at it immediately,
  - approach closer for better identification and use his short-range missiles on it if it is an enemy (with 20% chance he will be downed himself),

- approach close enough for positive identification and attack with his aircraft gun (with 50% chance he will be downed himself).

b) Procedures:

- i) As defined above, the pilot must be the decision maker, choosing one of the four alternatives.
- ii) The time frame for this decision is the next few seconds, during which one of the alternatives (which are considered to be exhaustive and mutually exclusive) must be selected.
- iii) This is a multiple-stage problem. If one alternative is considered "to delay", a second decision point will be reached. At this point, the pilot must decide to use his short-range missile, or to delay further and use his gun.
- iv) The alternatives open to the pilot are listed above.
- v) The events (world states) are:
  - the approaching aircraft is either enemy or friendly,
  - the pilot either downs the approaching aircraft or is himself downed (if it is an enemy). We will make the simplifying assumption that, as a result, his carrier is destroyed.
- vi) This is a decision under risk. The probabilities of the above events are based on

the stage of the decision problem, as noted above; i.e., the probability that the pilot himself is downed is zero if he fires immediately, is 20% if he delays and uses a short-range missile, and is 50% if he delays and uses his gun. The probability that he fires on a friendly aircraft is 70% if he fires now, 10% if he delays, and zero if he waits for positive identification.

vii) There are eight possible outcomes:

- the aircraft is friendly and he chooses to continue on patrol (cost: nothing);
- the aircraft is hostile, he continues on patrol, and his carrier is attacked and destroyed (cost: \$2 billion);
- the aircraft is friendly and he fires his long-range missile at it (cost: \$1 million for the missile + \$25 million for the destroyed friendly aircraft = \$26 million);
- the aircraft is hostile and he destroys it with his long-range missile (cost: \$1 million for the missile);
- the aircraft is friendly, the target detection system says it is hostile, and he destroys it with his short-range missile (cost: \$26 million);
- the aircraft is hostile, and he has an 80% chance of surviving to destroy it with his short-range missile and a 20% chance both he and his ship will be destroyed (expected cost, based on probabilities:  $[(0.8)(\$1 \text{ million})]$  that he kills the

enemy] + [ (0.2) (\$2025 million) that he and ship are killed] = \$406 million);

- the aircraft is friendly, and he kills it with his gun (cost: \$25 million, except this event will occur with zero probability);
- the aircraft is hostile, and he has a 50% chance of surviving to kill it (at negligible cost) and a 50% chance it destroys both him and his ship (expected cost, based on probabilities: [ (0.5) (\$2025 million) ] = \$1012 million).

viii) For purposes of this study, money will be considered to be the payoff, with minimum cost to be considered the utility value and MOE.

ix) Aspiration level for this problem will be an expected loss (based on probabilities of occurrence) no greater than \$500 million--a highly artificial situation, on the surface. However, it is convenient for demonstration of this technique.

x) Both a payoff matrix (Figure 8.1) and a decision tree (figure 8.2) are useful to structure the situation for analysis (although each shows essentially all of the information--in different forms). From the matrix we can determine that none of the alternatives exhibits dominance over any other. Thus they all will be retained for consideration.

|  |  | <u>Probability</u> | <u>Enemy</u> | <u>Friendly</u> |
|--|--|--------------------|--------------|-----------------|
|  |  | Stage 1            | 0.3          | 0.7             |
|  |  | Stage 2            | 0.9          | 0.1             |
|  |  | Stage 3            | 1.0          | 0.0             |

| <u>Probability</u><br><u>enemy</u><br><u>downs him</u> | <u>Alternative</u><br><u>actions</u> | <u>State-of-the-world</u><br><u>Cost, \$M</u> |                 | <u>EMV</u> |
|--------------------------------------------------------|--------------------------------------|-----------------------------------------------|-----------------|------------|
|                                                        |                                      | <u>Enemy</u>                                  | <u>Friendly</u> |            |
| 0.0                                                    | Do nothing                           | 2000                                          | 0               | -600       |
| 0.0                                                    | Shoot now                            | 1                                             | 26              | -18.5      |
| 0.2                                                    | Short-range missile                  | 406                                           | 26              | -368       |
| 0.5                                                    | Aircraft gun                         | 1012                                          | 25              | -1012      |

Figure 8.1 Payoff Matrix for the Example Problem

xi) Although the problem could be evaluated in several ways, it will be useful to use the concept of expected monetary value (EMV) here. Remembering that cost is a negative value, we make use of the expected costs calculated above for each of the eight outcomes. The cost of that outcome is multiplied by the probability of that outcome occurring, to obtain an expected outcome value. Then these expected values are summed for a given alternative.

- do nothing:  $[(0.3 \text{ probability it is hostile})(2000 \text{ cost})] + [(0.7 \text{ probability it is friendly})(0 \text{ cost})] = -600 \text{ EMV};$
- shoot now:  $[(0.3 \text{ probability it is hostile})(1 \text{ cost})] + [(0.7 \text{ probability it is friendly})(26 \text{ cost})] = -18.5 \text{ EMV};$

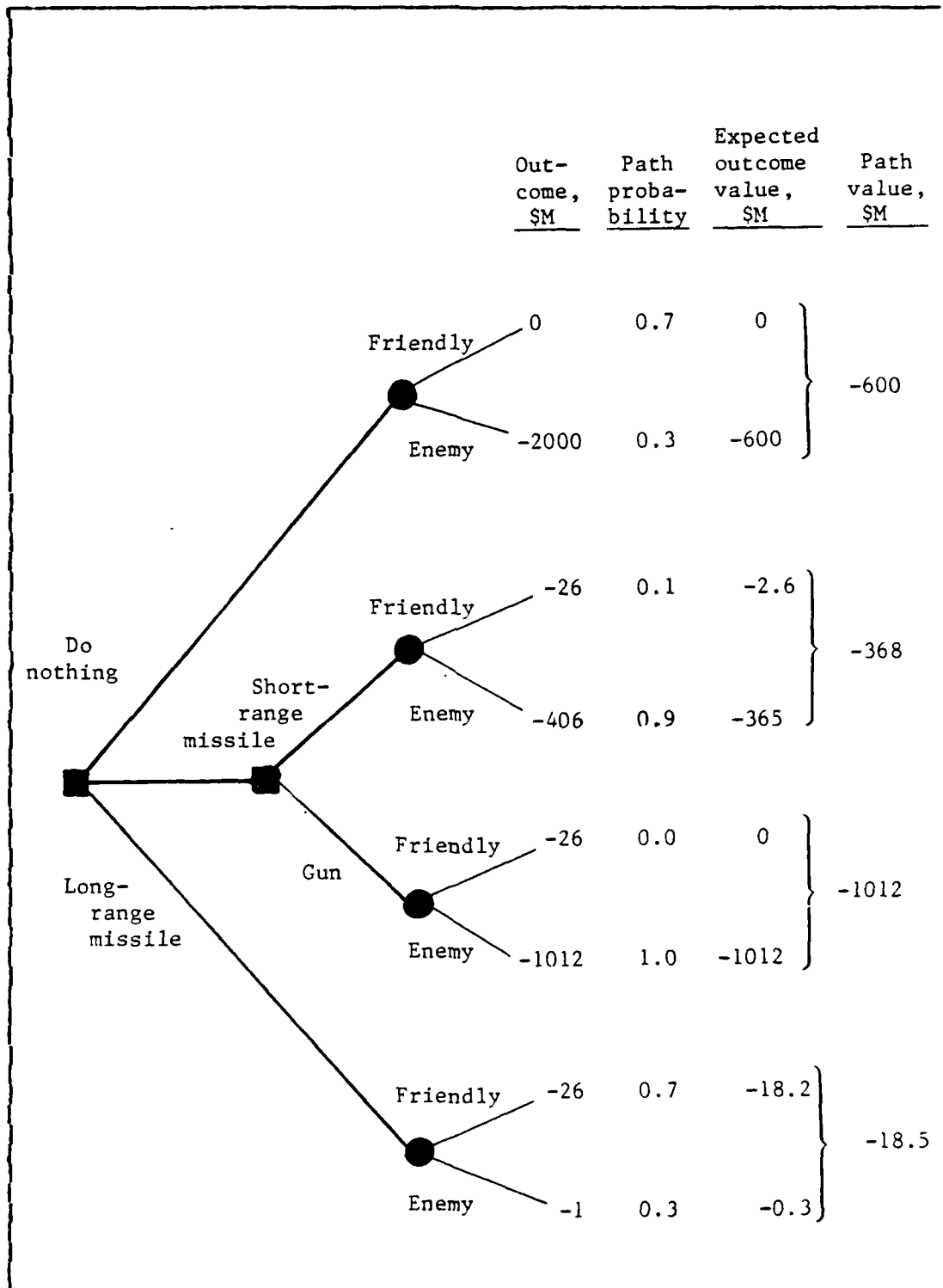


Figure 8.2 Decision Tree for the Example Problem

- use short-range missile:  $[(0.9 \text{ probability it is hostile})(406 \text{ cost})] + [(0.1 \text{ probability it is friendly})(26 \text{ cost})] = -368 \text{ EMV};$
- use gun:  $[(1.0 \text{ probability it is hostile})(1012 \text{ cost})] + [(0.0 \text{ probability it is friendly})(25 \text{ cost})] = -1012 \text{ EMV}.$

xii) Making use of our aspiration level, we now can see that two alternatives are satisfactory: shoot now (at an expected cost of \$18.5 million), or use the short-range missile (expected cost of \$368 million).

- c) Caveat: This example points up the importance of choosing a good measure of effectiveness for determining the usefulness of a decision. Obviously shooting down a friendly aircraft is considerably less desirable than cost/value alone would indicate. Some multi-attribute utility probably should be developed that would include such concepts as morale and the loss of life.

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- b) Puscheck, H.C. "Sequential Decision Making in a Conflict Environment", Human Factors, Vol. 14, No. 6, 1972, pp. 561-571. A two-sided wargame simulation was developed, to study game-playing strategy. Four decision-making models also were developed, to play one side of the game.
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Concepts from stochastic estimation theory are used to develop a theory of human decision making that employs optimal stochastic estimators with short-term and long-term memory models and estimates tradeoffs.

- d) Slovic, Paul, Fischhoff, Baruch, and Lichtenstein, Sarah. Behavioral Decision Theory. Decisions and Designs, Inc., McLean, VA, September 1976 (AD-A036 744).

A survey of the field to determine what is known, what it is good for, and what else must be learned, with emphasis on research on how people do make decisions versus how they should make decisions.

## 7. REFERENCES AND TEXTS:

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A clear and complete short text on how to use decision analysis techniques. Good section on multi-attribute utility problems, though more mathematical than Raiffa's explanations.

- b) Raiffa, Howard. Decision Analysis: Introductory Lectures on Choices Under Uncertainty. Reading, Mass.: Addison-Wesley Publishing Company, 1968.

A delightful short text, very readable. Excellent continuing example used to demonstrate the procedures. Includes choices under risk, as well as those under uncertainty.

- c) Sheridan, T.B., and Farrell, W.R. Man-Machine Systems: Information, Control, and Decision Models of Human Performance. Cambridge, Mass.: The MIT Press, 1974, 1981.

Discussion of utilities, of dynamic decision making, and of games, in a theoretical framework.

- d) Williams, J.D. The Compleat Strategyst. New York: McGraw-Hill Book Company, 1954.

This is the classic primer on game theory and game strategy. Intended for the non-mathematician, it is pleasant reading, while being a comprehensive look at the uses of payoff matrices.

## B. SIGNAL DETECTION THEORY MODELS

1. PURPOSE OF MODEL/TECHNIQUE: Describing the probability or percent of time that two classes of events will be discriminated by an observer. A relative operating characteristic (ROC) curve is used to show

a cross plot of hit rate versus false-alarm rate, for a given situation and observer population.

For example, a series of observers may be shown fuzzy CRT images of ships, in a laboratory, and told to identify each as friendly or hostile. Their responses are classed as: correct identification as hostile (hit), correct identification as not hostile (correct rejection), incorrect response as non-hostile (miss), and incorrect response as hostile (false alarm). The proportions of the various responses under laboratory conditions are used to prepare a ROC curve. This then will be used to predict how good ship identification performance will be under real-world conditions with similarly fuzzy images from TV system on a tactical fighter aircraft.

2. MATHEMATICAL TOOLS REQUIRED OR USEFUL:

- a) Algebra
- b) Probability theory
- c) Descriptive statistics
- d) Graphs and plots

3. HUMAN FACTORS APPLICATIONS:

- a) Describing individual differences, such as the relative proportion of hits, misses, etc., observed in given population groups, or individual differences in sensitivity versus decision criteria.
- b) Describing systems, such as the relative proportion of hits, misses, etc., observed in operators using two different systems.
- c) Designing systems, when a choice must be made between systems, based on observers' relative accuracy of discrimination with them.

- d) Evaluating human performance, where it is necessary for observers to meet some criterion or aspiration level which can be described and defined easily in signal detection theory terms.

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The ability of mine workers to discriminate visually between dangerous and safe rock conditions was examined. Responses to stimuli were analysed in terms of signal detection theory, and results indicate that experience and skills training improve performance.
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An accident causation model, based on the theory of signal detection, is developed. Differences in drivers' visual perception of curves is used, along with driving experience.
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Signal detection theory was used to study changes in pilot decision making behavior as a function of training time. Pilot performance was separated into distinct and theoretically orthogonal measures of sensitivity/accuracy ( $d'$ ) and response criterion ( $\beta$ ).
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The assumptions, procedures, limitations, and practical considerations relevant to signal detection theory are summarized, and application to cognitive processes is described.
- e) Young, J.M. The Effect of Signal Incidence Upon Detectability. Tracor, Inc., Austin, TX, April 1968 (TRACOR-68-591-U, AD-A071 770).  
An experimental determination of probability of detection, as a function of signal incidence, showed that the probability of detection decreases linearly but remains finite, as signal incidence is reduced and approaches zero.

5. REFERENCES AND TEXTS:

- a) Sheridan, T.B., and Farrell, W.R. Man-Machine Systems: Information, Control, and Decision Models of Human Performance. Cambridge, Mass.: The MIT Press, 1974, 1987.

A complete theoretical treatment of the subject, with a little information on applications.

- b) Welford, A.T. Skilled Performance: Perceptual and Motor Skills. Glenview, Ill.: Scott, Foresman and Company, 1976.

Clearly written, brief discussion of the theory and its application to decision making and performance.

## IX. SUMMARY

This thesis is intended as a primer for human factors engineers who wish to understand and make use of applicable models and techniques used in operations research. Nineteen of these techniques are listed here. Seven are discussed in detail, including illustrative examples related to human factors and to military systems. The other 12 are described briefly. Possible uses are noted, and sources of further information provided.

An extensive literature search was conducted as part of this study. It is interesting that numerous reports and other publications had keywords indicating that operations research and human factors were being combined. In actuality, however, these reports usually involved one or the other; rarely were both tied together. The logical pairing of these fields was pointed out in 1970 by DeGreene [Ref. 47], yet little progress has been made in the intervening years. And what has been done mostly is written by operations research analysts, and is unreadable by most (mathematically unsophisticated) human factors engineers.

The most valuable thing obtained from this study is strong evidence that many operations research techniques indeed can be useful in modeling human performance. Markov chains, queueing processes, and simulations all provide useful insights, along with linear programming, networks, and distribution models. The human factors engineer is strongly encouraged to consider whether one or more of these might be useful to him, as he goes about his job of describing people and systems, designing new systems, and evaluating performance.

Perhaps the most useful (and overlooked) techniques, overall, derive from decision theory. These models are relatively easy to use. Algebra and some understanding of logic and sets is useful, but otherwise little mathematical sophistication is required. Yet the straightforward development of payoff matrices and decision trees is a marvelous way to clarify a set of alternatives, and enable selection of one that will be satisfactory. It is highly recommended.

## APPENDIX A

### GLOSSARY

Air Combat Maneuvering (ACM): air battles between two or more fighter aircraft.

Algorithm: a set of logical and mathematical operations performed in an orderly, specific sequence, usually using a computer.

Analogy: viewing a new problem as if it were an old problem for which one has insight or a solution, in order to use available tools.

Analysis: the separation of a whole into its component parts, usually in order to understand its nature and to determine its essential features.

Analyst: a person who uses the techniques and tools of analysis.

Arithmetic operators: symbols which indicate a process that is to be carried out on two or more numbers or other characters, or a relationship between them, such as +, -,  $\leq$ , or an integration sign.

Continuous: variables which can take on a continuum of possible values.

Descriptive model: a model giving a description of a system or process, usually in mathematical or other symbolic terms.

Deterministic: a model or process that give an "exact" answer, that is, one that yields a number or numbers as its end product.

Discrete: variables which can take on only a finite or countable number of values.

Event: any subset of the set of all possible outcomes or occurrences in an experiment, study, or any other process being followed.

Exhaustive: enumeration of all possible states or events in a situation of interest.

Human Factors (HF): the study of human capabilities and limitations in performing work activities, plus application of this knowledge to design of equipment, facilities, and environments, and to the enhancement of capabilities through training.

Human Factors Engineering (HFE): a subcategory of human factors which emphasizes design of equipment, facilities, and environments to match the capabilities and limitations of people.

Iteration: the sequence of operations (usually in an algorithm) leading to a new and (hopefully) better solution.

**Linear:** a "straight-line" relationship among variables, so that contributions of the variables are proportional to their values, constant over the possible range of values, and additive.

**Man-machine system:** an entity consisting of human and non-human components which exists to carry out some purpose which transcends the individual purposes of these components.

**Mathematical tool:** a mathematical procedure which does not, in and of itself, answer a systems or organization question, but which is needed in order to use an operations research technique.

**Measure Of Effectiveness (MOE):** criteria of overall system performance used to evaluate proposals and designs; usually measurable, numerical values when possible.

**Mechanistic:** another term for deterministic.

**Model:** more or less abstract representation (physical, mathematical, and/or verbal) of a system or subsystem, used to define that system sufficiently well to answer questions about it using various techniques.

**Mutually exclusive:** enumeration of a set of states or events which do not overlap (are orthogonal).

**Operations analysis:** analysis of the operation of an existing system; often used interchangeably with systems analysis.

**Operations Research (OR):** Application of the techniques of the behavioral sciences and mathematics to models, in order to make tradeoffs in, solve problems of, or make decisions about complex problems (usually concerning organizations or systems).

**Operations research technique:** a procedure that clarifies a specific question about a system, condition, or event, or that gives a quantitative answer to such a question, through operations on a model.

**Optimum:** the most favorable value obtainable, or the best possible solution to a problem, within given constraints; one that maximizes some measure of benefit or minimizes some measure of cost.

**Paradigm:** a model or analogy which is widely accepted and recognized within a given field.

**Prescriptive model:** a model that prescribes a course of action needed to obtain a desired outcome.

**Probabilistic:** a model or process that yields probabilities of occurrence as its end product.

**Programmable method:** an orderly, step-wise approach to solution of a problem, laid out in a logical sequence (not directly related to computer programming).

**Quantitative:** the degree or level of some quality or attribute, including numerical values, probabilities, and ordinal comparisons.

**Sensitivity analysis:** evaluation of how a given optimum solution would change if input data values were changed;

used to determine the range within which input values can vary and still yield a satisfactory solution.

State: the condition or status of an object of interest, as a result of its initial conditions and events which have occurred subsequently.

Stochastic: a time-related probabilistic model or process.

Stopping rule: carefully specified conditions, used during the iteration process of obtaining better solutions, in order to recognize when the present solution is "good enough", and iterations should stop.

System: an assemblage of constituents that interact to fulfill a common purpose, transcending the individual purposes of the components.

Systems analysis: the scientific discipline of analysing systems by examining their component parts and the relationships among them, in order to solve system problems.

Systems engineering: application of scientific and engineering knowledge to planning, design, evaluation, and construction of systems.

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