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MATHEMATICAL PROBLEMS IN STABILITY CONTROL AND  
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AMHERST DEPT OF MATHEMATICS AND STATISTICS..

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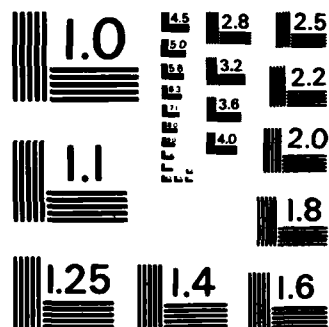
W A ROSENKRANTZ 11 JUN 85 AFOSR-TR-85-0715 F/G 17/2

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| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)<br>Network modelling and analysis, exponential back-off, slotted ALOHA, exponen-<br>tial martingales and point processes, stability.                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                    |                                             |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>Current research is focused on three problems: (1) stability analysis of the<br>exponential back-off protocol, (2) a diffusion approximation for the steady<br>state distribution of the slotted ALOHA protocol, and (3) a Poisson limit<br>theorem for a load balancing protocol. Work began on applying Stochastic<br>Catastrophe Theory to the study of the exponential back-off protocol when<br>there are a finite number of users. Work is also planned on a theoretical<br>analysis of the diffusion approximation to the slotted ALOHA protocol. |                                                                                                                                    |                                             |

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Title: Mathematical Problems in Stability, Control and Reliability of Random Access Communication Systems

Principal Investigator: Professor Walter A. Rosenkrantz

### I. Publications

1. Weak convergence of a sequence of Queueing and Storage Processes to a Singular Diffusion, in Proceedings of Seminaire International sur la modelisation et les methodes d'evaluation de Performance, Lecture Notes in Control and Information Sciences, v. 60, pp. 257-272. Published by Springer-Verlag.
2. Some Theorems on the Instability of the Exponential Back-off Protocol, Performance '84, Proceedings of the 10th International Symposium on Computer Performance, pp. 199-205, Published by North-Holland, Amsterdam. New York. Oxford.
3. An Operator Method for Computing the Asymptotics of a Collision Resolution Interval, AFOSR82-0167, Technical Report No. 9.
4. Construction of Exponential Martingales for Counting Processes, AFOSR82-0167, Technical Report No. 10.

### II. Coupling Activities i.e. Conferences, Lectures, etc.

1. I presented a paper entitled "Some Theorems on the Instability of the Exponential Back-off Protocol" to the Performance '84 Symposium held in Paris, France, December 19 - 21, 1984. At the symposium I met and consulted with Dr. R. Nelson (IBM), Dr. Guy Fayolle (INRIA, France), Dr. D. Mitra (Bell Labs), and L. Flatto (Bell Labs).
2. With the assistance of some colleagues in Computer Science (J. Kurose) and Electrical Engineering (D. Towsley and E. Geraniotis) a "network modelling and analysis" seminar was organized here at the University of Massachusetts in February 1985. In addition to presenting my own paper "An Operator Method for Computing the Asymptotics of a Collision Resolution Interval" I invited several consultants to lecture at our seminar including: S. Lavenberg (IBM), R. Nelson (IBM), L. Flatto (Bell Labs), and D. Mitra (Bell Labs).

### III. Professional Personnel Associated with Research Effort

1. Mr. W. Rising is working on (i) a stability analysis of the exponential back-off protocol, and (ii) an approximation for the steady state distribution of the slotted Aloha protocol (with a finite number of users) due to R. Nelson. The approximation seems to work much better than it should and the question is why? Can error bounds be obtained? Satisfactory answers to either of these problems will result in a nice thesis and a Ph.D. for Mr. Rising.

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2. Mr. Steve Krone was supported by the grant for the 1984-1985 academic year. Since he is a first-year graduate student, most of his time was spent taking courses in order to improve his technical competence, particularly in probability and statistics. He plans to take two qualifying exams in August 1985.

#### IV. Status of Current Research

Current research is focused in three problems: (i) stability analysis of the exponential back-off protocol; (ii) a diffusion approximation for the steady state distribution of the slotted Aloha protocol due to R. Nelson (see "The Stochastic Cusp,...Catastrophes as Manifest in a Simple Communications Model", Performance '84, pp. 207-224), and (iii) a Poisson limit theorem for a load balancing protocol (D. Towsley, private communication).

- (i) We have shown elsewhere that the exponential back-off protocol is unstable for an arrival rate  $\lambda \geq \lambda_c = .72$ . Professor F. P. Kelley (Statistical Laboratory, Cambridge University, Cambridge, England) has improved my estimate by showing that  $\lambda_c = .56$  (private communication) and Dr. Fayolle of INRIA, France has obtained an even better lower bound but I have not yet seen the details. I will, however, be working with him in France beginning in September 1985 at which time I expect to learn more. In the meantime Dr. R. Nelson has suggested to my student, W. Rising, that he might profitably apply Stochastic Catastrophe Theory to the study of the exponential back-off protocol when there are a finite number of users. Numerical simulations hint at bistability but much more remains to be done.
- (ii) In a purely formal way Dr. Nelson has devised a diffusion approximation for the stationary distribution of the slotted aloha protocol which seems to work very well. We propose to give a theoretical analysis of this approximation with a view to understanding how and when it works.
- (iii) Limit theorems for the univariate point processes have been obtained by T. Brown in Math. Proc. Camb. Phil. Soc. (1981) and by Kabanov, Liptser, Shiryaev in Stochastics (1980) using martingale methods. We propose to extend these results to a multivariate countering process which arises naturally in a load balancing protocols. This extension leads to problems of independent mathematical interest e.g. construction of exponential martingales for counting processes, etc.

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