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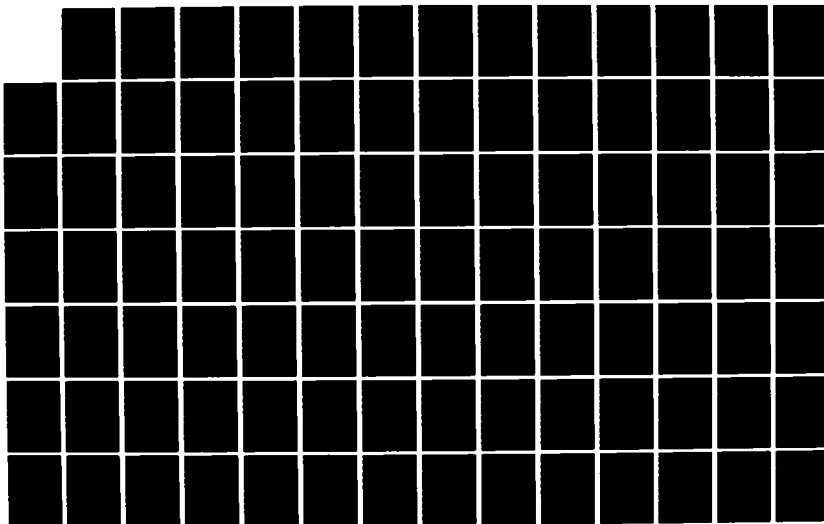
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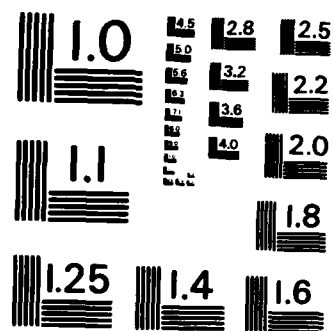
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EFFICIENCY/EQUITY ANALYSIS OF WATER RESOURCES
PROBLEMS--A GAME THEORETIC APPROACH

By

Elliot Kin Ng, Major, USAF

Ph.D., University of Florida, 1985, 160 pages

ABSTRACT

Successful regional water resources planning involves an efficiency analysis to find the optimal system that maximizes benefits minus costs, and an equity analysis to apportion project costs. Traditionally, these two problems have been treated separately.

A reliable total enumeration procedure is used to find the optimal system for regional water network problems. This procedure is easy to understand and can be implemented using readily available computer software. Furthermore, the engineer can use realistic cost functions or perform detailed cost analysis and, also, examine good suboptimal systems. In addition, this procedure finds the optimal system for each individual and each subgroup of individuals; hence, an equity analysis can be accomplished using the theory of the core from cooperative n-person game theory on the optimal system as well as good suboptimal systems.

A rigorous procedure using core conditions and linear programming is described to unambiguously measure an individual's minimum cost and maximum cost as a basis for

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EFFICIENCY/EQUITY ANALYSIS OF WATER RESOURCES
PROBLEMS--A GAME THEORETIC APPROACH

By

ELLIOT KIN NG

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
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To my parents
and
my wife, Eileen,
and children, Matthew, Michelle, Michael

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Abstract of Dissertation Presented to the Graduate School
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EFFICIENCY/EQUITY ANALYSIS OF WATER RESOURCES
PROBLEMS--A GAME THEORETIC APPROACH

By

Elliot Kin Ng

August, 1985

Chairman: James P. Heaney

Major Department: Environmental Engineering Sciences

Successful regional water resources planning involves an efficiency analysis to find the optimal system that maximizes benefits minus costs, and an equity analysis to apportion project costs. Traditionally, these two problems have been treated separately. This dissertation incorporates efficiency analysis and equity analysis into a single regional water resources planning model.

A reliable total enumeration procedure is used to find the optimal system for regional water network problems. This procedure is easy to understand and can be implemented using readily available computer software. Furthermore, the engineer can use realistic cost functions or perform detailed cost analysis and, also, examine good suboptimal systems. In addition, this procedure finds the optimal system for each individual and each subgroup of individuals; hence, an equity

analysis can be accomplished using the theory of the core from cooperative n-person game theory.

Game theory concepts are used to perform an equity analysis on the optimal system as well as good suboptimal systems. For any system, an equitable cost allocation exists if a core exists. However, if a game is not properly defined, even a cost allocation in the core may be inequitable.

A rigorous procedure using core conditions and linear programming is described to determine the core bounds. An individual's lower core bound and upper core bound unambiguously measure the individual's minimum cost and maximum cost, respectively. Traditional approaches for quantifying minimum cost and maximum cost assume that either a regional system involving the grand coalition is built or all the individuals will go-it-alone. However, this rigorous procedure accounts for the possibility that a relatively attractive system involving subgroups may form. Furthermore, this rigorous procedure gives a general quantitative definition of marginal cost and opportunity cost. Once the minimum cost and maximum cost for each individual are determined, a basis for equitable cost allocation is available.

Finally, efficiency analysis and equity analysis are not separable problems but are related by the economics of all the opportunities available to all individuals in a project.

CHAPTER 1 INTRODUCTION

In situations where multiple purposes and groups can take advantage of economies of scale in production and/or distribution costs, a regional water resources system is an attractive alternative to separate systems for each purpose and each group. However, a regional system imposes complex economic, financial, legal, socio-political, and organizational problems for the water resources professionals. This dissertation examines two problems associated with regional water resources planning that are typically treated separately, yet are closely related.

The first problem involves performing an efficiency analysis to determine the economically efficient or optimal regional system that maximizes benefits minus costs. Once the optimal regional system is determined, a major task still remains to allocate project costs; therefore, an equity analysis must be performed to apportion project costs in an equitable manner. This second problem is viewed from the perspective of each purpose and each group because they must each be convinced that the optimal regional system is their best alternative; otherwise, voluntary participation will be difficult. No doubt, each purpose's and each

group's decision to participate in the optimal regional system depends on its allocated cost, and not necessarily on what is best for the region.

The prevailing belief is that efficiency analysis and equity analysis are separate problems and, therefore, research has either focused entirely on efficiency analysis or equity analysis. Research on efficiency analysis has mainly been on the application of partial enumeration techniques to find optimal regional systems, while research on equity analysis has continued to explore the application of concepts from cooperative game theory to allocate project costs. The purpose of this dissertation is to integrate efficiency analysis and equity analysis into a single regional water resources planning model characterized by economies of scale. The model to be presented incorporates a total enumeration procedure along with concepts from cooperative game theory for efficiency/equity analysis. The specific application is to determine the least cost regional water supply network and to determine a "fair" allocation of costs among the multiple users.

Chapter 2 reviews selected works on efficiency analysis and equity analysis of water resources problems. Chapter 3 presents a reliable total enumeration procedure for efficiency analysis of regional water supply network problems. However, unlike traditional partial enumeration techniques used for efficiency analysis that give only the optimal

solution, this procedure also gives all the optimal solutions for each user and each subgroup of users which are necessary information to perform an equity analysis using concepts from cooperative game theory. In addition, this procedure gives all the suboptimal solutions. Chapter 4 shows how the information from the total enumeration procedure is used to perform an equity analysis of not only the optimal solution, but also "good" suboptimal solutions. Chapter 5 reveals how efficiency analysis and equity analysis are related. Finally, Chapter 6 summarizes the results and conclusions.

CHAPTER 2 LITERATURE REVIEW

Efficiency Analysis

During the past two decades, the problem of finding the economically efficient or optimal regional water system has been extensively modeled as a mathematical optimization problem. A review of selected works on efficiency analysis of regional water systems that includes Converse (1972), Graves et al. (1972), McConagha and Converse (1973), Yao (1973), Joeres et al. (1974), Bishop et al. (1975), Jarvis et al. (1978), Whitlatch and ReVelle (1976), Brill and Nakamura (1978), and Phillips et al. (1982) indicates a variety of partial enumeration techniques, e.g., nonlinear programming, for finding optimal regional systems. These optimal regional systems can be a least cost system or a system that maximizes benefits minus costs. Generally, regional water resources planning problems exhibit economies of scale in cost and, therefore, involve nonlinear concave cost functions. Consequently, to a great extent, the selection of the partial enumeration optimization technique to apply to a particular problem depends on the characterization of the nonlinear concave cost functions. For instance, linear programming can be applied if the nonlinear

concave cost functions are represented by linear approximations.

Equity Analysis

Unfortunately, successful regional planning is not merely knowing the optimal regional system but must also include an equity analysis to find an acceptable allocation of costs among the participants. Otherwise, the optimal system will be difficult to implement. Of the publications cited in the preceding paragraph, only McConagha and Converse (1973) dealt with both efficiency and equity in regional water planning. In addition to presenting a heuristic procedure for finding the least cost regional wastewater treatment facility for seven cities, they evaluated the equity of several cost allocation procedures. Although they recognized that an equitable cost allocation should not charge any city or subgroup of cities more than the cost of an individual treatment facility, they did not include the possibility of subgroup formation in their analysis.

Giglio and Wrightington (1972) introduced concepts from cooperative game theory as a way to consider the possibility of subgroup formation in allocating costs of water projects. However, their treatment of cooperative game theory was incomplete. Therefore, they concluded that the game theory approach rarely yields a unique cost allocation

and proceeded to recommend the separable costs, remaining benefits (SCRB) method or methods based on measure of pollution. Shortly thereafter, several researchers applied popular unique solution concepts from game theory like the Shapley value and the nucleolus to allocate the costs of regional water systems. Heaney et al. (1975) applied the Shapley value to find an equitable cost allocation of common storage units for storm drainage for pollution control among competing users. Suzuki and Nakayama (1976) applied the nucleolus to assign costs for a water resources development along Japan's Sakawa and Sagami Rivers. Loehman et al. (1979) used a generalization of the Shapley value to allocate the costs of a regional wastewater system involving eight dischargers along the lower Meramec River near St. Louis, Missouri.

Subsequently, Heaney (1979) established that the fairness criteria used for allocating costs in the water resources field and the concepts used in cooperative game theory are equivalent. Moreover, Straffin and Heaney (1981) showed that a conventional method for allocating costs used by water resources engineers is identical to a unique solution concept used by game theorists. More recently, Young et al. (1982) compared proportionality methods, game theoretic methods, and the SCRB method for allocating cost and concluded that the game theoretic methods may be too complicated while the SCRB method may give inequitable cost

allocations. Meanwhile, Heaney and Dickinson (1982) revealed why the SCRB method may fail to give equitable cost allocations and proposed a modification of the SCRB method that uses game theory concepts along with linear programming to insure an equitable cost allocation can be found if one exists.

The possibilities of using concepts from cooperative game theory as a basis for allocating costs of water projects continue to develop. In fact, concepts from cooperative game theory are gaining acceptance in other fields as well. Researchers in accounting are looking toward cooperative game theory as a possible solution to the arguments by Thomas (1969, 1974) that any cost allocation scheme in accounting is arbitrary and hence not fully defensible. Recent works by Jensen (1977), Hamlen et al. (1977, 1980), Callen (1978), and Balachandran and Ramakrishnan (1981) applied concepts from cooperative game theory to evaluate the equity of existing and proposed cost allocation schemes in accounting. Meanwhile, in economics, concepts from cooperative game theory are frequently used as a basis for evaluating subsidy-free and sustainable pricing policies for decreasing cost industries, e.g., the work of Loehman and Whinston (1971, 1974), Faulhaber (1975), Sorenson et al. (1976, 1978), Zajac (1978), Panzar and Willig (1977), Faulhaber and Levinson (1981), and Sharkey (1982b).

Conclusions

Three conclusions can be made from reviewing the literature on efficiency analysis and equity analysis of regional water resources planning. First, there is a gap in the research to jointly examine efficiency and equity in regional water resources planning. In spite of a continual effort to find economically efficient regional water systems and equitable cost allocation procedures, no published work incorporates both efficiency analysis and equity analysis in a single regional water resources planning model using realistic cost functions. Heaney et al. (1975) and Suzuki and Nakayama (1976) used linear cost models while Loehman et al. (1979) used conventional cost curves. Secondly, the cost allocation literature in the water resources field has consistently allocated the costs of treatment and piping together even though federal guidelines suggest that piping cost be allocated separately from treatment cost to the responsible users (Loehman et al., 1979; U.S. Environmental Protection Agency, 1976). Finally, the cost allocation literature has dealt with allocating the cost of the optimal system. However, situations in practice may require that "good" suboptimal systems be considered; therefore, an acceptable cost allocation procedure should be able to allocate the costs of several systems under consideration in an equitable manner. These three conclusions formed the basis for the research undertaken in this dissertation.

Chapter 3 begins integrating efficiency analysis and equity analysis by searching for a computational procedure to simultaneously perform an efficiency analysis and calculate all the necessary information to perform an equity analysis using concepts from cooperative game theory.

CHAPTER 3 EFFICIENCY ANALYSIS

Introduction

The importance of both efficiency analysis and equity analysis in planning regional water resources systems is well recognized. Over the years, researchers have applied methods ranging from simple cost-benefit analysis to sophisticated mathematical programming techniques to search for economically efficient or optimal regional water resources systems. Yet, the implementation of regional systems is difficult unless an equitable financial arrangement is found to allocate project costs among individuals (or participants) in a project. Until recently, a theoretically sound basis for allocating costs has eluded the water resources professional. However, there is increasing interest in using the theory of the core from cooperative n-person game theory as a basis for allocating costs, e.g., see Suzuki and Nakayama (1976), Bogardi and Szidarovsky (1976), Loehman et al. (1979), Heaney and Dickinson (1982), and Young et al. (1982). The theory of the core is based on principles of individual, subgroup, and group rationality. This means that no individual or subgroup of individuals should be allocated a cost in excess of the cost of nonparticipation,

while total cost must be apportioned among all individuals. The cost of nonparticipation is simply the cost that each individual and each subgroup of individuals must pay to independently acquire the same level of service by the most economically efficient means. As a result, to evaluate efficiency/equity for a regional system with n individuals, it is necessary to determine $2^n - 1$ optimal solutions.

Although the close association between efficiency analysis and equity analysis is recognized, there have been few attempts to incorporate these two analyses in regional water resources planning. A typical efficiency analysis usually ends with determining the optimal solution for a problem without addressing cost allocation, and a typical equity analysis begins by assuming the $2^n - 1$ optimal solutions are available to accomplish the cost allocation. This disjointed approach to efficiency/equity analysis is fostered by a belief that these two problems are independent (James and Lee, 1971; Loughlin, 1977). Furthermore, reliable techniques for finding the $2^n - 1$ optimal solutions to accomplish an efficiency/equity analysis of most problems encountered in actual practice are unavailable.

This chapter begins by evaluating the applicability of partial and total enumeration techniques for finding the $2^n - 1$ optimal solutions for problems with different types of cost functions. Subsequently, a computational procedure is described to examine a regional water supply network problem

wherein we need to find the economic optimum and a "fair" allocation of costs among the individuals in the project. In order to do the cost allocation we need to find the costs of the optimal systems for each individual and each subgroup of individuals since these costs are going to be the basis for cost allocation.

Partial Enumeration Techniques

The difficulty of finding the optimal solution for a particular problem depends on the nature of the cost functions. Generally, a cost function can be classified as either linear, convex, concave, S-shape, or irregular (see Figure 3-1). To find the optimal solution for problems with either linear or convex cost functions is straightforward using readily available and reliable linear programming codes. Accordingly, a vast body of overlapping theoretical results is available from classical economics and operations research, e.g., convex programming, for finding the optimal solution to problems with convex cost functions. However, problems with linear and convex cost functions are unable to characterize the economies of scale in cost typically encountered in regional water resources planning.

The concave cost function is generally used to represent economies of scale, and several partial enumeration techniques are available for dealing with this cost

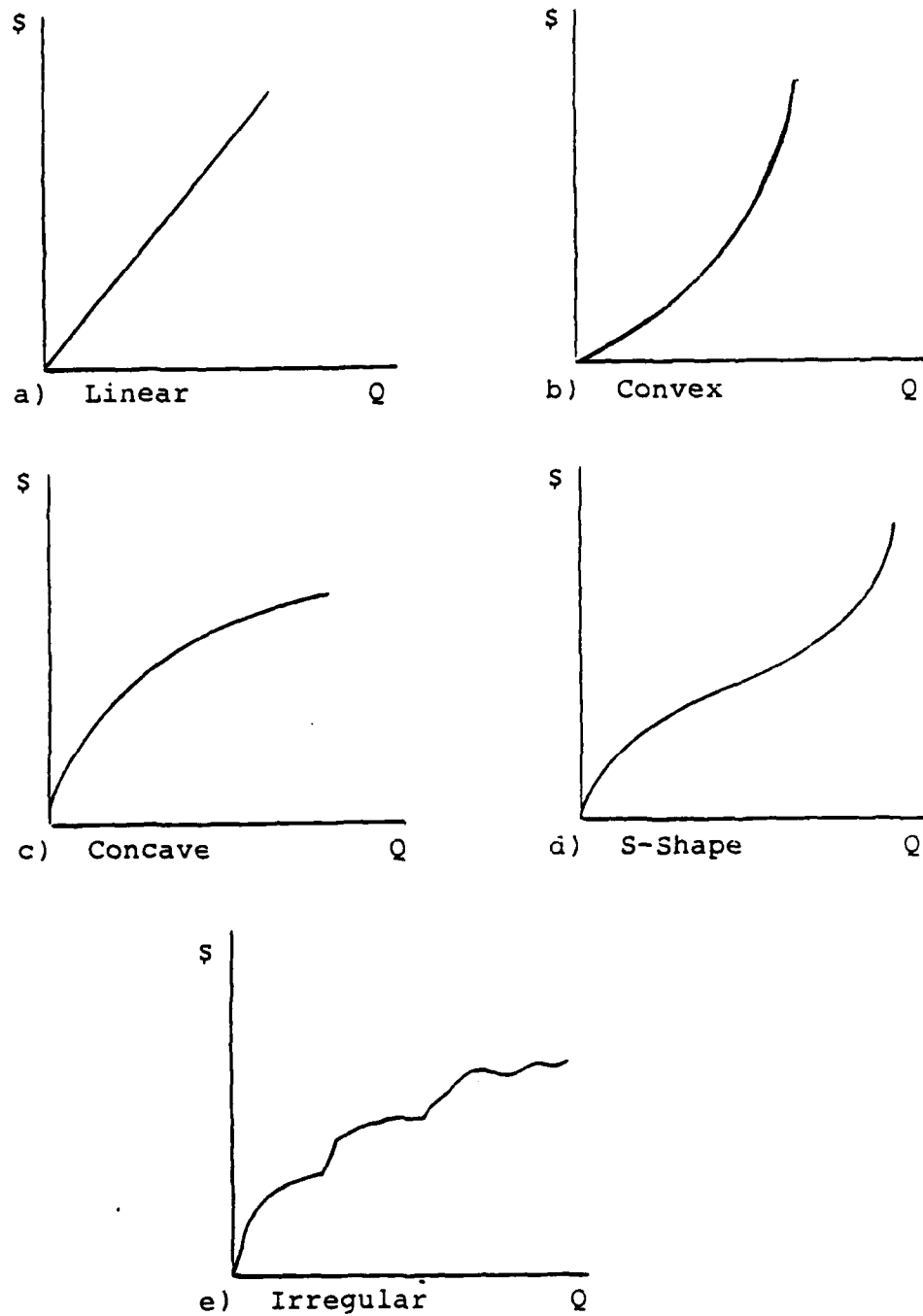


Figure 3-1. Types of Cost Functions.

function. One approach surveyed by Mandl (1981) is separable programming which takes advantage of readily available linear programming codes by using a piecewise linear approximation of the concave cost function. Unfortunately, this approach is rather tedious to use and guarantees only a local optimal solution. A second approach is to retain the natural concave cost function and apply a general nonlinear programming code. However, according to surveys by Waren and Lasdon (1979) and Hock and Schittkowski (1983), general nonlinear programming codes may converge to local optima and may be subject to other failures, e.g., termination of code. A final approach used by Joeres et al. (1974) and Jarvis et al. (1978) is to approximate the concave cost function with several fixed-charge cost functions and apply a mixed-integer programming code. This approach guarantees a globally optimal solution, but standard mixed-integer programming codes are expensive to use. More importantly, unresolved problems remain as to how to properly define a fixed charge problem. If the fixed charge formulation is used because it is computationally expedient, then the resulting cost estimates may distort the cost allocation procedure. Given the current status of partial enumeration techniques for finding the optimal solutions to perform efficiency/equity analysis for problems with concave cost functions, one can conclude that other methods must be used. Obviously, this conclusion applies

to problems with S-shape and irregular cost functions as well.

Total Enumeration Techniques

Total enumeration techniques can be used to find the optimal solution for a problem regardless of the types of cost functions involved. The ability to handle irregular cost functions is especially important because this type of cost function is frequently used by state-of-the-art cost estimating models like CAPDET, i.e., Computer Assisted Procedure for Design and Evaluation of Wastewater Treatment Systems (U. S. Army Corps of Engineers, 1978) and MAPS, i.e., Methodology for Areawide Planning Studies (U. S. Army Corps of Engineers, 1980). For example, in MAPS, the cost function for constructing a force main is composed of separate cost functions for pipes, excavation, appurtenances, and terrain. Furthermore, each of these cost functions is based on site-specific conditions. For instance, the cost function for pipe includes the cost of purchasing, hauling, and laying the pipe and depends on the material, diameter, length, and maximum pressure. No doubt, the composite site-specific cost function for a force main may be nonlinear, nonconvex, multimodel, and discontinuous.

Another advantage with a total enumeration technique is that it presents and ranks all of the alternative solutions. Unlike partial enumeration techniques which only

present the optimal solution for consideration, total enumeration techniques allow examination of suboptimal solutions which may be preferable when factors other than cost are considered. For example, proven engineering design or socio-political values are difficult to incorporate into an optimization model even if the problem is well defined, so the optimal solution may be so unrealistic that another solution must be selected.

Depending on the size of the problem, a possible drawback with total enumeration techniques may be the computational effort to enumerate all possible solutions. However, for some problems, total enumeration may be the only meaningful approach. For these problems, the challenge with using a total enumeration approach is to find ways to reduce the computational effort by applying mathematical techniques or engineering considerations. After a discussion on modeling network problems as digraphs, a total enumeration procedure that does not require extensive computational effort to find the least cost network for each individual and each group of individuals is presented.

Modeling Network Problems as Digraphs

Consider a situation wherein an existing water supply source, S , is going to serve n users with demands of Q_1 , Q_2 , . . . , Q_n , respectively. Assume that the water source is able to supply the total demand by the n users without

facility expansion except for a new regional water network. Furthermore, consider a particular system with three users that can be served directly by the source, and engineering considerations, e.g., gravity flow, have determined that it is feasible to send water from user 1 to both user 2 and user 3, and from user 2 to user 3. For this particular system, assume the total cost function for constructing a pipeline is rather simple. From Sample (1983), the total cost function for constructing a pipeline is characterized by economies of scale and can be expressed as a linear function of distance and a nonlinear function of flow; or

$$C = aQ^bL \quad (3-1)$$

where C = total cost of pipeline, dollars
 Q = quantity of flow, mgd
 L = length of pipeline, feet, and
 a, b = parameters, $0 < b < 1$.

Given this situation, the objective of the regional water authority is to determine the least cost water network for each user and each group of users in order to perform efficiency/equity analysis.

This problem can be modeled as a digraph or directed graph (see Figure 3-2) consisting of nodes to represent the source and users, and directed arcs to represent all

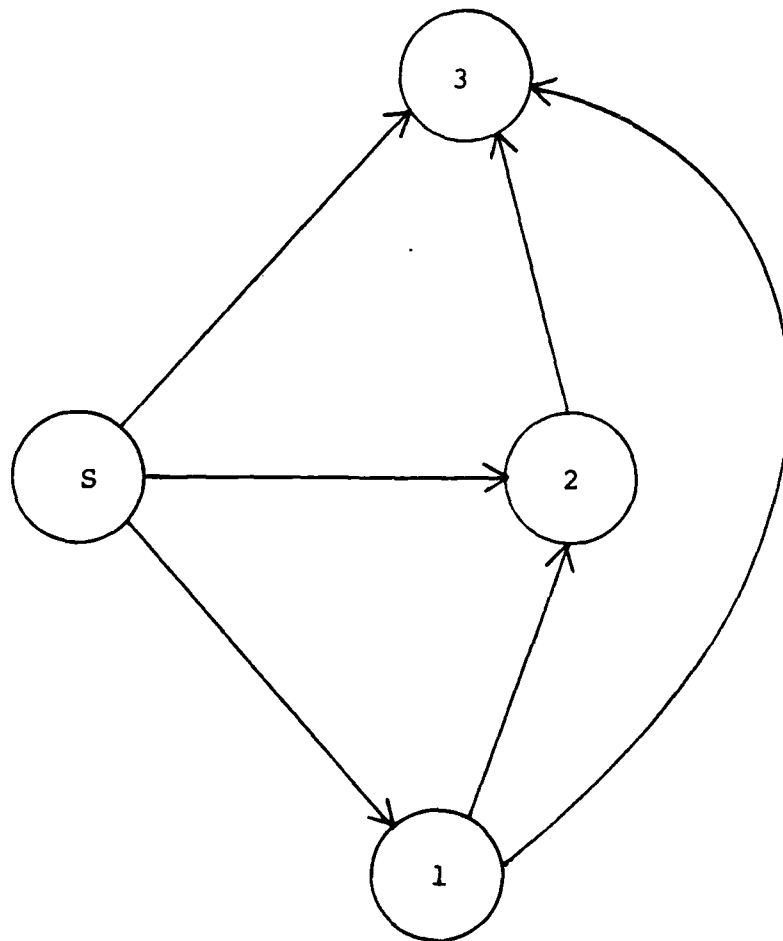


Figure 3-2. Example Digraph Representing a Regional Water Network Problem for Three Users.

possible interconnecting pipelines. If water can be sent in either direction between two users, then the pipeline is represented by two oppositely directed arcs. Consequently, any regional water network problem can be modeled by a digraph.

Before continuing, a few brief definitions and concepts are necessary since the nomenclature used in the network and graph theory literature is not standardized. A digraph or directed graph, $D(X,A)$, consists of a finite set of nodes, X , and a finite set of directed arcs, A . A directed arc is denoted by (i,j) where the direction of the arc (shown by an arrow) is from node i to node j ; node i is called the initial node and node j is called the terminal node. A subdigraph of $D(X,A)$ has a set of nodes that is a subset of X but contains all the arcs whose initial and terminal nodes are both within this subset. A path from node i to node j is simply a sequence of directed arcs from node i to node j . An elementary path is a path that does not use the same node more than once. A circuit is an elementary path with the same initial and terminal node. A directed tree or an arborescence is a digraph without a circuit for which every node, except the node called the root, has one arc directed into it while the root node has no arc directed into it. A spanning directed tree of a digraph is a directed tree that includes every node in the digraph. If a cost, $C(i,j)$ is associated with every arc

(i,j) of a digraph, then the cost of a directed tree is defined as the sum of the costs of the arcs in the directed tree. Finally, a minimum spanning directed tree of a digraph is the spanning directed tree of the digraph with the least cost. For the reader desiring more information regarding networks and graphs, numerous texts are available, e.g., Christofides (1975), Minieka (1978), and Robinson and Foulds (1980).

The problem of finding the least cost water network for each user and each group of users is the same as finding the minimum spanning directed tree rooted at node S for all possible subdigraphs as well as the digraph shown in Figure 3-2. In general, not every digraph has a spanning directed tree; however, for a realistic problem one can assume a pipeline is available to serve all individuals participating in a regional system. Thus, a spanning directed tree exists for digraphs representing realistic regional water network problems.

Although algorithms are found in Gabow (1977) and Camerini et al. (1980a, 1980b) for finding the minimum spanning directed tree or the K best spanning directed trees, these algorithms assume a linear cost model in which the cost on each arc is given prior to initiating the algorithm. As a result, these algorithms are not applicable to problems with nonlinear costs on each arc. That is, the cost along each arc cannot be determined in advance

because the cost is a function of the quantity of flow along the arc; yet, the quantity of flow along the arc is a function of the path in which the arc belongs.

The Total Enumeration Procedure

The procedure for enumerating and calculating the costs of all the spanning directed trees for all possible subdigraphs as well as the digraph is based on recognizing that a large number of spanning directed trees of a digraph can be constructed from specific spanning directed trees of subdigraphs. These specific spanning directed trees are characterized by one arc emanating from the root node and are referred to as "essential spanning directed trees." In contrast, "inessential spanning directed trees" are characterized by more than one arc emanating from the root node. The procedure sequentially calculates the costs of essential spanning directed trees for subdigraphs with increasing number of nodes, until the costs of essential spanning directed trees are calculated for all possible subdigraphs and for the digraph. Meanwhile, the cost of each inessential spanning directed tree for all possible subdigraphs as well as the digraph is calculated simply by summing the costs of essential spanning directed trees of subdigraphs that are associated with each arc emanating from the root node of the inessential spanning directed tree. That is, each arc emanating from the root node belongs to an

essential spanning directed tree of a subdigraph. By applying this procedure the costs of all the spanning directed trees can be systematically enumerated for all possible subdigraphs as well as the costs of all the spanning directed trees for the digraph. As a result, the least cost network for each user and each group of users is found.

In the following discussion, "n-node" means the number of nodes, not including the root node, is n ; e.g., an i -node digraph or subdigraph consists of $i+1$ nodes if the root node is counted. The total enumeration procedure for the n -node digraph is summarized by the flow diagram shown in Figure 3-3.

Step 1 begins the procedure for evaluating all subdigraphs consisting of the root node and one other node, i.e., the 1-node subdigraphs.

Step 2 initializes a count of the number of combinations of i -node subdigraphs evaluated.

Step 3 generates all possible combinations of i -node subdigraphs from the n -node digraph. The number of possible combinations is $\binom{n}{i}$. For example, the 3-node digraph shown in Figure 3-2 has $\binom{3}{2}$ or three possible 2-node subdigraphs, i.e., subdigraphs consisting of the following sets of nodes $\{S,1,2\}$, $\{S,1,3\}$, and $\{S,2,3\}$.

Step 4 selects one i -node subdigraph not previously selected and enumerates all of its spanning directed trees. A spanning directed tree may not exist in a case where a path does not exist from the root node to every node in the

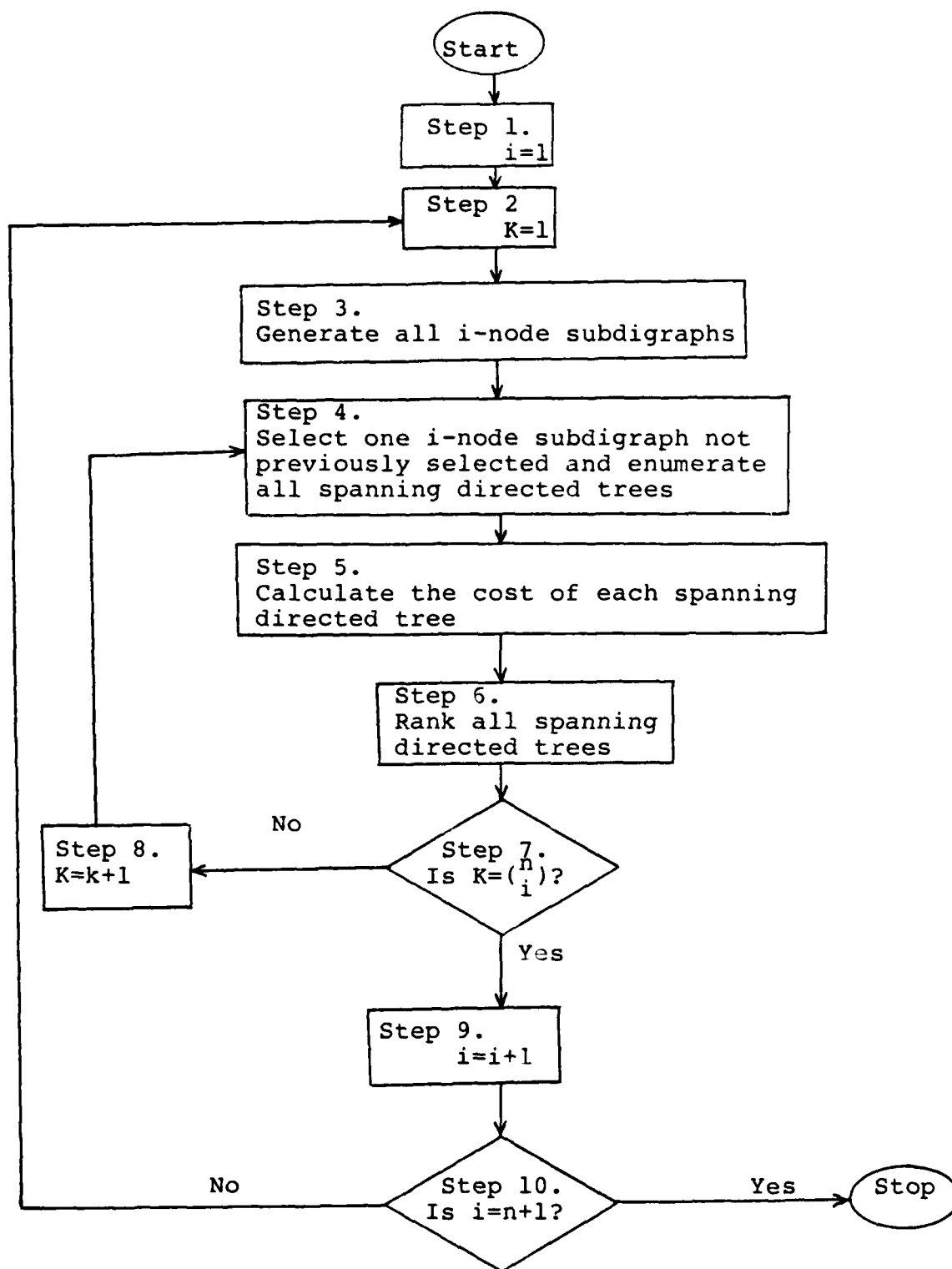


Figure 3-3. Flow Diagram of Total Enumeration Procedure for n-Node Digraph

i-node subdigraph, i.e., not every node in the i-node subdigraph has an arc directed into it.

Actually, only the essential spanning directed trees need to be enumerated. The enumeration of inessential spanning directed trees is simply done by finding all possible combinations of i-node digraphs from the entire set of essential spanning directed trees enumerated previously, i.e., all essential spanning directed trees for all possible subdigraphs of the i-node subdigraph. This process substantially reduces the effort involved in enumerating all the spanning directed trees for an i-node subdigraph because a large number of spanning directed trees are inessential. If the i-node subdigraph is unusually large and dense, algorithms are available in Chen and Li (1973), Christofides (1975), and Minieka (1978) for generating spanning directed trees.

If necessary, a procedure in Chen (1976) can be used to compute the number of spanning directed trees of an i-node subdigraph or an n-node digraph. A directed tree matrix, M , is defined for a digraph, where m_{ii} equals the number of arcs directed into node i and m_{ij} is equal to the negative of the number of arcs in parallel from node i to node j . The number of spanning directed trees rooted at node S for the digraph defined by M is given by the determinant of the minor submatrix resulting from deleting the S th row and

column of M . Applying this procedure to the 3-node digraph in Figure 3-2 gives the following directed tree matrix.

$$\begin{array}{c}
 \begin{array}{ccccc}
 & S & 1 & 2 & 3 \\
 S & \left(\begin{array}{cccc}
 0 & -1 & -1 & -1 \\
 0 & 1 & -1 & -1 \\
 0 & 0 & 2 & -1 \\
 0 & 0 & 0 & 3
 \end{array} \right)
 \end{array}
 \end{array}$$

The determinant of the minor submatrix resulting from deleting the S th row and column is six, so there are six spanning directed trees rooted at node S for this digraph.

Step 5 calculates the cost of each spanning directed tree enumerated in Step 4. The cost for each essential spanning directed tree is calculated independently. However, the cost for each inessential spanning directed tree is simply calculated by summing the costs of essential spanning directed trees of subdigraphs calculated previously that are associated with the arcs emanating from the root node. For inessential spanning directed trees the costs can be calculated along with the enumeration process described in Step 4.

Step 6 ranks all the spanning directed trees for the i -node subdigraph according to cost. The minimum spanning directed tree is the least cost network for the users associated with the set of nodes for the i -node subdigraph.

Step 7 checks the counter to see if all possible combinations of i -node subdigraphs have been evaluated. If not, Step 8 advances the counter by one before returning to Step 4 to evaluate another i -node subdigraph. If all of the possible combinations of i -node subdigraphs have been evaluated, the procedure goes to Step 9 and begins the evaluation of subdigraphs with $i+1$ nodes.

Step 10 checks if the n -node digraph has been evaluated. If not, the procedure returns to Step 2 and proceeds to evaluate the subdigraphs with $i+1$ nodes; otherwise, the procedure terminates.

The total enumeration procedure is illustrated in Table 3-1 using the regional water network problem modeled by the 3-node digraph shown in Figure 3-2.

During the first iteration all combinations of 1-node subdigraphs are evaluated. For this simple case three combinations, i.e., $\binom{3}{1} = 3$, are evaluated. Furthermore, each combination has only one spanning directed tree, and the one spanning directed tree is essential. As a result, the cost of the spanning directed tree for each combination must be calculated. Obviously each spanning directed tree is the least cost network for the associated user. During the second iteration, three combinations, i.e., $\binom{3}{2} = 3$, of 2-node subdigraphs are evaluated. In this case, each combination has two spanning directed trees, but the cost of only one spanning directed tree needs

Table 3-1. Example of Total Enumeration Procedure for 3-Node Digraph

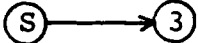
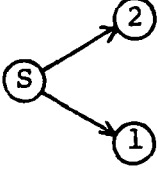
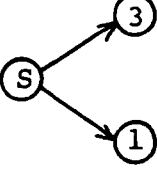
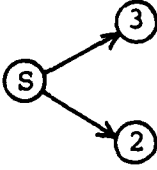
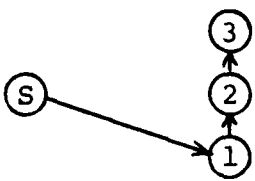
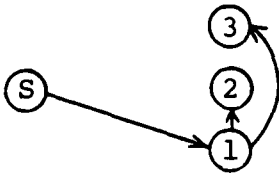
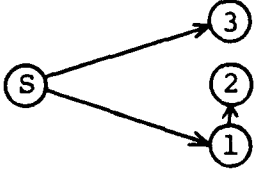
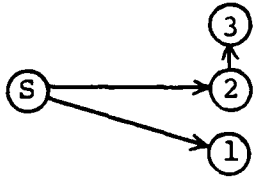
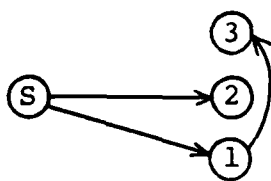
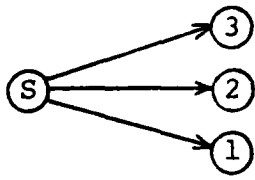
Iteration i	i-Node Subdigraphs	Spanning Directed Trees for i-Node Subdigraph	Are Spanning Directed Trees Essential?
i=1	{S, 1}		Yes
	{S, 2}		Yes
	{S, 3}		Yes
i=2	{S, 1, 2}		Yes
			No
	{S, 1, 3}		Yes
			No
	{S, 2, 3}		Yes
			No

Table 3.1. Continued.

Iteration i	i -Node Subdigraphs	Spanning Directed Trees for i -Node Subdigraph	Are Spanning Directed Trees Essential?
$i=3$	$\{S, 1, 2, 3\}$		Yes
			Yes
			No
			No
			No
			No

to be calculated. The cost of the inessential spanning directed tree is simply found by summing the costs of the corresponding essential spanning directed trees calculated during the first iteration. The minimum spanning directed tree for each combination is the least cost network for the associated group of users. Finally, for the third iteration, i.e., $i=n$, the 3-node digraph is being evaluated. This 3-node digraph has six spanning directed trees, and these six spanning directed trees can be enumerated by inspection.

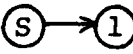
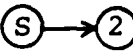
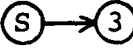
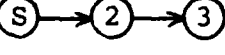
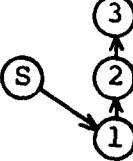
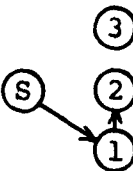
The four inessential spanning directed trees can be enumerated by simply finding all possible combinations of 3-node digraphs from the essential spanning directed trees generated during the first and second iterations. Thus, only two independent calculations are necessary to find the costs of the essential spanning directed trees. Meanwhile, the cost of the four inessential spanning directed trees is calculated simply by summing the costs of essential spanning directed trees for subdigraphs previously calculated during the first two iterations. For example, in Table 3-1, the cost for the inessential spanning directed tree consisting of the set of arcs $\{(S,3), (S,1), (1,2)\}$ is determined by summing the costs of the two essential spanning directed trees consisting of the sets of arcs $\{(S,3)\}$ and $\{(S,1), (1,2)\}$ associated with the two subdigraphs consisting of the sets of nodes $\{S,3\}$ and $\{S,1,2\}$, respectively. Therefore, eight independent calculations are

necessary to find the costs of the six spanning directed trees for the digraph, and only two of the six spanning directed trees are essential. In fact, the eight independent calculations enable us to find all 2^n-1 or seven optimal solutions necessary to perform efficiency/equity analysis. Table 3-2 shows that the number of independent calculations necessary to find the cost of all the spanning directed trees for all possible subdigraphs is simply equal to the number of independent calculations to find the cost of all the spanning directed trees for the digraph less the number of essential spanning directed trees for the digraph. Consequently, for our 3-node digraph, six independent calculations are necessary to find the optimal solution for each user and each subgroup of users. For the balance of this chapter, the optimal solution for each user and each subgroup of users will be referred to as the 2^n-2 optimal solutions. Finally, all suboptimal solutions are enumerated for all possible subdigraphs as well as for the digraph.

Computational Considerations

Although the number of independent calculations necessary to find the costs of all the spanning directed trees for all possible subdigraphs as well as the digraph is uniquely determined by the configuration of the digraph, we can get a sense of the computational effort by examining the

Table 3-2. The Number of Independent Calculations to Find the Costs of Spanning Directed Trees for All Possible Subdigraphs.

Independent Calculation	Is Independent Calculation Used to Find the Costs of Spanning Directed Trees for the Digraph?	Is Independent Calculation Used to Find the Costs of Spanning Directed Trees for All Possible Subdigraphs?
	Yes	Yes
	Yes	Yes
	Yes	Yes
	Yes	Yes
	Yes	Yes
	Yes	Yes
	Yes	No
	Yes	No
Total Number of Yes	8	6

three digraphs shown in Figure 3-4. For the 3-node digraph, six independent calculations are necessary to find the costs of the four spanning directed trees for the digraph, and only one of the four spanning directed trees is essential. More importantly, 12 calculations are necessary to find the seven optimal solutions, but only 6 of the 12 calculations (50%) are independent. Furthermore, only five independent calculations are necessary to find the 2^{n-2} optimal solutions. For the 4-node digraph, 10 independent calculations are necessary to find the cost of the eight spanning directed trees for the digraph, and only one of the eight spanning directed trees is essential. For this digraph, 33 calculations are necessary to find the 15 optimal solutions, but only 10 of the 33 calculations (30%) are independent. Moreover, only nine independent calculations are necessary to find the 2^{n-2} optimal solutions. Finally, for the 5-node digraph, 19 independent calculations are necessary to find the costs of the 24 spanning directed trees for the digraph, but only 2 of the 24 spanning directed trees are essential. In this case, 109 calculations are necessary to find the 31 optimal solutions, but only 19 of the 109 calculations (17%) are independent. From these 19 independent calculations, only 17 are necessary to find the 2^{n-2} optimal solutions. As we can see, summarized in Table 3-3, a large number of the spanning directed trees of a digraph are inessential.

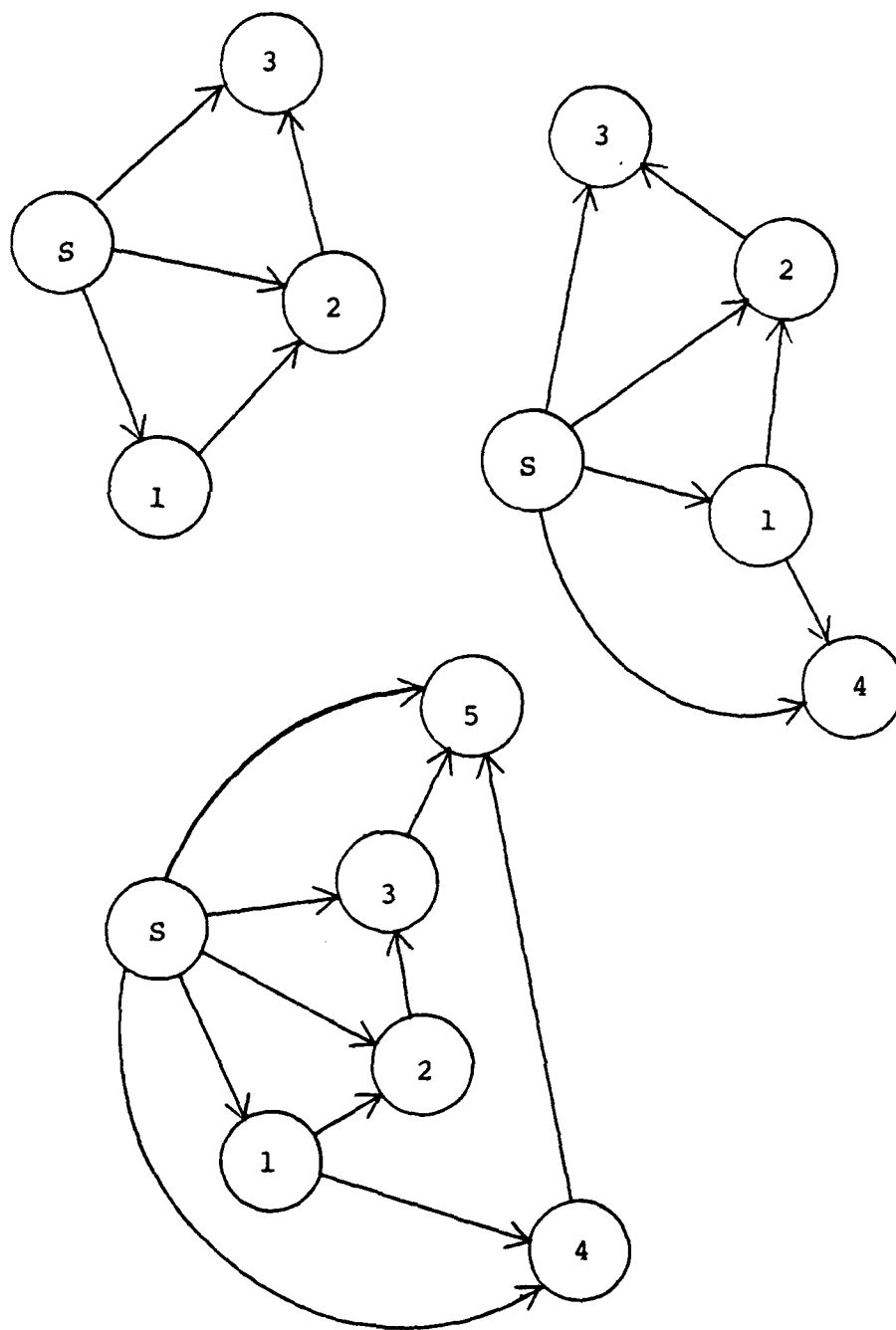


Figure 3-4. Examples of 3,4,5-Node Digraphs.

Table 3-3. Summary of Computational Effort for Digraphs Shown in Figure 3-4.

Digraph	2^{n-1} Optimal Solutions	Number of Spanning Directed Trees	Number of Inessential Spanning Directed Trees	Number of Calculations to Find 2^{n-1} Optimal Solutions	Number of Independent Calculations to Find 2^{n-1} Optimal Solutions (%)	Number of Independent Calculations to Find 2^{n-2} Optimal Solutions
3-node	7	4	3	12	6 (50%)	5
4-node	15	8	7	33	10 (30%)	9
5-node	31	24	22	109	19 (17%)	17

Also, the percentage of independent calculations decreases as the number of nodes for a digraph increases.

The 5-node digraph in Figure 3-4 shows that the actual number of independent calculations necessary to determine the 31 optimal solutions to perform efficiency/equity analysis of a regional water network problem involving five users is rather small. In fact, a regional water network serving five users may be considered a fairly large network. As larger systems form, increases in transactions costs because of multiple political jurisdictions, growing administrative complexity, etc., may eventually offset the gains from a regional system. In any case, real regional water network problems probably involve fairly small and sparse networks. That is, large networks can usually be broken down into smaller networks for analysis based on natural geographical and hydrological features, political boundaries, etc. Also, in actual problems there may not be that many choices for routing pipelines. Thus, the number of independent calculations necessary to calculate the $2^n - 1$ optimal solutions for a realistic regional water network should not be unreasonable.

One of the advantages of using this total enumeration procedure is that it can be accomplished on a personal computer using readily available software. Thus, decision makers involved with planning and negotiating a regional water network can have easy access to information to aid the

decision-making process. For instance, the procedure can be implemented using the extremely "user friendly" Lotus 1-2-3 spreadsheet software package. Lotus 1-2-3 has the mathematical functions to handle calculations involving nonlinear cost functions or involving detailed cost analysis. A sample Lotus 1-2-3 printout is shown in Table 3-4 for a hypothetical water network problem modeled by the 3-node digraph shown in Figure 3-2. This printout should be self-explanatory. The top portion of the printout contains the data for the problem, and the bottom portion is the calculations associated with the total enumeration procedure. The sorting capabilities of Lotus 1-2-3 allow automatic ranking of all the feasible solutions according to cost. Moreover, the Lotus 1-2-3 electronic spreadsheet automatically recalculates all values associated with a formula whenever a new value is entered or an existing value is changed. This automatically gives the total enumeration procedure the capability for sensitivity analysis. For example, the set of all feasible solutions ranked according to cost can be evaluated as the economies of scale, as represented by the value of b in equation (3-1), is varied over a specific range of values. Thus, for a regional network problem of realistic size, all the feasible solutions can be enumerated using a spreadsheet software package.

Table 3-4. Efficiency Analysis of a Three-User Water Supply Network with Nonlinear Cost Function Using Lotus 1-2-3.

Data			
Distance : $L(i,j)$ is the distance in feet from i to j			
$L(S,1)=$	17000	$L(S,3)=$	30250
$L(S,2)=$	26000	$L(1,3)=$	19670
$L(1,2)=$	13100	$L(2,3)=$	15500
Demand : $Q(i)$ is the demand in mgd for user i			
$Q(1)=$	1	$Q(2)=$	6
$Q(3)=$		$Q(3)=$	3
Cost Function: $a(Q^b)L$			
$a=$	38	$b=$	3.51
Calculations With Total Enumeration Procedure			
$C(i..j)[x]$ = Cost of network $[x]$ for $i..j$; $C(i..j)$ = Least cost for $i..j$			
$C(1)[S1]=$ 6460000 $C(2)[S2]=$ 2463848. $C(3)[S3]=$ 2012936.			
$C(12)[S1,12]=$	2984140.	$C(12)=$	2984140.
$C(12)[S1;S2]=$	3109848.		
$C(13)[S1,13]=$	2618975.	$C(13)=$	2618975.
$C(13)[S1;S3]=$	2658986.		
$C(23)[S2,23]=$	4061294.	$C(23)=$	4061294.
$C(23)[S2;S3]=$	4476835.		
$C(123)[S1,12,23]=$	4648439.		
$C(123)[S1,12;S3]=$	4997126.		
$C(123)[S1,12,13]=$	4640756.	$C(123)=$	4640756.
$C(123)[S1,13;S2]=$	5082824.		
$C(123)[S1;S2,23]=$	4707294.		
$C(123)[S1;S2;S3]=$	5122835.		
Sort $C(123)$ in ascending order			
Paths	Cost		
$C(123)[S1,12,13]=$	4640756.		
$C(123)[S1,12;S3]=$	4648439.	BEST	
$C(123)[S1;S2,23]=$	4707294.	$C(123)=$	4640756
$C(123)[S1,12;S3]=$	4997126.		
$C(123)[S1,13;S2]=$	5082824.		
$C(123)[S1;S2;S3]=$	5122835.		

Summary

A total enumeration procedure for finding the optimal solutions necessary for efficiency/equity analysis of realistic regional water network problems is presented. The procedure can be easily understood and applied by engineers with little knowledge or experience in operations research techniques. Furthermore, the procedure allows the engineers to handle all problems regardless of the types of cost function involved or to perform detailed cost analysis. Finally, if the optimal solution is impractical for implementation, all suboptimal solutions ranked according to cost are readily available for consideration.

CHAPTER 4 EQUITY ANALYSIS

Introduction

Proposed regional water resources systems involve multiple purposes and groups who must somehow share the cost of the entire project. The project may focus on construction of a large dam which serves numerous purposes such as water supply, flood control, and recreation. Also, canals from the dam direct the water to nearby users. A significant portion of the total cost of this project may involve elements which serve more than one purpose and/or group. These costs are referred to as joint or common costs. In such cases, it is possible to find the optimal or the most economically efficient regional system, i.e., the one that maximizes benefits minus costs. However, a major effort remains to somehow apportion the project cost in an equitable manner. In fact, the importance of the financial analysis to apportion project cost is not limited to the optimal system but includes any other integrated systems being considered for implementation as well.

This chapter examines principles of cost allocation using concepts from cooperative n-person game theory. An

example regional water network is used to illustrate these principles.

Cost Allocation for Regional Water Networks

A hypothetical situation similar to options contained in the West Coast Regional Water Supply Authority's master plan for Hillsborough, Pasco, and Pinellas counties in Florida (Ross et al., 1978) is now considered. Phase I (1980-1985) of the plan recommends the use of groundwater from existing and newly developed well fields to satisfy water demands in the tri-county area. For this hypothetical problem, assume that an existing well field is the most high quality and cost effective water supply source (S) available for three counties (1, 2, and 3) with projected demands of 1, 6, and 3 million gallons per day (mgd), respectively. The demand for each county is based on projected population growth and average per capita demand over a period of 5 years (see Table 4-1). Assume that the existing well field is currently operating below its capacity of 20 mgd and can satisfy the additional 10 mgd demanded by the three counties. In addition, assume that no facility expansion is required except for a new regional water network. Furthermore, each county can be served directly by the well field, and engineering considerations, e.g., gravity flow, have determined that water can be sent from county 1 to both county 2 and county 3, and from county 2 to county 3. The

Table 4-1. Projected Population Growth and Projected Average Per Capita Demand.

County	Projected Population Growth	Projected Average Per Capita Demand (gal/cap-day)	Projected Additional Demand (mgd)
1	8,000	125	1
2	40,000	150	6
3	18,750	160	3
Total	66,750	---	10
Weighted Average	---	150	--

lengths of all possible interconnecting pipelines are shown in Figure 4-1. For our hypothetical problem, assume that the total cost of constructing a pipeline has strong economies of scale and is $C = 38Q^{.5}L$, where C is total cost of pipeline in dollars, Q is quantity of flow in mgd, and L is the length of pipeline in feet.

Given the problem just described, the cost of a pipeline serving county 1 alone is \$646,000; the cost of a pipeline serving county 2 alone is \$2,420,095; and the cost of a pipeline serving county 3 alone is \$1,990,992. The total cost for three individual pipelines is \$5,057,087. However, when the costs for all the options available to these three counties are enumerated using the procedure outlined in the preceding chapter, we see that the counties can do better by cooperating (see calculations in Appendix A using Lotus 1-2-3). There may be a slight difference between the numbers used in the text and the numbers in Appendix A because of rounding off. Also, cost data are only significant to the nearest thousand dollars.

If the three counties cooperate, they can construct the least cost or optimal network consisting of pipelines from the well field to county 1, from county 1 to county 2, and from county 2 to county 3 (see Table 4-2). This optimal network costs \$4,556,409 and represents a savings of 9.9% or \$500,678 when compared with the cost for three individual pipelines. Obviously, constructing the optimal network is

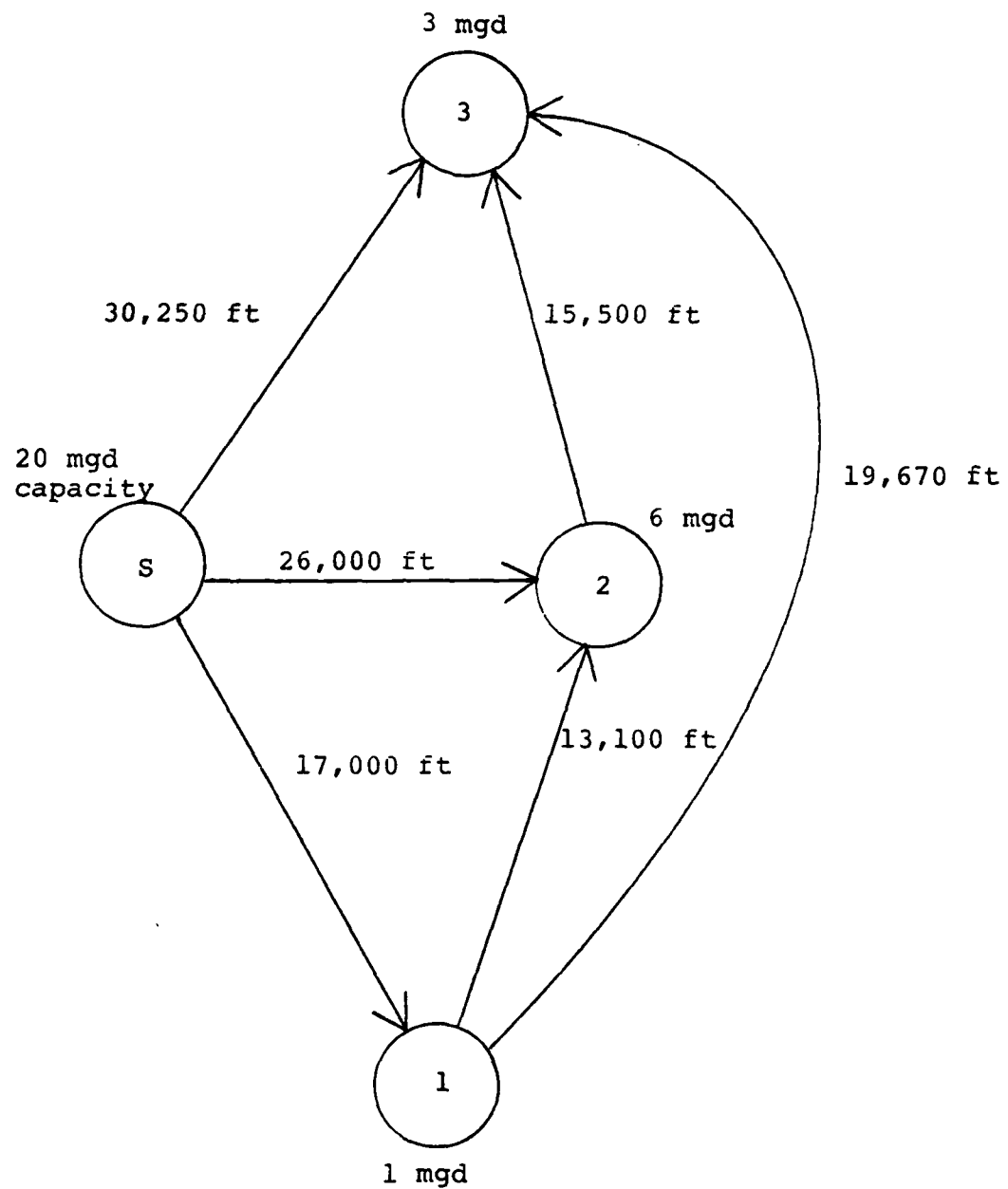
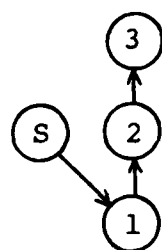


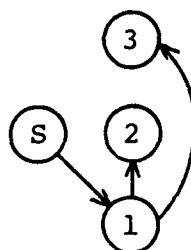
Figure 4-1. Lengths of Interconnecting Pipelines.

Table 4-2. The Costs and Percent Savings for All Options.

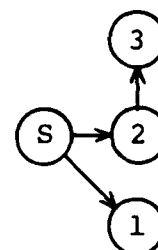
Option (Rank)	Cost (\$)	Savings (%)
1	4,556,409	9.90
2	4,556,826	9.89
3	4,630,177	8.44
4	4,919,503	2.72
5	5,006,734	1.00
6	5,057,087	0



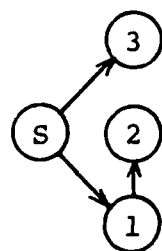
(1)



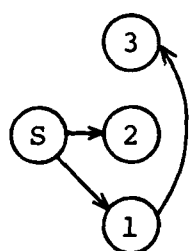
(2)



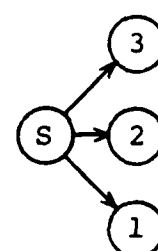
(3)



(4)



(5)



(6)

in the best interest of the three counties, but to implement this least cost network, an equitable way to allocate the cost among the three counties must be found. This financial problem is known as a cost allocation problem. The complexity is introduced because the counties share common pipes.

Criteria for Selecting a Cost Allocation Method

Several sets of criteria for selecting a cost allocation method are found in the literature. For the water resources field, criteria for allocating costs date back to the Tennessee Valley Authority (TVA) project in 1935 when prominent authorities were brought together to address the cost allocation problem. They developed the following set of criteria for allocating costs (Ransmeier, 1942, pp. 220-221):

1. The method should have a reasonable logical basis. It should not result in charging any objective with a greater investment than the fair capitalized value of the annual benefit of this objective to the consumer. It should not result in charging any objective with a greater investment than would suffice for its development at an alternate single purpose site. Finally, it should not charge any two or more objectives with a greater investment than would suffice for alternate dual purpose or multiple purpose improvement.
2. The method should not be unduly complex.
3. The method should be workable.
4. The method should be flexible.
5. The method should apportion to all purposes present at a multiple purpose enterprise a share in the overall economy of the operation.

This set of criteria developed for the water resources field is similar to the following set of criteria proposed by Claus and Kleitman (1973) for allocating the cost of a network:

1. The method must be easy to use and understandable to users. They must be able to predict the effects of changes in their service demands.
2. The method must have stability against system breakup. It should not be an advantage to one or more users to secede from the system. Thus, there are limits to which a method can subsidize one user or class of user at the expense of others.
3. It is desirable, though not necessary, that the costing be stable under evolutionary changes in the system or under mergers of users.
4. It is again desirable that the method should preserve the substance and appearance of non-discrimination among users.
5. If the method represents a change from present usage it is desirable that transition to the new method be easy.

From these two sets of criteria, the most important criterion for selecting a method to allocate the cost of a regional water network is the method's ability to ensure stability or prevent breakup of the network. That is, the method should not allocate cost in a manner whereby an individual or a subgroup of individuals can acquire the same level of service by a less expensive alternative. Otherwise, the individual or subgroup of individuals will consider their allocated cost inequitable or unfair and secede from the regional network for a less expensive alternative.

Heaney (1979) has expressed these fairness criteria for an equitable cost allocation mathematically as follows:

$$1) \quad x(i) \leq \text{minimum} [b(i), c(i)] \quad \forall i \in N \quad (4-1)$$

where $x(i)$ = cost allocated to individual i ,
 $b(i)$ = benefit of individual i ,
 $c(i)$ = the alternative cost to individual i of independent action, and
 N = set of all individuals; i.e.,
 $N = \{1, 2, \dots, n\}$.

This criterion simply means that individual i should not be charged a cost greater than the minimum of individual i 's benefit and alternative cost for independent action.

$$2) \quad \sum_{i \in S} x(i) \leq \text{minimum} [b(S), c(S)] \quad \forall S \subset N \quad (4-2)$$

where $c(S)$ = alternative cost to subgroup S of independent action, and
 $b(S)$ = benefit of subgroup S .

This second criterion extends the first criterion to include subgroup of individuals as well. These two fairness criteria are now used to evaluate some simple and seemingly fair cost allocation schemes for our regional water network problem. Throughout this chapter, we will assume for our regional water network problem that each county's and each

subgroup of counties' alternative cost of independent action is less than or equal to each county's and each subgroup of counties' benefits, respectively; i.e.,

$$\begin{aligned} c(i) &= \text{minimum } [b(i), c(i)] & \forall i \in N, \text{ and} & (4-3) \\ c(S) &= \text{minimum } [b(S), c(S)] & \forall S \subset N. \end{aligned}$$

Ad Hoc Methods

Over the years, many ad hoc methods have been proposed or used to apportion the costs of water resources projects (Goodman, 1984). In general, ad hoc methods used in the water resources field for allocating costs can be described as follows: allocate certain costs that are considered identifiable to an individual directly and prorate the remaining costs, i.e., total project cost less the sum of all identifiable costs, among all the individuals in the project by some physical or nonphysical criterion. Mathematically, this can be expressed as follows:

$$x(i) = x(i)_{id} + \psi(i) \cdot rc \quad (4-4)$$

where

- $x(i)$ = cost allocated to individual i ,
- $x(i)_{id}$ = costs identifiable to individual i ,
- $\psi(i)$ = prorating factor for individual i , and
- rc = remaining costs, i.e.,

$$c(N) - \sum_{i \in N} x(i)_{id}.$$

Furthermore, the requirement that $\sum_{i \in N} \psi(i) = 1.0$ should be obvious.

James and Lee (1971) summarize 18 ways for allocating the costs of water projects depending on the definition of identifiable costs and the basis for prorating the remaining costs (see Table 4-3). Basically, the differences among these 18 methods are the following three ways of defining identifiable costs: 1) zero, 2) direct or assignable costs, or 3) separable costs; and the following six ways of prorating remaining costs: 1) equal, 2) unit of use, 3) priority of use, 4) net benefit, 5) alternative cost, or 6) the smaller of net benefit or alternative cost. The next two sections analyze the effects of defining identifiable costs as either zero or direct costs. A detailed treatment of separable costs, i.e., the difference between total project costs with and without an individual, is given in the section on the separable costs, remaining benefits method.

Defining Identifiable Costs as Zero

The simplest way to allocate costs is to define identifiable costs as equal to zero and prorate total project cost by some physical or nonphysical criterion. For example, population and demand are two ways to prorate total project

Table 4-3. Cost Allocation Matrix.

Basis for Prorating Remaining Costs	Definition of Identifiable Cost		
	A. Zero	B. Direct Cost	C. Separable Cost
a. Equal	Aa	Ba	Ca
b. Unit of Use	Ab	Bb	Cb
c. Priority of Use	Ac	Bc	Cc
d. Net Benefit	Ad	Bd	Cd
e. Alternative Cost	Ae	Be	Ce
f. Smaller of d. or e.	Af	Bf	Cf

Source: Modified from James and Lee, 1971, p. 533.

cost (Young et al., 1982). Using these two ways to prorate the cost of the optimal network for our regional water network problem gives the following cost allocations (see calculations in Table 4-4 and Table 4-5):

Proportional to Population

County 1	\$ 546,769
County 2	2,733,845
County 3	<u>1,275,795</u>
	\$4,556,409

Proportional to Demand

County 1	\$ 455,641
County 2	2,733,845
County 3	<u>1,366,923</u>
	\$4,556,409

Although these cost allocations are simple to calculate and easy to understand, they fail to implement the optimal network because county 2 considers these cost allocations unfair. In contrast to counties 1 and 3, county 2 loses money by being allocated a cost in excess of its go-it-alone costs using either of these two methods. Consequently, county 2 would rather acquire a pipeline by itself than cooperate with counties 1 and 3 to construct the optimal network. The principal failure with these proportionality

Table 4-4. Cost Allocation of Optimal Network Based on Population.

County i	Population	Percent of Total Population	Allocated Cost (\$) $x(i)$	Go-It-Alone Cost (\$) $c(i)$	Is $x(i) < c(i)$?
1	8,000	12	546,769	646,000	Yes
2	40,000	60	2,733,845	2,420,095	No
3	18,750	28	1,275,795	1,990,992	Yes
Total	66,750	100	4,556,409	5,057,087	---

Table 4-5. Cost Allocation of Optimal Network Based on Demand.

County i	Demand (mgd)	Percent of Total Demand	Allocated Cost (\$) $x(i)$	Go-It-Alone Cost (\$) $c(i)$	Is $x(i) < c(i)$?
1	1	10	455,641	646,000	Yes
2	6	60	2,733,845	2,420,095	No
3	3	30	1,366,923	1,990,992	Yes
Total	10	100	4,556,409	5,057,087	---

methods is that they do not recognize explicitly each individual's contribution to total project cost.

Defining Identifiable Costs as Direct Costs

A way to recognize each individual's contribution to total project cost is by defining identifiable costs as those costs that can be directly assigned, and prorating the remaining costs by some physical or nonphysical criterion such as use or number of individuals; i.e.,

$$x(i) = x(i)_{\text{direct}} + \psi(i) \cdot rc \quad (4-5)$$

where $x(i)_{\text{direct}}$ = direct cost or assignable cost to individual i .

Although this direct costing approach intuitively seems fair, inequitable and unpredictable cost allocations can result. To illustrate, two direct costing methods are applied to our regional water network problem.

A common approach to allocating remaining costs is by some physical measure of each individual's use of the common facilities; this method is generally referred to as the use of facilities method (Loughlin, 1977; Goodman, 1984). This traditional method is easy to understand and apply because quantitative information on a physical measure of use is generally available. In the water resources field, use can be measured in terms of the storage capacity and/or the

quantity of water flow provided by the common facilities. For our regional water network problem, the flow to each county is the obvious measure of use to apportion the costs of common pipelines since the assumed cost function depends on the flow. In the case of the optimal network, the only direct cost is the cost of the pipeline from county 2 to county 3 serving county 3, and the use of facilities method gives the following cost allocation (see calculations in Table 4-6).

County 1	\$ 204,283
County 2	2,221,299
County 3	<u>2,130,827</u>
Total	\$4,556,409

Unfortunately, this cost allocation does not implement the optimal network because county 3 can do substantially better by going alone, i.e., \$1,990,992 versus paying \$2,130,827.

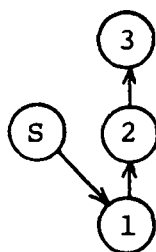
In addition to giving an inequitable cost allocation for the optimal network, the use of facilities method can promote noncooperation if other networks are also being considered. Table 4-7 shows the cost allocations for all possible options available to the three counties using the use of facilities method. Suppose the "second best" network or option 2 is also being considered by the counties. The second best network consists of the pipelines from the well field to county 1, from county 1 to county 2, and from county 1 to county 3. This second best network costs

Table 4-6. Cost Allocation of Optimal Network with Use of Facilities Method

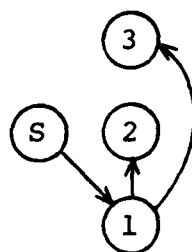
Pipeline	S-1	1-2	2-3	Total Cost (\$)	Go-It-Alone Cost (\$) C(i)
Length (ft)	17,000	13,100	15,500	---	---
Q (mgd)	10	9	3	---	---
Pipeline Cost (\$)	2,042,832	1,493,400	1,020,177	4,556,409	---
Cost for County 1 (\$) Q=1 mgd	204,283	0	0	204,283	646,000
Cost for County 2 (\$) Q=6 mgd	1,225,699	995,600	0	2,221,299	2,420,095
Cost for County 3 (\$) Q=3 mgd	612,850	497,800	1,020,177	2,130,827	1,990,992

Table 4-7. Cost Allocation for the Use of Facilities Method.

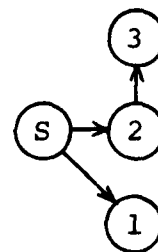
Option (Rank)	Cost Allocation to County i (\$)			$\Sigma x(i)$ (\$)	Is Cost Allocation Equitable?
	County 1 $x(1)$	County 2 $x(2)$	County 3 $x(3)$		
1	204,283	2,221,299	2,130,827	4,556,409	No $x(3) > c(3)$
2	204,283	2,445,055	1,907,488	4,556,826	No $x(2) > c(2)$
3	646,000	1,976,000	2,008,177	4,630,177	No $x(3) > c(3)$
4	244,165	2,684,346	1,990,992	4,919,503	No $x(2) > c(2)$
5	323,000	2,420,095	2,263,639	5,006,734	No $x(3) > c(3)$
6	646,000	2,420,095	1,990,992	5,057,087	---



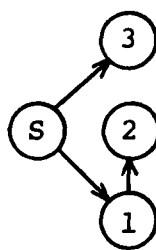
(1)



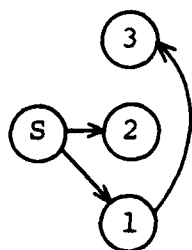
(2)



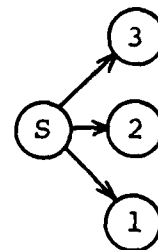
(3)



(4)



(5)



(6)

\$4,556,826 or \$417 more than the optimal network; so, both networks are essentially comparable in cost, and either network might be considered the least cost network. In fact, the second best network becomes the optimal network if the economies of scale or the value of b in the cost function is .51 instead of .50 (see Table 3-4). Nevertheless, applying the use of facilities method to this second best network gives the following cost allocation.

County 1	\$ 204,283
County 2	2,445,055
County 3	<u>1,907,488</u>
	\$4,556,826

In this case, the cost allocation fails to implement the second best network because county 2 is better off going alone, i.e., paying \$2,420,095 rather than \$2,445,055. Furthermore, if we examine the cost allocation for the optimal network and the second best network, another problem is evident. Although the costs for the two networks are \$417 apart, the difference in costs between the two networks for county 2 and county 3 is enormous. Consequently, this cost allocation method imposes another obstacle for the counties to cooperate and implement either one of the two networks. County 2 strongly opposes the second best network because of its substantially higher cost while county 3 strongly opposes the optimal network for the same reason. This problem is even more serious when more options are considered by the counties. Table 4-7 indicates tremendous

differences in allocated cost for each county depending on the network, thereby making cooperation very difficult. This situation shows the danger for individuals to simply accept the least cost network without carefully examining all of their options if the use of facilities method for allocating costs is chosen.

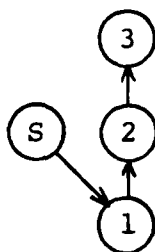
Another simple way to prorate the remaining costs is to divide it equally among the individuals associated with the common facilities (see calculations for optimal network in Table 4-8). Table 4-9 shows the cost allocations using this egalitarian approach and indicates that none of the cost allocations for options with savings are equitable. At first glance, the cost allocation for option 5 appears equitable because each county is charged a cost less than or equal to its go-it-alone cost. However, closer examination reveals that counties 1 and 2 can do better as a coalition. They can construct a pipeline from the well field to county 1 and from county 1 to county 2, i.e., option 4, for \$2,928,511 rather than pay the sum of their costs for option 5, i.e., \$3,066,095. Unfortunately, a transition from option 5 to option 4 causes county 1 to lose money, i.e., \$854,577 for option 4 versus \$646,000 for option 5. To further complicate matters, option 5 only gives a 1% savings and requires county 1 to cooperate with county 3 to build a pipeline without getting any savings.

Table 4-8. Cost Allocation of Optimal Network with Direct Costing/Equal Apportionment of Remaining Costs Method.

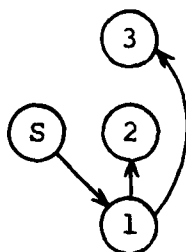
Pipeline	S-1	1-2	2-3	Total Cost (\$)	Go-It-Alone Cost (\$) c(i)
Length (ft)	17,000	13,100	15,500	---	---
Q (mgd)	10	9	3	---	---
Pipeline Cost (\$)	2,042,832	1,493,400	1,020,177	4,556,409	---
Cost for County 1 (\$) Q=1 mgd	680,944	0	0	680,944	646,000
Cost for County 2 (\$) Q=6 mgd	680,944	746,700	0	1,427,644	2,420,095
Cost for County 3 (\$) Q=3 mgd	680,944	746,700	1,020,177	2,447,821	1,990,992

Table 4-9. Cost Allocation for Direct Costing/Equal Apportionment of Remaining Costs Method

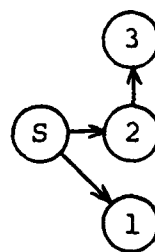
Option (Rank)	Cost Allocation to County i (\$)			$\Sigma x(i)$ (\$)	Is Cost Allocation Equitable?
	County 1	County 2	County 3		
	x(1)	x(2)	x(3)		
1	680,944	1,427,644	2,447,821	4,556,409	No $x(1) > c(1)$ $x(3) > c(3)$
2	680,944	1,900,300	1,975,582	4,556,826	No $x(1) > c(1)$
3	646,000	1,482,000	2,502,177	4,630,177	No $x(3) > c(3)$
4	854,577	2,073,934	1,990,992	4,919,503	No $x(1) > c(1)$
5	646,000	2,420,095	1,940,639	5,006,734	No $x(1) + x(2) > c(12)$
6	646,000	2,420,095	1,990,992	5,057,087	--



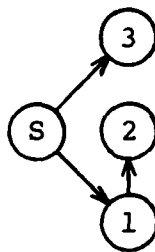
(1)



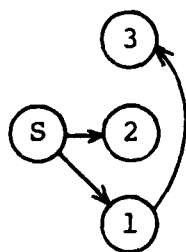
(2)



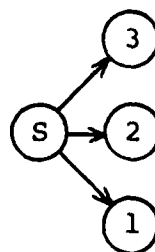
(3)



(4)



(5)



(6)

Given these observations, the stability of option 5 as a regional water network is at best questionable. Again, if the allocated costs for counties 2 and 3 for the optimal network are compared to the second best network, a similar situation like the one discussed for the use of facilities method exists. That is, counties 2 and 3 face substantially different costs for these two networks with comparable costs.

Thus, assigning direct costs does not help eliminate inequitable cost allocations. In fact, direct costing methods can impose additional obstacles to cooperation. This occurs because the assignment of direct costs depends on the configuration of the facilities. For instance, the cost of the pipeline from county 2 to county 3 for our regional water network problem can be a direct cost or a joint cost depending on the network. The cost of the pipeline is a direct cost for county 3 if the second best network, i.e., option 2, is being considered; yet, the cost of the pipeline is a joint cost for counties 2 and 3 if the optimal network, i.e., option 1, is being considered. These changes in the cost classification for the pipeline from county 2 to county 3 contribute to the tremendous difference in the cost allocations for counties 2 and 3 for the two comparable cost networks. This situation indicates an additional criterion not addressed by Claus and Kleitman (1973) for selecting a procedure to allocate network cost.

The cost allocation procedure should be independent of network configuration; otherwise, the cost allocation procedure can promote noncooperation if more than one network is being considered.

In summary, two approaches for allocating costs in the water resources field have been examined: 1) allocate total project cost in proportion to a physical or nonphysical criterion; or 2) allocate assignable costs directly and prorate the remaining costs by a physical or nonphysical criterion. In general, these two approaches are simple to apply and easy to understand. In fact, these two approaches are currently accepted cost allocation methods used in accounting (Kaplan, 1982). However, these two approaches are unable to consistently provide an equitable cost allocation when an equitable cost allocation exists, i.e., sometimes these methods work and sometimes they fail. Furthermore, methods attempting to assign costs directly may be influenced by the configuration of the facilities and may discourage cooperation when more than one configuration is being considered. This is particularly evident for our regional water network problem. For a theoretically sound method that is able to find an equitable cost allocation if one exists and is not influenced by the configuration of the facilities, concepts from cooperative n-person game theory are necessary.

Cooperative Game Theory

Game theory has been with us since 1944 when the first edition of The Theory of Games and Economic Behavior by John Von Neumann and Oskar Morgenstern appeared. In particular we are interested in games wherein all of the players voluntarily agree to cooperate because it is mutually beneficial. Furthermore, games are studied in three forms or levels of abstraction. The extensive form requires a complete description of the rules of a game and is generally characterized by a game tree to describe every player's move. A game in normal form condenses the description of a game into sets of strategies for each player and is represented by a game matrix. However, most efforts in cooperative game theory have been with games in characteristic function form whereby the description of a game is in terms of payoffs rather than rules or strategies. The characteristic function form appears to be the most appropriate for studying coalition formation which is an essential feature in cooperative games. Also, cooperative games can be of three types depending on whether the game is defined in terms of costs, savings, or values. To keep the notation as simple as possible, only cost games will be discussed. Introductory and intermediate material on cooperative game theory can be found in Schotter and Schwodiauer (1980), Jones (1980), Luce and Raiffa (1957), Lucas (1981), Rapoport (1970), Shubik (1982), and Owen (1982).

Concepts of Cooperative Game Theory

Let $N = \{1, 2, \dots, n\}$ be the set of players in the game. Associated with each subset of S players in N is a characteristic function c , which assigns a real number $c(S)$ to each nonempty subset of S players. For cost games, the characteristic function, $c(S)$, can be defined as the least cost or optimal solution for the S -member coalition if the $N-S$ member or complementary coalition is not present. However, depending on how the problem is defined, alternative definitions for $c(S)$ may be required. For example, Sorenson (1972) presents the following four alternative definitions for the characteristic cost function:

$c_1(S)$ = value to coalition if S is given preference over $N-S$.

$c_2(S)$ = value of coalition to S if $N-S$ is not present,

$c_3(S)$ = value of coalition in a strictly competitive game between coalition S and $N-S$, and

$c_4(S)$ = value of coalition to S if $N-S$ is given preference.

If $c(S)$ can be defined as the least cost solution for coalition S if $N-S$ is not present, then the cost game is naturally subadditive; i.e.,

$$c(S) + c(T) \geq c(S \cup T) \quad S \cap T = \emptyset, S, T \subset N \quad (4-6)$$

where $\emptyset$ is the empty set; and S and T are any two disjoint subsets of N . Subadditivity is a natural consequence of $c(S)$ because the worst S and T can do as a coalition is the cost of independent action; i.e.,

$$c(S) + c(T) = c(S \cup T) \quad S \cap T = \emptyset, S, T \subset N. \quad (4-7)$$

A coalition in which the players realize no savings from cooperation is said to be inessential.

General reasons why subadditivity exists are discussed by Sharkey (1982a). The primary reason why subadditivity exists for our regional water network problem is because of the economies of scale in pipeline construction cost. For a single output cost function, $C(q)$, economies of scale is defined by

$$C(\lambda q) \leq \lambda C(q) \quad (4-8)$$

where q = output level, and for all λ such that $1 < \lambda \leq 1 + \epsilon$, ϵ is a small positive number.

This definition means that the average costs are declining in the neighborhood of the output q . From Sharkey (1982a), economies of scale is sufficient but not a necessary condition for subadditivity. Subadditivity is a more general

condition which allows for both increasing marginal cost and increasing average cost over some range of outputs.

Solution concepts for cooperative cost games involve the following three general axioms of fairness (Heaney and Dickinson, 1982; Young et al., 1982):

1) Individual Rationality: Player i should not pay more than his go-it-alone cost, i.e.,

$$x(i) \leq c(i), \quad \forall i \in N, \quad (4-9)$$

where $x(i)$ is the allocated cost or the charge to player i .

2) Group Rationality: The total cost of the grand coalition, $c(N)$, must be apportioned among the N players; i.e.,

$$\sum_{i \in N} x(i) = c(N). \quad (4-10)$$

3) Subgroup Rationality: This final axiom extends the notion of individual rationality to include subgroups, i.e., no subgroup or subcoalition S should be apportioned a cost greater than its go-it-alone cost, or

$$\sum_{i \in S} x(i) \leq c(S), \quad \forall S \subset N. \quad (4-11)$$

The set of solutions or charges satisfying the first two axioms is called the set of imputations, while the

additional restriction of the third axiom defines what is known as the core of the game. For subadditive cost games the set of imputations is not empty, but the core may be empty. Shapley (1971) has shown that the core always exists for convex games. A cost game is convex if

$$c(S) + c(T) \geq c(S \cup T) + c(S \cap T) \quad S \cap T \neq \emptyset, \forall S, T \subseteq N \quad (4-12)$$

or equivalently, convexity can be written as

$$c(S \cup i) - c(S) \geq c(T \cup i) - c(T) \quad S \subset T \subseteq N - \{i\}, i \in N. \quad (4-13)$$

Convexity simply means the incremental cost for player i to join coalition T is less than or equal to the incremental cost for player i to join a subset of T . This notion of convexity is analogous to economies of scale and implies the game has a particular form of increasing returns to scale in coalition size. As will be shown, the more attractive the game, i.e., larger savings in project costs, the greater the chance that the game is convex; whereas, if the game is less attractive, i.e., lower savings in project costs, the potential for a nonconvex game or an empty core game is greater.

To illustrate the concept of the core, assume a three-person cost game with the following characteristic function values:

$$\begin{array}{lll}
 c(1) & = & 35 \\
 c(12) & = & 66 \\
 c(2) & = & 45 \\
 c(13) & = & 75 \\
 c(3) & = & 50 \\
 c(23) & = & 87 \\
 c(123) & = & 100
 \end{array}$$

This game is subadditive so each player has an incentive to cooperate; i.e.,

$$\begin{aligned}
 c(1) + c(2) + c(3) &\geq c(123) \\
 c(1) + c(23) &\geq c(123) \\
 c(2) + c(13) &\geq c(123) \\
 c(3) + c(12) &\geq c(123) \\
 c(1) + c(2) &\geq c(12) \\
 c(1) + c(3) &\geq c(13) \\
 c(2) + c(3) &\geq c(23).
 \end{aligned}$$

Furthermore, this game is convex; i.e.,

$$\begin{aligned}
 c(12) + c(13) &\geq c(123) + c(1) \\
 c(12) + c(23) &\geq c(123) + c(2) \\
 c(13) + c(23) &\geq c(123) + c(3).
 \end{aligned}$$

Using the three general axioms of fairness, the core conditions are as follows:

$$\begin{aligned}
 x(1) &\leq 35 \\
 x(2) &\leq 45 \\
 x(3) &\leq 50 \\
 x(1) + x(2) &\leq 66 \\
 x(1) + x(3) &\leq 75 \\
 x(2) + x(3) &\leq 87 \\
 x(1) + x(2) + x(3) &= 100.
 \end{aligned}$$

The first three conditions determine the upper bounds on $x(i)$, $i = 1, 2, 3$, while the last four conditions determine the lower bounds on $x(i)$, $i = 1, 2, 3$, i.e.,

$$c(123) - c(23) = 13 \leq x(1) \leq 35 = c(1)$$

$$c(123) - c(13) = 25 \leq x(2) \leq 45 = c(2)$$

$$c(123) - c(12) = 34 \leq x(3) \leq 50 = c(3)$$

For a three-person game, graphical examination of the core conditions and the nature of the charge vectors is possible using isometric graph paper (Heaney and Dickinson, 1982). As shown on Figure 4-2, each player is assigned a charge axis. The plane of triangle ABC, with vertices $(100, 0, 0)$, $(0, 100, 0)$, and $(0, 0, 100)$, represents points satisfying group rationality (axiom 2); whereas, the smaller triangle abc represents the set of imputations satisfying both individual rationality (axiom 1) and group rationality (axiom 2). The vertices a, b, and c represent the charge vectors: $[35, 15, 50]$, $[5, 45, 50]$, and $[35, 45, 20]$, respectively. Line ab represents the upper bound for player 3, i.e., $x(3) = c(3)$, where $c(123) - c(3)$ is allocated between players 1 and 2. As we move along line ab from point a to point b, the allocation to player 1 decreases from $c(1)$ to $c(123) - c(2) - c(3)$, i.e., from 35 to 5, while the allocation to player 2 increases from $c(123) - c(1) - c(3)$ to $c(2)$, i.e., from 15 to 45. Similar explanations can be given for lines bc and ac. A more restrictive set of solutions satisfying subgroup rationality (axiom 3),

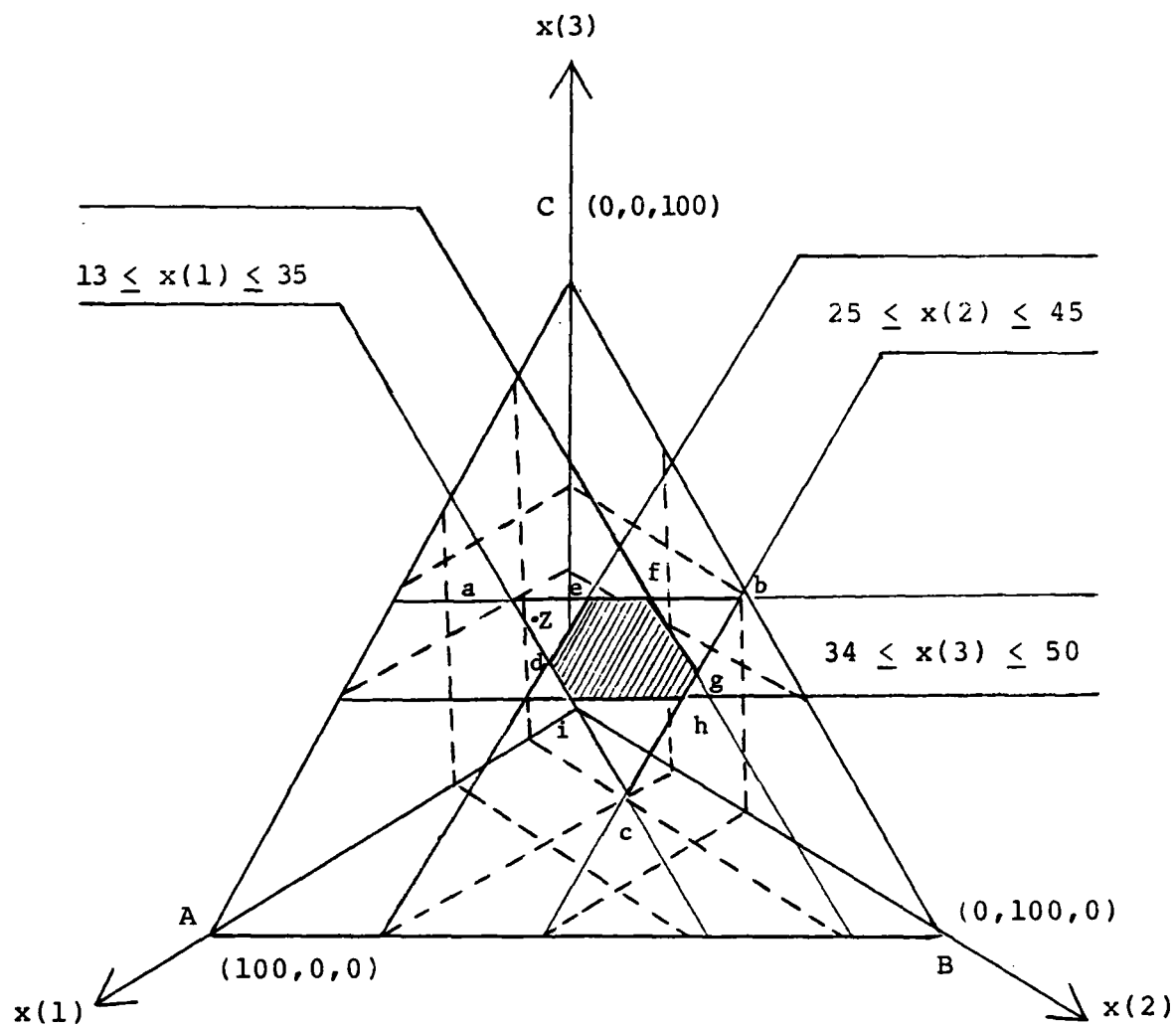


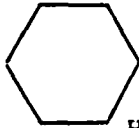
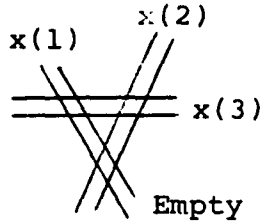
Figure 4-2. Geometry of Core Conditions for Three-Person Cost Game Example.

the shaded area on triangle abc, is the core for this game. The geometry of the core for this convex game is a hexagon. Line de represents the lower bound for player 2 or the set of charges where $c(13)$ is allocated between player 1 and player 3 with the remainder, $c(123) - c(13)$, going to player 2. Similar explanations can be given for lines fg and hi which are the lower bounds for players 1 and 3, respectively; and for lines id, gh, and ef which are the upper bounds for players 1, 2, and 3, respectively.

If an allocation lies outside the core, an inequitable situation prevails. For instance, point Z in Figure 4-2 allocates player 2 a cost less than its lower bound, $c(123) - c(13)$, which means $c(13)$ increases or the cost allocated to players 1 and 3 increases. Clearly, player 1 and player 3 can do better by forming their own two-person coalition rather than subsidizing player 2.

As mentioned earlier, the convexity of a game and its attractiveness are related. This relationship is illustrated in Table 4-10. When the costs for the two-person coalitions progressively decrease, there is less incentive for forming the grand coalition so the core becomes progressively smaller and the game becomes progressively more nonconvex. As a consequence of the core conditions for a three-person sub-additive cost game, a condition can be derived to determine if a core exists. From subgroup rationality and group rationality, we have the following conditions:

Table 4-10. Core Geometry for Three-Person Cost Game Example.

Characteristic Function $c(1) = 35, c(2) = 45,$ $c(3) = 50, c(123) = 100$			$\Sigma c(ij)$	Geometry of Core
$c(12)$	$c(13)$	$c(23)$		
66	75	87	228	 Hexagon
61	73	86	220	 Pentagon
59	71	85	215	 Trapezoid
58	70	80	208	 Triangle
56	68	76	200	 Point
55	65	72	192	 Empty

Source: Modified from Fischer and Gately, 1975, p. 27a.

$$\begin{aligned}
 x(1) + x(2) &\leq c(12) \\
 x(1) + x(3) &\leq c(13) \\
 x(2) + x(3) &\leq c(23) \\
 x(1) + x(2) + x(3) &= c(123)
 \end{aligned}
 \tag{4-14}$$

Summing the subgroup rationality conditions gives

$$2 \cdot [x(1) + x(2) + x(3)] \leq c(12) + c(13) + c(23). \tag{4-15}$$

If the group rationality conditions are substituted into the above equation, then we have the following condition to determine if a core exists:

$$2 \cdot c(123) \leq c(12) + c(13) + c(23). \tag{4-16}$$

Therefore, in Table 4-10, the core exists as long as the sum of the two-person coalitions is greater than 200 or twice the value of the grand coalition. When the sum of the two-person coalitions equals 200, the core reduces to a unique vector, i.e., $\bar{X} = [24, 32, 44]$. Finally, when the sum of the two-person coalition is less than 200, then the core is empty. Unfortunately, for larger games there is no simple condition for checking the existence of a core; however, as we will see later, a check can be made using linear programming.

Unique Solution Concepts

The three axioms of fairness defining the core of the game significantly reduce the set of admissible solutions. Unless the core is empty or is a unique vector, an infinite number of possible equitable charge vectors remain to be considered, so additional criteria are needed to select a unique charge vector. Numerous methods are available for selecting a unique charge vector; but the two most popular methods discussed in the literature are the Shapley value (Shapley, 1953; Heaney, 1983b; Shubik, 1962; Heaney et al., 1975; Littlechild, 1970) and the nucleolus (Schmeidler, 1969; Kohlberg, 1971; Suzuki and Nakayama, 1976).

Shapley value. The Shapley value for player i is defined as the expected incremental cost for the coalition of adding player i . Thus, each player pays a cost equal to the incremental cost incurred by the coalition when that player enters. Since the coalition formation sequence is unknown, the Shapley value assumes an equal probability for all sequences of coalition formation, i.e., the probability of each player being the first to join is equal, as are the probabilities of joining second, third, etc. For an n person game there are $n!$ orderings. The six sequences of coalition formation for a three-person game are as follows:

(123)	(213)	(231)
(132)	(312)	(321)

Therefore, the Shapley value or the cost to player 1 for a three-person game is

$$\begin{aligned}\phi(1) = & 1/3 c(1) + 1/6 [c(12) - c(2)] + 1/6 [c(13) - c(3)] \\ & + 1/3 [c(123) - c(23)].\end{aligned}\quad (4-17)$$

Player 1 has 1/3 probability of entering the coalition as the first player and 1/3 probability of entering the coalition as the last player. In addition, player 1 has 1/6 probability of entering the coalition after player 2 and 1/6 probability of entering the coalition after player 3.

Notice that $[c(S+i) - c(S)]$ is the incremental cost of adding player i to the S coalition.

The general formula for the Shapley value for player i is

$$\phi(i) = \sum_{S \subset N} \alpha_i(S) [c(S) - c(S - \{i\})] \quad (4-18)$$

where
$$\alpha_i(S) = \frac{(s-i)! (n-s)!}{n!}$$

s is the number of players in coalition S ,

$n!$ is the total number of possible sequences of coalition formation,

$(s-1)!$ is the number of arrangements for those players before S , and

$(n-1)!$ is the number of arrangements for those players after S .

For example, for $i = 1$, $n = 3$:

$$\alpha_1(1) = 0!2!/3! = 1/3$$

$$\alpha_1(12) = 1!1!/3! = 1/6$$

$$\alpha_1(13) = 1!1!/3! = 1/6$$

$$\alpha_1(123) = 2!0!/3! = \underline{1/3}$$

$$\text{Total} \quad 1.0$$

Note that

$$\sum_{i \in N} \phi(i) = c(N). \quad (4-19)$$

Furthermore, if the game is convex, the Shapley value lies in the center of the core (Shapley, 1971).

The Shapley value is criticized for several reasons. It may fall outside the core for nonconvex games, and it may be computed even when the core does not exist (Hamlen, 1980). Furthermore, the Shapley value is computationally burdensome for large games. For an n -person game, the Shapley value for each player requires the computation of 2^{n-1} coefficients and incremental costs. For example, an eight player game requires 128 coefficients and incremental costs to calculate the charge for each player.

Loehman and Whinston (1976) attempted to reduce the computational burden of the Shapley value by relaxing the assumption that all sequences of coalition formation are equally likely. This generalized Shapley value allows using a priori information to eliminate impossible sequences of coalition formation. Unfortunately, when Loehman et al. (1979) applied the generalized Shapley value to an eight-player regional wastewater management problem, they got a solution outside the core (Heaney, 1983a).

Littlechild and Owen (1973) developed the simplified Shapley value for games wherein the characteristic function is a cost function with the property that the cost of any subcoalition is equal to the cost of the largest player in the subcoalition. Although Littlechild and Thompson (1977) demonstrated the computational ease of the simplified Shapley value in their case study of airport landing fees consisting of 13,572 landings by 11 different types of aircraft, the use of the simplified Shapley value is restricted to games with these special properties.

Before calculating the Shapley value for our regional water network problem, the total enumeration procedure described in the preceding chapter is used to find the following characteristic cost function values (see Appendix A):

$$\begin{array}{lll} c(1) = 646,000 & c(2) = 2,420,095 & c(3) = 1,990,992 \\ c(12) = 2,928,511 & c(13) = 2,586,638 & c(23) = 3,984,177 \end{array}$$

and

$$c^1(123) = 4,556,409$$

$$c^2(123) = 4,556,826$$

$$c^3(1,23) = 4,630,177$$

$$c^4(12,3) = 4,919,503$$

$$c^5(13,2) = 5,006,734$$

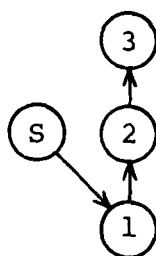
$$c^w(1,2,3) = 5,057,087$$

where $c^k(hi,j)$ is the cost of the k^{th} best regional water network consisting of pipelines from the well field to county h, from county h to county i, and from the well field to county j. Also, $c^w(1,2,3)$ is the cost for each county to go-it-alone. The cost allocation associated with the k^{th} best regional water network, i.e., the k^{th} network game, is simply found by setting $c(N)$ equal to $c^k(N)$.

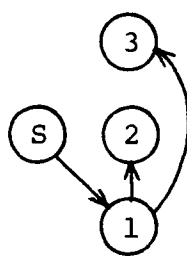
The Shapley values for all options available to the three counties are calculated in Appendix A and summarized in Table 4-11. Appendix A also checks whether each Shapley value satisfies core conditions. All of the network games in this example are nonconvex. Table 4-11 shows that the cost allocations for the optimal and the second best networks, i.e., the first two options, satisfy all core conditions; therefore, these cost allocations are in the core and are considered equitable. Furthermore, unlike the cost allocations using the direct costing methods discussed earlier, the cost allocations for these two comparable cost

Table 4-11. Cost Allocation for Three-County Example Using the Shapley Value.

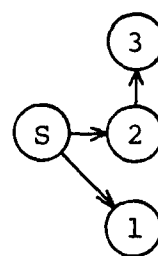
Option (Rank)	Cost Allocation to County i (\$)			$\Sigma x(i)$ (\$)	Is Cost Allocation In Core? (From Ap- pendix A)
	County 1 $x(1)$	County 2 $x(2)$	County 3 $x(3)$		
1	590,087	2,175,905	1,790,417	4,556,409	Yes
2	590,226	2,176,044	1,790,556	4,556,826	Yes
3	614,677	2,200,494	1,815,006	4,630,177	No
4	711,119	2,296,936	1,911,448	4,919,503	No
5	740,196	2,326,013	1,940,525	5,006,734	No
6	646,000	2,420,095	1,990,992	5,057,087	--



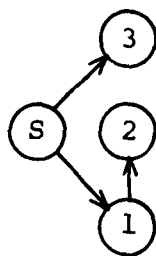
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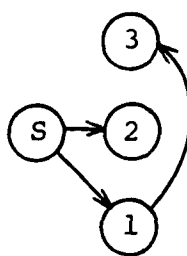
(2)



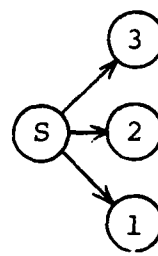
(3)



(4)



(5)



(6)

networks are nearly identical. The Shapley value divided the additional \$417 for the second best network equally among the counties. Option 3 illustrates the failure of the Shapley value to consistently give a core solution for nonconvex games. As shown in Appendix A, the cost allocation for option 3 fails to satisfy subgroup rationality for the coalition consisting of county 2 and county 3; i.e.,

$$x(2) + x(3) > c(23). \quad (4-20)$$

Moreover, options 4 and 5 illustrate Shapley values for games with an empty core. The nonexistence of the core for network games with options 4 and 5 can be determined by using other game theory methods, e.g., nucleolus. A close examination of the core conditions for network games with options 4 and 5 reveals these games are no longer subadditive. By defining $c(N)$ as $c^k(N)$, $c(N)$ is no longer the least cost or optimal solution for the grand coalition. Consequently, the k^{th} best network game is not naturally subadditive even though $c^k(N)$ may be less expensive than $c^w(N)$. In any event, because network games with options 4 and 5 are not subadditive, there is no incentive to cooperate. Therefore, options 4 and 5 no longer need to be considered by the counties.

Nucleolus. The other popular method to obtain a unique charge vector is to find the nucleolus. For a cost game,

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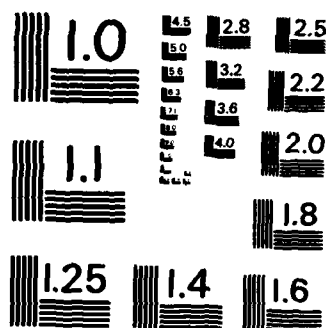
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the fairness criterion used by the nucleolus is based on finding the charge vector which maximizes the minimum savings of any coalition.

For each imputation in the core of a cost game, a vector in R^{2^n} is defined. The components of this vector are arranged in increasing order of magnitude and are defined by

$$e(S) = c(S) - \sum_{i \in S} x(i) \quad \forall S \subset N. \quad (4-21)$$

The imputation whose vector in R^{2^n} is lexicographically the largest is called the nucleolus of the cost game. Given two vectors, $\bar{X} = (x_1, \dots, x_n)$ and $\bar{Y} = (y_1, \dots, y_n)$, $\bar{X}$ is lexicographically larger than $\bar{Y}$ if there exists some integer k , $1 < k < n$, such that $x_j = y_j$ for $1 \leq j < k$ and $x_k > y_k$ (Owen, 1982). Basically, $e(S)$ represents the minimum savings of coalition S with respect to charge vector $\bar{X}$. Obviously, the coalition with the least savings objects to charge vector $\bar{X}$ most strongly, and the nucleolus maximizes this minimum savings over all coalitions.

The nucleolus can be found by solving at most $n-1$ linear programs (Kohlberg, 1972; Owen, 1974, 1982), where the first linear programming problem is

$$\begin{aligned} &\text{maximize } e(1) \\ &\text{subject to } e(1) + x(i) \leq c(i) \quad \forall i \in N \end{aligned} \quad (4-22)$$

$$e(1) + \sum_{i \in S} x(i) \leq c(S) \quad \forall S \subseteq N$$

$$\sum_{i \in N} x(i) = c(N)$$

$$x(i), e(1) \geq 0$$

The nucleolus is calculated by sequentially solving for $e(1)$, then $e(2)$, $e(3)$, etc., where $e(i)$ is the i^{th} smallest savings to any coalition.

Unlike the Shapley value, the nucleolus always is in the core for games with nonempty core. In fact, the nucleolus is always unique. However, the nucleolus is criticized because it cannot be written down in explicit form (Spinetto, 1975), and that it is difficult to compute and use in practice (Gugenheim, 1983). Probably the most difficult problem with using the nucleolus is the acceptance of its notion of fairness as opposed to other prevailing notions of fairness without generating unending controversies and debates. The nucleolus is generally considered to be analogous to Rawls' (1971) welfare criteria: the utility function of the least well off individual is maximized. Other notable notions of fairness include (1) Nozick's (1974) procedural approach to justice, and (2) Varian's (1975) or Baumol's (1982) definition of

equitable distribution whereby no one prefers the consumption bundle of anyone else.

Calculating the nucleolus for our regional water network problem using the linear programming problem (4-22) gives the results summarized in Table 4-12. Equitable cost allocations are given for the first three options, and the cost allocations for the optimal and the second best networks are essentially the same. The additional \$417 for the second best network is apportioned as follows:

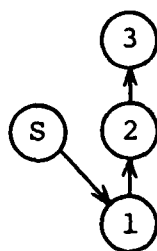
County 1	\$209
County 2	104
County 3	<u>104</u>
Total	\$417

No cost allocations are given for options 4 and 5 because these network games have empty cores. That is, the linear programming problem (4-22) is infeasible. Finally, Table 4-12 reveals that each of the three counties has an incentive to cooperate in order to implement the cheapest regional water network.

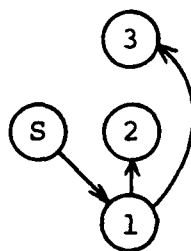
Propensity to disrupt. Another unique solution concept worth mentioning because of its intuitive appeal is the concept of an individual player's "propensity to disrupt." Gately (1974) defined an individual player i 's propensity to disrupt as a ratio of what the other players would lose if player i refused to cooperate over how much player i would lose by not cooperating. Mathematically, player i 's

Table 4-12. Cost Allocation for Three-County Example Using the Nucleolus.

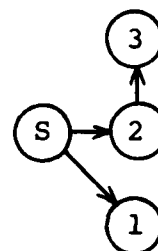
Option (Rank)	Cost Allocation to County i (\$)			$\Sigma x(i)$ (\$)	Is Cost Allocation In Core?
	County 1 $x(1)$	County 2 $x(2)$	County 3 $x(3)$		
1	609,116	2,144,583	1,802,710	4,556,409	Yes
2	609,325	2,144,687	1,802,814	4,556,826	Yes
3	646,000	2,163,025	1,821,152	4,630,177	Yes
4	--	--	--	--	--
5	--	--	--	--	--
6	646,000	2,420,095	1,990,992	5,057,087	--



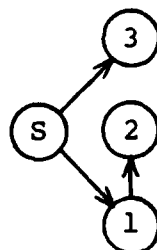
(1)



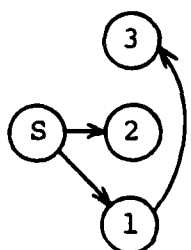
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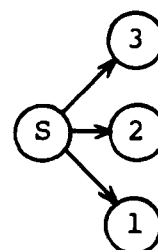
(3)



(4)



(5)



(6)

propensity to disrupt, $d(i)$, a charge vector, $\bar{X} = [x(1), \dots, x(n)]$, which is in the core is

$$d(i) = \frac{c(N-i) - \sum_{j \neq i} x(j)}{c(i) - x(i)} \quad (4-23)$$

The higher the propensity to disrupt, the greater a player's threat to the coalition; e.g., $d(i) = 10$ implies player i could impose a loss of savings to the other players 10 times as great as the loss of savings to player i . As an illustration, the propensity to disrupt is calculated for each of the counties using the nucleolus for the optimal network of our regional water network problem: $\bar{X} = [609,116; 2,144,583; 1,802,710]$.

$$d(1) = \frac{c(23) - x(2) - x(3)}{c(1) - x(1)} = 1.0$$

$$d(2) = \frac{c(13) - x(1) - x(3)}{c(2) - x(2)} = .63$$

$$d(3) = \frac{c(12) - x(1) - x(2)}{c(3) - x(3)} = .93$$

The calculations show that none of the counties have a strong threat against the other two counties with the nucleolus charge vector. County 1 could impose a loss to the other two counties which equals the loss imposed on itself, while, county 2's or county 3's departure would hurt the departing county more than it would hurt the remaining two counties.

Gately suggested equalizing each player's propensity to disrupt as a final cost allocation solution. Subsequently, Littlechild and Vaidya (1976) have generalized Gately's concept of an individual player's propensity to disrupt to include a coalition S 's propensity to disrupt. That is, a coalition S 's propensity to disrupt is defined as the ratio of what the complementary coalition, $N-S$, stands to lose over what the coalition S itself stands to lose for a given charge vector. More recently, Charnes et al. (1978) and Charnes and Golany (1983) refined these propensity to disrupt concepts into a unique solution concept which appears to have some empirical support. Finally, Straffin and Heaney (1981) have shown that Gately's propensity to disrupt is exactly the alternative cost avoided method first proposed during the TVA project in 1935. The alternative cost avoided method is discussed in the section on the separable costs, remaining benefits method.

Empty Core Solution Concepts

Examining games with an empty core is an active area of research. An empty core implies that no equitable cost allocation exists, and results from games wherein the additional savings from forming the grand coalition is relatively small. That is, the savings resulting from forming smaller coalitions are almost as much as the savings from forming the grand coalition. Therefore, proposed solution

concepts generally seek to relax the bounds on subgroup rationality until a "quasi" or "anti" core is created. Table 4-13 lists four methods for finding a charge vector for games with an empty core.

In any case, given the modest amount of economic gain for games with an empty core, it may be more advantageous to forego the grand coalition in favor of smaller coalition formations as suggested by Heaney (1983a). Furthermore, engineering projects tend to have a large proportion of the costs common to all participants; consequently, one would expect these games to be very attractive and games with an empty core to be fairly rare. Nevertheless, the game theory approach does alert us that a problem exists in allocating costs for such cases.

Cost Allocation in the Water Resources Field

Straffin and Heaney (1981) showed that the criteria of fairness as expressed by equations (4-1) and (4-2) associated with cost allocation proposed by the TVA experts in the 1930's paralleled the development of the concepts of individual and subgroup rationality found in cooperative game theory. Given that full costs have to be recovered, the core conditions are equivalent to the fairness criteria for allocating cost originally proposed by the TVA experts. Therefore, current practice for allocating costs in the water resources field should require the solution be in the core of a game.

Table 4-13. Empty Core Solution Methods.

Method	Approach	Source
1. Least Core or β -Core	Relax $c(S)$	Shapley and Shubik, 1973; Young et al., 1982; Williams, 1982, 1983
2. Weak Least Core or α -Core	Relax $c(S)$	Shapley and Shubik, 1973; Young et al., 1982; Williams, 1982, 1983
3. Minimum Cost, Remaining Savings	Relax $c(S)$	Heaney and Dickinson, 1982
4. Homocore	Relax $c(N)$	Charnes and Golany, 1983

Separable Costs, Remaining Benefits Method

As discussed previously, ad hoc methods generally used in the water resources field allocate certain costs that are considered identifiable costs directly, and prorate the remaining costs by some criterion. The primary difference among these ad hoc methods is how identifiable costs are defined. We have already shown that defining identifiable costs as either zero or direct costs does not insure an equitable or core solution. We will now discuss several methods whereby identifiable costs are defined as separable costs.

The recommended cost allocation method in the water resources field in the United States is the separable costs, remaining benefits (SCRB) method (Federal Inter-Agency River Basin Committee, 1950; Loughlin, 1977, 1978; Rossman, 1978; Heggen, 1980; Goodman, 1984). This method assigns each individual (or purpose) in a joint venture its separable costs and a share of the remaining costs in proportion to the remaining benefit, i.e., the minimum of the benefit or alternative cost less separable costs. Separable costs for individual i are defined as the difference between the cost of the joint venture with and without individual i . Separable costs include both the direct cost attributable to the entering individual and the incremental costs associated with a larger project because of the inclusion of another individual. Mathematically, separable costs are

$$sc(i) = c(N) - c(N-i) \quad \forall i \in N \quad (4-24)$$

where $sc(i)$ = separable costs to individual i ,
 $c(N)$ = least cost system associated with group N , and
 $c(N-i)$ = least cost system associated with subgroup $N-i$.

After the separable costs for each individual have been allocated, the remaining costs to be assigned are called nonseparable costs (ncs), or

$$nsc = c(N) - \sum_{i \in N} sc(i). \quad (4-25)$$

For the SCRB method the nonseparable costs are prorated on the basis of the remaining benefits; therefore, this prorated share is

$$\beta(i) = \frac{[\min [b(i), c(i)] - sc(i)]}{\sum_{i \in N} \{\min [b(i), c(i)] - sc(i)\}} \quad (4-26)$$

where $b(i)$ = benefit of individual i
 $c(i)$ = alternative cost of individual i if i acts independently, and
 $\beta(i)$ = prorating factor for the SCRB method.

The total charge to the i^{th} individual is

$$x(i) = sc(i) + \beta(i) \cdot nsc. \quad (4-27)$$

The total charge, $c(N)$, is

$$c(N) = \sum x(i) = \sum sc(i) + \sum \beta(i) \cdot nsc \quad (4-28)$$

where $\sum \beta(i) = 1.0$.

The alternative justifiable expenditure method and the alternative cost avoided method are two variants of the SCRB method frequently mentioned in the water resources literature. The alternative justifiable expenditure method is recommended when data are not available for separable costs. Then, separable costs are defined as direct costs, and the nonseparable costs equal total project cost less the sum of all direct costs and are distributed in proportion to the remaining benefits. The alternative justifiable expenditure method is equivalent to the Louderback-Moriarity method recently proposed by Balachandran and Ramakrishnan (1981) in the accounting literature. The alternative cost avoided method is equivalent to Gately's propensity to disrupt with separable cost defined as before, but the nonseparable costs are distributed in proportion to the

alternative cost avoided, i.e., the difference between the alternative cost for the single purpose project and the separable costs.

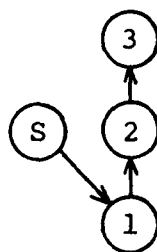
The SCRB method is now applied to our regional water network problem. The SCRB calculations are in Appendix A, and the results are summarized in Table 4-14. Like the nucleolus, the SCRB method gives equitable cost allocations for the first three options, and the three counties have an incentive to cooperate to implement the cheapest regional water network. Furthermore, Appendix A shows the SCRB method gives cost allocations for games without a core, i.e., solutions for network games for options 4 and 5 just like the Shapley value. Again, the cost allocations for the optimal network and the second best network are essentially the same. The additional \$417 for the second best network is allocated as follows:

County 1	\$211
County 2	89
County 3	<u>117</u>
Total	\$417

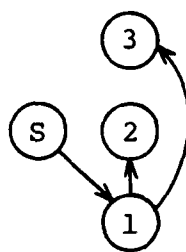
For our regional water network problem, each county's and each subgroup of counties' alternative cost of independent action is less than or equal to each county's and each subgroup of counties' benefits; therefore, the SCRB method is identical to the alternative cost avoided method and Gately's propensity to disrupt method. For example, if

Table 4-14. Cost Allocation for Three-County Example Using the SCRB Method.

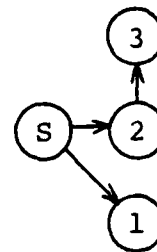
Option (Rank)	Cost Allocation to County i (\$)			$\Sigma x(i)$ (\$)	Is Cost Allocation In Core? (From Ap- pendix A)
	County 1 $x(1)$	County 2 $x(2)$	County 3 $x(3)$		
1	604,369	2,165,958	1,786,082	4,556,409	Yes
2	604,580	2,166,047	1,786,199	4,556,826	Yes
3	646,000	2,178,677	1,805,500	4,630,177	Yes
4	--	--	--	--	--
5	--	--	--	--	--
6	646,000	2,420,095	1,990,992	5,057,087	--



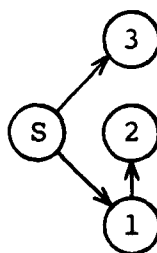
(1)



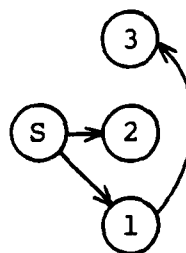
(2)



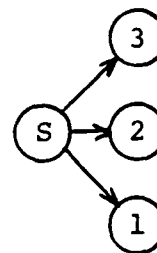
(3)



(4)



(5)



(6)

the SCRB charge vector for the optimal network is used: $\bar{X} = [604,369; 21,165,958; 1,786,082]$, the propensity to disrupt is $d(i) = .77$, $i = 1,2,3$. Since $d(i) < 1$, $i = 1,2,3$, each county is only a weak threat to the other two counties.

Minimum Costs, Remaining Savings Method

Although the SCRB method is recommended practice, it ignores subgroup rationality for projects with more than three individuals (Giglio and Wrightington, 1972; Young et al., 1982). The SCRB method only considers information on coalitions of size 1, $(N-1)$, and N . As a result, the SCRB solution may be outside the core. Recently, Heaney and Dickinson (1982) showed the SCRB method may use infeasible upper bounds for apportioning the nonseparable costs in addition to ignoring information on subgroup rationality. As an improvement they proposed the minimum costs, remaining savings (MCRS) method as a generalized SCRB method. With the inclusion of all available information on subgroup rationality, the MCRS method uses linear programming to determine the minimum and maximum feasible costs for each individual. The feasible costs are then used as bounds to prorate the nonseparable costs just like the SCRB method; however, the feasible bounds now ensure the core conditions are met if the core is not empty. If the game is convex, the MCRS and the SCRB methods are identical.

Mathematically, the MCRS method can be stated as

$$x(i) = x(i)_{\min} + \beta(i) \cdot nsc \quad (4-29)$$

$$\text{where } nsc = c(N) - \sum_{i \in N} x(i)_{\min}$$

$$\beta(i) = \frac{[x(i)_{\max} - x(i)_{\min}]}{\left\{ \sum_{i \in N} [x(i)_{\max} - x(i)_{\min}] \right\}}$$

and $x(i)_{\min}$ and $x(i)_{\max}$ are found by solving the following 2n linear programs:

$$\begin{aligned} & \max \text{ or } \min \quad x(i) \\ & \text{subject to} \quad x(i) \leq c(i) \quad \forall i \in N \quad (4-30) \\ & \quad \quad \quad \sum_{i \in S} x(i) \leq c(S) \quad \forall S \subset N \\ & \quad \quad \quad \sum_{i \in N} x(i) = c(N) \\ & \quad \quad \quad x(i) \geq 0 \quad \forall i \in N \end{aligned}$$

In addition, the MCRS method can be used to determine if a game has a core by checking whether the linear programming problem (4-30) is feasible. If the linear programming problem (4-30) is infeasible, the game has an empty core. To find a unique charge vector to games with an empty core

using the MCRS method, the bounds or characteristic cost function values for the S-member coalitions are relaxed until a core appears. This procedure can be formulated by the following linear programming problem:

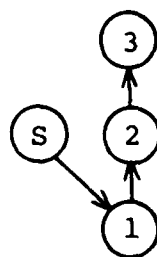
$$\begin{aligned}
 & \text{minimize } \theta \\
 & \text{subject to } x(i) \leq c(i) \quad \forall i \in N \\
 & \sum_{i \in S} x(i) - \theta c(S) \leq c(S) \quad \forall S \subset N \quad (4-31) \\
 & \sum_{i \in N} x(i) = c(N) \\
 & x(i) \geq 0
 \end{aligned}$$

The MCRS method is now applied to our regional water network problem. The calculations are contained in Appendix A and the results are summarized in Table 4-15. Table 4-15 shows equitable cost allocations for the first three options. Again, the cost allocations for the optimal network and the second best network are essentially the same. The additional \$417 for the second best network is allocated as follows:

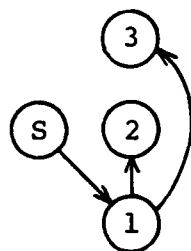
County 1	\$217
County 2	1
County 3	<u>199</u>
Total	\$417

Table 4-15. Cost Allocation for Three-County Example Using the MCRS Method.

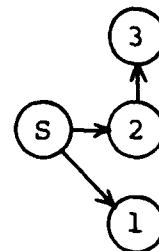
Option (Rank)	Cost Allocation to County i (\$)			$\Sigma x(i)$ (\$)	Is Cost Allocation In Core? (From Ap- pendix A)
	County 1 $x(1)$	County 2 $x(2)$	County 3 $x(3)$		
1	606,861	2,151,206	1,798,342	4,556,409	Yes
2	607,078	2,151,207	1,798,541	4,556,826	Yes
3	646,000	2,163,025	1,821,152	4,630,177	Yes
4	--	--	--	--	--
5	--	--	--	--	--
6	646,000	2,420,095	1,990,992	5,057,087	--



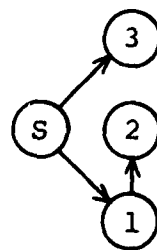
(1)



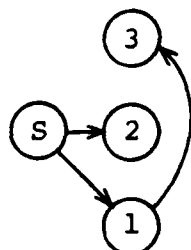
(2)



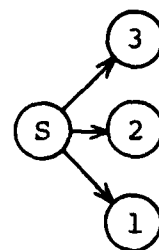
(3)



(4)



(5)



(6)

Furthermore, the MCRS cost allocations in Table 4-15 encourage the counties to cooperate to construct the cheapest regional water network possible because the cost for each county progressively decreases with decreasing total project cost.

Since none of the network games are convex, the MCRS solutions are different from the SCRB solutions because the MCRS method uses actual core bounds rather than nominal core bounds to apportion the nonseparable cost. For instance, the actual core bounds and the nominal core bounds for the optimal network game, shown in Table 4-16, illustrate the major difference between the MCRS method and the SCRB method. That is, the SCRB method distorts the allocation of the nonseparable cost by using infeasible bounds.

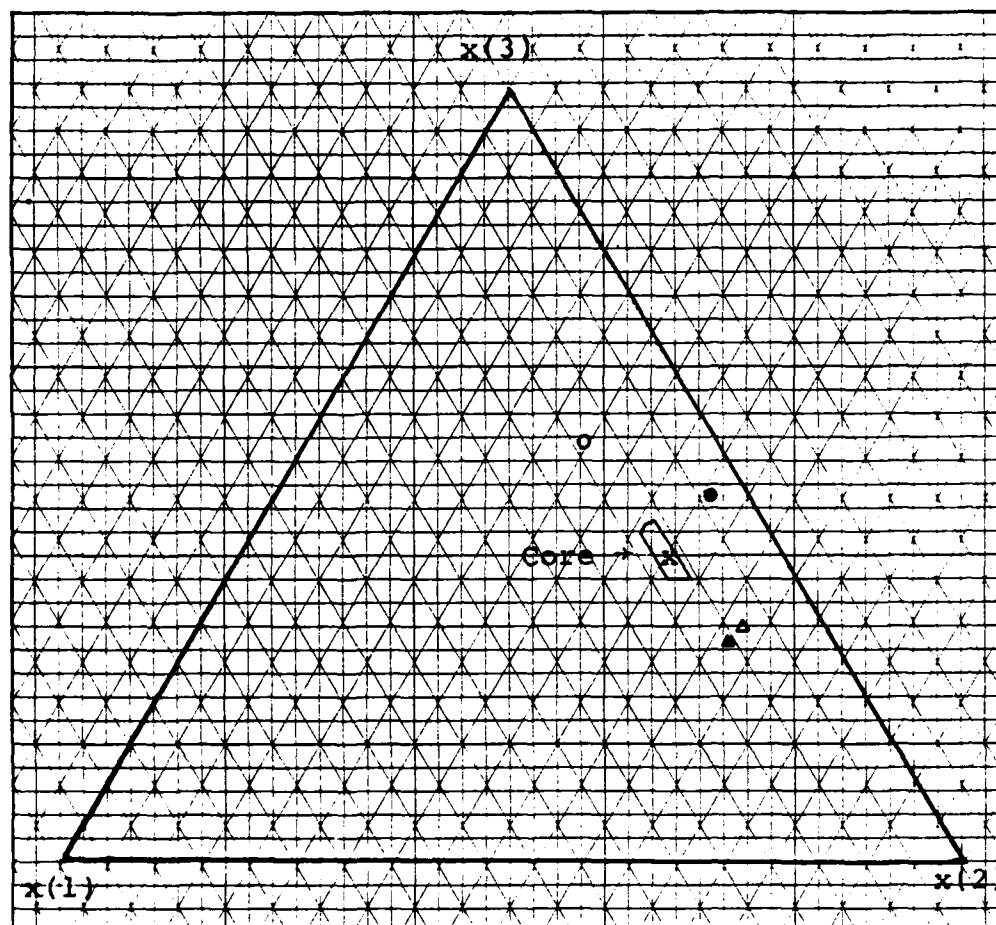
Allocating Cost Using Game Theory Concepts

The k^{th} Best System

The cores for the optimal network game and the second best network game of our regional water network problem is shown in Figures 4-3 and 4-4 along with charge vectors for some of the cost allocation methods we discussed. By comparing Figures 4-3 and 4-4, we can see summarized in Table 4-17 that the SCRB and the game theory methods not only give equitable cost allocations for the two comparable cost networks, but each of these methods also gives almost

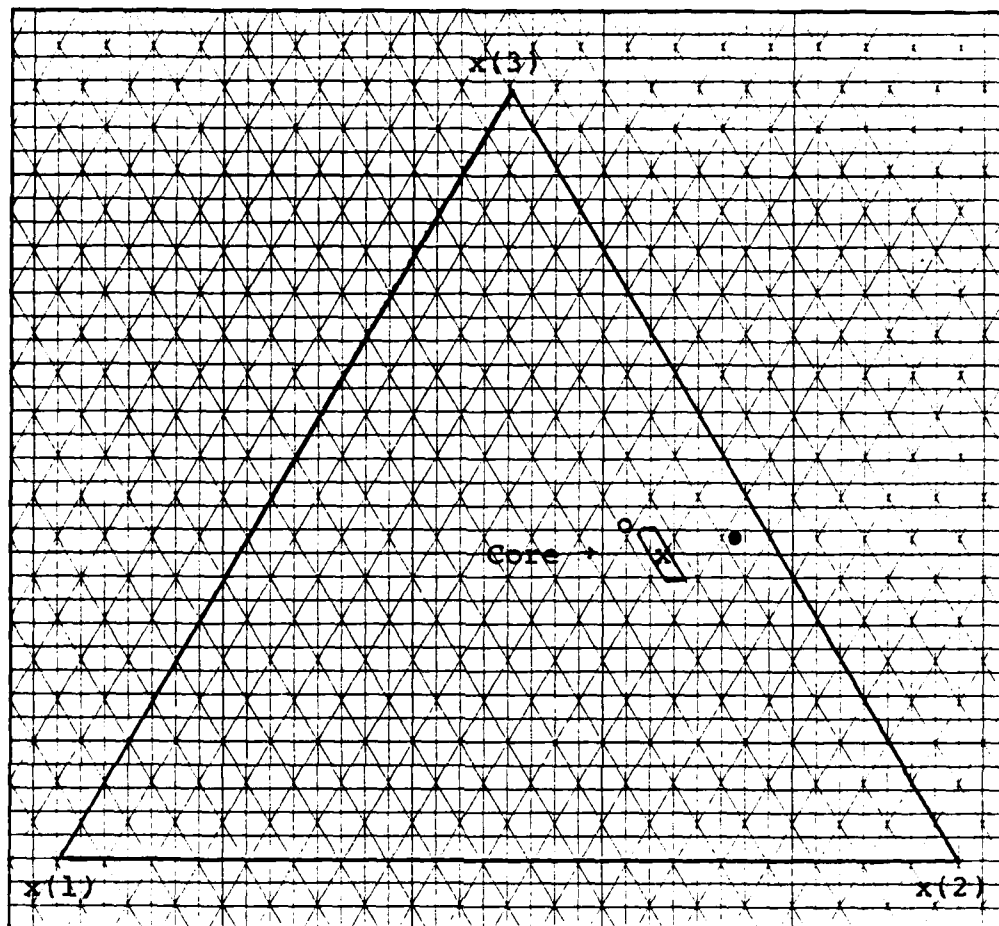
Table 4-16. Nominal Versus Actual Core Bounds for Optimal Network Game.

Nominal Core Bounds		
Player i	Lower Bound = $c(N) - c(N-1)$	Upper Bound = $c(i)$
1	572,232	646,000
2	1,969,771	2,420,095
3	1,627,898	1,990,992
Actual Core Bounds		
Player i	Lower Bound From LP (4-30) with Objective Function Min $x(i)$	Upper Bound From LP (4-30) with Objective Function Max $x(i)$
1	572,232	646,000
2	1,969,771	2,356,279
3	1,627,898	1,990,992



- ▲ Proportional to Population
- △ Proportional to Demand
- Use of Facilities
- Direct Costing/Equal Apportionment of Remaining Costs
- x Shapley Value, Nucleolus, SCRB (Gately's Propensity to Disrupt), MCRS

Figure 4-3. Core for the Optimal Network Game
($C(N) = \$4,556,409$).



- Use of Facilities
- Direct Costing/Equal Apportionment of Remaining Costs
- x Shapley Value, Nucleolus, SCRB (Gately's Propensity to Disrupt), MCRS

Figure 4-4. Core for the Second Best Network Game
($C(N) = \$4,556,826$).

Table 4-17. Cost Allocations for the Optimal Network and the Second Best Network (\$).

Method	Network	County 1	County 2	County 3
Shapley	1. Optimal	590,087	2,175,905	1,790,417
	2. Second Best	590,226	2,176,044	1,790,556
	Difference (2 - 1)	139	139	139
Nucleolus	1. Optimal	609,116	2,144,583	1,802,710
	2. Second Best	609,325	2,144,687	1,802,814
	Difference (2 - 1)	209	104	104
SCRB (Gately's Propensity to Disrupt)	1. Optimal	604,369	2,165,958	1,786,082
	2. Second Best	604,580	2,166,047	1,786,199
	Difference (2 - 1)	211	89	117
MCRS	1. Optimal	606,861	2,151,206	1,798,342
	2. Second Best	607,078	2,151,207	1,798,541
	Difference (2 - 1)	217	1	199

identical cost allocations for the two comparable cost networks.

The game theory approach is able to give equitable solutions for the two comparable cost networks because the cost of independent action for each county and each subgroup of counties is recognized by the core conditions, i.e., the values of the characteristic cost function. Furthermore, the game theory approach is able to give nearly identical cost allocations for the two comparable cost networks because the core conditions for the two comparable cost networks are essentially the same except for the group rationality condition or the value of $c(N)$, i.e.,

Core Conditions for the Optimal
Network Game

$$\begin{aligned} x(1) &\leq 646,000 \\ x(2) &\leq 2,420,095 \\ x(3) &\leq 1,990,992 \\ x(1) + x(2) &\leq 2,928,511 \\ x(1) + x(3) &\leq 2,586,638 \\ x(2) + x(3) &\leq 3,984,177 \\ x(1) + x(2) + x(3) &= 4,556,409 \end{aligned}$$

Core Conditions for the Second
Best Network Game

$$\begin{aligned} x(1) &\leq 646,000 \\ x(2) &\leq 2,420,095 \\ x(3) &\leq 1,990,992 \end{aligned}$$

$$\begin{aligned}
 x(1) + x(2) &\leq 2,928,511 \\
 x(1) + x(3) &\leq 2,586,638 \\
 x(2) + x(3) &\leq 3,984,177 \\
 x(1) + x(2) + x(3) &= 4,556,826
 \end{aligned}$$

The second best network naturally has a higher value for $c(N)$, i.e., $c^1(N) < c^2(N)$; consequently, the second best network game is less attractive than the optimal network game in terms of cost. Therefore, the core of the second best network game is naturally smaller and more nonconvex than the core of the optimal network game, but this "reduced core" is a subset of the core for the optimal network game. This is shown by examining the following actual core bounds for the two comparable cost network games:

Actual Core bounds for Optimal Network

$$\begin{aligned}
 572,232 &\leq x(1) \leq 646,000 \\
 1,969,771 &\leq x(2) \leq 2,356,279 \\
 1,627,898 &\leq x(3) \leq 1,990,992
 \end{aligned}$$

Actual Core Bounds for Second Best Network

$$\begin{aligned}
 572,649 &\leq x(1) \leq 646,000 \\
 1,970,188 &\leq x(2) \leq 2,355,862 \\
 1,628,315 &\leq x(3) \leq 1,990,992
 \end{aligned}$$

The core for the second best network has slightly higher lower bounds and slightly lower upper bounds compared with the core for the optimal network.

This natural reduction in the core as $c(N)$ increases from $c^{k-1}(N)$ to $c^k(N)$ is also shown in Figure 4-5 by examining nominal core bounds. Since the individual rationality core conditions are identical for the $(k-1)^{th}$ best network game and the k^{th} best network game, the nominal upper core bounds (NUB) for both of these network games are identical, i.e., $NUB^{k-1}(i) = NUB^k(i) = c(i)$, $\forall i \in N$. Furthermore, since the subgroup rationality core conditions are identical for the $(k-1)^{th}$ best network game and the k^{th} best network game, the values of $c(N-i)$, $\forall i \in N$, for both of these network games are identical. Consequently, as $c(N)$ increases from $c^{k-1}(N)$ to $c^k(N)$, the nominal lower core bounds (NLB), i.e., $c(N) - c(N-i)$, are increasing by the same value for all individual i . Thus, Figure 4-5 shows that the increasing values of the nominal lower core bounds as $c(N)$ increases from $c^{k-1}(N)$ to $c^k(N)$ is responsible for the reduction in the core.

The financial viability of the k^{th} best system can now be determined. As the value of $c(N)$ increases progressively for the k^{th} best system, the core progressively reduces until the core possibly becomes empty. Therefore, all k^{th} best systems associated with games with a core can be considered financially viable since an equitable cost allocation can be found. However, for all k^{th} best systems associated with games with an empty core, either an empty core cost allocation procedure is necessary, or these

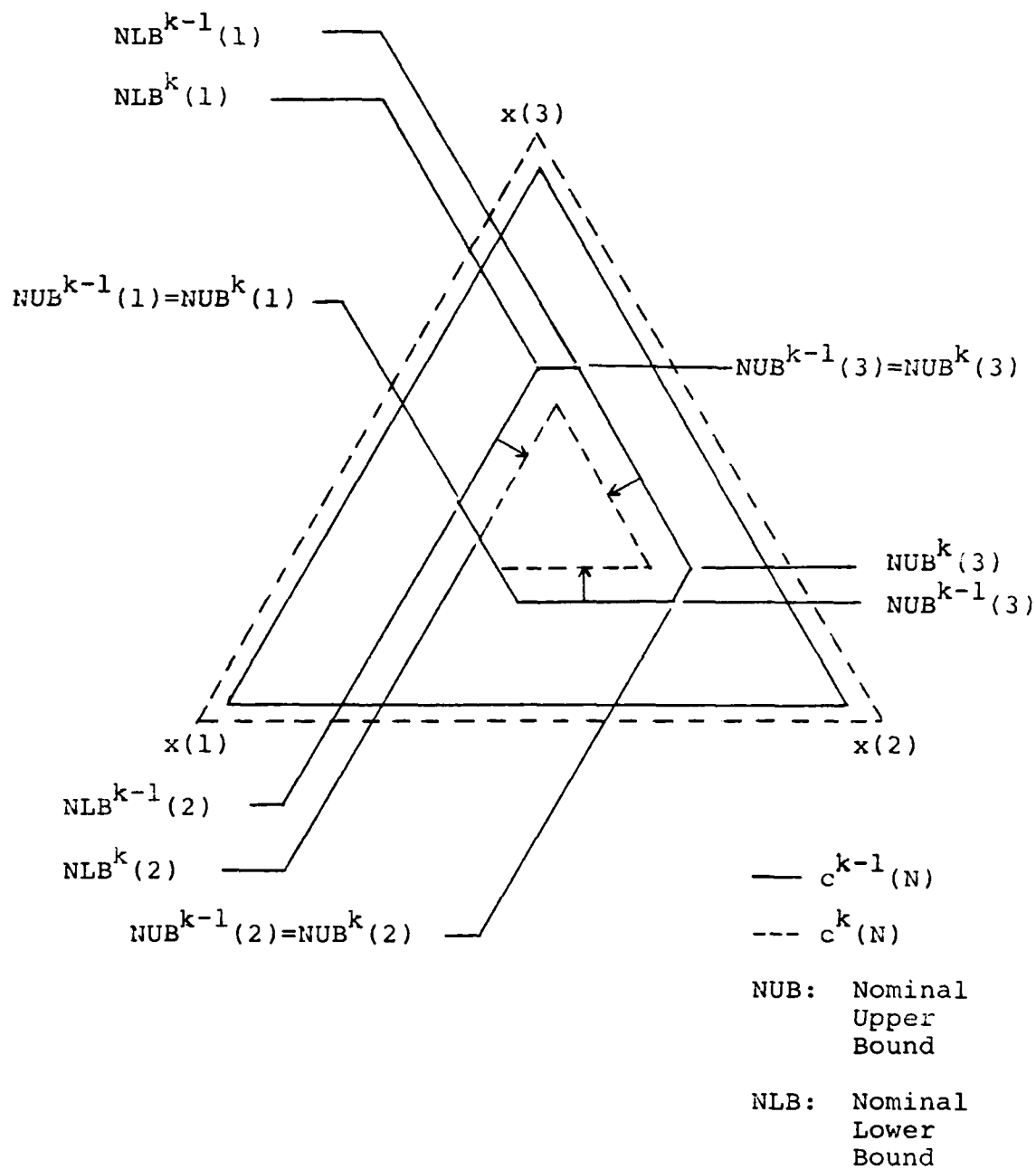


Figure 4-5. Reduction in Core as $c(N)$ Increases from $c^{k-1}(N)$ to $c^k(N)$.

systems should not be considered because of the minimal economic gain or the loss of subadditivity.

The Dummy Player

In allocating the cost for option 3 (see Table 4-2) of our regional water network problem, there may be a temptation to simply treat this network as a two-person game involving counties 2 and 3 rather than a three-person game that also includes county 1. The cost allocation for two-person games is very simple. As shown by Heaney and Dickinson (1982), the core for a two-person game is a line and always exists. Furthermore, the two-person game is always convex, so the SCRB solution and the MCRS solution are identical and are located at the center of the core.

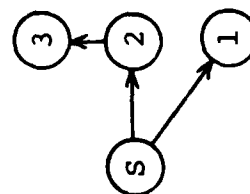
If option 3 is treated as a two-person game, the cost allocation is trivial. County 1 simply pays its go-it-alone cost while counties 2 and 3 share the saving equally between themselves (see calculations in Table 4-18), i.e.,

County 1	\$ 646,000
County 2	2,206,640
County 3	<u>1,777,537</u>
	\$4,630,177

Although this cost allocation satisfies core conditions regardless of whether option 3 is treated as a two-person game or a three-person game, there is a considerable difference between this solution and the solutions by the SCRB and

Table 4-18. Cost Allocation for Option 3 as a Two-Person Game Using the SCRB Method.

County i	$sc(i)=c(N)-c(N-i)$	$\beta(i)$	$nsc=c(N) - \sum sc(i)$	Allocated Cost (\$) $x(i)=sc(i)+\beta(i) \cdot nsc$
2	1,993,185	1/2	426,910	2,206,640
3	1,564,082	1/2	426,910	1,777,537



$$c(2) = 2,420,095$$

$$c(3) = 1,990,992$$

$$c(23) = 3,984,177$$

the MCRS methods when option 3 is treated as a three-person game. This is shown in Table 4-19. Whether option 3 can be properly treated as a two-person game is important to know since county 2 and county 3 face substantially different costs.

To determine whether county 1 can be excluded in the cost allocation for option 3, the core for option 3 as a three-person game is examined. The core conditions for option 3 as a three-person game are as follows:

$$\begin{array}{ll}
 \text{L1:} & x(1) \leq 646,000 \\
 \text{L2:} & x(2) \leq 2,420,095 \\
 \text{L3:} & x(3) \leq 1,990,992 \\
 \text{L4:} & x(1) + x(2) \leq 2,928,511 \\
 \text{L5:} & x(1) + x(3) \leq 2,586,638 \\
 \text{L6:} & x(2) + x(3) \leq 3,984,177 \\
 \text{L7:} & x(1) + x(2) + x(3) = 4,630,177
 \end{array}$$

The core bounds for this three-person game are found by using linear programming problem (4-30). As expected, Table 4-20 shows county 1 simply pays its go-it-alone cost, i.e., \$646,000. However, Table 4-20 indicates core constraint L4 is binding for $x(2)_{\max}$ and core constraint L5 is binding for $x(3)_{\max}$. Therefore, constraints L4 and L5 involving subcoalitions with county 1 are essential for determining the core for option 3 as a three-person game. Consequently, county 1's participation cannot be ignored when

Table 4-19. Comparing Cost Allocations for Option 3 as Two-Person Game and Three-Person Game Using the SCRB and MCRS Methods.

Approach (Method)	Cost Allocation to County i (\$)			Is Cost Allocation in Core of Two- Person Game?	Is Cost Allocation in Core of Three- Person Game?
	County 1 x(1)	County 2 x(2)	County 3 x(3)		
Two-Person (SCRB or MCRS)	646,000	2,206,640	1,777,537	Yes	Yes
Three-Person (SCRB)	646,000	2,178,677	1,805,500	Yes	Yes
Three-Person (MCRS)	646,000	2,163,025	1,821,152	Yes	Yes

Table 4-20. Core Bounds for Option 3 as a Three-Person Game.

Core Bound	Shadow Price for Core Constraint from LP (4-30)						
	L1	L2	L3	L4	L5	L6	L7
$x(1)_{\max} = 646,000$	+1						
$x(2)_{\max} = 2,282,511$				+1		+1	-1
$x(3)_{\max} = 1,940,638$					+1	+1	-1
$x(1)_{\min} = 646,000$						-1	+1
$x(2)_{\min} = 2,043,539$					-1		+1
$x(3)_{\min} = 1,701,666$				-1			+1

determining the cost allocation for counties 2 and 3 for option 3.

Suppose county 1 is a dummy player who contributes no savings to any coalition. This means subcoalitions (12) and (13) are inessential; i.e.,

$$c(12) = c(1) + c(2) = 3,066,095, \text{ and}$$

$$c(13) = c(1) + c(3) = 2,636,922.$$

If county 1 is a dummy player, the core conditions for option 3 as a three-person game can be rewritten as follows:

$$L1: \quad x(1) \leq 646,000$$

$$L2: \quad x(2) \leq 2,420,095$$

$$L3: \quad x(3) \leq 1,990,992$$

$$L4: \quad x(1) + x(2) \leq 3,066,095$$

$$L5: \quad x(1) + x(3) \leq 2,636,992$$

$$L6: \quad x(2) + x(3) \leq 3,984,177$$

$$L7: \quad x(1) + x(2) + x(3) = 4,630,177$$

Table 4-21 shows the core bounds for this game by solving linear programming problem (4-30). Again, county 1 simply pays its go-it-alone cost. However, Table 4-21 indicates core constraints L4 and L5 involving subcoalitions with county 1 are no longer binding for $x(2)_{\max}$ and $x(3)_{\max}$, respectively. The binding core constraint for $x(2)_{\max}$ is L2 and for $x(3)_{\max}$ is L3. Although Table 4-21 indicates core

Table 4-21. Core Bounds for Option 3 as a Three-Person Game with County 1 as a Dummy Player.

Core Bound	Shadow Price for Core Constraint from LP (4-30)						
	L1	L2	L3	L4	L5	L6	L7
$x(1)_{\max} = 646,000$	+1						
$x(2)_{\max} = 2,420,095$		+1					
$x(3)_{\max} = 1,990,992$			+1				
$x(1)_{\min} = 646,000$						-1	+1
$x(2)_{\min} = 1,993,185$					-1		+1
$x(3)_{\min} = 1,564,082$				-1			+1

constraints L4 and L5 are binding for $x(2)_{\min}$ and $x(3)_{\min}$, respectively, L4 and L5 are identical to L2 and L3, respectively, because $x(1) = 646,000$. Consequently, when county 1 is a dummy player, the three-person game can be reduced to a two-person game with the following core conditions:

$$L2: \quad x(2) \leq 2,420,095$$

$$L3: \quad x(3) \leq 1,990,992$$

$$L6: \quad x(2) + x(3) = 3,984,177$$

Comparing Table 4-21 and Table 4-22 reinforces this result because the core bounds for the two-person game are identical to the core bounds for the three-person game with county 1 as a dummy player.

In summary, an n -person game can be reduced to an $(n-1)$ -person game only if the player removed from the game is a dummy player, i.e., a player who contributes no savings to any coalition. Otherwise, the cost allocation may be distorted even if it satisfies the core conditions for both the n -person game and the $(n-1)$ -person game.

Comparing Methods

The methods we discussed are compared in Table 4-23. For any particular problem, any of these methods may successfully find equitable cost allocation. However, the most suitable method for allocating cost in the water resources field appears to be the MCRS method. Although the game theory and the SCRB methods all give equitable cost

Table 4-22. Core Bounds for Option 3 as a Two-Person Game.

Core Bound	Shadow Price for Core Constraint from LP (4-30)		
	L2	L3	L6
$x(2)_{\max} = 2,420,095$	+1		
$x(3)_{\max} = 1,990,992$		+1	
$x(2)_{\min} = 1,993,185$		-1	+1
$x(3)_{\min} = 1,564,082$	-1		+1

Table 4-23. Comparison of Methods Discussed for Allocating Costs of Water Resources Projects.

Feature	Method					MCRS
	Proportionality	Direct Costing	Shapley Value	Nucleolus	SCRB	
1. Will always get core solution for convex game	No	No	Yes	Yes	Yes	Yes
2. Will always get core solution for nonconvex game	No	No	No	Yes	No	Yes
3. Applicable to empty core game	No	No	No	No	No	Yes
4. Can determine if core exists	No	No	No	Yes	No	Yes
5. Calculations easy for one or more systems	Yes	Yes	No	No	Yes	Yes
6. Independent of system configuration	Yes	No	Yes	Yes	Yes	Yes
7. Similar to methods used in water resources field	Yes	Yes	No	No	Yes	Yes
8. Easy to understand	Yes	Yes	Yes	No	Yes	Yes
9. Currently accepted accounting method	Yes	Yes	No	No	No	No
Total Number of Yes	5	4	3	4	5	8

allocation if the game is convex, this convexity check may be burdensome for large games. In any case, both the Shapley value and the SCRB method may break down when the game is nonconvex. This is especially undesirable when suboptimal systems are evaluated since these games will naturally increase in nonconvexity with increasing value of $c(N)$. In contrast, both the nucleolus and the MCRS methods give core solutions regardless of the degree of nonconvexity of a game. However, the MCRS method has an advantage over the nucleolus method in the water resources field because the MCRS method extends the presently recommended SCRB method. This avoids controversies over the acceptance of a different fairness criterion with using the nucleolus method. Moreover, while both the MCRS and nucleolus methods require solving multiple linear programming problems, the MCRS method is much easier to solve. The constraint sets for each of the $2n$ linear programming problems for the MCRS method are identical, whereas the nucleolus requires changing the constraint set for each of the possible $n-1$ linear programming problems. This makes the calculation of the nucleolus much more complex, especially when several systems are involved. However, even if several systems are involved, the sets of constraints are essentially unchanged except for the value of $c(N)$ when using the MCRS method. Finally, the MCRS method is easier to understand and explain to the eventual

decision makers which is also a criterion for selecting a cost allocation method.

Summary

Several intuitively appealing ad hoc methods for allocating cost fail to give an equitable solution when an equitable solution exists. Moreover, in situations where several facility configurations are being considered, some ad hoc methods encourage noncooperation because these methods are not independent of the configuration of the facility. In order to overcome these shortcomings with ad hoc methods, concepts from cooperative game theory are necessary. The basis for determining whether the k^{th} best system is financially viable is the existence of the core. This is because an equitable cost allocation exists to implement the system. However, a core solution may be "inequitable" if caution is not taken to include all nondummy players. Several game theoretic methods for allocating cost are examined, but the most suitable method for allocating cost in the water resource field appears to be the MCRS method. This conclusion is based on

- 1) reliability of finding an equitable cost allocation;
- 2) simplicity of computing the cost allocation for one or more systems;
- 3) adaptability to recommended methodology;
- and 4) ease of understanding.

CHAPTER 5 EFFICIENCY/EQUITY ANALYSIS

Introduction

The cost allocation literature is replete with terminologies like opportunity cost, alternative cost, and marginal cost to represent the maximum or minimum amount that an individual should be charged. However, the precise meaning of these terms is obscured because no procedure for measurement is usually given. This chapter outlines a rigorous procedure to unambiguously quantify an individual's maximum cost and minimum cost for equity analysis. Furthermore, efficiency analysis and equity analysis are shown to be related by the costs of all opportunities available to all individuals in a project.

Before the results in this chapter are discussed, recall that the computational effort used by the total enumeration procedure described in Chapter 3 to find $c(N)$ is concurrently used to find $c(i)$ and $c(S)$ with little additional effort. That is, the independent calculations are not only used to find $c(N)$ but are also used to find $c(i)$ and $c(S)$. This important aspect of the procedure is illustrated in Table 5-1 using our three-county regional water network problem.

Table 5-1. Using Independent Calculations From the Total Enumeration Procedure to Find $c(i)$, $c(S)$, and $C(N)$ for the Three-County Regional Water Network Problem.

Independent Calculation:

$(S1); (S2); (S3); (S1,12); (S1,13); (S2,23);$
 $(S1,12,23); (S1,12,13)$

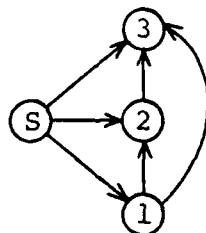
Efficiency Analysis for $c(N)$:

$c(123) = \text{minimum } [(S1,12,23); (S1,12,13);$
 $(S1,12) + (S3); (S1,13) + (S2);$
 $(S1) + (S2,23); (S1) + (S2) + (S3)]$

Efficiency Analysis for $c(i)$ and $c(S)$:

$c(1) = (S1) \quad c(2) = (S2) \quad c(3) = (S3)$
 $c(12) = \text{minimum } [(S1,12); (S1) + (S2)]$
 $c(13) = \text{minimum } [(S1,13); (S1) + (S3)]$
 $c(23) = \text{minimum } [(S2,23); (S2) + (S3)]$

Note: (S_i, i_j) represents the cost of the water network consisting of pipelines from the well field to county i and from county i to county j .



Maximum Cost

Opportunity cost (or alternative cost) is a concept used in economics to define the true cost of any action and is measured by the cost of the next best alternative that must be forgone when an action is taken (Nicholson, 1983). As a result, there is an opportunity cost for each individual associated with a regional water project because each individual must forego the opportunity to acquire the same level of service by either going-it-alone or joining a subcoalition. Although the costs for an individual to go-it-alone and for each subcoalition an individual can join to acquire the same level of service can be measured, an individual's opportunity cost cannot be determined just from these costs. This is because there is no way of specifying an individual's next best alternative without also knowing the individual's cost of joining each subcoalition. This means the cost of each subcoalition an individual can join must also be allocated. To further complicate matters, an individual's opportunity for joining a subcoalition depends on the opportunities available to the other individuals in a regional water project as well. For example, in a three-person game not every individual can join a two-person subcoalition. Kaplan (1982) correctly states that opportunity costs are extremely important for decision making, yet difficult to measure because opportunity costs arise from transactions not executed. However, since opportunity

cost is the cost of an individual's next best alternative, the principle of individual rationality requires that the opportunity cost be the limit on how much an individual can be charged for joining the regional system. Otherwise, the individual can obviously do better by paying for his next best alternative rather than joining the regional system. Consequently, the maximum cost for individual i is equivalent to individual i 's opportunity cost, and the maximum charge for individual i can be stated mathematically as follows:

$$x(i) \leq x(i)_{\text{maximum}} \quad \forall i \in N \quad (5-1)$$

where $x(i)$ = cost allocated to individual i ,
 $x(i)_{\text{maximum}}$ = maximum cost for individual i given individual i 's opportunity to go-it-alone or join a subcoalition, and
 N = set of all individuals; i.e.,
 $N = \{1, 2, \dots, n\}.$

This condition simply means individual i should be charged a cost less than or equal to individual i 's cost of going-it-alone or joining a subcoalition. The maximum cost for individual i is now unambiguously quantifiable as the upper core bound for individual i , and is found by using the procedure outlined for the MCRS method to find the upper core bound for individual i , i.e.,

$$x(i)_{\text{maximum}} = \text{maximize } x(i) \quad (5-2)$$

$$\text{subject to} \quad x(i) \leq c(i) \quad \forall i \in N$$

$$\sum_{i \in S} x(i) \leq c(S) \quad \forall S \subset N$$

$$\sum_{i \in N} x(i) = c(N)$$

$$x(i) \geq 0 \quad \forall i \in N$$

The interpretation of the upper core bound is now clear. The upper core bound for individual i is found by considering all the opportunities available to all individuals participating in a regional system. These opportunities, represented by the core conditions, include the possibility of all the individuals going-it-alone or some combination of subcoalition formation. Without knowing what will eventually take place, the core of a game accounts for all possibilities by representing all possible feasible solutions. These feasible solutions are found by taking the convex combination of the feasible core bounds. The feasible upper core bound determined from linear programming problem (5-2) represents the maximum cost for individual i . Any charges exceeding $x(i)_{\text{maximum}}$ for individual i represent solutions outside the core, and, consequently, individual i can do better by going-it-alone or joining a subcoalition.

The upper core bounds for the optimal network game of our regional water network problem are now examined. This

is a nonconvex game whereby at least one subcoalition formation is relatively attractive in comparison to the grand coalition. From Table 5-2, the upper core bound for county 2 indicates county 2's maximum cost is \$2,356,279 which is not simply the cost for county 2 of independent action, i.e., \$2,420,095. The difference between the value of county 2's maximum cost, $x(2)_{\text{maximum}}$, and go-it-alone cost, $c(2)$, is due to the consideration by the core of not only county 2's opportunities for acquiring the same level of service by going-it-alone or joining a subcoalition, but also similar opportunities for county 1 and county 3 as well. To find which opportunities are relevant in determining $x(2)_{\text{maximum}}$, linear programming problem (5-2) can be solved. Table 5-2 indicates core constraints L4 and L6 representing subcoalitions (12) and (23), respectively, are binding when solving for $x(2)_{\text{maximum}}$. This result means the opportunities of forming subcoalitions (12) and (23) limit how much county 2 can be charged. However, when linear programming problem (5-2) is solved for $x(1)_{\text{maximum}}$ and $x(3)_{\text{maximum}}$, Table 5-2 indicates that the only binding core constraints are L1 and L3, respectively. Therefore, county 1's and county 3's maximum costs are their respective go-it-alone costs. Linear programming problem (5-2) is an unambiguous and rigorous procedure to refute any claims by county 1 and/or county 3 that their maximum cost is less than $c(1)$ and $c(3)$, respectively, because of opportunities

Table 5-2. Efficiency/Equity Analysis of the Optimal Network.

Efficiency Analysis			Equity Analysis		
Network	Savings (%)	Cost (\$)	Core Condition	Binding Constraint	
				max	min
				$x(1)$ $x(2)$ $x(3)$	$x(1)$ $x(2)$ $x(3)$
$c(123)$	9.9	$c(123) = 4,556,409$	L1: $x(1) \leq c(1)$	x	
$c(1,23)$	8.4	$c(1)+c(23) = 4,630,177$	L2: $x(2) \leq c(2)$		
$c(12,3)$	2.7	$c(3)+c(12) = 4,919,503$	L3: $x(3) \leq c(3)$	x	
$c(13,2)$	1.0	$c(2)+c(13) = 5,006,734$	L4: $x(1)+x(2) \leq c(12)$	x	x
$c(1,2,3)$	0	$c(1)+c(2)+c(3) = 5,057,087$	L5: $x(1)+x(3) \leq c(13)$		x
			L6: $x(2)+x(3) \leq c(23)$	x	x
			L7: $x(1)+x(2)+x(3) = c(123)$		x

Note: $x(1)_{\max} = 646,000$; $x(2)_{\max} = 2,356,279$; $x(3)_{\max} = 1,990,992$; $x(1)_{\min} = 572,232$; $x(2)_{\min} = 1,969,771$; $x(3)_{\min} = 1,627,898$
 $c(1) = 646,000$; $c(2) = 2,420,095$; $c(3) = 1,990,992$; $sc(1) = 572,232$; $sc(2) = 1,969,771$; $sc(3) = 1,627,898$

to join a subcoalition. For example, even though subcoalition (13) is an essential coalition, Table 5-2 indicates that subcoalition (13) is never a factor in determining the maximum cost for any individuals in the game.

Another aspect of properly quantifying maximum cost is in calculating the amount of savings from a regional water project. Normally, the amount of savings is based on each individual's go-it-alone cost; i.e.,

$$\text{Savings (\%)} = 100 - \left[\frac{c(N)}{\sum_{i \in N} c(i)} \times 100 \right] \quad \forall i \in N. \quad (5-3)$$

However, equation (5-3) assumes that either the regional water project involving the grand coalition is built or all the individuals will go-it-alone and, unfortunately, does not consider the possibility that a relatively attractive regional water project involving subcoalitions may be formed. To account for the possibility of relatively attractive subcoalition formations, the amount of savings from a regional water project should be defined in terms of maximum cost as defined by linear programming problem (5-2); i.e.,

$$\text{Savings (\%)} = 100 - \left[\frac{c(N)}{\sum_{i \in N} x(i)_{\text{maximum}}} \times 100 \right] \quad \forall i \in N. \quad (5-4)$$

For our regional water network problem, the amount of savings from the optimal network is 9.9% if equation (5-3) is used and is 8.7% if equation (5-4) is used. This result indicates failure to consider the possibility of attractive subcoalition formations can lead to an overestimation of maximum cost and, consequently, an overestimation of the amount of savings from a regional water project.

For convex games, an individual's maximum cost is simply the individual's go-it-alone cost. A game is convex if none of the subcoalitions are attractive relative to the grand coalition. As discussed in Chapter 4, the nominal core bounds and the actual core bounds are identical if a game is convex. As a result, the maximum cost for each individual in a convex game is simply the individual's go-it-alone cost; i.e., $x(i)_{\text{maximum}} = c(i)$. This means in a convex game none of the subcoalition core constraints for linear programming problem (5-2) are binding; therefore, none of the individuals in a convex game can claim a maximum cost less than their go-it-alone cost because of opportunities to join a subcoalition.

In summary, individual i 's upper core bound represents individual i 's maximum cost. If a game is convex, $x(i)_{\text{maximum}}$ is simply equal to individual i 's go-it-alone cost, i.e., $c(i)$; but, if a game is nonconvex, $x(i)_{\text{maximum}}$ must be determined from linear programming problem (5-2). Whether a game is convex or nonconvex can be determined

either by a convexity check using equation (4-12) or (4-13), or by comparing the nominal core bounds with the actual core bounds determined from linear programming problem (4-30). If a game is large, convexity check using equation (4-12) or (4-13) can be burdensome, and using linear programming problem (4-30) is more practical. In any event, determining whether a game is convex or nonconvex requires knowing the least cost system or characteristic cost function for each subcoalition, i.e., $c(S)$. Furthermore, if a game is nonconvex, the importance of each subcoalition for equity analysis cannot be determined until linear programming problem (5-2) is solved. As we can see, efficiency analysis and equity analysis are related because the maximum costs which determine the maximum charges for the individuals in a regional project are found by considering the economics of all opportunities available to all individuals in the project.

Minimum Cost

A corresponding interpretation of an individual's lower core bound can be given. The lower core bound represents the minimum cost assignable to individual i for joining the grand coalition based on considering all the opportunities for all individuals in a project. Therefore, the minimum charge for individual i can be expressed mathematically as follows:

$$x(i) \geq x(i)_{\text{minimum}} \quad \forall i \in N \quad (5-5)$$

where $x(i)$ = cost allocated to individual i ,

$x(i)_{\text{minimum}}$ = minimum cost to individual i to join the grand coalition, and

N = set of all individuals; i.e.,
 $N = \{1, 2, \dots, n\}$.

This condition simply means individual i should be charged a cost greater or equal to individual i 's cost of joining the grand coalition. The minimum cost for individual i is found by using the procedure outlined for the MCRS method to find the lower core bound for individual i , i.e.,

$$x(i)_{\text{minimum}} = \text{minimize } x(i) \quad (5-6)$$

$$\text{subject to} \quad x(i) \leq c(i) \quad \forall i \in N$$

$$\sum_{i \in S} x(i) \leq c(S) \quad \forall S \subset N$$

$$\sum_{i \in N} x(i) = c(N)$$

$$x(i) \geq 0 \quad \forall i \in N$$

Any charges less than $x(i)_{\text{minimum}}$ for individual i mean individual i is not paying for its minimum cost to join the

grand coalition and represent solutions outside the core whereby individual i is being subsidized.

The accepted practice of using separable cost or marginal cost as the minimum cost is based on the marginality principle that every individual should be charged at least the additional cost of being served (Young et al., 1982) and assumes that the minimum cost to join a coalition is last. The practicability of this assumption is troubling because not every individual can join a coalition last. In fact, a coalition may form without any clear understanding of the sequence of formation. Furthermore, Heaney (1979) has shown that the assumption is only true if a game is convex; i.e.,

$$c(N) - c(N - \{i\}) \leq c(S) - c[(S) - \{i\}] \quad \forall S \subset N. \quad (5-7)$$

Fortunately, linear programming problem (5-6) eliminates any ambiguities in defining or quantifying the minimum cost for each individual in a project. Table 5-2 indicates that the minimum cost for each county of the optimal network for our regional water network problem is equal to each county's separable cost, i.e., the binding core constraints for $x(i)_{\text{minimum}}$ is $c(N)$ and $c(N - \{i\})$.

In summary, individual i 's lower core bound represents individual i 's minimum cost. If a game is convex,

$x(i)_{\text{minimum}}$ is simply equal to individual i 's separable cost or marginal cost, i.e.,

$$x(i)_{\text{minimum}} = sc(i) = c(N) - c(N - \{i\}) \quad \forall i \in N. \quad (5-8)$$

For nonconvex games, $x(i)_{\text{minimum}}$ may or may not be identical to the separable cost or marginal cost for individual i ; therefore, linear programming problem (5-6) must be solved.

Fairness Criteria

The fairness criteria for an equitable cost allocation expressed by equations (4-1) and (4-2) can now be simply and explicitly stated in terms of the core bounds, i.e.,

$$x(i)_{\min} \leq x(i) \leq x(i)_{\max} \quad \forall i \in N. \quad (5-9)$$

This condition embodies the fairness criteria expressed by equations (4-1) and (4-2), yet clearly defines the range of costs that each individual i can be charged without violating individual rationality and/or subgroup rationality. Moreover, the lower and upper core bounds for each individual can be unambiguously quantified and interpreted as each individual's minimum cost and maximum cost, respectively. Finally, equation (5-9) can simplify the cost allocation procedure because the minimum cost for each individual is already determined, and only the remaining

costs need to be allocated. Equation (4-4) can now be rewritten as follows:

$$x(i) = x(i)_{\min} + \psi(i) \cdot rc \quad \forall i \in N \quad (5-10)$$

where $x(i)$ = cost allocated to individual i ,

$x(i)_{\min}$ = minimum $x(i)$ from linear programming problem (5-6),

$\psi(i)$ = prorating factor for individual i , and

rc = remaining costs, i.e.,

$$c(N) - \sum_{i \in N} x(i)_{\min}.$$

Therefore, any cost allocation procedure that is agreeable to the individuals participating in a regional project can be used to apportion the remaining costs as long as inequalities (5-9) are satisfied. For example, the remaining costs might be prorated in proportion to a measure of use.

Summary

The interpretation of the core bounds is now clear. The lower core bound is a measure of minimum cost, while the upper core bound is a measure of maximum cost. The procedure for unambiguously measuring these costs is the same procedure used in the MCRS method for finding the core bounds. However, to determine the core bounds, an efficiency analysis is necessary to find the costs of all

opportunities available to all individuals in a project. Once the core bounds are found, they can be used as simple guidelines for allocating costs.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

The motivation for this dissertation is based on the following three conclusions from reviewing the literature on efficiency analysis and equity analysis of regional water resources planning. First, no published work has incorporated efficiency analysis and equity analysis into a single regional water resources planning model using realistic cost functions. Secondly, the allocation of piping cost has not been examined separately from treatment cost. Thirdly, the cost allocation literature has not dealt with situations whereby good suboptimal systems are considered along with the optimal system.

The first conclusion establishes the primary purpose of this dissertation. That is, to integrate efficiency analysis and equity analysis into a single water resources planning model using realistic cost functions. The selection of a regional water network problem is obviously based on the second conclusion. Finally, the third conclusion helped initiate a search for a reliable computational procedure to find good suboptimal systems that ultimately led to developing a total enumeration procedure for efficiency/equity analysis of regional water network problems.

A major task with integrating efficiency analysis and equity analysis is finding a computational procedure. The principles of individual, subgroup, and group rationality from cooperative game theory give a theoretically sound basis for equity analysis. Consequently, successful efficiency/equity analysis depends on having a reliable method for finding not only the optimal regional system, but also the optimal system for each individual and each subgroup of individuals. Basically, either a partial enumeration or a total enumeration approach can be used to find these optimal solutions. Reliable partial enumeration techniques can be used for problems with well defined cost functions; but, for the types of cost function generally encountered in actual practice a total enumeration technique must be used.

A reliable total enumeration procedure for finding the least cost water supply network for each individual, each subgroup of individuals, and the region is described. This procedure is easy to understand and use, and allows the engineer to use realistic cost functions or to perform detailed cost analysis. More importantly, the computational effort used by this procedure to find the optimal regional system can be concurrently used to find the optimal system for each individual and each subgroup of individuals with little additional effort. Furthermore, this procedure naturally gives all the suboptimal systems; therefore, good

suboptimal regional systems can be examined when factors other than cost are considered.

Once a reliable method is available to find the optimal solutions, equity analysis can be accomplished using concepts from cooperative game theory. The financial viability of any system is based on the existence of a core because an equitable cost allocation can be found if a core exists. As the costs of suboptimal systems increase, the core naturally reduces in size until the core possibly becomes empty. Any system with an empty core is considered not financially viable because of the minimal economic gain or the loss of subadditivity.

In comparing several ad hoc and game theory methods for allocating costs, the MCRS method appears to be the most suitable method for the water resources field. More importantly, the MCRS method gives a procedure for finding the core bounds by simply using the core conditions along with linear programming.

The lower core bound and the upper core bound for an individual provide unambiguous measures of the individual's minimum cost and maximum cost, respectively. These costs are found by considering all the opportunities, represented by the core conditions, available to all individuals in a project. If the core conditions do not account for all the opportunities for each individual in a project to form an essential subcoalition, then the core bounds may be

distorted. In such cases, even a cost allocation in the core may be inequitable.

Whether a game is convex or nonconvex is essential to how minimum cost, maximum cost, and savings are determined. The traditional approach is to assign minimum cost as the cost to join a coalition last, to assign maximum cost as the go-it-alone cost, and to calculate savings with respect to go-it-alone cost. These traditional approaches are acceptable only if a game is convex. For nonconvex games, these traditional approaches overlook opportunities to form good subcoalitions and, therefore, may distort the analysis by overestimating minimum cost, maximum cost, and savings. If a game is nonconvex, the minimum cost should be found by using linear programming problem (5-6), and the maximum cost should be found by using linear programming problem (5-2). Moreover, savings should be calculated with respect to the maximum cost as defined by linear programming problem (5-2).

By knowing each individual's minimum cost and maximum cost, a basis for finding an equitable cost allocation is available. Since the minimum cost for each individual is already determined, decision makers simply need to agree on a method to apportion remaining costs without exceeding any individual's maximum cost.

In summary, efficiency analysis and equity analysis in regional water resource planning are not separable problems. An efficiency analysis is necessary to find an

optimal or a good suboptimal system, but to implement this desirable system an equity analysis must be accomplished. Yet, accomplishing an equity analysis depends on an efficiency analysis to find the optimal system for each individual and each subgroup of individuals to account for all opportunities available to all individuals in the project. Otherwise, each individual's minimum cost and maximum cost cannot be properly determined. Therefore, an efficiency analysis is incomplete unless it also provides the necessary information to accomplish an equity analysis.

Finally, some thoughts for further research generated during the course of this dissertation are listed.

1. If different cost functions are used for different individuals in a regional project, a consistent set of accounting procedures is necessary to insure the cost functions are based on a comparable set of cost data. What is a consistent set of accounting procedures?

2. In any practical application, total project costs are not known precisely until after the project has been completed; therefore, a legitimate concern is how cost overruns are to be allocated.

3. Determine mathematical procedures to compute the total number of calculations to enumerate all $2^n - 1$ optimal solutions for any given digraph and to compute how many of these calculations are independent calculations.

4. Set up a computer model for detailed cost analysis, e.g., MAPS, and apply the procedures described in this dissertation to perform an efficiency/equity analysis of a real regional water network problem.

5. After a regional network is in place, how should the cost of expanding the network to serve new users be allocated.

6. A method to quantify transactions cost is necessary to better evaluate the financial gains of a regional system.

APPENDIX
EFFICIENCY/EQUITY ANALYSIS OF A THREE-COUNTY
REGIONAL WATER NETWORK WITH NONLINEAR
COST FUNCTION

Appendix A contains the efficiency/equity calculations for the three-county cost game example using Lotus 1-2-3. The results are presented as follows:

Table 1 Efficiency Calculations With Total Enumeration Procedure
 Table 2 Cost Allocation for Option 1
 Table 3 Cost Allocation for Option 2
 Table 4 Cost Allocation for Option 3
 Table 5 Cost Allocation for Option 4
 Table 6 Cost Allocation for Option 5
 Table 7 Template Used for Calculations

Data

Distance : $L(i,j)$ is the distance in feet from i to j

$L(S,1)= 17000$ $L(S,3)= 30250$ $L(1,3)= 19670$

$L(S,2)= 26000$ $L(1,2)= 13100$ $L(2,3)= 15500$

Demand : $Q(i)$ is the demand in mgd for player i

$Q(1)= 1$ $Q(2)= 6$ $Q(3)= 3$

Cost Function: $a(Q^b)L$ $a= 38$ $b= 0.5$

Table 1 Calculations With Total Enumeration Procedure
 $C(i..j)[x]=$ Cost of network $[x]$ for $i..j$; $C(i..j)=$ Least cost for $i..j$

$C(1)[S1]= 646000$ $C(2)[S2]=2420095.$ $C(3)[S3]=1990992.$

$C(12)[S1,12]= 2928511.$ $C(12)= 2928511.$

$C(12)[S1;S2]= 3066095.$

$C(13)[S1,13]= 2586638.$ $C(13)= 2586638.$

$C(13)[S1;S3]= 2636992.$

$C(23)[S2,23]= 3984177.$ $C(23)= 3984177.$

$C(23)[S2;S3]= 4411088.$

$C(123)[S1,12,23]= 4556409.$ 0

$C(123)[S1,12;S3]= 4919503.$ 0

$C(123)[S1,12,13]= 4556826.$ 0 $C(123)= 4556409.$

$C(123)[S1,13;S2]= 5006734.$ 0

$C(123)[S1;S2,23]= 4630177.$ 0

$C(123)[S1;S2;S3]= 5057088.$ 0

Sort $C(123)$ in ascending order (Yes=1, No=0):

Paths	Cost	Convex
$C(123)[S1,12,23]=$	4556409.	0
$C(123)[S1,12,13]=$	4556826.	0 BEST
$C(123)[S1;S2,23]=$	4630177.	0 $C(123)= 4556409$
$C(123)[S1,12;S3]=$	4919503.	0
$C(123)[S1,13;S2]=$	5006734.	0
$C(123)[S1;S2;S3]=$	5057088.	0

Table 2 Cost Allocation For C(123) = 4556409

Calculate the Shapley value (SV) for the best C(123):

SV(1)= 592087.3 SV(2)= 2175904. SV(3)= 1790416.

Sum of SV(1)+SV(2)+SV(3)= 4556409

Check core conditions: (core test valid for subadditive game only)

IS core empty (yes=1,no=0)? 0

Does the Shapley value satisfy the core conditions?

$$SV(1) \subset C(1)$$
$$SV(2) \subset C(2)$$
$$SV(3) \subset C(3)$$
$$SV(1)+SV(2) \leq C(12) \quad 1$$
$$SV(1)+SV(3) < C(13) \quad 1$$
$$SV(2)+SV(3) < C(23) \quad 1$$

Calculate the SCRB value (X) for the best C(123):

Nominal Core Bounds:

572231.0 <X(1)< 646000

1969770. <X(2)< 2420095.

1627897. <X(3)< 1990992.

$$X(1) = 604369.0 \quad X(2) = 2165957. \quad X(3) = 1786082.$$

Sum of $X(1)+X(2)+X(3)=$ 4556409

IS core empty (yes=1,no=0)? 0

Does the SCRB value satisfy the core conditions?

$$X(1) < C(1) \quad 1$$

```

X(2)<C(2)

```

$$X(3) \subset C(3)$$

$X(1)+X(2)<C(12)$	1
-------------------	---

$$X(1) + X(3) < C(13) \quad 1$$

$x(2)+x(3) < c(23)$	1
---------------------	---

Calculate the MCRS value (M) for the best C(123):

Actual Core Bounds Determined From LP:

572232 <M(1)< 646000

1969771 <M(2)< 2356279

1627898	<M(3)<	1990992
---------	--------	---------

$$M(1) = 606860.3 \quad M(2) = 2151236. \quad M(3) = 1793342.$$

Sum of $M(1)+M(2)+M(3)=$ 4556409

IS core empty (yes=1,no=0)?

Does the MCRS value satisfy the core conditions?

$$N(1) < C(1) \quad 1$$
$$M(2) \subset C(2)$$
$$M(3) < C(3) \quad 1$$
$$M(1)+M(2) \leq C(12)$$
$$M(1)+M(3) < C(13)$$
$$M(2)+M(3) < C(23) \quad 1$$

Table 3 Cost Allocation For C(123) = 4556826

Calculate the Shapley value (SV) for the best C(123):

SV(1)= 590226.3 SV(2)= 2176043. SV(3)= 1790555.

Sum of SV(1)+SV(2)+SV(3)= 4556826

Check core conditions: (core test valid for subadditive game only)

IS core empty (yes=1,no=0)? 0

Does the Shapley value satisfy the core conditions?

SV(1)<C(1) 1

SV(2)<C(2) 1

SV(3)<C(3) 1

SV(1)+SV(2)<C(12) 1

SV(1)+SV(3)<C(13) 1

SV(2)+SV(3)<C(23) 1

Calculate the SCRB value (X) for the best C(123):

Nominal Core Bounds:

572648.0 <X(1)< 646000

1970187. <X(2)< 2420095.

1628314. <X(3)< 1990992.

X(1)= 604580.4 X(2)= 2166046. X(3)= 1786199.

Sum of X(1)+X(2)+X(3)= 4556826

IS core empty (yes=1,no=0)? 0

Does the SCRB value satisfy the core conditions?

X(1)<C(1) 1

X(2)<C(2) 1

X(3)<C(3) 1

X(1)+X(2)<C(12) 1

X(1)+X(3)<C(13) 1

X(2)+X(3)<C(23) 1

Calculate the MCBS value (M) for the best C(123):

Actual Core Bounds Determined From LP:

572649 <M(1)< 646000

1970188 <M(2)< 2355862

1628315 <M(3)< 1990992

M(1)= 607077.0 M(2)= 2151207. M(3)= 1798541.

Sum of M(1)+M(2)+M(3)= 4556826

IS core empty (yes=1,no=0)? 0

Does the MCBS value satisfy the core conditions?

M(1)<C(1) 1

M(2)<C(2) 1

M(3)<C(3) 1

M(1)+M(2)<C(12) 1

M(1)+M(3)<C(13) 1

M(2)+M(3)<C(23) 1

Table 4 Cost Allocation For C(123) = 4630177

Calculate the Shapley value (SV) for the best C(123):

SV(1)= 614676.6 SV(2)= 2200494. SV(3)= 1815006.
Sum of SV(1)+SV(2)+SV(3)= 4630177

Check core conditions: (core test valid for subadditive game only)

IS core empty (yes=1,no=0)? 0
Does the Shapley value satisfy the core conditions?
SV(1)<C(1) 1
SV(2)<C(2) 1
SV(3)<C(3) 1
SV(1)+SV(2)<C(12) 1
SV(1)+SV(3)<C(13) 1
SV(2)+SV(3)<C(23) 0

Calculate the SCRB value (X) for the best C(123):

Nominal Core Bounds:

645999.0 <X(1)< 646000
2043538. <X(2)< 2420095.
1701665. <X(3)< 1990992.
X(1)= 645999.4 X(2)= 2178677. X(3)= 1805499.
Sum of X(1)+X(2)+X(3)= 4630177

IS core empty (yes=1,no=0)? 0
Does the SCRB value satisfy the core conditions?

X(1)<C(1) 1
X(2)<C(2) 1
X(3)<C(3) 1
X(1)+X(2)<C(12) 1
X(1)+X(3)<C(13) 1
X(2)+X(3)<C(23) 1

Calculate the MCBS value (M) for the best C(123):

Actual Core Bounds Determined From LP:

646000 <M(1)< 646000
2043539 <M(2)< 2282511
1701666 <M(3)< 1940638
M(1)= 646000 M(2)= 2163025 M(3)= 1821152
Sum of M(1)+M(2)+M(3)= 4630177

IS core empty (yes=1,no=0)? 0
Does the MCBS value satisfy the core conditions?

M(1)<C(1) 1
M(2)<C(2) 1
M(3)<C(3) 1
M(1)+M(2)<C(12) 1
M(1)+M(3)<C(13) 1
M(2)+M(3)<C(23) 1

Table 5 Cost Allocation For C(123) = 4919503

Calculate the Shapley value (SV) for the best C(123):

SV(1)= 711118.6 SV(2)= 2296936. SV(3)= 1911448.

Sum of SV(1)+SV(2)+SV(3)= 4919503

Check core conditions: (core test valid for subadditive game only)

IS core empty (yes=1,no=0)? 1

Does the Shapley value satisfy the core conditions?

SV(1)<C(1) 0

SV(2)<C(2) 1

SV(3)<C(3) 1

SV(1)+SV(2)<C(12) 0

SV(1)+SV(3)<C(13) 0

SV(2)+SV(3)<C(23) 0

Calculate the SCRB value (X) for the best C(123):

Nominal Core Bounds:

935325.0 <X(1)< 646000

2332864. <X(2)< 2420095.

1990991. <X(3)< 1990992.

X(1)= 449026.7 X(2)= 2479483. X(3)= 1990992.

Sum of X(1)+X(2)+X(3)= 4919503

IS core empty (yes=1,no=0)? 1

Does the SCRB value satisfy the core conditions?

X(1)<C(1) 1

X(2)<C(2) 0

X(3)<C(3) 0

X(1)+X(2)<C(12) 1

X(1)+X(3)<C(13) 1

X(2)+X(3)<C(23) 0

Table 6 Cost Allocation For C(123) = 5006734

Calculate the Shapley value (SV) for the best C(123):

SV(1)= 747195.6 SV(2)= 2326013. SV(3)= 1940525.

Sum of SV(1)+SV(2)+SV(3)= 5006734

Check core conditions: (core test valid for subadditive game only)

IS core empty (yes=1,no=0)? 1

Does the Shapley value satisfy the core conditions?

SV(1)<C(1) 0

SV(2)<C(2) 1

SV(3)<C(3) 1

SV(1)+SV(2)<C(12) 0

SV(1)+SV(3)<C(13) 0

SV(2)+SV(3)<C(23) 0

Calculate the SCRB value (X) for the best C(123):

Nominal Core Bounds:

1022556. <X(1)< 646000

2420095. <X(2)< 2420095.

2078222. <X(3)< 1990992.

X(1)= 605116.4 X(2)= 2420095. X(3)= 1981521.

Sum of X(1)+X(2)+X(3)= 5006734

IS core empty (yes=1,no=0)?	1
Does the SCRB value satisfy the core conditions?	
X(1)<C(1)	1
X(2)<C(2)	0
X(3)<C(3)	1
X(1)+X(2)<C(12)	0
X(1)+X(3)<C(13)	1
X(2)+X(3)<C(23)	0

Table 7 Template Used for Calculations

A	B	C	D	E	F	G	H
Data							203
							204
Distance : L(i,j) is the distance in feet from i to j							205
L(S,1)=	17000	L(S,3)=	30250	L(1,3)=	19670		206
L(S,2)=	26000	L(1,2)=	13100	L(2,3)=	15500		207
Demand : Q(i) is the demand in mgd for player i							208
Q(1)=	1	Q(2)=	6	Q(3)=	3		209
Cost Function: a(Q^b)L							210
		a=	38	b=	0.5		

Calculations With Total Enumeration Procedure

C(i..j)[x]= Cost of network [x] for i..j ; C(i..j)= Least cost for i..j

C(1)[S1]=	646000	C(2)[S2]=	2420095.	C(3)[S3]=	1990992.	215
C(12)[S1,12]=	2928511.	C(12)=	2928511.			217
C(12)[S1;S2]=	3066095.					218
C(13)[S1,13]=	2586638.	C(13)=	2586638.			220
C(13)[S1;S3]=	2636992.					221
C(23)[S2,23]=	3984177.	C(23)=	3984177.			223
C(23)[S2;S3]=	4411088.					224
C(123)[S1,12,23]=	4556409.	0				226
C(123)[S1,12;S3]=	4919503.	0				227
C(123)[S1,12,13]=	4556826.	0	C(123)=	4556409.		228
C(123)[S1,13;S2]=	5006734.	0				229
C(123)[S1;S2,23]=	4630177.	0				230
C(123)[S1;S2;S3]=	5057088.	0				231

Sort C(123) in ascending order (Yes=1, No=0):		233
Paths	Cost	Convex
C(123) [S1, 12, 23]=	4556409.	0
C(123) [S1, 12, 13]=	4556826.	0 BEST
C(123) [S1; S2, 23]=	4630177.	0 C(123) = 4556409
C(123) [S1, 12; S3]=	4919503.	0
C(123) [S1, 13; S2]=	5006734.	0
C(123) [S1; S2; S3]=	5057088.	0
		241
Cost Allocation For C(123) = 4556409		243
Calculate the Shapley value (SV) for the best C(123):		245
SV(1)= 590087.3 SV(2)= 2175904. SV(3)= 1790416.		246
Sum of SV(1)+SV(2)+SV(3)= 4556409		247
Check core conditions: (core test valid for subadditive game only)		
IS core empty (yes=1, no=0)?	0	250
Does the Shapley value satisfy the core conditions?		251
SV(1) < C(1)	1	252
SV(2) < C(2)	1	253
SV(3) < C(3)	1	254
SV(1)+SV(2) < C(12)	1	255
SV(1)+SV(3) < C(13)	1	256
SV(2)+SV(3) < C(23)	1	257
		258
Calculate the SCRB value (X) for the best C(123):		260
Nominal Core Bounds:		261
572231.0 < X(1) < 646000		262
1969770. < X(2) < 2420095.		263
1627897. < X(3) < 1990992.		264
X(1)= 604369.0 X(2)= 2165957. X(3)= 1786082.		265
Sum of X(1)+X(2)+X(3)= 4556409		266
IS core empty (yes=1, no=0)?	0	268
Does the SCRB value satisfy the core conditions?		269
X(1) < C(1)	1	270
X(2) < C(2)	1	271
X(3) < C(3)	1	272
X(1)+X(2) < C(12)	1	273
X(1)+X(3) < C(13)	1	274
X(2)+X(3) < C(23)	1	275
		276
Calculate the MCRS value (M) for the best C(123):		278
Actual Core Bounds Determined From LP:		279
572232 < M(1) < 646000		280
1969771 < M(2) < 2356279		281
1627898 < M(3) < 1990992		282
M(1)= 606860.3 M(2)= 2151206. M(3)= 1793342.		283
Sum of M(1)+M(2)+M(3)= 4556409		284

IS core empty (yes=1,no=0)?	0	286
Does the MCRS value satisfy the core conditions?		287
M(1)<C(1)	1	288
M(2)<C(2)	1	289
M(3)<C(3)	1	290
M(1)+M(2)<C(12)	1	291
M(1)+M(3)<C(13)	1	292
M(2)+M(3)<C(23)	1	293
		294

Partial Listing of Cell Formulas

Total Enumeration Procedure:

B215: +\$E\$210*(\$C\$209^\$G\$210)*\$B\$206
 D215: +\$E\$210*(\$E\$209^\$G\$210)*\$B\$207
 F215: +\$E\$210*(\$G\$209^\$G\$210)*\$D\$206
 C217: +\$E\$210*(((\$C\$209+\$E\$209)^\$G\$210)*\$B\$206+\$E\$210*(\$E\$209^\$G\$210)*\$D\$207
 G217: @MIN(\$C\$217..\$C\$218)
 C218: +\$B\$215+\$D\$215
 D226: +\$E\$210*(((\$C\$209+\$E\$209+\$G\$209)^\$G\$210)*\$B\$206+\$E\$210*(\$E\$209+\$G\$209)^\$G\$210*\$D\$207+\$E\$210*(\$G\$209^\$G\$210)*\$F\$207
 D227: +\$C\$217+\$F\$215
 D228: +\$E\$210*(((\$C\$209+\$E\$209+\$G\$209)^\$G\$210)*\$B\$206+\$E\$210*(\$E\$209^\$G\$210)*\$D\$207+\$E\$210*(\$G\$209^\$G\$210)*\$F\$206
 D229: +\$C\$220+\$D\$215
 D230: +\$B\$215+\$C\$223
 D231: +\$B\$215+\$D\$215+\$F\$215
 E235: @IF(+\$G\$217+\$G\$220>=\$D\$226+\$E\$215#AND#+\$G\$217+\$G\$223>=\$D\$226+\$D\$215#AND#+\$G\$220+\$C\$223>=\$D\$226+\$F\$215

Shapley value:

B246: 1/3*\$B\$215+1/6*(\$G\$217-\$D\$215)+1/6*(\$G\$220-\$F\$215)+1/3*(\$G\$237-\$G\$223)
 D247: +\$B\$246+\$D\$246+\$F\$246
 E250: @IF(+\$G\$217+\$G\$220+\$G\$223>=2*\$G\$237,0,1)
 E252: @IF(+\$B\$246<=\$B\$215,1,0)
 E253: @IF(+\$D\$246<=\$D\$215,1,0)
 E254: @IF(+\$F\$246<=\$F\$215,1,0)
 E255: @IF(+\$B\$246+\$D\$246<=\$C\$217,1,0)
 E256: @IF(+\$B\$246+\$F\$246<=\$G\$220,1,0)
 E257: @IF(+\$D\$246+\$F\$246<=\$G\$223,1,0)

SCRB:

C262: +\$G\$237-\$G\$223
 E262: +\$B\$215
 B265: ((\$B\$215-\$C\$262)/(\$B\$215-\$C\$262+\$D\$215-\$C\$263+\$F\$215-\$C\$264))*(\$G\$237-\$C\$262-\$C\$263-\$C\$264)+\$C\$262

MCRS:

B283: ((\$E\$280-\$C\$280)/(\$E\$280-\$C\$280+\$E\$281-\$C\$281+\$E\$282-\$C\$282))*(\$G\$237-\$C\$280-\$C\$281-\$C\$282)+\$C\$280

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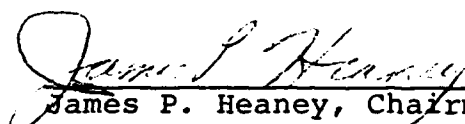
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BIOGRAPHICAL SKETCH

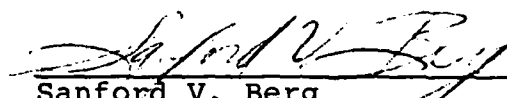
Elliot Kin Ng was born August 9, 1950, in San Francisco, California, the son of Wah Hin Ng and Kit Har Yan. In 1968, he graduated from Lowell high school in San Francisco. He entered the University of California, Berkeley, in 1968 and received a Bachelor of Science in electrical engineering in 1972 and a Master of Science in electrical engineering in 1974. Subsequently, he spent two years working for Bechtel Incorporated, San Francisco, California, as an electrical engineer for the chemical and refinery division. He was commissioned in the U.S. Air Force as a captain in 1976. He held assignments at USAF School of Aerospace Medicine, Brooks AFB, Texas (Environmental, Safety and Facility Manager), USAF Occupational and Environmental Health Laboratory, Brooks AFB, Texas (Consultant, Water Resources Engineer), and USAF Hospital, Wurtsmith AFB, Michigan (Bioenvironmental Engineer). During his assignments at Brooks AFB, Texas, he received a Master of Science degree in environmental management from the University of Texas, San Antonio. He was selected by the Air Force Institute of Technology (AFIT) in 1981 for doctoral training in environmental engineering. In 1982, he entered the University of Florida to pursue the Doctor of Philosophy

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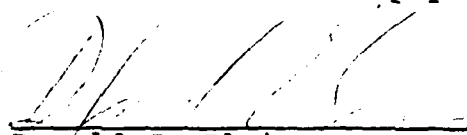
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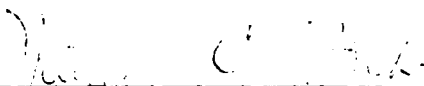
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This dissertation was submitted to the Graduate Faculty of the College of Engineering and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1985

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