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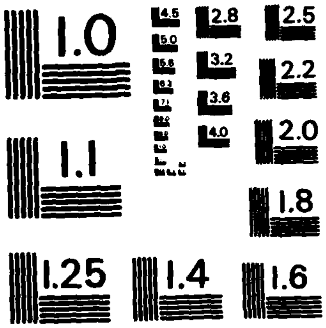

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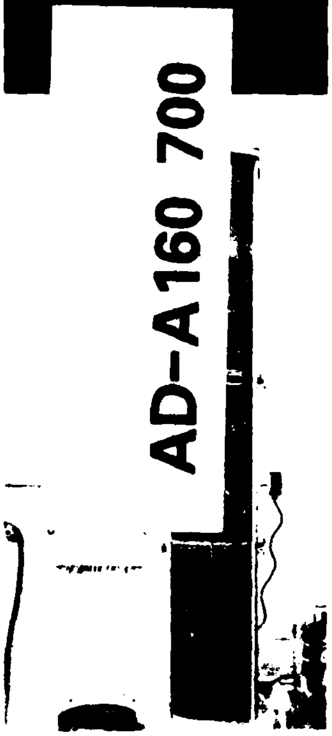

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BOSEF: BEAM ON SWELLING ELASTIC FOUNDATION

by

James P. Stewart

Department of Civil Engineering
Syracuse University
Syracuse, New York 13210

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Final Report

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20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

This study developed an improved methodology for studying the behavior of moderately loaded mat foundations on expansive soils. A computer program BOSEF (Beam on Swelling Elastic Foundation) was prepared that includes modelling of the following phenomena:

a. Two-dimensional steady-state moisture flow through foundation soils;

b. Computation of subgrade distortion resulting from changes of soil

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PREFACE

This study was conducted under Department of the Army Project AT22, Task Area AO, Work Unit 010, "Mat Foundations for Intermediate and Heavy Military Structures," sponsored by the Office, Chief of Engineers (OCE), US Army. Mr. A. F. Muller was the OCE Technical Monitor.

The computer program BOSEF (Beam on Swelling Elastic Foundation) was developed and the report was prepared by Dr. James P. Stewart, Department of Civil Engineering, Syracuse University, while working at the US Army Engineer Waterways Experiment Station (WES) on a 10-week Battelle Research Institute Summer Faculty Research grant during 1984. The work was performed under the direct supervision of Dr. L. D. Johnson, Research Group, Soil Mechanics Division (SMD), Geotechnical Laboratory (GL), and under the general supervision of Mr. C. L. McAnear, Chief, SMD, and Dr. W. F. Marcuson III, Chief, GL. Dr. P. F. Hadala, Assistant Chief, GL, Mr. R. W. Peterson, Soils Research Facility, SMD, and Mr. J. M. Andersen, Engineering Group, SMD, reviewed the report and provided many helpful comments. Ms. Joyce Walker and Mr. Robert Baylot, Publications and Graphic Arts Division, WES, provided editorial assistance.

COL Tilford C. Creel, CE, and COL Robert C. Lee, CE, were Commanders and Directors of WES during the conduct of the study and preparation of this report; Mr. Fred. R. Brown was Technical Director. COL Allen F. Grum, USA, was Director of WES at the time of publication of this report; Dr. Robert W. Whalin was Technical Director.

CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
degrees Fahrenheit	5/9	Celsius degrees*
feet	0.3048	metres
inches	25.4	millimetres
kips (force) per inch	175.1268	kilonewtons per metre
kips (force) per square foot	47.88026	kilopascals
miles (US statute)	1.609347	kilometres
pounds (force)	4.448222	newtons
pounds (force) per square inch	6,894.757	pascals
pounds (mass)	0.4535924	kilograms
pounds (mass) per cubic inch	16.01846	kilograms per cubic metre
pounds (mass) per cubic inch	27,679.9	kilograms per cubic metre
square inches	6.4516	square centimetres

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*To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use the following formula: C = (5/9)(F - 32). To obtain Kelvin (K) readings, use: K = (5/9)(F - 32) + 273.15.

are not suitable for the design of large, moderately loaded mat foundations. The size, stiffness, and load pattern strongly influence the performance of the structures. The "worst case" approach used for the more lightly loaded and smaller residential mat foundations may be overly conservative for a large mat. In addition, greater engineering effort can usually be justified for larger industrial structures than for smaller residential structures.

Purpose and Scope

5. The goal of this research was to develop a rational technique to analyze the behavior of a large, moderately loaded mat foundation on expansive soil. As simplification to render the problem tractable for the 10-week work period, the behavior of a mat foundation was modelled by a beam on a Winkler subgrade. The interaction between soil and moisture was modelled as a 2-dimensional steady state moisture flow. The technique is suitable for extension to a more general condition as well. The scope of this study includes:

(a) review literature pertinent to expansive soils and analysis of mat foundations;

(b) develop a technique for determining the design displacements, moments and shears of a moderately loaded mat foundation on an expansive soil that explicitly accounts for moisture flow through the soil;

(c) code the proposed technique for computer solution;

(d) test the computer program to verify its usefulness;

(e) recommend directions for future research in this problem area; and

(f) present results in a written report.

6. This report is presented in several major sections. Part II presents a literature review of previously published work pertinent to the analysis of a beam or mat on an expansive soil subgrade. Part III describes the methodology and computer program BOSEF. Part IV presents the results of several example problems using the program BOSEF. Part V recommends directions for future research. A listing of the computer program is given in Appendix A. Copies of computer runs for some of the case studies are given in Appendix B.

ture changes in the underlying soil will not be uniform. The nonuniform moisture change can cause several inches of differential movement between mat center and edge. Differential foundation movements of this magnitude can cause and have caused severe structural distress.

10. The amount of differential movement that develops across a mat foundation depends on several factors. First, it depends on the rigidity of the mat itself. A rigid mat, by definition, will not be subject to the differential movements that create structural distress. In practice, it may not be economical to construct mats that are sufficiently rigid to eliminate troublesome differential movements. Most mats constructed by the Corps of Engineers are flexible. Second, the location and magnitude of loads on the mat influence the nature of the differential movements. Third, the differential movements of the foundation can be influenced by the degree to which the underlying soils expand and contract because of moisture changes. Fourth, differential movements depend also on the distribution of soil moisture changes beneath the mat with respect to the initial conditions. Identical mat foundations on identical soils could behave differently if one was constructed after a wet season and one after a dry season. A fifth factor is the rate at which moisture can be transmitted through the foundation soils. The moisture shielding action of a mat foundation may be small if moisture is quickly transported through the foundation soils and around the edges of the mat.

11. The direction of movement of a mat foundation on expansive soil depends on the net change in soil moisture with respect to the initial moisture conditions. If a net moisture increase develops, the soil expands and vice versa. It is probable that moisture levels will vary with time and may show increases at some locations beneath a mat and decreases at other locations. Also a net increase in soil moisture may develop at some times during the year while a net decrease may develop at other times. Therefore mat displacements, shears and moments can change significantly throughout the year, and may even reverse themselves from wet season to dry season.

12. Since the foundation movements associated with soil expansion depend on relative moisture changes, pre-construction moisture conditions

by many authors (for example Hillel 1982, Richards 1974). These mechanisms include capillary and electrostatic attractive forces. Other systems such as the atmosphere can also attract water. This attraction can be described in terms of suction. The atmosphere with a low relative humidity has a great drying power, and therefore is characterized as having a high suction. If the atmosphere has a sufficiently low relative humidity to have a higher value of suction than nearby soil, moisture will flow out of the soil into the air. If the soil and surrounding atmosphere have the same values of suction, there will be no net movement of moisture from one medium to the other so the soil and air are said to be in moisture equilibrium.

16. The quantitative description of soil suction provides a basis for measuring suction with a psychrometer or a wet bulb/dry bulb thermometer (Lytton, 1977):

$$\tau = \frac{RT}{mg} \ln \left(\frac{H}{100}\right) \qquad (2)$$

where τ = soil suction measured in units pF

R = universal gas constant = 8.31×10^7 ergs/mole·°C

T = absolute temp, °Kelvin

m = mass of one mole of water, 18.02 gm/mole

g = gravitational constant, 980.66 cm/sec^2

H = relative humidity, %, of atmosphere in moisture equilibrium with soil

From Equation 2 it can be seen that soil with a suction of zero is in moisture equilibrium with an atmosphere with relative humidity of 100%. Values of soil suction corresponding to various conditions are presented in Table 1 in units of pF, psi, tsf and cm of water. The units pF correspond to the log to the base 10 of the value of soil suction expressed in cm of water.

17. Lytton (1977) states that the mechanism by which moisture is transported through expansive soils is poorly understood. It is not clear whether the dominant mode of transport acts on the adsorbed water, the vapor phase of water, or liquid phase. The rate at which moisture is transported under a given suction gradient is shown by Richards (1974) in Figure 1 as a function of the suction state of the soil. Lytton (1977) and Lytton

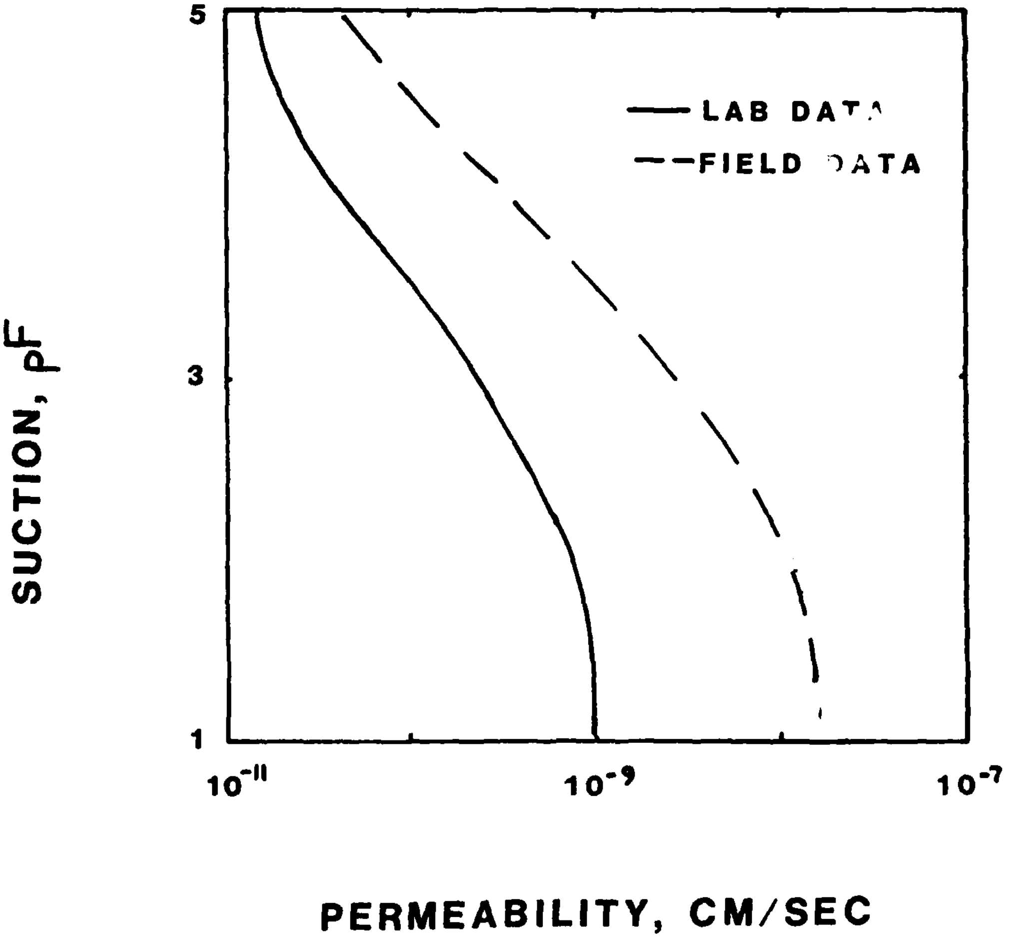


Figure 1. Typical relationship between coefficient of permeability and soil suction (after Richards, 1974)

z = elevation potential

Ω = overburden potential

For lightly loaded slabs and pavements the overburden potential may be small enough to neglect. For high values of soil suction, the gravitational component of the potential function may be relatively small and neglected.

20. Johnson and Desai (1975) and Johnson (1978) used the finite difference method to solve equations similar to Equation 3 to yield the distribution of soil moisture with time.

Beam on Winkler Subgrade

21. The displacements, moments, and shears in a mat foundation can be estimated by analyzing the behavior of a beam on a Winkler subgrade (Scott, 1981). The mat can be broken into a series of strips or beams to be analyzed separately. Alternately, the mat can be modelled as a slab on a Winkler or elastic subgrade. The beam approach involves simpler computations and has been preferred in the past although the slab approach offers a more reasonable approximation. The advantage of the beam analysis is becoming less and less significant because better computer programs are being developed and computer costs are plunging. Engineering practice for design of structures on mat foundations includes finite element structural analysis assuming a Winkler subgrade soil. Fraser and Wardle's (1975) FOCALS program models a stiffened mat on an elastic subgrade using a finite element approach, but such programs appear to require excessive effort for routine design applications.

Soil Heave

22. The computation of subgrade distortion resulting from changes in moisture alone has been recently studied by several investigators. Earlier work concentrated on determining the maximum potential vertical soil heave on the basis of soil index tests or swell tests (Vijayvergiya and Ghazzaly, 1973). Since the earlier works do not explicitly account for changes in soil moisture, they are not appropriate for use in a procedure that carefully analyzes the flow of moisture through the underlying soil and were not considered.

Distorted Subgrade Model

27. The distorted subgrade model was suggested by Lytton and Meyer (1971). Refinements have been suggested subsequently by Pitt (1982) and Wray (1978). The basis of the method is that the shape of the surface of a subgrade beneath a loaded mat foundation can be simply described by:

$$y = cx^m \tag{6}$$

in which y = vertical distance below point of maximum heave
c = constant
x = horizontal distance from center of slab
m = mound exponent

Values of differential structural movement can be determined if two other parameters can be estimated in addition to c and m. They are (a) the value of maximum subgrade heave at the edge or center of the slab (it is recommended to consider both); and (b) the point where soil and mat separate, described by the edge liftoff distance shown in Figure 2. The edge liftoff distance is related to the seasonal penetration of moisture change.

28. Values of the four mound parameters listed above must be estimated to determine differential mat movement from reference tables presented by Wray (1978). Wray also presents guidelines for choosing the values of these parameters, although he states that the edge parameter is the most difficult parameter to evaluate and no method is known to predict a reliable value. Therefore, it may be difficult to use the distorted subgrade models to make reliable predictions of mat performance in practice.

Coupled Moisture Flow and Elastic Deformation Models

29. A procedure that explicitly accounts for moisture flow and soil deformation has been suggested by Richards (1974). Other methods are described in a general way by Lytton (1977). Mitchell (1980) suggests a simplified method that accounts for moisture flow and soil deformation, but his suggested analysis method is not a coupled moisture flow elastic deformation model per se. Mitchell's method is considered in this section because his 1980 paper lays all the groundwork for a simple moisture flow-elastic deformation model that could be simple enough to use in practice.

30. The true coupled solutions for moisture flow and soil deformation under loading and moisture change are based on a 3-dimensional consolidation model for saturated soils by Biot (1941). These solutions often require a great deal of engineering effort and rely on soil parameters that are not routinely or economically available to foundation designers. Furthermore, the saturation assumption is not often satisfied. Therefore these methods have not been found useful in practice.

31. Mitchell (1980) published closed form solutions to the moisture flow problem beneath a slab. These solutions necessarily corresponded to cases that could be described by relatively simple boundary conditions. Soil-structure interaction was modelled as a beam on a distorted Winkler subgrade. The difference between Mitchell's method and other subgrade distortion models is that Mitchell computed the shape of the distorted subgrade using basic soil parameters and changes in moisture instead of using an empirical shape. Mitchell computed moisture changes and beam soil interactions to accurately estimate the shape of the distorted subgrade beneath lightly loaded slabs thus demonstrating the potential power of the model.

32. Mitchell's method was criticized by Pitt (1982) as being too simplistic. The boundary conditions used for Mitchell's closed form solutions were necessarily simple, as were the loading conditions and soil characterization. The design procedure suggested by Mitchell does not appear to be significantly different from the subgrade distortion models. The importance of Mitchell's paper is that it shows that relatively simple rational models for moisture flow and soil structure interaction have the potential to predict reasonable trends for mat foundation behavior that is consistent with observations.

Significant Findings

33. The literature review indicates that Mitchell's method appears to provide a rational, although simplistic, procedure for prediction of moisture diffusion beneath mat foundations. The edge penetration distance of seasonal soil moisture change beneath the mat is an important parameter for simplified mat design methods. Design moments, shears, and displacements depend on the value chosen for this parameter. It is difficult for designers to choose the edge penetration distance for the appropriate soil

PART III. COMPUTER PROGRAM BOSEF

Methodology for Analysis

34. Previous attempts at modelling soil structure interaction on an expansive subgrade have fallen into two categories. The first type of analysis assumes a distorted shape of the subgrade. The shears and moments are computed as the loaded foundation conforms to the subgrade (for example, Lytton, 1971). The second type of analysis couples moisture flow and mechanical interaction (for example Richards, 1974; Mitchell, 1980). A coupled analysis solves for both moisture flow and mechanical interaction. This approach requires a computer program and can require significant input effort and laboratory testing to determine the proper input parameters. The great effort involved apparently limits the applicability of coupled analyses to practice.

35. This project was initially envisioned as modifying the CON2D program (Duncan et al., 1981) to do coupled moisture flow and deformation analyses. During the writer's stay at WES, however, CON2D was being rewritten and was not available for modification. Therefore, it was chosen to devise a fresh approach to the analysis of beams on expansive subgrades. The desired approach would be computationally simple enough for direct use in practice yet rigorous enough for performing parametric research studies.

36. The analysis methodology developed for this study considers two problems that are solved independently: (a) moisture flow and the associated soil heave and (b) foundation deformation as a beam on elastic foundation. In actuality, the solution to the first problem, soil heave, depends on the change in vertical stress at each point below the foundation or the results of the second problem. The results of the second problem, beam on elastic foundation analysis, depends on the distortion of the subgrade from swelling, or the results of the first problem. For computational simplicity, each problem is solved separately and an iterative approach is followed until the solutions to the two problems are consistent. Figure 3 illustrates the methodology in general terms.

37. The method proposed here is similar in principle to the method developed by Mitchell (1980). Pitt (1982) contends that Mitchell's model is too simplistic for practical design of foundation mats. Mitchell obtained closed form solutions for several moisture flow boundary conditions. The

closed form nature of these solutions required that the boundary conditions be simple. The Mitchell solution accounts for mechanical interaction of a uniformly loaded beam on a distorted subgrade but ignores the important effect of increased vertical stress from the loaded beam on the magnitude of soil swell. This writer agrees with Pitt that Mitchell's procedure is simplistic, but nevertheless it has great potential because it explicitly models moisture-soil-structure interaction and is considerably more simple to use than previous coupled approaches. The method adopted here is essentially to solve the steps of the Mitchell methodology using numerical methods. By using numerical modelling, the Mitchell method is no longer limited to simplistic boundary conditions, simplistic loading conditions, and simplistic soil property characterization.

Moisture Flow

38. The results of the moisture flow analysis computed by BOSEF correspond to the case in which an impervious foundation slab of great length is placed on the surface of an expansive soil layer. The computed values of soil suction correspond to those that would exist if the moisture flow boundary conditions were applied for a long period of time. As discussed later, this case is not usually realistic and BOSEF should be modified to solve the moisture flow under transient conditions corresponding to seasonal change. The simplistic steady-state flow model was adopted because during the initial stages of this investigation it appeared to be a reasonable first approximation for long-term conditions beneath a large mat.

39. BOSEF models moisture flow through soil assuming:

(a) plane diffusion

(b) isotropic, homogeneous flow

(c) steady state flow

The flow under these conditions is described by the LaPlace Equation

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (7)$$

in which u = potential field causing flow, e.g. soil suction or a combination of soil suction and other factors

x, y = horizontal and vertical coordinates respectively

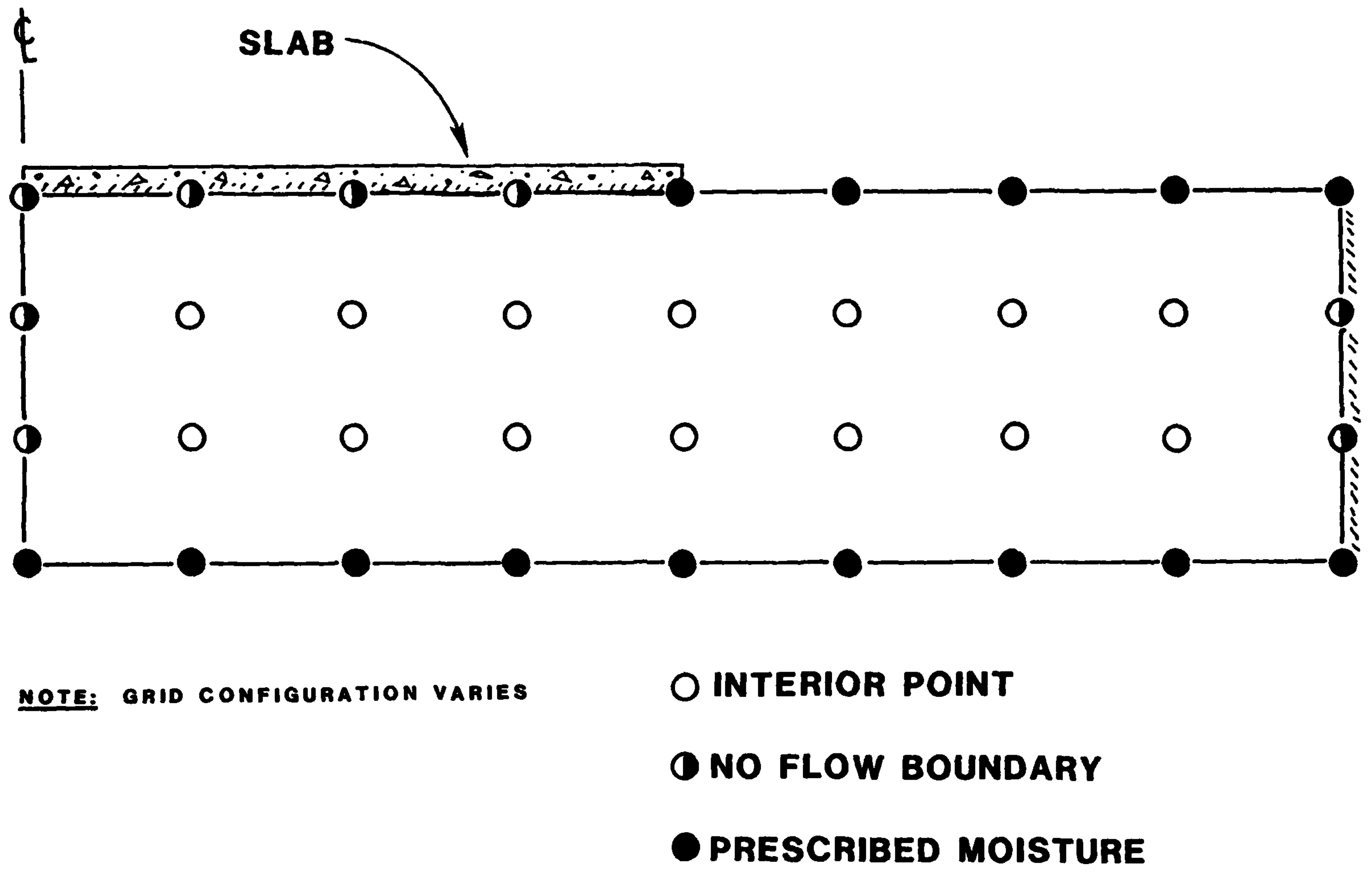


Figure 4. Moisture flow problem: finite difference idealization for two-dimensional moisture flow (plane flow)

e_o = initial void ratio, typically about 1 for many expansive soils

τ_{insitu} = insitu soil suction potential

α = compressibility factor ranging from about 0.33 to 1.0 (see Johnson, 1982)

σ_m = mean normal total stress = $(\sigma_v + 2K_T\sigma_v)/3$

σ_v = vertical total stress

K_T = coefficient of horizontal total earth stress.

Subscripts "o" and "f" in Equation 10 refer to initial and final conditions, respectively. The value of soil suction measured in the lab, τ_{lab}, reflects the stress relief of sampling since $\sigma_o = 0$ in the lab. Therefore,

$$\tau_{lab} \simeq (\tau_{insitu} + \alpha\sigma_m)_o \tag{11}$$

Equation 10 is rewritten in the form used by BOSEF:

$$S = C_\tau \left[\frac{H}{1+e_o}\right] \log \left[\frac{\tau_{lab}}{(\tau_{insitu} + \alpha\sigma_m)_f}\right] \tag{12}$$

Equation 12 requires values of the final mean total normal stress, σ_m. Computed values of swell, S, will therefore depend on the value of the contact stress between the foundation and underlying soil since the contact stress influences the value of mean total stress. The value of contact stress is part of the solution to the overall problem and not generally known until after the problem has been solved. For this reason, trial values of the contact stresses between the foundation and soil are initially input to BOSEF. If the trial values are not consistent with the values eventually computed by BOSEF, the problem is resolved using modified values of the contact stress. The procedure iterates until the assumed contact stresses and computed contact stresses are consistent.

46. The vertical stress beneath any point on the foundation is computed from the assumed contact stress, q, plus the total overburden stress. The increase in vertical total stress, $\Delta\sigma_v$, at a point below a loaded area is given by Craig (1978) for plane strain:

$$\Delta\sigma_v = \frac{q}{\pi}\left[\alpha + \sin\alpha\,(\cos\ (\alpha\ + 2\beta\ ))\right] \tag{13}$$

in which q = uniform contact stress of loaded area

α, β = angles defined in Figure 5 (in radians).

BOSEF computes the increase in stress at a point below a loaded beam by summing the $\Delta\sigma_v$ determined for each loaded beam segment.

Beam on Winkler Subgrade

47. After estimating the moisture changes and resultant subgrade distortion, the third step performed by BOSEF is to estimate foundation displacements, moments and shears by assuming the foundation to be a beam on an elastic foundation. Several references were useful for this type of analysis. Probably the most useful, were Scott (1981, Chapter 5), Lytton and Meyer (1971), and Bowles (1975).

48. If one simplifying assumption is valid, the solution is relatively straight forward. The assumption is that the foundation material can be described by exactly one constant parameter, k, the coefficient of subgrade reaction, where

$$k = \frac{q}{\delta} = \text{constant} \tag{14}$$

in which q = contact stress at soil surface

δ = displacement at soil surface.

A foundation subgrade described only by the value of the coefficient of subgrade reaction is called a Winkler subgrade. The Winkler subgrade can best be visualized as a series of vertical springs, each one acting independently with a spring constant equal to k. Figure 6a illustrates a beam on the Winkler subgrade.

49. An ideal elastic foundation material would be described by exactly two constants, E and ν, Young's modulus and Poisson's ratio respectively. Therefore, it can be anticipated that the Winkler assumption does not correctly model an ideal elastic material. This is shown in Figure 6b and c.

50. The fact that the Winkler subgrade does not correctly model the case of an ideal elastic subgrade has been advanced as motivation for using a more complex representation of the foundation called a coupled-Winkler foundation (Lytton and Meyer, 1971). For this representation, the foundation subgrade is represented by two parameters, k and a coupling coefficient between adjacent springs. The coupling coefficient can be thought analogously to a cable stretched between Winkler springs (Scott, 1981). For a beam, values of k and the coupling coefficient can be found to virtually duplicate the beam's behavior on an elastic subgrade with constants E and ν.

51. The coupling coefficient for the coupled-Winkler analysis is

difficult to evaluate in practice. Results nearly equivalent to those obtained by the coupled-Winkler approach can be obtained by judiciously varying the value of the coefficient of subgrade reaction, k, across a Winkler subgrade. The Winkler subgrade beneath the beam can be made to approximate an ideal elastic subgrade if softer values of k are used beneath the center of the beam and stiffer values are used around the edge. It should be noted that this variation of k is appropriate for soils that are approximately elastic, such as lightly loaded stiff clays. The appropriate variation of k for foundations on sand tends toward the opposite; stiffer values of k may be used beneath the center and softer values near the edges reflecting variations in soil confinement. The "correct" choice of k and its variation beneath a particular foundation may be open to interpretation.

52. For the reasons described above, it was felt that the simple representation of a beam on a Winkler subgrade should be incorporated into BOSEF. Commonly used procedures for choosing the coefficient of subgrade reaction are well documented (Terzaghi, 1955; Department of the Navy, 1982). Also, computed values of moments and shears are usually not particularly sensitive to refined values of the coefficient of subgrade reaction. BOSEF can be easily modified to account for any desired variation of k from center to edge of the beam.

53. The differential equation governing a beam on a Winkler subgrade is:

$$EI \frac{d^4y}{dx^4} + ky - q = 0 \qquad (15)$$

in which EI = flexural stiffness of beam, modulus times moment of inertia
y = vertical displacement
k = value of Winkler subgrade stiffness
q = loading on beam

Equation 15 has been solved in closed form (Hetenyi, 1946; Scott, 1981). Closed form solutions applicable to general loading conditions and to non-linear or non-uniform values of Winkler subgrade stiffness are quite complex and of limited use. Equation 15 is solved in BOSEF using the finite difference method. The beam is discretized into a group of 1-dimensional elements, each indicated by a node, as shown in Figure 7. Equation 15 is written for each node m on the beam in finite difference form using the recursion formula:

$$y_{m-2} - 4y_{m-1} + \left(6 + \frac{kh^4}{EI}\right)y_m - 4y_{m+1} + y_{m+2} = \frac{Q_m h^3}{EI} \qquad (16)$$

in which Q_m = equivalent nodal load at node m; and,

h = spacing between nodes.

54. Equation 16 results in one equation at each node m on the beam. To allow Equation 16 to be written for nodes at the end of the beam, four imaginary nodes must be considered - two at each end as shown in Figure 7. To develop a set of equations with matching numbers of equations and unknowns, four more equations must be developed. This can be done by considering equations corresponding to boundary conditions at the end of the beam.

$$EI\frac{d^2y}{dx^2} = 0 \qquad (17a)$$

$$EI\frac{d^3y}{dx^3} = \pm Q_e \qquad (17b)$$

in which Q_e = load applied to end of beam.

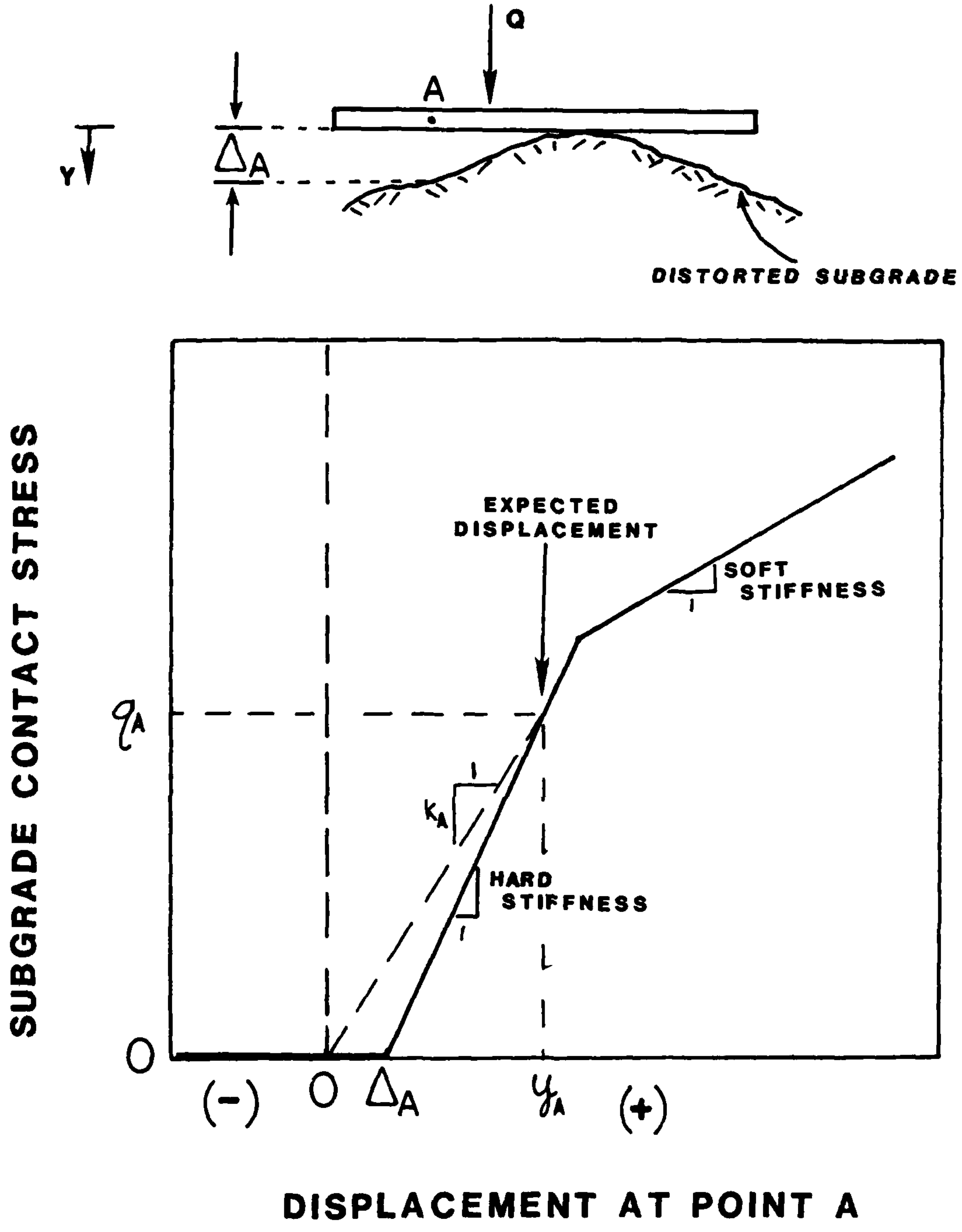
Equations 17a and 17b correspond to the condition of moment and shear at the end of the beam, respectively. The (±) in Equation 17b is assigned so that a downward load on the left end of the beam causes (+) shear and at the right end (-) shear. Equations 17a and 17b are written at both end points of the beam according to the recursion formulas:

$$y_{m-1} - 2y_m + y_{m+1} = 0 \qquad (18a)$$

$$y_{m-2} - 2y_{m-1} + 2y_{m+1} - y_{m+2} = \frac{\pm Q_e h^3}{EI} \qquad (18b)$$

For a beam modelled by n nodes, Equations 16 and 18 yield a set of n+4 equations with n+4 unknowns and are solved implicitly with Gaussian elimination.

55. For the formulation described above to be valid for a beam on a distorted subgrade, the value of k used in Equation 16 must depend on the displacement of the beam. If the beam displaces such that it is not in contact with the subgrade, the appropriate value of k is zero. Since the displacement, y, at any node is the problem solution, values of displace-



$$K_A = \frac{q_A}{y_A} = \text{WINKLER MODULUS AT POINT A}$$

Figure 8. Development of Winkler modulus from stress displacement curve for use in program BOSEF

(g)	HEAVE:	Computes magnitude of heave or settlement of soil surface due to changes in moisture and total stress.
(h)	SOLVE:	Solves a banded augmented matrix representing a set of simultaneous equations.
(i)	FILLM:	Sets up the coefficient matrix for beam on Winkler subgrade analysis.
(j)	MODULUS:	Determines appropriate secant value of the coefficient of subgrade reaction for use by the subroutine FILLM.

A flow chart showing the approximate execution sequence of BOSEF is shown in Figure 9.

60. BOSEF is written in FORTRAN 77 to run on the Honeywell Timesharing System (TSS) at Waterways Experiment Station. Based on limited experience with the program, practical problems can be executed on this system within a few minutes and at a cost of less than $2.00.

PART IV. CASE STUDIES

61. BOSEF was tested by examining four case studies. The first two cases involve beams on non-expansive soil. The remaining two case studies are comparisons of the BOSEF prediction and the observed displacements of actual mat foundations on expansive soils.

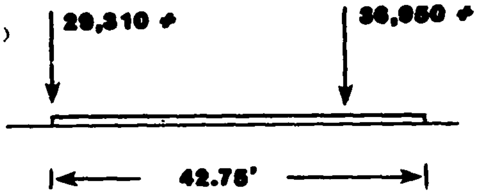
Beam on Winkler Subgrade (Bowles, 1975)

62. The purpose of this case study was to verify that BOSEF yielded results consistent with other analysis methods for a beam on a non-expansive soil. Published displacements, shears, moments and contact stresses are compared with the results from the BOSEF analysis in Figure 10. The only significant differences occur in the computed values of shear in the vicinity of the right hand load, and to a lesser extent the moment values. The explanation for the difference is thought to result from differences in the discretization of the problem and that Bowles does not consider a true point loading on the beam. By inspection, it should be apparent that the BOSEF solution is more intuitively satisfying for the loading condition shown since there is an implied discontinuity in the shear diagram at the point of loading. Sample input and output for this problem are presented in Figure 11.

Beam on Winkler Subgrade (Scott, 1981)

63. Scott (1981) presents a closed form solution for the center displacement of beam on a Winkler subgrade with a concentrated load applied at the center. BOSEF solutions for a beam with a concentrated load at the center on non-expansive and expansive subgrades are compared with Scott's solution in Figure 12. Scott's solution for the center displacement of the beam on a non-expansive soil and the BOSEF solution for the same case yield virtually identical results.

64. In Figure 12 the BOSEF solution for the beam on a swollen subgrade of expansive soil is also shown. Important differences in the solution for the expansive and non-expansive cases can be seen. First, on the expansive subgrade, BOSEF predicts that the edge of the beam will lift off the ground. It should be interjected that BOSEF does not make adjustment to the moisture flow problem in this case. It would be desirable to do so



```
 FRN
BEAM ON SWELLING ELASTIC FOUNDATION (BOSEF)
UNITS=FT, LBS  PER FT OF WIDTH
PRINT INTERMEDIATE RESULTS?  1=YES  0=NO
=0
 *DIFFUSION PROBLEM INPUT:
   NV,DY,NH,DX,NS
=6,1,10,2.375,5
  BSB,   BST
=400,400
 *BEAM PROBLEM INPUT:
 NO. OF NODES IN BEAM (INCL 4 IMAGINARY NODES),  EI
=24,5.63E8
INPUT NODAL LOADS  -- REAL NODES ONLY
=29310,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,36950,0,0
ASSUMED CONTACT STRESSES -- REAL NODES ONLY
=5000,5000,5000,5000,5000,5000,5000,5000,5000,5000
=5000,5000,5000,5000,5000,5000,5000,5000,5000,5000
INPUT HARD,SOFT STIFFNESS
=152000
=152000
INPUT NV VALUES OF LAB MEAS SOIL SUCTIONS  TOP FIRST
=400,490,580,670,760,850
NV VALUES,INITIAL VOID RATIO AT EACH DEPTH (TOP FIRST)
=1,1,1,1,1,1
INPUT    SUCTION INDEX,KO,ALPHA,UNIT WT
=0,1,1,90
-------------------------- ------------------------------

SOLUTION CONVERGED AFTER  2 ITERATIONS

*FINAL RESULTS
UNITS = FT    LBS    PER FT OF BEAM WIDTH
DISP:  (+)=SETTLEMENT    (-)=HEAVE
 1      2          3          4          5        6         7           8
NOD    HEAV        RDIS       ADIS       ACST     CCST      MOM         SHR
 3      0.         0.0279     0.0279     4243.    4243.          3.    -29309.
 4      0.         0.0266     0.0266     4041.    4041.     -27191.    -25167.
 5      0.         0.0253     0.0253     3846.    3846.     -50332.    -21221.
 6      0.         0.0241     0.0241     3665.    3665.     -69633.    -17469.
 7      0.         0.0230     0.0230     3503.    3503.     -85270.    -13882.
 8      0.         0.0221     0.0221     3364.    3364.     -97397.    -10447.
 9      0.         0.0214     0.0214     3250.    3250.    -106164.     -7143.
10      0.         0.0208     0.0208     3166.    3166.    -111683.     -3933.
11      0.         0.0205     0.0205     3112.    3112.    -114029.      -794.
12      0.         0.0203     0.0203     3088.    3088.    -113270.      2307.
13      0.         0.0204     0.0204     3096.    3096.    -109416.      5400.
14      0.         0.0206     0.0206     3132.    3132.    -102470.      8513.
15      0.         0.0210     0.0210     3197.    3197.     -92389.     11679.
16      0.         0.0216     0.0216     3286.    3286.     -79111.     14919.
17      0.         0.0223     0.0223     3396.    3396.     -62551.     18260.
18      0.         0.0232     0.0232     3524.    3524.     -42591.     21723.
19      0.         0.0241     0.0241     3663.    3663.     -19106.     25316.
20      0.         0.0250     0.0250     3807.    3807.       8041.     10575.
21      0.         0.0260     0.0260     3949.    3949.       2045.     -4021.
22      0.         0.0269     0.0269     4091.    4091.          0.         0.
1=NODE 2=HEAVE 3=DISP WRT MAX HEAVE 4=DISP WRT ORIG SURF
5=ASSUMED CONTACT STRESS  6=COMPUTED CONTACT STRESS
7=MOMENT PER FT OF BEAM WIDTH  8=SHEAR PER FT OF WIDTH
```

Figure 11. Input and output for BOSEF program for Figure 10

because the boundary condition of no through flow assumed by BOSEF is not reasonable where the beam has lifted off the ground. Second, as expected, the displacement trends are somewhat similar for the two cases, but not identical. Third, the BOSEF solution shows the beam on expansive soil to have both upward and downward displacements with respect to the initial condition. Program results for the expansive case are shown in Figure 13.

Swinburne Tests

65. Mitchell (1980) presents the results of observations of displacements of a 7.4-m-square lightly-loaded test slab on expansive soil. The original data were published by Holland (1978) for tests performed at Swinburne College in Australia.

66. The slab was constructed at the end of the dry season on top of a lift of sand. During the subsequent wet seasons, the area was flooded which permanently moistened the sand. Attempting to model this slab with BOSEF exposed the following inadequacies in the program:

(a) BOSEF could not model the constant soil suction applied beneath the slab year round by the moist bed of sand between the slab and subgrade

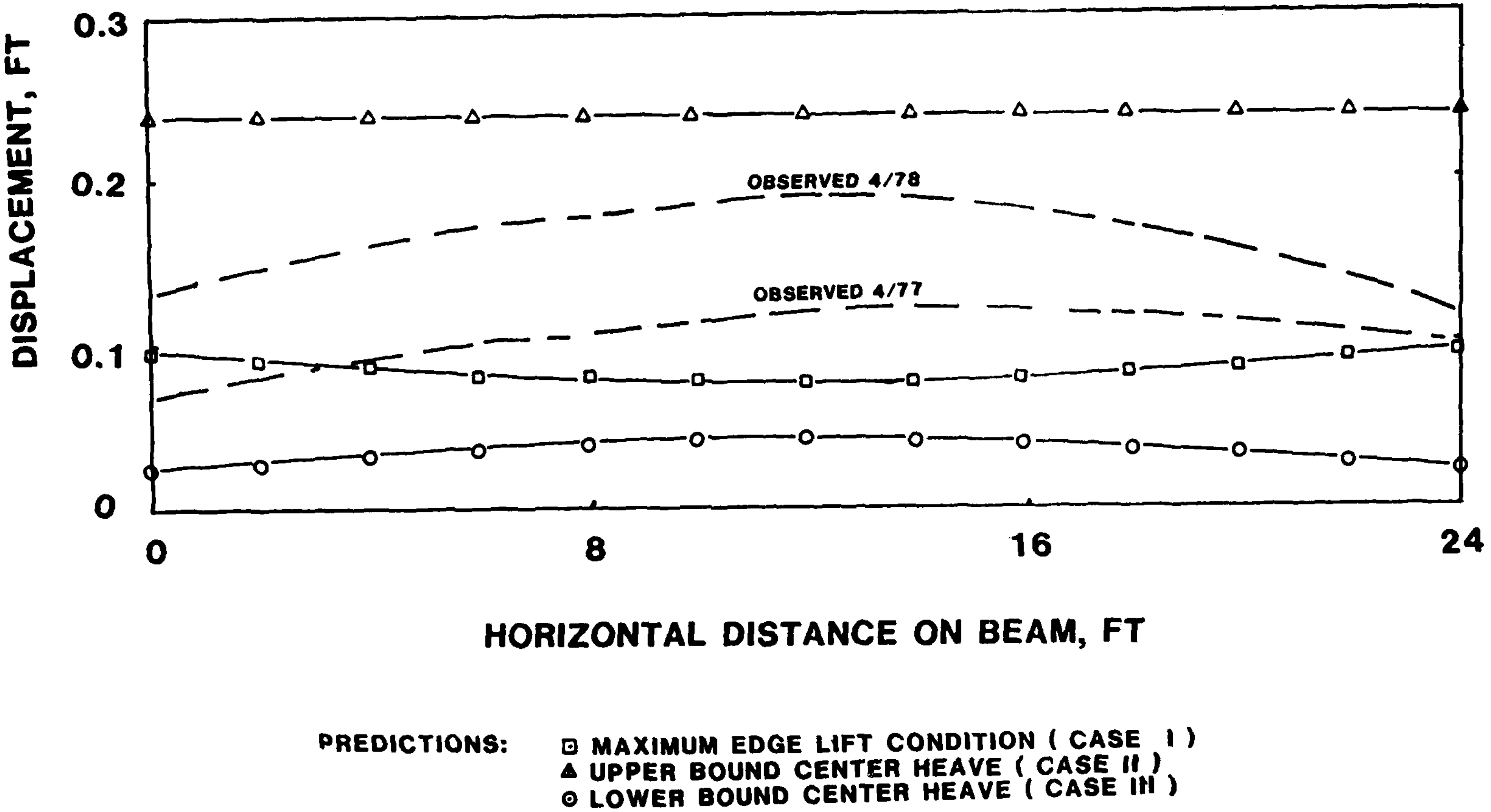
(b) BOSEF could not model the transient moisture boundary conditions applied to the soil surface during wet and dry seasons.

BOSEF could, however, adequately model 3 limiting moisture conditions for the Swinburne test slab.

67. The results of BOSEF analyses for these 3 limiting cases are shown in Figure 14. The final moisture regimes for the three cases are listed below:

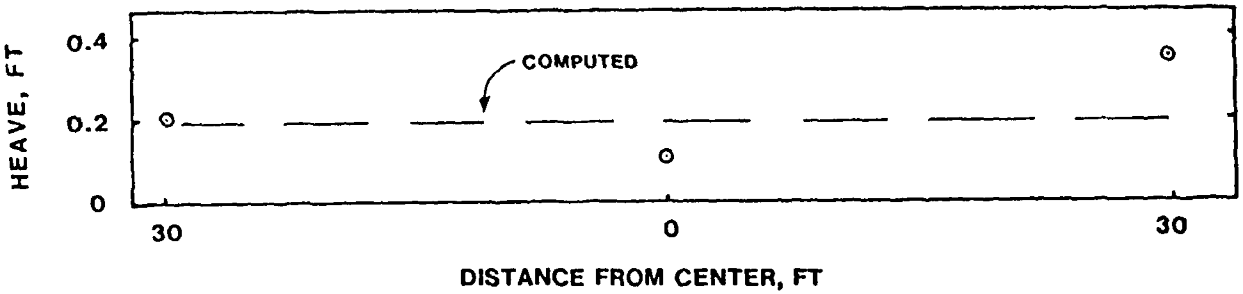
(I) Steady-state moisture flow corresponding to continued edge wetting around the slab but drying through the subsoil: this condition reasonably models the edge displacements since continued edge wetting simulates the moisture supplied by the moist sand beneath the slab. However, because the drying at depth assumed by BOSEF for this case, heave at the center of the slab is underestimated.

(II) Low values of final soil suction corresponding to soil moisture after prolonged periods of flooding. This case overestimates moisture availability and the displacements of the slab since

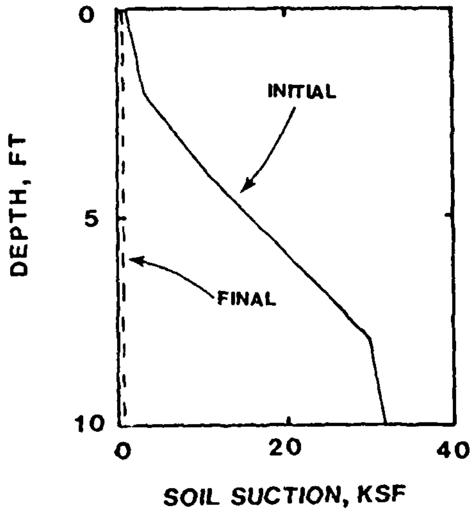


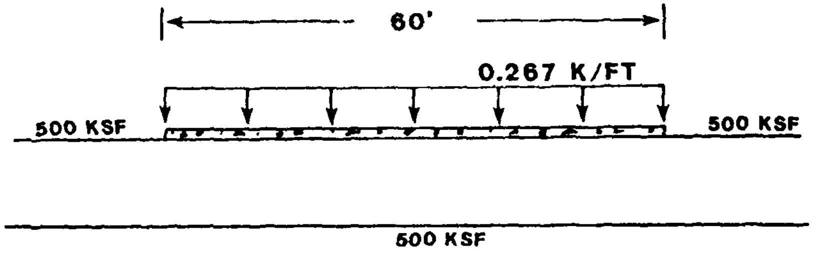
SEE TEXT FOR EXPLANATION OF CASES

Figure 14. Comparison of predicted and observed mat displacements using program BOSEF, Swinburne tests on 24-ft-square slab



A. COMPARISON OF OBSERVED AND PREDICTED HEAVE





C. FINAL BOUNDARY SUCTION VALUES

B. SOIL MOISTURE PROFILE

Figure 15. Comparison of predicted and observed mat displacement using program BOSEF, Pest Manage Facility slab

> "...differential movement of expansive clay foundations were largely out of control of the designer and owner. These factors included: location and growth of trees, shrubs, and flowerbeds; broken drain and sewer pipe; down hill creep; unloading due to cuts; cut and fill with poorly compacted downhill fill; depth of water table; antecedent and subsequent moisture conditions; and micro-geologic variables such as cracks, fissures, decayed roots, slip lines and planar pores".

These factors can hardly be expected to be accurately accounted for in foundation analysis since they are uncertainties at design time. Nevertheless, if suitable moisture flow boundary conditions and soil properties can be established for the factors listed above, BOSEF would be a rational method for exploring the nature of the differential movements that could develop.

73. This methodology does not require a choice of edge penetration distance. Instead, other parameters, namely soil properties and environmental moisture conditions, are used to estimate the behavior of the foundation. The proposed methodology appears to be rational but is limited by our ability to determine and quantify pertinent soil properties and environmental conditions. Progress has been made in recent years to estimate reliability these parameters, although it is apparent that more research is needed.

the determination of reliable edge penetration parameters. Simply these parameters describe the distance from the edge of the mat that seasonal moisture variations are important. It is quite likely that seasonal moisture variations are unimportant beneath the center of a large mat, but the seasonal variations are probably critical near the edge.

(d) Correlations of soil moisture diffusivity with soil index properties or standardized test methods for predicting soil moisture diffusivity are necessary before a procedure such as BOSEF can be used in practice to model transient moisture flow. Testing and research should be undertaken to develop the required data base. This work is important because the fundamental problem for moisture flow analyses is the determination of the value of soil moisture diffusivity, i.e. the parameter describing how quickly moisture moves through the soil. Field measurements and reliable laboratory techniques to measure this parameter are needed.

(e) Probabilistic characterization of initial soil suctions, soil suction index, and moisture diffusivity is needed. Since moderately loaded mat foundations may be several hundred feet long, the variation of soil swelling characteristics needs to be explored. It could be that variation of soil properties is nearly as important in determining mat displacements, moments and shears as moisture variations.

(f) The BOSEF model should be extended to that of a plate on a Winkler subgrade with a 3-D moisture flow model. Some of the simplicity and computational economy of BOSEF would be compromised by this refinement, but studies should be done to ascertain how well the 2-dimensional flow model and one-dimensional beam model describe a realistic mat foundation.

76. An interim solution to the complex problem of analysis of mats on swelling, elastic subgrades is to analyze the worst case situation that occurs over the long-term. The proposed modified version of program BOSEF can be used to determine the worst case for given input parameters. Relatively simple design equations can subsequently be developed through parametric analysis to assist the design of mat foundations. Such design equations, however, may contain soil input parameters that have not usually been determined for design.

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APPENDIX A:

PROGRAM LISTING FOR BOSEF

```
590 SUBROUTINE INPUT
600C******************************************************************
610C  READS INPUT DATA
620C******************************************************************
630 PARAMETER M=10,MM=25,NO=50,NN=10
640 COMMON/AREA1/NV,DY,NH,DX,NS,BSB,BST,F(NO,NO),K,IFLAG
650 COMMON/AREA2/P(MM),PA(MM),XK(MM,2),N,EI,S
660 COMMON/AREA5/Y(MM),SINIT(NN),EO(NN),CT,XKO,ALPHA,GAMMA
670C  INPUT VARIABLES FOR DIFFUSION PROBLEM
671 PRINT,'BEAM ON SWELLING ELASTIC FOUNDATION (BOSEF)'
672 PRINT,'UNITS=FT, LBS  PER FT OF WIDTH'
673 PRINT,'PRINT INTERMEDIATE RESULTS?  1=YES  0=NO'
674 READ,IFLAG
680 PRINT,' *DIFFUSION PROBLEM INPUT:'
690 PRINT,'   NV,DY,NH,DX,NS'
700 READ,NV,DY,NH,DX,NS
710 PRINT,'  BSB,   BST'
720 READ,BSB,BST
730C  INPUT VARIABLES FOR BEAM BENDING PROBLEM
740 PRINT,' *BEAM PROBLEM INPUT:'
750 PRINT,' NO. OF NODES IN BEAM (INCL 4 IMAGINARY NODES),  EI'
760 READ,N,EI
770 S=(((2*NS)-2)*DX)/(N-5)
780 PRINT,'INPUT NODAL LOADS  -- REAL NODES ONLY'
790 READ,(P(II),II=3,N-2)
800 PRINT,'ASSUMED CONTACT STRESSES -- REAL NODES ONLY'
810 READ,(PA(II),II=3,N-2)
820 PRINT,'INPUT HARD,SOFT STIFFNESS'
830C  HARD STIFFNESS FOR FIRST .1 DISP,  SOFT STIFF FOR LARGER DISP
840 READ,XK(3,1),XK(3,2)
850 DO 10 I=4,N-2
860 XK(I,1)=XK(3,1)
870 XK(I,2)=XK(3,2)
880 10 CONTINUE
890 PRINT,'INPUT NV VALUES OF LAB MEAS SOIL SUCTIONS  TOP FIRST'
900 READ,(SINIT(I),I=1,NV)
910 PRINT,'NV VALUES,INITIAL VOID RATIO AT EACH DEPTH (TOP FIRST)'
920 READ,(EO(I),I=1,NV)
930 PRINT,'INPUT    SUCTION INDEX,KO,ALPHA,UNIT WT'
940 READ,CT,XKO,ALPHA,GAMMA
945 PRINT,'----------------------------------------------------------'
946 PRINT,' '
950 RETURN
960 END
```

```
1610 I=NS+NH-1
1620 F(I,I-1)=2*C
1630 F(I,I+NH)=B
1640 F(I,K+1)=-1*B*BST
1650C**  (NODES (NS+NH) TO ((NS-1)+(NV-3)*NH) REMAINING NODES EXCEPT
1660C**                        LAST NUMBERED ROW
1670C**  I CORRESPONDS TO NODE NUMBER
1680
1690 DO 250 II=3,NV-2
1700C * LEFTMOST NODE IN ROW II OF FD MESH
1710 I=(II-1)*NH-NS
1720 F(I,I-NH)=B
1730 F(I,I+NH)=B
1740 F(I,I+1)=2*C
1750 III=I+1
1760 IIII=I+NH-2
1770C * INTERIOR NODES  ROW II
1780 DO 240 I=III,IIII
1790 F(I,I-NH)=B
1800 F(I,I-1)=C
1810 F(I,I+1)=C
1820 F(I,I+NH)=B
1830 240 CONTINUE
1840C * RIGHTMOST NODE  ROW II
1850 I=(NS+((II-1)*NH))-1
1860 F(I,I-NH)=B
1870 F(I,I-1)=2*C
1880 F(I,I+NH)=B
1890 250 CONTINUE
1900C (NS+((NV-3)*NH)) TO ((NS-1)+(NV-2)*NH)))  BOTTOM NUMBERED ROW
1910 I=(NS+((NV-3)*NH))
1920C LEFTMOST NODE IN BOTTOM NUMBERED ROW, IE ROW(NV-1)
1930 F(I,I-NH)=B
1940 F(I,I+1)=2*C
1950 F(I,K+1)=-1*B*BSB
1960C INTERIOR NODES IN BOTTOM NUMBERED ROW
1970 II=I+1
1980 III=NS+((NV-2)*NH)-2
1990 DO 300 I=II,III
2000 F(I,I-NH)=B
2010 F(I,I-1)=C
2020 F(I,I+1)=C
2030 F(I,K+1)=-1*B*BSB
2040 300 CONTINUE
2050C LAST NUMBERED NODE
2060 I=NS+((NV-2)*NH)-1
2070 F(I,I-NH)=B
2080 F(I,I-1)=2*C
2090 F(I,K+1)=-1*B*BSB
2100C  FD MATRIX IS COMPLETE AND READY FOR SOLUTION
2110 CALL SOLVE (K,NH,F)
2120C  VALUE OF POTENTIAL AT NODE N IS IN F(N,K+1)
2125 IF(IFLAG.EQ.0)GO TO 900
2130C PRINT SUCTIONS
2140 PRINT,' '
2150 PRINT,'*DIFFUSION PROBLEM: '
2160 PRINT,'COMPUTED SUCTION VALUES AT DIFFUSION NODES'
2170 DO 700 I=1,K,4
2180 PRINT 699,I,I+3,F(I,K+1),F(I+1,K+1),F(I+2,K+1),F(I+3,K+1)
2190 699 FORMAT(I5,'-',I2,4F9.2)
2200 700 CONTINUE
2210 900 RETURN
```

```
2770 L=J
2780 IF(BC.GE.HC(J)-.0001.AND.BC.LE.HC(J+1)+.0001)GO TO 250
2790 240 CONTINUE
2800 PRINT,'SOMETHING'S WRONG WITH THE INTERPOLATION SCHEME'
2810 GO TO 300
2820 250 DO 290 JJ=1,NV
2830C NODE I ON BEAM IS BETWEEN HORIZ COORDS HC(J) & HC(J+1) IN FD MESH
2840C JJ=ROW NUMBER FOR DIFFUSION MESH
2850C INTERPOLATE TO FIND VALUE OF SUCTION AT NV POINTS BELOW NODE I
2860 CALL INTERP(HC(L+1),HC(L),SS(JJ,L+1),SS(JJ,L),BC,BB(JJ,I))
2870 290 CONTINUE
2880 300 CONTINUE
2890C ******* PRINT VALUES OF INTERPOLATED SOIL SUCTION
2900 IF(IFLAG.EQ.1)PRINT,'INTERPLTD SOIL SUCTIONS BENEATH BEAM'
2905 IF(IFLAG.EQ.0)GO TO 600
2910 DO 490 J=1,N
2920 PRINT 498,J,(BB(I,J),I=1,NV)
2930 498 FORMAT(I3,10F8.0)
2940 490 CONTINUE
2950 600 RETURN
2960 END
2970 SUBROUTINE INTERP(XU,XL,YU,YL,XI,YI)
2980C*******************************************************************
2990C  LINEAR INTERPOLATION
3000C FINDS YI BY LINEAR INTERPOLATION
3010 YI=YL+((XI-XL)/(XU-XL))*(YU-YL)
3020 RETURN
3030 END
```

```
3520 SUBROUTINE HEAVE
3530C*******************************************************************
3540C COMPUTES SURFACE HEAVE BELOW LOADED BEAM
3550C  HEAVE IS COMPUTED AT BEAM NODES 3 TO N-2 ONLY
3560C HEAVE BELOW IMAGINARY NODES IS EXTRANEOUS
3570C   XKO - KO
3580C   D(I) - DEPTH VECTOR FOR ROWS OF FD MESH POINTS BELOW BEAM
3590C   CT  - SUCTION INDEX
3600C   EO(I) - INITIAL VOID RATIO VS DEPTH
3610C   SINIT(I) - INITIAL SUCTION AS MEASURED IN LAB
3620C  SINIT(I)= FIELD SUCTION +OVERBURDEN (TOTAL STRESS)
3630C   ALPHA - ALPHA, COMPRESSIBILITY FACTOR
3640C   GAMMA - UNIT WT OF SOIL
3650C   BB(I,J) - FINAL SUCTIONS, INTERPOLATED AT POINTS DIRECTLY BELOW BE
3660C  Y(J)   -  HEAVE CALCUTED AT BEAM NODES
3670C  DSIG(I,J) - CHANGE IN STRESS BELOW NODE ON BEAM
3680C  I - ROW OR DEPTH       J- COLUMN OR BEAM NODE NO.
3690C*****************************************************************
3700 PARAMETER M=10,MM=25,NO=50,NN=10
3710 DIMENSION TI(NN),D(NN)
3720 COMMON/AREA1/NV,DY,NH,DX,NS,BSB,BST,F(NO,NO),K,IFLAG
3730 COMMON/AREA2/P(MM),PA(MM),XK(MM,2),N,EI,S
3740 COMMON/AREA3/BB(M,MM)
3750 COMMON/AREA4/DSIG(M,MM)
3760 COMMON/AREA5/Y(MM),SINIT(NN),EO(NN),CT,XKO,ALPHA,GAMMA
3770C** COMPUTES SWELL FOR EACH NODE AT EACH LAYER
3780C**  T=SUCTION
3790C*  SWELL=CT*H/(1+EO)*LOG((TERM INIT)/(TERM FINAL))
3800C     IN WHICH
3810C                TERM=(INSITU SUCTION)+(ALPHA*(MEAN STRESS))
3820 TI(1)=DY/2
3830 TI(NV)=DY/2
3840 D(1)=DY/4
3850 D(NV)=(NV-1)*DY-DY/4
3860 DO 10 I=2,NV-1
3870 TI(I)=DY
3880 D(I)=(I-1)*DY
3890 10 CONTINUE
3900 DO 100 J=3,N-2
3910 Y(J)=0
3920 DO 99 I=1,NV
3930 TINIT=(SINIT(I))
3940 TFINAL=(BB(I,J)+ALPHA*(1+2*XKO)/3*(GAMMA*D(I)+DSIG(I,J)))
3950 SWELL=(CT*TI(I)/(1+EO(I)))*ALOG10((TINIT/TFINAL))
3960 Y(J)=Y(J)+SWELL
3970 99 CONTINUE
3980 100 CONTINUE
3985 IF(IFLAG.EQ.0)GO TO 300
3990 PRINT,'*BEAM PROBLEM'
4000 PRINT,' NODES     COMPUTED HEAVE (-)=SETTLEMENT'
4010 DO 200 I=3,N-2,5
4020 PRINT 201,I,I+4,(Y(J),J=I,I+4)
4030 201 FORMAT(I3,'-',I3,5F8.3)
4040 200 CONTINUE
4050 300 RETURN
4060 END
```

```
4370 SUBROUTINE FILLM
4380C****************************************************************
4390C FILL FD MATRIX FOR BEAM ON ELASTIC FNDTN ANALYSIS
4400C ACCORDING TO THE FOLLOWING SCHEME
4410C         (1)    (-4)   (6+YK(I)*S**4)/EI     (-4)    (1)
4420C THEN SOLVE FOR BEAM DISPL
4430C ITERATE UNTIL COMPUTED CONTACT STRESS = ASSUMED CONT STRESS
4440C NOTE ASSUMED CONTACT STRESS WAS USED TO DETERMINE MODULUS
4450C YK=SECANT MODULUS
4460 PARAMETER M=10,MM=25,NO=50,NN=10
4470 DIMENSION G(NO,NO),YK(MM),A(MM,6),PAC(MM)
4475 COMMON/AREA1/NV,DY,NH,DX,NS,BSB,BST,F(NO,NO),K,IFLAG
4480 COMMON/AREA2/P(MM),PA(MM),XK(MM,2),N,EI,S
4490 COMMON/AREAS/Y(MM),SINIT(NN),EO(NN),CT,XKO,ALPHA,GAMMA
4500 COMMON/AREA6/DISP(MM),XHEAVM
4510C G=FD MATRIX      YK=COEFF OF SUBGRD REACTION INTERP FRUM A
4520C A=TABLE A = LOAD DISP VALUES FOR NONLINEAR DISTORTED SUBGRADE
4530C PAC-VALU OF BEAM-SOIL CONT STR COMPUTED BY THIS ROUTINE
4540C P=VALUES OF NODAL LOADS
4550C  PA= VALUES OF BEAM-SOIL CONTACT STRESSASSUMED FOR CURRENT ITER
4560C  XK=VALUES OF HARD AND SOFT STIFFNESSES
4570C N,EI,S= # NODES  EI  SPACING
4580C DISP=DISP W.R.T. XHEAVM FOR BEAM ON ELAS FNDN
4590C XHEAVM= MAX VALUE OF HEAVE COMPUTED BY SUBR HEAVE
4600C NUMITER= # ITER FOR BEAM PROB
4610C Y=VAL OF HEAVE INPUT TO THIS SUBR;; CHNGED BELOW TO XHEAVM-Y
4620C SINIT = INITIAL SUCTION  (FIELD VALUE)
4630C EO= INITIAL VOID RATIO
4640C CT=SUCTION INDEX
4650C XKO=HORIZ EARTH PRESS COEFF
4660C ALPHA=COMPRESS COEFF (PERCENT SWELL IN VERT DIR)
4670C GAMMA=TOTAL UNIT WT OF SOIL
4680 NUMITER=20
4690 XHEAVM=-1.
4695 K=N+1
4700 DO 20 I=3,N-2
4710 XHEAVM=AMAX1(Y(I),XHEAVM)
4720 20 CONTINUE
4730 DO 25 I=3,N-2
4740 Y(I)=XHEAVM-Y(I)
4750 25 CONTINUE
4760 NITER=0
4770 50 CALL MODULUS (XK,PA,A,Y,YK,N)
4780 NITER=NITER+1
4790 G(1,N+1)=-2*P(3)*(S**3)/EI
4800 G(2,N+1)=0
4810 G(N-1,N+1)=0
4820 G(N,N+1)=2*P(N-2)*(S**3)/EI
4830 DO 100 I=3,N-2
4840 G(I,I-2)=1
4850 G(I,I-1)=-4
4860 G(I,I)=6+(YK(I)*(S**4)/EI)
4870 G(I,I+1)=-4
4880 G(I,I+2)=1
4890C NODAL LOADS
4900 G(I,N+1)=P(I)*(S**3)/EI
4910 IF(I.EQ.3.OR.I.EQ.N-2)G(I,N+1)=0
4920 100 CONTINUE
4930 G(1,1)=1
4940 G(1,2)=-2
4950 G(1,4)=2
```

```
5450 SUBROUTINE MODULUS(XK,PA,A,Y,YK,N)
5460C********************************************************************
5470C  MODULUS DETERMINES SECANT MOD FROM TABLE A
5480C Y(I)= (MAXIMIUM HEAVE BELOW BEAM)-(HEAVE AT POINT I)
5490C  XK(I,1 AND 2) = HARD AND SOFT STIFFNESS AT EACH NODE I
5500C PA(I)=CONTACT STRESS AT NODE I
5510C DISP= INTERPOLATED DISPLACEMENT FROM P-Y CURVE FOR CONTACT STRESS P
5520 PARAMETER M=10,MM=25,NO=50,NN=10
5530C YK(I) = SECANT MODULUS OF SUBGRADE REACTION
5540C A=P-Y  ABLE
5550C N=NO. OF NODES, INCLUDING 4 IMAGINARY NODES
5560 DIMENSION XK(MM,2),PA(MM),Y(MM),A(MM,6),YK(MM)
5570C P-Y TABLE = A MATRIX
5580C   ROW COLUMN     1        2            3      4        5       6
5590C         1           0       HEAVE,NODE 1  .1*XK   ,1+HEAVE  10*XK  1
 5595
5600 DO 20 I=3,N-2
5610 A(I,2)=Y(I)
5620 A(I,3)=.1*XK(I,1)
5630 A(I,4)=A(I,2)+.1
5640 A(I,5)=A(I,3)+10*XK(I,2)
5650 A(I,6)=A(I,4)+10
5660 20 CONTINUE
5670 DO 100 I=3,N-2
5680C FIND INTERPOLATION POINTS FOR P-Y DATA
5690 IF(PA(I).LE.A(I,1))GO TO 70
5700 IF(PA(I).LE.A(I,3))GO TO 80
5710 IF(PA(I).LE.A(I,5))GO TO 90
5720 PRINT,' DISPLACEMENT OUT OF BOUNDS'
5730 GO TO 250
5740 70 YK(I)=0
5750 GO TO 100
5760 80 CALL INTERP(A(I,3),A(I,1),A(I,4),A(I,2),PA(I),DISP)
5770C COMPUTE SECANT MODULUS YK(I)
5780 IF(DISP.LT..00001)YK(I)=0
5790 IF(DISP.LT..00001)GO TO 100
5800 YK(I)=PA(I)/DISP
5810 GO TO 100
5820 90 CALL INTERP(A(I,5),A(I,3),A(I,6),A(I,4),PA(I),DISP)
5830C COMPUTE SECANT MODULUS, YK(I)
5840 IF(ABS(DISP).LT..001)YK(I)=0
5850 IF(ABS(DISP).LT..001)GO TO 100
5860 YK(I)=PA(I)/DISP
5870 100 CONTINUE
5880 250 RETURN
5890 END
```

APPENDIX B:

BOSEF INPUT AND OUTPUT FOR SWINBURNE TEST AND PEST MANAGEMENT FACILITY

```
BEAM ON SWELLING ELASTIC FOUNDATION (BOSEF)
UNITS=FT, LBS  PER FT OF WIDTH
PRINT INTERMEDIATE RESULTS?  1=YES  0=NO
=0
 *DIFFUSION PROBLEM INPUT:
   NV,DY,NH,DX,NS
=6,1,10,3,5
  BSB,    BST
=4000,800
 *BEAM PROBLEM INPUT:
 NO. OF NODES IN BEAM (INCL 4 IMAGINARY NODES),  EI
=17,1.7E7
INPUT NODAL LOADS  -- REAL NODES ONLY
=170,340,340,340,340,340,340,340,340,340,340,340,170
ASSUMED CONTACT STRESSES -- REAL NODES ONLY
=170,170,170,170,170,170,170,170,170,170,170,170,170
INPUT HARD,SOFT STIFFNESS
=25000,25000
INPUT NV VALUES OF LAB MEAS SOIL SUCTIONS  TOP FIRST
=15000,11970,8380,6400,4410,4450
NV VALUES,INITIAL VOID RATIO AT EACH DEPTH (TOP FIRST)
=1,1,1,1,1,1
INPUT    SUCTION INDEX,KO,ALPHA,UNIT WT
=.12,1,1,90
-----------------------------------------------------------


*FINAL RESULTS
UNITS = FT    LBS    PER FT OF BEAM WIDTH
DISP:  (+)=SETTLEMENT    (-)=HEAVE
 1      2          3          4          5       6       7          8
NOD   HEAV        RDIS       ADIS       ACST    CCST    MOM        SHR
 3     0.1226     0.0309    -0.0918    775.    772.       -0.      -170.
 4     0.0976     0.0361    -0.0865    276.    277.     1204.       709.
 5     0.0861     0.0410    -0.0816    112.    112.     2837.       758.
 6     0.0797     0.0453    -0.0773     59.     59.     4238.       589.
 7     0.0769     0.0486    -0.0740     71.     72.     5193.       379.
 8     0.0751     0.0507    -0.0720     77.     77.     5755.       188.
 9     0.0743     0.0513    -0.0713     76.     76.     5943.        -0.
10     0.0751     0.0507    -0.0720     77.     77.     5755.      -188.
11     0.0769     0.0486    -0.0740     71.     72.     5193.      -379.
12     0.0797     0.0453    -0.0773     59.     59.     4238.      -589.
13     0.0861     0.0410    -0.0816    112.    112.     2837.      -758.
14     0.0976     0.0361    -0.0865    276.    277.     1204.      -709.
15     0.1226     0.0309    -0.0918    775.    772.        0.       170.
1=NODE 2=HEAVE 3=DISP WRT MAX HEAVE 4=DISP WRT ORIG SURF
5=ASSUMED CONTACT STRESS  6=COMPUTED CONTACT STRESS
7=MOMENT PER FT OF BEAM WIDTH  8=SHEAR PER FT OF WIDTH

*
```

SWINBURNE TESTS — CASE I

```
 FRN
BEAM ON SWELLING ELASTIC FOUNDATION (BOSEF)
UNITS=FT, LBS  PER FT OF WIDTH
PRINT INTERMEDIATE RESULTS?  1=YES  0=NO
=0
 *DIFFUSION PROBLEM INPUT:
   NV,DY,NH,DX,NS
=6,1,10,3,5
  BSB,   BST
=800,800
 *BEAM PROBLEM INPUT:
 NO. OF NODES IN BEAM (INCL 4 IMAGINARY NODES),  EI
=17,1.7E7
INPUT NODAL LOADS  -- REAL NODES ONLY
=170,340,340,340,340,340,340,340,340,340,340,340,170
ASSUMED CONTACT STRESSES -- REAL NODES ONLY
=170,170,170,170,170,170,170,170,170,170,170,170,170
INPUT HARD,SOFT STIFFNESS
=25000,25000
INPUT NV VALUES OF LAB MEAS SOIL SUCTIONS  TOP FIRST
=15000,11970,8380,6400,4410,4450
NV VALUES,INITIAL VOID RATIO AT EACH DEPTH (TOP FIRST)
=1,1,1,1,1,1
INPUT    SUCTION INDEX,KO,ALPHA,UNIT WT
=.12,1,1,90
----------------------------------------------------------


*FINAL RESULTS
UNITS = FT    LBS    PER FT OF BEAM WIDTH
DISP:  (+)=SETTLEMENT    (-)=HEAVE
 1      2           3         4          5       6         7          8
NOD   HEAV        RDIS      ADIS       ACST    CCST      MOM        SHR
 3      0.2433    0.0095   -0.2361    293.    180.         0.      -170.
 4      0.2404    0.0101   -0.2356    191.    121.        20.       -39.
 5      0.2450    0.0106   -0.2350    104.    250.      -156.        -8.
 6      0.2456    0.0112   -0.2344     93.    281.       -12.       183.
 7      0.2414    0.0118   -0.2338    148.    190.       576.       313.
 8      0.2362    0.0123   -0.2333    216.     73.      1242.       236.
 9      0.2341    0.0125   -0.2331    243.     32.      1519.        -0.
10      0.2362    0.0123   -0.2333    216.     73.      1242.      -236.
11      0.2414    0.0118   -0.2338    148.    190.       576.      -313.
12      0.2456    0.0112   -0.2344     93.    281.       -12.      -183.
13      0.2450    0.0106   -0.2350    104.    250.      -156.         8.
14      0.2404    0.0101   -0.2356    191.    121.        20.        39.
15      0.2433    0.0095   -0.2361    293.    180.         0.       170.
1=NODE 2=HEAVE 3=DISP WRT MAX HEAVE 4=DISP WRT ORIG SURF
5=ASSUMED CONTACT STRESS  6=COMPUTED CONTACT STRESS
7=MOMENT PER FT OF BEAM WIDTH  8=SHEAR PER FT OF WIDTH

*
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SWINBURNE TESTS — CASE II **PG 2 OF 2**

```
 REM PEST MANAGEMENT FACILITY
PEST MANAGEMENT FACILITY
*FRN
BEAM ON SWELLING ELASTIC FOUNDATION (BOSEF)
UNITS=FT, LBS  PER FT OF WIDTH
PRINT INTERMEDIATE RESULTS?  1=YES  0=NO
=0
 *DIFFUSION PROBLEM INPUT:
   NV,DY,NH,DX,NS
=6,2,10,4,5
  BSB,   BST
=500,500
 *BEAM PROBLEM INPUT:
 NO. OF NODES IN BEAM (INCL 4 IMAGINARY NODES),  EI
=25,2.59E7
INPUT NODAL LOADS  -- REAL NODES ONLY
=200,400,400,400,400,400,400,400,400,400,400,400,400
=400,400,400,400,400,400,400,200
ASSUMED CONTACT STRESSES -- REAL NODES ONLY
=355,305,272,253,242,233,230,227,225,225,223,225,225,227,230,232,233
=242,253,272,305,355
INPUT HARD,SOFT STIFFNESS
=5000,5000@@@@@@@@2000,2000
INPUT NV VALUES OF LAB MEAS SOIL SUCTIONS  TOP FIRST
=100,300,10400,20600,30800,31000
NV VALUES,INITIAL VOID RATIO AT EACH DEPTH (TOP FIRST)
=1,1,1,1,1,1
INPUT    SUCTION INDEX,KO,ALPHA,UNIT WT
=.1,1,1,100
-------------------------------------------------------

SOLUTION CONVERGED AFTER  4 ITERATIONS

*FINAL RESULTS
UNITS = FT    LBS    PER FT OF BEAM WIDTH
DISP:  (+)=SETTLEMENT    (-)=HEAVE
 1      2          3         4          5      6       7         8
NOD   HEAV         RDIS       ADIS      ACST    CCST    MOM       SHR
 3      0.3308     0.1388   -0.1919    277.    278.       0.     -200.
 4      0.3216     0.1391   -0.1917    260.    260.      35.       30.
 5      0.3176     0.1393   -0.1914    252.    252.      96.       40.
 6      0.3157     0.1396   -0.1912    249.    249.     163.       41.
 7      0.3146     0.1398   -0.1910    247.    247.     227.       38.
 8      0.3140     0.1400   -0.1908    247.    246.     284.       33.
 9      0.3136     0.1402   -0.1906    246.    246.     332.       27.
10      0.3134     0.1403   -0.1905    246.    246.     370.       20.
11      0.3132     0.1404   -0.1904    246.    246.     397.       14.
12      0.3132     0.1405   -0.1903    246.    246.     413.        7.
13      0.3131     0.1405   -0.1903    246.    246.     419.       -0.
14      0.3132     0.1405   -0.1903    246.    246.     413.       -7.
15      0.3132     0.1404   -0.1904    246.    246.     397.      -14.
16      0.3134     0.1403   -0.1905    246.    246.     370.      -20.
17      0.3136     0.1402   -0.1906    246.    246.     332.      -27.
18      0.3140     0.1400   -0.1908    246.    246.     284.      -33.
19      0.3146     0.1398   -0.1910    247.    247.     227.      -38.
20      0.3157     0.1396   -0.1912    249.    249.     163.      -41.
21      0.3176     0.1393   -0.1914    252.    252.      96.      -40.
22      0.3216     0.1391   -0.1917    260.    260.      35.      -30.
23      0.3308     0.1388   -0.1919    277.    278.       0.      200.
1=NODE 2=HEAVE 3=DISP WRT MAX HEAVE 4=DISP WRT ORIG.SURF
5=ASSUMED CONTACT STRESS  6=COMPUTED CONTACT STRESS
7=MOMENT PER FT OF BEAM WIDTH  8=SHEAR PER FT OF WIDTH
```

PEST MANAGEMENT FACILITY