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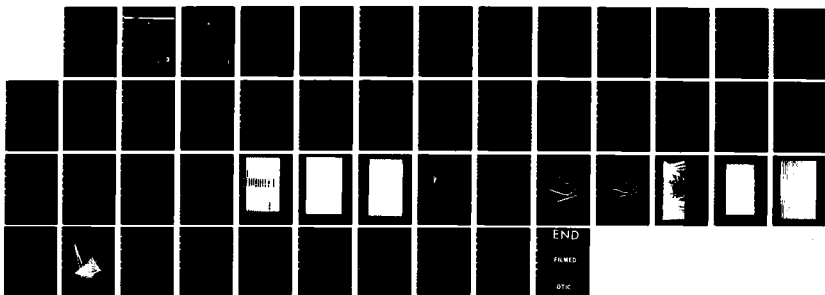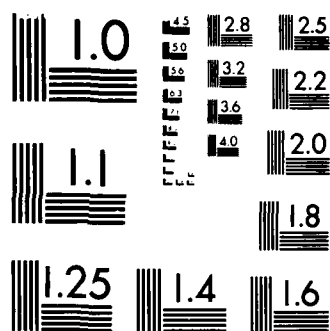   GOLDEN CENTER FOR WAVE PHENOMENA   N BLEISTEIN ET AL.
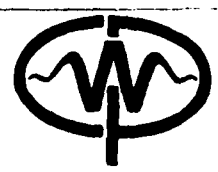UNCLASSIFIED 01 OCT 85 CWP-037 N00014-84-K-0049   F/G 8/11   NL

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PROGRESS REPORT: OCTOBER 1, 1985

of the

Selected Research Program of the Office of Naval Research

at the

Center for Wave Phenomena, Colorado School of Mines

Principal Investigators

Norman Bleistein, Jack K. Cohen,

Frank G. Hagin, John A. DeSanto

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CENTER FOR WAVE PHENOMENA

Introduction

This is a progress report on the current status of the research program
of the Center for Wave Phenomena at the Colorado School of Mines. There are
presently four faculty members and six graduate students supported by this
program; three additional students are working on research projects with
Center faculty. The Center derives its research support from four sources:
The Selected Research Opportunities Program of the Office of Naval Research
(SRO), The Consortium Project on Inverse Methods for Complex Structures (CP)
supported by eleven energy companies -- Amoco, Conoco, Digicon, Geophysical
Company of Norway, Marathon, Mobil, Phillips, Sun, Texaco, Union and Western
Geophysical -- the Ocean Acoustics Program of the Office of Naval Research
(OA) and the Colorado School of Mines (through reduced teaching loads over
and above the reduction supported by research contracts).

The People

The four faculty members are Norman Bleistein, Jack K. Cohen, John A.
DeSanto and Frank G. Hagin. The Selected Research Opportunities project
partially supports Bleistein, Cohen, DeSanto and Hagin. The Ocean Acoustics
project partially supports DeSanto. The Consortium project partially
supports Bleistein, Cohen and Hagin. The School of Mines partially supports
all four members.

At present, the graduate students and their support are as follows: Linda Boden (OA), Paul Docherty (SRO), Peter Kaczkowski (SRO), Thomas Jorden (CP), Brian Sumner (CP) and Michael Sullivan (CP). During the previous academic year the supported graduate students included Shelby Worley (SRO), Kingsley Smith (CP) and Jason Kao (CP) but did not include Jorden.

Related Activities

<u>Educational</u>

In the past year, the Center for Wave Phenomena faculty memebers have taught or are teaching a wide spectrum of courses in applied mathematics:

Applied Complex Variables

Applied Functional Analysis

Linear Algebra

Linear Systems

Mathematical Seismology

Numerical Analysis

Ocean Acoustics

Potential Theory

Partial Differential Equations

Principles of Applied Mathematics

Seismic Data Processing

Seismic Inverse Methods

Several students have completed Master's degrees during the past year under the guidance of Center for Wave Phenomena faculty. These theses have previously been sent to the Office of Naval Research and a brief abstract of each is in the annotated bibliography attached as an appendix to this progress report.

- Kingsley Smith's thesis was "Seismic Tomography in Boreholes Using an Algebraic Reconstruction Technique."

- Paul Docherty's thesis was "A Fast Ray Tracing Routine for Laterally Inhomogeneous Media." Docherty is continuing work on his Ph. D. degree with us.

- Isabelle Leroux's thesis was "Qualitative sign-bit Processing."

- Paul Violette's thesis was "Analysis of Two-Parameter Constant Background Born Inversion for Acoustic Synthetic Data." Violette is now pursuing a Ph. D. at Harvard.

Computing

During the past year, Gould Corporation has made a major gift to the Colorado School of Mines of two computers (models 6050 and 9750) and peripherals valued at approximately $700,000. Frank Hagin played a major role in the negotiations for this donation. CWP is using the 9750 for all program development and testing.

The Center for Wave Phenomena has made contacts with three domestic manufacturers of supercomputers. CRAY Corporation has donated computer time and assistance in converting a 2.5D inversion code to CRAY format. A successful benchmark run has been performed which demonstrated a speed-up factor of 500 over a conventional VAX 780. ETA Corporation has also offered us computer time and has benchmarked the same code on a CYBER 205. They plan to repeat the benchmark on a prototype ETA. Denelcor has likewise made an offer of computer time and plans to benchmark the 2.5D computer code.

The Office of Naval Research has made supercomputer time available to us on the Naval Research Laboratory's CRAY in Washington. Later this fall, we plan to benchmark a newly developed experimental version of a fully 3D inversion code on this machine.

Invited Lectures and Papers, Other Lectures

Norman Bleistein gave an invited plenary lecture at the joint SIAM/SEG/SPE meeting on Mathematical Methods for the Extraction of Energy Resources from the Earth, in Houston in January, 1985. He also gave a lecture at a meeting arranged for representatives of the energy industry held at the Colorado School of Mines in May, 1985.

Bleistein completed an invited article for the Encyclopedia of Engineering and Science to be published by Academic Press. He has also completed an invited article for a special issue on inversion of IEEE Proceedings. This article is co-authored with Samuel Gray. He has also completed an invited article based on the talk presented at the Houston

meeting. That article will appear in a special proceedings volume.

Jack Cohen gave an invited lecture at the joint SIAM/SEG/SPE meeting in Houston in January, 1985. He also gave invited lectures at the research centers of Mobil Oil and Union Oil and at S-Cubed (Systems, Science and Software).

Bleistein presented two lectures and was co-author of one other presentation at the Fall, 1984, meeting of the Society of Exploration Geophysicists. Cohen and Hagin were co-authors of one of those presentations. Bleistein also gave a paper at the International Meeting of the European Association of Exploration Geophysicists in Budapest in June, 1985.

John DeSanto's chapter "Ocean Acoustics" appeared in the Encyclopedia of Physics published by Van Nostrand Reinhold. He has also completed three invited papers on various aspects of rough surface scattering. The first, on a spectral formulation of the problem, will appear in the Journal of the Optical Society of America. The second, a long review article, will appear in Progress In Optics, edited by Emil Wolf, and the third, on recent results in the field will appear in a new journal, the Journal of Wave Material Interactions. Both the latter were done in collaboration with G. S. Brown of VPI. He has given an invited lecture at the multiple scattering workshop at Penn State where he chaired the surface scattering session, and will present the John Wright Memorial Lecture at the Naval Research Laboratory in Washington in October.

In the past year we have had two visitors.

Dr. George Frisk from the Woods Hole Oceanographic Institution who lectured in DeSanto,s Ocean Acoustics course on using plane wave reflection coefficients to do ocean bottom inversion.

Dr. David Stickler from the Courant Institute of Mathematical Sciences who also lectured in DeSanto's course on theoretical methods for ocean bottom inversion. Dr. Stickler will be joining CWP on a permanent basis in January.

Research Background and Current Status

There have been two major research projects in inverse scattering under the Selected Research Opportunities Program. The first of these is reflector imaging for seabed mapping and seismic exploration. The second is ocean profile inversion. We will discuss those programs here in that order. The first project was carried out under the direction of the principal investigators Bleistein, Cohen and Hagin; the second project was led by DeSanto.

Reflector Imaging for Seabed Mapping and Seismic Exploration

Our research group is committed to the practical solution of inverse problems. We have been developing stable algorithms for inverse problems for over ten years. In this section, we give a short summary of our work prior to the period covered by the SRO (i.e. prior to Fall 1983) and then we continue with a more detailed account of the last two years.

Our early research in inverse problems was motivated by the results of N. N. Bojarski [1974] who formulated a fundamental integral equation which became our main tool for studying the inverse source problem. Our work in this area was reported in Bleistein and Cohen [1977a].

Simultaneously, we began research on another theory of Bojarski's [1967], addressing the problem of imaging a scattering obstacle from high frequency far field scattering data [Bleistein 1976, Mager and Bleistein 1978, Cohen and Bleistein, 1979a]. We soon began applying these results to

the problem of imaging flaws in solids [Bleistein and Cohen, 1977b, 1980] which arises in non-destructive testing. While the non-destructive testing aspect of inversion has not been pursued by our group during the last few years, other researchers have achieved success by implementing our methods [Langenberg, Brück and Fischer 1983, Höller, Langenberg and Schmitz 1984, Langenberg, Fischer, Berger and Weinfurter, 1985].

During 1976 we began a fruitful line of research on the problem of inversion of subsurface layering in a half space. This model is applicable to both the sound speed estimation problem in the seabed and to the seismic exploration problem. Our work in this area began with a formulation involving plane wave sources [Cohen and Bleistein, 1977] and was followed by work employing the more realistic model of point source probes [Cohen and Bleistein, 1979b]. This latter paper is often cited since it gave a practical algorithm for the seismic "backscatter" or "zero-offset" problem. Here, we inverted for a perturbation of sound speed, based on a constant background sound speed. In this setting, the problem is amenable to high frequency asymptotic analysis. This paper has become the basis for further development by both us and other researchers.

Our method has come to be known as "Born inversion" because the perturbation approach is similar to the Born approximation in potential scattering. Although small variations in sound speed is a basic premise of this method, we have found that it has broader applicability. In particular, we have applied our algorithm to Kirchhoff approximate data from a single reflector in a constant background medium. We find by asymptotic analysis that, when the background velocity is chosen as the velocity in the

upper medium, the method will properly locate the reflector and accurately estimate reflection strength for any size jump in velocity across the reflector. This type of verification has persisted throughout our work as we have extended our method to more complex background structure and to various source/receiver configurations used in practice.

Although we began our analysis by seeking perturbations in the sound speed itself, we have modified our output so that it produces an array Dirac delta functions with support on each of the surfaces of discontinuity of the velocity field. These surfaces are just the reflectors in the subsurface. The scaling of each delta function is proportional to the reflection strength of that reflector. We call the Dirac delta function with support on the surface the singular function of the surface [Cohen and Bleistein, 1979a, Bleistein, 1984a, Bleistein, Cohen and Hagin, 1985a] and we call the array of scaled singular functions the reflectivity function of the subsurface.

This approach to the inverse problem is motivated by the fact that seismic surveys, on land or over the seabed, produce bandlimited data which are also high frequency data for most of the length scales of interest. From near zero offset high frequency data, one cannot detect trends in the earth parameters, but only discontinuities. Such discontinuities are most easily detected as bandlimited delta functions. In Cohen and Bleistein [1979a], we developed a rigorous asymptotic theory for the transition from bandlimited high frequency data for a function to determination of the singular functions of its surfaces of discontinuities.

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In 1978 Frank Hagin joined the research group and began exploring the stability of the class of inversion algorithms being developed by Bleistein and Cohen. We had empirically observed the stability of these algorithms. However, since an integral equation of the first kind was being inverted, the theoretical issue of stability had to be addressed in order to lay a foundation for continued work. We soon recognized that our inversion equation was more closely related to the well-conditioned problem of inverting Fourier transforms than to the ill-conditioned problem of inverting compact operators. In Hagin [1980, 1981a, 1981b] and Gray and Hagin [1982] the stability issue was definitively laid to rest for the one-dimensional inverse problem. Moreover, many of the concepts developed in one dimension carried over to the three dimensional approach of Bleistein and Cohen [Hagin and Gray, 1984]. This research also introduced the theme of variable reference speed which has become important in our current work.

In summary, prior to the commencement of the SRO grant in Fall 1983 the group was well grounded in the basics of inversion techniques as applied to the wave equation in simple three dimensional settings. They had developed a research level computer program for inverting "backscatter" data to determine perturbations from a constant reference background. In addition to these accomplishments, clear direction was seen for several lines of research; these were outlined in the SRO proposal.

We now describe our progress during the two years Fall, 1983 to Fall, 1985 in removing the limitations of the pre-existing inversion algorithm.

The constant reference speed assumption has important applications.

However, the algorithm produced inversions that deteriorated unacceptably for structures whose cumulative velocity change was large. Recursive applications of the algorithm can alleviate this problem to a degree by using different reference velocities in different regions. A better solution is to develosp inversiou algorithms which allow the _ab initio_ inclusion of as much of the known velocity structure as possible. Such variable reference speed schemes hold the potential for improved accuracy and economy.

Secondly, the backscatter experiment, although an important theoretical model, can only be approximated by the standard "stacking" of the actual data. There are well known situations for which this approximation is poor.

As a first attempt to improve the inversion obtained in the case of large overall variations of the background velocity, we developed a post-processing scheme which corrected for large velocity variations. In this approach it is necessary to find regions in which several major reflectors are nearly parallel (not necessarily horizontal). In such regions, the algorithm described in Hagin and Cohen [1984] can be used to correct for the major errors inherent in the constant reference speed algorithm. When applicable, this algorithm provides an inexpensive way to refine the inversion and can provide dramatic improvement in both location and parameter estimates.

An example of this method is provided in Figures 1 and 2 taken from the Hagin and Cohen paper. Figure 1 shows the output of the constant reference inversion algorithm. One can see from the figure that the first reflector

and velocity increment are properly reconstructed. The output is seen to degrade with depth, both in reflector mapping and velocity estimation. Figure 2 shows the results of applying the refinement algorithm and the fit is seen to be nearly perfect, despite the fact that the velocity increment is more than 150%. Furthermore, the cpu time for the refinement is only about 2% of the cpu time for the initial inversion.

As suggested above, inversion schemes which allow a variable reference speed have clear advantages over those which are based on one or more constant reference speeds. In Bleistein and Gray [1985], such an algorithm was derived for the case of a depth-dependent reference velocity, c(z). A key step in the derivation was the early use of the "high frequency" assumption (which was already being used at the implementation stage, as noted above).

The philosophy of this paper now pervades our entire research program. An inversion algorithm is an integration over source/receiver pairs in which the kernel of the integral operator uses ray-theoretic (WKBJ) traveltimes and amplitudes consistent with the background reference speed. The output is the reflectivity function of the subsurface. This output is a reflector map in which the amplitude provides a means for estimating reflection strength.

The computer code developed by Gray has become a production line code at Amoco. It is particularly well suited for imaging flanks of salt domes in otherwise horizontally stratified media, such as in the Gulf of Mexico. Figures 3A-C, taken from the Bleistein and Gray paper demonstrate this

capability. Figure 3A is a geologic model of a salt dome intruding into an otherwise horizontally stratified geologic structure. Figure 3B shows a zero offset time section for this model generated using a finite difference scheme developed by Dan Whitmore at Amoco Production Company. Figure 3C is the result of applying Gray's program to this synthetic data. It can be seen that the flank of the structure is well defined up to the vertical. This program does not image reflectors beyond vertical because, at the time of implementation, turned rays were not incorporated into the code. In contrast, a constant background inversion would steepen slanting images in the time section Figure 3B but could not bring that reflector to near vertical. The improvement in location of the $c(z)$ algorithm can be traced ultimately to the bending of rays along their trajectory, allowing the observed data to be projected back along curved trajectories to their origin on the reflectors. The lack of refraction in a constant background algorithm will mislocate the reflectors by projecting the data back along straight ray paths. The new algorithm requires only a modest increase in cpu time over the constant background code [Bleistein and Gray, 1985]; the cpu times for inversion with a $c(z)$ background are comparable to the computer codes for k-f migration following Stolt [1978].

While the Bleistein-Gray $c(z)$ algorithm produced improved reflector mapping, it was soon discovered that it did not provide the correct magnitude of jump in velocity in the case of a curved reflector. Cohen and Hagin were simultaneously studying the $c(z)$ reference problem from a somewhat different point of view. Late in 1984 they succeeded in finding an inversion operator which correctly estimated the jump in velocity across a single reflector given accurate synthetic high frequency data. The structure

of this inversion operator was similar to the Bleistein-Gray operator which meant that the virtues of the code for that algorithm carried over to the new one. However, the derivation of the new algorithm put the determination of inversion operators on a new footing. In this approach the general problem of finding an inversion operator was reduced to that of finding a suitable "completeness" relation [Cohen and Hagin, 1985]. This idea suggested a systematic approach to developing further inversion algorithms, for example, extensions to $c(x,z)$ and $c(x,y,z)$ reference velocities and to offset source/receiver configurations.

Recently [Beylkin, 1985], a paper which greatly systematized the description of inversion operators has appeared. Beylkin's result is framed in the context of pseudo-differential operators and generalized Radon transforms. However, the key insight was compatible with the approach taken by Cohen and Hagin. With Beylkin's technique one can directly obtain, in principle, the required completeness representation for virtually all cases of interest for the acoustic wave equation. The result of this key step is consistent with the high frequency assumptions used in our approach. Thus, we are able to apply our singular function theory [Cohen and Bleistein, 1979b] to determine the reflectivity function of the subsurface. That is, we obtain an output in terms of the singular functions of the subsurface reflectors with amplitudes proportional to the reflection strength of each reflector.

There are still fundamental issues to be resolved in the new approach. In particular one must evaluate a certain Jacobi determinant; this is usually non-trivial. In fact, two issues arise here. The first is the

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reduction of this determinant to a form amenable to tractable computation. The second is that when the determinant vanishes, the current inversion theory breaks down. Nonetheless, the reduction of the inversion problem to the analysis of a determinant is a significant theoretical advance which allows the focusing of effort on the central issue of each inverse problem being studied.

In Cohen, Bleistein and Hagin [1985] the $c(z)$ backscatter algorithm discussed above is extended to a general $c(x,y,z)$ background algorithm in which the observation surface can be curved instead of planar, as in the earlier work. In this same report we presented the inversion algorithms for the source/receiver common source case and for the common receiver case with the same general $c(x,y,z)$ background velocity function. These latter results and the earlier common offset algorithm for constant reference described in Cohen and Sullivan [1985] are our first inversions which dispense with the zero offset (backscatter) assumption.

Combining this new approach with the results on "two-and-a-half dimensional" problems presented in Bleistein [1984b], we have derived computationally feasible solutions. The term "two-and-a-half dimensional" (2.5D) characterizes the assumption that the earth parameters depend only on depth and a single transverse variable while the wave propagation is three dimensional. The former assumption is mandated by the conventional deployment of sources and receivers along only a single line on the surface. Much of the migration/inversion literature uses two dimensional wave propagation. However, this conflicts with the three dimensional sources used in actual data acquisition and so we have always derived inversion

algorithms for three dimensional wave propagation.

In the 2.5D case, the necessary computations are straightforward to carry out for constant background velocity, $c(z)$, and even for $c(x,z)$ background velocity. Thus, we are able to write down inversion algorithms for the following source/receiver configurations: (i) common source or (ii) common receiver, and (iii) fixed (common) offset between source and receiver [Bleistein, Cohen and Hagin, 1985b]. In all three cases, the upper surface can be curved. This last feature may reduce the need for certain types of "static" corrections. Furthermore, we can show that the output will produce a reflector map of the subsurface and an estimate of reflection strength for all configurations.

The theory and computer implementation of the zero offset $c(z)$ background inversion is an ongoing project which will lead to Sumner's Ph.D. thesis. Sumner's research continues the analysis introduced in the Cohen and Hagin $c(z)$ project discussed above. The algorithm has been tested on both synthetic and real data. As a check, this algorithm was run on the same data set shown in Figure 3. The reflector mapping is virtually identical.

Another graduate student, Docherty [1984, 1985], has been engaged in a project to develop a ray tracing algorithm in a $c(x,z)$ medium. Building on Keller and Perozzi [1983] and Fawcett [1983], Docherty has developed an algorithm and computer code to do ray tracing in a model consisting of a number of layers of constant velocity bounded by general surfaces. This ray tracing capability will be of use to our group in several ways including

building more realistic synthetics, studying caustics and providing migrations and inversions in geometries too complex for either constant background or c(z) background inversion schemes.

Figure 4 shows a sample of the ray tracing provided by Docherty's code. Multiple reflections of rays from a point source are depicted. The user prescribes the layering, the order of intersection with the layers, an initial point and a final point. In this case, the ordering of the layers demanded multiple reflections. The method is iterative and therefore becomes particularly efficient when a family of nearby rays is of interest. Each ray provides a first iterate for the location of the adjacent ray.

Docherty has already implemented a migration algorithm based on his ray program and our theory. Figure 5A shows a model with three layers and an initial velocity of 6000 ft/sec and increments of 1000 ft/sec at each interface. Zero offset data was generated by a finite difference scheme. Figure 5B depicts the output of Docherty's algorithm when a nearly correct background velocity is chosen. Gaps in the output arise from both specular ray paths which reach the surface outside the receiver array and ray paths which have passed through caustics in the subsurface. At this stage, Docherty's implementation does not include rays that have passed through caustics.

Proper location of the lowest layer confirms the validity of an algorithm which properly accounts for refractions. As a comparison, Figure 5C shows the output from a conventional constant background migration. This shows the need for a $c(x,z)$ algorithm in complex media.

Figure 6A shows a salt dome model which ray paths from one particular horizon. Figure 6B is output from Docherty's algorithm applied to synthetic data for this model. Figure 6C is the output for the same data from the c(z) algorithm. Both algorithms give comparable results in the horizontal layers and on the flanks of the salt dome. However, Docherty's algorithm more accurately depicts the horizontal layer directly below the salt dome. The reason for this is that a c(z) background velocity cannot characterize the lateral changes in velocity across the salt dome. This research project is being carried out under the guidance of Bleistein and Gray and is Docherty's Ph.D. project.

Another student, Sullivan [1984, 1985] has developed an alternative modeling program based on the wave equation datuming method of Berryhill [1979]. Sullivan has extended that work in two ways. First, he has introduced a correct two-and-one-half dimensional amplitude adjustment based on Bleistein [1984b]. Second, he has developed a hybrid ray-theoretic Kirchhoff technique which allows him to account for multiple transmission and reflection effects in the theory and in the resulting computer algorithm.

Figure 7 shows backscatter output from a single synclinal reflector as produces by the fundamental program. The syncline is sufficiently deep that the rays reflected from it form a caustic below the observation surface -- a buried focus -- and a triplication of responses on the traces near the center of the figure. The short time response near the top of the figure reproduces the source wavelet. The later response, which has passed through

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the caustic, exhibits the well-known phase shifting of such responses and the impulse of the source wavelet has been transformed into a doublet.

Figure 8 depicts a ray tracing model of the hybrid method that is being developed. Rays are "shot" from the source point at the left and refract and reflect to arrive back at the first curved interface. Geometrical optics or ray theory is used to estimate the contribution to the scattered field at this surface due to these trajectories and also to estimate the Green's function at this surface. Now a Kirchhoff integral with appropriate Kirchhoff approximation is used to evaluate the contribution to the observed field due to this sequence of refractions and reflections.

On the one hand, this technique requires an integration over one reflecting surface. On the other hand, it does not require determination of the actual geometrical optics ray path from source to receiver. There is therefore a trade-off in computer time between this approach and Docherty's approach to modeling.

Sullivan's modeling technique will be used to generate synthetic data for testing of inversion algorithms. Direct modeling of this sort is also used to test the output of migration and inversion algorithms on real data. That is, a model based on the output of the inversion algorithm is produced, synthetic data for the model is generated and this data is compared to the original field data. This work is being carried out under the guidance of Bleistein and Cohen as part of Sullivan's Ph.D. thesis project.

In addition, Sullivan is working on a project of inversion of Kirchhoff

approximate data in a constant background medium for a fixed offset source/receiver array (Sullivan and Cohen, 1985) . The solution for the reflectivity function provides an independent check on the result which has evolved from Beylkin's theory as modified by us. Sullivan has started the development of a computer code to test this theory.

Ocean Profile Inversion

DeSanto, with a graduate student, Linda Boden, is engaged in a research project on ocean soundspeed profile inversion, in the case of a soundspeed which is dependent on depth, only. This inversion problem is fundamentally different from the seismic problem. The source(s) and receiver(s) are separated laterally while the variation being sought is a function of the vertical variable, nearly orthogonal to the ray paths of propagation of the signals.

An integral equation for the perturbation of the soundspeed from a reference speed is formulated. For this class of problems, the reference speed cannot be taken to be a constant because the wave propagation for such a case deviates too much from the propagation in the ocean. Instead, the reference speed must characterize the nonmonotonicity of the ocean, including some interior minimum of soundspeed. Then the reference fields have the proper "turning" and "trapping" properties to be close to the total field.

The integral equation for the perturbation in soundspeed is then to be solved asymptotically. However, the asymptotic nature of the kernel changes

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as a function of the relative heights of the source, the receiver and the minimum of the reference soundspeed. For the simplest case of a reference made up of two linear segments, there are five separate regions in which the asymptotic inversion of the integral equation must be carried out separately.

The full analysis of an example problem is complete for the five ocean layers mentioned above. In all layers we are able to treat the case of an arbitrary depth dependent profile in both the direct or inverse problems. The integral equation involves an integration with respect to a depth variable z'. The kernel of the integral equation is itself given by an integral with respect to a separation variable δ. The kernel must be evaluated asymptotically in the high frequency limit. This has involved several asymptotic techniques even in each layer. For example in the upper and lower layers the asymptotics depends on the position of the minimum branch point in the integral in δ. In these regions this also corresponds to the index of refraction value which varies with z'. In addition there is a different asymptotic evaluation depending on the slope of the profile either as input (for either the direct problem or a guess at the inverse problem) or as the profile corrections are generated. The simple statement that we can treat an arbitrary depth-dependent profile thus involves considerable asymptotics. This problem has been solved, and included in the computational implementation.

In two of the regions, one must address the coalescence of two critical points of the δ-integral. The resulting asymptotics lead to an integral which must be evaluated numerically. Further analysis on this integral

enabled us to express it as the Laplace transform of the Airy function. No
tables of this were available, and we had the additional complication that
the Laplace transform argument was complex.

A major analytical simplification occurred when we were able to develop
a generalized asymptotic inversion procedure in the spirit of the method
proposed by Beylkin [1985]. As in the seismic inversion work, there is
still a computationally difficult determination of a Jacobian of a
transformation. This involved essentially taking the second derivative of a
complicated phase term and tabulating the results to use in the inversion
integral in frequency.

In a test problem for this method, a soundspeed profile was chosen in
which the total refractive index contained a perturbation from two piecewise
linear sections, which were taken to be the reference index of refraction.
The perturbation occured only in the upper section of the linear index.

The observed field from an impulsive point source was calculated both
for the full index of refraction and for the reference index of refraction.
The difference between these two provides the data for the inversion
integral equation.

The inversion of the integral equation actually provides a solution for
$n'(z)$ rather than for $n(z)$. The solutions obtained to date by this method,
properly locate the perturbation. However, $n'(z)$ exhibits an overshoot of
thetype usually associated with Gibbs phenomenon. Methods of integration of
$n'(z)$ which minimize this affect are presently being investigated.

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Cohen, J. K., F. G. Hagin, and N. Bleistein, "A preliminary report on some recent results in Born inversion," Res. Rep. CWP-031, Center for Wave Phenomena, Colo. Sch. of Mines, 1985.

Cohen, J. K. and F. G. Hagin, "Velocity inversion using a stratified reference," *Geophysics,* vol. 50, no. 11, p. (in press), 1985. Center for Wave Phenomena Research Report, CWP-021, 1984.

Docherty, Paul, "A fast ray tracing routine for laterally inhomogeneous media," Res. Rep. CWP-018, Center for Wave Phenomena, Colo. Sch. of Mines, 1985. Presented at the 1984 SEG meeting.

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Hagin, F. G., "On the construction of well-conditioned systems for Fredholm I problems by mesh adapting," *J. Comput. Phys.,* vol. 35, pp. 154-169, 1980.

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Hagin, F. G., "A stable approach to solving one-dimensional inverse problems," *SIAM J. Appl. Math.,* vol. 40, pp. 439-453, 1981b.

Hagin, F. G. and S. H. Gray, "Travel time(like) variables and the solution of velocity inverse problems," *Geophysics,* vol. 49, no. 6, pp. 758-766, 1984.

Hagin, F. G. and J. K. Cohen, "Refinements to the linear velocity inversion theory," *Geophysics,* vol. 49, no. 2, pp. 112-118, 1984. Center for Wave Phenomena Research Report, CWP-003, 1983. See errata in June, 1984 Geophysics.

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Langenberg, K. J., D. Bruck, and M. Fischer, "Inverse scattering algorithms," in New procedures in nondestructive testing, Proceedings of the German-U.S. workshop, Fraunhofer Institute, Saarbrucken, Germany, August 30 - September 3, 1982, ed. P. Holler, Springer Verlag, New York, 1985.

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Stolt, R. H., "Migration by Fourier Transform," Geophysics, vol. 43, no. 1, pp. 23-48, 1978.

Sullivan, M. F., "Kirchhoff modeling via wave equation datuming," Res. Rep. CWP-017, Center for Wave Phenomena, Colo. Sch. of Mines, 1984.

Sullivan, M. F. and J. K. Cohen, "Pre-stack Kirchhoff inversion of common offset data," Res. Rep. CWP-027, Center for Wave Phenomena, Colo. Sch. of Mines, 1985.
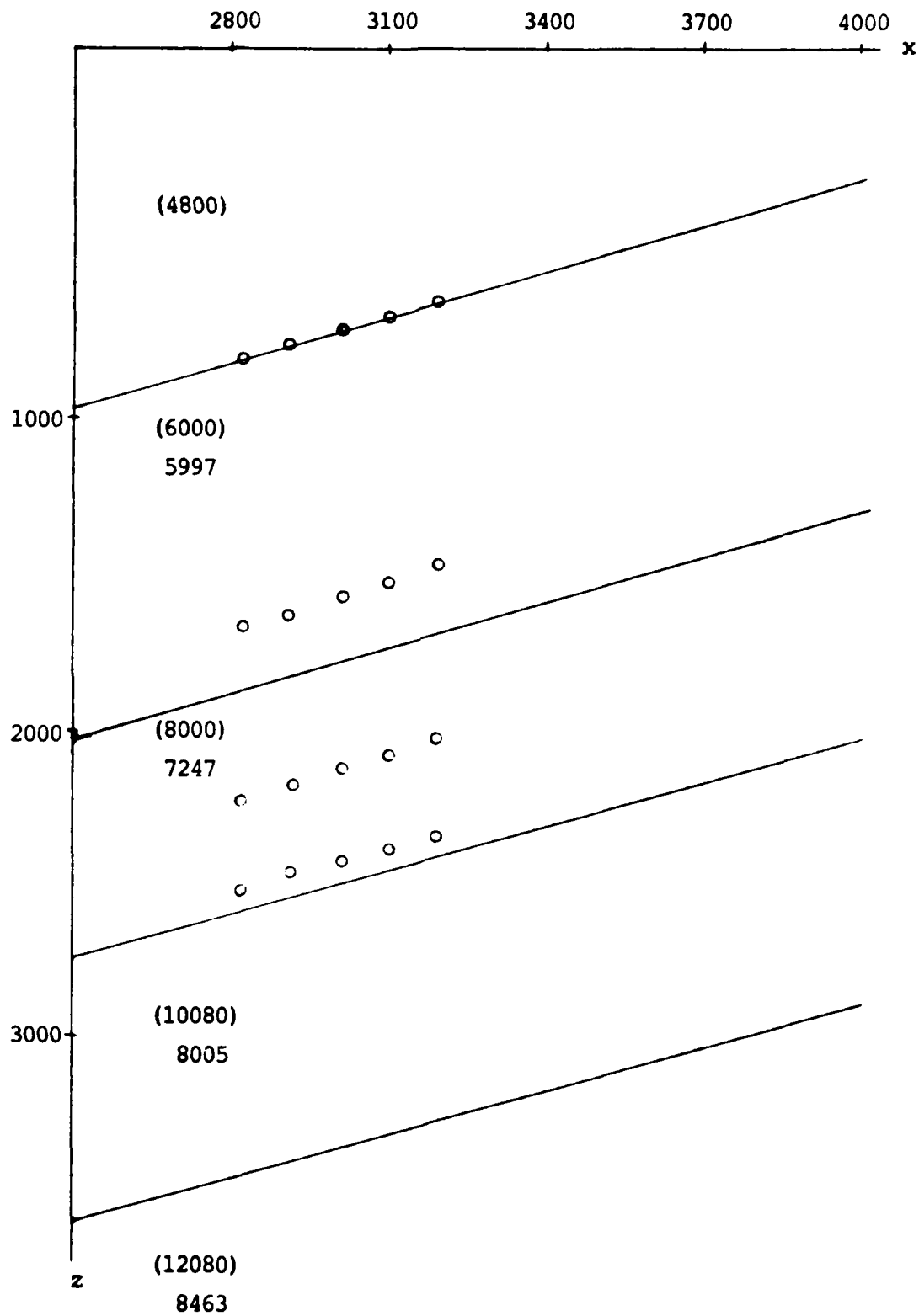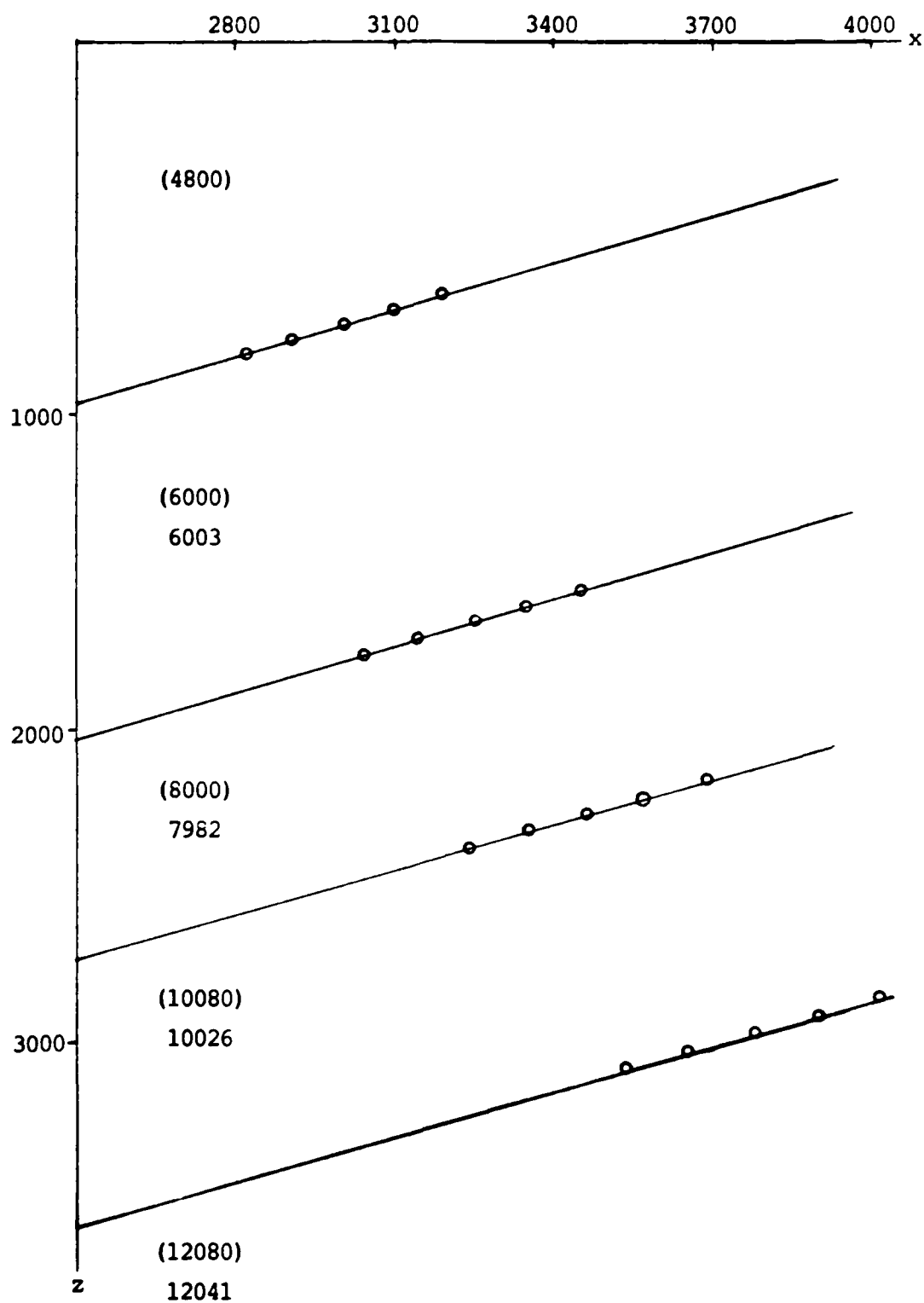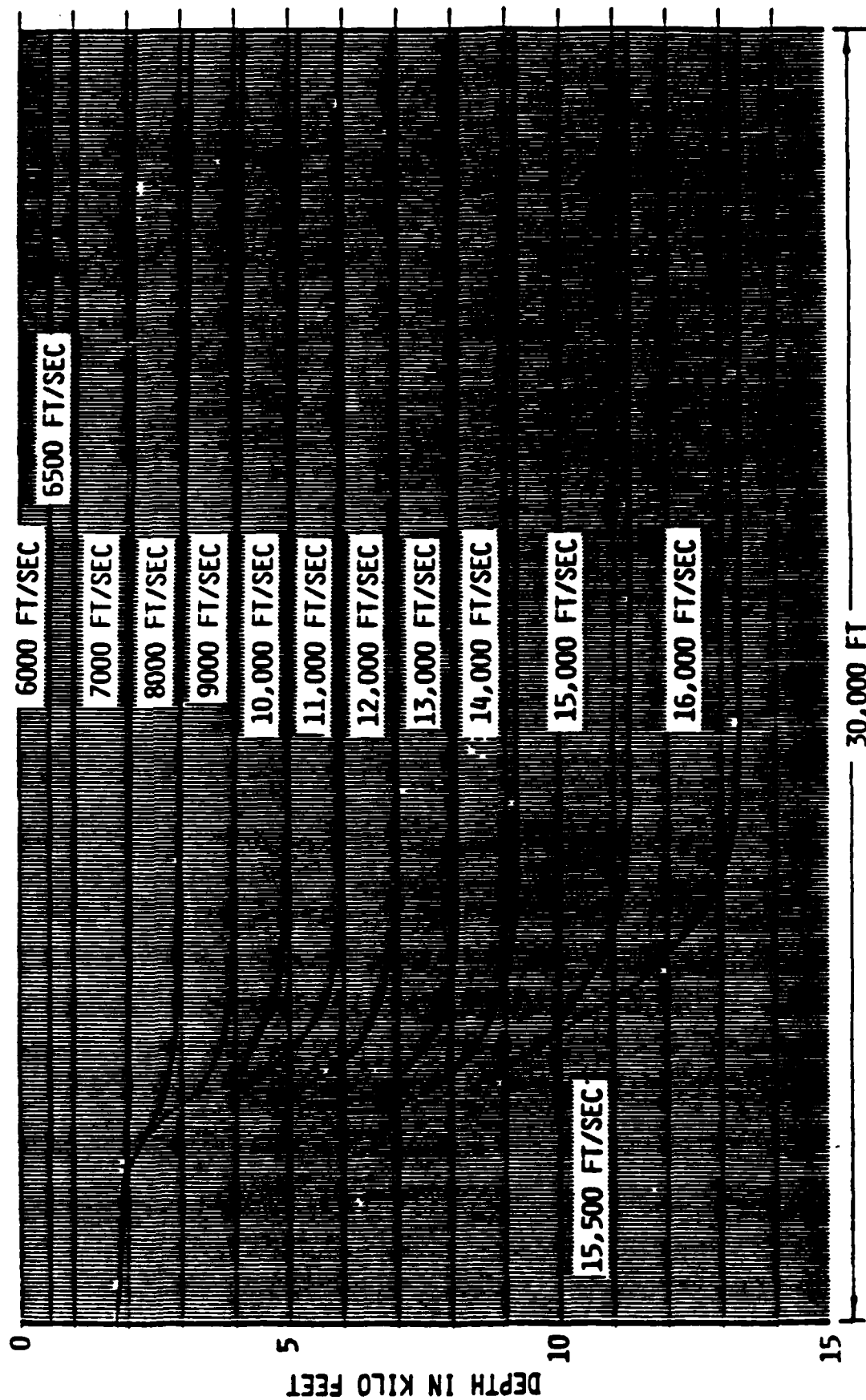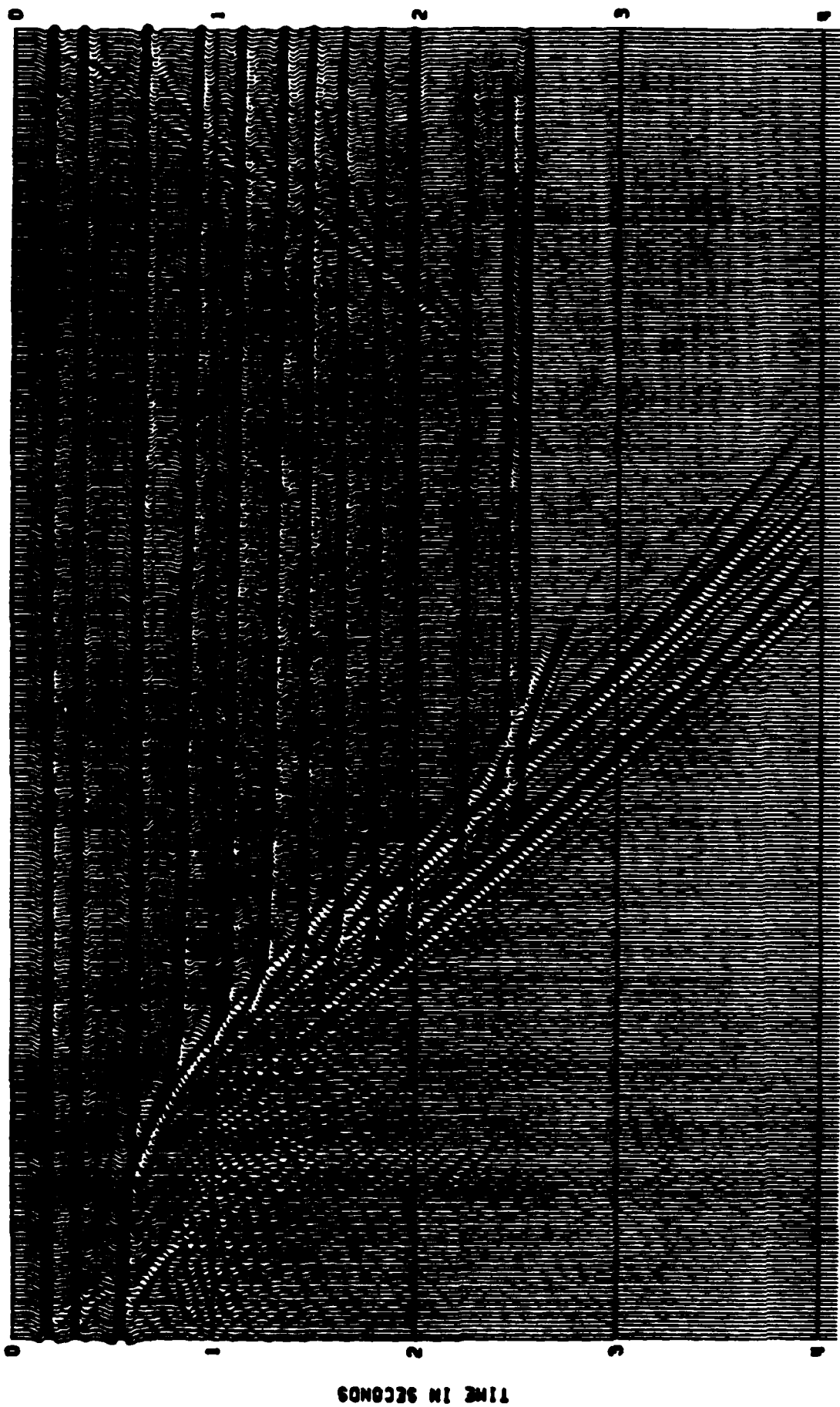
FIGURE 1

FIGURE 2

Geologic Model

6000 FT/SEC
6500 FT/SEC
7000 FT/SEC
8000 FT/SEC
9000 FT/SEC
10,000 FT/SEC
11,000 FT/SEC
12,000 FT/SEC
13,000 FT/SEC
14,000 FT/SEC
15,000 FT/SEC
16,000 FT/SEC
15,500 FT/SEC

30,000 FT

DEPTH IN KILO FEET

0

5

10

15

FIGURE 3A

FIGURE 3B

TIME IN SECONDS

FIGURE 3C

DEPTH IN FEET
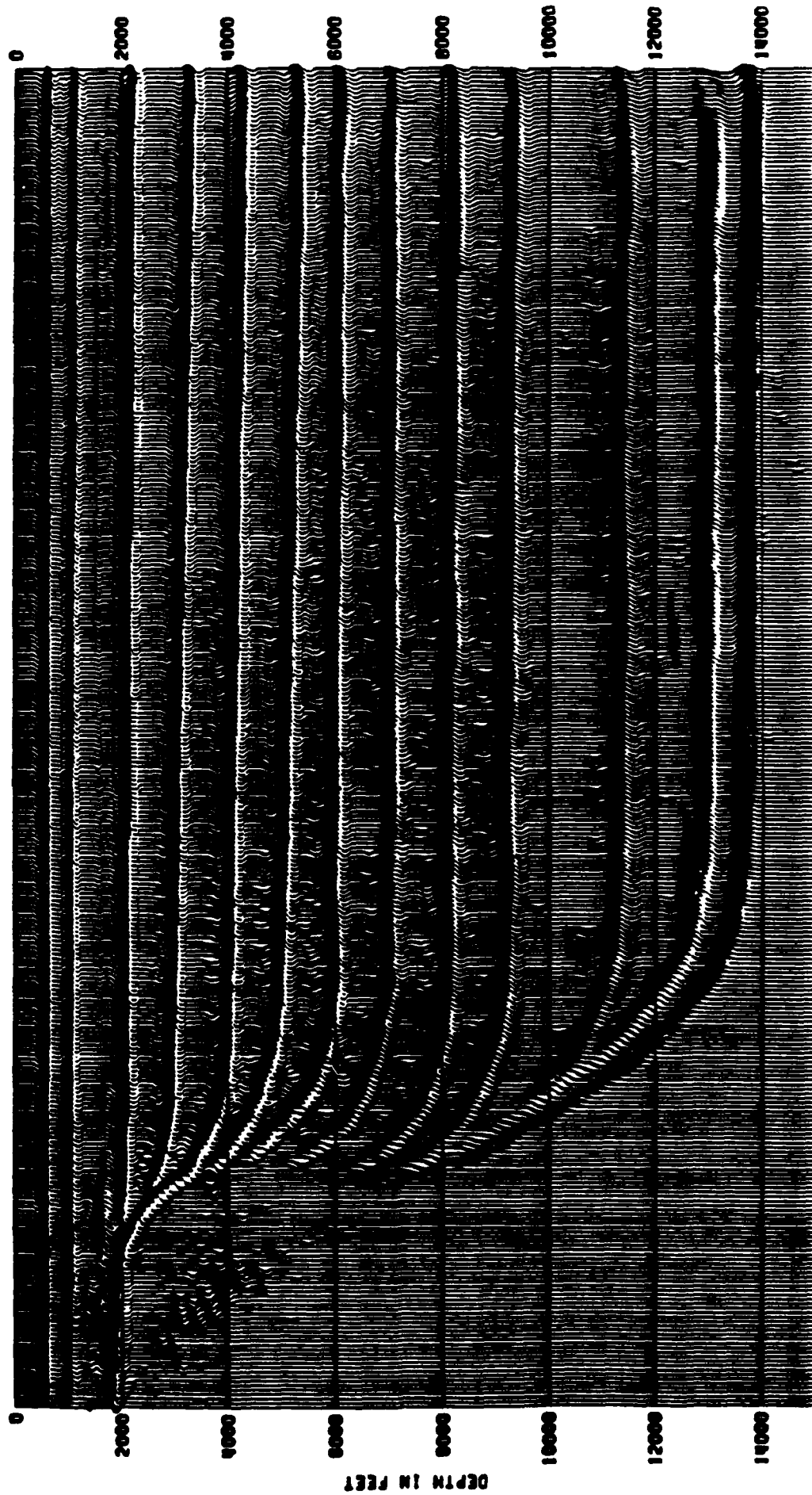
RECEIVER CONTINUATION

FIGURE 4

FIGURE 5A

DEPTH IN FEET
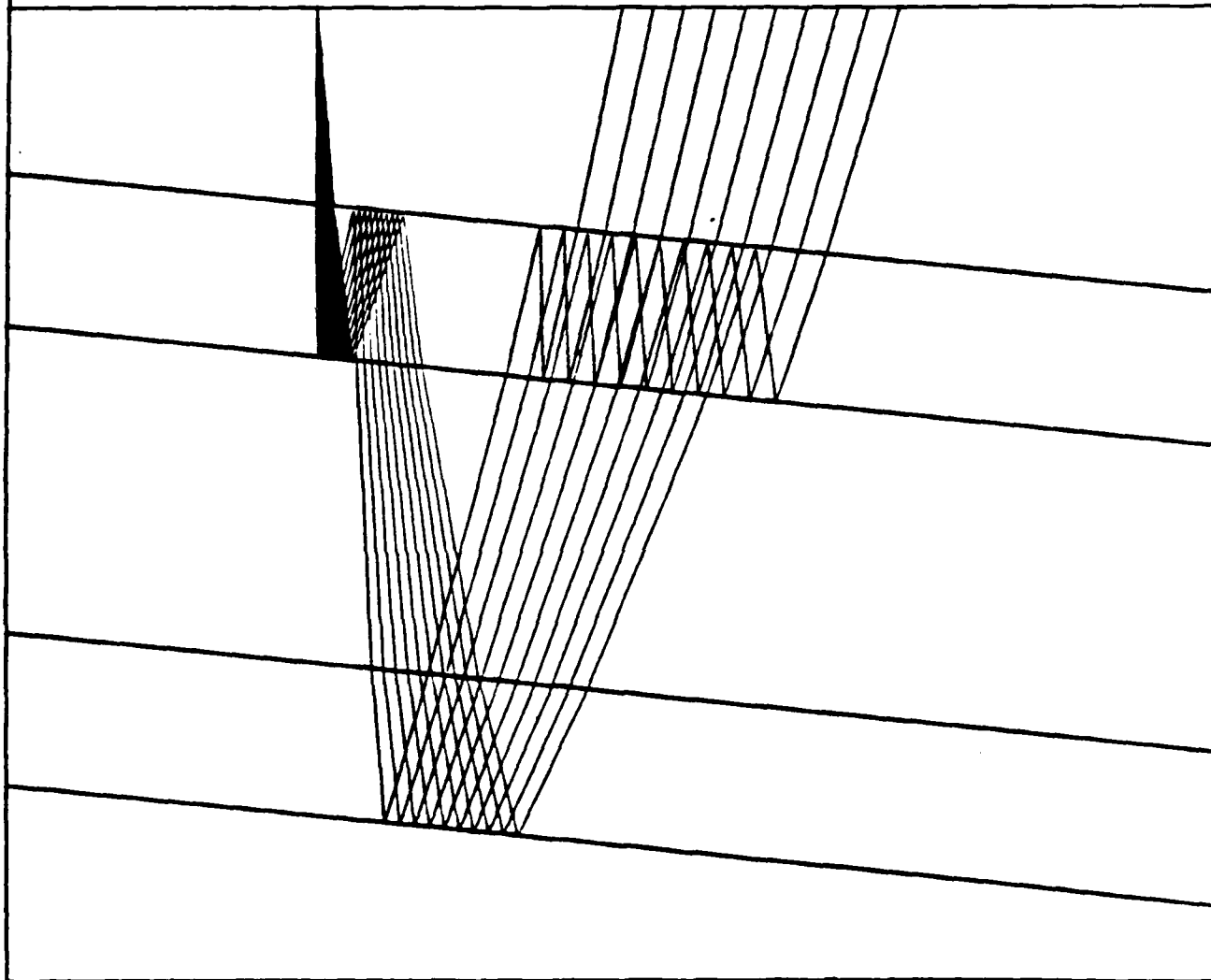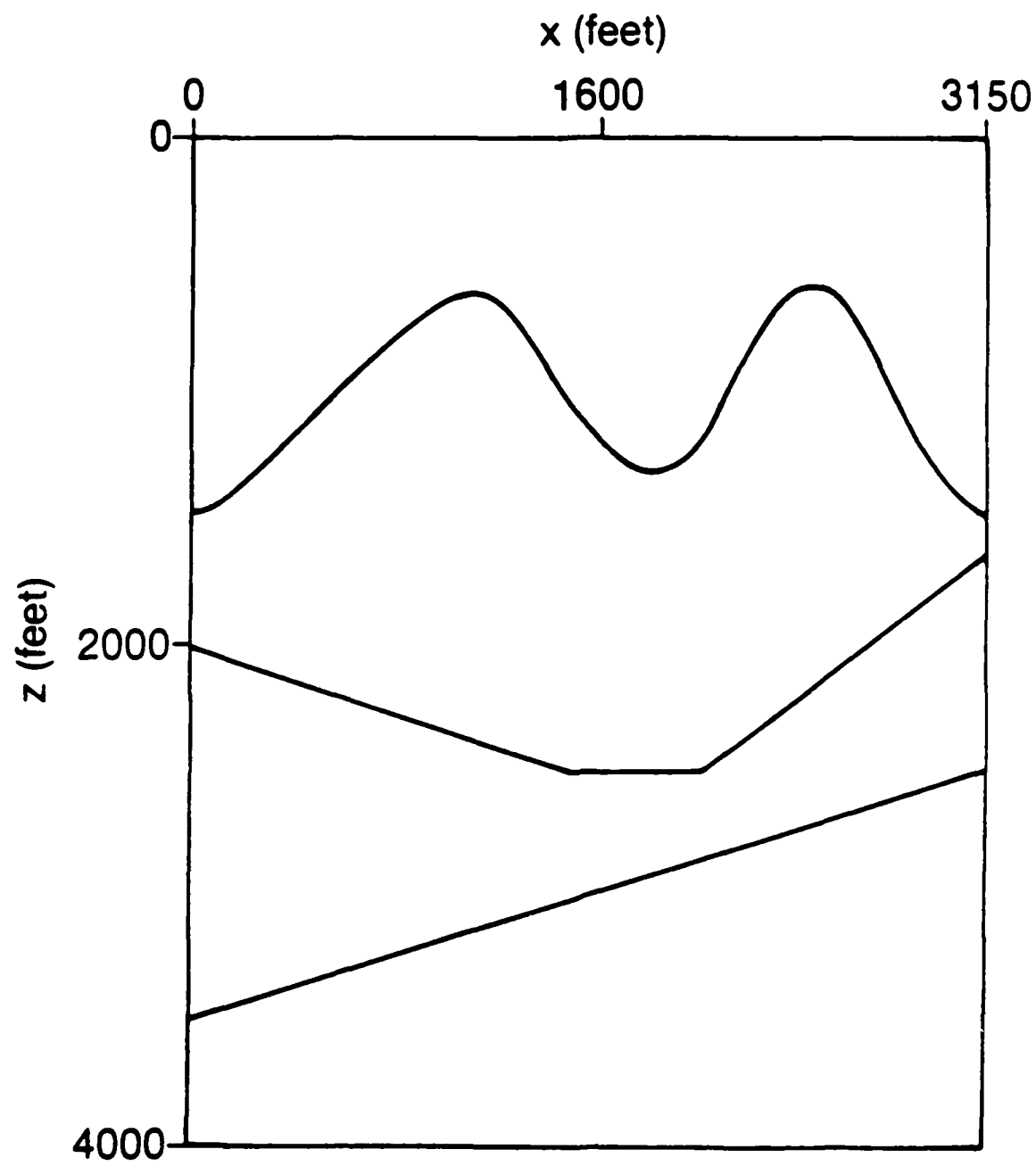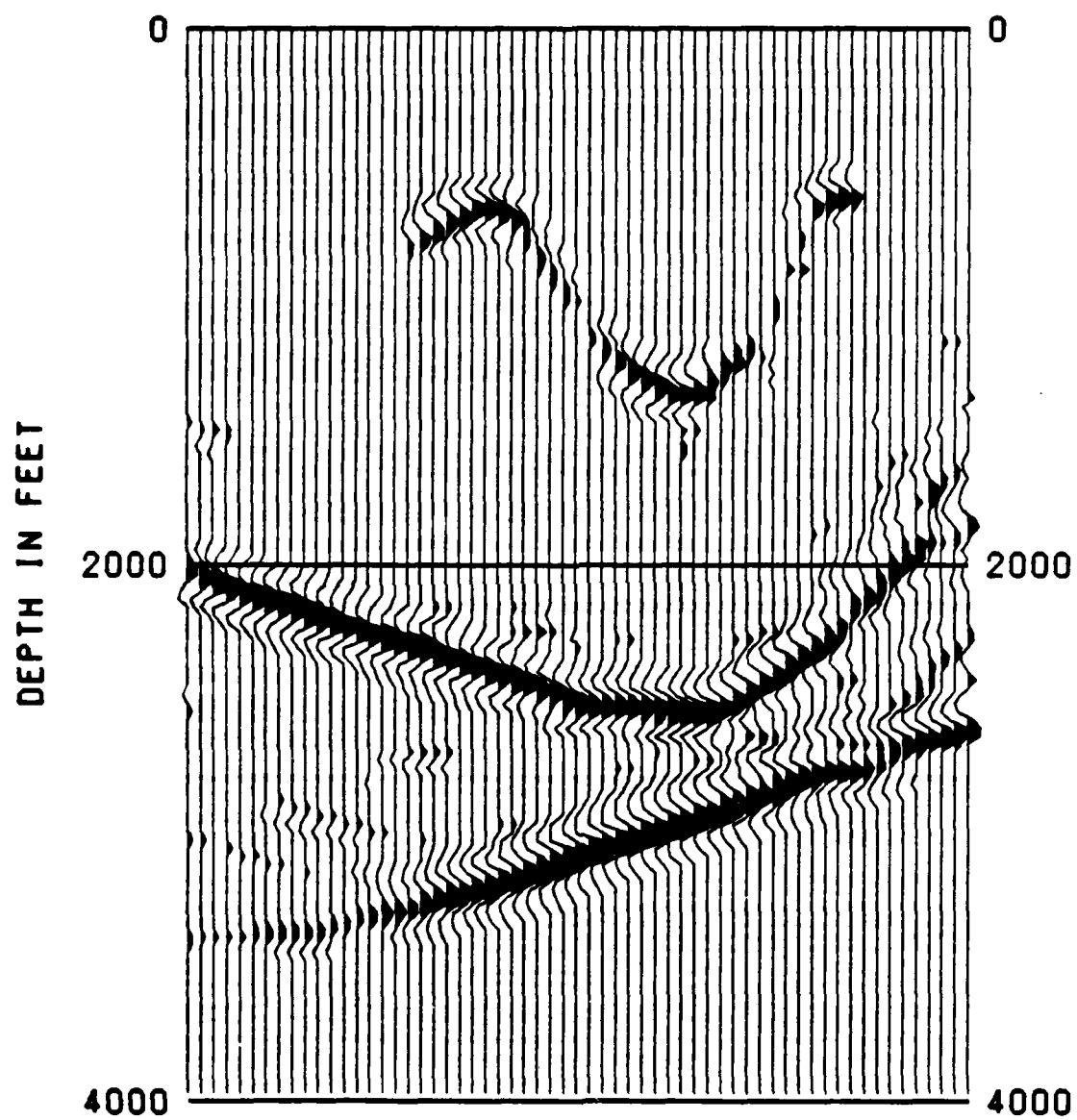
FIGURE 5B

FIGURE 5C

FIGURE 6A

DEPTH IN FEET

FIGURE 6B

FIGURE 66

DEPTH IN FEET

SYNCLINE

MODEL 1

FIGURE 7

TIME (SECONDS)

1.70 1.90 2.10 2.30 2.50 2.70

(0.0) COMMON SHOT ACQUISITION GEOMETRY

(10000.3750)

MODEL 1
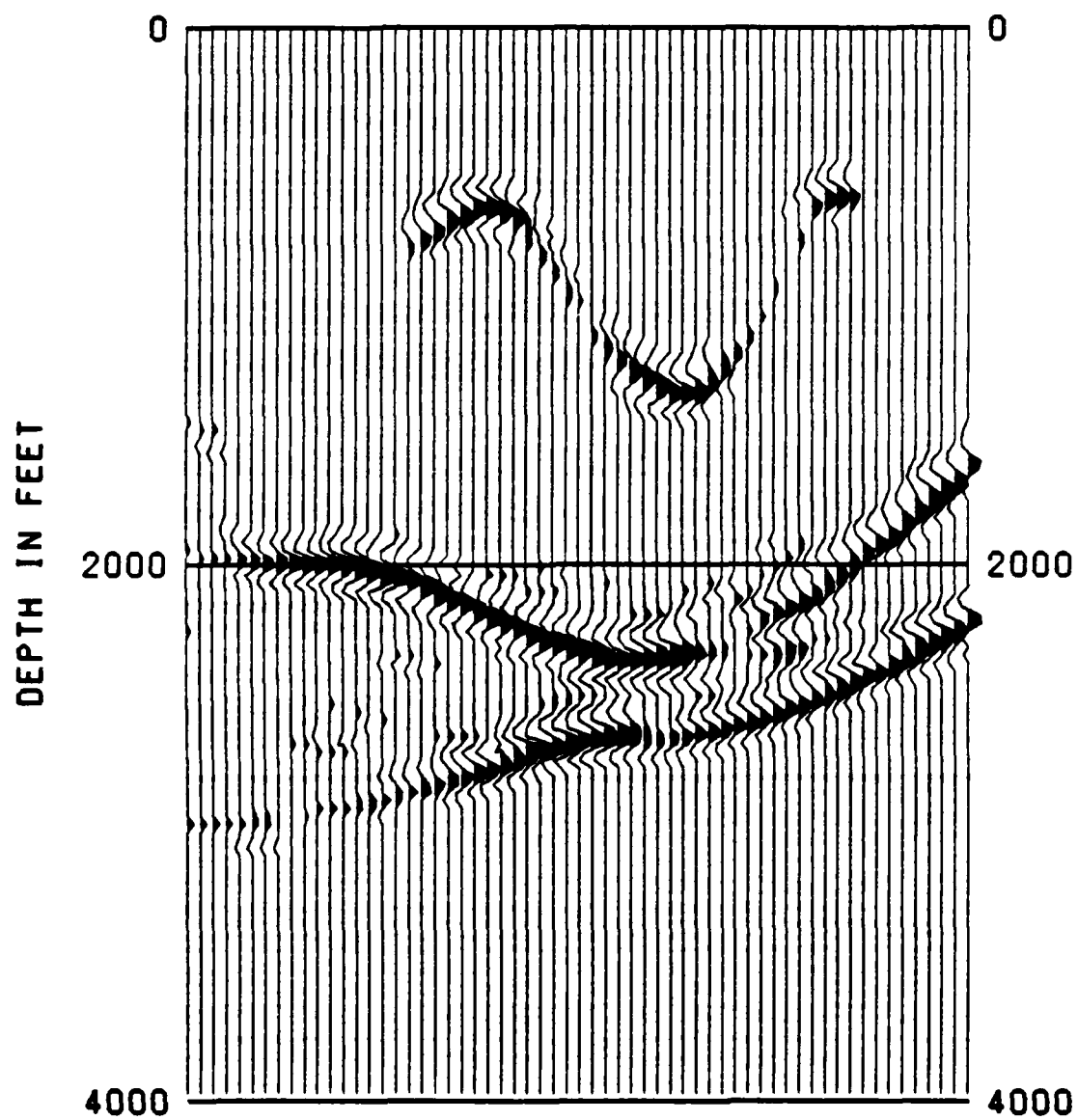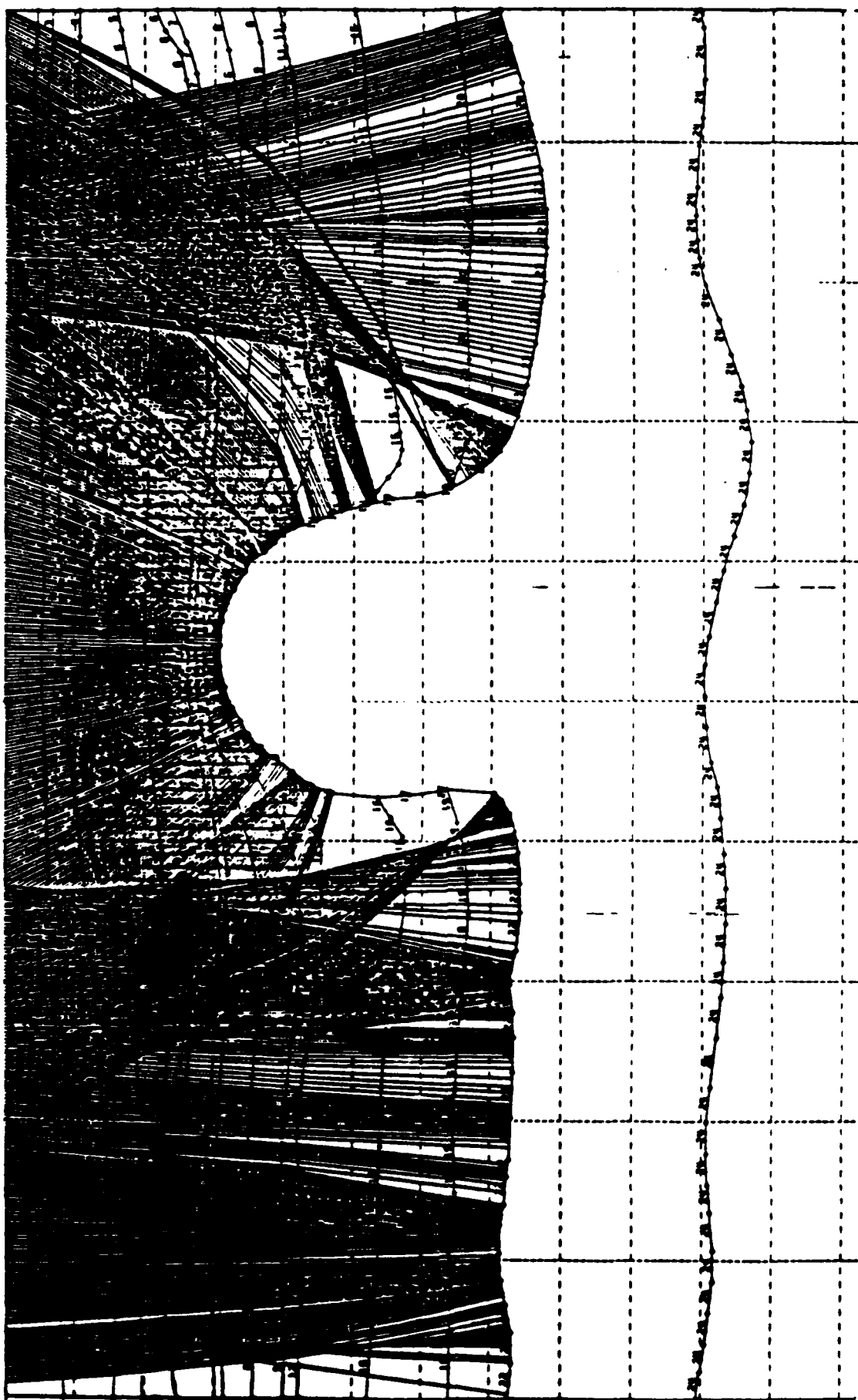
FIGURE 8

CHRONOLOGY OF PAPERS AND REPORTS

DeSanto, J. A., "Oceanic sound speed profile inversion," *IEEE J. Oceanic Eng.*, vol. OE-9, no. 1, pp. 12-17, 1984. Center for Wave Phenomena Research Report, CWP-001, 1983.

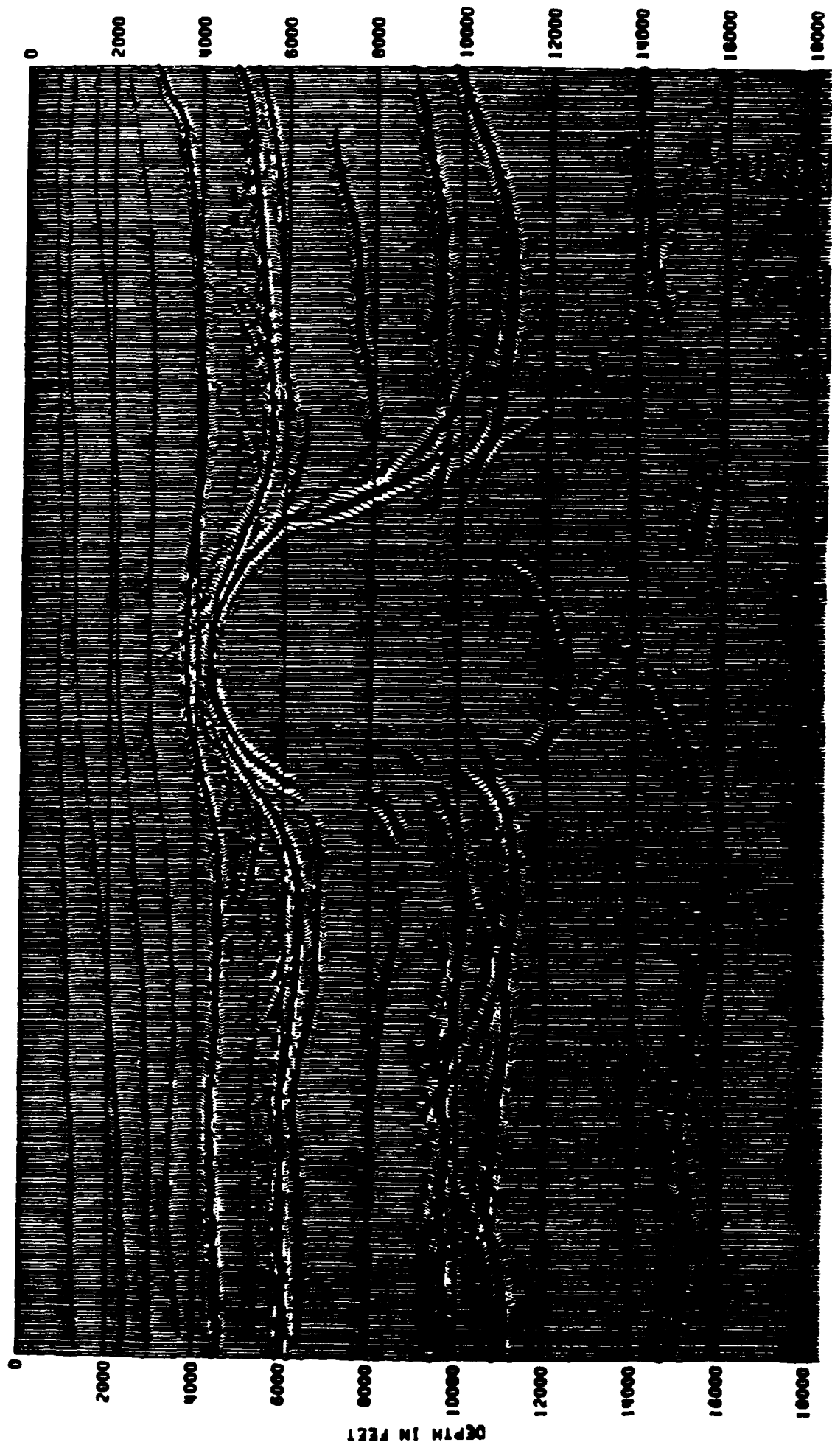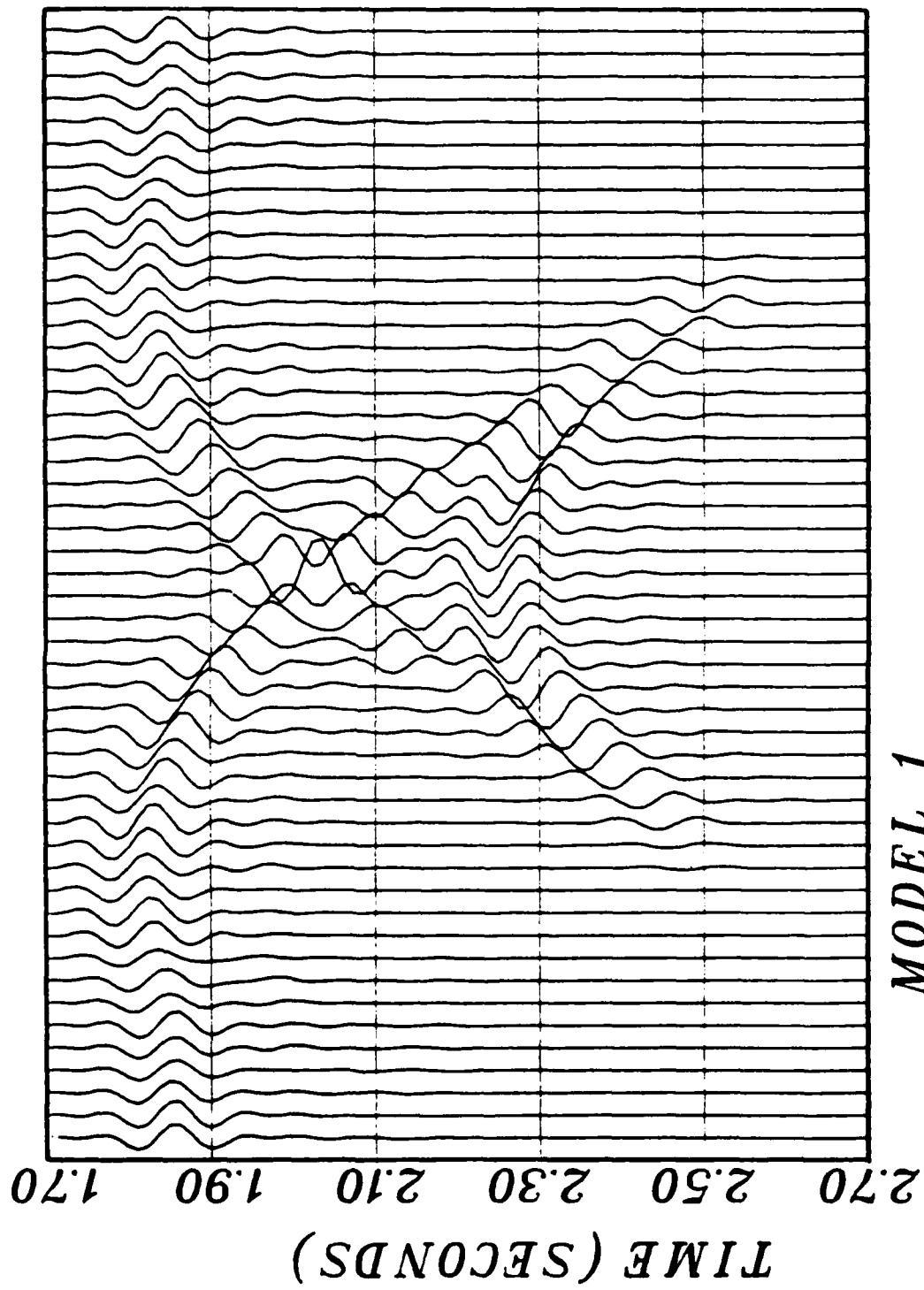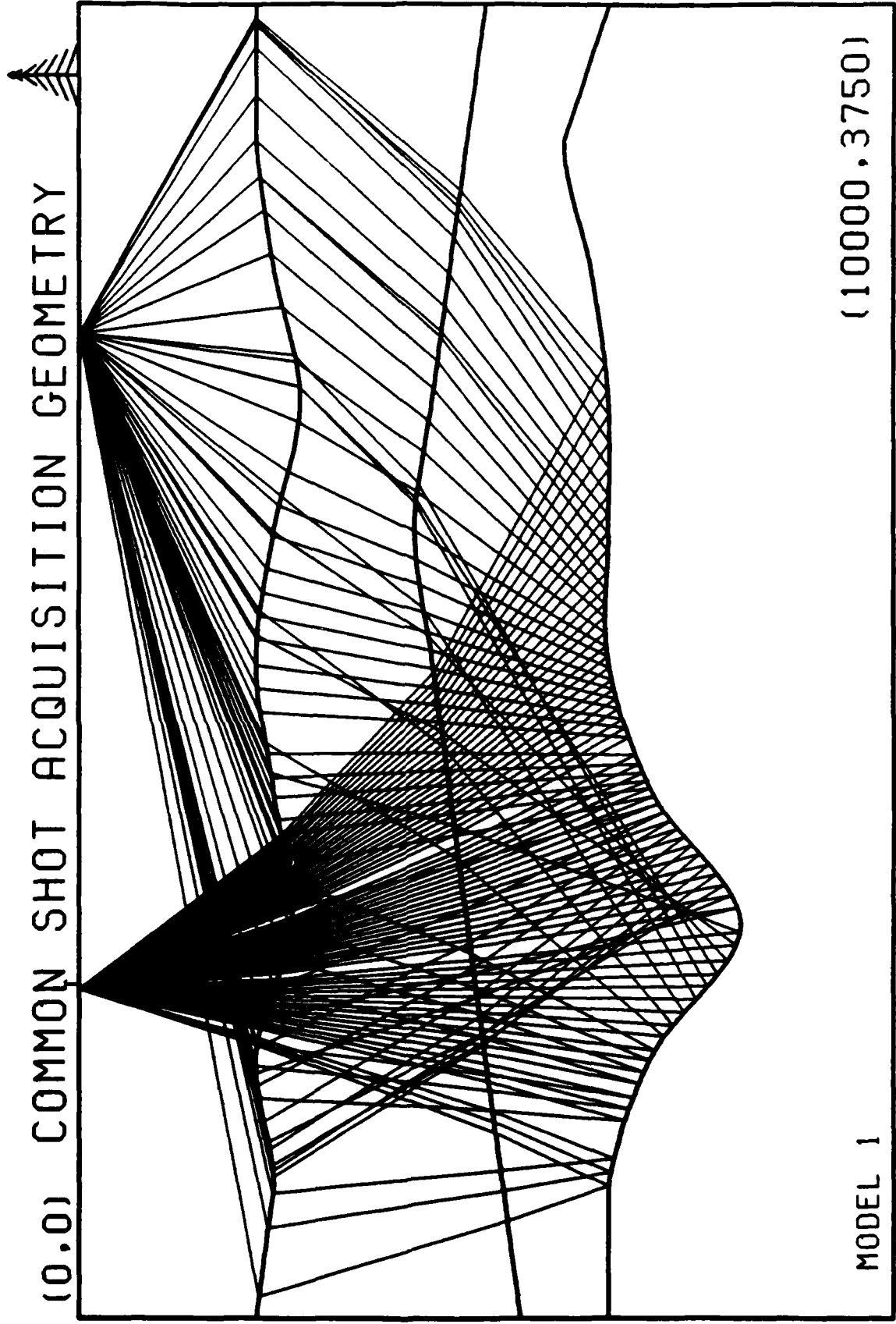Develops a modal (full-wave) method to predict ocean sound speed profiles from propagated acoustic field data. The method assumes a point source of sound in the ocean and uses as data the values of the transmitted acoustic field at the receiver. The method admits the use of realistic input profiles (eg. arbitrary functions of depth) as initial data for an iteration scheme.

Lahlou, M., J. K. Cohen, and N. Bleistein, "Highly accurate inversion methods for three-dimensional stratified media," *SIAM J. Appl. Math.*, vol. 43, pp. 726-758, 1983. Center for Wave Phenomena Research Report, CWP-002, 1983.

Treats the case of variable reference speed inversion in the 1.5D case. Handles discontinuous reference speed. The results are useful for highly accurate forward modeling and were used for this purpose in Hagin and Cohen (1984). This paper was published in special SIAM issue dedicated to Joe Keller.

Hagin, F. G. and J. K. Cohen, "Refinements to the linear velocity inversion theory," *Geophysics*, vol. 49, no. 2, pp. 112-118, 1984. Center for Wave Phenomena Research Report, CWP-003, 1983.

The scattering model derived by Lahlou et al, was used to develop a post-processing algorithm for the zero-offset, constant reference speed algorithm of Cohen and Bleistein. The velocity jump estimates of the original Cohen-Bleistein algorithm gradually degraded with depth. A major factor contributing to this degradation was the continued use of the constant reference velocity at depths where it greatly differed from the true velocity. The "refinement" consisted of picking portions of the inverted section where a dominant dip could be discerned and then systematically updating the computation of dips and velocity estimates on the basis of the output of the original Cohen-Bleistein algorithm. The refinement is an extremely low-cost computation compared with the original inversion. It's efficacy was demonstrated on synthetic sections with up to 20% Gaussian noise added to the signal.

Bleistein, N., J. K. Cohen, and F. G. Hagin, "Computational and asymptotic aspects of velocity inversion," *Geophysics*, vol. 50, no. 8, pp. 1253-1265, 1985. Center for Wave Phenomena Research Report, CWP-004, 1984.

Describes the practical aspects of implementing the zero-offset, constant reference speed algorithm of Cohen and Bleistein, including the use of their singular function theory in developing graphical inversion output. Discusses the exploitation of the high frequency nature of field data to simplify and speed up the computer processing. Also discusses the role of other practical constraints such as causality and spatial aliasing. These lessons largely carry over to more complex

situations involving one or more of variable reference velocity, non-zero-offset, or more detailed propagation models.

Worley, S. C. and J. K. Cohen, "Spatial-temporal aliasing and the wave equation," Res. Rep. CWP-005, Center for Wave Phenomena, Colo. Sch. of Mines, 1984.

A tutorial on the interplay of aliasing phenomena and wave propagation. Includes a geometric interpretation of spatial aliasing and details various strategies for avoiding spatial aliasing.

Bleistein, N., J. K. Cohen, J. A. DeSanto, and F. G. Hagin, "Project review on geophysical and ocean sound speed profile inversion," in Inverse problems of acoustic and elastic waves, ed. F. Santosa, Y. Pao, W. W. Symes, C. Holland, pp. 236-249, SIAM, Philadelphia, 1984. Center for Wave Phenomena Research Report, CWP-006, 1984.

Center for Wave Phenomena presentation at the Cornell Symposium sponsored by the Office of Naval Research in connection with the Selected Opportunities Program.

Bleistein, N. and S. H. Gray, "An extension of the Born inversion method to a depth dependent reference profile," Res. Rep. CWP-007, Center for Wave Phenomena, Colo. Sch. of Mines, 1984. To appear in Geophys. Prosp., 1986.

The first extension of our inversion methods to a non-constant reference velocity. The case treated was a depth-only dependent profile, $c(z)$. In contrast to Lahlou, et al (1983), this algorithm, like the 1979 inversion algorithm, produces a full 3D perturbation correction. The derivation introduced the theme of exploiting the high frequency assumption at an early stage instead of first deriving a wide band algorithm and then simplifying it in the case of high pass data. It subsequently turned out that the inversion derived did not give the correct velocity jump estimate for dipping or curved reflectors. However, the geometric locations of the reflectors were obtained correctly. The importance of this paper was in establishing a new approach, subsequently greatly refined, to developing inversion formulae.

Cohen, J. K. and F. G. Hagin, "Born inversion with a stratified reference velocity," Res. Rep. CWP-008, Center for Wave Phenomena, Colo. Sch. of Mines, 1984. Superceded by Geophysics article and CWP-021.

Developed an algorithm along the lines of the Bleistein-Gray 1984 algorithm. However, this new algorithm produce correct velocity-jump estimates for curved reflectors. The reason that this report was not submitted for publication was that subsequently a much more elegant and powerful method for its derivation was discovered (CWP-021). The version in this unpublished report had the blemish of appealing to the use of the high frequency Kirchhoff scattering formula to derive the amplitude of the inversion kernel.

Sumner, B., "A Fortran 77 self-sorting mixed-radix fast Fourier transform package," Res. Rep. CWP-009, Center for Wave Phenomena, Colo. Sch. of Mines, 1984.

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Discusses an extremely efficient version of the FFT tailored to our typical application. In our inversion work we will often perform hundreds of FFT's all on time series of a common length. The FFT discussed here is based on an algorithm due to Temperton (1983). It has the useful feature of not requiring that the time series have a power of two length, thus we avoid the wasteful processing involved in processing a trailing sequence of zeroes appended to the time series merely in order to convert the length of the series to a power of two.

Gray, S. H. and N. Bleistein, "Seismic imaging and inversion," Res. Rep. CWP-011, Center for Wave Phenomena, Colo. Sch. of Mines, 1985. To appear in IEEE.

Survey article discussing the relation of Born inversion to other methods.

Smith, K. L., "Acoustic tomography in boreholes using an algebraic reconstruction technique," Res. Rep. CWP-012, Center for Wave Phenomena, Colo. Sch. of Mines, 1984. Master's thesis.

Investigates the use of a technique expounded by Mason (1983) in the context of vertical seismic profiling (VSP).

DeSanto, J. A., "Some computational problems in ocean acoustics," Res. Rep. CWP-013, Center for Wave Phenomena, Colo. Sch. of Mines, 1984. To be published in The Proceedings of the Computational Ocean Acoustics Workshop, Yale University, in Comp. and Maths. with Applics.

Summarizes the status of several problems occuring in the propagation and scattering of acoustic waves in the ocean: the scattering of acoustic energy from random and periodic surfaces as models of the ocean surface and bottom, and the inversion of the soundspeed profile using wideband propagated field data. Computational problems in each area are discussed.

Bleistein, N., "Two-and-one-half dimensional in-plane wave propagation," Res. Rep. CWP-014, Center for Wave Phenomena, Colo. Sch. of Mines, 1984. Submitted to Geophys. Prosp.

Exposes a number of useful results about the 2.5D forward problem (the usual geometry for field data). Includes the ray theory for the 2.5D case (3D point sources, "in-plane") and relates it to that for the 2D wave equation (line sources). In addition, results are given for the 2.5D forward modeling problem.

Mager, R. D., "Asymptotic construction of a procedure for plane-wave synthesis and migration," Res. Rep. CWP-015, Center for Wave Phenomena, Colo. Sch. of Mines, 1984.

Returns to a theme introduced in Cohen and Bleistein (1977), namely the use of plane wave (instead of point source) probes. In the intervening years, the development of "slant stack" and "tau-p" techniques (Diebold and Stoffa, 1981; Stoffa et al, 1981; Tatum, 1984; Treitel et al, 1982) has renewed interest in this type of inversion.

Discusses the synthesis and inversion of plane waves from field data.

Bleistein, N., J. K. Cohen, F. G. Hagin, and J. A. DeSanto, "Progress Report: October 1, 1984 of the Selected Research Program of the Office of Naval Research at the Center for Wave Phenomena, Colorado School of Mines," Res. Rep. CWP-016, Center for Wave Phenomena, Colo. Sch. of Mines, 1984.

Sullivan, M. F., "Kirchhoff Modeling via Wave Equation Datuming," Res. Rep. CWP-017, Center for Wave Phenomena, Colo. Sch. of Mines, 1984.

Applies a result of Berryhill (1979) to the efficient creation of scattering data. Here, to gain computational efficiency, only the single layer 2.5D case is treated.

Docherty, Paul, "A fast ray tracing routine for laterally inhomogeneous media," Res. Rep. CWP-018, Center for Wave Phenomena, Colo. Sch. of Mines, 1985. Presented at the 1984 SEG meeting.

Extends the ideas of Keller and Perozzi (1983) and Fawcett (1983) to create an algorithm suitable for finding large numbers of raypaths and traveltimes. This algorithm will play an important role in both creating modeling data and in implementing inversion schemes in media with laterally varying reference velocity.

Bleistein, N., J. K. Cohen, F. G. Hagin, J. A. DeSanto, and R. D. Mager, "Project Review, December 1, 1984, Consortium Project on Seismic Inverse Methods for Complex Structures," Res. Rep. CWP-019, Center for Wave Phenomena, Colo. Sch. of Mines, 1984.

Cohen, J. K. and F. G. Hagin, "Velocity inversion using a stratified reference," _Geophysics_, vol. 50, no. 11, 1985 (in press). Center for Wave Phenomena Research Report, CWP-021, 1984.

Gives an improved derivation of the inversion formula for the report, "Born Inversion with a Stratified Velocity" discussed above. This publication version derives the inversion formula on the basis of imposing a "completeness" relation of the type occuring in classical transform theory. Then the computation involving the use of Kirchhoff data which was vital to the previous derivation becomes merely a verification of the algorithm. The completeness relation theme has subsequently proved useful in deriving other inversions. The extension from constant reference velocity to stratified reference velocity represented by the Bleistein-Gray and the Cohen-Hagin algorithms was an important step in the practical application of the inversion method since the trend of a real earth section can now be included, making the assumption of a small perturbation much more reasonable. This algorithm should be particularly well suited to inversion of the near surface seabed in cases where high frequency data is collected from a region of roughly horizontal sedimentary layers.

Cohen, J. K., "Programming standards," Res. Rep. CWP-022, Center for Wave Phenomena, Colo. Sch. of Mines, 1985.

DeSanto, J. A., "Relation between the connected diagram and smoothing
    methods for rough surface scattering," Res. Rep. CWP-023, Center for
    Wave Phenomena, Colo. Sch. of Mines, 1985. Submitted to J. Math.
    Phys.

    Extension of previous work by the author on connected diagram methods.
    Here it is shown that the smoothing method applied to the Lippman-
    Schwinger equation for stochastic scattering from a rough surface is
    not fully equivalent to the connected diagram expansion. In fact,
    they give the same result only to second order in the surface
    interaction.

DeSanto, J. A., "Ocean acoustics," in The encyclopedia of physics, third
    edition, ed. R. M. Besancon, pp. 836-840, Van Nostrand & Reinhold
    Inc., 1984. Center for Wave Phenomena Research Report, CWP-024, 1985.

    Discusses the physical properties of the ocean surface, volume and
    bottom as well as ocean variability in general. Contains an overview
    of sound scattering and propagation methods in the ocean.

Bleistein, N., J. K. Cohen, J. A. DeSanto, and F. G. Hagin, "Project
    Review, May 8, 1985, Consortium Project on Seismic Inverse Methods for
    Complex Structures," Res. Rep. CWP-025, Center for Wave Phenomena,
    Colo. Sch. of Mines, 1985.

DeSanto, J. A., "Exact spectral formalism for rough surface scattering," J.
    Opt. Soc. Am., Dec. 1985. Center for Wave Phenomena Research Report,
    CWP-026

    Derives the exact spectral amplitudes of the scattered and transmitted
    fields for a plane wave incident on an arbitrary rough surface in one
    dimension. Results are stated in terms of integrals over values of
    the field and its normal derivative on the interface.

Sullivan, M. F. and J. K. Cohen, "Pre-stack Kirchhoff inversion of common
    offset data," Res. Rep. CWP-027, Center for Wave Phenomena, Colo. Sch.
    of Mines, 1985.

    This report represented our first treatment of non-backscattered data.
    Actual field data is collected in offset mode and requires "stacking"
    in order to simulate backscattered data. The stacking process
    inevitably degrades the amplitude information, thus algorithms which
    can perform "inversion before stack" are desirable. In this paper,
    the inversion was obtained using a constant reference velocity.
    The inversion presented here is based on the Kirchhoff integral
    equation (Cohen and Bleistein, 1979b; Bleistein, 1984a; Bleistein,
    1984b) instead of the Born integral equation used in our previous
    work. The Kirchhoff equation approach has the advantage of
    immediately producing the "singular function" of the surface, while
    the Born approach must appeal to Fourier transform results to convert
    its natural output of the "characteristic function" to the desired
    singular function.

Leroux, I., "Qualitative analysis of sign-bit processing," Res. Rep. CWP-

028, Center for Wave Phenomena, Colo. Sch. of Mines, 1985. Master's thesis.

Adapted the Cohen-Bleistein constant reference algorithm to the case where only the sign of each datum is recorded instead of the full floating amplitude. It was shown that even with this severe data reduction, the algorithm succeeds in accurately picking reflectors out of noise for a series of synthetic data sets. Furthermore, it was shown that a significant amount of amplitude information resides in the reduced data.

Violette, Paul B., "Analysis of two-parameter constant background Born inversion for acoustic synthetic data," Res. Rep. CWP-029, Center for Wave Phenomena, Colo. Sch. of Mines, 1985. Master's thesis.

Attempts to recover two parameters (density and velocity) using a technique expounded by Clayton and Stolt (1981). Here, the simple case of a constant reference was examined. Unfortunately, the results were largely negative; this technique appears to be very sensitive, even with synthetic data.

DeSanto, J. A. and G. S. Brown, "Analytical techniques for multiple scattering from rough surfaces," in _Progress in optics, vol. 23_, ed. E. Wolf, North-Holland Publishing Co. Inc., Amsterdam, 1986. Center for Wave Phenomena Research Report, CWP-030

A review paper on the status of analytical methods in rough surface scattering, with particular emphasis on multiple scattering techniques. It includes a discussion of surface roughness, surface and scattered field statistics, as well as the relation of single scattering techniques to various multiple scattering methods. The paper describes such multiple scattering techniques as the Lippman-Schwinger equation, diagrammatic methods, the k-space formalism and the smoothing method. Applications are made to both the acoustic and the electromagnetic theory. Contains an extensive bibliography.

Cohen, J. K., F. G. Hagin, and N. Bleistein, "A preliminary report on some recent results in Born inversion," Res. Rep. CWP-031, Center for Wave Phenomena, Colo. Sch. of Mines, 1985.

Combine the completeness relation theme of Cohen and Hagin, and the work of Beylkin (1984, 1985) to develop a number of new inversion formulas. We treat the case of zero-offset with a completely arbitrary background as well as the offset cases of single-source-many receivers and single-receiver-many-sources. These algorithms give the inversion in terms of quantities which are obtained by tracing rays.

Bleistein, N., J. K. Cohen, and F. G. Hagin, "A note on 2.5D Born inversion," Res. Rep. CWP-032, Center for Wave Phenomena, Colo. Sch. of Mines, 1985.

Continues the work of CWP-031 for the 2.5D, $c(x,z)$, case. Algorithms involving only ray tracing are given for the common source, common receiver and common offset configurations. It should be noted that the first two of these are not specializations of the 3-D analogs and that

the third was previous derived only for constant background (CWP-027).
Includes discussion of extracting velocity jumps and reflectivity.

DeSanto, J. A. and G. S. Brown, "Some recent results in rough surface
scattering," Res. Rep. CWP-033, Center for Wave Phenomena, Colo. Sch. of
Mines, 1985. To be published in Proceedings of a Conference on "Multiple
Scattering of Waves in Random Media and Random Rough Surfaces",
Pennsylvania State University.

A brief overview of recent theoretical developments in rough surface
scattering. Discusses the relation between the connected diagram
expansion and the smoothing method, the development of the stochastic
Fourier transform approach, the use of smoothing method techniques to
derive an algebraic solution for the amplitude of the coherent wave
and finally, the derivation of exact expressions for field spectral
amplitudes.

Bleistein, N., "An introduction to the mathematical theory of wave
phenomena," in _Encyclopedia of physical science and technology_,
Academic Press Inc., New York, 1986. Center for Wave Phenomena
Research Report, CWP-034, 1985.

Review article discussing the common features of wave motion in
various media.

DeSanto, J. A., "Impedance at a rough waveguide boundary," _Wave Motion_,
vol. 7, pp. 307-318, 1985. Center for Wave Phenomena Research Report,
CWP-035, 1985.

Derives the impedance at the randomly rough boundary of an ocean
waveguide to second order in the surface interaction. The result is a
rational approximation in terms of the waveguide Green's function and
the statistical properties of the surface. The sound speed in the
waveguide is an arbitrary function of depth.

Bleistein, N., "Progress on an inverse method for seabed mapping and
seismic exploration," Res. Rep. CWP-036, Center for Wave Phenomena,
Colo. Sch. of Mines, 1985. To be published in SIAM Review.

Review article for an invited lecture at the Houston joint SIAM-SEG
meeting.

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