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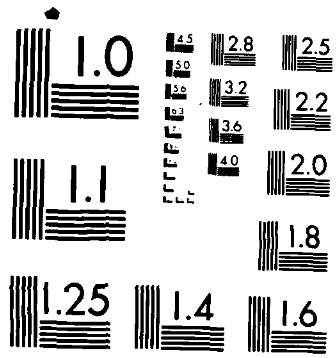

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EXISTENCE OF RANDOM VARIABLES WITH
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by

S. Ramaswamy

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EXISTENCE OF RANDOM VARIABLES WITH
VALUES IN THE DUAL OF A NUCLEAR SPACE

by

S. Ramaswamy*

Abstract

The aim of this article is to apply some results of L. Schwartz's theory of radonifying maps to prove existence theorems for infinite dimensional valued random variables. As a consequence, we deduce some known results in this direction due to K. Itô, M. Perez-Abreu C., and T. Bojdecki and L.G. Gorostiza.

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1. Preliminaries

Before we state and prove our main result, we need the following definitions and propositions from L. Schwartz [3], in the Chapter XIII, pp. 4 and 5.

Definition 1. Let (Ω,F,P) be a probability space. Let E be a locally convex Hausforff topological vector space and let E' be its dual. Let f be a linear random function from E' to $L^{\circ}(\Omega,F,P)$. f is said to be *decomposed* if $\exists$ a measurable mapping ϕ from Ω to E such that for all $\xi \in E'$,

$$\xi \circ \phi = f(\xi).$$

Definition 2. Let E and G be two Banach spaces. Let u be a continuous linear mapping of E in G. The map $t_u : G' \to E'$ is said to be *p-decomposing* $(0 \le p \le \infty)$ if for every linear random function $f : E' \to L^p(\Omega,F,P)$, the composite $f \circ t_u$ from G' to $L^p(\Omega,F,P)$ is decomposed by a mapping ϕ from Ω to G, $\phi \in L^p(\Omega,F,P;G)$ (with ess sup $\|\phi\| < \infty$ in the case when $p = \infty$).

Proposition (XIII, 3;2). Let E,G be Banach spaces. Let u be a continuous linear mapping of E in G. Then u is p-radonifying $(p > 0)$ if and only if $t_u : G' \to E'$ is p-decomposing.

We also need the fact that if T is a Hilbert-Schmidt operator from one Hilbert space to another, then it is p-radonifying for all $p > 0$. This is proved in Chapter XII, p. 2 in [3].

2. Existence theorem

We first prove a simple proposition.

Proposition 1. Let H_1 and H_2 be two Hilbert spaces and let T be a Hilbert-Schmidt operator from H_1 to H_2. Then, T is p-decomposing for any $p > 0$. Further, if f is a continuous linear random function from H_2 to $L^2(\Omega,F,P)$, then the composite $f \circ T$ is decomposed by a mapping $X : \Omega \to H_1$ such that $X \in L^2(\Omega,F,P:H_1)$ with

$$\int \|X(w)\|^2 dP(w) \le \|f\|^2 \|T\|_2^2$$

where $\|T\|_2$ is the Hilbert-Schmidt norm of T.

Proof. Since T is Hilbert-Schmidt, its transpose ${}^tT$ from H_2 to H_1 is also Hilbert-Schmidt. Hence, it is p-radonifying for any $p > 0$. Hence, by the proposition (XIII, 3;2) mentioned above, its transpose ${}^t({}^tT)$ which is T is p-decomposing for any $p > 0$. In particular it is 2-decomposing.

Let f be a continuous linear random function from H_2 to $L^2(\Omega, \ ,P)$. Then $f \circ T$ is decomposed by a mapping $X : \Omega \to H_1$ such that $\int \|X(w)\|^2 dP(w) < \infty$.

Let $(\phi_i)_{i \in I}$ be an orthonormal basis for H_1. Then, for all $w \in \Omega$,

$$\|X(w)\|^2 = \sum_{i \in I} |\langle X(w), \phi_i \rangle|^2 = \sup_{\substack{J \\ J \text{ finite} \\ J \subset I}} \sum_{i \in I} |\langle X(w), \phi_i \rangle|^2 .$$

As the family $(\Sigma_{i \in J} |\langle X(\cdot), \phi_i \rangle|^2)_J$, $J \subset I$, J finite, of functions on Ω, is directed increasing, by Lebesgue's monotone convergence theorem, we have

$$\int \|X(w)\|^2 dP(w) = \sup_{\substack{J \\ J \text{ finite} \\ J \subset I}} \int \sum_{i \in J} |\langle X(w), \phi_i \rangle|^2 dP(w)$$

$$= \sup_{\substack{J \\ J \text{ finite} \\ J \subset I}} \sum_{i \ J} \int |\langle X(w), \phi_i \rangle|^2 dP(w)$$

$$= \sum_{i \in I} \int |\langle X(w), \phi_i \rangle|^2 dP(w).$$

As $f \circ T$ is decomposed by X,

$$\langle X(\cdot),\phi_i\rangle = f(T(\phi_i)) \quad \text{for all } i \in I.$$

Hence, for all $i \in I$,

$$\int |\langle X(w),\phi_i\rangle|^2 dP(w) = \int |f(T(\phi_i))(w)|^2 dP(w) \le \|f\|^2 \|T(\phi_i)\|^2.$$

Hence,

$$\sum_{i\in I} \int |\langle X(w),\phi_i\rangle|^2 dP(w) \le \|f\|^2 \sum_{i\in I} \|T(\phi_i)\|^2 \le \|f\|^2 \|T\|_2^2. \qquad \text{QED}$$

Theorem 1. Let E be a nuclear space. Let E' be its dual. Let ϕ be a continuous positive-definite bilinear form on E. Then, there exists a probability space (Ω,F,P) and a random variable $X : \Omega \to E'$ such that for all $x \in E$, the real-valued random variable X_x defined as $X_x = x \circ X$ is Gaussian with mean zero and the covariance kernel of the process $(X_x)_{x\in E}$ is ϕ.

Proof. Since ϕ is a positive-definite kernel on E, $\exists$ a real-valued Gaussian process $(X_x)_{x\in E}$ on a probability space (Ω,F,P) with mean zero and with covariance kernel ϕ.

Since ϕ is bilinear, it is easy to see that the mapping f from E to $L^2(\Omega,F,P)$ taking x to X_x is linear. Further, f is continuous, as ϕ is continuous. Hence, E being nuclear, $\exists$ neighborhoods U,V of (0), U,V both convex, balanced and closed, $V \subset U$, $\hat{E}_V$ and $\hat{E}_U$ both Hilbert spaces such that the canonical map $\phi_{U,V}$ from $\hat{E}_V$ to $\hat{E}_U$ is Hilbert-Schmidt and such that f admits a factorization $\psi \circ \phi_{U,V} \circ \phi$

$$E \xrightarrow{\phi_V} \hat{E}_V \xrightarrow{\phi_{U,V}} \hat{E}_U \xrightarrow{\psi} L^2(\Omega,F,P)$$

where ψ is a continuous linear and ϕ_V is the canonical mapping.

As $\phi_{U,V}$ is Hilbert-Schmidt, by Proposition 1, it is 2-decomposing. Hence, $\psi \circ \phi_{U,V}$ is decomposed by a mapping Y from Ω to $\hat{E}'_V$ such that $Y \in L^2(\Omega, F, P : \hat{E}'_V)$ with $\|Y\| \leq \|\psi\| \, \|\phi_{U,V}\|_2$.

Let X be the mapping from Ω to E' defined as $X = {}^t\phi_V \circ Y$. Then, as Y decomposes $\psi \circ \phi_{U,V}$, X decomposes $\psi \circ \phi_{U,V} \circ \phi_V$ which is f. Therefore, for all $x \in E$, we have $x \circ X = f(x) = X_x$ as elements of $L^\circ(\Omega, F, P)$. QED

Remark. As the image of E in $\hat{E}_V$ under the map ϕ_V is dense, the transpose map ${}^t\phi_V$ from $\hat{E}'_V$ to E' is an injection. Hence, $\hat{E}'_V$ can be thought of as a subspace of E' algebraically. Hence, the E'-valued random variable X of the above theorem is actually $\hat{E}'_V$-valued.

3. Application to known results

We now deduce theorem 3.1 of K. Itô in [2], concerning the existence of $\mathcal{S}'_{p+2}$ regularizations, from our proposition 1.

To deduce this, we have only to prove that the canonical inclusion from $\mathcal{S}_{p+2}$ to $\mathcal{S}_p$ is Hilbert-Schmidt with Hilbert-Schmidt norm $(\frac{\Pi^2}{8})^{1/2}$. This is done as follows.

We consider $\mathcal{S}_p$ as a sequence space consisting of all the sequences $a = (a_n)_{n \in \mathbb{N}}$ such that $\Sigma_{n=1}^{\infty} |a_n|^2 (2n+1)^p < \infty$.

Let for all $n \in \mathbb{N}$, e^n be the sequence $(e_1^n, e_2^n, \ldots, e_i^n, \ldots)$ where $e_i^n = \delta_{ni}$. Then it is easily seen that the sequence $(f^n)_{n \in \mathbb{N}}$ of elements of $\mathcal{S}_{p+2}$ where $f^n = \dfrac{e^n}{\|e^n\|_{p+2}}$ is an orthonormal basis for $\mathcal{S}_{p+2}$.

Now

$$\|f^n\|_p^2 = \frac{\|e_n\|_p^2}{\|e^n\|_{p+2}^2} = \frac{(2n+1)^p}{(2n+1)^{p+2}} = \frac{1}{(2n+1)^2} .$$

Hence

$$\sum_{n=1}^{\infty} \|f_n\|_p^2 = \sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} = \frac{\Pi^2}{8} .$$

This shows that the canonical inclusion from $\mathcal{S}_{p+2}$ to $\mathcal{S}_p$ is Hilbert-Schmidt with Hilbert-Schmidt norm $(\frac{\Pi^2}{8})^{1/2}$.

The theorem 1 and the remark following it give immediately as corollary the existence for all $t \in \mathbb{R}_+$ of a Φ'-valued random variable (actually a H_{-q} random variable, $q \in \mathbb{N}$, independent of t) which is proved in theorem 4.1.1 of [4]. There, Φ is a countably Hilbert nuclear space.

In the same way, the existence of a $S'(\mathbb{R}^d)$-valued random variable W_t, for all $t \in \mathbb{R}_+$ in theorem 2.4 of [1] will follow provided we prove the continuity of the bilinear form $(\phi,\psi) \to \int_0^t \langle Q_u\phi,\psi\rangle du$ on $S(\mathbb{R}^d) \times S(\mathbb{R}^d)$. This follows from the following proposition.

Proposition 2. Let E,F be Frechet spaces. Let for all $u \in \mathbb{R}_+$, Q_u be a continuous linear map from E to F', F' being provided with the topology $\sigma(F',F)$. Let further, for all $(x,y) \in E \times F$, the function $u \to \langle Q_u x,y\rangle$ be cadlag. Then for all $t \in \mathbb{R}_+$, the bilinear form

$$(x,y) \to \int_0^t \langle Q_u x,y\rangle du$$

is continuous on $E \times F$.

Proof. Since E and F are Frechet, to prove that a bilinear form is continuous, sufficient to prove that it is separately continuous. Therefore, we shall prove that for all $y \in F$, the linear mapping $x \to \int_0^t \langle Q_u x,y\rangle du$ is continuous on E. Analogously, it will follow that for all $x \in E$, the linear map $y \to \int_0^t \langle Q_u x,y\rangle du$ is continuous on F.

Let $y \in F$ be fixed. Let $(x_n)_{n\in\mathbb{N}}$ be a sequence of elements of E such that $x_n \to 0$. Then, for all u, $0 \le u \le t$, $\langle Q_u x_n, y\rangle \to 0$. Hence the convergence of the integrals $\int_0^t \langle Q_u x_n, y\rangle du$ to zero will follow from the dominated convergence theorem, in case we prove that

$$\sup_{n\in\mathbb{N}} \sup_{\substack{u \\ 0\le u\le t}} |\langle Q_u x_n, y\rangle| < \infty.$$

Now for all u, $0 \le u \le t$, the linear map f_y^u from E to $\mathbb{R}$ defined as $f_y^u(x) = \langle Q_u x, y\rangle$ is continuous. As for all (x,y), the real-valued function $u \to \langle Q_u x, y\rangle$ is cadlag, $\sup_{\substack{u \\ 0\le u\le t}} |\langle Q_u x, y\rangle| < \infty$. Hence, the family of linear maps $(f_y^u)_{0\le u\le t}$ is pointwise bounded. As E is Fréchet, it is barreled and hence by the theorem of Banach-Steinhaus, the family $(f_y^u)_{0\le u\le t}$ is equicontinuous. Hence there exists a neighborhood U of (0) such that

$$\sup_{\substack{x \\ x\in U}} \sup_{\substack{u \\ 0\le u\le t}} |f_y^u(x)| \le 1.$$

That is

$$\sup_{\substack{x \\ x\in U}} \sup_{\substack{u \\ 0\le u\le t}} |\langle Q_u x, y\rangle| \le 1.$$

As $x_n \to 0$, $\exists$ N such that $x_n \in U$ for all $n \ge N$. Hence

$$\sup_{\substack{n \\ n\in\mathbb{N}}} \sup_{\substack{u \\ 0\le u\le t}} |\langle Q_u x_n, y\rangle| < \infty.$$ QED

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