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DETERMINATION OF THE MODULATION TRANSFER FUNCTION OF LINE SCANNED INFRARED IMAGING SYSTEMS

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S U M M A R Y

A computer model to determine the Modulation Transfer Function of a line scanned imaging system is discussed. The model can incorporate the effects of the atmospheric path, the system optics and the display as well as the detector's resolution limits. It can be used with single element or SPRITE detectors and its application to a design task is discussed.

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1. INTRODUCTION

The theoretical assessment of the performance of a line scanned imaging system is an important part of the design and evaluation of such systems, provides the means to choose from competing design approaches in the developmental stage and may provide information to facilitate the assessment of working equipment. The performance of an imaging system depends on a number of complex sub-systems each of which may degrade the performance of the overall system. The effectiveness of an imaging system in performing a task cannot readily be predicted using any simple parameter to describe the system performance since in general the task involves the presentation of complex, spatially modulated two-dimensional information to the observer. In performing the assessment of an imaging system a complex set of factors must be analysed, and the results of this analysis may then provide a useful guide to the effectiveness of a given design approach.

Thermal sensitivity and spatial resolution dominate the performance of infrared imaging systems; Minimum Resolvable Temperature Difference (MRTD) is used as a figure of merit combining these parameters in the assessment of the performance of infrared imaging systems. The MRTD relates these two parameters to the eye response and provides a measure of system performance with respect to the detection of equivalent bar targets. The determination of the MRTD requires a knowledge of the overall system Modulation Transfer Function (MTF) as well as the sensitivity and noise performance of the system. Other figures of merit may be defined and in general, to be of use, these require a knowledge of the MTF of the system.

The MTF of a system provides a useful measure of the system's performance and can be used to calculate other performance parameters. The MTF can be calculated using a mathematical model of each of the system components, the performances of which are then combined to determine the overall system response. To be effective a system model must take account of all sources of image degradation, and knowledge of the performance of the component parts of a system may allow the designer to optimise the system configuration to meet a specified design criterion, or it may facilitate the determination of the causes of degraded performance during evaluation. A model of an imaging system which can provide the overall MTF from the component MTFs is therefore necessary in the evaluation of a system design.

This document describes a mathematical model of a line scanned imaging system which allows the calculation of the MTF of the system. The model has been developed to aid in the analysis and design of an imaging system which may use discrete single-element detectors or more advanced detectors using charge sweepout effects (the 'SPRITE' detectors developed by the UK Royal Signals and Radar Establishment). The components of the model represent the individual sections of the imaging system and the effects of component changes on system performance can thus be monitored. The model has been implemented in a computer program and its use in determining the design parameters for a digital frame store to be used with a slow scan imaging system is discussed. The model allows the differences in spatial resolution capabilities of the two detector types to be accommodated in the design and its use with the differing configurations is illustrated.

2. SYSTEM MODEL AND APPLICATIONS

The analysis of an imaging system involves the manipulation of the description of spatial and temporal processes which occur in the image acquisition and display. The formulation of the problem must therefore take into account the differences between temporal (electronic) and spatial (optical) responses. Time varying signals can be tailored through the use of electronic filters to be sufficiently band limited to reduce the degradation due to effects such as aliasing to an insignificant degree; however the optical response of line scanned imaging systems cannot be so readily modified and the analysis must account for spatial aliasing where necessary.

The mathematical methods used to analyse the system performance via the MTF are those of linear filter theory, and their application is based on a number of assumptions, primarily that the signal processing is linear and that the imaging process is spatially invariant. Neither of these conditions are consistently satisfied in practical systems but the approximations are used to allow the analysis to be undertaken. Thermal imaging systems with discrete detectors are dominated by the detector performance, characterised by the detector angular subtense, and the display characteristics. The assumption of isoplanatic image formation is usually acceptable if the detector response (or more usually the display medium response) is the limiting factor controlling system performance.

The conventional means of assessment of imaging systems is to develop a measure of performance, such as the MRTD or Noise Equivalent Temperature Difference (NETD), and to use the parameter determined as an indicator of the system quality. Parameters such as MRTD can be used to predict detection performance of "equivalent bar targets" but are not an adequate descriptor for imaging systems since they do not provide a measure of 'image quality'. For example, MRTD cannot be used to predict the image spatial characteristics. Resolution in a gross sense is indicative of the MTF limits but provides no information on the shape of the MTF curve which can be related to image quality in any predictable manner. The correlation of image quality with MTF has been investigated extensively and it has been found that the MTF can provide a measure (non-specific in the case of particular target types) upon which some guide to image quality can be developed. The MTF can be used to calculate two parameters which correlate well with subjective measures of image quality and task performance, as well as providing the basis for the MRTD calculations. These are the Equivalent Line Number and Modulation Transfer Function Area (MTFA).

The equivalent line number, N_e, is defined by

$$N_e = \int_0^{\infty} [T(f)]^2 \, df$$

where T(f) is the system modulation transfer function and has been found to be useful in subjectively ranking image quality(ref.1). A 'barely detectable' change in image sharpness can be related to a 5% change in N_e, thus N_e provides some measure of 'image quality'.

The MTFA (originally called Threshold Quality factor TQF) is defined as

$$\text{MTFA} = \int_0^{f_1} [T(f) - M_D(f)] \, df$$

where f_1 is the limiting resolution of the system for a sine wave target and $M_D(f)$ is the threshold detectability. It has been shown that MTFA is strongly related to subjective estimates of image quality(ref.2) and to the capacity of observers to extract information from images. The MTFA has been found to be a valid predictor of overall system performance for raster scanned systems although it does not provide a valid base for prediction of task specific performance. To be useful the threshold modulation ($M_D(f)$) must be determined for the given system and operating parameters with due regard to the image signal to noise ratio. At this stage of development of the model described here this is not possible.

With the advent of digital processing of images (both real-time or off-line) there arises a need to characterise the effect on image quality of the sampling and quantisation processes inherent in such systems. The availability of two dimensional sensor arrays exacerbates the problem of finding a suitable description of image quality and imaging system performance since the normal tools of linear filter theory as usually applied in one scan dimension become inadequate. The system model developed applies only to one dimensional systems but has been implemented in a computer program which will provide the basis for further extension of system assessment to accommodate the concepts of information theory and apply the technique to two dimensional systems. With suitable modification the model can be used to determine MRTD.

The components of a typical imaging system model are shown in figure 1. These are idealised in that the determination of the MTF of the system does not require the inclusion of noise sources in the model. A complete analysis of the performance of the system leading to the calculation of the MRTD or the information density would require the use of the complete model (figure 2) including noise sources but determination of the resolution capability and equivalent line number is the only function of the current program.

The imaging process converts the continuous spectral radiance field in the object space, $L(x,y,\lambda)$ into a brightness distribution on a display medium, $D(x,y)$. The radiance field is modified by the atmosphere and the optical components to be focussed onto the detector which transforms the incident radiation into a time varying electrical signal. The self capacitance of the detector and the detector preamplifier characteristics are modelled by the electronic filter which follows the detector and can be used to include the overall effects of the limited passband of any further electronics.

The complete system model, which is shown in figure 2, includes a noise input at the detector which represents the background noise in the radiance distribution, the detector noise and the input-referred amplifier noise. The detector signals are band limited by the detector and preamplifier (which is modelled by the electronic filter following the detector). The signal is then passed through an anti-aliasing filter, sampled and quantised. Figure 2 includes noise contributions due to the aliasing induced by the sampling circuit and quantisation noise. The final stage of the model includes the post sampling (or reconstruction) filter and the display stage.

As currently implemented the program calculating the transfer function does not include the effects of the sampling circuits or any noise contributions since it merely calculates the overall MTF and equivalent line number. The program input specifies the blur characteristics of the atmosphere and the optics as equivalent Gaussian pulse spread functions and handles the display similarly. The filters are specified in terms of the number of real poles (passive filters) and their 3 dB points. Passive filters are used because it has been found that complex filter poles tend to produce unacceptable overshoot on edge transients, which can visually degrade an image.

The model accommodates two types of detectors, single element detectors and the recently developed 'SPRITE' devices which include analogue signal processing on the detector using swept charge integration. The latter devices have an MTF which differs from the older detectors due to carrier diffusion in the detector and this is accounted for in the program.

The calculation of the resolution capabilities of the system does not take into account the effects of aliasing due to the detector or any following sampling circuitry. These do not alter the MTF below the cutoff frequency as determined by the detector response or the sampling frequency but merely create a corruption of the image signal (figure 3) which will affect the MRTD and the image information density. A complete assessment of the system should therefore include these effects, as being potentially significant contributions to the performance. The contribution of the sampling circuitry involves the aliasing of noise components as well as any frequency components which may lie above the Nyquist limit in the detector response if the sampling frequency is incorrectly chosen.

3. SYSTEM COMPONENTS

3.1 Atmospheric turbulence effects

The image of a point source of radiation is distorted by passage of the radiation through the atmosphere. Macroscopic variations in the refractive index of the atmosphere over a line-of-sight may cause a variation in the apparent direction of a target. These are usually time varying and in the case of the visible spectrum can be seen with the eye (eg mirages). However the blurring of the image of a point source is due to much smaller scale phenomena and in the model used here is assumed to be adequately represented by a Gaussian pulse spread function. The model uses the pulse spread function specified by the 1/e half width of the spread function generated on the detector surface. Atmospheric blur is not often considered in imaging system analysis and the program implementing the model can have the pulse spread function width specified as zero thus neglecting atmospheric effects.

3.2 Optical components

The contribution of the optical components of an imaging system to the overall resolution of the system is the amplitude of the Optical Transfer Function of the optical system. The effects of both aberrations and diffraction in the optical system contribute to the dispersion of an image of a point source. If an imaging system produces a point spread function $g(x,y)$ with two dimensional optical transfer function $G(\nu_x, \nu_y)$ and further if $g(x,y)=l(x).l(y)$ which enables the assumption:

$$G(\nu_x, \nu_y) = L(\nu_x) L(\nu_y)$$

and $g(x,y)$ is circularly symmetric then the point spread function and the Optical Transfer Function are Gaussian(ref.4).

This is often a reasonable approximation for imaging systems and provides a simple means of including the effect of the optical components in the resolution model for a line scanned imager. In the model this is implemented by specifying the half width of the point spread function (to the 1/e point) produced on the detector surface by the optical components of the system. If the optical system consists of a number of components the assumption of a Gaussian pulse spread function (and therefore OTF) is even less in error since by the central limit theorem the convolution of a number of functions tends to a normal curve.

3.3 The detector

The detector performs the transformation of the object scene radiance distribution projected via the optical system onto the detector into an electrical signal which can then be processed. A single element detector scanning an object scene produces an MTF of the form:

$$D(\omega) = \sin(\pi f_S/f_D)(\pi f_S/f_D)^{-1} \tag{1}$$

where f_S is the spatial frequency input to the system and f_D is a cutoff frequency determined by the detector size and the focal length of the optical system. The transfer function $D(\omega)$ has a zero at an input spatial frequency of f_D and further zeroes at multiples of f_D (figure 3(a)). For input frequencies greater than f_D the transfer function as given by $D(\omega)$ is non-zero; however the spatial integration produced by the physical size of the detector produces 'aliasing' of spatial frequencies greater than f_D into the frequency band up to f_D (the 'baseband', figure 3(b)).

The effect of the aliasing produced by the finite detector size is difficult to predict for random scenes. For scenes with high frequency structures the aliasing produces an interference pattern ('Moire' fringes) which can mask small targets and distort the images of larger targets. The class of imaging systems generally of interest is designed to perform the functions of target detection and recognition. The former generally involves targets of small extent and the latter is dependent on recognition of image detail; both functions may be impaired by aliasing. It is therefore important to minimise the effects of aliasing for while the simple form of the transfer function $D(\omega)$ implies resolution beyond the spatial frequency limit set by f_D this is not achieved and the image is in fact degraded because the aliased frequencies represent a corruption of the image. Aliassing effects can be minimised by reducing the detector readout zone dimensions, or by restrictions on the spatial bandwidth of the optical system.

The 'SPRITE' detector(ref.5) exhibits two mechanisms which limit the spatial resolution of a system employing the detector. By sweeping the focussed image along the detector at a velocity equal to the drift velocity of the photo-generated carriers which is produced by a bias field along the detector strip (figure 4), the detector effectively provides an integration of an image point. The image information is then extracted by monitoring the conductivity modulation produced in a small readout zone at the end of the detector due to the drift of the photo-generated carriers.

The readout zone imposes a cutoff frequency on the spatial response in the same way that the physical dimensions of the detector do for a single element detector, ie a cyclic distribution of incident radiation with a period equal to the readout zone dimensions will produce no modulation of the conductivity of the readout zone and therefore no output signal. This results in a component of the MTF of the form of equation (1). The exact shape of the readout zone is a complicated function of the shape of the ohmic contacts used to define the zone and of the shape of the detector in the region of the zone. The distribution of the bias field through the detector as set by these factors determines the effective readout zone shape and therefore its spatial frequency response.

The exact shape and therefore the MTF of the readout zone are not accessible but a function of the form $D(\omega)$ as for a single element detector provides an adequate model for the effects of the readout zone where f_D is determined by the effective readout zone length.

The second mechanism which limits the spatial resolution of the 'SPRITE' detector is due to the diffusion of the photo-generated carriers away from the generation site. This diffusion is superimposed on the drift due to the bias field and therefore 'smears' the image information. For a long detector (such that the detector length is much greater than $v_D\tau$ where v_D is the carrier drift velocity due to the applied bias and τ is the carrier recombination lifetime) the pulse spread function due to diffusion alone has the form $\exp(-x/Q_a)$ where Q_a is the ambipolar diffusion length which approximates Q_p, the hole diffusion length, for n type HgCdTe. The carrier spread between the 1/e points ($2Q_a$) defines the nominal resolution limit and the function $\exp(-x/Q_a)$ thus gives the MTF due to diffusion which is of the form $(1 + f_S^2 Q_a^2)^{-1}$ where f_S is the input spatial frequency.

If the detector bias and/or the detector length are such that the transit time of the carrier is much less than the lifetime the effects of diffusive spread are decreased and the pulse spread function becomes sharper than $\exp(-x/Q_a)$ and the diffusive MTF alters. The determination of the MTF due to diffusion requires the solution of the diffusion equation for the carrier distribution in the detector. The program implementing the model performs the required computation to determine the pulse spread function for a given detector length, bias voltage and material parameters and from this determines the diffusive MTF using a discrete Fourier Transform. Appendix I details the solution of the diffusion equation and its application to the determination of the diffusive MTF component.

The implementation of the model provides facilities to calculate the MTF of a single element detector or a 'SPRITE' detector given the necessary detector parameters and system constants.

3.4 Processing electronics

The design of the electronics used to amplify the detector output and ultimately drive the display medium are subject to restrictions based on the necessity to ensure that the optimum system performance consistent with minimum added noise is achieved, while not degrading the resolution of the imager. The principal limit on the bandwidth of the electronics and the noise performance is the self capacitance of the detector and the inherent noise of the detector combined with the bandwidth limitations and input referred noise of the preamplifier (usually a high gain stage).

The use of electronic filters with complex poles can lead to unacceptable overshoot on transient signals ('ringing') which may be found to be objectionable and in severe cases may mask small targets located near larger targets. The principal electronic components affecting system bandwidth should have frequency responses determined by simple 'real' poles in the transfer functions. This requirement conflicts with the requirement that the bandwidth be restricted to minimise noise (or in the case of sampling circuits aliasing) since the frequency response of such systems 'rolls-off' more slowly than the frequency response of systems with complex poles. The need to minimise 'ringing' may be more important if the noise bandwidth is limited and the optics/detector response is the major restriction on resolution.

The overall electronics passband and the pre-filtering and post-filtering for the sampling circuitry are then specified in terms of the number of passive poles and their temporal frequencies. The temporal frequencies are translated to spatial frequencies in the program so that filter bandwidths are related to the MTF due to optical and display components.

3.5 Sampling and quantisation

The sampling process in general modifies the transfer function by aliasing high frequency components into the system passband. The overall process of sampling and reconstruction can significantly modify the input signal if the sampling rate is not correctly chosen. Since the detector spatial extent defines the upper limit to the spatial frequency response of the system by the first zero in its transfer function it is assumed that the sampling rate of any system will be designed so that it is at least twice the detector cutoff frequency. In this case the major effect of any sampling system is to increase the baseband noise by aliasing of any residual noise in the upper sampling bands. If this criterion of band limiting is not met then the sampling function forces a sharp cutoff in the frequency response at the sampling frequency and may create significant aliasing at lower frequencies. This possible corruption of the video signal is not accounted for here. Quantisation adds a further noise component (wideband) with well defined statistics but does not affect the MTF. It is therefore ignored in the MTF calculation.

3.6 Display

The usual display medium for imaging systems is a CRT screen and the characteristics of the phosphor and the electron beam used to excite the phosphor determine the effect of the display on overall system performance. In general the spot shape produced on a CRT screen is treated as a Gaussian pulse or a superposition of two Gaussian pulses. The determination of pulse shape can be difficult but experience indicates that a Gaussian spot acceptably models the CRT response. Specifying the half width (to the 1/e point), the screen dimension in the direction of scan and the display field in the direction of scan allows the CRT response to be included as part of the overall system response.

3.7 Total system MTF

The combination of the component MTFs can then be used to produce an overall system MTF which is used to calculate equivalent line number N_e and from the Demand Modulation Function or threshold detectability, $M_D(f)$, the MTFA can be determined. If the sources of noise and the detector responsivity are known, then the NETD and MRTD can be calculated from a knowledge of the system transfer function(ref.1). The noise characteristics are then used in the determination of the information density which, as a measure of the information content of the reconstructed image, leads to a figure of merit which can be used to account for image degradation due to blurring, aliasing, quantisation and noise.

4. IMPLEMENTATION OF THE MODEL

The model of the imaging system is implemented in a program written in VS FORTRAN to run on an IBM 370/3033 computer. The program accepts inputs which specify the system parameters. The detector and display characteristics, atmospheric and optical blurring and electronics transfer functions are used as the input to calculate the responses of the components of the model. For the case of a system using SPRITE detectors the user may specify the detector characteristics or allow the program to assume default values of the characteristics for a typical detector.

Figure 5 shows a simplified flow diagram of the program. The program uses a set of arrays to store MTF data for each of the system components. After initialising these arrays and default system parameters, the parameters required are obtained from the user via terminal input (the program runs interactively). All calculations are done in the spatial frequency domain and the MTFs due to atmospheric blurring and optical component blurring are calculated first.

The diffusion MTF due to a SPRITE detector is then calculated if necessary. This is done by calculating the pulse spread function profile for the detector with the characteristics as specified and with an applied bias field. A Fast Fourier Transform is then used to calculate the MTF due to carrier diffusion in the detector. The MTF due to the readout zone/detector size is then calculated.

The transfer functions due to the electronics components (the detector rolloff and the pre- and post-sampling filters) are then calculated after transforming temporal frequencies of the electronics responses to the spatial domain.

The MTF due to the display (CRT) spot size is calculated and a composite system MTF then formed by multiplying the system component MTFs. The composite and component MTFs are then plotted either at the terminal (if a graphics terminal is being used) or at a remote plotter.

To assess the relative effectiveness of particular design configurations it is often advantageous to have a single figure of merit which when calculated for different designs can be used as a basis for comparison. The determination of the system MTF is a necessary step in the calculation of any useful figure of merit and a full analysis of the performance of an imaging system should provide an estimate of MRTD.

The scope of the model at its present stage of development precludes any attempt to calculate MRTD or MTFA but the program does calculate N_e as a figure of merit to provide a point of comparison between competing designs. The determination of MTFA requires a further extension of the model since it is dependent on the observers noise related demand modulation function. It is proposed that the present program be used as a basis for extension of the work to incorporate the effects of noise and observer performance and thus provide a sounder basis for the comparison of candidate designs.

5. CONCLUSION

The analysis of an imaging system in terms of its Modulation Transfer Function is central to the determination of the performance of such systems and a model for a line scanned imaging system is discussed. The model has been implemented in a computer program which has been used to determine the design parameters of a frame storage system for a scan- converter. The application of the analysis of the system MTF to the determination of minimum resolvable temperature difference and information density as figures of merit for imaging systems is discussed, and its use in the design of a digital frame store for use with a 'SPRITE' based imaging system presented.

The 'SPRITE' detector provides an increase in performance over a single element detector, but its use requires that the system designer match the detector operating conditions to the physical scanning mechanism of an imaging system. The detector geometry and operating conditions affect the sensitivity and resolution of the system. Adjusting the bias voltage of the detector can alter the detector diffusive line spread function and thus the detector MTF. The MTF is also affected by the detector readout area geometry. The model developed here allows the determination of the resultant system MTF for various configurations, and the results presented indicate that improvements in system MTF can be achieved by using a smaller readout area or increasing the detector bias.

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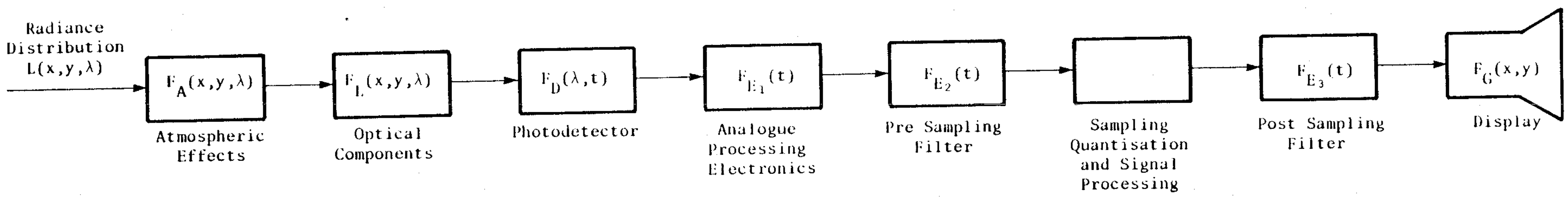


Figure 1. Idealised imaging system model

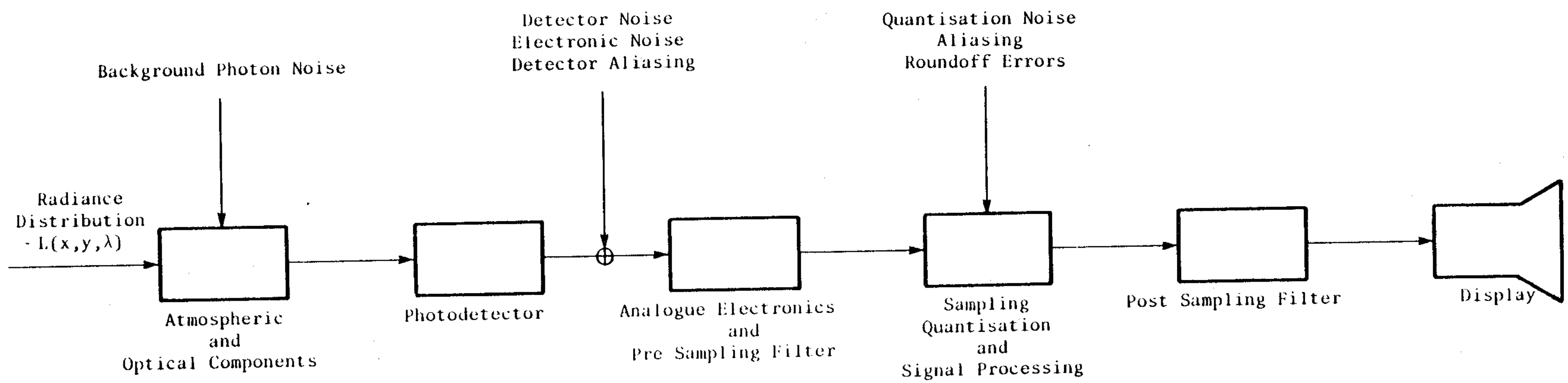
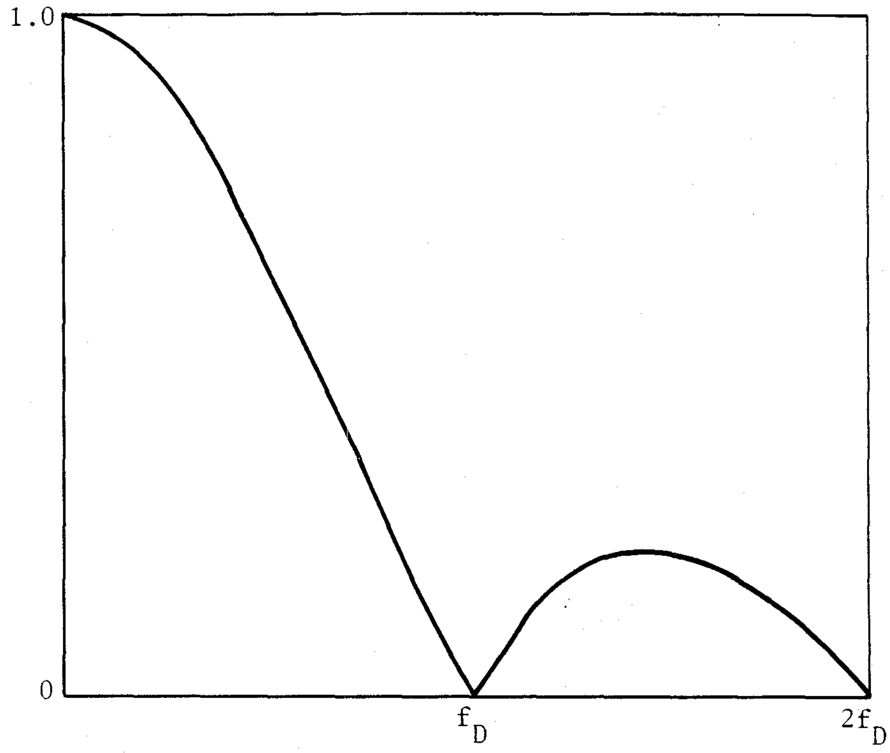
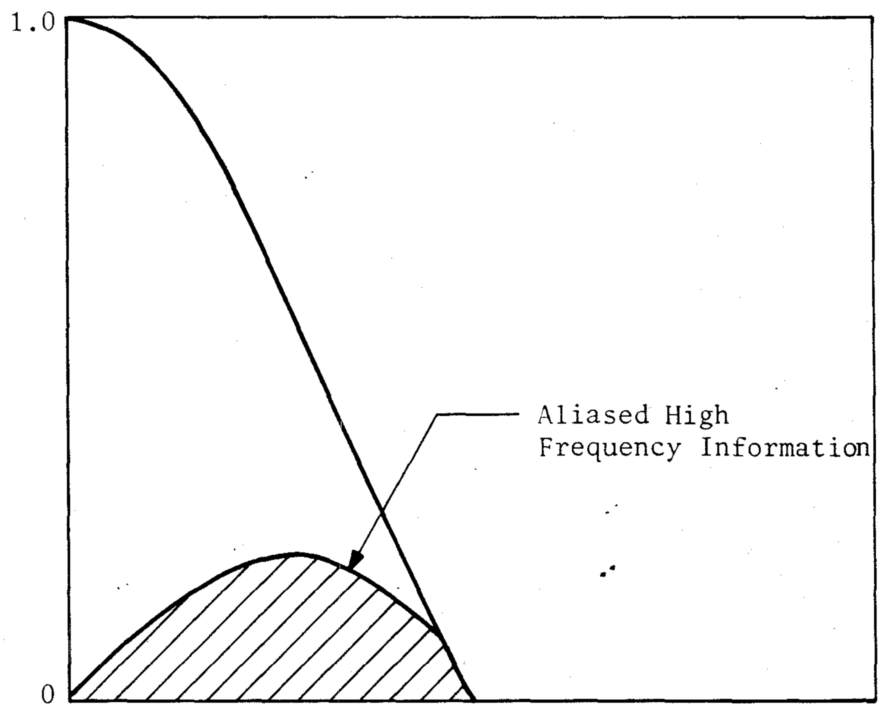


Figure 2. Imaging system model with noise sources

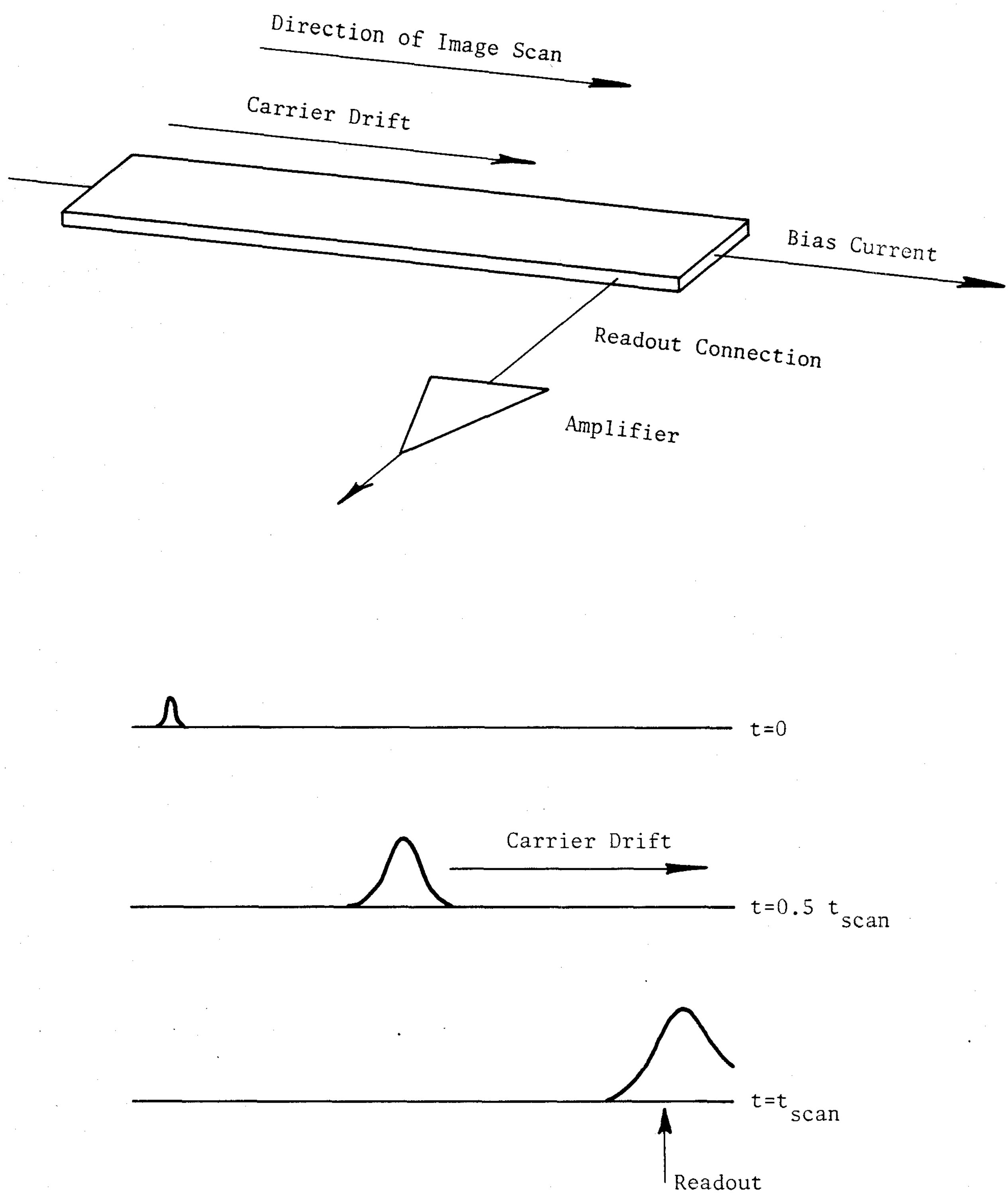


(a) Theoretical detector transfer function $D(w) = Sin(\pi f_s/f_D).(\pi f_s/f_D)^{-1}$



(b) Actual spectrum of detector output

Figure 3. Effect of aliasing due to detector spatial frequency cutoff



Integration of Signal with Image Scan

Figure 4. The SPRITE detector

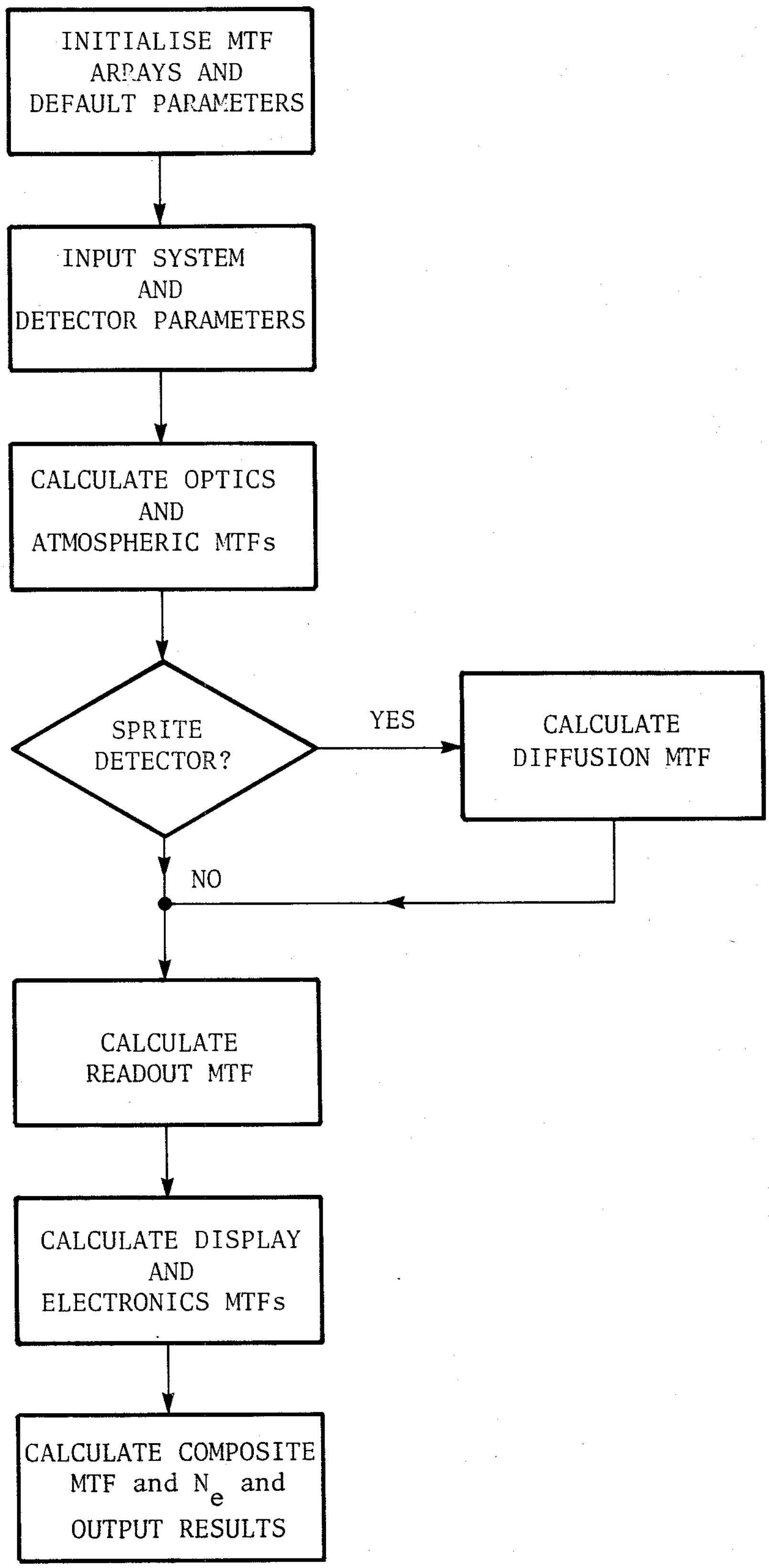


Figure 5. Simplified program flow chart

APPENDIX I

SPATIAL RESOLUTION OF A 'SPRITE' DETECTOR

The 'SPRITE' infrared detector implements a time delay and integration function on a scanned radiance distribution by sweeping the induced photocarrier along the detector at the scan velocity. This results in improved detectivity, D*, since the detector responsivity is improved by the ratio of the length of the 'SPRITE' sensitive region to the sensitive region of a conventional single element detector (this is an approximation only since recombination of photocarriers and diffusion away from the generation site will limit the improvement in responsivity).

The spatial resolution of a single element detector is limited by the angular subtense of the detector in the object space (α) and is limited to a spatial frequency of $1/\alpha$ (usually expressed in cycles per radian in the object space). The radiant energy incident on the 'SPRITE' detector is converted into a conductivity modulation which is detected by a pair of contacts defining a 'readout zone' at the end of the detector (figure I.1). Spatial detail in the carrier distribution traversing the readout zone which is of the order of the dimensions of the readout zone cannot be resolved and the MTF of the readout zone exhibits the same form as the MTF of a single element detector where the cutoff frequency is determined by the length of the readout zone and the focal length of the optical system, in effect by the angular subtense of the readout zone in the object space.

If the drift velocity of the photo-generated carriers in the detector is V_D the transit time of the carriers traversing the readout zone is

$$t = L_R/V_D$$

where L_R is the readout zone length.

In operation the drift velocity of the carriers in the detecor is adjusted, by the bias voltage applied, to match the scan rate through the object space. This means that the carrier distribution traversing the readout zone in time t arises from an object distribution of angular dimension L_R/f where f is the system focal length as long as the drift velocity matches the scanning velocity of the system. Spatial frequencies exceeding this frequency are aliased into the frequency band below the cutoff frequency (in much the same way as occurs with a single element detector) and this cutoff frequency then defines the fundamental limit to the spatial resolution of the 'SPRITE' detector.

If this were the only mechanism limiting the detector resolution then the line spread function due to the detector would be a rectangular pulse of angular width L_R/f. However, due to the diffusion of photo-generated carriers during their transit of the length of the detector element (which is at least 10 times the length of the readout zone) the line spread function is degraded. The 'SPRITE' spatial response can then be modelled as due to two mechanisms, a diffusion component which produces an initial line spread function and the readout component which is then convolved with the spread function due to diffusion, since they act serially. The overall MTF of the 'SPRITE' detector is then readily obtained by multiplying the MTFs of the diffusion and readout effects.

To calculate the Modulation Transfer Function due to diffusion effects the pulse spread function must be determined. If the image scan rate matches the carrier drift velocity due to the imposed bias then the carrier distribution which traverses the readout zone is the cumulative distribution of the carriers generated during the time the image traverses the detector.

To determine the pulse spread function the continuity equation for photo-generated minority carriers must be solved. This equation is(ref.7);

$$\frac{\partial p}{\partial t} = G_p - U_p - \frac{1}{q}\nabla . J_p \qquad (I.1)$$

where p is the hole concentration, G_p is the hole generation rate and U_p is the recombination rate and J_p is the current density. For the one dimensional case with low injection this becomes:

$$\frac{\partial p}{\partial t} = G_p - \frac{p-p_o}{\tau_p} - p\mu_p \frac{\partial E}{\partial x} - \mu_p E \frac{\partial p}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} \qquad (I.2)$$

where E is the applied electric field and the other symbols have their usual meaning, p_o is the equilibrium carrier density, τ_p the carier lifetime and μ_p the carrier mobility. For no applied electric field this is:

$$\frac{\partial p}{\partial t} = G_p - \frac{p-p_o}{\tau_p} + D_p \frac{\partial^2 p}{\partial x^2} \qquad (I.3)$$

Consider the irradiation of the detector by a localised pulse of very short (much less than the carrier lifetime), duration radiation. This is the initial condition for the equation and can be approximated as a δ function to represent the initial distribution of excess carriers. If p_o is a constant then we can write, using p^1 for excess carrier density:

$$\frac{\partial p^1}{\partial t} = G_p - \frac{p^1}{\tau_p} + D_p \frac{\partial^2 p^1}{\partial x^2} \qquad (1.4)$$

and after the initial illumination $G_p = 0$, so

$$\frac{\partial p^1}{\partial t} = - \frac{p^1}{\tau_p} + D_p \frac{\partial^2 p^1}{\partial x^2} \qquad (I.5)$$

Substitution of $p^1 = ue^{-t/\tau_p}$ gives(ref.6)

$$\frac{\partial u}{\partial t} = D_p \frac{\partial^2 u}{\partial x^2} \qquad (I.6)$$

with initial conditions $u(x,f) = \delta(x)$ the solution of which is given by:

$$u(x,t) = \int_{-\infty}^{\infty} G(x, \xi, t)\ \delta\ (\xi)\ d\xi \qquad (I.7)$$

where

$$G(x,\xi,t) = (2\sqrt{D_p\ \pi\ t})^{-1} \exp\left[- \frac{(x-\xi)^2}{4\ D_p\ t} \right] \qquad (I.8)$$

Therefore with the given initial conditions:

$$u = (2\sqrt{D_p\ \pi\ t})^{-1} \exp\left[\frac{-x^2}{4\ D_p\ t}\right] \qquad (I.9)$$

and

$$p^1 = (2\sqrt{D_p\ \pi\ t})^{-1} \exp\left[\frac{-x^2}{4\ D_p\ t} - \frac{t}{\tau_p}\right] \qquad (I.10)$$

The minority carrier density is:

$$p = (2\sqrt{D_p\ \pi\ t})^{-1} \exp\left[\frac{-x^2}{4\ D_p\ t} - \frac{t}{\tau_p}\right] + p_o \qquad (I.11)$$

which is a Gaussian pulse which diffuses from its origin and decays with time due to recombination.

To determine the pulse spread function the carrier distribution for continuous illumination in time must be determined. The illumination is a δ function in space and if the carrier generation rate is N_p the total carrier population at any time is the cumulative distribution of all carriers generated, therefore:

$$p = \int_o^t N_p\ (2\sqrt{D_p\ \pi\ t})^{-1} \exp\left[\frac{-x^2}{4\ D_p\ t} - \frac{t}{\tau_p}\right] dt + p_o \qquad (I.12)$$

This can be manipulated by changing the variables to yield:

$$p = N_p (2\sqrt{D_p\ \pi\ t})^{-1} \exp\left(\frac{-x}{\sqrt{D_p\ \tau_p}}\right) \int_o^{2\sqrt{D_p\ t}} z^{-\frac{1}{2}} \exp\left[\frac{-(z-\sqrt{\tau_p}\ x)^2}{2\sqrt{D_p}\ \tau_p\ z}\right] dz + p_o \qquad (I.13)$$

which does not have an easily manipulated analytic solution.

In the limit as τ approaches infinity p approaches the form:

$$p = \text{Const.}\exp(-x/L_p) \qquad (I.14)$$

where $L_p = (D_p \tau_p)^{\frac{1}{2}}$ which is the limiting solution for the carrier distribution arising from steady state injection(ref.7). To determine the pulse spread function equation (I.13) is integrated numerically over the time period defined by the carrier transit time through the detector. The application of a bias field superimposes a drift velocity on the diffusion characteristics. This does not alter the line spread function shape but merely results in the carrier distribution moving along the detector. A computer program has been written to perform the numerical integration and then determine the MTF using a Fast Fourier Transform.

The integration is performed in discrete time steps (ignoring the singularity at the time origin) and the pulse shape evolves from strictly Gaussian for very short times to exponential for long transit times (in agreement with the previous conclusions) with the width of the pulse spread function being determined by the diffusion length L_p (figure I.2). Typical results for the MTF of a 'SPRITE' detector showing the diffusion and readout zone components and the composite MTF are shown in figure I.3.

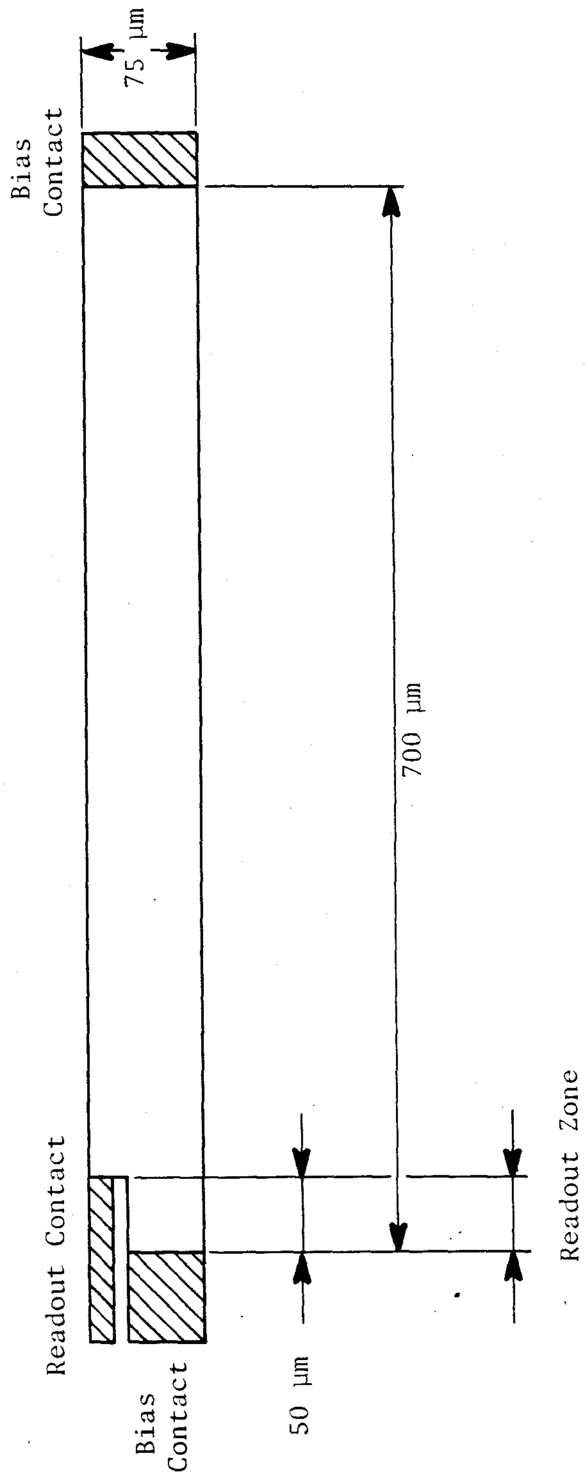


Figure 1.1 SPRITE detector geometry

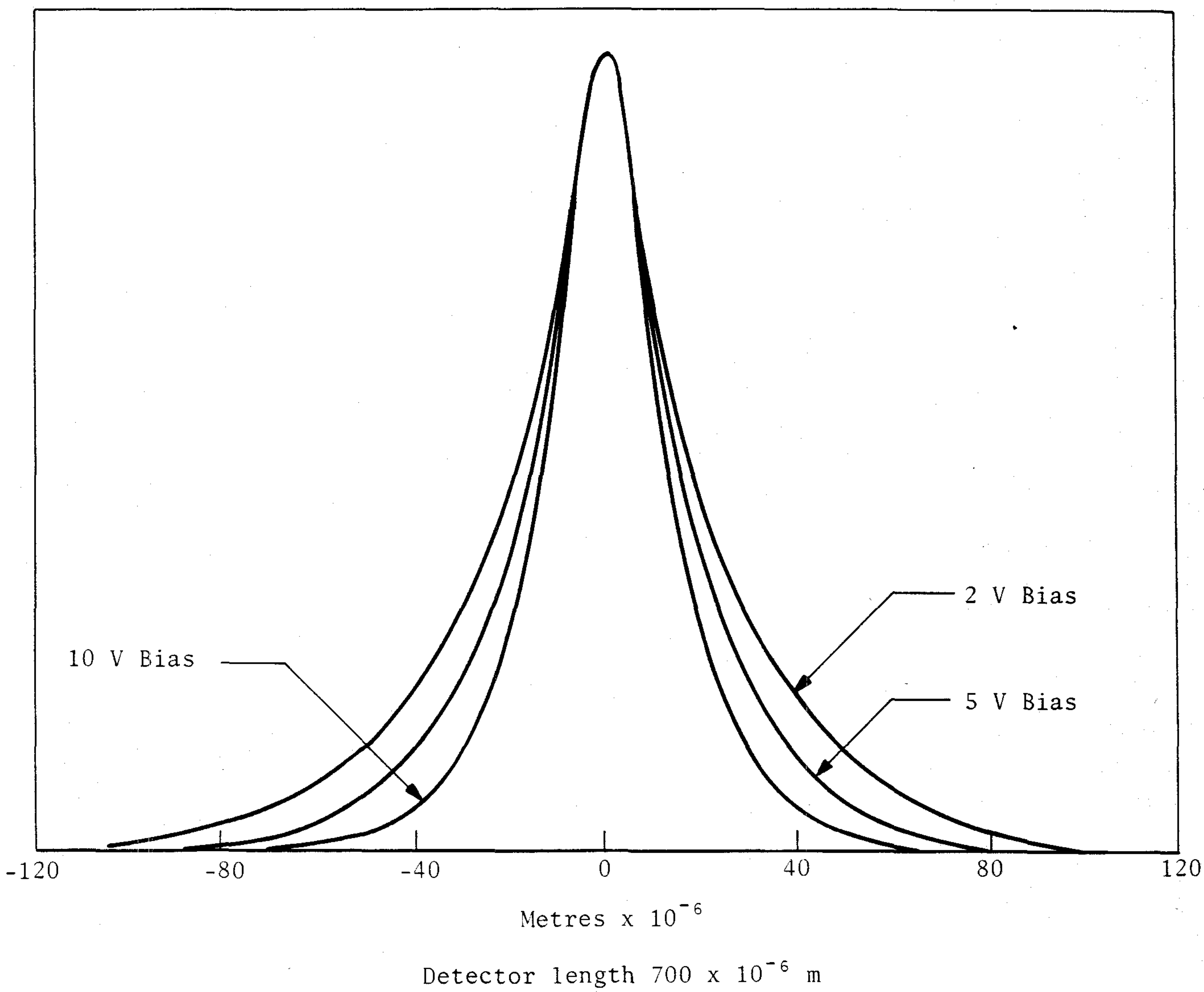


Figure I.2 SPRITE detector line spread function

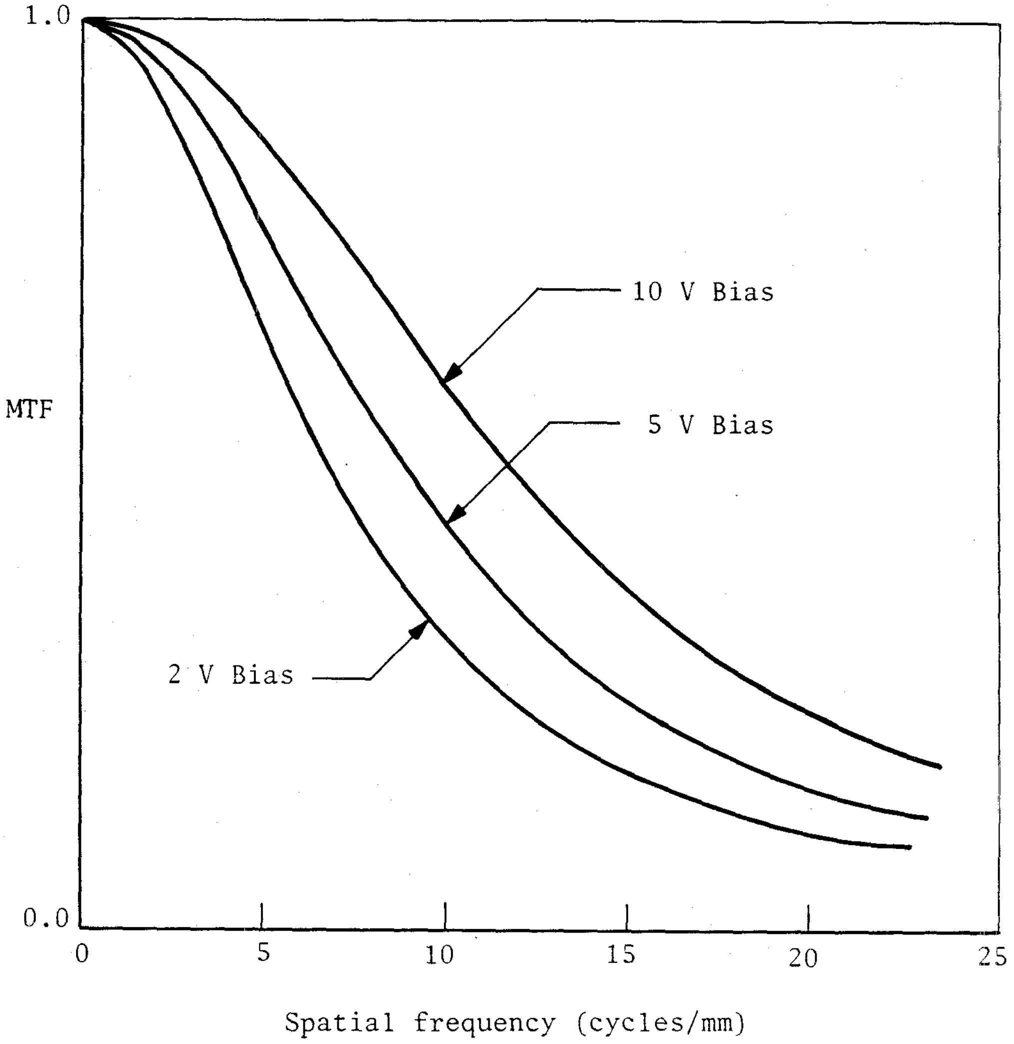


Figure I.3 SPRITE detector diffusion MTFs

APPENDIX II

APPLICATION TO DESIGN OF A SCAN CONVERTER

High resolution, wide field of view imaging devices create unique problems in display formatting since the format of the field of view does not match the format of the commonly available television technology and can exceed the resolution capability of such technology. The information capability of such wide field of view systems is very high and when the image is produced by mechanical scanning employing a limited number of detectors, the mechanical limitations and the available bandwidth limit the framing rate of such systems. To achieve a wide field of view with large information capacity and to present this to an observer at a framing rate which does not produce some discomfort or reduction in the observer's detection ability, requires some means of scan-conversion to increase the framing rate.

The resolution limitations of available display technologies further require that some form of 'zoom' facility within the field of view be available to ensure that the observer has access to all the information capacity of the system. This zoom facility may be difficult to implement with a mechanical scanning facility or may involve costly optical systems to achieve. To provide a zoom capability, digital scan conversion may be used and this would also satisfy the flicker free display requirements discussed above. This would allow the display format to be adjusted to suit the capabilities of common (and therefore cheap) display technologies and allow the display of either a full field of view or a higher resolution segment of the complete field of view thus allowing the observer to "roam through" the complete field of view at will.

Segmented display formatting may be necessary to effectively display all the information which potentially can be gathered. It is proposed that this can be performed electronically using digital storage techniques which may later incorporate processing to facilitate detection of targets and allow automatic target tracking.

The design specifications for such a system are developed here. Two parameters are of key significance in the design of a frame store for a line scanned image system - these are the horizontal resolution capabilities of the system and the frame format (the field of view and the number of lines per frame). The resolution capability of a system plays a major part in determining the target detection capabilities of the system. For an infrared imaging system the sensitivity and resolution define the system performance. As requirements of the frame store are not affected by the sensitivity of the system, the resolution is then the major system parameter affecting the design.

In order to perform complete scan conversion from a low frame rate to 25 Hz frame rate one complete frame of an image must be stored. Scan conversion is then achieved by reading the data out from the memory at a rate which differs from that used to acquire the data, and can be used to achieve display formatting which would not be possible with alternative technology. Two possible ways of utilising the wide field of view image are:

(a) single mode display in which either the complete wide field of view or a high resolution (within the display capabilities) segment of it are displayed, and

(b) composite mode wherein the complete field of view and a high resolution segment are displayed together.

Using as an example an imager with a field of view of $54^\circ \times 6^\circ$, optimum use of a 4 x 3 format display is made in the zoom mode with a field of view of $8^\circ \times 6^\circ$ with option (a). The composite display (option (b)) could be used to display the full field of view ($54^\circ \times 6^\circ$) occupying a narrow strip at the top of the frame with a $10^\circ \times 6^\circ$ expanded segment of the image below this to fully utilise the display area. Option (b) in this mode does require a much greater bandwidth (approximately double) but may provide better operator response.

To achieve either operating mode requires storage of a complete frame. Display technology cannot match the information bandwidth available and therefore the complete frame may be stored at lower resolution, together with the zoom segment at higher resolution, as an alternative to storing the complete frame at the highest resolution commensurate with the detector and optics performance. The tradeoffs between these options will be dependent on the sampling rate and the format chosen and the fact that the information content may exceed the observer's interpretive capability. The sampling rate is determined by the information bandwidth of the signals which may be extracted from the detector.

With a detector readout zone dimension of 50×10^{-6} m and an optical system focal length of 250 mm the first 'zero' of the detector transfer function occurs at 5000 cycles/rad and with a readout zone dimension of 35×10^{-6} m this occurs at 7140 cycles/rad. Figures II.1 and II.2 illustrate the MTFs for systems with these detector dimensions and an optical system producing a blur spot of 15×10^{-6} m on the detector surface (the blur spot size is the 'half width' to the 1/e point on the spot profile). It can be seen from these figures the effect of the blur spot and the diffusion effects in a 'SPRITE' detector (assumed to be operating as a 'long' detector with the transit time greater than minority carrier lifetime although the optimum bias level will reduce the transit time and hence diffusive spread) limit the useable information to 4500 and 5500 cycles/rad at the 1% MTF level for readout zone dimensions of 50×60^{-6} and 35×10^{-6} m respectively.

With a display system of sufficient resolution so as not to degrade the information, these frequencies define the required sampling rates of 9000 samples/rad and 11 000 samples/rad. The effect of sampling the 35×10^{-6} readout zone output at 9000 samples/rad is to introduce a component in the MTF below 4500 cycles which is due to spatial frequencies in the range 4500 to 7140 cycles/rad. Spatial frequencies above the 'zero' of the detector function do not give rise to significant aliasing by the detector due to the attenuation by the optics and diffusion in the 'SPRITE' detector. The aliasing due to undersampling is less than 1% of the peak MTF and may be tolerated but its effect on the probability of detection of small targets cannot be predicted. It does imply the loss of some performance since the effect of aliasing is to introduce distortion which may mask small targets.

Assuming that a sampling rate of 9000 samples/rad can be achieved current display technology does not permit the presentation of information with such spatial frequency content for wide fields of view (eg 54.0°). The information from a much narrower field of view (eg 18°) could be displayed on a display with horizontal dimensions of 0.25m and a spot size of 50×10^{-6} m. For both detectors the effect of the CRT spot size restricts the MTF to approximately 3000 cycles/rad at the 1% point in the wide field of view mode implying a sampling rate of 6000 samples/rad would be adequate to preserve the information bandwidth for this part of the task.

The MTFs shown in figures II.1 and II.2 also show the effect of the diffusion effects, inherent in the operation of the 'SPRITE' detector, on the aliasing of high frequency information in the image by the detector since the diffusion MTF virtually eliminates spatial frequencies above the detector cutoff. The diffusion of photogenerated carriers in the 'SPRITE' detector reduces the peak amplitude of the aliased signal component from 20 % of the normalised MTF to approximately 1.5%. The blur due to the optics reduces this even further to less than 0.5%. For a conventional single element detector with the same optical system the peak aliased signal would be 4% of the normalised MTF.

The 'SPRITE' detector MTFA may be up to 25% less than an equivalent single element detector (neglecting blur due to optical components and the display etc) but offers considerably better detectivity due to the signal integration within the body of the detector and due to the reduced noise component introduced by aliasing. When the MTF loss due to limited display capability is considered the performance of a 'SPRITE' based imager may not be significantly different in terms of the MTFA alone and will be better when the noise performance is considered.

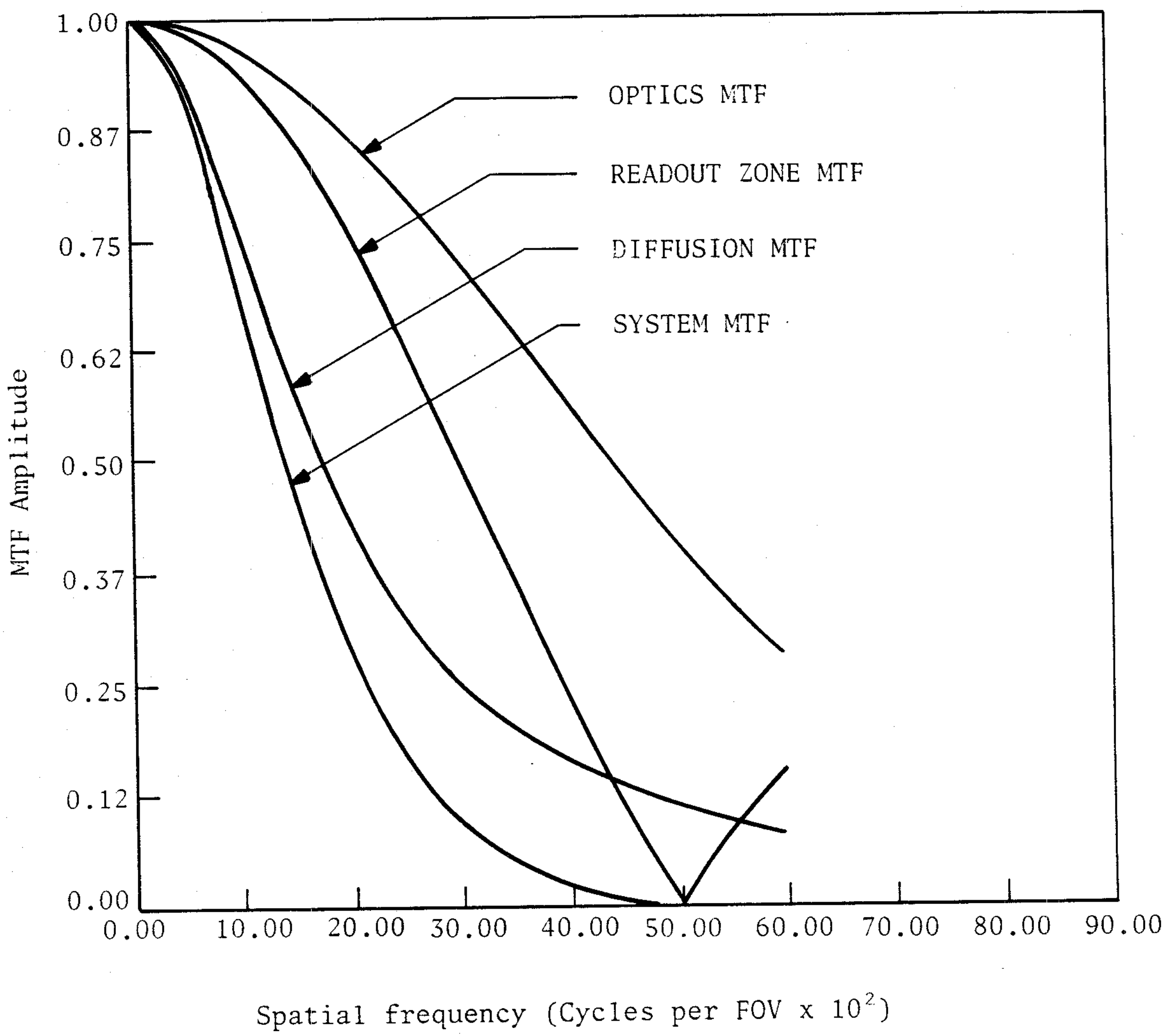


Figure II.1 SPRITE detector MTF with 50 x 10^{-6} m readout zone

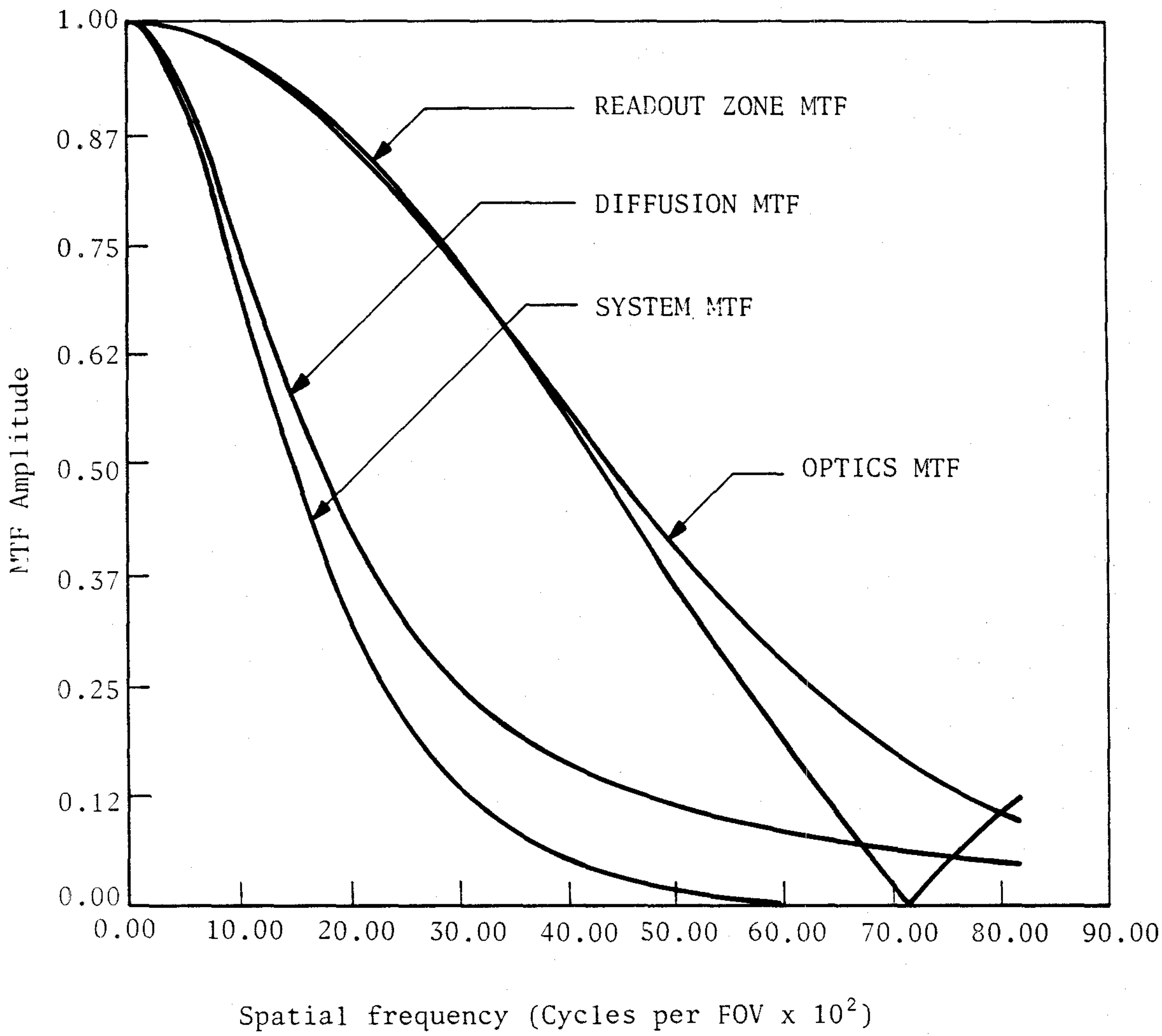


Figure II.2 SPRITE detector MTF with 35 x 10^{-6} m readout zone

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A computer model to determine the Modulation Transfer Function of a line scanned imaging system is discussed. The model can incorporate the effects of the atmospheric path, the system optics and the display as well as the detector's resolution limits. It can be used with single element or SPRITE detectors and its application to a design task is discussed.

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