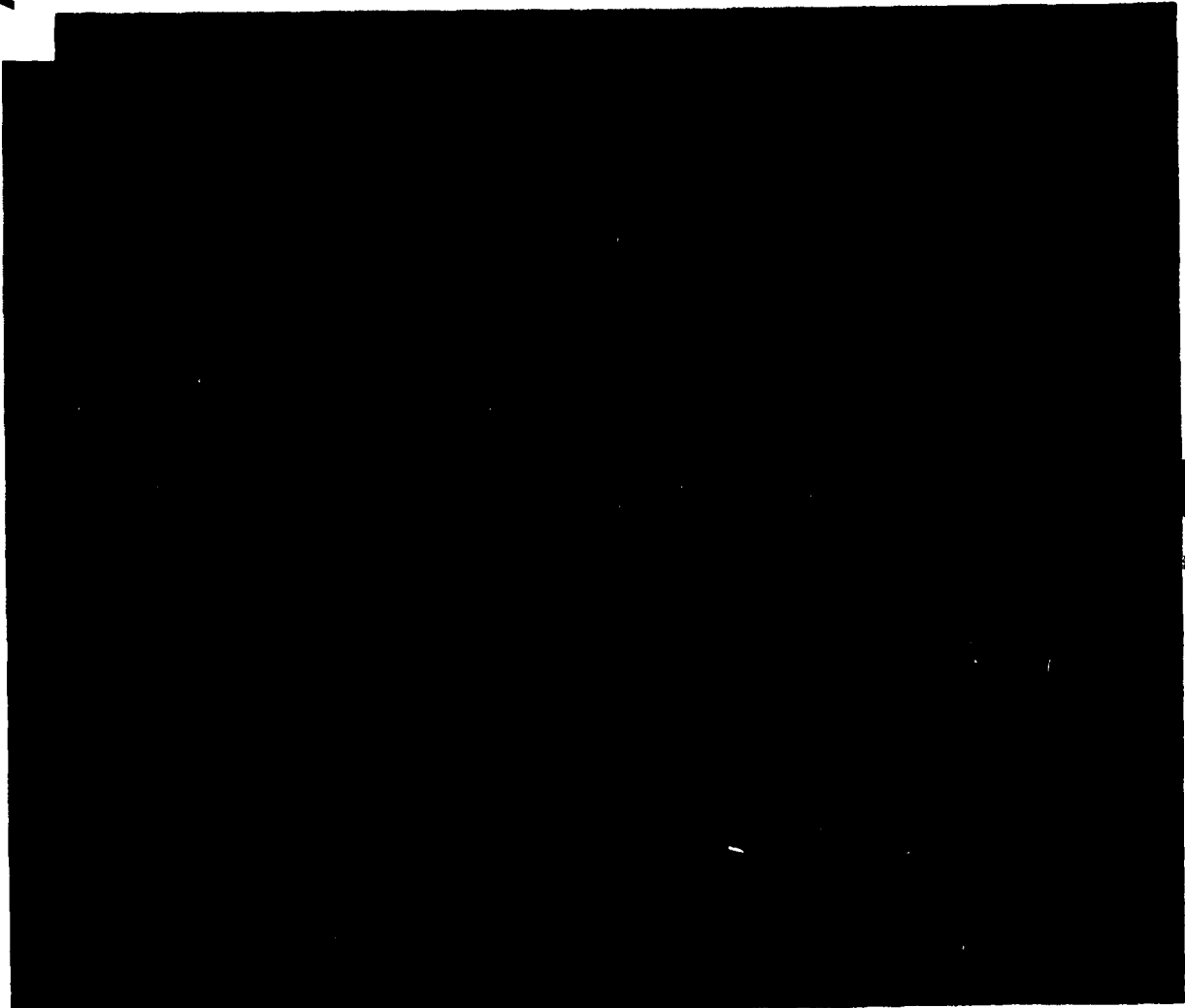MICROCOPY RESOLUTION TEST CHART
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MULTI-INPUT / MULTI-OUTPUT
SENSITIVITY POINTS TUNING

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MULTI-INPUT/MULTI-OUTPUT SENSITIVITY POINTS TUNING

BY

STEPHEN THINK HUNG

B.S., University of Tennessee, 1983

THESIS

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for the degree of Master of Science in Electrical Engineering
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TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Periodic tuning is a virtually essential aspect in the maintenance of almost any physical plant. One method of such tuning is that known as sensitivity points turning method. The theory of this method and application to single-input/single-output (SISO) systems have been fairly well developed. This study investigates the application of sensitivity points turning concepts to the tuning of multi-input/multi-output (MIMO) systems.

The concept of sensitivities in feedback systems was originally introduced by Bode in the 1940's [1]. Methodologies for using sensitivity information for optimal tuning of system parameters date back to the late 1950's and early 1960's and include work done by Meissinger [2,3], Margolis and Leondes [4,5], and numerous others (see [6,7]). These works all involve the use of system sensitivities in order to provide a searchless means of computing the gradient of a given cost function which possesses some extremal value that is related in some way to some desired characteristic performance of the system under tuning.

Included in this class of optimization techniques are the iterative parameter optimization techniques, such as those of Meissinger [2,3], Brunner [8], Roberts [9], and Kokotovic [10]. The choice and generation of pertinent sensitivities for use in such techniques are topics treated in works by, among others, Kokotovic [10], and Wilkie and Perkins [11,12]. The sensitivity points tuning process, developed by Kokotovic [10], is an iterative process which obtains sensitivities in a straightforward manner and then adjusts parameters in an optimizing fashion.

1.1. Sensitivity Points Tuning Concepts

Consider the system depicted in Figure 1. In this arrangement, which occurs frequently in practical applications of control systems, compensator blocks $K_1(\alpha_i,s)$ and $K_2(\beta_j,s)$ of known structure are used for control of system blocks $W_1(s)$ and $W_2(s)$, whose structures need not be necessarily known.

For a given input $r(s)$ and some small deviations $\Delta\alpha_i$ and $\Delta\beta_j$ from some nominal parameter values α_i^0 and β_j^0, respectively, a Taylor series expansion of the output response yields

$$y(\alpha_i^0 + \Delta\alpha_i \, , \, \beta_j^0 + \Delta\beta_j \, , \, s)$$

$$= y(\alpha_i^0 \, , \, \beta_j^0 \, , \, s) + \sum_{i=1}^{n} \frac{\partial}{\partial\alpha_i} y(\alpha_i^0 \, , \, s)\Delta\alpha_i + \ldots$$

$$+ \sum_{j=1}^{m} \frac{\partial}{\partial\beta_j} y(\beta_j^0 \, , \, s)\Delta\beta_j + \ldots \tag{1}$$

where

$$\frac{\partial}{\partial\alpha_i} y(\alpha_i,s) \triangleq \text{sensitivity of the output } y(\alpha_i,\beta_j,s) \text{ with respect}$$

to the parameter α_i at $\alpha_i = \alpha_i^0$, and

$$\frac{\partial}{\partial\beta_j} y(\beta_j,s) \triangleq \text{sensitivity of the output } y(\alpha_i,\beta_j,s) \text{ with respect}$$

to the parameter β_j at $\beta_j = \beta_j^0$.

Note that (1) may be rewritten as

$$y(\alpha_i^0 + \Delta\alpha_i \, , \, \beta_j^0 + \Delta\beta_j \, , \, s)$$

$$= y(\alpha_i^0 \, , \, \beta_j^0 \, , \, s) + \nabla_\alpha y(\alpha_i^0 \, , \, \beta_j^0 \, , \, s) \cdot \Delta\alpha + \ldots$$

$$+ \nabla_\beta y(\alpha_i^0 \, , \, \beta_j^0 \, , \, s) \cdot \Delta\beta + \ldots \tag{2}$$
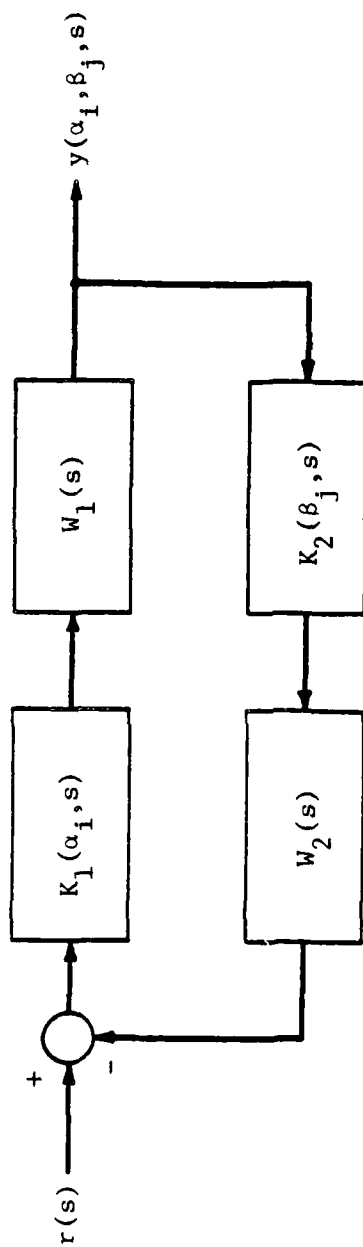
Figure 1. SISO system with system blocks $W_1(s)$ and $W_2(s)$ and known compensator blocks $K_1(\alpha_i,s)$ and $K_2(\beta_j,s)$.

where

$$\alpha = [\alpha_1 \ldots \alpha_n]^T \quad,$$

$$\Delta\alpha = [\Delta\alpha_1 \ldots \Delta\alpha_n]^T \quad,$$

$$\beta = [\beta_1 \ldots \beta_m]^T \quad,$$

and

$$\Delta\beta = [\Delta\beta_1 \ldots \Delta\beta_m]^T \quad.$$

If the expansion is truncated at the first-order term and $y(\alpha_i^0, \beta_j^0, s)$ is taken to be the nominal response of the system, then the derivation of the system response $y(\alpha_i + \Delta\alpha_i, \beta_j + \Delta\beta_j, s)$ from the nominal response $y(\alpha_i^0, \beta_j^0, s)$, denoted as the error $e(s)$, may be approximated by

$$e(s) \approx \nabla_\alpha y(\alpha_i^0, \beta_j^0, s) \cdot \Delta\alpha$$

$$+ \nabla_\beta y(\alpha_i^0, \beta_j^0, s) \cdot \Delta\beta \qquad . \qquad (3)$$

If $\alpha' = \alpha^0 + \Delta\alpha$, $\beta' = \beta^0 + \Delta\beta$, and $\Delta\alpha$ is known to be small, then the approximation in (3) may be rewritten as

$$e(s) \approx \nabla_\alpha y(\alpha_i', \beta_j', s) \cdot \Delta\alpha$$

$$+ \nabla_\beta y(\alpha_i', \beta_j', s) \cdot \Delta\beta \qquad . \qquad (4)$$

The approximation made in (4) implies that knowledge of the sensitivities $\frac{\partial}{\partial\alpha_i} y(\alpha_i', s)$ and $\frac{\partial}{\partial\beta j} y(\beta_j', s)$ would allow one to approximate appropriate values of $\Delta\alpha_i$ and $\Delta\beta_j$, subtract them from the values of α_i' and β_j' resident in the system, and reduce the amount of error $e(s)$. The resultant $e(s)$ would have a magnitude approximately that of the higher-order terms ignored in the approximation of $e(s)$ made in (4). The magnitude

of this resultant e(s) would be, therefore, smaller than that of the original
e(s). Repetition of this process through several iterations should reduce
the magnitude of e(s) to some minimal value (the presence of measurement
noise may make it impossible to reduce the magnitude of e(s) to zero).
Note that, due to the Laplace integral's property of differentiability
with respect to system parameters, this process is also directly applicable
in the time domain.

The sensitivities required for this procedure may be obtained in
the following manner. Refer again to the system diagrammed in Figure 1.
The system transfer function and sensitivities are

$$y(\alpha_i, \beta_j, s) = \frac{W_1(s)K_1(\alpha_i, s)}{1 + W_1(s)K_1(\alpha_i, s)W_2(s)K_2(\beta_j, s)} \cdot r(s) \tag{5}$$

$$\frac{\partial}{\partial \alpha_i} y(\alpha_i', s) = -\frac{W_1^2(s)K_1(\alpha_i', s)W_2(s)K_2(\beta_j', s)}{(1 + W_1(s)K_1(\alpha_i', s)W_2(s)K_2(\beta_j', s))^2} \cdot \frac{\partial}{\partial \alpha_i} K_1(\alpha_i', s) \cdot r(s)$$

$$+ \frac{W_1(s)\frac{\partial}{\partial \alpha_i} K_1(\alpha_i', s)}{1 + W_1(s)K_1(\alpha_i', s)W_2(s)K_2(\beta_j', s)} \cdot r(s)$$

$$= \frac{1}{1 + W_1(s)K_1(\alpha_i', s)W_2(s)K_2(\beta_j', s)} \cdot \frac{\frac{\partial}{\partial \alpha_i} K_1(\alpha_i', s)}{K_1(\alpha_i', s)} \cdot$$

$$\cdot y(\alpha_i', \beta_j', s) \tag{6}$$

$$= \frac{1}{1 + W_1(s)K_1(\alpha_i', s)W_2(s)K_2(\beta_j', s)} \cdot \frac{\partial}{\partial \alpha_i} \ln K_1(\alpha_i', s)$$

$$\cdot y(\alpha_i', \beta_j', s) \tag{7}$$

$$\frac{\partial}{\partial \beta_j} y(\beta_j', s) = - \frac{W_1^2(s) K_1^2(\alpha_i', s) W_2(s)}{(1 + W_1(s) K_1(\alpha_i', s) W_2(s) K_2(\beta_j', s))^2} \cdot \frac{\partial}{\partial \beta_j} K_2(\beta_j', s) \cdot r(s)$$

$$= - \frac{W_1(s) K_1(\alpha_i', s)}{1 + W_1(s) K_1(\alpha_i', s) W_2(s) K_2(\beta_j', s)} \cdot W_2(s) \frac{\partial}{\partial \beta_j} K_2(\beta_j', s) y(\alpha_i', \beta_j', s).$$

$$(8)$$

$$= - \frac{W_1(s) K_1(\alpha_i', s)}{1 + W_1(s) K_1(\alpha_i', s) W_2(s) K_2(\beta_j', s)} \cdot W_2(s) K_2(\beta_j', s)$$

$$\cdot \frac{\frac{\partial}{\partial \alpha_i} K_2(\alpha_i', s)}{K_2(\alpha_i', s)} \cdot y(\alpha_i', \beta_j', s)$$

$$= - \frac{W_1(s) K_1(\alpha_i', s)}{1 + W_1(s) K_1(\alpha_i', s) W_2(s) K_2(\beta_j', s)} \cdot W_2(s) K_2(\beta_j', s)$$

$$\cdot \frac{\partial}{\partial \beta_j} \ln K_2(\beta_j', s) \cdot y(\alpha_i', \beta_j', s) \qquad . \qquad (9)$$

The reason for using the sensitivity filters $\frac{\partial}{\partial \alpha_i} \ln K_1(\alpha_i', s)$ and $\frac{\partial}{\partial \beta_j} \ln K_2(\beta_j', s)$ in (7) and (9), respectively, instead of $\frac{\partial}{\partial \alpha_i} K_1(\alpha_i', s)$ and $\frac{\partial}{\partial \beta_j} K_2(\beta_j', s)$, as in (6) and (8), is to avoid using higher-order sensitivity filters when the parameter is in the denominator of the compensator transfer function.

Example: Let

$$K_2(\beta_j', s) = \frac{f_z(s)}{f_p(\beta_j', s)} \qquad (10)$$

where $f_z(s)$ and $f_p(\beta_j',s)$ are polynomials in the Laplace operator describing the zeros and poles, respectively, of $K_2(\beta_j',s)$. Then

$$\frac{\partial}{\partial\beta_j} K_2(\beta_j',s) = - \frac{f_z(s)}{(f_p(\beta_j',s))^2} \frac{\partial}{\partial\beta_j} f_p(\beta_j',s) \qquad (11)$$

where

$$\frac{\partial}{\partial\beta_j} \ell n\, K_2(\beta_j',s) = \frac{1}{K_2(\beta_j',s)} \frac{\partial}{\partial\beta_j} K_2(\beta_j',s)$$

$$= - \frac{f_z(s)}{f_p(\beta_j',s)} \frac{\partial}{\partial\beta_j} f_p(\beta_j',s) \qquad . \qquad (12)$$

The order of the filter in (11) is lower than that in (10) by the order of $f_p(\beta_j',s)$

      The signals described in (7) and (9) may be obtained as diagrammed in Figure 2. The points S_1 and S_2 are known as the "sensitivity points" of the system at which appropriate signals may be easily picked off for use in calculating sensitivites.

1.2. Parameter Deviation Estimation

      The approximations of the parameter deviations $\Delta\alpha_i$ and $\Delta\beta_j$ may be obtained in any number of ways in order to satisfy whatever error minimization criteria that are specified. One of the more common tuning goals is that of finding $\Delta\alpha_i$ and $\Delta\beta_j$ to minimize the magnitude of difference between the system response and a nominal reference response over the entire response period T_R. For this purpose, one may use a very common cost function; namely,
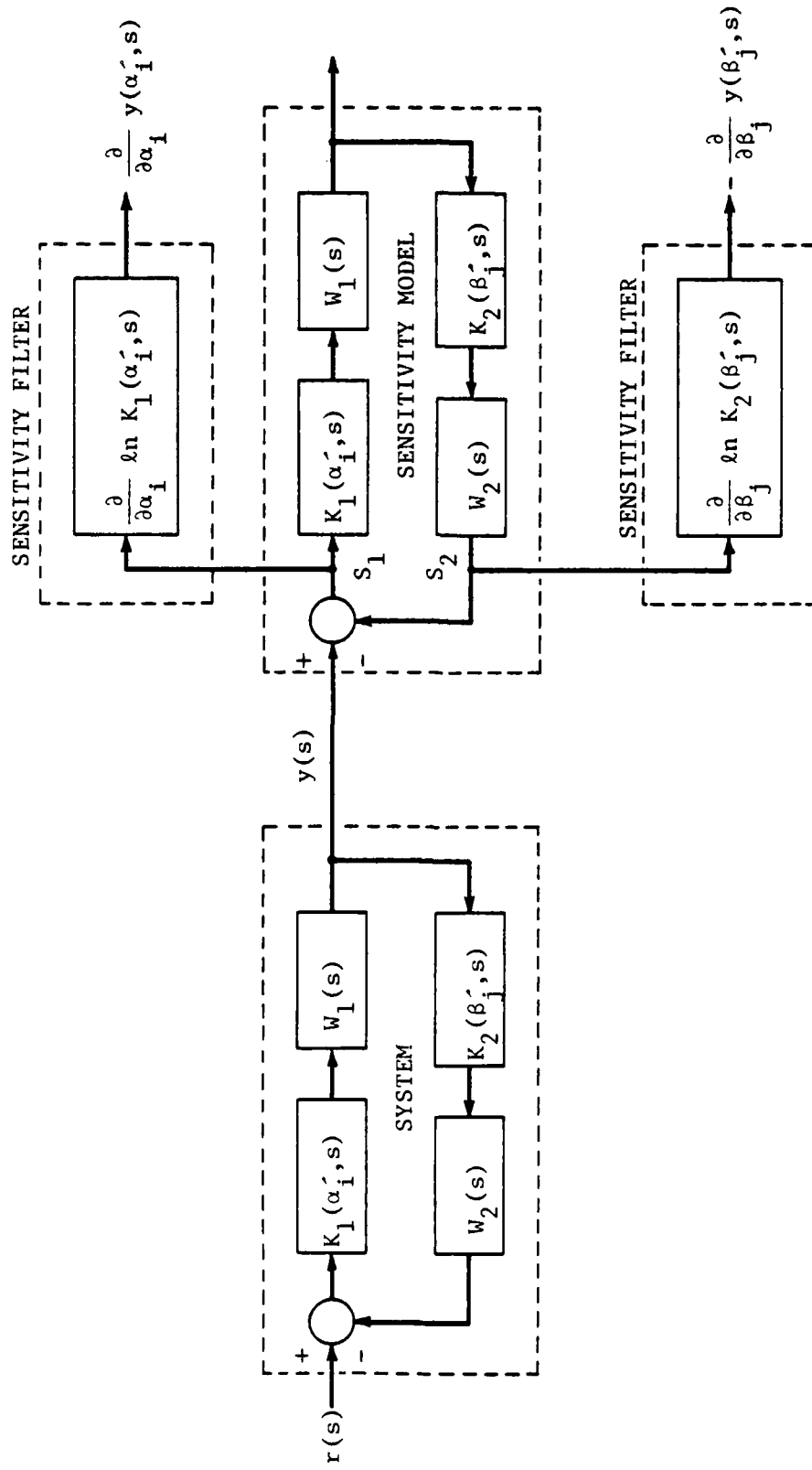
Figure 2. Configuration for obtaining sensitivities of the system diagrammed in Figure 1.

that of rms error over the specified response period. The cost function
may be expressed as

$$J = \int_0^{T_R} (e(t))^2 dt \qquad . \qquad (13)$$

In discrete time, (11) becomes

$$J = \sum_{k=0}^{N} (e(kT))^2 \qquad , \qquad (14)$$

where $NT = T_R$.

Now let

$$\gamma = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad \text{and} \quad \Delta\gamma = \begin{bmatrix} \Delta\alpha \\ \Delta\beta \end{bmatrix} \qquad .$$

If a time-domain analogy is made of the error approximation
expressed in (4), the tuning goal in each iteration becomes that of finding

$$\Delta\gamma^* = \arg\min_{\Delta\gamma} \left[\sum_{k=0}^{N} (e(kT) - \nabla_\gamma y(\alpha_i, \beta_j, kT) \cdot \Delta\gamma)^2 \right] \qquad (15)$$

in discrete time, or

$$\Delta\gamma^* = \arg\min_{\Delta\gamma} \left[\int_0^{T_R} (e(t) - \nabla_\gamma y(\alpha_i, \beta_j, t) \cdot \Delta\gamma)^2 dt \right] \qquad (16)$$

in continuous time.

If data processing were to be performed by a digital computer or
microprocessor, the tuning objective in each iteration would be that stated
in (15), which may be interpreted as a minimization problem in a Euclidean
vector space of order N+1. In this context, (15) may be written as

$$\Delta\gamma^* = \arg\min_{\Delta\gamma} \; \|S\Delta\gamma - \mu\| \quad , \tag{17}$$

where

$$S = \begin{bmatrix} \nabla_\gamma y(\alpha_i,\beta_j,0) \\ \vdots \\ \nabla_\gamma y(\alpha_i,\beta_j,kT) \\ \vdots \\ \Delta_\gamma y(\alpha_i,\beta_j,NT) \end{bmatrix}$$

and

$$\mu = \begin{bmatrix} e(0) \\ \vdots \\ e(kT) \\ \vdots \\ e(NT) \end{bmatrix} \quad .$$

The solution for (17) may be given by

$$\Delta\gamma^* = (S^T S)^{-1} S^T \mu \tag{18}$$

if $(S^T S)^{-1}$ exists (implying that no linear dependencies exist between the columns of S) [13]. Note that for $n+m \leq 2$, (18) always applies; for $n+m \geq 3$, the possibility of linear dependencies between columns of S arises and a more complex computation than (18) must be undertaken in order to ensure that a suitable solution to (17) may be obtained.

1.3. Notes

Up to this point, this discussion has mentioned only the use of a first-order method of parameter approximation. Higher-order methods for approximation may be used [14], but generally do not yield significantly

superior performance. In addition, higher-order methods usually require longer computation times in each iteration.

A second, more important point to be made here is that the sensitivity model used to obtain the pertinent sensitivities need not be the system itself or a copy of the system, as might be inferred by (7) and (9), and as depicted in Figure 2. If a full model of the system is available (in either digital or analog form), this model may be used as the sensitivity model. In addition, since the parameter deviations of interest are only being approximated at each iteration (as opposed to one single exact calculation for the entire process), a sufficiently descriptive reduced-order model may be substituted as a sensitivity model if the full-order model of the system is inappropriately large or too complex for such use. If the reduced-order model is well-chosen, the performance of the sensitivity points tuning process will not be significantly degraded by such approximation. If the chosen reduced-order model is suspect, however, two changes may be made to the procedure outlined above in order to reduce effects of using such a mode.

1. The sensitivity point may be located in the original system, as depicted in Figure 3, instead of the reduced-order model, whereupon only the input and output of the reduced-order sensitivity model need be accessible. This eliminates the need for the internal structure of the sensitivity model to be identical to that of the system. Matching only of input-output characteristics of the sensitivity model and system would, therefore, be sufficient in formulating a viable sensitivity model.

Figure 3. Alternate configuration for obtaining sensitivities of the system diagrammed in Figure 1.
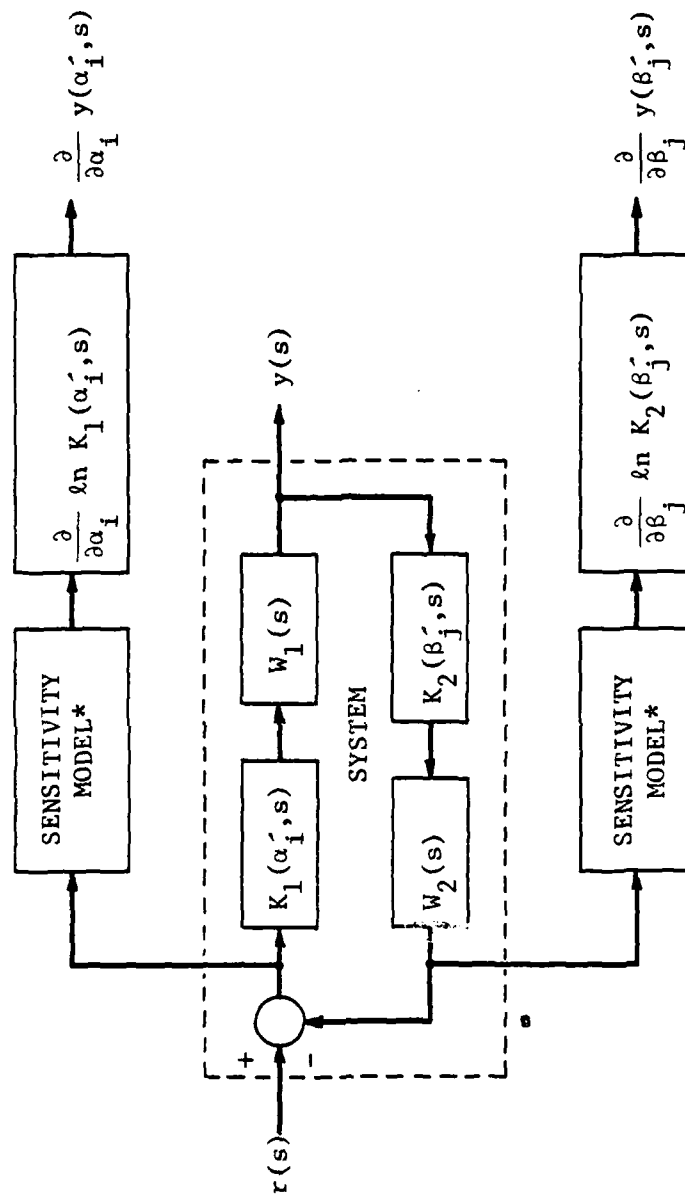
*Model may be of reduced order.

2. The estimation of the parameter deviation vector $\Delta\gamma^*$ may be made using a steepest descent estimate of the form

$$\Delta\gamma^* = cS^T\mu \quad , \quad c \in \mathbb{R}^1 \quad ,$$

instead of the least-squares solution given in (18). This update form is less sensitive to inaccuracies in the sensitivity matrix S. In addition, if c is appropriately chosen, this steepest descent update will tend to have faster initial convergence. One disadvantage of this form is that, with constant c, the updates may oscillate about the optimal solution. One possible solution to this problem is to utilize the steepest descent estimate for several initial iterations, and then use the least-squares estimate when in the neighborhood of the optimal solution in order to ensure convergence.

1.4. Subjects for Discussion

In the ensuing discussion, the concepts of the sensitivity points method will be extended to apply to multiple-input/multiple-output (MIMO) systems. The Mimo sensitivity points method will then be used to examine, first of all, techniques of tuning MIMO systems in a rather general sense, and, then, the applicability of the sensitivity points method to decentralized tuning of MIMO systems. Examples and simulations will be included to help illustrate points to be made. Finally, an example of a realized system will be given and discussed.

CHAPTER 2

SENSITIVITY POINTS TUNING IN MULTI-INPUT/MULTI-OUTPUT SYSTEMS

In this chapter, the sensitivity points tuning concepts described in Chapter 1 for single-input/single-output (SISO) systems will be extended to apply to multiple-input/multiple-output (MIMO) systems. Techniques for sensitivity points tuning of MIMO systems will then be discussed.

2.1. MIMO Sensitivity Points

The deviations for the sensitivities of MIMO systems progress in a manner quite similar to that described in Chapter 1 for SISO systems. Consider the very general MIMO system depicted in block diagram form in Figure 4. This system has an input vector $R(s)$ of dimension d_R; output vector $y(\alpha,\beta,s)$ of dimension d_y; system feedforward block $W_1(\alpha,s)$ with transfer function matrix of dimension $d_y \times d_R$, and with adjustable parameter vector $\alpha = [\alpha_1,\ldots,\alpha_n]^T$; and system feedback block $W_2(\beta,s)$ with transfer function matrix of dimension $d_R \times d_y$ and with adjustable parameter vector $\beta = [\beta_1,\ldots,\beta_m]^T$. The transfer function of the system is

$$y(\alpha,\beta,s) = [I + W_1(\alpha,s)W_2(\beta,s)]^{-1}W_1(\alpha,s)R(s) \qquad . \qquad (19)$$

If one were to follow a notation convention analogous to that used in the SISO sensitivity derivation, one would find the corresponding MIMO sensitivities to be
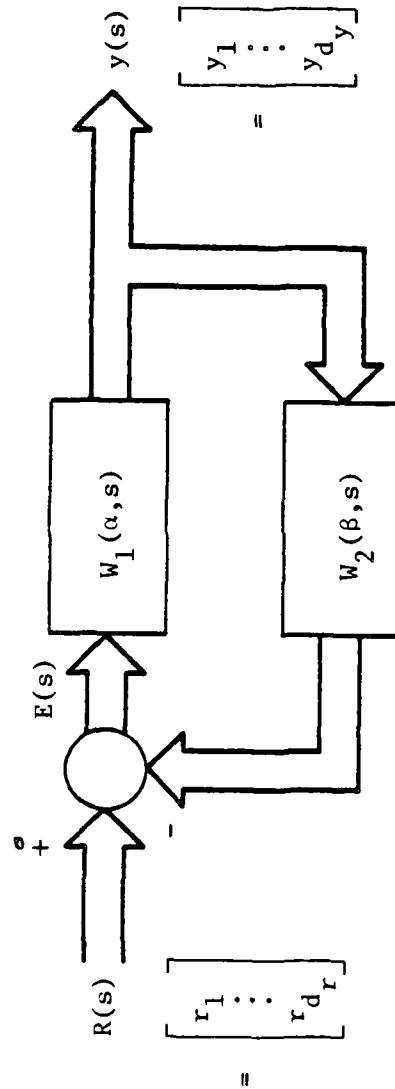
15

Figure 4. A general multi-input/multi-output (MIMO) feedback system.

$$\frac{\partial}{\partial \alpha_i} y(\alpha',\beta',s) = - [I + W_1(\alpha',s)W_2(\beta',s)]^{-1} \frac{\partial}{\partial \alpha_i} (I + W_1(\alpha',s)W_2(\beta',s))$$

$$\cdot \; [I + W_1(\alpha',s)W_2(\beta',s)]^{-1} W_1(\alpha',s)R(s)$$

$$+ [I + W_1(\alpha',s)W_2(\beta',s)]^{-1} \frac{\partial}{\partial \alpha_i} W_1(\alpha',s)R(s)$$

$$= [I + W_1(\alpha',s)W_2(\beta',s)]^{-1} \frac{\partial}{\partial \alpha_i} W_1(\alpha',s) \cdot E(s) \qquad (20)$$

where

$$E(s) = R(s) - W_2(\beta',s)y(\alpha',\beta',s) \qquad ;$$

and

$$\frac{\partial}{\partial \beta_j} y(\alpha',\beta',s) = - [I + W_1(\alpha',s)W_2(\beta',s)]^{-1} W_1(\alpha',s) \cdot \frac{\partial}{\partial \beta_j} W_2(\beta',s)$$

$$\cdot \; [I + W_1(\alpha',s)W_2(\beta',s)]^{-1} W_1(\alpha',s)R(s)$$

$$= - [I + W_1(\alpha',s)W_2(\beta',s)]^{-1} W_1(\alpha',s) \frac{\partial}{\partial \beta_j} W_2(\beta',s)y(\alpha,\beta',s) \qquad .$$

$$(21)$$

A realization for obtaining the signals described by (20) and (21) is diagrammed in Figure 5. If one were to refer back to the SISO realization depicted in Figure 2, one would see that a commutation of the sensitivity filter and sensitivity model in Figure 2 would result in a degenerate form of the realization shown in Figure 5.

The lack of commutativity in the MIMO case has one major implication; namely, that only one sensitivity filter at a time may be used in the sensitivity-generating process. In the SISO case, commutativity of terms allows for the placement of multiple filters on the output of the
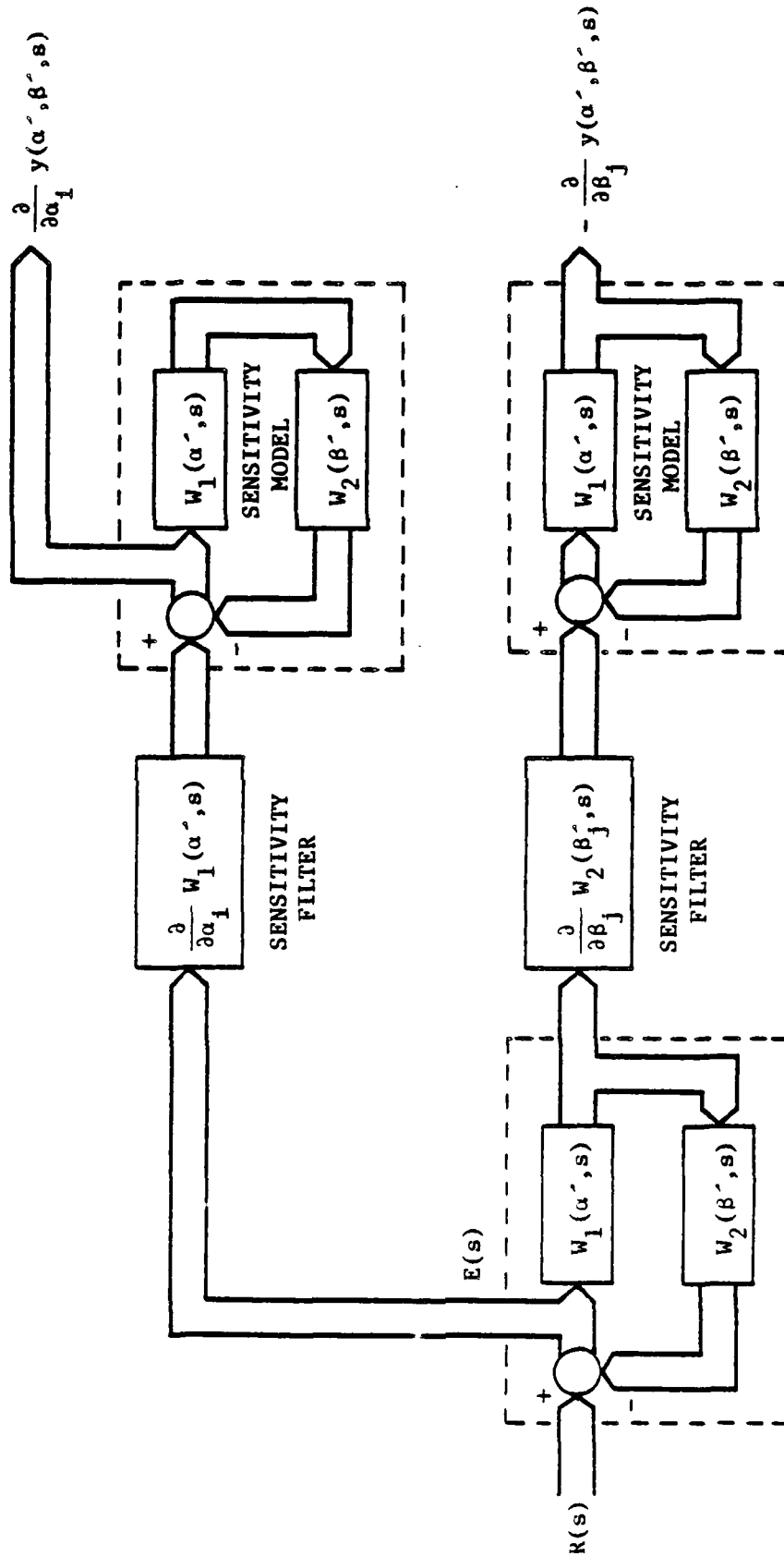
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Figure 5. Configuration for obtaining sensitivities of the system diagrammed in Figure 4.

sensitivity model. Sensitivities with respect to several different parameters may, therefore, be obtained simultaneously. Since no such commutativity exists in the MIMO case, however, and since only one input vector signal may be applied to the sensitivity model input at any time, only the sensitivities with respect to one parameter will be generated at any given time. This parameter is the parameter corresponding to the sensitivity filter in use. If the sensitivities with respect to p parameters are desired, then the process of inputting y(s) (or E(s), depending upon which is applicable) into a sensitivity filter-sensitivity model pair must be executed p times.

If one were to use the MIMO system depicted in Figure 6, one would find the system transfer function to be

$$y(\alpha,\beta,s) = [I + W_1(s)K_1(\alpha,s)W_2(s)K_2(\beta,s)]^{-1}W_1(s)K_1(\alpha,s)R(s) \quad . \qquad (22)$$

The corresponding system sensitivities would be

$$\frac{\partial}{\partial\alpha_i}y(\alpha',\beta',s) = -[I + W_1(s)K_1(\alpha',s)W_2(s)K_2(\beta',s)]^{-1}$$

$$\cdot W_1(s)\frac{\partial}{\partial\alpha_i}K_1(\alpha',s)W_2(s)K_2(\beta',s)$$

$$\cdot [I + W_1(s)K_1(\alpha',s)W_2(s)K_2(\beta',s)]^{-1}W_1(s)K_1(\alpha',s)R(s)$$

$$+ [I + W_1(s)K_1(\alpha',s)W_2(s)K_2(\beta',s)]^{-1}W_1(s)\frac{\partial}{\partial\alpha_i}K_1(\alpha',s)R(s)$$

$$= [I + W_1(s)K_1(\alpha',s)W_2(s)K_2(\beta',s)]^{-1}W_1(s)\frac{\partial}{\partial\alpha_i}K_1(\alpha',s)E(s)$$

$$(23)$$

$$R(s) = \begin{bmatrix} r_1 \\ \vdots \\ r_{d_r} \end{bmatrix}$$

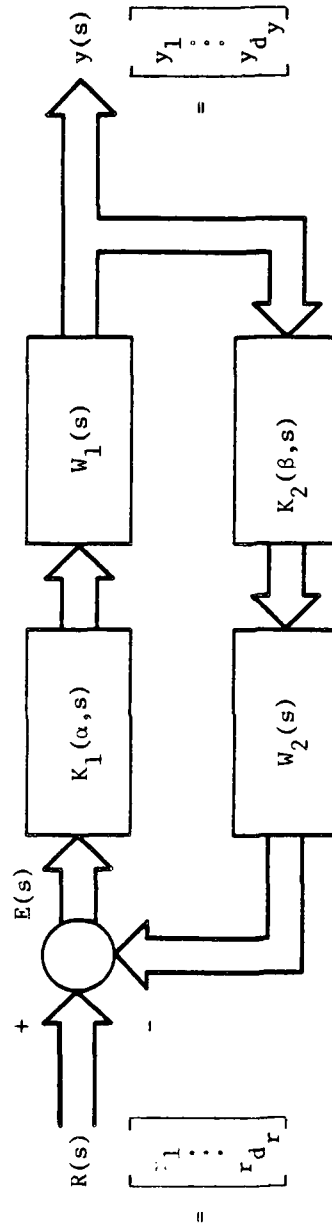$$y(s) = \begin{bmatrix} y_1 \\ \vdots \\ y_{d_y} \end{bmatrix}$$

Figure 6. MIMO system with system blocks $W_1(s)$ and $W_2(s)$ and known compensator blocks $K_1(\alpha,s)$ and $K_2(\beta,s)$.

where

$$E(s) = R(s) - W_2(s)K_2(\beta\hat{\,},s)y(\alpha\hat{\,},\beta\hat{\,},s)$$

and

$$\frac{\partial}{\partial \beta_j} y(\alpha,\beta,s) = - [I + W_1(s)K_1(\alpha,s)W_2(s)K_2(\beta,s)]^{-1}$$

$$\cdot W_1(s)K_1(\alpha,s)W_2(s) \frac{\partial}{\partial \beta_j} K_2(\beta,s)$$

$$\cdot [I + W_1(s)K_1(\alpha,s)W_2(s)K_2(\beta,s)]^{-1} W_1(s)K_1(\alpha,s)R(s)$$

$$= - [I + W_1(s)K_1(\alpha,s)W_2(s)K_2(\beta,s)]^{-1} W_1(s)K_1(\alpha,s)$$

$$W_2(s) \frac{\partial}{\partial \beta_j} K_2(\beta,s)y(\alpha,\beta,s) \quad . \tag{24}$$

A realization for obtaining the signals described by (23) and (24) is diagrammed in Figure 7. An alternate realization of (23) and (24) is shown in Figure 8. The attractiveness of this alternate realization lies in the fact that the system itself may be used as the sensitivity model without any modifications such as the addition of auxiliary inputs shown in the realization of Figure 7.

In addition, if a reduced-order model were to be used as the sensitivity model, it would only need to match the input/output characteristics of the system.

Use of the alternate realization, however, is limited to those systems where $K_1^{-1}(\alpha,s)$ exists and is realizable, and where $W_2(s)$ is readily replicable. In either case, as in the more general realization shown in Figure 5, a lack of commutativity implies that only the sensitivities
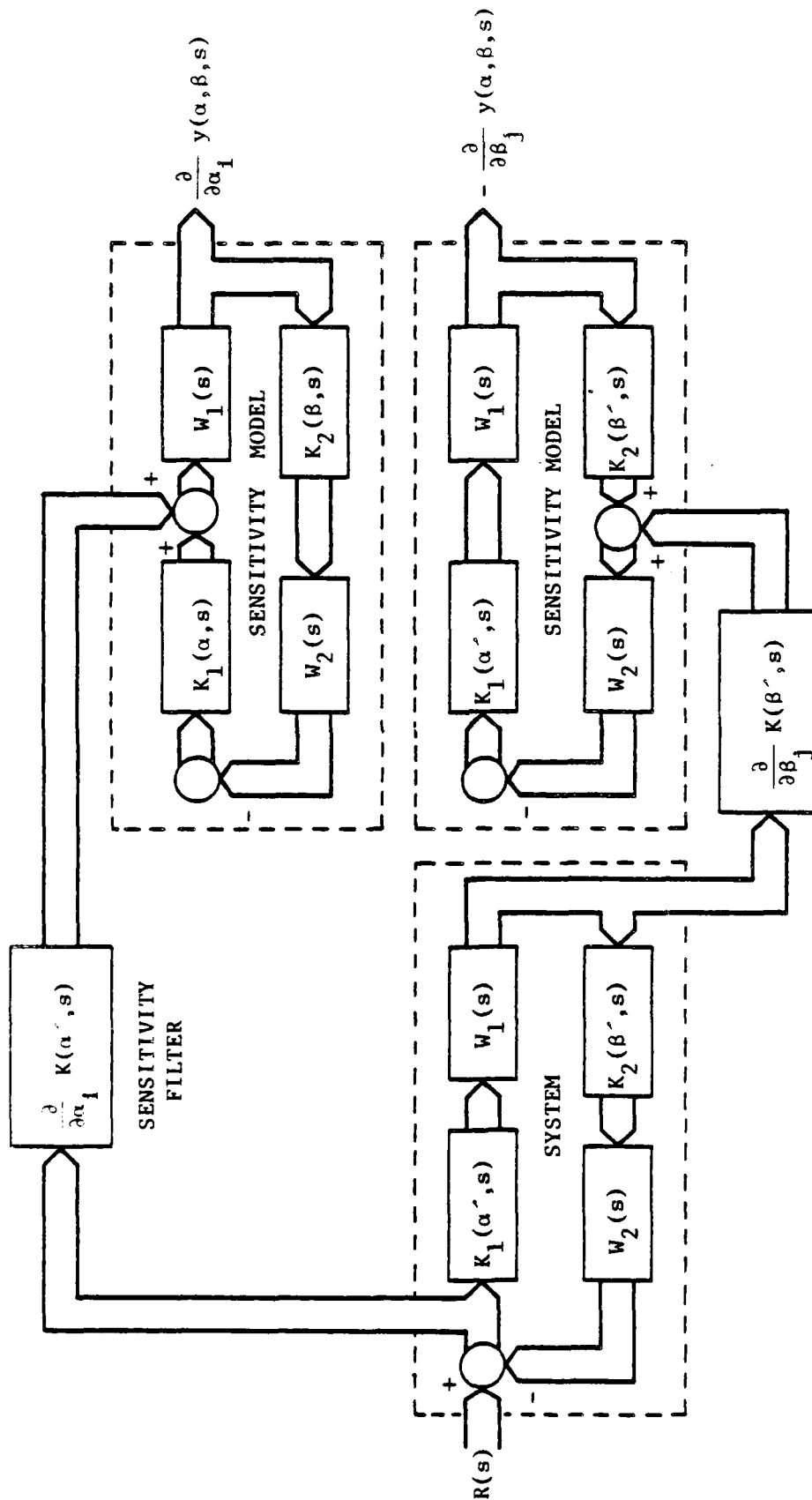
21

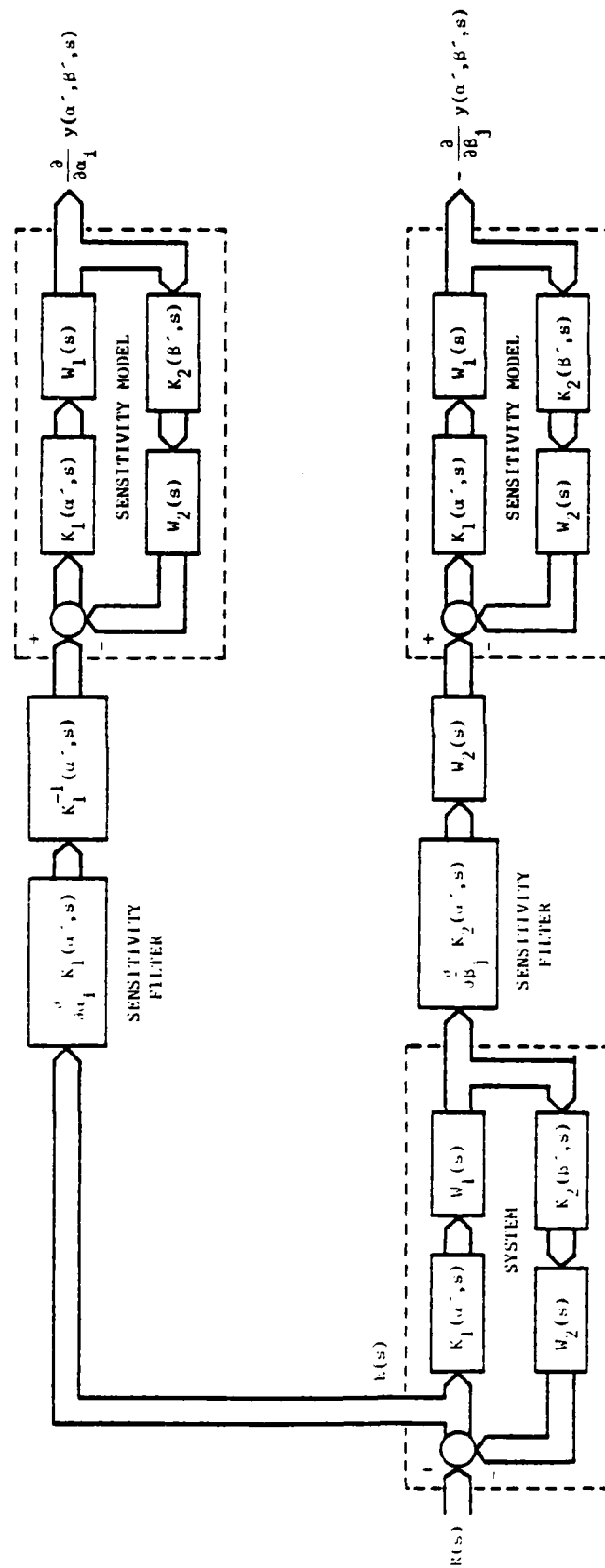Figure 7. Configuration for obtaining sensitivities of the system diagrammed in Figure 6.

Figure 8. Alternate configuration for obtaining sensitivities of the system diagrammed in Figure 6.

with respect to one parameter may be obtained at any one time, since only a single sensitivity filter may be used in series with the sensitivity model. One possible solution to this problem would be to use multiple sensitivity models (or multiple approximate sensitivity models), which would allow for the extraction of sensitivities with respect to as many parameters as the number of sensitivity models used.

2.2. The MIMO Sensitivity Filter

At this point, it may be appropriate to discuss the interpretation of the MIMO sensitivity filter, $\frac{\partial}{\partial(\cdot)} K(\cdot,s)$. In the SISO case, realization of the sensitivity filter amounted to, quite simply, the realization of an SISO transfer function. In the MIMO case, however, the transfer function of the sensitivity filter is a matrix $K(\cdot,s)$ whose elements may be denoted as $K_{ij}(\cdot,s)$ (i denoting row assignment, j denoting column assignment); $\frac{\partial}{\partial(\cdot)} K(\cdot,s)$ is simply a matrix of the same dimension as $K(\cdot,s)$ whose i-j-th element is $\frac{\partial}{\partial(\cdot)} K_{ij}(\cdot,s)$.

Example: Let

$$K = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 5 \\ 6 & 7 & \frac{1}{s+5} \end{bmatrix} \quad ;$$

then

$$\frac{\partial}{\partial 5} K(5,s) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$\frac{\partial}{\partial \xi} K(\xi, s) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\dfrac{1}{(s+\xi)^2} \end{bmatrix} \quad .$$

The realization of this is quite straightforward: for each i and j, take the jth element of the signal drawn from the system, pass it through a filter whose transfer function is $\frac{\partial}{\partial(\cdot)} K_{ij}(\cdot, s)$, and input the resultant signal as the ith element of the input vector for the sensitivity model. If $\frac{\partial}{\partial(\cdot)} K(\cdot, s)$ contains multiple elements in its ith row, one must, quite simply, sum the signals emanating from those filters corresponding to elements in the ith row of $\frac{\partial}{\partial(\cdot)} K(\cdot, s)$ and use this summed signal as the ith element of the input vector to the sensitivity model.

2.3. MIMO Parameter Deviation Estimation

The basic concepts described for the estimation of parameter deviation in the SISO case are still applicable in the MIMO case. The basic task is still that of finding

$$\Delta\gamma^* = \arg \min_{\gamma} \ \| S\Delta\gamma - \mu \| \quad .$$

In the MIMO setting, however,

$$S = \begin{bmatrix} \nabla_\gamma\, y_1\, (\alpha',\beta',0) \\ \vdots \\ \nabla_\gamma\, y_1\, (\alpha',\beta',NT) \\ \nabla_\gamma\, y_2\, (\alpha',\beta',0) \\ \vdots \\ \nabla_\gamma\, y_2\, (\alpha',\beta',NT) \\ \vdots \\ \nabla_\gamma\, y_{d_y}\, (\alpha',\beta',0) \\ \vdots \\ \nabla_\gamma\, y_{d_y}\, (\alpha',\beta',NT) \end{bmatrix} \qquad ,$$

and

$$\mu = \begin{bmatrix} e_1\ (0) \\ \vdots \\ e_1\ (NT) \\ e_2\ (0) \\ \vdots \\ e_2\ (NT) \\ \vdots \\ e_{d_y}\ (0) \\ \vdots \\ e_{d_y}\ (NT) \end{bmatrix} \qquad ,$$

where

$$e_i = y_i(\alpha',\beta',t) - y_i(\alpha^0,\beta^0,t) \qquad . \tag{25}$$

With S and μ defined in this fashion, $\Delta\gamma^*$ may be estimated with the same methods used in the SISO case.

It should be noted at this point that none of the outputs need necessarily be tuned with respect to all parameters. If, for instance, one wished to tune output y_i by using all parameters in γ <u>except</u> for some β_j, all that one would need to do would be to set $\frac{\partial}{\partial\beta_j} y_i(t) = 0$ for all $t = [0,T_R]$. This would set $\frac{\partial}{\partial\beta_j} y_i(t)\Delta\beta_j = 0$, meaning that it would have no effect in the tuning process. The estimation would, therefore, ignore $\Delta\beta_j$ in approximating $\Delta\gamma^*$.

2.4. MIMO Tuning Techniques

If one were to assume that the MIMO system to be tuned was, in fact, a network of SISO or MIMO subsystems, then tuning techniques would fall into two general categories: centralized tuning and decentralized tuning.

2.4.1. Centralized tuning

In this scenario, subsystems of the overall system are interconnected by inter-subsystem feedback as well as any coupling inherent in the plant. As depicted in Figure 9, the main purpose of a centralized tuner would be to maintain the inter-subsystem feedback elements in a way that would assure overall system stability. These particular feedback elements have less effect on subsystem dynamics than local feedback elements, but may become important if inherent coupling is strong. Otherwise, a perturbing input to one
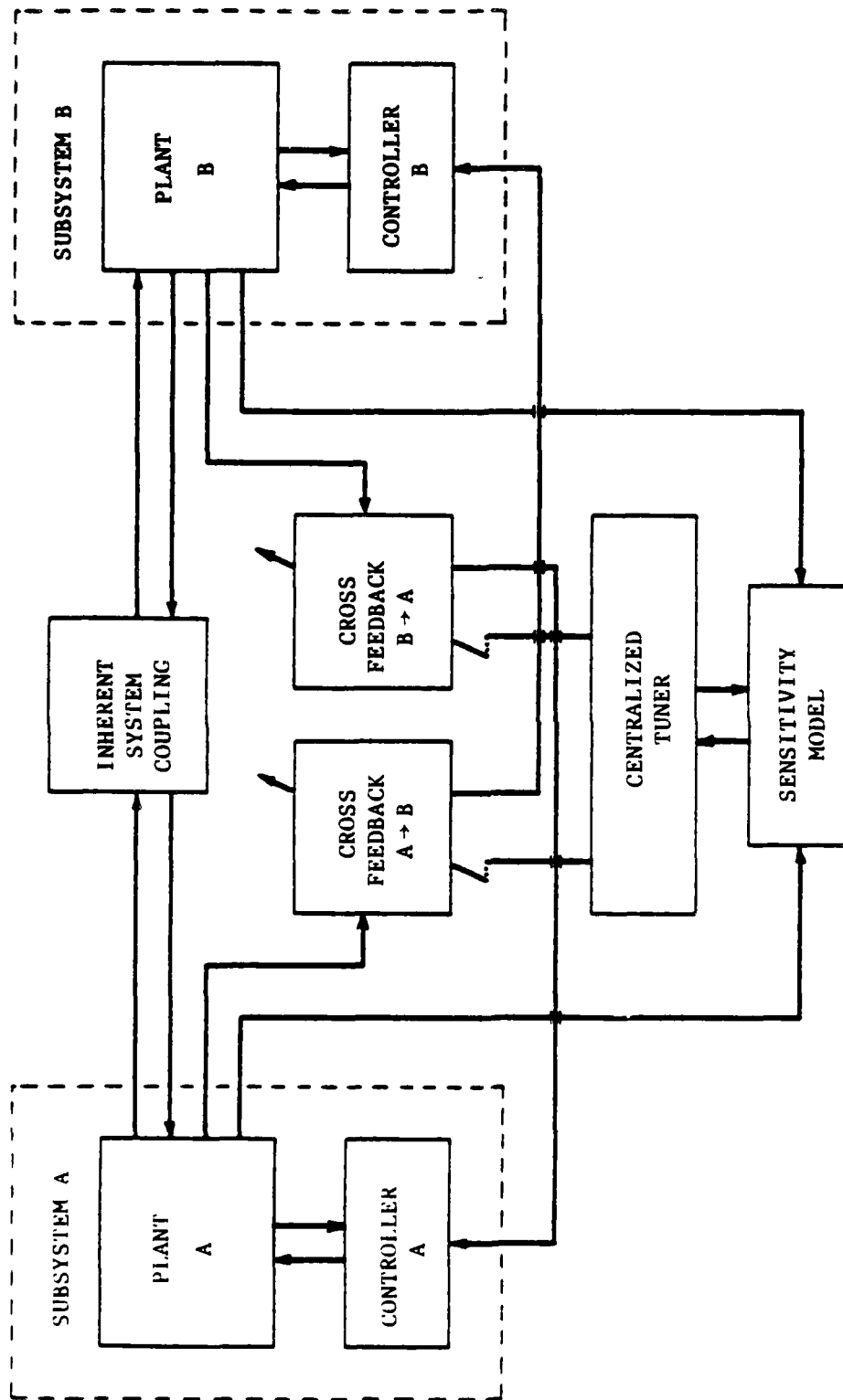
Figure 9. Centralized tuning scenario.

subsystem may excite resonances in some less-damped subsystems via inherent coupling, which may then result in loss of overall system stability.

Example: Magnetic supension system. Consider the magnetic suspension system sketched in Figure 10. The objective is to maintain the bar at a nominal distance h_o and to keep it level. The linearized state equations of the system are

$$
\begin{bmatrix} \delta \dot{x}_1 \\ \delta \dot{x}_2 \\ \delta \dot{x}_3 \\ \delta \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2b & 0 & -b & 0 \\ 0 & 0 & 0 & 1 \\ -b & 0 & 2b & 0 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \\ \delta x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\phi & -\theta \\ 0 & 0 \\ -\theta & -\phi \end{bmatrix} \begin{bmatrix} \delta r_1 \\ \delta r_2 \end{bmatrix}
$$

and

$$
\begin{bmatrix} \delta y_1 \\ \delta y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \delta x
$$

where

$$
\phi = L^2 q + 3Lq \quad ,
$$

$$
\theta = L^2 q - 3Lq \quad ,
$$

$$
q = \frac{1}{h_o L} \sqrt{\frac{2\alpha g}{m}} \quad ,
$$

and

$$
b = \frac{2g}{h_o} \quad .
$$

Notice that the system may be treated as two weakly coupled subsystems. Note also that these subsystems appear in the system matrix as diagonal blocks, which allows for a feedback controller matrix of the form
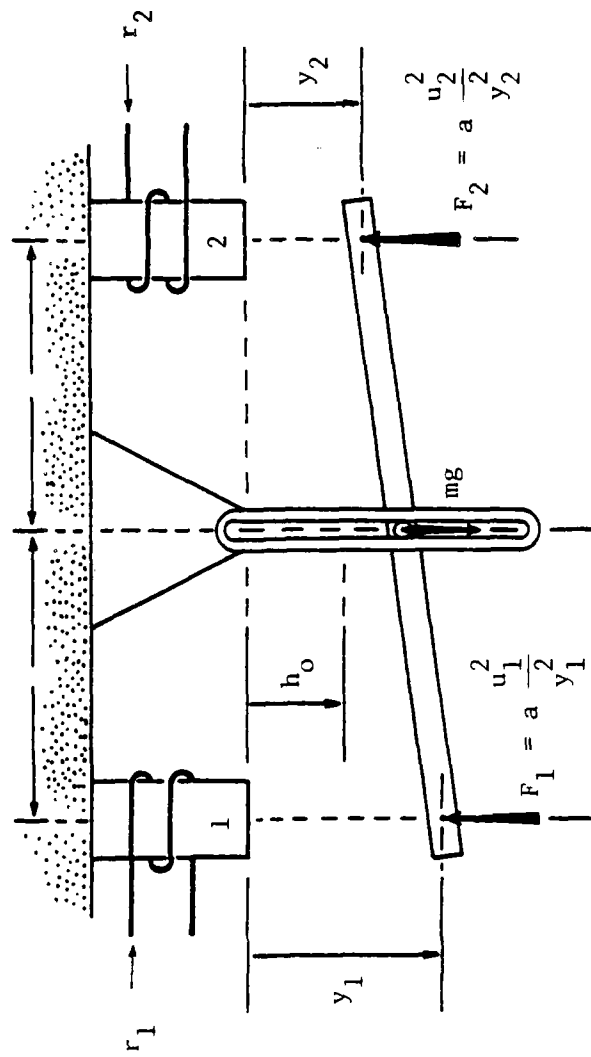
Figure 10. Magnetic suspension system.

$$
K = \begin{bmatrix} k_{11} & k_{12} & \vdots & k_{13} & k_{14} \\ \hline k_{21} & k_{22} & \vdots & k_{23} & k_{24} \end{bmatrix} \quad .
$$

In this controller form, those elements in the diagonal blocks are associated with local feedback and the off-diagonal elements govern cross-feedback. Assume now that $a = 1$, $m = 2$, $g = 1$, $h_o = 1$, $L = 2$, and that the sampling interval is 50 msec. Assume further, for the sake of simplicity, that $k_{11} = k_{23} = 0.9$, $k_{12} = k_{24} = 0.2$, and $k_{14} = k_{22} = 0$, so we will be tuning with just the two cross feedback elements k_{13} and k_{21}. This describes an unstable fourth-order system. Computer simulation will demonstrate that, for a step disturbance applied to subsystem 1 with $k_{13} = k_{21} = -1$, this system is unstable. Tuning k_{13} and k_{21} by the sensitivity points method with a second-order reference step response (poles at $s = -2 \pm j3.46$) results in convergence of k_{13} and k_{21} to stability in 12 iterations. Responses change from unstable to 32% overshoot in y_1 and 150% overshoot in y_2 (tuning inputs: unit step into subsystem 1 (left), half step into subsystem 2 (right)). The tuning history of this particular run is given in the Appendix as Table I. The effectiveness of a centralized tuner is, therefore, quite clear and may be critical, as demonstrated by this simple example.

2.4.2. Decentralized tuning

Consider now the problem of decentralized tuning at each subsystem. In contrast to centralized tuning, local tuners may use only locally available signals for the tuning process, as depicted in Figure 11. This implies that the task of local tuning is limited to the tuning of only local
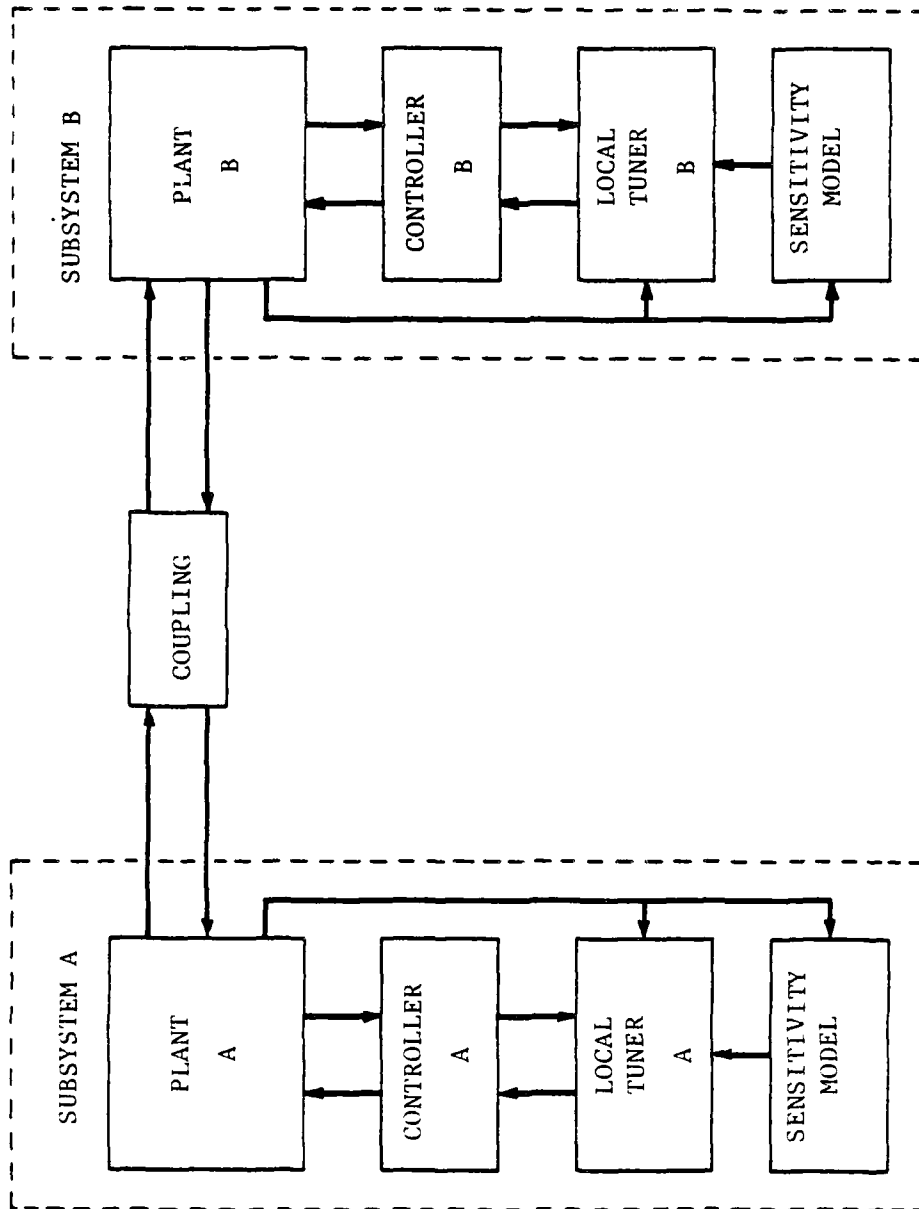
31

Figure 11. Decentralized tuning scenario.

compensators (which may be in either the feedforward or feedback path).
Consequently, only those parameters in the local compensators need be
tuned.

One major issue in decentralized tuning is whether or not exact
tuning is possible from the local level. If the full model of the overall
system is available to the subsystem as a sensitivity model, such tuning
is possible.

In order to visualize why this is true, assume that the transfer
function matrix of the system may be written in a form such that each
subsystem lies within a block on the main diagonal. This would result in
the placement of local compensator terms in blocks of the main diagonal
of their respective transfer function matrices, which would correspond to
the location of their respective subsystem blocks in the main system
transfer function matrix. The structure of the MIMO sensitivity filter
(discussed in Section 2.2) would, then, ensure that only local signals
would be needed for local tuning, irrespective of the level of coupling to
other subsystems and irrespective of inputs to other subsystems. This
implies that, if the full model of the system is available as the sensitivity
model, no approximation of sensitivities will be made; the exact sensitivities
described in (20) and (21) (or (23) and (24)) will be generated instead.
Availability of such exact information ensures that exact tuning from the
local level is, indeed, possible.

Irrespective of whether or not a full system model is available as
a sensitivity model, decentralized tuning may be performed under any of
several schemes. All of such schemes, however, seem to be one or the other
or a combination of two basic methods.

1. **Alternating tune-up.** In this scheme, subsystems are individually tuned one at a time. If the subsystem to be tuned is treated as a SISO system, it may be tuned by the SISO sensitivity points tuning procedure. All other subsystems are assumed to be at some respective operating point, but no assumptions are made about the inputs to those subsystems not being tuned.

2. **Joint tuning.** In this scheme, the entire network of subsystems is treated as one large MIMO system. Each subsystem is given its own tuning input, and all inputs are applied together as the input to the MIMO system. All subsystems are tuned simultaneously; the overall tune-up time for the entire network may, therefore, be reduced.

Computer simulations using these schemes were performed with various permutations of step and delayed-step inputs and confirmed that exact local tuning can be performed. Tuning histories for these two schemes are given in Tables II and III of the Appendix.

CHAPTER 3

DECENTRALIZED TUNING OF MIMO SYSTEMS WITH SIMPLIFIED SENSITIVITY MODELS

Given an MIMO system, consisting of coupled subsystems, consider again the problem of decentralized tuning of individual subsystem controller sections. As depicted in Figure 11, local tuners have only locally available signals for tuning information.

It has been established in Chapter 2 that, using the MIMO sensitivity points tuning procedure, exact local tuning at a particular subsystem may be obtained irrespective of coupling or of the size and timing of inputs to other subsystems if a full model of the overall plant is available at the subsystem for use as a sensitivity model. Such a full model, however, may not be readily available for such use. The problem to be addressed, then, is whether or not a subsystem can be satisfactorily tuned if only a greatly simplified model of the overall system is available for use as a sensitivity model.

In order to assess limitations of decentralized tuning with simplified sensitivity models, three tuning situations have been examined.

1. _Tuning with the rest of the plant tuned_. In this case, the subsystem of interest is tuned up from some detuned state, while all of the other subsystems in the plant are operating at their respective fully _tuned_ states. Simulation will show that, with this particular situation, tuning with a simplified sensitivity model is as effective as tuning with a full model of the plant as the sensitivity model.

2. <u>Alternate tuning</u>. This situation involves cooperation between subsystems. Subsystems are tuned one after the other in a tuning "run" through the entire plant. This process is then repeated until successive tuning runs yield no change in the respective parameters being tuned in each subsystem controller. The only limitation of this tuning process is that, while a particular subsystem is being tuned in the midst of a tuning run, the rest of the plant must be stable.

3. <u>Tuning with the rest of the plant mistuned</u>. This situation, like Situation 1, does <u>not</u> involve cooperation between subsystems. For the purposes of this thesis, the term "mistuned" will have the connotation that some subsystem <u>not</u> being tuned has been detuned to such an extent that the subsystem being tuned tunes to parameter values different from those arrived at when using a full model of the plant for the sensitivity model. This situation arises when one or both of two conditions are met:

(a) The rest of the plant is unstable; or,

(b) The rest of the plant has a natural undamped frequency below some low frequency which is specific to each overall plant and may be specific ot each subsystem.

Example: Magnetic suspension system. Consider, again the magnetic suspension described in Chapter 2. The controller matrix is, again,

$$K = \left[\begin{array}{cc|cc} k_{11} & k_{12} & k_{13} & k_{14} \\ \hline k_{21} & k_{22} & k_{23} & k_{24} \end{array} \right] \quad .$$

As before, those elements in the diagonal blocks are associated with local feedback and the off-diagonal elements govern cross-feedback. Since we are dealing with decentralized tuning, assume that the off-diagonal terms are zeros. For the values of the suspension model parameters, assume that $a = 1$, $m = 2$, $g = 1$, $h_o = 1$, $L = 2$, and that the sampling interval is 50 msec. Tuning k_{11}, k_{12}, k_{23}, and k_{24} by the sensitivity points method with the full plant model as the sensitivity model and with a second-order reference step response (double poles at $s = -3.0$) results in convergence of parameters to $k_{11} = k_{23} = 1.16$ and $k_{12} = k_{24} = 0.68$. Assume for each subsystem that its respective simplified sensitivity model is defined by those portions of the plant's system, control, and output matrices which pertain to that particular subsystem and local feedback about that subsystem. For the left subsystem (Subsystem 1), then, the sensitivity model is defined by

$$
\begin{bmatrix} \dot{\sigma}_{11} \\ \dot{\sigma}_{12} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2b - \phi k_{11} & -k_{11} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ -\phi \end{bmatrix} \mu
$$

and

$$
S = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{12} \end{bmatrix} \quad .
$$

The right subsystem (Subsystem 2) sensitivity model is similarly defined.

Detuning Subsystem 1 alone to $k_{11} = 0.5$ and $k_{12} = 1.5$ and tuning result in a demonstration of Situation 1 (tuning with the rest of the plant tuned). The sensitivity points tuning procedure yields convergence in 4

iterations to $k_{11} = 1.15$ and $k_{12} = 0.69$. The tuning history for this tune-up is shown in the Appendix as Table IV.

Detuning Subsystem 1 to $k_{11} = 0.5$ and $k_{12} = 1.5$, detuning Subsystem 2 to $k_{23} = 0.25$ and $k_{24} = 0.1$, and tuning up the two subsystems in alternate fashion result in a demonstration of the alternate tuning scenario of Situation 2. The sensitivity points tuning procedure yields covergence in 16 iterations in 2 "runs" to $k_{11} = 1.2$, $k_{12} = 0.70$, $k_{23} = 1.11$, $k_{24} = 0.66$. The tuning history for this tune-up is shown in the Appendix as Table V.

Consider now the case of tuning with the rest of the plant mistuned (Situation 3). For this example, let us tune Subsystem 1 while Subsystem 2 is tuned to some arbitrary state in order to observe what Subsystem 2 must be tuned to for it to be considered mistuned. The poles of "the rest of the plant" in this case are those of Subsystem 2, and may be calculated from the sensitivity model system matrix of Subsystem 2 (in this particular example). Now let us assume the following about all of the tune-ups associated with this particular situation:

1. The tuning reference input to Subsystem 1 is a unit step.

2. The reference response for Subsystem 1 is a unit step response of a second-order system with poles at $s = -3$.

3. The disturbance input to Subsystem 2 is 0.5 step delayed by 0.6 sec.

A summary table of initial and final values of k_{11} and k_{12}, corresponding values of k_{23} and k_{24}, and corresponding resultant undamped natural frequencies of Subsystem 2 is given in the Appendix as Table VI. Respective tuning histories are shown in the Appendix as Table VII. As can be seen in Table VI, undamped natural frequencies in Subsystem 2 of 2

or above result in k_{11} and k_{12} converging upon values in the neighborhood of those attained when tuning with a full model of the plant as the sensitivity model, while undamped natural frequencies of 1.414 and below result in final values which are much different.

CHAPTER 4

CONCLUSION

A general framework has been presented for the tuning of MIMO systems by use of the MIMO sensitivity points method. Examination and computer simulation of the MIMO sensitivity points method have revealed that, for a system composed of weakly coupled subsystems, decentralized tuning may be satisfactorily performed even if only simplified sensitivity models are available for use. In addition, it has been established that, if the full system model is available for use as a sensitivity model, exact tuning of a subsystem from the local level is possible, irrespective of coupling to other subsystems or inputs to other subsystems. One natural extension of these results which may be of interest for further study is that of the application of MIMO sensitivity points principles for truly adaptive (instead of periodic) on-line tuning.

APPENDIX

TUNING HISTORIES OF TUNING EXAMPLES

TABLE I.   CENTRALIZED TUNING EXAMPLE: TUNING HISTORY

FOR THIS RUN: SUBSYSTEM 1 INPUT = UNIT STEP;
                SUBSYSTEM 2 INPUT = 0.5 STEP.

CROSS-FEEDBACK GAINS: K21 AND K13

ITERATION I	K21	K13
0	-1.00000	-1.00000
1	-0.81544	-0.79887
2	-0.61201	-0.57941
3	-0.32785	-0.25119
4	-0.07316	+0.29346
5	-0.04473	+2.06399
6	-0.04061	+2.51908
7	-0.03874	+2.49874
8	-0.03907	+2.55895
9	-0.03870	+2.53242
10	-0.03889	+2.55015
11	-0.03877	+2.53984
12	-0.03884	+2.54607

TABLE II. DECENTRALIZED TUNING EXAMPLE: ALTERNATING TUNE-UP TUNING HISTORY

LOCAL FEEDBACK GAINS:
K11 AND K12 CORRESPOND TO SUBSYSTEM 1
K23 AND K24 CORRESPOND TO SUBSYSTEM 2

ITERATION I	K11	K12	K23	K24

FOR THIS RUN: REFERENCE INPUT = UNIT STEP;
DISTURBANCE INPUT = 0.5 STEP APPLIED TO
OTHER SUBSYSTEM INPUT.

I	K11	K12	K23	K24
0	+1.00	+1.00	+1.00	+1.00
1	+1.16	+0.69	+1.00	+1.00
2	+1.15	+0.70	+1.00	+1.00
3	+1.15	+0.70	+1.00	+1.00
4	+1.15	+0.70	+1.17	+0.68
5	+1.15	+0.70	+1.15	+0.69
6	+1.15	+0.70	+1.15	+0.69

FOR THIS RUN: REFERENCE INPUT = UNIT STEP;
DISTURBANCE INPUT = 0.5 STEP DELAYED
BY 0.6 SEC APPLIED TO INPUT OF
OTHER SUBSYSTEM INPUT.

I	K11	K12	K23	K24
0	+0.60	+1.40	+2.30	+0.35
1	+0.60	+1.40	+0.14	+1.00
2	+0.60	+1.40	+0.34	+1.33
3	+0.60	+1.40	+0.79	+1.23
4	+0.60	+1.40	+1.14	+0.85
5	+0.60	+1.40	+1.15	+0.70
6	+0.60	+1.40	+1.14	+0.70
7	+0.60	+1.40	+1.14	+0.70
8	+1.10	+0.91	+1.14	+0.70
9	+1.16	+0.67	+1.14	+0.70
10	+1.15	+0.69	+1.14	+0.70
11	+1.15	+0.69	+1.14	+0.70

FOR THIS RUN: REFERENCE INPUT = UNIT STEP DELAYED BY
0.6 SEC;
DISTURBANCE INPUT = 0.5 STEP APPLIED TO
OTHER SUBSYSTEM INPUT.

I	K11	K12	K23	K24
0	+0.60	+1.40	+2.30	+0.35
1	+1.17	+0.81	+2.30	+0.35
2	+1.17	+0.66	+2.30	+0.35
3	+1.16	+0.68	+2.30	+0.35
4	+1.16	+0.68	+2.30	+0.35
5	+1.16	+0.68	+0.17	+1.00
6	+1.16	+0.68	+0.42	+1.21
7	+1.15	+0.68	+0.90	+1.03
8	+1.16	+0.68	+1.16	+0.68
9	+1.16	+0.68	+1.15	+0.67
10	+1.16	+0.68	+1.15	+0.67

TABLE III. DECENTRALIZED TUNING EXAMPLE: JOINT TUNING
            TUNING HISTORY

LOCAL FEEDBACK GAINS:
K11 AND K12 CORRESPOND TO SUBSYSTEM 1
K23 AND K24 CORRESPOND TO SUBSYSTEM 2

ITERATION I	K11	K12	K23	K24

FOR THIS RUN: SUBSYSTEM 1 INPUT = UNIT STEP;
               SUBSYSTEM 2 INPUT = UNIT STEP DELAYED
                    BY 0.6 SEC.

I	K11	K12	K23	K24
0	+0.60	+1.30	+1.60	+0.35
1	+1.05	+0.92	+0.95	+0.75
2	+1.12	+0.69	+1.09	+0.68
3	+1.12	+0.70	+1.11	+0.66
4	+1.12	+0.70	+1.11	+0.66

FOR THIS RUN: SUBSYSTEM 1 INPUT = UNIT STEP DELAYED
                    BY 0.6 SEC;
               SUBSYSTEM 2 INPUT = 0.6 STEP.

I	K11	K12	K23	K24
0	+0.60	+1.30	+1.60	+0.35
1	+1.11	+0.84	+0.90	+0.79
2	+1.14	+0.65	+1.05	+0.74
3	+1.14	+0.67	+1.08	+0.71
4	+1.14	+0.67	+1.08	+0.71

TABLE IV.   TUNING EXAMPLE FOR DECENTRALIZED TUNING WITH
            SIMPLIFIED SENSITIVITY MODELS

FOR THIS RUN: REFERENCE INPUT   = UNIT STEP;
                DISTURBANCE INPUT = 0.5 STEP APPLIED TO
                    SUBSYSTEM 2 INPUT;
                REFERENCE RESPONSE: SECOND-ORDER STEP
                    RESPONSE (POLES AT S=-3).

            LOCAL FEEDBACK GAINS:
        K11 AND K12 CORRESPOND TO SUBSYSTEM 1
        K23 AND K24 CORRESPOND TO SUBSYSTEM 2

ITERATION
    I   K11   K12   K23   K24

    0   +0.50   +1.50   +1.16   +0.68
    1   +1.06   +0.99   +1.16   +0.68
    2   +1.17   +0.66   +1.16   +0.68
    3   +1.15   +0.70   +1.16   +0.68
    4   +1.15   +0.69   +1.16   +0.68

TABLE V. TUNING HISTORY: ALTERNATING TUNE-UP WITH SIMPLIFIED
SENSITIVITY MODEL

FOR THIS RUN: SUBSYSTEM 1 INPUT = UNIT STEP;
SUBSYSTEM 2 INPUT = UNIT STEP DELAYED
BY 0.6 SEC.
REFERENCE RESPONSES: SECOND-ORDER STEP
RESPONSES (POLES AT S=-3).

LOCAL FEEDBACK GAINS:
K11 AND K12 CORRESPOND TO SUBSYSTEM 1
K23 AND K24 CORRESPOND TO SUBSYSTEM 2

ITERATION I	K11	K12	K23	K24
0	+0.50	+1.50	+0.25	+0.10
1	+0.47	+1.18	+0.25	+0.10
2	+0.48	+1.23	+0.25	+0.10
3	+0.48	+1.23	+0.25	+0.10
4	+0.48	+1.23	+0.28	+0.22
5	+0.48	+1.23	+0.35	+0.42
6	+0.48	+1.23	+0.49	+0.66
7	+0.48	+1.23	+0.72	+0.83
8	+0.48	+1.23	+0.97	+0.79
9	+0.48	+1.23	+1.07	+0.70
10	+0.48	+1.23	+1.07	+0.70
11	+0.89	+1.06	+1.07	+0.70
12	+1.12	+0.73	+1.07	+0.70
13	+1.12	+0.70	+1.07	+0.70
14	+1.12	+0.70	+1.07	+0.70
15	+1.12	+0.70	+1.11	+0.66
16	+1.12	+0.70	+1.11	+0.66

TABLE VI. DECENTRALIZED TUNING EXAMPLE: SUMMARY TABLE OF
RESULTS FROM VARIOUS TUNINGS

| SUBSYSTEM 1 | | SUBSYSTEM 2 | | | |
| FEEDBACK GAINS | | FEEDBACK GAINS | | SUBSYSTEM POLES | UNDAMPED NATURAL FREQUENCY (RAD/S) |
INITIAL K11 K12	FINAL* K11 K12	K23	K24	S =	
0.50	0.85	0.25	0.10	-1.00	1.000
1.50	0.93			-1.00	
-	-	-	-	-	-
2.00	0.87	0.30	0.00	+j1.414	1.414
0.30	0.82			-j1.414	
-	-	-	-	-	-
2.00	1.09	0.50	0.00	+j2.449	2.449
0.30	0.68			-j2.449	
-	-	-	-	-	-
0.50	1.04	0.40	0.0	+j2.000	2.000
1.50	0.69			-j2.000	
-	-	-	-	-	-
0.50	1.05	0.40	0.141	-1.414 + j1.414	2.000
1.50	0.73			-1.414 - j1.414	
-	-	-	-	-	-
0.50	1.06	0.40	0.20	-2.000	2.000
1.50	0.74			-2.000	

* WITH A FULL PLANT AS SENSITIVITY MODEL,
    FINAL K11 = 1.16, FINAL K12 = 0.69.

TABLE VII. DECENTRALIZED TUNING EXAMPLE: TUNING HISTORIES FOR
            VARIOUS TUNINGS

FOR THESE RUNS: SUBSYSTEM 1 INPUT = UNIT STEP;
                 SUBSYSTEM 2 INPUT = 0.5 STEP DELAYED
                 BY 0.6 SEC.
                 REFERENCE RESPONSES: SECOND-ORDER STEP
                 RESPONSES (POLES AT S=-3).

              LOCAL FEEDBACK GAINS:
     K11 AND K12 CORRESPOND TO SUBSYSTEM 1
     K23 AND K24 CORRESPOND TO SUBSYSTEM 2

ITERATION I	K11	K12	K23	K24
0	+0.50	+1.50	+0.25	+0.10
1	+0.87	+1.00	+0.25	+0.10
2	+0.85	+0.93	+0.25	+0.10
3	+0.85	+0.93	+0.25	+0.10
0	+2.00	+0.30	+0.30	0.00
1	0.006	+0.83	+0.30	0.00
2	+0.05	+1.23	+0.30	0.00
3	+0.38	+1.31	+0.30	0.00
4	+0.83	+0.91	+0.30	0.00
5	+0.88	+0.79	+0.30	0.00
6	+0.87	+0.82	+0.30	0.00
7	+0.87	+0.82	+0.30	0.00
0	+2.00	+0.30	+0.50	0.00
1	+0.50	+0.79	+0.50	0.00
2	+0.77	+0.90	+0.50	0.00
3	+1.03	+0.76	+0.50	0.00
4	+1.09	+0.67	+0.50	0.00
5	+1.09	+0.68	+0.50	0.00
6	+1.09	+0.68	+0.50	0.00
0	+0.50	+1.50	+0.40	0.00
1	+1.09	+0.76	+0.40	0.00
2	+1.04	+0.68	+0.40	0.00
3	+1.04	+0.69	+0.40	0.00
4	+1.04	+0.69	+0.40	0.00
0	+0.50	+1.50	+0.40	+0.14
1	+1.04	+0.92	+0.40	+0.14
2	+1.06	+0.71	+0.40	+0.14
3	+1.05	+0.73	+0.40	+0.14
4	+1.05	+0.73	+0.40	+0.14
0	+0.50	+1.50	+0.40	+0.20
1	+1.03	+0.94	+0.40	+0.20
2	+1.07	+0.72	+0.40	+0.20
3	+1.06	+0.74	+0.40	+0.20
4	+1.06	+0.74	+0.40	+0.20

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