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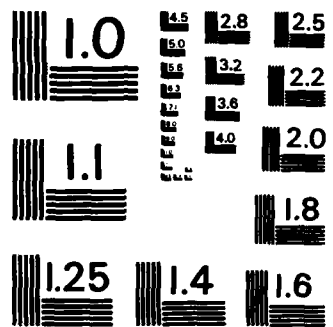
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STRESS-STRAIN BEHAVIOR AND FAILURE OF UNIAXIAL COMPOSITES IN

COMBINED COMPRESSION AND SHEAR - I

S.B. BATDORF AND ROBERT W.C. KO

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This paper considers the effects of increasing fiber tilt with increasing load in uniaxial composites, culminating in kink band formation. The fibers themselves are assumed to remain elastic. Nonlinearities result from changing fiber tilt and yielding of the matrix. Compressive stress-strain relations and failure are treated from a unified viewpoint. Comparisons are made between theory and experiment. Originator supplied keywords. include:		

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STRESS-STRAIN BEHAVIOR AND FAILURE OF UNIAXIAL COMPOSITES IN COMBINED COMPRESSION AND SHEAR - I

INTRODUCTION

Uniaxially reinforced composites are generally weaker in compression than in tension. Thus in stress states in which they are balanced against each other, such as bending and thermal stressing, failure usually occurs in compression. In addition, these composites are more nonlinear in compression. Thus compressive behavior is in a sense more important to understand, but at present the converse situation prevails.

It would be nice to have a single unified theory embracing both the stress-strain behavior and the final failure for all stress states. This is probably not achievable since more than one failure mechanism is involved, depending on the stress state. But it is reasonable to strive for such a unified theory for stress ranges within which a single failure mode governs.

The present paper is intended to be a step in that direction. Numerous studies have been made of failure in pure compression parallel to the fibers. It is generally agreed that the observed failure stresses can only be explained on the assumption that initial misalignments are present in the fibers. Such misalignments must of course affect not only the failure load, but also the stress-strain behavior; this latter effect, however, has usually not been included as part of the investigation. Off-axis compressive stress-strain behavior has also been studied, but generally not coupled to a failure theory. Instead the failure envelop is usually found by some unrelated approach, like an energy-based or even an empirical formulation of some type.

A number of factors combine to make the problem difficult. One is that the elastic constants of highly anisotropic materials can be very sensitive to the precise orientation of the material axes, especially when they nearly coincide with the principal stress axes. In such cases the standard assumption in linear elastic theory that the strain does not affect the stress distribution no longer applies. In addition the composite itself is anisotropically nonlinear, even in the absence of material axis rotations.

The purpose of the present investigation is to devise a unified theory for the stress-strain behavior and ultimate failure of a uniaxial composite in combined compression and shear. To keep things simple, we treat here mainly small deformations. We start with a brief review of previous work on compressive failure.

PREVIOUS WORK ON COMPRESSIVE FAILURE

The published work on compressive failure is too voluminous to review here, so we touch on only a few of the highlights. In a pioneering treatment of the problem, Rosen [1] analyzed failure by microbuckling. In the shear mode of failure, all fibers were considered to be deformed sinusoidally in phase with each other. In the extensional mode, fibers deformed sinusoidally but neighboring fibers were 180° out of phase. The shear mode was shown to lead to a lower buckling stress, and the theoretical compressive failure stress was shown to be equal to or greater than the shear modulus of the material. Experiments on conventional composites led to failure stresses an order of magnitude lower, and this was attributed to the influence of initial fiber deformations. Experimental confirmation of this interpretation was furnished by Greszczuk [2], who showed that a nearly perfect composite (carefully aligned round metal bars in a polymeric matrix) failed close to the theoretical buckling load.

Budiansky [3] developed a theory for failure of composites in the kink mode, a mode which will be discussed later. Assuming an elastic-perfectly plastic matrix he found for the compressive buckling stress

$$\sigma_f = \tau_y / (\phi + \gamma_y) \quad (1)$$

where τ_y is yield shear stress of the composite, γ_y is yield shear strain, and ϕ is the initial misalignment angle. He was able to account in considerable detail for the general configuration and orientation of the kink band that forms as a result of compressive collapse. Here we shall not concern ourselves with these post-failure phenomena, but concentrate on estimating the failure stress itself and on the events preceding failure (such as the nonlinear stress-strain behavior).

Hahn and Williams [4] developed a theory for sinusoidal buckling of a composite with a strain-hardening matrix, and compared the results with experimental data on a number of different matrix materials. Their theoretical failure stress was given by the expression

$$\sigma_f = \frac{V_F G_{LT} \gamma_{LT}}{\gamma_{LT} + \pi f_0 / l} \quad (2)$$

where f_0 and l are the initial amplitude and wavelength of the sinusoidal deformation, G_{LT} is the shear secant modulus of the composite, and γ_{LT} is the average shear strain at failure. V_F is the fiber volume ratio.

KINK MODE FAILURE OF A VOLUME ELEMENT

According to Hahn and Williams [4] failure of uniaxially reinforced composites under axial compression usually occurs in the kink mode. Fig. 1 taken from a study by Evans and Adler of kink failure in carbon-carbon [5] shows the post-buckling appearance of the kink band. Here we shall analyze the behavior of a single element of such a band.

Fig. 2 shows such an element. The solid lines show its initial configuration while the dashed lines represent its shape after loads are applied. In the loading frame of reference the forces F_V and F_H result in an applied compressive stress σ and an applied shear stress τ , respectively. We now consider what will be the corresponding stresses in the material frame of reference.

The general result for such an axis transformation is, in indicial notation

$$\sigma'_{ij} = l_{im} \sigma_{mn} l_{jn} \quad (3)$$

Here the l 's are direction cosines and repeated indices are summed. Assuming that the angle $\alpha_0 + \gamma$ between the two frames of reference is small, and neglecting terms of second order, we obtain for the compression and shear in the material frame of reference

$$\sigma' = \sigma - 2(\alpha_0 + \gamma)\tau \quad (4)$$

$$\tau' = \tau + \sigma(\alpha_0 + \gamma) \quad (5)$$

For the remainder of this paper, primes will be used to designate stresses or strains in the material frame of reference, and unprimes the same quantities in the load frame of reference. α_0 and γ are measures of the angle between the two frames of reference, and are therefore the same in both, just as the direction cosines are the same in both.

Solving (5) for σ we obtain

$$\sigma = \frac{\tau' - \tau}{\alpha_0 + \gamma} \quad (6)$$

The maximum stress the element can support occurs when

$$\frac{d\sigma}{d\gamma} = 0 \quad (7)$$

If the applied shear stress τ is held constant

$$\frac{d\sigma}{d\gamma} = -\frac{d\tau'}{d\gamma} / (\alpha_0 + \gamma) + (\tau' - \tau) / (\alpha_0 + \gamma)^2 \quad (8)$$

Combining (6), (7) and (8) we obtain for the failure stress

$$\sigma_f = \frac{\tau_f' - \tau}{\alpha_0 + \gamma_f} \quad (9)$$

Thus the compressive failure stress of the kink element is the tangent shear modulus of the material taken at the failure point. Previous investigators have recognized this relationship for the case of pure compression, but its extension to combined loads (i.e., $\tau \neq 0$) is believed to be new.

The preceding proof of the failure criterion assumed that the applied shear stress is held constant. A similar brief proof shows that it applies equally well for proportional loading. Actually the criterion is very general, but a general proof is more cumbersome. More insight into how to evaluate the kink buckling stress for various loading paths will result from a study of the graphical construction discussed in the next section.

To find the compressive strain in the applied stress frame of reference we assume this quantity is the sum of two components, one resulting from the elasticity of the fibers and the other caused by their changing tilt. Thus

$$\epsilon = \epsilon_1 + \epsilon_2 \quad (10)$$

where

$$\epsilon_1 = \sigma / E \quad (11)$$

and

$$\begin{aligned} \epsilon_2 &= \cos \alpha_0 - \cos (\alpha_0 + \gamma) \\ &\approx \alpha_0 \gamma + 0.5 \gamma^2 \end{aligned} \quad (12)$$

Before leaving this section, we note that in pure compression (9) becomes

$$\sigma_f = \tau_f' / (\alpha_0 + \gamma_f) \quad (13)$$

In the elastic-perfectly plastic case this reduces to Budiansky's solution, Eq.(1). In the strain-hardening case we note that the maximum initial tilt in sinusoidally deformed fibers is $\pi \delta_0 / \ell$. Thus our solution agrees with that of Hahn and Williams, Eq. (2), except for their factor V_F which seems to us to be unjustified.

GRAPHICAL REPRESENTATION

Finding the compressive and shear strains when the applied stresses and the initial fiber tilt are known is greatly facilitated by making use of a simple graphical construction. The general case is illustrated in Fig. 3. In this figure the ordinate is shear stress and the abscissa measures an angle. In the right hand quadrant we have shown the shear stress-strain curve as measured in the material principal axis system.

Consider the sloping line extending from the point $(-\alpha_0, \tau)$ to a point on the curve. The slope of this line is given by the expression

$$m = \frac{\tau' - \tau}{\alpha_0 + \gamma} \quad (14)$$

If we replace m by σ in (14) we find that (14) and (6) are equivalent. This means that our graphical construction gives the shear stress and strain in the material axis system immediately when we run a line of slope σ to the shear stress-strain curve from the point whose abscissa is the negative of the initial fiber tilt and whose ordinate is the applied shear stress.

As we increase the compressive stress σ the slope increases, but it can only increase to the locus shown dashed in the figure. The dashed line is the tangent to the curve, and is the point of neutral stability. Lines of smaller slope intersect the stress-strain curve at two different points. The first intersection represents a stable stress state, while the second represents an unstable one.

SPECIAL CASES

To see how this all works, let us apply it to a couple of special cases. The simplest is the case of pure compression of a single kink element. In pure compression the sloping line starts from the point $(-\alpha_0, 0)$. First we consider what happens when not only the fibers but also the matrix is elastic. Then the shear stress-strain curve is a straight line with slope G . From (6) it follows that

$$\sigma = G \gamma / (\alpha_0 + \gamma) \quad (15)$$

or

$$\gamma = \alpha_0 \sigma / (G - \sigma) \quad (16)$$

Thus γ and σ are nonlinearly related, even for a purely elastic material.

So are ϵ and σ . Using the relation

$$\epsilon = \sigma/E + \alpha_0 \gamma + 0.5 \gamma^2 \quad (17)$$

we obtain

$$\begin{aligned} \epsilon &= \sigma/E + \frac{\alpha_0^2 \sigma}{G - \sigma} + \frac{\alpha_0^2 \sigma^2}{2(G - \sigma)^2} \\ &= (1/E + \alpha_0^2/G) \sigma + \frac{3}{2} \frac{\alpha_0^2 \sigma^2}{G^2} + \text{higher order} \end{aligned} \quad (18)$$

We note that ϵ_1 is linearly related to σ , while ϵ_2 has both a linear and a nonlinear component. Since the material is elastic, the latter is a geometric nonlinearity, resulting from the changing stiffness caused by the changing fiber tilt.

Of course if the material is inelastic, as is usually the case, a material nonlinearity is present also, and the compressive stress-strain relation is even more nonlinear. If this makes an analytical approach difficult or impractical to carry out, we can simply find γ as a function of σ graphically and apply (17) to obtain the compressive stress-strain relation.

We note that most treatments of the stress-strain behavior in off-axis loading of uniaxial composites are unable to account for the nonlinearity at zero degree fiber inclination in terms of the constituent materials because they assume perfect geometry. Actually the composite should be regarded as composed of many elements with varying initial inclinations, some positive and some negative, with the zero degree case corresponding to a mean inclination equal to zero. Also we note that in the elastic case, within the limitations of small deflection theory, even in the presence of initial fiber tilt compressive instability doesn't occur till $\sigma = G_T = G$, where G_T is tangent shear modulus. This means that initial tilting and nonlinear matrix properties are both needed to account for compressive failure stresses far smaller than G , confirming the findings of previous investigators.

We next consider proportional loading. Here we assume that the applied shear stress is proportional to the applied compressive stress, i.e.,

$$\tau = k \sigma \quad (19)$$

By referring to Fig. 4, it is readily seen that in this case the sloping lines representing differing values of σ all pass through the point $(-\alpha_0, 0)$. Also where the abscissa is $-\alpha_0$ the ordinate of such a sloping line is the applied shear stress τ . Thus it is convenient to regard

the point $(-k - \alpha_0, 0)$ as a pivot through which all sloping lines pass, as in Fig. 4.

COMPARISON WITH EXPERIMENT

A limited comparison with experiment is attempted herein, with two main purposes in mind. One is to illustrate the theory just outlined. The second is to make a tentative judgment concerning the validity and utility of the theory. We will deduce the nonlinear compressive stress-strain behavior from the shear stress-strain relation for two different materials, an orthogonal weave carbon-carbon and a uniaxial graphite/epoxy.

The shear stress-strain relation in the carbon-carbon case was obtained from a cylinder torsion test and is shown in Fig. 5. The compressive stress-strain relation is shown in Fig. 6. Making use of the fact that the failure stress (about 20 ksi) is equal to the slope of the tangent line from the point $(-\alpha_0, 0)$ to the shear stress-strain curve, we deduce that $\alpha_0 = 0.033$. This value of initial fiber tilt leads to a somewhat larger nonlinear component in the compression curve than that observed in experiments. Best agreement between theory and experiment was obtained by assuming $\alpha_0 = 0.030$, about 10% less. This could be because all tilted elements contribute to the total strain, whereas failure may be precipitated by the most highly inclined elements in a domino-type effect. Or it may be simply that the shear stress-strain curve and the compressive stress-strain curve were obtained from different specimens, so that perfect correlation is not to be expected.

We now apply the same procedure to a graphite/epoxy, the properties of which have been obtained by Daniel et al [6]. The experimental shear stress-strain curve is shown in Fig. 7, together with the line whose slope is equal to the measured compressive failure stress. This construction leads to a value of initial tilt $\alpha_0 = 0.040$. Fig. 8 compares the calculated nonlinear component of compressive strain with the experimental one. It is evident that the calculated nonlinearity is much smaller than that observed. No plausible combination of α_0 and γ could be found that would result in a nonlinearity approaching that obtained experimentally.

In searching for an explanation for the surprising disparity between these two examples, we note two important differences. First, in the case of carbon-carbon, a tangent line of slope equal to the maximum compressive load actually can be constructed; in the case of the graphite/epoxy it cannot, because the shear stress-strain curve ends before its slope drops to σ_f . Second, in the graphite/epoxy one would expect the initial tilt in the constituent fibers to vary all the way from zero to some maximum value, with zero

being the most probable. In orthogonal weave carbon-carbon one would anticipate a comparatively small spread in initial tilt around some mean absolute value, with zero tilt very unlikely. For both of these reasons we would expect the theory to work better for the carbon-carbon.

It is not entirely clear what happens physically in the graphite/epoxy at the moment of compressive failure. The nonlinear component of strain is small compared to the elastic strain in both the composite and the neat resin [7]. Perhaps the most highly tilted fibers tear loose from the surrounding matrix and are thereby decoupled from the rest of the composite. This type of action might be expected to result in additional compressive strain of an inelastic nature. It must be admitted, however, that such an explanation is at present both qualitative and speculative.

DISCUSSION

Budiansky [3] has pointed out that although his equation for the maximum compressive load a composite can support (1) was initially developed for elastic-perfectly plastic materials, it is also a good approximation for certain classes of strain-hardening materials. By and large, these are materials with a very definite yield point and a shear stress-strain curve in which most of the change in slope occurs in a rather localized portion of the curve just above the yield. It is not suitable for materials in which the change in slope is spread out over most of the curve, like those discussed in this paper (see Figs. 5 and 7).

The theory presented herein treats nonlinear stress-strain behavior and failure from a unified viewpoint. On the basis of limited experimental evidence it would appear that the theory works well for materials capable of surviving large shear strains but poorly for materials subject to shear failure at low strains. It may be that microbuckling is not the principal cause of failure in the latter case.

For the treatment of pure compression a one-element model sufficed. This is because the shear strain induced by pure compression was simply ignored. Actually, of course, the shear strain in one element in a composite is balanced by the shear strain generated elsewhere by an element tilted in the opposite direction. Thus for the analysis of combined compression and shear a two-element model is needed. This more complex case is the subject of Part II of this paper.

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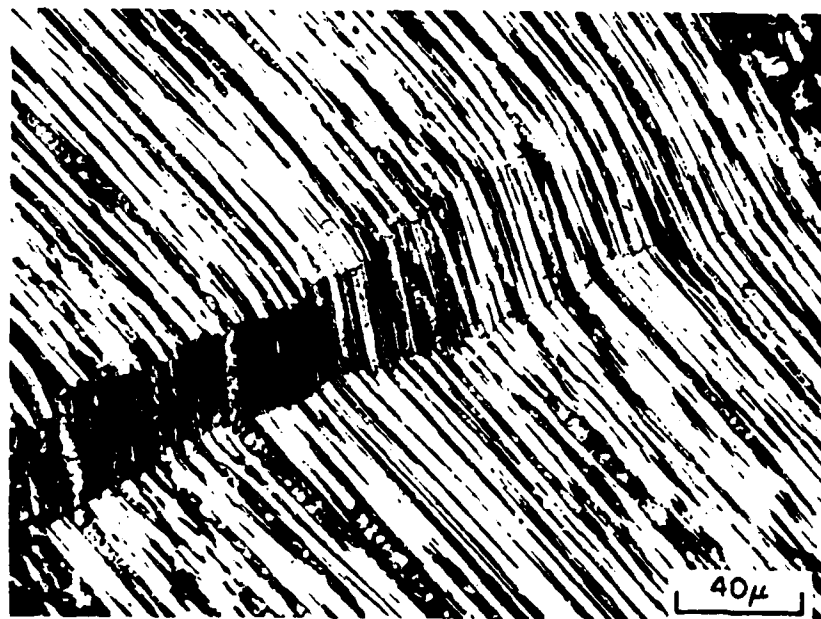


Fig. 1 Partially formed kink. Photograph taken by A.G. Evans and W.F. Adler and published in *Acta Met.* 26, 725-738 (1978).

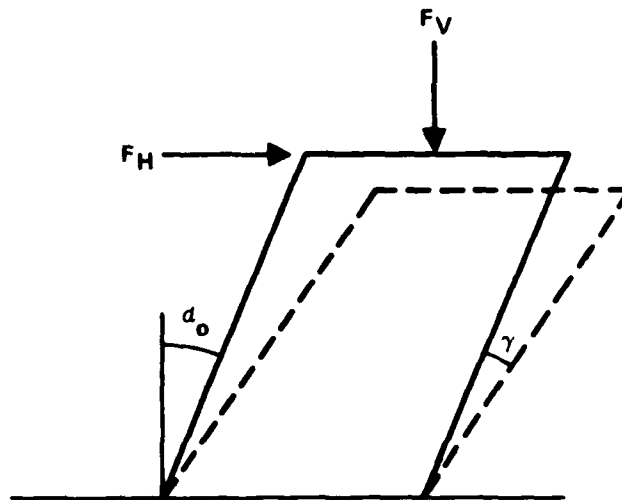


Fig. 2 Tilted kink element deforming under applied loads.

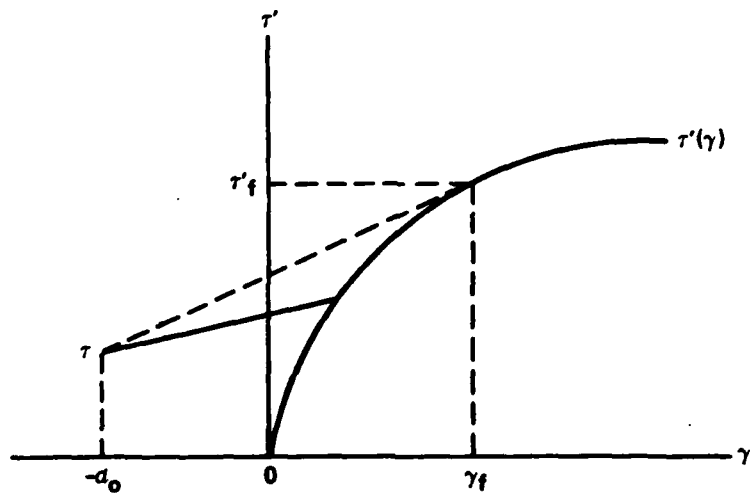


Fig. 3 Graphical method of finding shear stress and strain in principal axis system of composite (note $\gamma' = \gamma$).

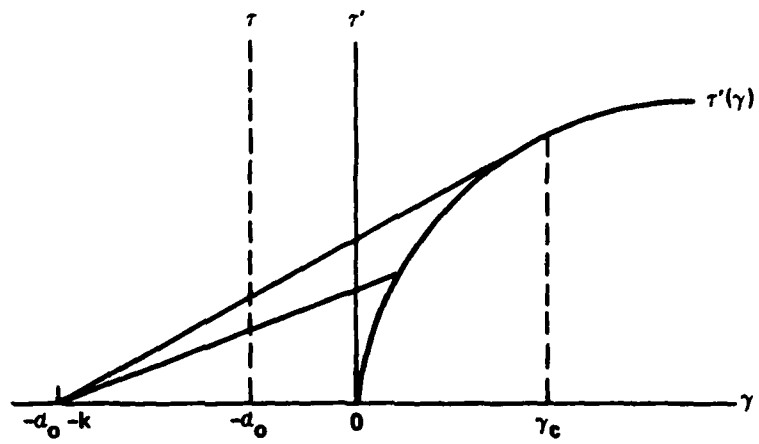


Fig. 4 Graphical method for proportional loading, $\tau = k\sigma$.

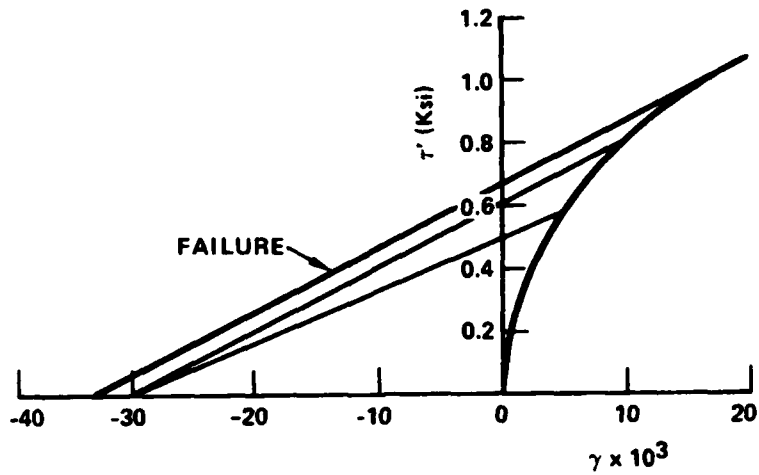


Fig. 5 Initial tilt deduced from carbon-carbon failure stress:
 $\alpha_0 = 0.033$. Initial tilt giving best agreement with
 compressive stress-strain curve: $\alpha_0 = 0.030$.

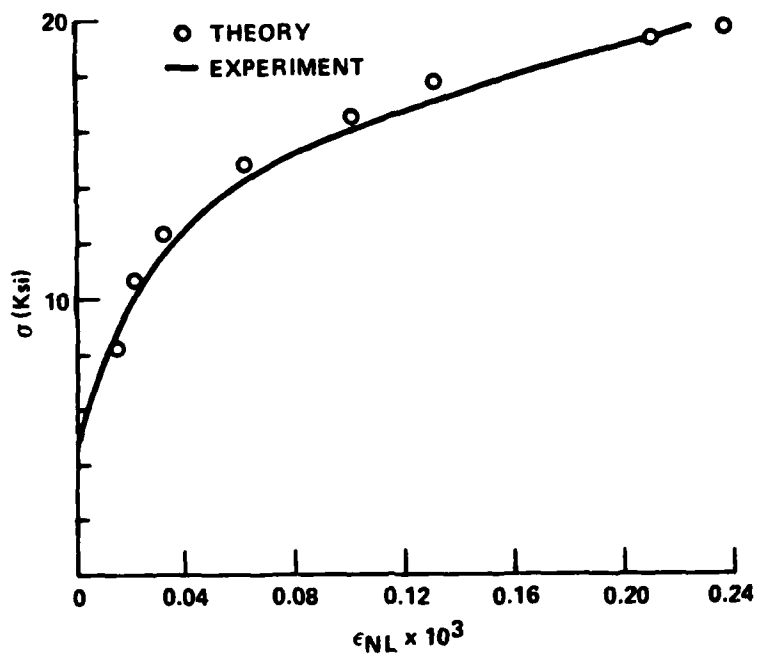


Fig. 6 Nonlinear component of compressive strain in carbon-carbon composites - theory vs. experiment.

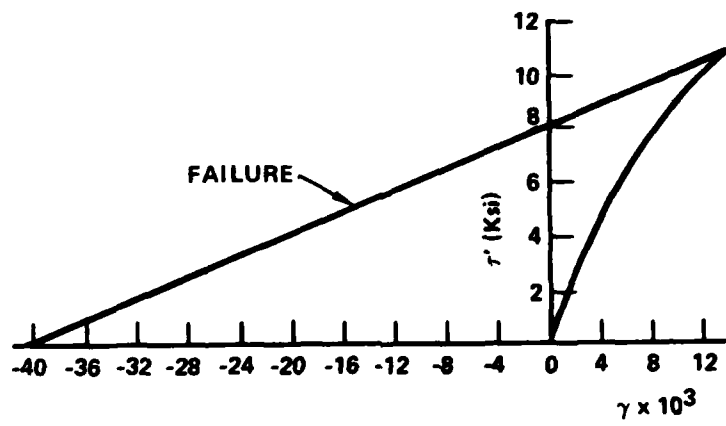


Fig. 7 Graphical analysis of graphite/epoxy failure.

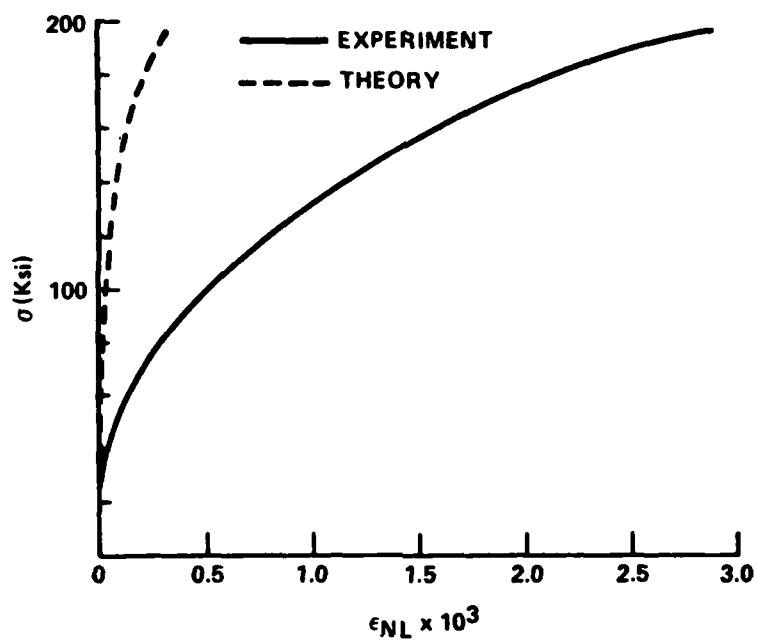


Fig. 8 Nonlinear component of compressive strain in graphite/epoxy - theory vs. experiment.

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