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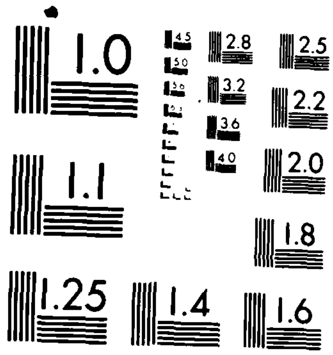


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FINAL REPORT

TO

OFFICE OF NAVAL RESEARCH

# THREE-DIMENSIONAL FLOW IN COMPRESSORS AND CHANNELS

Contract N00014-79-C-0285
NR 094-395

1 March 1979 to 30 September 1985

Principal Investigators

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# I. INTRODUCTION

This project was begun in 1979 as a study of three dimensional transonic flows through channels and between compressor blades. In the latter problem, the blades were to be lightly loaded. In 1981, a three year study with the broadened goal of studying flow problems in turbomachines was initiated. Specifically, the work was to include a continuation of that in progress on three-dimensional transonic flows through a lightly loaded compressor blade row, supersonic flow over a compression ramp with a turbulent boundary layer, and consideration of transonic flows over heavily loaded blades in a compressor, starting with a two-dimensional cascade and then going to a three-dimensional rotor if the calculations for the cascade were successful. The work in heavily loaded cascades was to build on the experience gained in the lightly loaded case. The work on the compression ramp has application at blade-shroud interfaces in transonic and supersonic flows. A brief discussion of each of those problems follows in the next section.

Asymptotic methods of analysis have been employed in all the problems to be described, with numerical methods of solution used as needed in some of the inner regions of the flow fields and to illustrate results for example problems.

Five graduate students, one post-doctoral fellow, one visiting professor, and one professor from the faculty of the Department of Aerospace Engineering have worked with the two principal investigators (see last section); two PhD theses, one presently unfinished but nearing completion, have resulted, as well as publications mentioned later.

# II. SUMMARY OF ACCOMPLISHMENTS

The work done under this contract falls naturally into two main categories, namely inviscid transonic flows through channels and compressors and supersonic flow over a compression ramp. They are considered separately in what follows.

INVISCID TRANSONIC FLOW THROUGH CHANNELS AND COMPRESSOR ROTORS

This work actually began with a preliminary study of a transonic shear flow in a three-dimensional rectangular channel. Two of the opposing walls were shaped to form a flow constriction similar to that found between two blades in a rotor with zero stagger. The remaining two walls were parallel. Thus, this was a model problem for flow through a three-dimensional rotor, the shear flow representing the gradient in the relative flow velocity resulting from the change in tangential velocity component between hub and tip, the two parallel walls representing the hub and tip shrouds, and the aforementioned constriction representing the blade surfaces. This work was published(1) in a volume containing the proceedings of an ONR workshop, edited by this author and Professor Platzer of the Naval Postgraduate School. The results found for the case considered indicated that the three dimensional transonic flow between thin blades in a compressor rotor could be described in a manner similar to that for two-dimensional channel flow; i.e. to lowest order, the perturbation to the incoming flow is one-dimensional with two or three-dimensional effects being found in second order solutions.

During the year preceding the contract work covered in this report, the problem of transonic flow through a three-dimensional compressor with lightly loaded blades was formulated in asymptotic terms during a sabbatical leave taken by one of the principal investigators (T.C.A.). The general formulation allowed completely subsonic, completely supersonic, or mixed flows (velocities supersonic at tip and subsonic at hub) to be considered. However, it quickly became clear that general analytical solutions would not be forthcoming and that much would be gained by consideration of the cascade (therefore two-dimensional) problem, both subsonic and supersonic, and extended analysis of the channel flow mentioned above. Both of these analyses were initiated a few months before the contract covered by this report began and were continued under this contract, along with the analysis of the three-dimensional rotor. The cascade work allowed study of the various regions which arise in the application of asymptotic techniques to the periodic flow patterns found when an infinite number of evenly spaced blades are considered, but in a simple two-dimensional geometry. As it turned out, solutions found for two of the regions were directly useful in the three-dimensional rotor problem, so more than education was gained from the study of the cascade problem. The analysis of shear flow in a channel allowed three-dimensional effects to be studied, again in a simple geometry, and showed how passage shock waves which don't fill the passage, because the flow is mixed, may be treated.

## Transonic cascade flow

The first problem studied, then, was flow through a cascade. Both subsonic and supersonic flows, each being in the transonic regime, were considered with the blade thickness to chord ratio, angle of attack, and blade camber all being small compared to one: thus, the results hold for lightly loaded blades. It was found that the flow could be subdivided into four general regions, as indicated in the sketch in Figure 1. Thus in Figure 1a, region A is the channel flow region, and region B is the near field region in which the governing equations are the same as those for region A; however since only one blade wall is present, the boundary condition represented by the other wall must be replaced by a matching condition. This matching condition is found from the solution to the flow field in the outer region, depicted in Figure 1b. In this outer region, the scale, in the direction perpendicular to the blades, is very large compared to the chord. To this scale, the distance between the blades (i.e. the blade spacing) is negligible

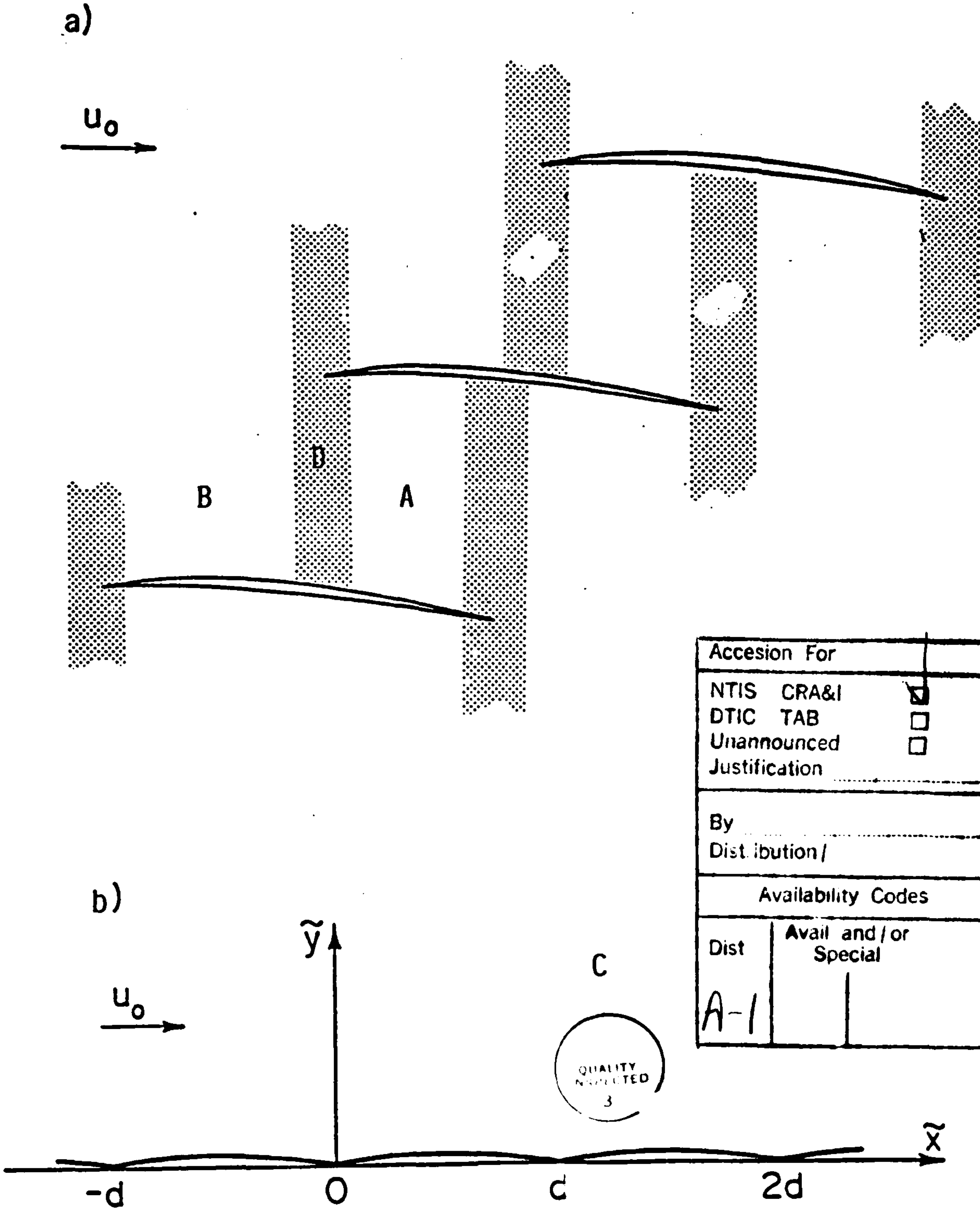




Figure 1. Asymptotic flow description.

a) Various regions between and near blades.
b) Approximate flow problem at large distances.

and the cascade appears to have collapsed upon itself such that only that part of the blade between the leading edge and the leading edge of the next blade is visible. Hence, the flow problem is that of a uniform flow at infinity (corresponding to the relative flow far ahead of the blades) passing over a scalloped wall, as shown in Figure 1b. Because there are singularities at the corners of the wall, corresponding to the leading edges of the blades, direct matching of the outer, near field, and channel solutions is not possible. Hence it is necessary to consider an inner region D (Figure 1a) which includes the leading edge. For the supersonic case, an additional far field region is required, to account for the curvature of the shock waves and Mach waves. It should be noted that there is an outer region downstream of the cascade also, and a corresponding inner region enclosing the trailing edge of each blade as in Figure 1a. Composite solutions, made up of the analytical solutions from each region, comprise the final results. A publication[2] covering this work, and a report[3] including details not contained in Reference 2, presented these solutions. Data from numerical examples were given in the form of blade pressure distributions and lines of constant Mach number. In addition, analytical results were given for the calculation of the exit angle of the flow downstream of the cascade and for the unique incidence angle of the incoming flow when it is supersonic.

Transonic shear flow in a channel

The second problem on which work was done during the first half of this contract period was the completion of the analysis of the model flow problem mentioned earlier, that of transonic shear flow through a three-dimensional rectangular channel. The various classes of flow which may be considered are characterized by the ratio R of the order of the difference in flow velocity across the channel due to the shear profile to the order of the change in flow velocity induced by the flow constriction modelling the blade shapes. When this ratio is very small, well known two-dimensional channel flow results hold. Analyses for the case when this ratio is of order one, the condition for which the greatest three-dimensional effects occur, and for the case when this ratio is large compared to unity are presented in a publication[4] and report[5]. Solutions are given in analytical form. As in the preliminary study mentioned earlier, it was found that three-dimensional effects occur first in second order, variations to the incoming shear flow being one-dimensional to first order. More importantly, an understanding of the manner in which back pressure acts upon a shock wave in a shear flow is found for $R = 1$. Thus, if the shear flow is such that supersonic flow does not extend completely across the channel, then if a shock wave forms, it too does not extend across the channel. When the back pressure is changed, the signals from downstream can now proceed upstream of the wave and thus change its shape as well as its location. The question is whether one or the other of these effects is more important. It was shown that to lowest order, changes in back pressure change the location of the shock wave in much the same way as in one-dimensional nozzle flow. The shape of the shock wave may indeed be changed, but these variations are of higher order. Finally, detailed conditions for choking of the flow were given.

In the case of a highly sheared incoming flow ($R >> 1$), where one would expect the effects of the constriction to give small changes to the incoming flow, it was found that the flow still could be choked if the average Mach number of the incoming flow were near unity. Solutions for a specific case were presented.

Each of the cases considered in this work[4,5] gives insight into similar flow fields found in three-dimensional transonic compressor rotors.

Transonic flow through a compressor rotor with lightly loaded blades

As alluded to earlier, analytical solutions for the flowfield about a three-dimensional compressor rotor are quite difficult, in general, even for lightly loaded blades. When the velocity at the blade tip is subsonic, so that all of the relative incoming flow is subsonic, the difficulty lies only in the complexity of the problem; there are many regions to consider, solutions for some of which must be found numercially, and a composite solution must be formed using solutions from each of these regions. When the tip velocity is supersonic, so that mixed flow results in the rotor, more fundamental problems arise, associated with the formation, reflection, and decay of shock waves from that part of the leading edge of the blade over which the relative velocity is supersonic. Moreover, there are problems in understanding the flow in the far field, as will be seen.

In general, the number of regions needed for the three-dimensional rotor is the same as those used for the cascade flow. Thus, at any radius, the region may be pictured as sketched in Figure 1. However, there are several significant differences, insofar as the solutions within the regions are concerned.

In the channel flow region, the solutions follow the pattern found in the work on three-dimensional channel flow. That is, the incoming flow relative to the blades is basically a shear flow, varying with the radius. The first order perturbation from this flow depends only upon the coordinate in the main flow direction; i.e., to lowest order this flow is one dimensional. Dependence upon the remaining two independent variables, i.e. a three-dimensional effect, is found in the second and higher order perturbations. Solutions for the rotor are considerably more complex than those for the model channel problem because of the geometrical asymmetry caused by the stagger of the blades, and by the fact that there are additional perturbation functions to be found; for example there is a "three halves order" term, also one dimensional, between the first and second order terms and corresponding "n + ½ order" terms throughout the expansion. Nevertheless, the fundamental solutions are similar to those found in the channel flow analysis.

Aside from the fact that in these regions a radial direction must be accounted for, there is a significant difference between the cascade and three-dimensional rotor solutions in the regions downstream of the cascade. Thus, in the downstream outer region, and in the inner regions which enclose the trailing edges, the vortex sheets which emanate from each trailing edge must be accounted for. Because the blades are lightly loaded, these vortex sheets remain in the plane in which they began. The velocity components parallel and perpendicular to these planes are continuous, but the velocity potential jumps across each sheet. At any radius, the jump across each sheet is the same because of the periodicity of the flow, but the jump in potential varies with radius and the radial velocity is discontinuous across each sheet. In the two dimensional cascade problem, discontinuities in the potential occur across lines extending downstream from the trailing edges of the blades. However, the velocity components, which are all that are desired, are continuous. In the three-dimensional case,, the vortex sheets form an important, additional complexity.

The most difficult aspect of the solution for the three-dimensional compressor rotor is the aforementioned complication introduced by the reflecting shock waves in the mixed flow case. First of all, for the parameter range considered, the flow in the outer region is two-dimensional to lowest order, for subsonic or supersonic tip velocities. The governing equation obtained for this lowest order solution is a typical Prandtl-Glauert equation, written in terms of the average (over the span of the blade) Mach number of the incoming flow. Thus, if $M_{av}^2 - 1 < 0$ the solution corresponds to that for subsonic flow over the scalloped wall shown in Figure 1b, found already for the cascade problem, but now written in terms of the rotor parameters. If $M_{av}^2 - 1 > 0$, the solution corresponds to that for supersonic flow over the scalloped wall, again known from the cascade problem solution. Now, in either of these cases, the incoming flow can be mixed, so that at each blade leading edge the flow is subsonic over part of the span and supersonic over the remainder. The result obtained for the outer region indicates that when $M_{av}^2 - 1 < 0$, the shock waves weaken sufficiently that in the average the flow behaves as a subsonic flow. When $M_{av}^2 - 1 > 0$, on the other hand, these shock waves from the supersonic leading edges persist and are seen as two dimensional waves, evidently filling the radial distance between hub and tip, in the outer region. Since the waves originate at the leading edge, the description of their propagation, including reflections from the tip shroud and the sonic surface, is part of the analysis of the inner region about the blade leading edge. It should be noted that because the blades are thin and the flow is transonic, the waves are weak; in fact, the local wave angles are Mach angles to lowest order.

In an effort to understand these results, studies of both the outer and inner regions, for the case where shock waves occur, were begun. Considerable effort was expanded upon the problem by both principal investigators, Dr. R. L. Enlow, visiting from the University of Otago, New Zealand, and Professor Alfred Kluwick, on sabbatical from the Technical University of Vienna, Austria. The work done by Dr. Enlow[6] was presented at the Second Australian Mathematics Convention.

Because the flow relative to the blades has a gradient, the angle of the shock wave relative to the chord line increases as radius decreases; this is accentuated by the blade twist. This would indicate that the lire marking the intersection of the shock wave and the tip shroud (the line along which the shock wave reflects) diverges from the line showing the direction of the shock wave where it leaves the leading edge (with zero strength) at the sonic point. If this occurred, the leading shock wave would become more tilted as it propagated farther from the blade. However, analysis of the leading shock wave, involving the calcuation of bi-characteristics, shows that the line marking the intersection of the shock wave with the tip shroud is not straight, but curves to bring it in the same direction as that taken by the shock wave at the sonic point. This change in geometry tends to support the idea that the shock wave is becoming two-dimensional as it propagates away from the blade. However, other constraints then come into play, indicating that the first wave and each of its reflections are finite in length but that each propagates a bit farther, leading to the final multiple wave form which apparently affects the flow as a two-dimensional shock wave would. However it has not been possible either to fill in the details of the manner in which the wave progresses or to ascertain the overall strength of the multiple reflected waves. Some indications of the way in which the reflected waves form have been given by solutions valid near a blade; however it has not been possible to go farther than this analytically, in spite of the time and effort spent on the analysis. Recently, it became apparent that it might be possible to obtain some of this information using numerical techniques in the inner region. Hence, the analysis of the problem when the tip velocity is supersonic is being pursued by the graduate student (Mr. Hemant Kamath) who aided in the solutions for the subsonic tip case as part of his training. The programming for the numerical work in the inner region has been completed and solutions with subsonic tip velocities are being run to check the programs. In addition, the program used to form the overall composite solution is being revamped and improved, and example runs are being made. Next, examples with supersonic tip velocities will be considered in the inner region at the leading edge. When this program is running properly, the number of mesh points and the extent of the calculation in a direction away from the blades will each be increased systematically in an attempt to follow the propagation of the leading edge and reflected waves. These is a distinct possibility that this can be achieved in this formulation because only the inner region is being considered; the solutions to which the solution in this inner region must match are all known, and so all mesh points available, i.e. the complete capability of the computer, can be focussed on the inner region. Results should be helpful not only in understanding the shapes of the shock waves, but in understanding noise propagation.

A paper(7) is being written for the case where tip velocities are subsonic. A PhD thesis(8) will be written on the problem associated with the mixed flow case, i.e. with supersonic tip velocities.

Transonic cascade flow
-heavily loaded blades

This study was begun in an effort to ascertain whether the methods used in analysing flow over lightly loaded blades could be extended to cover flow over heavily loaded blades. In this problem, the blades may be thin, but their thickness, curvature of the camber line, and angle of attack are all large enough that the transonic similarity parameter is of order unity, whereas it was large for the lightly loaded blades.

The first results found were very interesting in that it appeared that two solutions were possible for the supercritical flow region which generally occurs in the near field region, (Region B in Figure 1a). That is, in the near field region, subsonic incoming flow can accelerate through sonic velocity and form a closed area of supersonic flow surrounded by subsonic flow. In the asymptotic formulation used, this involved consideration of another inner region containing the "bubble" of supercritical flow. In one of the solutions found for this supercritical inner region, an analytical solution, no shock wave could occur. In the other solution, which has to be found by integrating the nonlinear small disturbance equation numerically, a shock wave was possible. For each solution, there was a different length scale in the flow direction. Hence, it appeared that if it were possible to find the conditions under which one solution appeared and not the other, it might be possible to predict conditions, and thus design for them, under which shock-free flows are possible. Presently such solutions are found using algorithms which essentially are based on trial and error, and don't give fundamental reasons for the occurrence of one solution or another.

For the heavily loaded blade case, it was found also, that many more terms were necessary in the asymptotic expansions to achieve the same accuracy obtained in the lightly loaded case. This simply served to complicate the computations needed to test ideas in the analysis of the supercritical region. Hence, it was decided to consider the simpler case of a single airfoil, with the idea that methods developed for it could be extended to the cascade.

The two possibilities found for solutions valid in the supercritical region, found previously for the cascade, were reproduced for the single airfoil. It was shown that the existence of the shock-free solution depends upon the existence of a certain positive constant, which in turn must be found by matching with the outer solution. Now, the outer solution, in this case, is that valid for a uniform flow at critical Mach number over an airfoil rather than over a scalloped wall as was the case for the cascade. Nevertheless, the governing equation for the lowest order term is nonlinear for heavily loaded airfoils and the general solution is not known. Hence, the solution for the next higher order term, which is dependent upon the lowest order term and which is the solution necessary to find the desired constant, cannot be found in general either. However, it appears that the solution is an eigenfunction, the magnitude of which is given either by the magnitude of the difference between the actual and critical flow Mach numbers or by the difference between the actual and critical flow angles of attack. That is, for each angle of attack there is a critical Mach number; small variations from these critical conditions can be made either by holding the Mach number constant and varying the angle of attack or vice versa. Moreover, based on the general results obtained so far it appears that the manner in which the critical region forms is as follows, if one holds flight Mach nubmer fixed at its critical value and increases the angle of attack from the critical value:

a) a shock free critical region forms

b) the critical region decreases in size and a shock wave may form.

Of course, these results are preliminary; e.g. it may well be that there are conditions under which the range of angles of attack for (a) to take place are so small as to make a shock free region essentially impossible to form. Much more remains to be done. The motivation for doing more is the possibility of finding conditions for shock free supercritical flow, of course.

Although nothing more appears possible insofar as analytical work is concerned, it may be possible to use a combined asymptotic and numerical method. This would involve finding outer solutions numerically, at critical conditions. This may involve more than at first appears necessary from the viewpoint that it may be important to keep the spacing of the mesh points in the proper asymptotic ratios. In addition, one and perhaps two higher order solutions would also be necessary. In spite of the problems associated with the intial formulation of the problem, it appears that much can be learned about shock free airfoils and cascades by carrying out this combined analytical and numerical approach. Unfortunately, because of the work which had to be done to formulate the problem properly and thus to gain an udnerstanding of what was necessary, the end of the contract was reached before any of the extensive numerical computations needed could be done. It is hoped that this work will be completed in the future.

Finally, it may be noted that the work covered in this section was described in an annual report[9]. There, it was said that the heavily loaded cascade work was the subject of a PhD thesis, and the lightly loaded cascade work, computations for which were being used as a training guide for the student, was not a thesis project. In the interim, two points became clear. First, the work on the lightly loaded cascade proved more difficult then suspected and it appeared interesting results could be obtained numerically for the supersonic tip case, and second, the analysis of the heavily loaded cascade proved to be very difficult after the first easily obtained solutions were found. Hence, the heavily loaded cascade was dropped as a PhD thesis topic, and instead worked upon by the principal investigator. The student continued on the lightly loaded blade problem and plans to finish this work within the present academic year.

## SUPERSONIC TURBULENT BOUNDARY LAYER AT A COMPRESSION RAMP

When a turbulent boundary layer at supersonic speed encounters a shallow compression corner, details of the local mean flow are determined by an interaction between the boundary layer and an oblique shock wave. For an unseparated flow, the shock wave forms at a distance from the corner which is quite small in comparison with the boundary-layer thickness, and the initial rise in pressure is very steep. The subsequent more gradual pressure increase continues for a distance of perhaps a few boundary-layer thicknesses, depending on the local Mach number.

In these regions near the corner, the mean fluid acceleration has much larger magnitude than in the undisturbed boundary layer, and the pressure gradient is much larger than the perturbation in the force due to Reynolds shear stresses. Changes in the mean flow properties may then be described approximately by inviscid-flow equations, except at points in a thinner sublayer, very close to the surface. This formulation has been used previously in studies of the closely related flow problem of interaction at transonic speeds between an unseparated turbulent boundary layer and a normal shock wave. Other turbulent boundary-layer interactions which have been studied in this way include the subsonic flow at a trailing edge and the incompressible flow over a shallow bump.

The present investigation was motivated largely by the work of Roshko and Thomke,[10] which included measurements of surface pressure for a wide range of Mach numbers and corner angles. They also demonstrated that numerical calculations by the method of characteristics agree very closely with experimental data for most of the gradual part of the pressure rise. This agreement provides strong support for the use of an inviscid-flow approximation. However, their calculation introduced a supersonic slip velocity at the wall, and an estimate of the "slip Mach number" was required[10,11].

The purposes of the present work have been to obtain analytical solutions for the portion of the pressure rise calculated numerically by Roshko and Thomke, to explore in a systematic way the implications of an asymptotic inviscid-flow description at smaller distances from the corner, and to attempt a prediction of the surface shear-stress distribution, all for unseparated flow.

The mean velocity profile in the undisturbed boundary layer is described by the velocity-defect law and the law of the wall, suitably modified for compressible flow. Solutions have been obtained using the method of matched asymptotic expansions. Outer solutions, for a transverse length scale equal to the boundary-layer thickness, have been derived in supersonic, hypersonic, and transonic small-disturbance limits. A number of different intermediate solutions have been found for smaller distances form the corner. Finally, the solution in an appropriate sublayer limit allows calculation of the wall shear-stress distribution downstream of the corner.

Neither the supersonic nor the hypersonic solution alone gives good agreement with the data of Roshko and Thomke. A composite supersonic-hypersonic solution leads to improved but still not satisfactory agreement, with an error which grows as the distance from the corner decreases. The inaccuracy arises because these solutions use the external-flow velocity as a first approximation to the mean velocity within the boundary layer, and is related to the logarithmic behavior of the undisturbed velocity profile. By a careful study of intermediate limits of the equations, still better agreement with experiment can be achieved. The supersonic-hypersonic solution is thereby modified, with the help of "supersonic intermediate solutions," so as to remain uniformly valid over an extended range which includes points much closer to the corner.

Calculation of the pressure according to this result does not require the introduction of a "slip Mach number." Rather, the solution simply requires substitution of values for x and for the parameters. This modified solution for the surface pressure gives excellent agreement with experiment for a Mach number $M_\infty \approx 5$ and a corner angle $\varepsilon \approx 15^\circ$. The Reynolds number based on boundary-layer thickness in this case is about $5 \times 10^6$. Similar agreement is found for $M_\infty \approx 4$ and $\varepsilon \approx 15^\circ$. Agreement with a numerical solution obtained by Roshko and Thomke using the method of characteristics is equally good. In a comparison at $M_\infty \approx 3$ and $\varepsilon \approx 10^\circ$, however, the theoretical prediction lies about 8% below the experimental values over a similar range of distance.

At a still lower Mach number, $M_\infty \approx 2$, for $\varepsilon \approx 5^\circ$, the data show a completely different trend. A rather high maximum pressure is reached slightly downstream of the corner, and the pressure then decreases toward the final value. For parts of the boundary layer where the Mach number M has low supersonic values, the incoming waves, due to reflection of the shock wave, are expansions rather than compressions; in the linear approximation the change occurs when $M = \sqrt{2}$. Roshko and Thomke suggested that the reversal of the pressure gradient at the surface may be associated with the sign change of the reflected waves. However, the present solutions show that this effect occurs over a far smaller length scale than is shown by the experiments. This conclusion is based on detailed derivations of "transonic intermediate solutions" corresponding to points in the boundary layer where M is close to 1. An inviscid-flow description therefore does not seem capable of reproducing the measured pressure distribution for this case. It does seem possible, however, that the presence of a shallow separation bubble, perhaps of length comparable with the boundary-layer thickness, might lead to a pressure distribution of the form observed.

The largest terms in the "outer" solutions given earlier are derived from inviscid-flow equations, and therefore can not be expected to contain enough information for calculation of changes in the wall shear stress. Instead, the flow details must also be studied in a sublayer where the changes in turbulent stresses are important. This sublayer plays the role of a new, thinner boundary layer, in an inviscid rotational external flow described by the outer solutions. From a different view, the Reynolds stress in the very thin wall layer is nearly in equilibrium with the local value of the wall shear stress, and can not be expected to match with the Reynolds stress in the outer part of the boundary layer, which depends primarily on upstream history. Instead, the perturbations in the wall-layer solution and in the outer solution are to be matched with the perturbations in the sublayer. This sublayer has been called a "Reynolds-stress sublayer" or a "blending layer."

An approximate solution has been obtained using a Prandtl mixing-length representation. The result for one case has been compared with experiment(12). Except for points very close to the corner, the form of the shear stress is predicted accurately, but the theoretical values are everywhere roughly 10% too high.

A PhD dissertation covering this work has been completed by S. Agrawal(13) and has been described in a journal article(14).

## References


1. Adamson, T. C., Jr., "Three-Dimensional Transonic Shear Flow in a Channel," Transonic Flow Problems in Turbomachinery, Edited by T. C. Adamson, Jr. and M. F. Platzer, Hemisphere Publishing Corp., 1977, pp. 70-77.

2. Messiter, A. F., and Adamson, T. C. Jr., "Transonic Small Disturbance Theory for Lightly Loaded Cascades," AIAA Journal, 19, 1981, pp. 1047-1054.

3. Messiter, A. F., and Adamson, T. C., Jr., "Transonic Small Disturbance Theory for Lightly Loaded Cascades," Technical Report N00014-79-C-0285-T2, February 1981.

4. Adamson, T. C., Jr., and Sichel, M., "Transonic Shear Flow in a Three-Dimensional Channel," Journal of Fluid Mechanics, 123, 1982, pp. 443-457.

5. Adamson, T. C., Jr., and Sichel, M., "Transonic Shear Flow in a Three-Dimensional Channel," Technical Report N00014-79-C-0285-T1, January 1981.

6. Enlow, R.L., "Wavefront Propagation in Non-Uniform Transonic Flow," presented at Second Australian Mathematics Convention, University of Sydney, Australia, May 1981.

7. Adamson, T. C., Jr., Kamath, H., and Messiter, A.F., "Transonic Flow Through a Lightly Loaded Compressor Rotor, I; Subsonic Tip Velocities," in preparation.

8. Kamath, H., "Transonic Flow Through a Lightly Loaded Compressor Rotor with Supersonic Tip Velocities," PhD Thesis in progress.

9. Adamson, T. C., Jr., and Messiter, A. F., "Flow Problems in Turbomachines," Annual Report No. 1, Contract N00014-79-C-0285; Nr094-395, 15, April 1983.

10. Roshko, A. and Thomke, G.J., "Supersonic Turbulent Boundary-Layer Interaction with a Compression Corner at Very High Reynolds Number," Proc. ARL Symp. on Viscous Interaction Phenomena in Supersonic and Hypersonic Flow, Univ. of Dayton Press, Dayton, Ohio, 1969, pp. 109-138.

11. Rosen, R., Roshko, A. and Pavish, D. L., "A Two-Layer Calculation for the Initial Interaction Region of an Unseparated Supersonic Turbulent Boundary Layer with a Ramp," AIAA Paper 80-0135, AIAA 18th Aerospace Sciences Meeting, Pasadena, CA, 1980.

12. Settles, G. S., Fitzpatrick, T. J., and Bogdonoff, S. M., "Detailed Study of Attached and Separated Compression Corner Flow-Fields in High Reynolds Number Supersonic Flow," AIAA Journal, 17, 1979, pp. 579-585.

13. Agrawal, S., "Asymptotic Theory of An Unseparated Supersonic Turbulent Boundary Layer at a Compression Corner," PhD Thesis, University of Michigan, Ann Arbor, MI, 1983.

14. Agrawal, S., and Messiter, A. F., "Turbulent Boundary-Layer Interaction with a Shock Wave at a Compression Corner," Journal of Fluid Mechanics, 143, 1984, pp. 23-46.

## III. INDEX OF TECHNICAL REPORTS

1. Adamson, T. C., Jr., and Sichel, M., "Transonic Shear Flow in a Three-Dimensional Channel," Technical Report N00014-79-C-0285-T1, January 1981.

2. Messiter, A.F., and Adamson, T. C., Jr., "Transonic Small-Disturbance Theory for Lightly Loaded Cascades," Technical Report N00014-79-C-0285-T2, February 1981.

## IV. INDEX OF PUBLICATIONS

1. Messiter, A. F., and Adamson, T. C., Jr., "Transonic Small-Disturbance Theory for Lightly Loaded Cascades," AIAA Journal, 19, 1981, pp. 1047-1054.

2. Adamson, T. C., Jr., and Sichel, M., "Transonic Shear Flow in a Three-Dimensional Channel," Journal of Fluid Mechanics, 123, 1982, pp. 443-457.

3. Enlow, R. L., "Wavefront Propagation in Non-Uniform Transonic Flow," presented at Second Australian Mathematics Convention, University of Sydney, Australia, May 1981.

4. Agrawal, S., "Asymptotic Theory of an Unseparted Supersonic Turbulent Boundary Layer at a Compression Corner," PhD Thesis, University of Michigan, Ann Arbor, MI, 1983.

5. Agrawal, S., and Messiter, A. F., "Turbulent Boundary-Layer Interaction with a Shock Wave at a Compression Corner," Journal of Fluid Mechanics, 143, 1984, pp. 23-46.

6. Adamson, T. C., Jr., Kamath, H., and Messiter, A. F., "Transonic Flow Through a Lightly Loaded Compressor Rotor, I; Subsonic Tip Velocities," in preparation.

7. Kamath, H., "Transonic Flow Through a Lightly Loaded Compressor Rotor with Supersonic Tip Velocities," PhD Thesis in progress.

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