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INPUT-OUTPUT CORRELATIONS AND MODEL PARAMETERS(U)
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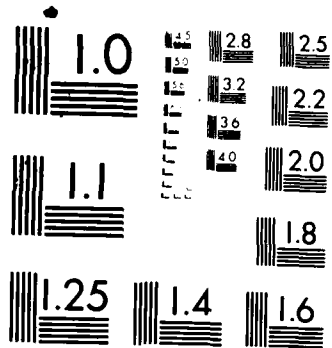
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FREQUENCY DOMAIN EXPERIMENTS: SPECTRAL AMPLIFICATION, INPUT-OUTPUT CORRELATIONS, AND MODEL PARAMETERS

by

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Abstract

A frequency domain approach to identifying a polynomial response model for a system is presented in Schruben and Cogliano, 1985. Frequency domain experiments can help identify important input factors (or parameters) and interactions in systems where one can change factor values during an experimental trial and observe the response at periodic intervals. An example of such a system is a simulation run on a computer. In this paper we consider the relationships between spectral amplification, input-output correlation, and factor values for such experiments. Conditions are also given for which the frequency domain approach is equivalent to ranking prospective terms in a response model by their correlation with the output.

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1. The Frequency Domain Approach

A goal of many experiments is to identify a model describing how a response depends on some factors. The conventional approach requires many experimental trials, with factor values prescribed by an experimental design and remaining fixed during each trial. Classical experimental designs, where each design point requires a separate trial, are quite expensive when there are several factors.

A frequency domain experimental approach is possible when one has control over the factors. Here the factors oscillate during a trial. In some classes of systems each important factor's oscillation induces response oscillations at predictable frequencies. Spectral analysis can be used to detect response oscillations, determine which factors or interactions are responsible, and thus identify a response model.

The goal of the frequency domain approach is to identify the important terms in a polynomial model of the response,

$$E(y) = \beta_0 + \sum_{j=1}^q \beta_j t_j \tag{1}$$

where y is the response
 t_1, \dots, t_q are possible terms in the polynomial model; they are products of non-negative integral powers of continuous factors x_1, \dots, x_p
 β_0, \dots, β_q are real-valued coefficients.

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An alternative, more modest goal could be to determine whether a linear model



is adequate or whether higher powers and interactions are present. This kind of information can help to focus attention on important factors, to screen unimportant factors from further consideration, and to design future studies more effectively.

The frequency domain approach can be used to identify a model for systems in which:

1. Parameter settings can be changed during an experimental trial.
2. The response can be observed at periodic intervals.
3. The response can be adequately modeled as a time-invariant linear combination of products of powers of the factors. That is: Given p factors observed at equally spaced intervals

$$x_{iT} \quad (i=1, \dots, p)$$

we can form q polynomial terms by taking products of powers of the factors

$$t_{jT} = \prod_{i=1}^p x_{iT}^{a_{ij}} \quad (j=1, \dots, q)$$

(a_{ij} some non-negative integer)

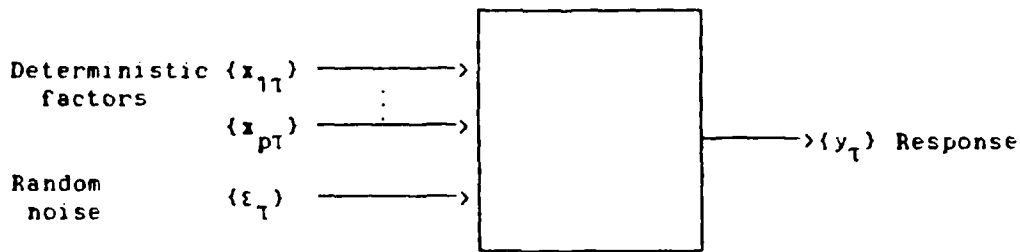
The response is a time-invariant linear combination (filter) of these terms and noise that is independent of the factors,

$$y_T = \sum_{j=1}^q \sum_{k=-\infty}^{\infty} h_{jk} t_{j(T-k)} + \varepsilon_T \quad (2)$$

Such a system can be thought of as a black box (Figure 1) with two types of input: deterministic factors, which may change during an experimental trial, and random noise. Whatever series of values are assigned to the factors, the rule for forming the response remains unchanged.

Figure 1

A System Suitable for the Frequency Domain Approach



A simulation computer program is an important special class of systems where the frequency domain approach is applicable. Simulation models are often computer programs, which can be written to change factor settings and periodically record the response. If the response can be modeled as a polynomial function of the factors (at least in the region of interest), then all three requirements are met and the frequency domain approach should identify a response model.

The next section of this paper gives the input-output spectrum relationship applicable to general polynomial response models. Section 3 gives the relationships between spectrum amplification, input-output correlations, and response model coefficients.

2. Inducing, Identifying, and Interpreting Oscillations

The frequency domain approach to model identification uses oscillation frequencies, not experimental trials, as the experimental unit. Since many oscillation frequencies are available in an experimental trial, many factors can be screened in one trial. This advantage can best be demonstrated in a linear system with added noise, in which the response is a linear function of one factor. This is a special case of the model of Equation 2.

Denote the linear system by

$$y_T = \sum_{h=-\alpha}^{\alpha} h_k x_{T-k} + \xi_T$$

where y_T , x_T , h_k , and ξ_T are as in Equation 2. The response spectrum $f_y(\omega)$ and the factor spectrum $f_x(\omega)$ are related by

$$f_y(\omega) = G^2(\omega) f_x(\omega) + f_{\xi}(\omega) \quad (3)$$

where $G(\omega)$, called the gain, describes how the linear system amplifies or attenuates oscillations at different frequencies. The gain is zero and there is no oscillation-induced peak if the response does not depend on the factor. Suppose the factor oscillates with amplitude α and frequency ω

$$x_T = \alpha \cos 2\pi\omega T.$$

Then the factor spectrum has a peak at ω . From Equation 3 the response spectrum also has a peak at ω , scaled by the gain at ω and masked by the noise. A response spectrum peak at ω is evidence that the factor affects the response; no peak is evidence that it does not.

Consider next a multiple-factor linear system

$$y_T = \sum_{j=1}^p \sum_{k=-\infty}^{\infty} h_{jk} x_j^{(1-k)} + \varepsilon_T$$

Here the response spectrum and the factor spectra, $f_{x_j}(\omega)$, are related by

$$f_y(\omega) = \sum_{j=1}^p G_j^2(\omega) f_{x_j}(\omega) + f_{\varepsilon}(\omega) \quad (4)$$

where each $G_j(\omega)$ is the gain for oscillations of x_j . Suppose each factor x_j oscillates at the frequency ω_j

$$x_{jT} = a_j \cos 2\pi\omega_j T. \quad (5)$$

Then the factor spectrum $f_{x_j}(\omega)$ has a peak at ω_j . If $G_j(\omega_j)$ is not zero, the response spectrum has a peak at ω_j . A response spectrum peak at ω_j is evidence that x_j affects the response; no peak is evidence that it does not.

As shown in the reference, the presence of a higher order term or factor interaction in a general polynomial response surface is equivalent to several additional linear terms. For example, a second order term (the square of a factor or a two factor interaction) in the response surface model has exactly the same effect as two additional linear terms. Each of these equivalent linear terms oscillate at one of the frequencies in the indicator frequency set that corresponds to the particular higher order term. These term indicator frequencies are easily determined and the experiment can be designed to make them unique. This means that the input-output spectrum relationship given by (4) is applicable to general polynomial response surfaces as well as the linear response case.

3. Spectrum Amplification, Input-Output Correlations, and Model Coefficients

The heights of spectrum peaks are related to the coefficients of the response model

$$E(y) = \beta_0 + \sum_{j=1}^q \beta_j t_j$$

To demonstrate this, start with the relationship between the response spectrum and the factor spectra in a multiple-factor linear system with added noise (Equation 4),

$$f_y(\omega) = \sum_{i=1}^p G_i^2(\omega) f_{x_i}(\omega) + f_z(\omega).$$

Since the gain $G_i(\omega)$ is scaled by the coefficient β_i , then

$$G_i(\omega) = \beta_i g_i(\omega)$$

for some function $g_i(\omega)$. Since x_i oscillates as a sinusoid with amplitude α_i and frequency ω_i , then

$$f_{x_i}(\omega) = \alpha_i^2 s(\omega_i)$$

where $s(\omega_i)$ is the spectrum of a sinusoid with frequency ω_i and amplitude 1; it has a sharp peak at ω_i . Substituting these into Equation 4,

$$f_y(\omega) = \sum_{i=1}^p \beta_i^2 g_i^2(\omega) \alpha_i^2 s(\omega_i) + f_z(\omega).$$

If we assume that the argument $2\pi\omega_i \tau$, modulo 2π , ($\tau = 1, \dots, n$) from

Equation 5 is sampled uniformly between 0 and 2π , then the variance of x_1 is

$$\sigma_{x_1}^2 = \sigma_1^2/2.$$

The experiment is designed so that input factors oscillate at different frequencies. If these frequencies are selected such that the correlation between factor values is zero, then the correlation between y_τ and $x_{1\tau}$ ($\tau = 1, \dots, n$) is

$$\rho_1 = \beta_1 \sigma_{x_1} / \sigma_y.$$

Substituting the above into the general response relationship, expression (4), gives

$$f_y(\omega) = 2\alpha_y^2 \sum_{i=1}^p \rho_i^2 g_i^2(\omega) s(\omega_i) + f_z(\omega).$$

This shows that the height of the peak at ω_1 is scaled by $2\alpha_y^2 \rho_1^2 g_1^2(\omega)$. The relative heights are proportional to $\rho_i^2 g_i^2(\omega)$. If the gain $g_i(\omega)$ is constant for every x_i and ω , then the heights of the peaks are proportional to ρ_i^2 . Therefore ranking the factors according to the heights of their peaks is equivalent to ranking the factors according to the magnitude of their correlations with the response. Techniques for dealing with non-uniform system gain are presented in the reference.

To compensate for the oscillation amplitudes, divide the heights of the peaks by the square of the amplitudes. These are proportional to

$$\frac{2\sigma_{y_1}^2 \beta_1^2(\omega)}{\sigma_1^2} = \frac{2\sigma_{y_2}^2 \beta_2^2(\omega)}{2\sigma_{x_1}^2} = \beta_1^2 \beta_2^2(\omega).$$

If the gain is constant, then these are proportional to β_1^2 . Therefore ranking the factors according to the heights of their peaks divided by the squares of their oscillation amplitudes is equivalent to ranking the factors according to the magnitude of their coefficients.

There are several technical problems with applying frequency domain experiments. These include system gain and random noise as well as the design problem of selecting input frequencies and amplitudes. Techniques for solving these problems and statistical methods for identifying significant response model terms appear in the reference. In that paper several examples are also presented.

reference:

Schruben, L. and V. J. Cogliano, "An Experimental Procedure for Simulation Response Surface Model Identification", Tech. Rpt. 669, SORIE, Cornell University, Ithaca, N.Y. 14853.

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