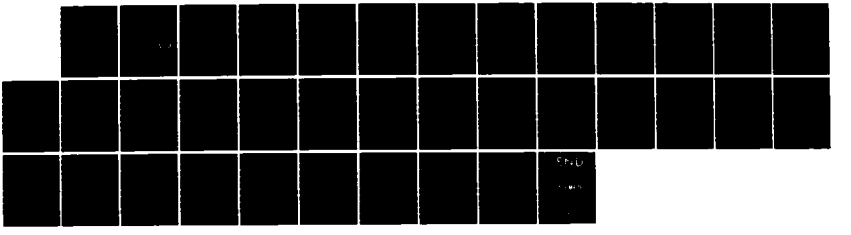
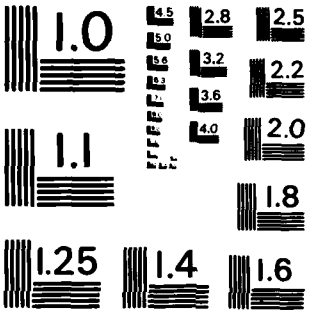
AD-A162 546 TIME AND FREQUENCY DEPENDENT BEHAVIOR OF A TWO DIMENSIONAL ELECTRON GAS A.. (U) GEORGIA UNIV ATHENS DEPT OF PHYSICS AND ASTRONOMY M H LEE ET AL. 05 NOV 85 1/1

UNCLASSIFIED TR-10 N00014-84-K-0365 F/G 9/3 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS - 1963 - A

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(6)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER: Technical Report No. 10	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): TIME AND FREQUENCY DEPENDENT BEHAVIOR OF A TWO DIMENSIONAL ELECTRON GAS AT LONG WAVELENGTHS		5. TYPE OF REPORT & PERIOD COVERED: Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): M. Howard Lee and J. Hong		8. CONTRACT OR GRANT NUMBER(s): N00014-84-K-0365
9. PERFORMING ORGANIZATION NAME AND ADDRESS: University of Georgia, Department of Physics, Athens, GA 30602		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR 051-861
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Department of the Navy, Arlington, VA 22217		12. REPORT DATE: November 5, 1985
		13. NUMBER OF PAGES: 33
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/ DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; its distribution is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DTIC ELECTE DEC 2 3 1985 S D B

18. SUPPLEMENTARY NOTES

To be published in Phys. Rev. B (15 Dec. 1985)

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Autocorrelation function, memory function, intrinsic conductivity, recurrence relations, metal-oxide-semiconductors.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Using the recently developed method of recurrence relations, we have studied the time and frequency dependent behavior of the two dimensional electronic system at long wavelengths. Several families of autocorrelation functions including the relaxation and memory functions are obtained exactly. In addition, other linear response quantities e.g. the density-density response function and dynamic structure factor are obtained. This work contains a detailed application of the method of recurrence relations.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AD-A162 546

DTIC FILE COPY

OFFICE OF NAVAL RESEARCH

Contract N00014-84-K-0365

Task No. NR 051-861

TECHNICAL REPORT No. 10

Time and Frequency Dependent Behavior of a Two Dimensional Electron Gas at Long Wavelengths

by

M. Howard Lee and J. Hong

Prepared for Publication

in

Physical Reviews Part B

University of Georgia
Department of Physics
Athens, Georgia

November 5, 1985

Reproduction in whole or in part is permitted for any purpose of the United States Government.

This document has been approved for public release and sale; its distribution is unlimited.

85 12 20 016

To appear in Phys. Rev. B (15 December 1985)

Time and Frequency Dependent Behavior of a Two Dimensional Electron Gas at Long Wavelengths

M. Howard Lee
Department of Physics, University of Georgia
Athens, GA 30602

J. Hong
Department of Physics Education
Seoul National University
Seoul 151, Korea

Abstract

Using the recently developed method of recurrence relations, we have studied the time and frequency dependent behavior of the two dimensional electronic system at long wavelengths. Several families of autocorrelation functions including the relaxation and memory functions are obtained exactly. In addition, other linear response quantities e.g. the density-density response function and dynamic structure factor are obtained. This work contains a detailed application of the method of recurrence relations.

PACS: 72.10 Bg, 72.20.-i, 73.40Qv

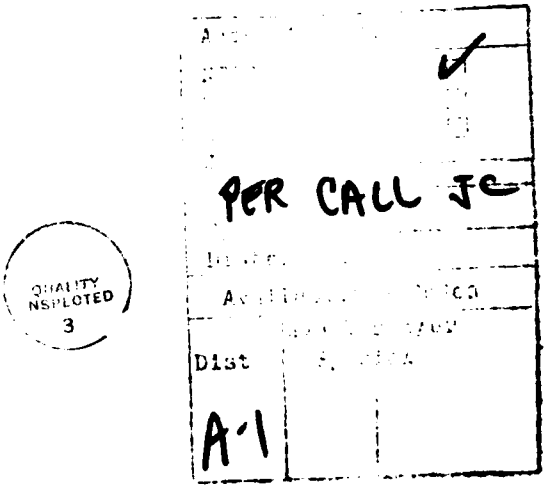
I. Introduction

The electron gas has had a long history as a model for metals and has attracted considerable theoretical efforts over the years.[1] Interest in this model has been substantially heightened recently by the physical realization of an essentially two dimensional electronic system in MOS, heterostructures and superlattices.[2,3] In addition, the realization that many-body effects are in some ways more pronounced in two dimensions than in three dimensions has further given impetus especially to theoretical efforts. The static properties of the two dimensional version of the electron gas model have already been extensively studied.[4] The correlation energy, for example, has been calculated by techniques which are basically similar to those employed for the three dimensional case.[5-10] The dynamic properties have also received some attention, but largely limited to the frequency-dependent density response function calculated by RPA and other perturbative techniques.[11-14] To our knowledge, there are as yet no measurements of the dynamic structure factor with which these calculations can be compared.

In the two dimensional electron gas model, one has an attractive possibility of finding exact time and frequency dependent solutions, normally precluded in the three dimensional model. Such solutions can contribute to our understanding of nonlinear behavior of fluids, e.g., memory effects and long time tails in the velocity autocorrelation function. These studies are attracting considerable current activities.[15]

The time evolution of a dynamical variable e.g., density-

fluctuation operator may be studied by the method of recurrence relations.[16,17] Given certain static properties known as the recurrants, one can obtain the exact time-dependent behavior of the density fluctuations, from which the associated relaxation and memory functions as well as the random force.[18] This method has already been successfully applied to several formal and physical models.[19] We present here a complete account of the time and frequency dependent behavior of the two dimensional electron gas at long wavelengths. A preliminary account of this work was previously reported.[20]



QUALITY INSPECTED 3

II. Time Evolution of Density Fluctuations in the Electron Gas at T=O

We consider the two dimensional electron gas model defined by the Hamiltonian H

$$H = \sum_k \varepsilon_k c_k^+ c_k + \tfrac{1}{2} \sum_{k,p,q} v_k c_{p+k}^+ c_{q-k}^+ c_p c_q \equiv H_o + V \qquad (1)$$

where c_k^+ & c_k are, respectively, the creation and annihilation operators at wave vector k, $\varepsilon_k = k^2/2m$, $v_k = 2\pi e^2/k$, where m and e are, respectively, the electron mass and charge, $\hbar = 1$. Our system is imposed under an external perturbing potential of the following form:

$$H_{ext} = \sum_k \rho_k(t)\ p_k\ e^{i\omega t} \qquad (2)$$

where p_k is the Fourier component of the external electric field sufficiently small as to permit the use of linear response theory, ω is the frequency of the field, ρ_k is the density fluctuation operator defined as

$$\rho_k = \sum_p c_p^+ c_{p-k} \qquad (3a)$$

and

$$\rho_k(t) = e^{iHt}\ \rho_k\ e^{-iHt} \qquad (3b)$$

with $\rho_k(O) = \rho_k$. Given an explicit knowledge of time dependence in $\rho_k(t)$, one can proceed to calculate e.g. relaxation function $(\rho_k(t), \rho_k)/(\rho_k,\rho_k)$, from which other physical quantities via linear-response relations. Here the inner product means the Kubo scalar product (see sec. III).

It is important to note that the calculations of time evolution are carried out with the static properties e.g. k fixed at some given value. In this work, we shall confine ourselves to $k/k_F << 1$, where k_F is the Fermi wave vector, such that the density fluctuations are those which occur very near the Fermi

surface. Physically it means that excitations in (1) are limited to electron-hole pairs only (i.e., no multipair excitations allowed). In this static regime ($k/k_F << 1$ and $T = 0$) the static susceptibility (ρ_k, ρ_k) and other static quantities of interest are known exactly.

III. Method of Recurrence Relations

If the electron gas is momentarily perturbed by an external field coupled to density fluctuations (2) in accordance with linear response theory, the system will undergo a relaxation process. The time evolution of density fluctuations satisfies the Heisenberg equation of motion viz.,

$$\dot{\rho}_k(t) = i\,[H,\ \rho_k(t)]. \tag{4}$$

One can put it in the form of the generalized Langevin equation[18]

$$d\rho_k(t)/dt + \int_0^t dt'\ M_k(t-t')\ \rho_k(t') = F_k(t) \tag{5}$$

where M and F are, respectively the memory function and generalized random force. This form has the advantage of introducing the two physical quantities. The formal solution for the time evolution of ρ_k is given by the method of recurrence relations[16].

According to this method, one constructs S a d-dimensional Hilbert space of ρ_k spanned by a set of orthogonal basis vectors $f_o\ f_1 \cdots f_{d-1}$. This physical space S is an abstract space realized by the Kubo scalar product

$$(X,Y) = \int_0^\beta du\ \langle e^{uH}\ X\ e^{-uH}\ Y^+\rangle - \langle X\rangle\langle Y^+\rangle \tag{6}$$

for every $X, Y \subset S$, where $\langle \ldots \rangle$ denotes the canonical ensemble average, + Hermitian conjugation and β is the inverse temperature. The method of recurrence relations shows that in such a space the basis vectors satisfy the following recurrence relations (RRI):

$$f_{\nu+1} = \dot{f}_\nu + \Delta_\nu\ f_{\nu-1}, \quad 0 \le \nu \le d-1 \tag{7}$$

where $\dot{f}_\nu = i\,[H,\ f_\nu]$ (8)

and

$$\Delta_\nu = (f_\nu, f_\nu)/(f_{\nu-1},\ f_{\nu-1}). \tag{9}$$

Here Δ_ν, a relative norm, will be referred to as the νth recurrant.

The boundary conditions require that $f_{-1} \equiv 0$ & $\Delta_o \equiv 1$.

By choosing f_o appropriately, one can readily obtain all other f_ν's by the RRI. Given these basis vectors for S, the solution of the Heisenberg equation of motion for $\rho_k(t)$ may be formally given an orthogonal expansion as

$$\rho_k(t) = \sum_{\nu=0}^{d-1} a_\nu(t) f_\nu \qquad (10)$$

where $a_\nu(t)$'s are real time-dependent functions i.e., auto-correlation functions. If in fact one chooses $f_o = \rho_k$, $a_o(t)$ corresponds to the relaxation function of linear response theory. The most important property for these functions is that they satisfy a recurrence relation (RRII):

$$\Delta_{\nu+1}\, a_{\nu+1}(t) = -\dot{a}_\nu(t) + a_{\nu-1}(t), \quad 0 \leq \nu \leq d-1 \qquad (11)$$

where $\dot{a}_\nu(t) = da_\nu(t)/dt$ and $a_{-1}(t) = 0$. With the choice $f_o = \rho_k$ the boundary conditions are that $a_o(t=0) = 1$, $a_\nu(t=0) = 0$ if $\nu \geq 1$. The above recurrence relation is realized by the recurrants Δ_ν's, which are static properties. For the two dimensional gas at long wavelengths ($k/k_F << 1$) and $T = 0$ they are exactly calculable. Hence, it may be possible to solve the RRII to obtain the auto-correlation functions $a_\nu(t)$.

The random force for ρ_k is a vector in a subspace of S. It may also be given an orthogonal expansion

$$F_k(t) = \sum_{\nu=1}^{d-1} b_\nu(t)\, f_\nu \qquad (12)$$

where $b_\nu(t)$'s are real time-dependent functions, of which $b_1(t)$ is related to the memory function $M_k(t) = \Delta_1\, b_1(t)$. These auto-correlation functions also satisfy a recurrence relation like RRII:

$$\Delta_{\nu+1}\, b_{\nu+1}(t) = -\dot{b}_{\nu}(t) + b_{\nu-1}(t), \quad 1 \leq \nu \leq d-1. \tag{13}$$

Observe that Δ_1 does not enter into (13). Hence, $b_\nu(t)$'s do not depend on the first recurrant. The boundary conditions require that $b_1(t=0) = 1$ & $b_\nu(t=0) = 0$ if $\nu \geq 2$. The two families of autocorrelation functions are related by a convolution relation

$$a_\nu(t) = \text{convol.}(b_\nu(t) * a_o(t)), \quad 1 \leq \nu \leq d-1. \tag{14}$$

Hence, if $a_o(t)$ is known, $a_\nu(t)$ is knowable by the RRII and $b_1(t)$ via convolution. The method of recurrence relations is exact. If the recurrants are exactly given in some static regime, the RRII yields exact time dependence in that static regime. For example, the response function, one of the autocorrelations $a_\nu(t)$'s, satisfies the moment sum rules exactly to all orders in the static regime where the recurrants are valid.

IV. Basis Vectors and Norms

By choosing $f_o = \rho_k$ we can directly obtain from the RRI all other basis vectors spanning S:

$$f_o = \rho_k \tag{15a}$$

$$f_1 = \dot{\rho}_k \tag{15b}$$

$$f_2 = \ddot{\rho}_k + \Delta_1 \rho_k \tag{15c}$$

$$f_3 = \dddot{\rho}_k + (\Delta_1 + \Delta_2)\, \dot{\rho}_k \tag{15d}$$

$$f_4 = \overset{4}{\rho}_k + (\Delta_1 + \Delta_2 + \Delta_3)\, \ddot{\rho}_k + \Delta_1 \Delta_3 \rho_k \tag{15e}$$

$$f_5 = \overset{5}{\rho}_k + (\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4)\dddot{\rho}_k + (\Delta_1 \Delta_3 + \Delta_1 \Delta_4 + \Delta_2 \Delta_4)\dot{\rho}_k \tag{15f}$$

etc, where the number of dots or simply the number above ρ_k refers to the number of times the commutator of ρ_k with H is being nested e.g., $\dddot{\rho}_k = i\,[H, \ddot{\rho}_k] = i^3\,[H, [H, [H, \rho_k]\,]\,] = \overset{3}{\rho}_k$.

The norms of these vectors are:

$$(f_o, f_o) = (\rho_k, \rho_k) = \chi_k \tag{16a}$$

$$(f_1, f_k) = (\dot{\rho}_k, \dot{\rho}_k) \tag{16b}$$

$$(f_2, f_2) = (\ddot{\rho}_k, \ddot{\rho}_k) - (\Delta_1\, (\dot{\rho}_k, \dot{\rho}_k) \tag{16c}$$

$$(f_3, f_3) = (\dddot{\rho}_k, \dddot{\rho}_k) - (\Delta_1 + \Delta_2)\, (\ddot{\rho}_k, \ddot{\rho}_k) \tag{16d}$$

$$(f_4, f_4) = (\overset{4}{\rho}_k, \overset{4}{\rho}_k) - (\Delta_1 + \Delta_2 + \Delta_3)\, (\dddot{\rho}_k, \dddot{\rho}_k) + \Delta_1 \Delta_3\, (\ddot{\rho}_k, \ddot{\rho}_k) \tag{16e}$$

$$(f_5, f_5) = (\overset{5}{\rho}_k, \overset{5}{\rho}_k) - (\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4)\, (\overset{4}{\rho}_k, \overset{4}{\rho}_k) + (\Delta_1 \Delta_3 + \Delta_1 \Delta_4 + \Delta_2 \Delta_4)\, (\dddot{\rho}_k, \dddot{\rho}_k) \tag{16f}$$

etc., where we have used the identity $(\overset{n}{\rho}, \overset{m}{\rho}) = -(\overset{n-1}{\rho}, \overset{m+1}{\rho}) = -(\overset{n+1}{\rho}, \overset{m-1}{\rho})$, n or $m \geq 1$.

Observe that to calculate f_ν or its norm, we need to know the norms of $f_{\nu-1}$, $f_{\nu-2}$, ...f_o. Thus, if we calculate the norms successively starting with the basal vector f_o, we can obtain all the basis vectors and their norms explicitly. For the electron gas at long wavelengths, these norms can be obtained

by working out the nested commutators of ρ_k and evaluating the inner products at T = 0. Observe also that $\dot{\rho}_k = i[H,\rho_k] = i\ [H_o, \rho_k] = \dot{\rho}_k^{(o)}$, where H_o is the noninteracting part of H. As a result, the noninteracting or ideal Hamiltonian will have an important input in the behavior of the recurrants.

Ideal system

Here we express k in units of k_F such that $\varepsilon_F = 1/2m$ where ε_F is the Fermi energy in two dimensions. In these units, the norms for the two dimensional electron gas are given below:

$$(f_o,f_o)^{(o)} = \chi_k^{(o)} = m/\pi, \qquad k \leq 2 \tag{17a}$$

$$(f_1,f_1)^{(o)} = k^2\ \varepsilon_F/\pi \tag{17b}$$

$$(f_2,f_2)^{(o)} = (k^4 + k^6)\ \varepsilon_F^3/\pi \tag{17c}$$

$$(f_3,f_3)^{(o)} = (k^6 + 4k^8 + 0(k^{10}))\ \varepsilon_F^5/\pi \tag{17d}$$

$$(f_4,f_4)^{(o)} = (k^8 + 10k^{10} + 0(k^{12}))\ \varepsilon_F^7/\pi \tag{17e}$$

$$(f_5,f_5)^{(o)} = (k^{10} + 20k^{12} + 0(k^{14}))\ \varepsilon_f^9/\pi \tag{17f}$$

etc. For the recurrants valid in the static regime of $k << 1$, we shall retain only the leading order in k:

$$\Delta_1^{(o)} = 2k^2\ \varepsilon_F^2 \tag{18a}$$

$$\Delta_2^{(o)} = \Delta_3^{(o)} = \Delta_4^{(o)} = \ldots\ k^2\ \varepsilon_F^2 \equiv \Delta \tag{18b}$$

In the two dimensional ideal electron gas at long wavelengths the norms are finite and non-vanishing for all ν's. Hence, the dimensionality of the Hilbert space of ρ_k is infinite ($d=\infty$).

Interacting systems

For the static regime of small k, we find that the norms are to the leading order in k as follows:

$$(f_o,f_o) = \chi_k^{(o)}/(1 + v_k\ \chi_k^{(o)}) \tag{19a}$$

$$(f_\nu,\ f_\nu) = (f_\nu,\ f_\nu)^{(o)}, \qquad \nu \geq 1. \tag{19b}$$

Hence, to the leading order in k,

$$\Delta_1 = 2\Delta + \Gamma \tag{20a}$$

$$\Delta_\nu = \Delta_\nu^{(o)} = \Delta, \ \nu \geq 1 \tag{20b}$$

where $\Gamma = (\omega_p^{cl})^2 = 2\pi\rho\ e^2\ k/m$, where ρ is the number density. For the two dimensional interacting electron gas at long wavelengths the dimensionality of the Hilbert space of ρ_k is also infinite.

V. Autocorrelation Functions for Ideal System

Using the recurrants(18a,b) obtained for the ideal electron gas, we can now obtain the autocorrelation functions $a_\nu^{(o)}(t)$ & $b_\nu^{(o)}(t)$ from the RRI valid at long wavelengths. Defining $\Delta = \frac{1}{2}\mu^2$, we can decompose the RRII as follows:

$$\frac{1}{2}\mu^2 a_1^{(o)}(t) = -\dot{a}_o^{(o)}(t) \tag{21a}$$

$$\frac{1}{2}\mu^2 a_{\nu+1}^{(o)}(t) = -\dot{a}_\nu^{(o)}(t) + a_{\nu-1}^{(o)}(t), \quad \nu \geq 1. \tag{21b}$$

We look for a function which satisfies the above equations. Let

$$a_\nu^{(o)}(t) = \lambda^{-\nu} U_\nu h_\nu(\lambda t) \tag{22}$$

where λ is a scale factor, U_ν is a coefficient, and h_ν is a function, all to be determined. The boundary condition $a_o^{(o)}(t=0) = 1$ imposes that $U_o = 1$ & $h_o(0) = 1$. Substituting (22) into (21a,b) we get

$$\frac{1}{2}\mu^2 \lambda^{-2} U_1 h_1 = -h_o' \tag{23a}$$

$$\frac{1}{2}\mu^2 \lambda^{-2}(U_{\nu+1}/U_{\nu-1}) h_{\nu+1} = -(U_\nu/U_{\nu-1}) h_\nu' + h_{\nu-1}, \quad \nu \geq 1, \tag{23b}$$

where $h' = dh(x)/dx$. Clearly the scale factor is $\lambda = \mu$, which somewhat simplifies (23a,b) to

$$\frac{1}{2} U_1 h_1 = -h_o' \tag{24a}$$

$$\frac{1}{2}(U_{\nu+1}/U_{\nu-1}) h_{\nu+1} = -(U_\nu/U_{\nu-1}) h_\nu' + h_{\nu-1}, \quad \nu \geq 1. \tag{24b}$$

Now the above may be compared with the recurrence relation for the Bessel function J_ν[21]

$$J_1 = -J_o' \tag{25a}$$

$$J_{\nu+1} = -2J_\nu' + J_{\nu-1}, \quad \nu \geq 1. \tag{25b}$$

If $U_\nu = 2^\nu$, (24a,b) become identically reduced to (25a,b). Hence,

$$a_\nu^{(o)}(t) = 2^\nu \mu^{-\nu} J_\nu(\mu t), \quad \nu > 0. \tag{26}$$

Observe that the solution satisfies the boundary condition that $a_\nu^{(o)}(t=0) = 0$, $\nu \geq 1$. Thus the RRII realized by the

recurrants of the two dimensional ideal electron gas is isomorphic to the recurrence relation for the Bessel functions. The short and long time behavior of the relaxation functions $a_\nu^{(o)}(t)$ is given directly by the asymptotic properties of the Bessel functions e.g.

$$a_o^{(o)}(t \to \infty) \sim t^{-\frac{1}{2}} \cos(\mu t - \tfrac{1}{4}\pi). \tag{27}$$

For dynamical analysis it will be useful to have the Laplace transform of the relaxation functions:

$$a_\nu^{(o)}(z) = \mathcal{T}\,[a_\nu^{(o)}(t)] \tag{28}$$

where $\mathcal{T}$ is the Laplace transform operator.

The autocorrelation functions $b_\nu^{(o)}(t)$ can be obtained by the convolution eq. (14) which when Laplace-transformed takes a simple form:

$$b_\nu(z) = a_\nu(z)/a_o'(z) \tag{29}$$

where $b_\nu(z) = \mathcal{T}\,[b_\nu(t)]$, $\nu \geq 1$. Using (28) we obtain directly,

$$b_\nu^{(o)}(z) = 2^\nu\, \mu^{-2}\, ((z^2 + \mu^2)^{\frac{1}{2}} - z)^\nu, \quad \nu \geq 1. \tag{30}$$

Hence,

$$b_\nu^{(o)}(t) = \mathcal{T}^{-1}\,[b_\nu^{(o)}(z)]$$

$$= 2^\nu\, \mu^{1-\nu}\, \nu\, J_\nu(\mu t/\mu t), \qquad \nu \geq 1. \tag{31}$$

[22]

The memory function for the two dimensional ideal electron gas becomes

$$M^{(o)}(t) = \Delta_1^{(o)} b_1^{(o)}(t) = \mu^2\, J_1(\mu t/\mu t) \tag{32}$$

$$\sim t^{-3/2} \cos(\mu t - 3\pi/4) \text{ if } t \to \infty. \tag{32a}$$

Other linear response quantities e.g., the response function $\chi_k(\omega)$, the dynamic structure factor $S_k(\omega)$ can now be directly obtained from the autocorrelation functions. From linear response theory (henceforth suppressing k dependence)

$$\tilde{\chi}(t) \equiv \chi(t)/\chi = -\frac{\partial}{\partial t} a_o(t), \quad t \geq 0. \tag{33}$$

Hence,

$$\tilde{\chi}^{(o)}(t) = \mu J_1(\mu t) \tag{34}$$

and

$$\tilde{\chi}^{(o)}(\omega) \equiv T\,[\chi^{(o)}(t)]_{z=i\omega}$$

$$= 1 - i\omega\,(\mu^2 - \omega^2)^{-\frac{1}{2}}, \quad 0 < \omega < \mu \tag{35a}$$

$$= 1 - \omega(\omega^2 - \mu^2)^{-\frac{1}{2}}, \quad \mu < \omega < \infty. \tag{35b}$$

This is equivalent to the well known result due to Stern[11] at long wavelengths. Also since $\pi\, S(\omega) = -\,\mathrm{Im}\,\chi(z = i\omega)$, we obtain directly

$$\tilde{S}^{(o)}(\omega) \equiv S^{(o)}(\omega)/\chi^{(o)}$$

$$= \omega(\mu^2 - \omega^2)^{-\frac{1}{2}}, \quad 0 < \omega < \nu \tag{36a}$$

$$= 0, \quad \mu < \omega < \infty, \tag{36b}$$

Observe that $S^{(o)}(\omega)$ has a power law singularity at $\omega = \mu-$, suggestive of an apparent collective excitation. The dynamic structure factor satisfies the standard moment sum rules to all orders at long wavelengths. In this static regime the time evolution in the two dimensional ideal electron gas is thus exactly and completely characterized.

VI. Autocorrelation Function for Interacting System

It was shown that for the two dimensional interacting electron gas at long wavelengths $\Delta_\nu = \Delta_\nu^{(o)}$, $\nu \geq 2$, although $\Delta_1 \neq \Delta_\nu^{(o)}$ (see eq. 20a,b). Since the autocorrelation functions $b_\nu(t)$ depend not on the first recurrants but on others only we must have $b_\nu(t) = b_\nu^{(o)}(t)$, $\nu \geq 1$. We take advantage of this simplification to obtain the autocorrelation functions $a_\nu(t)$ as follows: By taking the Laplace transform of the RRII, we get for $\nu = 1$,

$$\Delta_\nu \, b_1(z) = (a_o(z))^{-1} - z. \tag{37}$$

Using (30) for $b_1(z) = b_1^{(o)}(z)$, we get

$$a_o(z)/s\mu = \frac{(z^2 + \mu^2)^{\frac{1}{2}} - (1 - s)z}{\mu^2 + \alpha z^2} \tag{38}$$

where $s \equiv \Delta_1^{(o)}/\Delta_1 = \mu^2/(\mu^2 + 2\Gamma)$ (39)

$$\alpha \equiv 1 - (1 - s)^2 = (x^2 + \tfrac{1}{2})/(x^2 + \tfrac{1}{2})^2 \tag{40}$$

where $x^2 \equiv \Gamma/\mu^2$. Note that $0 \leq s \leq 1$ and hence $0 \leq \alpha \leq 1$. In both cases, the upper bound is reached in the ideal limit. By taking an inverse transform we have

$$a_o(t) = \frac{s\mu}{2\pi i} \int_c dz \, e^{zt} \, \frac{(z^2 + \mu^2)^{\frac{1}{2}} - (1 - s)z}{\mu^2 + \alpha z^2} \tag{41}$$

where the contour c runs along the imaginary axis. There are two types of singularities on the imaginary axis. There is a cut from $z = -i\mu$ to $z = i\mu$. In addition, there are a pair of isolated poles beyond the cut at $z = \pm i\alpha^{-\frac{1}{2}} \mu \equiv \pm i\omega_p$. In Appendix A we have evaluated this integral:

$$a_o(t) = A_s \sum_{n=o}^{\infty} (-\alpha)^n \, (\partial/\partial\mu t)^{2n} \, [J_1(\mu t)/\mu t] + A_p \cos \omega_p t \qquad (42)$$

where

$$A_s = 1 - (1 - \alpha)^{\frac{1}{2}} \qquad (42a)$$

$$A_p = 2 \, [(1 - \alpha)^{\frac{1}{2}} - (1 - \alpha)]/\alpha. \qquad (42b)$$

We can now check various boundary conditions on $a_o(t)$. In Appendix B, we show that $a_o(t=0) = 1$. In Appendix C, we show $a_o(t) = J_o(\mu t)$ at the ideal limit $\alpha = 1$. It is easy to show that as $\alpha \to 0$, $a_o(t) = (1 - \frac{1}{2}\alpha) \cos \omega_p t + \frac{1}{2}\alpha \, J_1(\mu t)/\mu t + O(\alpha^2)$. For $t \to \infty$, $a_o(t) \sim t^{-3/2} \cos(\mu t - 3\pi/4) + A_p \cos \omega_p t$. Also given $a_o(t)$, the other $a_\nu(t)$'s can be obtained from it with successive applications of the RRII, e.g., $\Delta_1 a_1(t) = - \dot{a}_o(t)$, $\Delta_2 a_2(t) = - \dot{a}_1(t) + a_o(t)$, etc.

In Appendix D, we have obtained a closed form expression for the Fourier transform of $a_o(t)$ using (42):

$$a_o(\omega) = \frac{A_s \, (\mu^2 - \omega^2)^{\frac{1}{2}}}{\pi(\mu^2 - \alpha\omega^2)}, \quad 0 < \omega < \mu \qquad (43a)$$

$$= \tfrac{1}{2} A_p [\delta(\omega - \omega_p) + \delta(\omega + \omega_p)], \quad \mu < \omega < \infty. \qquad (43b)$$

In Appendix E, we have obtained the response function $\tilde{\chi}_k(\omega) \equiv \chi_k(\omega)/\chi_k$ using (42): Again suppressing k dependence,

$$\text{Re} \; \tilde{\chi}(\omega) = 1 + \frac{A_s \, (1 - \alpha)^{\frac{1}{2}} \, \omega^2}{(\mu^2 - \alpha\omega^2)}, \quad 0 < \omega < \mu \qquad (44a)$$

$$= 1 + \frac{A_s \, [(1 - \alpha)^{\frac{1}{2}} \, (1 - \mu^2/\omega^2)^{\frac{1}{2}}]\omega^2}{(\mu^2 - \alpha\omega^2)}, \quad \mu < \omega < \alpha \qquad (44b)$$

$$-\text{Im} \; \tilde{\chi}(\omega) = \frac{A_s \, \omega(\mu^2 - \omega^2)^{\frac{1}{2}}}{(\mu^2 - \alpha\omega^2)}, \quad 0 < \omega < \mu \qquad (45a)$$

$$= \tfrac{1}{2} \pi A_p \, [\delta(\omega - \omega_p) + \delta(\omega + \omega_p)], \quad \mu < \omega < \infty. \qquad (45b)$$

Eqs. (43) and (45) satisfy the well-known relation: Im $\tilde{\chi}(\omega) = -\pi\omega\, a_o(\omega)$. For $\omega \to 0$, Re $\tilde{\chi}(\omega) = 1 + O(\omega^2)$; and for $\omega \to \infty$, Re $\tilde{\chi}(\omega) = O(\omega^{-2})$ as required. In the high frequency regime, Im $\tilde{\chi}(\omega)$ agrees with Rajagopal.[12] For $\alpha = 1$, eqs. 43-45 all reduce to the ideal results of sec. V.

The dynamic structure factor can be obtained by the relation $\pi S(\omega) = -$ Im $\chi(\omega)$. Hence, directly from (45) we have $\tilde{S}(\omega) \equiv S(\omega)/\chi$

$$= \frac{A_s\, \omega\, (\mu^2 - \omega^2)^{\frac{1}{2}}}{\pi\, (\mu^2 - \alpha\omega^2)}\,, \qquad 0 < \omega < \mu \tag{46a}$$

$$= \tfrac{1}{2} A_p\, \omega[\delta(\omega - \omega_p) + \delta(\omega + \omega_p)], \quad \mu < \omega < \infty\,. \tag{46b}$$

The general shape of the dynamic structure factor resembles the three dimensional version at the same small k, consisting of a broad single-particle spectrum and a sharp collective peak. However, the amplitude of the two dimensional single-particle spectrum is "soft", i.e., it grows with α. Hence, there is an interesting crossover behavior going from interacting to ideal, which is unique to two dimensions at small k.[23] The total area under the single-particle spectrum remains finite at all values of α including $\alpha = 1$. The dynamic structure factor (46) satisfies the moment sum rules to all orders exactly as in the ideal case.

VII. Intrinsic Conductivity

At a low temperature the frequency-dependent conductivity $\sigma_k(\omega)$ is generally understood to be largely due to impurities present in the system. The electron-electron interaction contributes to it hardly at all. This conventional notion ignores the possible existence of random currents arising entirely from density fluctuations. A random current can contribute to the frequency-dependent conductivity as may be seen from the Kubo conductivity formula, which may be termed an intrinsic conductivity. It appears that Kubo had speculated upon the existence of an intrinsic conductivity some years ago.[24]

The density fluctuations and the total current j_k are connected by the continuity equation,

$$\dot{\rho}_k(t) = - ik.j_k(t) \tag{47}$$

where the total current j_k includes a random current j_k^r. Kubo[24] showed that the random current is associated with the random force $F_k(t)$ such that

$$ik.j_k^r(t) = F_k(t). \tag{48}$$

Since our system contains no impurities, the Kubo formula assumes the following form:

$$\sigma_k(\omega) = e^2 \int_0^\infty dt\, e^{-i\omega t}\, (j_k^r(t),\, j_k^r)/(j_k^r, j_k^r) \tag{49}$$

Hence, using (48) one can calculate the intrinsic conductivity present in a pure, homogeneous system if the random force is known. This contribution must presumably be added to the conventional mechanisms to obtain the total frequency-dependent conductivity.

Using (13), (15), and (31), we can readily evaluate the random current: Suppressing k-dependence,

$$(j^r(t),\ j^r)/(j^r,j^r) = b_1(t) = 2\ J_1(\mu t)/\mu t\ . \qquad (50)$$

Hence, writing $\tilde{\sigma}\ (\omega) = \sigma\ (\omega)/\sigma$, where $\sigma = 2\pi e^2\ \rho/m\mu$, we obtain

$$\text{Re}\ \tilde{\sigma}\ (\omega) = (\mu^2 - \omega^2)^{\frac{1}{2}}/\mu, \qquad 0 < \omega < \mu$$

$$= 0, \quad \mu < \omega < \infty. \qquad (51)$$

$$-\text{Im}\ \tilde{\sigma}\ (\omega) = \omega/\mu, \qquad 0 < \omega < \mu$$

$$= \mu/(\omega + (\omega^2 - \mu^2)^1), \quad \mu < \omega < \infty. \qquad (52)$$

The real part of the conductivity satisfies the conductivity sum rule:[24]

$$\int_0^\infty d\omega\ \text{Re}\ \tilde{\sigma}_k(\omega) = \pi e^2\ \rho/2m. \qquad (53)$$

We note that the intrinsic conductivity exists in <u>low</u> frequencies only, abruptly vanishing beyond a critical frequency $\omega = \omega_c = \mu$ (see Re $\sigma_k\ (\omega)$). Detecting the intrinsic conductivity raises an interesting possibility.[25] In an impure system, e.g., metals, in addition to the intrinsic conductivity there is the contribution from impurity scattering. Ordinarily the latter is given by the Drude formula which is a high frequency approximation. Nonsmooth behavior in the vicinity of ω_c in an impure system would be an indication of merging of the two different physical mechanisms.

VIII. Dynamic RPA

It is well known that the response function may be put in the form

$$\chi_k(\omega) = \chi_k^{(o)}(\omega)/(1 + \Lambda_k(\omega)\ \chi_k^{(o)}(\omega)) \tag{54}$$

where $\chi_k^{(o)}(\omega)$ is the response function due to H_o and $\Lambda_k(\omega)$ is some function of V (see (1)). Various dynamic RPA theories are equivalent to taking $\Lambda_k(\omega) = \Lambda_k(0) \equiv \Lambda_k$.[26] For example, $\Lambda_k = v_k$ gives the simple RPA and $\Lambda_k = v_k(1 - G_k)$ gives the generalized RPA, where G_k is a local field term. For $k \ll 1$, $G_k \approx 0$.[27] Hence, at long wavelengths the two dynamic RPA theories are essentially the same.

Taking $\Lambda_k(\omega)$ to be frequency-independent is clearly an approximation, the validity of which may very well depend on the static regime. Several people [28-31] have obtained asymptotic conditions in three dimensions for $\Lambda_k(\omega)$ or equivalently for $G_k(\omega)$, where $\Lambda_k(\omega) = v_k\ (1 - G_k(\omega))$, sometimes referred to as a dynamic local field term. There are otherwise no exact general expressions known. Using the method of recurrence relations we have recently derived an expression for $\Lambda_k(z=i\omega)$:[32] Suppressing k dependence

$$\Lambda(z) = [\chi^{-1} - (\chi^{(o)})^{-1}] + \frac{z}{(f_1, f_1)}[(b_1(z))^{-1} - (b_1^{(o)}(z))^{-1}]$$

$$\equiv \Lambda + \lambda(z). \tag{55}$$

The first bracketed term Λ is z-independent. In general, $b_1(z) \neq b_1^{(o)}(z)$ owing to the fact that $\Delta_\nu \neq \Delta_\nu^{(o)}$, $\nu \geq 2$. A change in the memory function when interaction is turned on makes the local field term frequency-dependent.

In the two dimensional electron gas at long wavelengths, we have shown that $b_1(z) = b_1^{(o)}(z)$, since $\Delta_\nu = \Delta_\nu^{(o)}$, $\nu \geq 2$ (see sec. VI). That is, when density fluctuations are confined to the vicinity of the Fermi surface, the memory function is unaffected by the electron-electron interaction. Hence, in this static regime i.e., $k<<1$, $\Lambda_k(\omega) = \Lambda_k$ and the dynamic RPA theories are valid. This accounts for the RPA work of Holas et al.[33]

IX. Discussion

The method of recurrence relations treats the time evolution of dynamical variables as a geometric problem in Hilbert space. In this approach, the recurrants are basic elements of a realized Hilbert space whose structure contains the possible time evolution. The recurrence relations RRI and II are merely mathematical expressions of this geometric picture.

For the two dimensional electron gas at long wavelengths, the recurrants assume a very simple form making the structure of the Hilbert space very elementary. As a result, we were able to deduce an exact solution for the time evolution of density fluctuations occurring near the Fermi surface. Using this solution, we have also obtained other physical quantities via linear response relations. In some other static regimes e.g., $k \approx k_F$, the recurrants should have some other forms. Hence, we expect the time evolution of density fluctuations to be different.

Even when confined to the vicinity of the Fermi surface, the density fluctuations show persistent memory effects. There thus exists a random current and there is an intrinsic conductivity. Remarkably, the memory is shaped by quantum fluctuations and not by the electron-electron interaction. This has an unexpected consequence that the dynamic RPA theories are exactly valid in this static regime.

The time evolution of density fluctuations may be regarded as a Fermi sphere undergoing a restoring motion due to a slight initial displacement in momentum space. In the two dimensional

electron gas there are an infinite degrees of freedom for such a relaxation, giving rise to an infinite dimensions for the corresponding Hilbert space. The resulting density fluctuations are electron-hole scattering processes on which a collective process is superimposed. These processes are dimension-sensitive. In one dimensional electron gas, for example, the restoring motion is one dimensional and the single-particle and collective processes are bound together. For this system, the Hilbert space is also finite-dimensional making the relaxation always oscillatory.

Acknowledgments

The work at the University of Georgia was supported in part by the U. S. Department of Energy, Office of Basic Energy Sciences and the Office of Naval Research. The work at the Seoul National University was supported by the Korea Science and Engineering Foundation and the Research Institute for Basic Science, Seoul National University.

Appendix A. Relaxation function

We shall now evaluate the integration given below:

$$a_o(t) = \frac{s\mu}{2\pi i} \int_c dz\, e^{zt}\, \frac{(z^2+\mu^2)^{\frac{1}{2}} - (1-s)z}{z^2+\mu^2/\alpha} \quad . \tag{A1}$$

Since $\alpha<1$, there are isolated poles at $z = \pm i\mu\alpha^{-\frac{1}{2}} = \pm i\omega_p$, which are outside the cut bounded by $z = \pm i\mu$ on the imaginary axis. Hence the contributions from the isolated poles are straightforward to evaluate. The residue at $z = \pm i\omega_p$ is:

$$s/2\alpha \cdot [\sqrt{1 - (\mu/\omega_p)^2} + (1-s)]\, e^{\pm i\omega_p t} = [\sqrt{1-\alpha} - (1-\alpha)]e^{\pm i\omega_p t}. \tag{A2}$$

The integration about contour c_1 containing the cut is as follows:

$$s/2\pi i\mu \cdot \int_{c_1} dz\, e^{zt}\, \frac{(\mu^2 + z^2)^{\frac{1}{2}}}{1 + \alpha\, z^2/\mu^2}$$

$$= s/\mu\pi \cdot \int_{-\mu}^{\mu} dy\, \sqrt{\mu^2 - y^2}\, [1 + \alpha y^2/\mu^2 + \alpha^2 y^4/\mu^4 + \ldots]\, e^{iyt}$$

$$= s/\pi\mu \cdot \int_{-\mu}^{\mu} dy\, \sqrt{\mu^2 - y^2}\, [1 - \frac{\alpha}{\mu^2}(\frac{\partial}{\partial t})^2 + \frac{\alpha^2}{\mu^4}(\frac{\partial}{\partial t})^4 - \ldots]e^{iyt}$$

$$= s[1 - \frac{\alpha}{\mu^2}(\frac{\partial}{\partial t})^2 + \frac{\alpha^2}{\mu^4}(\frac{\partial}{\partial t})^4 - \ldots]J_1(\mu t)/\mu t \tag{A3}$$

where we have used an integral representation of the Bessel function J_1.

Combining the above results we have with the definitions (42a,b)

$$a_o(t) = A_s \sum_{\nu=0}^{\infty} (-\alpha)^{\nu}\, (\frac{\partial}{\partial \mu t})^{2\nu}\, J_1(\mu t)/\mu t + A_p \cos \omega_p t. \tag{A4}$$

Appendix B. Proof that $a_o(t=0) = 1$

We shall consider $t \to 0$ in our solution for $a_o(t)$ found in Appendix A:

$$a_o(t) = A_s \sum_{\nu=0}^{\infty} (-\alpha)^{\nu} \left(\frac{\partial}{\partial \mu t}\right)^{2\nu} J_1(\mu t)/\mu t + A_p \cos \omega_p t. \qquad \text{(B1)}$$

Using the expansion formula, we have

$$J_1(z)/z = \sum_{k=0}^{\infty} c_{2k}\, z^{2k}, \qquad \text{(B2)}$$

where

$$c_{2k} = (-)^k/(2^{2k+1}\, k!(k+1)!). \qquad \text{(B3)}$$

Hence,

$$\left(\frac{\partial}{\partial z}\right)^{2\nu} J_1(z)/z \Big|_{z=0} = 2\nu!\; c_{2\nu} \equiv g_{2\nu}. \qquad \text{(B4)}$$

Substituting these results in (B1), we obtain

$$a_o(0) = A_s \sum_{n=0}^{\infty} (-\alpha)^{\nu}\, g_{2\nu} + A_p. \qquad \text{(B5)}$$

Now one can show that

$$\sum_{\nu=0}^{\infty} \frac{\alpha^{\nu} 2\nu!}{2^{2\nu+1}\, \nu!(\nu+1)!} = \frac{1}{\alpha}\left(1 - \sqrt{1-\alpha}\right).$$

Using $A_s = 1 - \sqrt{1-\alpha}$ and $A_p = 2(\sqrt{1-\alpha} - (1-\alpha))/\alpha$, we find $a_o(0) = 1$.

Appendix C. Proof that $a_o(t) = a_o^{(o)}(t)$ when $\alpha = 1$

We shall consider $\alpha \to 1$ in our solution for $a_o(t)$ given below:

$$a_o(t) = A_s \sum_{\nu=0}^{\infty} (-)^{\nu} \left(\frac{\partial}{\partial \mu t}\right)^{2\nu} J_1(\mu t)/\mu t + A_p \cos \omega_p t. \qquad (C1)$$

For $\alpha \to 1$, we can write setting $A_s = 1$ and $A_p = 0$,

$$a_o(t) = \sum_{k=0}^{\infty} d_k(\alpha \to 1)\ (\mu t)^{2k}. \qquad (C2)$$

Although $d_k(\alpha)$ is complicated, it becomes simple when $\alpha = 1$:

$$d_k(1) = (-)^k/(2^{2k}\ (k!)^2). \qquad (C3)$$

Hence,

$$\lim_{\alpha \to 1} a_o(t) = \sum_{k=0}^{\infty} \frac{(-)^k\ (\mu t)^{2k}}{2^{2k}\ (k!)^2} = J_o(\mu t). \qquad (C4)$$

We recall that $a_o^{(o)}(t) = J_o(\mu t)$, completing our proof.

Appendix D. Frequency-Dependent Relaxation Function

We shall first calculate the frequency-dependent relaxation function due to single-particle excitations $a_o^s(\omega)$ via the formula

$$a_o^s(\omega) = 1/2\pi \cdot \int_{-\infty}^{\infty} dt\ e^{i\omega t - \varepsilon|t|}\ a_o^s(t) \qquad \text{(D1)}$$

where $\varepsilon \to 0$. Using (B1), we have

$$a_o^s(\omega) = A_s/2\pi \cdot \sum_{n=0}^{\infty} (-\alpha/\mu^2)^{\nu}\ \phi^{(2\nu)} \qquad \text{(D2)}$$

where

$$\phi^{(2\nu)} = \left(\frac{\partial}{\partial t}\right)^{2\nu} \int_{-\infty}^{\infty} dt\ e^{-i\omega t - \varepsilon|t|}\ J_1(\mu t)/\mu t, \qquad \text{(D3)}$$

and $\phi^{(o)} \equiv \phi \doteq 2(\mu^2 - \omega^2)^{\frac{1}{2}}, \quad 0<\omega<\mu$

$$= 0, \qquad \mu<\omega<\infty . \qquad \text{(D4)}$$

Now integrating by parts, we get $\phi^{(2\nu)} = (i\omega)^{2\nu}\ \phi$. Hence, using these results we obtain directly

$$a_o^s(\omega) = \frac{1}{\pi} A_s\ (\mu^2 - \omega^2)^{\frac{1}{2}}/(1 - \alpha\omega^2/\mu^2), \quad 0<\omega<\mu$$

$$= 0 \qquad \mu<\omega<\infty \qquad \text{(D5)}$$

The relaxation function due to the plasmon excitations $a_o^p(\omega)$ is straightforward:

$$a_o^p(\omega) = 1/2\ A_p(\delta(\omega - \omega_p) + \delta(\omega + \omega_p)). \qquad \text{(D6)}$$

Since $\omega_p = \mu/\alpha^{\frac{1}{2}}$, where $\alpha<1$, the plasmon contributes only for $\mu<\omega<\infty$.

Appendix E. The Response Function $\tilde{\chi}(\omega)$

We shall use the well-known relation given below to calculate the response function from the relaxation function,

$$\tilde{\chi}(\omega) = 1 - i\omega \int_0^\infty dt\, e^{-i\omega t - \varepsilon t}\, a_o(t) \qquad \text{(E1)}$$

where $\varepsilon \to 0$. From our solution, we can write:

$$a_o(t) \equiv A_s M(t) + A_p N(t). \qquad \text{(E2)}$$

One can immediately show that

$$N(\omega) = i\omega/(\omega_p^2 - \omega^2) + \pi\frac{1}{2}(\delta(\omega_r - \omega) + \delta(\omega_p + \omega)), \qquad \text{(E3)}$$

where $\omega_p = \mu/\alpha^{\frac{1}{2}}$. To calculate $M(\omega)$, we first define

$$\Gamma_{2\nu}(\omega) = \int_0^\infty dt\, e^{-i\omega t - \varepsilon t} \left(\frac{\partial}{\partial \mu t}\right)^{2\nu} J_1(\mu t)/\mu t \qquad \text{(E4)}$$

Then,

$$M(\omega) = \sum_{n=0}^{\infty} (-\alpha)^\nu\, \Gamma_{2\nu}(\omega). \qquad \text{(E5)}$$

Now integrating by parts, we find a recurrence relation for Γ's:

$$\Gamma_{2\nu} = -i\omega\, g_{2\nu-2} - \omega^2\, \Gamma_{2\nu-2}\,, \quad \nu > 1\,, \qquad \text{(E6)}$$

where $g_{2\nu} = \left(\frac{\partial}{\partial \mu t}\right)^{2\nu} J_1(\mu t)/\mu t$. The above recurrence relation can be used to sum (E5). By substituting (E6) in (E5), we get

$$M = \Gamma_o + \alpha\, \omega^2\, M/\mu^2 + i\alpha\omega \sum_{\nu=0}^{\infty} (-\alpha)^\nu\, g_{2\nu} \qquad \text{(E7)}$$

or

$$M = \frac{1}{1 - \alpha\omega^2/\mu^2}\, [\Gamma_o + i\alpha\omega \sum_{\nu=0}^{\infty} (-\alpha)^\nu\, g_{2\nu}]. \qquad \text{(E8)}$$

The sum has been already carried out (see Appendix B). Also Γ_o can be evaluated:

$$\Gamma_o = [(\mu^2 - \omega^2)^{\frac{1}{2}} - i\omega]/\mu^2 , \qquad 0<\omega<\mu$$

$$= -i(\omega - \sqrt{\omega^2 - \mu^2})/\mu^2 , \qquad \mu<\omega<\infty . \tag{E9}$$

Together

$$M = (\mu^2 - \alpha\omega^2)^{-1} \ (\sqrt{\mu^2 - \omega^2} \ - i\omega\sqrt{1 - \alpha}) , \qquad 0<\omega<\mu$$

$$= i(\mu^2 - \alpha\omega^2)^{-1} \ (\sqrt{\omega^2 - \mu^2} \ - \omega\sqrt{1 - \alpha}) , \qquad \mu<\omega<\infty . \tag{E10}$$

Combining (E3) and (E10), we get

$$\text{Re } \tilde{\chi} = 1 + \omega A_s \text{ Im } M + \omega A_p \text{ Im } N$$

$$= 1 + A_s \omega^2\sqrt{(1 - \alpha)}/(\mu^2 - \alpha\omega^2) \qquad 0<\omega<\mu$$

$$= 1 + A_s \omega^2(\sqrt{1 - \alpha} + \sqrt{1 - \mu^2/\omega^2})/(\mu^2 - \alpha\omega^2) , \quad \mu<\omega<\infty \tag{E11}$$

where we have used $A_p = 2A_s \sqrt{1 - \alpha}/\alpha$. Similarly,

$$-\text{Im } \tilde{\chi} = \omega A_s \text{ Re } M + \omega A_p \text{ Re } N$$

$$= A_s \omega \sqrt{\mu^2 - \omega^2}/(\mu^2 - \alpha\omega^2) , \qquad 0<\omega<\mu$$

$$= \tfrac{1}{2}\pi A_s \omega(\delta(\omega - \omega_p) + \delta(\omega + \omega_p)) , \qquad \mu<\omega<\infty . \tag{E12}$$

Observe that our results satisfy the relationship Im $\tilde{\chi} = -\pi\omega a_o$.

References

1. See e.g. G. D. Mahan, Many-Particle Physics (Plenum, NY 1981), Chap. 5.
2. See e. g. Electronic Properties of Two-dimensional Systems, edited by G. Dorda and P. J. Stiles (North-Holland, Amsterdam, 1978); also Surf. Sci., 98, (1980) and 113 (1982).
3. Two-dimensional Systems, Heterostructures and Superlattices, edited by G. Bauer, F. Kuchar and H. Heinreich (Springer-Verlag, Berlin 1984).
4. T. Ando, A. B. Fowler and F. Stern, Rev. Mod. Phys. 54, 437 (1982).
5. A. K. Rajagopal and J. C. Kimball, Phys. Rev. B 15, 2819 (1977).
6. D. L. Freeman, Solid State Commu. 26 289 (1978).
7. A. Ishihara and T. Toyoda, Phys. Rev. B 21, 3358 (1980).
8. L. C. Ioriatti and A. Ishihara, Z. Phys. B 44, 1 (1981).
9. M. Johnson, J. Phys. C 9, 3055 (1976).
10. S. Nagano, K. S. Singwi and S. Ohnishi, Phys. Rev. B 29, 1209 (1984).
11. F. Stern, Phys. Rev. Lett. 18, 549 (1967).
12. A. K. Rajagopal, Phys. Rev. B 15, 4264 (1977).
13. P. F. Maldague, Surf. Sci. 73 296 (1978).
14. A. Czachor, A. Holas, S. R. Sharma and K. S. Singwi, Phys. Rev. B 25, 2144 (1982).
15. See e. g. Nonlinear Fluid Behavior, edited by H. H. M. Hanley (North-Holland, Amsterdam 1983); Nonlinear Nonequilibrium Statistical Mechanics, Suppl. Prog. Theor. Phys. (Kyoto), No.64, 1978.

16. M. H. Lee, Phys. Rev. B 26, 2547 (1982), Phys. Rev. Lett. 49, 1072 (1982).

17. A method related to Ref. 16 was independently developed by P. Grigolini, G. Grosso, G. P. Parravicini and M. Sparpaglione, Phys. Rev. B 27, 7342 (1983); M. Giodano, P. Grigolini, D. Leporini and P. Marin, Phys. Rev. A 28, 2474 (1983).

18. M. H. Lee, J. Math. Phys. 24, 2512 (1983).

19. M. H. Lee, Phys. Rev. Lett. 51, 1227 (1983); M. H. Lee and J. Hong, Phys. Rev. B 30, 6767 (1984); M. H. Lee, J. Hong and N. L. Sharma, Phys. Rev. A 29, 1561 (1984); M. H. Lee, I. M. Kim, and R. Dekeyser, Phys. Rev. Lett. 52 579 (1984); J. Florencio and M. H. Lee, Phys. Rev. A 31, 3231 (1985).

20. M. H. Lee and J. Hong, Phys. Rev. Lett. 48, 634 (1982).

21. Handbook of Mathematical Functions, edited by M. Abramowitz and I. A. Stegum, National Bureau of Standard Applied Mathematical Series 55. (U. S. Department of Commerce, Washington, D. C. 1970).

22. The autocorrelation functions $b_\nu^{(o)}(t)$ can also be obtained directly from(14) by writing out the convolution equation:

$$a_\nu(t) = \int_o^t dt'\, b_\nu(t-t')\, a_o(t'), \quad \nu \leq 1.$$

Using (26) for $a_\nu(t)$, we get

$$2^\nu \mu^{-n} J_\nu(\mu t) = \int_o^t dt'\, b_\nu(t-t')\, J_o(\mu t')$$

which is precisely in the form of a well-known Bessel integral (see Ref. 21, pp 480). Hence, we obtain immediately

$$b_\nu^{(o)}(t) = 2^\nu \mu^{1-\nu} \nu J_\nu(\mu t)/\mu t, \quad \nu \geq 1.$$

23. M. H. Lee and J. Hong, Phys. Rev. B 26 2227 (1982).

24. R. Kubo, Rep. Prog. Phys. 29 255 (1966).

25. M. H. Lee and J. Hong. Physica 128B, 301 (1985).

26. See e. g. N. Iwamoto, Phys. Rev. A 30, 3289 (1984).

27. S. Ichimaru, Rev. Mod. Pyys. 54, 1017 (1982).

28. A. K. Rajagopal, Phys. Rev. A 6, 1239 (1972).

29. J. C. Kimball, Phys. Rev. A 7, 1648 (1973).

30. G. Niklasson, Phys. Rev. B 10, 3052 (1974).

31. A. A. Kugler, J. Stat. Phys. 12, 356 (1975).

32. J. Hong and M. L. Lee (To be published).

33. A. Holas, S. Nagano and K. S. Singwi, Phys. Rev. B 27, 5981 (1983).

END

FILMED

2-86

DTIC