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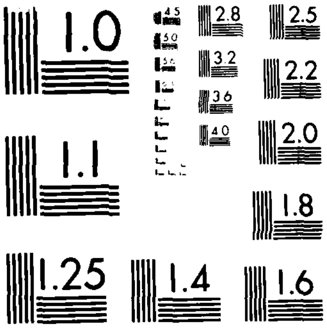

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Analysis of a Linear Antenna Two Wavelengths Long: Consideration of Options

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1 November 1985

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The far-zone electric field of a wire dipole antenna is numerically determined by the method of moments. Additional numerical methods that are used and compared are finite differences for computing a first derivative and L-V decomposition and the conjugate gradient method for solving lineae equations. These additional methods are compared in terms of their agreement with previously accepted results and their computer execution time and memory requirements.

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FIGURES

TABLES

I. INTRODUCTION

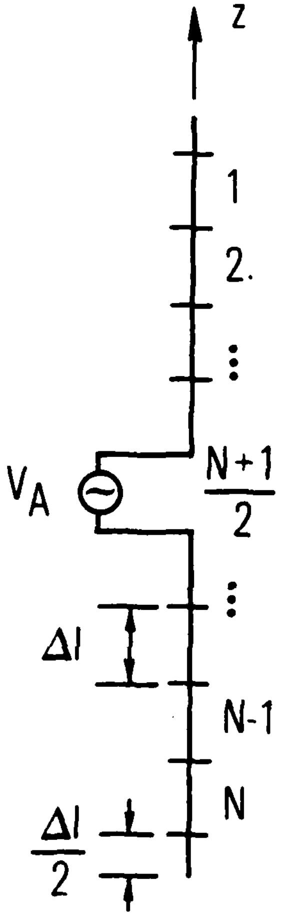
For many electromagnetic problems that require a rigorous solution for the electric field, a closed-form analysis is impractical because of the complexity of the integrals involved. On the other hand, the computer approximation of these problems is generally easy and accurate.

An approximation method that is extensively used in the community is the method of moments.[1] The method of moments reduces a nonlinear equation to a set of linear equations that can be solved by matrix methods such as Gaussian elimination and conjugate gradients.[2,3] In this report, the method of moments is applied to Pocklington's integral equation to approximate the electric field radiated by a wire dipole antenna two wavelengths long.

II. PROBLEM FORMULATION

In order to compute the far-field pattern from the dipole, the procedure is first to compute the current on the wire by the method of moments and then to integrate the current using the Green's function. The method of moments can be used to approximate the current on the wire as pulses at discrete positions on the wire. The distances between these pulses are arbitrary but, for programming ease, they should be uniform. The model that is developed for the dipole antenna is a wire divided into an integral number of segments with half-segments at the ends where the current is zero. The pulse approximation dictates that the values for the current and the voltage on each segment be constant. The current and voltage can be represented as twin arrays, with each element of an array corresponding to the value either of the current or the voltage on a segment. For the center-fed dipole antenna, as shown in Figure 1, the values for the elements of the voltage array are zero, except that the element corresponding to the center segment has the value of the voltage applied, V_A. The values of the elements of the current array are the unknowns in the dipole antenna problem.

The number of segments is critical to the resolution of the current. A rule of thumb is to select at least 10 segments per wavelength. For example, if the antenna is 0.5λ long, then at least five segments are required. The overall size of the dipole antenna to be analyzed is limited by the number of segments that can be treated practically. For antenna structures that are more general than a simple linear dipole, the two-dimensional current distribution, rather than axial dipole current, must be obtained. Thus, the overall size of the antenna structure is limited to designs that are small in terms of wavelengths.



$$(C) = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_{N-1} \\ C_N \end{pmatrix}$$

$$(V) = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ V_A \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

Figure 1. Center-Fed Dipole Antenna with N Segments. Array elements represent the current and voltage on the segments.

III. METHOD OF MOMENTS

The pulse model of the dipole antenna is implemented in the method of moments by using the pulse function

$$P(z,z_n) = \begin{cases} 1 & |z - z_n| < \frac{\Delta l}{2} \\ 0 & \text{elsewhere} \end{cases}$$

as a basis function. Because z_n denotes the center of the nth segment and Δl is the length of the segment, the pulse function is zero except at the nth segment. Other basis functions that are appropriate for modeling the dipole antenna are the sinusoidal and triangular functions. The equation to be linearized in the wire antenna problem is Pocklington's integral:

$$(k^2 + \frac{\delta^2}{\delta z^2}) \int_o^L C(z')G(z,z')dz' = -j\omega 4\pi\varepsilon_o E^i_{tan}(z)$$

where z', the integration point, is the position of the current on the wire; z, the observation point, is the position of the impressed electric field on the wire; and G(z,z') is the Green's function for a thin wire antenna of radius a:

$$G(z,z') = \frac{\exp\{-jk[a^2 + (z-z')^2]^{1/2}\}}{\sqrt{a^2 + (z - z')^2}}$$

The current on the wire, C(z'), is the unknown for which the moment method will approximate a solution. The observation points are selected to be on the surface of the wire, where the expression for the impressed electric field is valid, and the integration points are on the axis of the wire. The first step of the moment method is to write C(z') in terms of the basis function:

$$C(z') = \sum_{n=1}^{N} C_n P(z',z_n)$$

If we bring the second derivative inside the integral in accordance with Pearson,[4] the Pocklington's integral becomes

$$\sum_{n=1}^{N} C_n \int_{z_n - \frac{\Delta l}{2}}^{z_n + \frac{\Delta l}{2}} \left(k^2 G + \frac{\delta^2 G}{\delta z^2}\right) dz' = -j\omega 4\pi\varepsilon_o E^i_{tan}$$

This simplified equation can be linearized by using a Dirac delta function as the testing function. This changes Pocklington's equation to

$$\sum_{n=1}^{N} C_n \int_{z_n - \frac{\Delta l}{2}}^{z_n + \frac{\Delta l}{2}} \left\{k^2 G(z_m,z') + \left[\frac{\delta^2}{\delta z^2} G(z,z')\right]_{z=z_m}\right\} dz' = -j\omega 4\pi\varepsilon_o E^i_{tan}(z_m).$$

The second derivative can be evaluated two ways - analytically and by finite differences. The difference between the far-zone electric field of the analytical and finite differences is minimal, as is shown in the numerical results section of this report. However, the current at the center of the wire and, therefore, the input admittance, do not agree.

The resultant matrix equation, $[Z][C] = [\Delta l \cdot E]$, can be solved for the unknown current by standard matrix inversion methods, such as Gaussian elimination and L-U factorization or by the conjugate gradient method. The conjugate gradient method is discussed subsequently because it is not as universally used as the other methods and because it requires less memory.

IV. CONJUGATE GRADIENT METHOD

The conjugate gradient method is an iterative method that assumes a solution and, by computing the residual, adjusts the initial solution until the relative error is within the specified tolerance. The method is best used to minimize or maximize an optimization function but can also be used, as in this case, to solve a linear equation. The conjugate gradient algorithm is as follows:[2,3]

$$r_o = ZC_o - V$$

$$P_o = -b_{-1}Z^* r_o$$

$$C_{k+1} = C_k + t_k P_k$$

$$r_{k+1} = r_k + t_k Z P_k$$

$$P_{k+1} = P_k - b_k Z^* r_{k+1}$$

$$t_k = \frac{1}{\| ZP_k \|^2}$$

$$b_k = \frac{1}{\| Z^* \; r_{k+1} \|^2}$$

where

$V = [\Delta l \cdot E\ ]$

Z^* = the complex conjugate of the impedance matrix

The algorithm computes the residual array, r_k, at each iteration and makes adjustments to the current array, C_k, based on the residual array to minimize the relative error. The expression for the relative error is $\| ZC_k - V \| / |V|$. If the current is initially assumed to be zero at every segment, the error equals 1 at the first iteration and decreases monotonically, until the error is less than the prescribed tolerance. For the problem of the wire antenna two wavelengths long, a tolerance of 10^{-8} was used, and the algorithm converged within a reasonable number of iterations. A chart of the convergence rate is shown in the numerical results section of this report.

The memory requirement for the conjugate gradient method is 7N, as opposed to $2N^2 + 2$ for the IMSL library routine that was used to verify the results presented in this report.

V. NUMERICAL RESULTS

The numerical results reported in the tables and figures of this section are illustrations of the comparison of four combinations of second derivative methods with linear equation methods: (1) analytical derivative with conjugate gradients, (2) analytical derivative with L-U decomposition, (3) finite differences with conjugate gradients, and (4) finite differences with L-U decomposition. (The listings for the computer programs written in-house are given in the Appendix.) The combinations were applied to a theoretical wire antenna with a length of 2λ and a radius of 0.013λ, with 1 V applied to the center. Because an error tolerance of 10^{-8} was set for the conjugate gradient method and 10^{-14} for the L-U decomposition, the current, far-zone electric field, and input admittance calculated by each of these matrix methods for the same second derivative method are equal to at least eight significant figures. Because the numerical values calculated by the conjugate gradient method and L-U decomposition are equivalent, no distinction is made between them in Figures 2, 3, and 4 or in Tables 1 and 2.

Figures 2, 3, and 4 illustrate the trends of the current approximated by the analytical derivative and by finite differences as the number of segments is increased from 10 to 20 and then to 60. Assuming that the current approximated by 60 segments is close to an actual experimental sampling at those segments, we can deduce, from the plots of the approximations with 10 and 20 segments, that 10 segments is not sufficient and that 20 segments is sufficient. In fact, if we were able to superpose the plot with 20 segments over the one with 60 segments, we would see that they were very close in numerical value but that the one with 20 segments had less resolution.

The electric field, at a distance of 50 wavelengths, calculated from the approximations of the currents on 20 segments, is shown in Table 1. The differences in the electric fields are within 0.2 dB for the main lobes and 2 dB for the low-power angles.

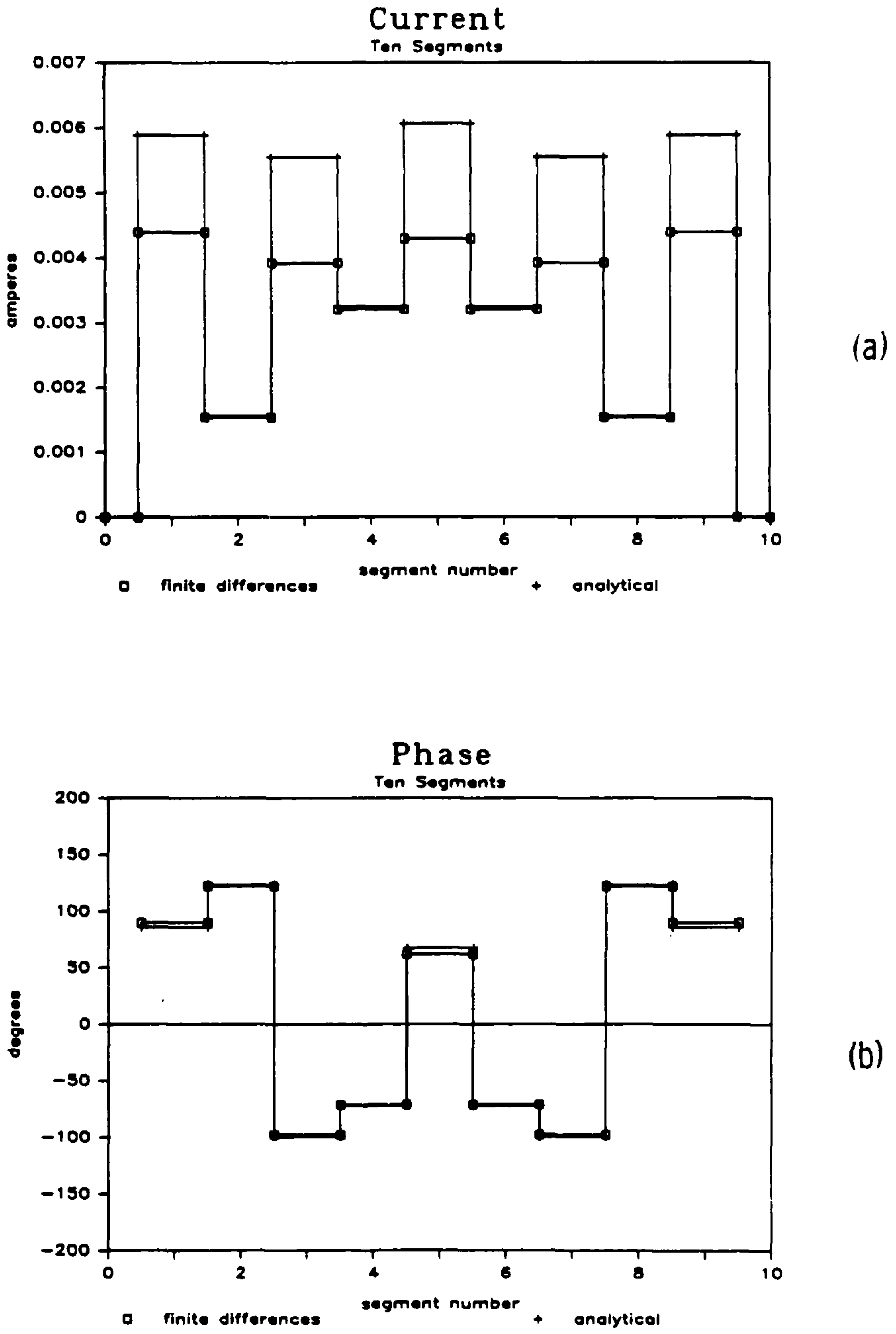


Figure 2. Ten Segment Approximation. Amplitude and phase of the current on a center-fed antenna two wavelengths long, 0.013 wavelengths in radius with 1 V applied.

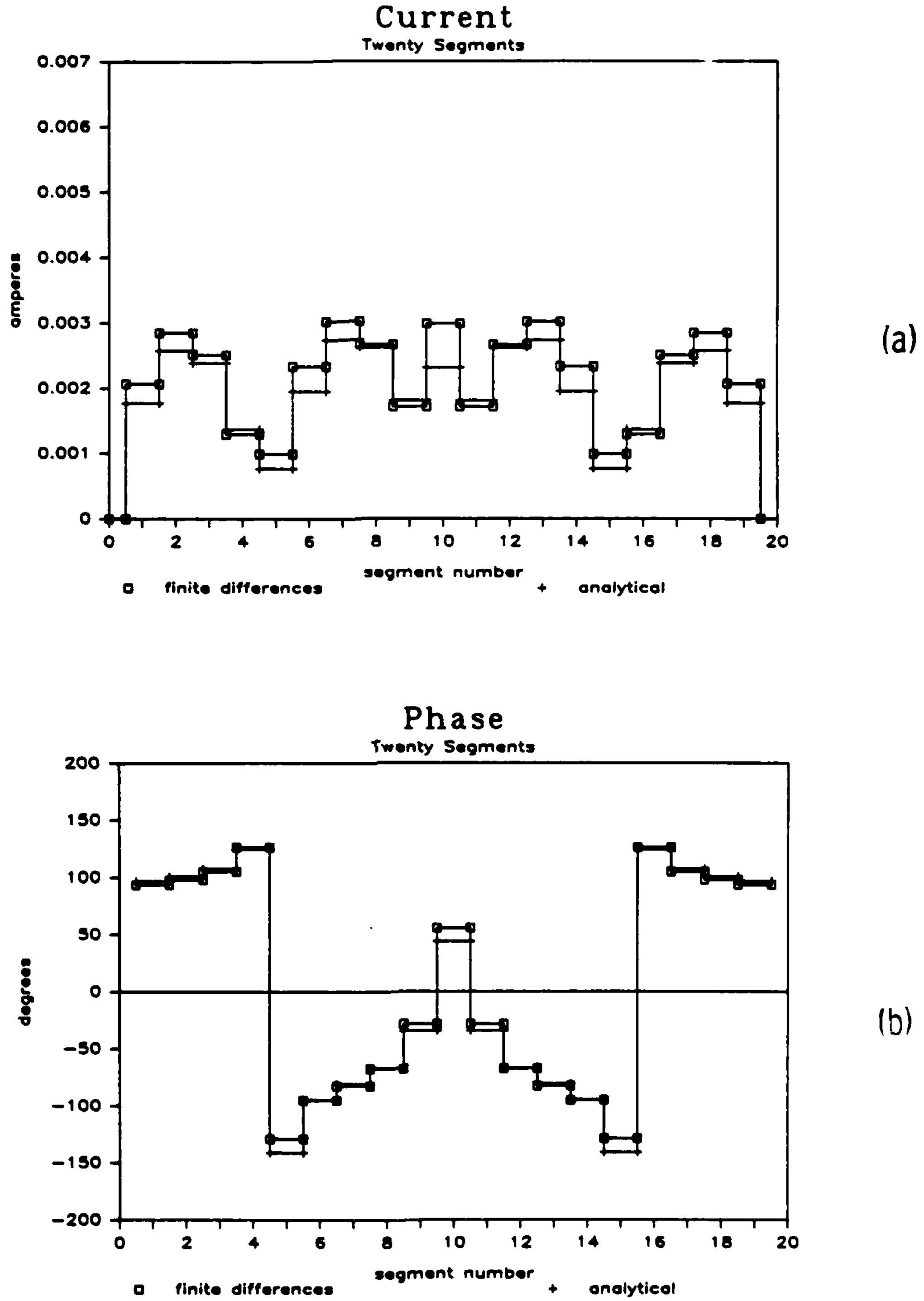


Figure 3. Twenty Segment Approximation. Amplitude and phase of the current on a center-fed antenna two wavelengths long, 0.013 wavelengths in radius with 1 V applied.

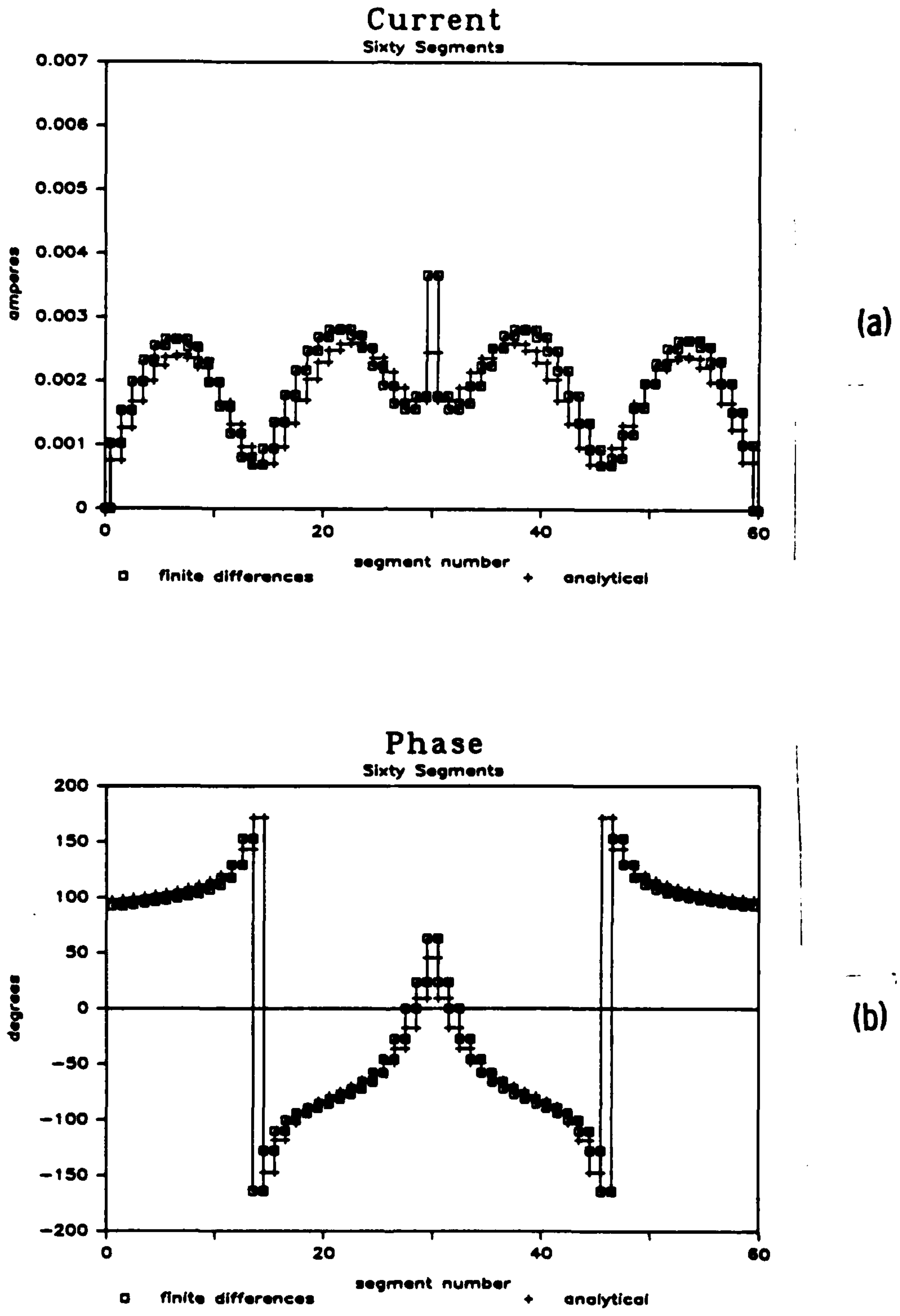


Figure 4. Sixty Segment Approximation. Amplitude and phase of the current on a center-fed antenna two wavelengths long, 0.013 wavelengths in radius with 1 V applied.

Table 1. Far-Zone Electric Field

Angle (deg)	Finite Differences (dB)	Analytical (dB)
10	-3.4	-5.3
20	1.0	+0.3
30	5.5	6.3
40	11.7	12.2
50	15.8	15.9
60	17.0	16.8
70	14.4	14.2
80	5.7	5.8
90	-2.3	-3.8

Although the agreement of the far-zone electric field is adequate, the admittance calculated with analytical derivatives and finite differences yields approximately 23 percent difference in magnitude. This magnitude difference for the 2λ antenna is in the same range of differences reported by Butler and Wilton[5] for a 1.5λ antenna. The value of the input admittance is important for matching. The result for 20 segments are given in Table 2.

Table 2. Input Admittance

REAL	
Finite differences	1.687 mmhos
Analytical	1.668 mmhos
IMAGINARY	
Finite differences	2.466 mmhos
Analytical	1.609 mmhos

The calculations were executed with ANSI Standard Fortran 77 on the CDC Cyber 176. Two matrix routines were used to find the solution of the matrix equation: (1) the conjugate gradient algorithm described in this report and (2) an IMSL library routine that uses L-U decomposition and iterative improvement. The execution time varied with the method used for the linear equation solution and the method used for evaluating the second derivative of the Green's function. For the second derivative evaluated by finite differences on a wire approximated by 20 segments, the times for different linear equation methods are given in Table 3.

Table 3. CPU Times for Finite Differences

Conjugate gradient	2.3 cpu sec
L-U decomposition	1.9 cpu sec

The times for different second derivative methods using the conjugate gradient method are given in Table 4.

Table 4. CPU Times for the Conjugate Gradient Method

Finite difference	2.3 cpu sec
Analytical derivative	5.3 cpu sec

The times given in Tables 3 and 4 are the total cpu times for calculating the current, input admittance, and far-zone electric field. Execution times for the conjugate gradient method were made with an initial current of zero at every segment. Because the conjugate gradient method is iterative, the execution time depends upon the residual computed from the initial current array.

For this antenna, for which the current is symmetric about the center segment, the conjugate gradient method arrived at a solution in N/2 iterations. In Figure 5, the logarithm of the relative error is shown versus the iteration number for 10, 20, and 60 segments. The relative error approaches the tolerance of 10^{-8} monotonically in each case.

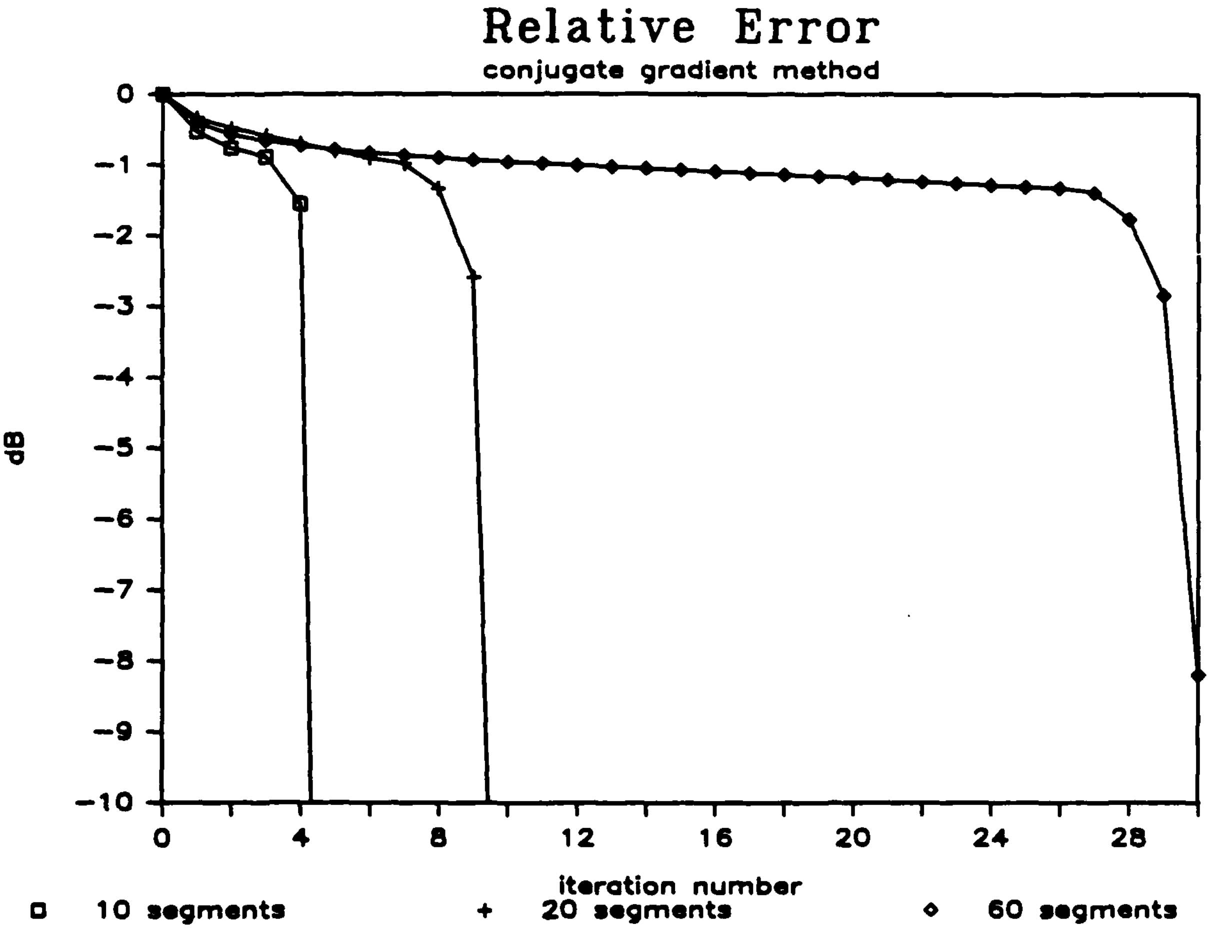


Figure 5. The Logarithm of the Relative Error Versus the Number of Iterations of the Conjugate Gradient Algorithm for 10, 20, and 60 Segments

VI. CONCLUSIONS

In this report, the numerical methods are discussed for analyzing a linear antenna using the method of moments. Within the method of moments there are several options to consider: which basis and testing functions to use, how to evaluate the second derivative, and which linear equation method to use. The criteria used in selecting the options may be as follows:

1. Agreement of numerical results with previously accepted results.
2. Ease of implementation.
3. Computer execution time.
4. Computer memory requirements.

A summary of the options discussed in this paper and how they meet the criteria is given subsequently.

A. ANALYTICAL DERIVATIVE VERSUS FINITE DIFFERENCES

The far-zone electric fields predicted by each of these methods for computing the second derivative are in relative agreement, but the input admittances and execution times differ. The finite difference calculation takes less cpu time, but the resultant input admittance differs by 23 percent from the admittance calculated with the analytical second derivative.

B. CONJUGATE GRADIENT VERSUS IMSL LIBRARY ROUTINE

The results of the conjugate gradient and IMSL routines for the current, admittance, and far field are equal, but the routines differ in memory requirements and execution times. The IMSL routine takes less execution time but requires more memory than the conjugate gradient routine.

Numerical results in this report should be valuable in matching the options to specific applications.

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APPENDIX

The three computer programs that are listed compute the current distribution and far-zone electric field for a center-fed antenna using Pocklington's equation and the method of moments. The programs run interactively with voltage source amplitude, phase and frequency, antenna length and radius, and far-zone distance as the input parameters.

Listing 1 evaluates the second derivative analytically and solves the matrix equation using the conjugate gradient algorithm.

Listing 2 evaluates the second derivative by finite differences and solves the matrix equation using the conjugate gradient algorithm.

Listing 3 evaluates the second derivative by finite differences and solves the matrix equation using an IMSL routine.

LISTING NO. 1

```
C
C          PROGRAM WANTCG CALCULATES THE CURRENTS ON A WIRE ANTENNA
C
C       WANTCG CALCULATES THE CURRENTS ON A WIRE ANTENNA WITH THE
C       SOURCE AT THE CENTER.  (C) IS THE CURRENT VECTOR, (V) IS THE
C      APPLIED VOLTAGE VECTOR AND OPERAT IS THE INTEGRAL
C      OPERATOR SUBROUTINE.  WANTCG SOLVES FOR THE
C      CURRENTS USING THE CONJUGATE GRADIENT METHOD.
      PROGRAM WANTCG(INPUT,TAPE5=INPUT,OUTPUT,TAPE6=OUTPUT)
      PARAMETER(NDIM = 70)
      PARAMETER(NDIM2 =-NDIM)
      DIMENSION CMOD(NDIM),CARG(NDIM),VMOD(NDIM),VARG(NDIM)
      COMPLEX C(NDIM),PSI(NDIM2:NDIM),V(NDIM),VA,COEFF,E,COEFF2
      COMPLEX R(NDIM),P(NDIM),Q(NDIM)
      REAL MU,L,K
      COMMON PI,K,MU,EPS,W
      OPEN(UNIT=4,FILE='CURRENT')
      REWIND 4
C
C        DECLARE CONSTANTS EPS AND MU, PERMITTIVITY AND PERMEABILITY
C
    5 EPS = 8.84194E-12
      MU = 1.256637E-06
      PI = 3.14159
C
C          READ IN FREQUENCY, LENGTH AND RADIUS OF THE WIRE, VOLTAGE
C          THE CENTER OF THE WIRE, AND THE NUMBER OF SEGMENTS.
C
   10 WRITE(6,900)
      READ *, F
      WRITE(6,905)
      READ *, L.A
      WRITE(6,910)
   15 READ *,VA
      WRITE(6,915)
      READ *, N
      WRITE(6,916)
  916 FORMAT(2X,'INPUT FAR-FIELD DISTANCE',/)
      READ *,RO
C
C         CALCULATE ANGULAR FREQUENCY, WAVE NUMBER, SEGMENT LENGTH
C
      N1 = N*(N+2)
   20 W = 2*PI*F
      K=W*SQRT(MU*EPS)
      DL = L/(FLOAT(N) + 1.)
C
C          ASSIGN VOLTAGE VECTOR
C
      CALL VOLTS(N,V,VA)
C
C
C          INITIALIZE CURRENT VECTOR
      DO 25 I=1,N
         C(I) = (0.,0.)
   25 CONTINUE
C          ASSIGN IMPEDANCE MATRIX
C
      CALL INITPSI(N,PSI,A,DL)
C
C          CALCULATE CURRENT VECTOR
```

```
C
      CALL CONGRAD(N,C,V,R,P,Q,PSI)
      DO 30 I = 1,N
      CMOD(I) = CABS(C(I))
      CI = AIMAG(C(I))
      CR = REAL(C(I))
      CARG(I) = ATAN2(CI,CR)*180/PI
      WRITE(6,920) I,CMOD(I),CARG(I)
   30 CONTINUE
      WRITE(4,925)N
  925 FORMAT(I3)
      WRITE(4,930)
  930 FORMAT('C, MODULUS OF ')
      WRITE(4,935)(CMOD(I),CMOD(I),I=1,N)
  935 FORMAT(50(E16.8,/))
      WRITE(4,940)
  940 FORMAT('C, ARGUMENT OF')
      WRITE(4,945)(CARG(I),CARG(I),I=1,N)
  945 FORMAT(50(F10.3,/))
      COEFF = (0.,1.)*SQRT(MU/EPS)*CEXP((0.,-1.)*K*R0)/(2.*PI*R0)
      DO 56 J=0,18
         THETA = 10.*J
         THETAR = PI/180.*THETA
         E = (0.,0.)
         PHI = K*COS(THETAR)
         COEFF2 = TAN(THETAR)*COEFF*SIN(0.5*DL*PHI)
         DO 57 I = 1,N
            R1 = DL*(FLOAT(I)-0.5*FLOAT(N+1))
            E = E + C(I)*COS(PHI*R1)
   57    CONTINUE
         E = COEFF2*E
         EMOD = CABS(E)
         WRITE(6,956)THETA,EMOD
         WRITE(4,956)THETA,EMOD
  956    FORMAT(2X,'THETA = ',F5.1,2X,'E = ',E10.4)
   56 CONTINUE
  900 FORMAT(2X,'INPUT FREQUENCY IN HERTZ',/)
  905 FORMAT(2X,'INPUT LENGTH AND RADIUS OF THE WIRE',/)
  910 FORMAT(2X,'INPUT COMPLEX VOLTAGE APPLIED TO THE CENTER OF THE',

     1' WIRE IN THE FORM (VR,VI)',/)
  915 FORMAT(2X,'INPUT THE NUMBER OF SEGMENTS = ODD INTEGER',/)
  920 FORMAT(2X,I3,5X,E12.4,5X,F7.2)
      STOP
      END
C
C
C           SUBROUTINE VOLTS ASSIGNS VOLTAGE VECTOR
C
C
      SUBROUTINE VOLTS(N,V,VA)
      COMPLEX V(N),VA
      I = 0
    5 IF(2*I+1.EQ.N)GO TO 10
      V(I+1) = (0.,0.)
      V(N-I) = (0.,0.)
```

```
      I = I + 1
      GO TO 5
   10 V(I+1)=VA
      RETURN
      END
C
C
C        CONGRAD ITERATES TO THE SOLUTION VECTOR C
C
C
      SUBROUTINE CONGRAD(N,C,V,R,P,O,PSI)
      COMPLEX C(N),V(N),R(N),P(N),O(N),PSI(-N:N)
      CALL SQUARE(N,V,V2)
      CALL OPERAT(N,C,PSI,O)
      J = 0
      DO 100 I=1,N
         R(I) = O(I)-V(I)
  100 CONTINUE
      CALL SQUARE(N,R,R2)
      ERROR = R2/V2
      IF(ERROR.LT.1E-8)GO TO 800
      CALL COPERAT(N,R,PSI,O)
      CALL SQUARE(N,O,B)
      IF(B.LT.1E-08)GO TO 2
      B = 1/B
      GO TO 3
    2 B = 1.E+08
    3 CONTINUE
      DO 200 I=1,N
         P(I) = -B*O(I)
  200 CONTINUE
      CALL OPERAT(N,P,PSI,O)
      CALL SQUARE(N,O,T)
      IF(T.LT.1E-8)GO TO 5
      T = 1/T
      GO TO 6
    5 T = 1.E+08
    6 WRITE(6,900)J,ERROR
  900 FORMAT(2X,'ITERATION',I3,2X,'ERROR = ',E10.4 )
    1 J = J+1
      DO 300 I=1,N
         C(I) = C(I)+T*P(I)
  300 CONTINUE
      CALL OPERAT(N,P,PSI,O)
      DO 400 I=1,N
         R(I) = R(I)+T*O(I)
  400 CONTINUE
      CALL OPERAT(N,C,PSI,O)
      DO 15 I=1,N
         O(I) = O(I) - V(I)
   15 CONTINUE
      CALL SQUARE(N,O,ERROR)
      ERROR = ERROR/V2
      IF(ERROR.LT.1E-8)GO TO 800
      CALL COPERAT(N,R,PSI,O)
```

```
1810         CALL SQUARE(N,O,B)
1820         IF(B.LT.1E-8)GO TO 18
1830         B = 1/B
1840         GO TO 19
1850    18   B = 1.E+08
1860    19   CONTINUE
1870         DO 500 I=1,N
1880            P(I) = P(I)-B*O(I)
1890   500   CONTINUE
1900         CALL OPERAT(N,P,PSI,O)
1910         CALL SQUARE(N,O,T)
1920         IF(T.LT.1E-8)GO TO 16
1930         T = 1/T
1940         GO TO 17
1950    16   T = 1.E+0  1960    17  WRITE(6,900)J,ERROR
1970       IF(J.LT.100)GO TO 1
1980         WRITE(6,930)
1990   930   FORMAT(2X,'FAILED TO CONVERGE AFTER 100 ITERATIONS')
2000         GO TO 801
2010   800   CONTINUE
2020         WRITE(6,900)J,ERROR
2030         WRITE(6,920)
2040   920   FORMAT(2X,'THE ANSWER IS')
2050   801   RETURN
2060         END
2070         SUBROUTINE GAUSS(FCN,D,R,U,N,PSI)
2080 C
2090 C           96 POINT GAUSSIAN RULE INTEGRATION
2100 C
2110         COMPLEX Z,PSI,FCN,PSI1,PSI2
2120         DIMENSIOAA(48),W(48)
2130         DATA A/.99968950E00, .99836438E00, .99598184E00, .99254390E00,
2140        1 .98805413E 00, .98251726E 00, .97593917E 00, .96832683E 00,
2150        1 .95968829E 00, .95003272E 00, .93937034E 00, .92771246E 00,
2160        1 .91507142E 00, .90146064E 00, .88689452E 00, .87138851E 00,
2170        1 .85495903E 00, .83762351E 00, .81940031E 00, .80030874E 00,
2180        1 .78036904E 00, .75960234E 00, .73803064E 00, .71567681E 00,
2190        1 .69256454E 00, .66871831E 00, .64416340E 00, .61892584E 00,
2200        1 .59303236E 00, .56651042E 00, .53938811E 00, .51169418E 00,
2210        1 .48345797E 00, .45470942E 00, .42547899E 00, .39579765E 00,
2220        1 .36569686E 00, .33520852E 00, .30436494E 00, .27319881E 00,
2230        1 .24174316E 00, .21003131E 00, .17809688E 00, .14597371E 00,
2240        1 .11369585E 00, .81297495E-01, .48812985E-01, .16276745E-01 /
2250         DATA W/.79679207E-03,.18539608E-02,.29107318E-02,.39645543E-02,

2260        1 .50142027E-02, .60585455E-02, .70964708E-02, .81268769E-02,
2270        1 .91486712E-02, .10160771E-01, .11162102E-01, .12151605E-01,
2280        1 .13128230E-01, .14090942E-01, .15038721E-01, .15970563E-01,
2290        1 .16885480E-01, .17782502E-01, .18660680E-01, .19519081E-01,
2300        1 .20356797E-01, .21172940E-01, .21966644E-01, .22737070E-01,
2310        1 .23483399E-01, .24204842E-01, .24900633E-01, .25570036E-01,
2320        1 .26212341E-01, .26826867E-01, .27412963E-01, .27970008E-01,
2330        1 .28497411E-01, .28994614E-01, .29461090E-01, .29896344E-01,
2340        1 .30299915E-01, .30671376E-01, .31010333E-01, .31316426E-01,
2350        1 .31589331E-01, .31828759E-01, .32034456E-01, .32206205E-01,
```

```
     1 .32343823E-01, .32447164E-01, .32516119E-01, .32550614E-01/
      NPOINT = 48
      PSI = (0.,0.)
      G = U/2./FLOAT(N)
      DO 2 I = 1,N
      Z = (0.,0.)
      DO 1 K = 1,NPOINT
      ARGA = G*(A(K) + 2*I-1.)
      ARGB = G*(-A(K) + 2*I-1.)
      Z = Z + (FCN(ARGA,D,R) + FCN(ARGB,D,R))*W(K)
    1 CONTINUE
      PSI = PSI + Z
    2 CONTINUE
      PSI1 = G*PSI
      PSI = (0.,0.)
      G = -G
      DO 4 I = 1,N
      Z = (0.,0.)
      DO 3 K = 1,NPOINT
      ARGA = G*(A(K) + 2*I-1.)
      ARGB = G*(-A(K) + 2*I-1.)
      Z = Z + (FCN(ARGA,D,R) + FCN(ARGB,D,R))*W(K)
  3   CONTINUE
      PSI = PSI + Z
  4   CONTINUE
      PSI2 = G*PSI
      PSI = PSI1 - PSI2
      RETURN
      END
C
C
C           INITPSI CALCULATES THE PSI VALUES
C
C
      SUBROUTINE INITPSI(N,PSI,A,DL)
      COMPLEX PSI(-N:N),CINT,GFUNCT,F
      REAL K,MU
      EXTERNAL CINT,AFUNCT,BFUNCT,GFUNCT
      COMMON PI,K,MU,EPS,W
      U=0.5*DL
      F = DL/((0.,-1.)*W*4.*PI*EPS)
      DO 1 I=-N,N
         D = I*DL
         CALL GAUSS(CINT,D,A,U,1,PSI(I))
      PSI(I) = F*PSI(I)
   1  CONTINUE
      RETURN
      END
C
C
C           CINT CALCULATES THE COMPLEX INTEGRAND
C
C
      COMPLEX FUNCTION CINT(ARG,D1,A1)
      COMPLEX GFUNCT,B,D
```

```
      REAL K
      COMMON PI,K
      A = K**2
      B = (0.,-1.)*K/(BFUNCT(ARG,D1,A1)**.5)
      C = -(1 + (K**2)*AFUNCT(ARG,D1))/BFUNCT(ARG,D1,A1)
      D = 3*(0.,1.)*K*AFUNCT(ARG,D1)/(BFUNCT(ARG,D1,A1)**1.5)
      E = 3*AFUNCT(ARG,D1)/(BFUNCT(ARG,D1,A1)**2)
      CINT = (A + B + C + D + E)*GFUNCT(ARG,D1,A1)
      RETURN
      END
C
C
      FUNCTION AFUNCT(ARG,D)
      AFUNCT = (ARG-D)**2
      RETURN
      END
C
C
      FUNCTION BFUNCT(ARG,D,A1)
      BFUNCT = AFUNCT(ARG,D) + A1**2
      RETURN
      END
C
C
C        GFUNCT CALCULATES THE GREEN/UNCTION
C
C
      COMPLEX FUNCTION GFUNCT(ARG,D,A1)
      REAL K
      COMMON PI,K
      R = SQRT(BFUNCT(ARG,D,A1))
      GFUNCT = CEXP((0.,-1.)*K*R)/R
      RETURN
      END
C
C
C        COPERAT IS THE ADJOINT OPERATOR
C
C
      SUBROUTINE COPERAT(N,R,PSI,O)
      COMPLEX PSI(-N:N),R(N),O(N)
    1 CONTINUE
      DO 200 I = 1,N
        O(I) = (0.,0.)
        DO 100 J=1,N
          M =I-J
          O(I) = O(I) + CONJG(PSI(M))*R(J)
  100   CONTINUE
  200 CONTINUE
      RETURN
      END
C
C
C       SQUARE MULTIPLIES A VECTOR BY ITS COMPLEX CONJUGATE
C
```

```
C
      SUBROUTINE SQUARE(N,O1,O2)
      COMPLEX O1(N)
      O2=0.
      DO 100 I=1,N
         O2 = O2 + O1(I)*CONJG(O1(I))
  100 CONTINUE
      RETURN
      END
C
C
C           OPERAT IS THE INTEGRAL OPERATOR
C
C
      SUBROUTINE OPERAT(N,C,PSI,O)
      COMPLEX PSI(-N:N),C(N),O(N)
      DO 200 I=1,N
         O(I) = (0.,0.)
         DO 100 J=1,N
            M = I-J
           O(I) = O(I) + PSI(M)*C(J)
  100    CONTINUE
  200 CONTINUE
      RETURN
      END
```

LISTING NO. 2

```
C
C         PROGRAM WANTFD CALCULATES THE CURRENTS ON A WIRE ANTENNA
C
C      WANTFD CALCULATES THE CURRENTS ON A WIRE ANTENNA WITH THE
C      SOURCE AT THE CENTER.  (C) IS THE CURRENT VECTOR, (V) IS THE
C     APPLIED VOLTAGE VECTOR AND OPERAT IS THE INTEGRAL
C     OPERATOR SUBROUTINE.  WANTFD SOLVES FOR THE
C     CURRENTS USING THE CONJUGATE GRADIENT METHOD.
      PROGRAM WANTFD(INPUT,TAPE5=INPUT,OUTPUT,TAPE6=OUTPUT)
      PARAMETER(NDIM = 70)
      PARAMETER(NDIM2 =-NDIM)
      DIMENSION CMOD(NDIM),CARG(NDIM),VMOD(NDIM),[illegible](NDIM)
      COMPLEX C(NDIM),PSI(NDIM2:NDIM),V(NDIM),[illegible],COEFF2
      COMPLEX R(NDIM),P(NDIM),Q(NDIM),[illegible]Z(NDIM2:NDIM)
      REAL MU,L,K
      COMMON PI,[illegible],W
      OPEN([illegible],FILE='CURRENT')
      [illegible]
C
C          DECLARE CONSTANTS EPS AND MU, PERMITTIVITY AND PERMEABILITY
C
    5 EPS = 8.84194E-12
      MU = 1.256637E-06
      PI = 3.14159
C
C         READ IN FREQUENCY, LENGTH AND RADIUS OF THE WIRE, VOLTAGE
C         THE CENTER OF THE WIRE, AND THE NUMBER OF SEGMENTS.
C
   10 WRITE(6,900)
      READ *, F
      WRITE(6,905)
      READ *, L,A
      WRITE(6,910)
   15 READ *,VA
      WRITE(6,915)
      READ *, N
      WRITE(6,[illegible])
 [illegible] FORMAT(1X,'INPUT FAR FIELD DISTANCE',[illegible])
      READ *,R[illegible]
C
C         CALCULATE ANGULAR FREQUENCY, WAVE NUMBER, SEGMENT LENGTH
C
      N1 = [illegible]
   20 W = 2*PI*F
      K=W*SQRT(MU*EPS)
      DL = L/FLOAT(N[illegible])
C
C         [illegible] VOLTAGE VECTOR
C
       CALL VOLTS(N,V,VA)
C
C
C         INITIALIZE CURRENT VECTOR
      DO 25 I=1,N
         C(I) = (0.,0.)
   25 CONTINUE
C         ASSIGN IMPEDANCE MATRIX
C
      CALL INITPSI(N,PSI,A,DL,K)
C
C         CALCULATE CURRENT VECTOR
```

```
C
      CALL CONGRAD(N,C,V,K,R,U,Z)
      DO 30 I = 1,N
      CMOD(I) = CABS(C(I))
      CI = AIMAG(C(I))
      CR = REAL(C(I))
      CARG(I) = ATAN2(CI,CR)*180/PI
      WRITE(6,920) I,CMOD(I),CARG(I)
   30 CONTINUE
      WRITE(4,925)N
  925 FORMAT(I3)
      WRITE(4,930)
  930 FORMAT('C, MODULUS OF ')
      WRITE(4,935)(CMOD(I),CMOD(I),I=1,N)
  935 FORMAT(50(E16.8,/))
      WRITE(4,940)
  940 FORMAT('C, ARGUMENT OF')
      WRITE(4,945)(CARG(I),CARG(I),I=1,N)
  945 FORMAT(50(F10.3,/))
      COEFF = (0.,1.)*SQRT(MU/EPS)*CEXP((0.,-1.)*K*R0)/(2.*PI*R0)
      DO 56 J=0,18
         THETA = 10.*J
         THETAR = PI/180.*THETA
         E = (0.,0.)
         PHI = K*COS(THETAR)
         COEFF2 = TAN(THETAR)*COEFF*SIN(0.5*DL*PHI)
         DO 57 I = 1,N
            R1 = DL*(FLOAT(I)-0.5*FLOAT(N+1))
            E = E + C(I)*COS(PHI*R1)
   57    CONTINUE
         E = COEFF2*E
         EMOD = CABS(E)
         WRITE(6,956)THETA,EMOD
         WRITE(4,956)THETA,EMOD
  956    FORMAT(2X,'THETA = ',F5.1,2X,'E = ',E10.4)
   56 CONTINUE
  900 FORMAT(2X,'INPUT FREQUENCY IN HERTZ',/)
  905 FORMAT(2X,'INPUT LENGTH AND RADIUS OF THE WIRE',/)
  910 FORMAT(2X,'INPUT COMPLEX VOLTAGE APPLIED TO THE CENTER OF THE',

     1 WIRE IN THE FORM (VR,VI)',/)
  915 FORMAT(2X,'INPUT THE NUMBER OF SEGMENTS = ODD INTEGER',/)
  920 FORMAT(2X,I3,5X,E12.4,5X,F7.2)
      STOP
      END
C
C
C         SUBROUTINE VOLTS ASSIGNS VOLTAGE VECTOR
C
C
      SUBROUTINE VOLTS(N,V,VA)
      COMPLEX V(N),VA
      I = 0
    5 IF(2*I+1.EQ.N)GO TO 10
      V(I+1) = (0.,0.)
      V(N-I) = (0.,0.)
```

```
1260          I = I + 1
1270          GO TO 5
1280   10     V(I+1)=VA
1290          RETURN
1300          END
1310 C
1320 C
1330 C            CONGRAD ITERATES TO THE SOLUTION VECTOR C
1340 C
1350 C
1360          SUBROUTINE CONGRAD(N,C,V,R,P,O,Z)
1370          COMPLEX C(N),V(N),R(N),P(N),O(N),Z(-N:N)
1380          CALL SQUARE(N,V,V2)
1390          CALL OPERAT(N,C,Z,O)
1400          J = 0
1410          DO 100 I=1,N
1420             R(I) = V(I)
1430   100    CONTINUE
1440          CALL SQUARE(N,R,R2)
1450          ERROR = R2/V2
1460          IF(ERROR.LT.1E-8)GO TO 800
1470          CALL COPERAT(N,R,Z,O)
1480          CALL SQUARE(N,O,B)
1490          IF(B.LT.1E-08)GO TO 2
1500          B = 1/B
1510          GO TO 3
1520    2     B = 1.E+08
1530    3     CONTINUE
1540          DO 200 I=1,N
1550             P(I) = -B*O(I)
1560   200    CONTINUE
1570          CALL OPERAT(N,P,Z,O)
1580          CALL SQUARE(N,O,T)
1590          IF(T LT.1E-8)GO TO 5
1600          T = 1/T
1610          GO TO 6
1620    5     T = 1.E+08
1630    6     WRITE(6,900)J,ERROR
1640   900    FORMAT(2X,'ITERATION ',I3,2X,'ERROR = ',E10.4)
1650    1     J = J+1
1660          DO 300 I=1,N
1670             C(I) = C(I)+T*P(I)
1680   300    CONTINUE
1690          CALL OPERAT(N,P,Z,O)
1700          DO 400 I=1,N
1710             R(I) = R(I)+T*O(I)
1720   400    CONTINUE
1730          CALL OPERAT(N,C,Z,O)
1740          DO 15 I=1,N
1750             O(I) = O(I) - V(I)
1760    15    CONTINUE
1770          CALL SQUARE(N,O,ERROR)
1780          ERROR = ERROR/V2
1790          IF(ERROR.LT.1E-8)GO TO 800
1800          CALL COPERAT(N,R,Z,O)
```

```
      CALL SQUARE(N,D,B)
      IF(B.LT.1E-8)GO TO 18
      B = 1/B
      GO TO 19
   18 B = 1.E+08
   19 CONTINUE
      DO 500 I=1,N
         F(I) = F(I)-B*U(I)
  500 CONTINUE
      CALL OPERAT(N,F,Z,U)
      CALL SQUARE(N,D,T)
      IF(T.LT.1E-8)GO TO 16
      T = 1/T
      GO TO 17
   16 T = 1.E+08
   17 WRITE(6,900)J,ERROR
      IF(J.LT.100)GO TO 1
      WRITE(6,930)
  930 FORMAT(2X,'FAILED TO CONVERGE AFTER 100 ITERATIONS')
      GO TO 801
  800 CONTINUE
      WRITE(6,900)J,ERROR
      WRITE(6,920)
  920 FORMAT(2X,'THE ANSWER IS ')
  801 RETURN
      END
      SUBROUTINE GAUSS(FCN,D,R,U,N,PSI)
C
C         96 POINT GAUSSIAN RULE INTEGRATION
C
      COMPLEX Z,PSI,FCN,PSI1,PSI2
      DIMENSION A(48),W(48)
      DATA A/.99968950E00, .99836438E00, .99598184E00, .99254390E00,
     1 .98805413E 00, .98251726E 00, .97593917E 00, .96832683E 00,
     1 .95968829E 00, .95003272E 00, .93937034E 00, .92771246E 00,
     1 .91507142E 00, .90146064E 00, .88689452E 00, .87138851E 00,
     1 .85495903E 00, .83762351E 00, .81940031E 00, .80030874E 00,
     1 .78036904E 00, .75960234E 00, .73803064E 00, .71567681E 00,
     1 .69256454E 00, .66871831E 00, .64416340E 00, .61892584E 00,
     1 .59303236E 00, .56651042E 00, .53938811E 00, .51169418E 00,
     1 .48345797E 00, .45470942E 00, .42547899E 00, .39579765E 00,
     1 .36569686E 00, .33520852E 00, .30436494E 00, .27319881E 00,
     1 .24174316E 00, .21003131E 00, .17809688E 00, .14597371E 00,
     1 .11369585E 00, .81297495E-01, .48812985E-01, .16276745E-01 /
      DATA W/.79679207E-03,.18539608E-02,.29107318E-02,.39645543E-02,

     1 .50142027E-02, .60585455E-02, .70964708E-02, .81268769E-02,
     1 .91486712E-02, .10160771E-01, .11162102E-01, .12151605E-01,
     1 .13128230E-01, .14090942E-01, .15038721E-01, .15970563E-01,
     1 .16885480E-01, .17782502E-01, .18660680E-01, .19519081E-01,
     1 .20356797E-01, .21172940E-01, .21966644E-01, .22737070E-01,
     1 .23483399E-01, .24204842E-01, .24900633E-01, .25570036E-01,
     1 .26212341E-01, .26826867E-01, .27412963E-01, .27970008E-01,
     1 .28497411E-01, .28994614E-01, .29461090E-01, .29896344E-01,
     1 .30299915E-01, .30671376E-01, .31010333E-01, .31316426E-01,
     1 .31589331E-01, .31828759E-01, .32034456E-01, .32206205E-01,
```

```
     1 .32343823E-01, .32447164E-01, .32516119E-01, .32550614E-01/
      NPOINT = 48
      PSI = (0.,0.)
      G = U/2./FLOAT(N)
      DO 2 I = 1,N
      Z = (0.,0.)
      DO 1 K = 1,NPOINT
      ARGA = G*(A(K) + 2*I-1.)
      ARGB = G*(-A(K) + 2*I-1.)
      Z = Z + (FCN(ARGA,D,R) + FCN(ARGB,D,R))*W(K)
    1 CONTINUE
      PSI = PSI + Z
    2 CONTINUE
      PSI1 = G*PSI
      PSI = (0.,0.)
      G = -G
      DO 4 I = 1,N
      Z = (0.,0.)
      DO 3 K = 1,NPOINT
      ARGA = G*(A(K) + 2*I-1.)
      ARGB = G*(-A(K) + 2*I-1.)
      Z = Z + (FCN(ARGA,D,R) + FCN(ARGB,D,R))*W(K)
  3   CONTINUE
      PSI = PSI + Z
  4   CONTINUE
      PSI2 = G*PSI
      PSI = PSI1 - PSI2
      RETURN
      END
C
C
C          INITPSI CALCULATES THE PSI VALUES
C
C
      SUBROUTINE INITPSI(N,PSI,A,DL,Z)
      COMPLEX PSI(-N:N),CINT,PSIA,PSIB,PSIC,PSID,PSIE,Z(-N:N)
      REAL K,MU
      EXTERNAL CINT
      COMMON PI,K,MU,EPS,W
      U=0.5*DL
      DO 1 I=-N,N
         D = I*DL
         CALL GAUSS(CINT,D,A,U,1,PSI(I))
  1   CONTINUE
      DO 200 J = 1,N
         DO 100 I = 1,N
            M = I-J
           PSIA = PSI(M)/DL
            PSIB = PSIA
            M = I-J+1
            PSIC = PSI(M)/DL
            M = I-J-1
            PSID = PSI(M)/DL
            PSIE = PSIA
      M = I-J
```

```
                Z(M) = (0.,1.)*W*MU*DL**2.*PSIA
                Z(M) = Z(M) + 1./W/(0.,1.)/EPS*(PSIB+PSIC+PSID+PSIE)
  100       CONTINUE
  200   CONTINUE
        RETURN
        END
C
C
C            CINT CALCULATES THE COMPLEX INTEGRAND
C
C
        COMPLEX FUNCTION CINT(ARG,D,A)
        REAL K
        COMMON PI,K
        R = SQRT(A**2 + (ARG-D)**2)
       CINT = CEXP((0.,-1.)*K*R)/(4*PI*R)
        RETURN
        END
C
C
C            COPERAT IS THE ADJOINT OPERATOR
C

        SUBROUTINE COPERAT(N,R,Z,O)
        COMPLEX R(N),O(N),Z(-N:N)
        DO 200 I = 1,N
           O(I) = (0.,0.)
           DO 100 J = 1,N
              M = I-J
              O(I) = O(I) + CONJG(Z(M))*R(J)
  100      CONTINUE
  200   CONTINUE
        RETURN
        END
C
C
C          SQUARE MULTIPLIES A VECTOR BY ITS COMPLEX CONJUGATE
C
C
        SUBROUTINE SQUARE(N,O1,O2)
        COMPLEX O1(N)
        O2=0.
        DO 100 I=1,N
           O2 = O2 + O1(I)*CONJG(O1(I))
  100   CONTINUE
        RETURN
        END
C
C
C            OPERAT IS THE INTEGRAL OPERATOR
C
C
        SUBROUTINE OPERAT(N,C,Z,O)
        COMPLEX C(N),O(N),Z(-N:N)
        DO 200 I=1,N
           O(I) = (0.,0.)
           DO 100 J = 1,N
              M = I-J
              O(I) = O(I) + Z(M)*C(J)
  100      CONTINUE
  200   CONTINUE
        RETURN
        END
```

```
                DO 100 J = 1,N
                   H = 1.0
                   [illegible]
      100      CONTINUE
      200   CONTINUE
            RETURN
            END
```

LISTING NO. 3

```
C
C          PROGRAM WANTMOM CALCULATES THE CURRENTS ON A WIRE ANTENNA
C
C        WANTMOM CALCULATES THE CURRENTS ON A WIRE ANTENNA WITH THE
C        SOURCE AT THE CENTER.  [Z] IS THE IMPEDANCE MATRIX, (C) IS
C        THE CURRENT VECTOR, AND (V) IS THE APPLIED VOLTAGE VECTOR.
C        WANTMOM SOLVES FOR THE CURRENTS USING THE METHOD OF MOMENTS.
C
      PROGRAM WANTMOM(INPUT,TAPE5=INPUT,OUTPUT,TAPE6=OUTPUT)
      PARAMETER(NDIM = 70)
      PARAMETER(NDIM2 = NDIM**2,NDIM3 = -NDIM,NDIM4 = NDIM*(NDIM+2))
      DIMENSION CMOD(NDIM),CARG(NDIM),VMOD(NDIM),VARG(NDIM)
      COMPLEX Z(NDIM2),C(NDIM),PSI(NDIM3:NDIM),V(NDIM),VA
      COMPLEX WA(NDIM4),WK(NDIM),COEFF,COEFF2,E
      REAL MU,L,K
      COMMON PI,K,MU,EPS,W
      OPEN(UNIT=4,FILE='CURRENT')
      REWIND 4
C
C          DECLARE CONSTANTS EPS AND MU, PERMITTIVITY AND PERMEABILITY
C
    5 EPS = 8.84194E-12
      MU = 1.256637E-06
      PI = 3.14159
C
C            READ IN FREQUENCY, LENGTH AND RADIUS OF THE WIRE, VOLTAGE
C            THE CENTER OF THE WIRE, AND THE NUMBER OF SEGMENTS.
C
   10 WRITE(6,900)
      READ *, F
      WRITE(6,905)
      READ *, L,A
      WRITE(6,910)
   15 READ *,VA
      WRITE(6,915)
      READ *, N
      )
  916 FORMAT(2X,'INPUT FAR-FIELD DISTANCE',/)
      READ *,RO
C
C           CALCULATE ANGULAR FREQUENCY, WAVE NUMBER, SEGMENT LENGTH
C
      N1 = N*(N+2)
   20 W = 2*PI*F
      K=W*SQRT(MU*EPS)
      DL = L/(FLOAT(N) + 1.)
C
C            ASSIGN VOLTAGE VECTOR
C
      CALL VOLTS(N,V,VA)
C
C            ASSIGN IMPEDANCE MATRIX
C
      CALL IMP(N,Z,A,DL,PSI)
C
C            CALCULATE CURRENT VECTOR
C
      CALL LEQS(Z,N,N1,V,WA,WK)
      DO 30 I = 1,N
      CMOD(I) = CABS(V(I))
      CI = AIMAG(V(I))
```

```
      CR = REAL(V(I))
      CARG(I) = ATAN2(CI,CR)*180/PI
      WRITE(6,920) I,CMOD(I),CARG(I)
   30 CONTINUE
      DO 40 I = 1,N
         C(I) = V(I)
   40 CONTINUE
      WRITE(4,925)N
  925 FORMAT(I3)
      WRITE(4,930)
  930 FORMAT('C, MODULUS OF')
      WRITE(4,935)(CMOD(I),CMOD(I),I=1,N)
  935 FORMAT(50(E16.8,/))
      WRITE(4,940)
  940 FORMAT('C, ARGUMENT OF')
      WRITE(4,945)(CARG(I),CARG(I),I=1,N)
  945 FORMAT(50(F10.3,/))
      COEFF = (0.,1.)*SQRT(MU/EPS)*CEXP((0.,-1.)*K*R0)/(2.*PI*R0)
      DO 56 J=0,18
         THETA = 10.*J
         THETAR = PI/180.*THETA
         E = (0.,0.)
         PHI = K*COS(THETAR)
         COEFF2 = TAN(THETAR)*COEFF*SIN(0.5*DL*PHI)
         DO 57 I = 1,N
            R1 = DL*(FLOAT(I)-0.5*FLOAT(N+1))
            E = E + C(I)*COS(PHI*R1)
   57    CONTINUE
         E = COEFF2*E
         EMOD = CABS(E)
         WRITE(6,956)THETA,EMOD
        WRITE(4,956)THETA,EMOD
  956    FORMAT(2X,'THETA = ',F5.1,2X,'E = ',E10.4)
   56 CONTINUE
 900  FORMAT(2X,'INPUT FREQUENCY IN HERTZ',/)
 905  FORMAT(2X,'INPUT LENGTH AND RADIUS OF THE WIRE',/)
  910 FORMAT(2X,'INPUT COMPLEX VOLTAGE APPLIED TO THE CENTER OF THE',

     1' WIRE IN THE FORM (VR,VI)' ,/)
 915  FORMAT(2X,'INPUT THE NUMBER OF SEGMENTS = ODD INTEGER',/)
  920 FORMAT(2X,I3,5X,E12.4,5X,F7.2)
      STOP
      END
C
C
C          SUBROUTINE VOLTS ASSIGNS VOLTAGE VECTOR
C
C
      SUBROUTINE VOLTS(N,V,VA)
      COMPLEX V(N),VA
      I = 0
    5 IF(2*I+1.EQ.N)GO TO 10
      V(I+1) = (0.,0.)
      V(N-I) = (0.,0.)
      I = I + 1
      GO TO 5
```

```
   10 V(I+1)=VA
      RETURN
      END
C
C
C           SUBROUTINE IMP ASSIGNS IMPEDANCE MATRIX
C
C
      SUBROUTINE IMP(N,Z,A,DL,PSI)
      COMPLEX Z(N,N),PSI(-N:N),PSIA,PSIB,PSIC,PSID,PSIE,CINT
      REAL K,MU
      EXTERNAL CINT
      COMMON PI,K,MU,EPS,W
C
      U = 0.5*DL
C      INTEGRATE FROM -U TO U TO GET PSIS
      DO 1 I = -N,N
         D = I*DL
C
C           D IS THE DISTANCE FROM INTEGRATING POINT, J TO OBVERSATION

C           POINT, I.
C
         CALL GAUSS(CINT,D,A,U,1,PSI(I))
    1 CONTINUE
      DO 200 J=1,N
    5 DO 100 I=1,N
      M = I-J
      PSIA = PSI(M)/DL
         PSIB = PSIA
   10 M = I-J+1
      PSIC = PSI(M)/DL
      M = I-J-1
      PSID = PSI(M)/DL
         PSIE = PSIA
   15 Z(I,J) = (0.,1.)*W*MU*DL**2.*PSIA
      Z(I,J) = Z(I,J) + 1./W/(0.,1.)/EPS*(PSIB-PSIC-PSID+PSIE)
C     WRITE(6,900) I,J,Z(I,J)
  100 CONTINUE
  200 CONTINUE
  900 FORMAT(2X,'Z(',I3,',',I3,') = ',2E15.8)
      RETURN
      END
C
C
C           CINT CALCULATES THE COMPLEX INTEGRAND
C
C
      COMPLEX FUNCTION CINT(D,A,ARG)
      REAL K
      COMMON PI,K
      R = SQRT(A**2 + (ARG-D)**2)
      CINT = CEXP((0.,-1.)*K*R)/(4*PI*R)
      RETURN
      END
C
```

```
1810 C          LINEAR EQUATION SOLUTION
1820 C
1830       SUBROUTINE LEQS(Z,N,N1,V,WA,WK)
1840       COMPLEX Z(N,N),V(N,1),WA(N1),WK(N)
1850       CALL LEQ2C(Z,N,N,V,1,N,0,WA,WK,IER)
1860       RETURN
1870       END
1880       SUBROUTINE GAUSS(FCN,D,R,U,N,PSI)
1890 C
1900 C        96 POINT GAUSSIAN RULE INTEGRATION
1910 C
1920       COMPLEX Z,PSI,FCN,PSI1,PSI2
1930       DIMENSION A(48),W(48)
1940       DATA A/.99968950E00, .99836438E00, .99598184E00, .99254390E00,
1950      1 .98805413E 00, .98251726E 00, .97593917E 00, .96832683E 00,
1960      1 .95968829E 00, .95003272E 00, .93937034E 00, .92771246E 00,
1970      1 .91507142E 00, .90146064E 00, .88689452E 00, .87138851E 00,
1980      1 .85495903E 00, .83762351E 00, .81940031E 00, .80030874E 00,
1990      1 .78036904E 00, .75960234E 00, .73803064E 00, .71567681E 00,
2000      1 .69256454E 00, .66871831E 00, .64416340E 00, .61892584E 00,
2010      1 .59303236E 00, .56651042E 00, .53938811E 00, .51169418E 00,
2020      1 .48345797E 00, .45470942E 00, .42547899E 00, .39579765E 00,
2030      1 .36569686E 00, .33520852E 00, .30436494E 00, .27319881E 00,
2040      1 .24174316E 00, .21003131E 00, .17809688E 00, .14597371E 00,
2050      1 .11369585E 00, .81297495E-01, .48812985E-01, .16276745E-01 /
2060       DATA W/.79679207E-03,.18539608E-02,.29107318E-02,.39645543E-02,

2070      1 .50142027E-02, .60585455E-02, .70964708E-02, .81268769E-02,
2080      1 .91486712E-02, .10160771E-01, .11162102E-01, .12151605E-01,
2090      1 .13128230E-01, .14090942E-01, .15038721E-01, .15970563E-01,
2100      1 .16885480E-01, .17782502E-01, .18660680E-01, .19519081E-01,
2110      1 .20356797E-01, .21172940E-01, .21966644E-01, .22737070E-01,
2120      1 .23483399E-01, .24204842E-01, .24900633E-01, .25570036E-01,
2130      1 .26212341E-01, .26826867E-01, .27412963E-01, .27970008E-01,
2140      1 .28497411E-01, .28994614E-01, .29461090E-01, .29896344E-01,
2150      1 .30299915E-01, .30671376E-01, .31010333E-01, .31316426E-01,
2160      1 .31589331E-01, .31828759E-01, .32034456E-01, .32206205E-01,
2170      1 .32343823E-01, .32447164E-01, .32516119E-01, .32550614E-01/
2180       NPOINT = 48
2190       PSI = (0.,0.)
2200       G = U/2./FLOAT(N)
2210       DO 2 I = 1,N
2220       Z = (0.,0.)
2230       DO 1 K = 1,NPOINT
2240       ARGA = G*(A(K) + 2*I-1.)
2250       ARGB = G*(-A(K) + 2*I-1.)
2260       Z = Z + (FCN(D,R,ARGA) + FCN(D,R,ARGB))*W(K)
2270     1 CONTINUE
2280 C     WRITE(6,990)I,K,FCN(D,R,ARGA),FCN(D,R,ARGB)
2290       PSI = PSI + Z
2300     2 CONTINUE
2310       PSI1 = G*PSI
2320   990 FORMAT(2X,'I=',I2,2X,'K=',I2,2X,'F(A)=',2E10.4,2X,'F(B)=',2E10.
4)
2330       PSI = (0.,0.)
2340       G = -G
2350       DO 4 I = 1,N



2360       Z = (0.,0.)
2370       DO 3 K = 1,NPOINT
2380       ARGA = G*(A(K) + 2*I-1.)
2390       ARGB = G*(-A(K) + 2*I-1.)
2400       Z = Z + (FCN(D,R,ARGA) + FCN(D,R,ARGB))*W(K)
2410   3   CONTINUE
2420       PSI = PSI + Z
2430   4   CONTINUE
2440       PSI2 = G*PSI
2450       PSI = PSI1 - PSI2
2460       RETURN
2470       END
```

```
2360       Z = (0.,0.)
2370       DO 3 K = 1,NPOINT
2380       ARGA = G*(A(K) + 2*I-1.)
2390       ARGB = G*(-A(K) + 2*I-1.)
2400       Z = Z + (FCN(D,R,ARGA) + FCN(D,R,ARGB))*W(K)
2410    3  CONTINUE
2420       PSI = PSI + Z
2430    4  CONTINUE
2440       PSI2 = G*PSI
2450       PSI = PSI1 - PSI2
2460       RETURN
2470       END
```

LABORATORY OPERATIONS

The Laboratory Operations of The Aerospace Corporation is conducting experimental and theoretical investigations necessary for the evaluation and application of scientific advances to new military space systems. Versatility and flexibility have been developed to a high degree by the laboratory personnel in dealing with the many problems encountered in the nation's rapidly developing space systems. Expertise in the latest scientific developments is vital to the accomplishment of tasks related to these problems. The laboratories that contribute to this research are:

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Chemistry and Physics Laboratory: Atmospheric chemical reactions, atmospheric optics, light scattering, state-specific chemical reactions and radiation transport in rocket plumes, applied laser spectroscopy, laser chemistry, laser optoelectronics, solar cell physics, battery electrochemistry, space vacuum and radiation effects on materials, lubrication and surface phenomena, thermionic emission, photosensitive materials and detectors, atomic frequency standards, and environmental chemistry.

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Materials Sciences Laboratory: Development of new materials: metal matrix composites, polymers, and new forms of carbon; nondestructive evaluation, component failure analysis and reliability; fracture mechanics and stress corrosion; analysis and evaluation of materials at cryogenic and elevated temperatures as well as in space and enemy-induced environments.

Space Sciences Laboratory: Magnetospheric, auroral and cosmic ray physics, wave-particle interactions, magnetospheric plasma waves; atmospheric and ionospheric physics, density and composition of the upper atmosphere, remote sensing using atmospheric radiation; solar physics, infrared astronomy, infrared signature analysis; effects of solar activity, magnetic storms and nuclear explosions on the earth's atmosphere, ionosphere and magnetosphere; effects of electromagnetic and particulate radiations on space systems; space instrumentation.

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