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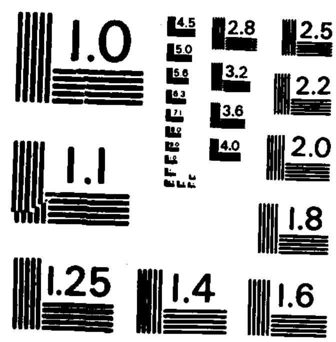

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POINTED BUBBLES RISING IN A TWO-DIMENSIONAL TUBE

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ABSTRACT

Pointed bubbles with a 120° angle at the apex rising in a two-dimensional tube are considered. A solution is found at a unique Froude number of 0.36.

AMS (MOS) Subject Classification: 76B10

Key Words: bubble, free surface flow

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SIGNIFICANCE AND EXPLANATION

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~~We~~ considers a periodic array of plane bubbles rising in a gravity field. This configuration can serve as a model for an advanced stage of Rayleigh-Taylor instability. Vanden-Broeck~~[3,8]~~ 9 solved the problem numerically and showed that two classes of solutions are possible. One class is characterized by a bubble profile with a continuous slope at the apex of the bubble whereas the other is characterized by the presence of a cusp at the apex.

In a recent paper Garabedian[2] 9 and Modi[4] 9 conjectured that solutions with a deg 120° angle at the apex might also exist.

In this ~~the present~~ paper a scheme is presented to compute such solutions. It is found that a solution exists at a unique Froude number of 0.36. This result does not support Garabedian's[2] conjecture that bubbles with a 120° deg angle at the apex exist for all values of F between 0.23 and 0.36.

Keywords: free surface flow.

POINTED BUBBLES RISING IN A TWO-DIMENSIONAL TUBE

Jean-Marc Vanden-Broeck

We consider a bubble rising at a constant velocity U in a two-dimensional tube of width h. We choose a frame of reference moving with the bubble and we assume that the bubble extends downwards without limit. We denote by 2γ the angle between the free streamlines at the apex of the bubble (see Figure 1). We shall neglect the effect of surface tension. Therefore the problem is characterized by the Froude number

$$F = U/(gh)^{1/2} . \tag{1}$$

Here g denotes the acceleration of gravity. This problem was considered before by Garabedian[1,2], Vanden-Broeck[3] and Modi[4].

Garabedian[1] presented analytical evidence that a solution with $\gamma = \frac{\pi}{2}$ exists for each value of the Froude number F smaller than a critical value F_c. Vanden-Broeck[3] solved the problem numerically and found $F_c = 0.36$. In addition Vanden-Broeck[3] showed numerically that a solution with $\gamma = 0$ (i.e. with a cusp at the apex) exists for each value of F greater than F_c.

Garabedian[2] and Modi[4] pointed out that solutions with $\gamma = \frac{\pi}{3}$ might also exist. The flow in the neighborhood of the apex would then be similar to the classical Stokes flow near the crest of the highest progressive wave. Other flows with an angle of 120° on the free surface have been found by Vanden-Broeck and Keller[5] and by Goh and Tuck[6].

In this paper we present numerical evidence that a solution with $\gamma = \frac{\pi}{3}$ exists only for $F = F_c \sim 0.36$.

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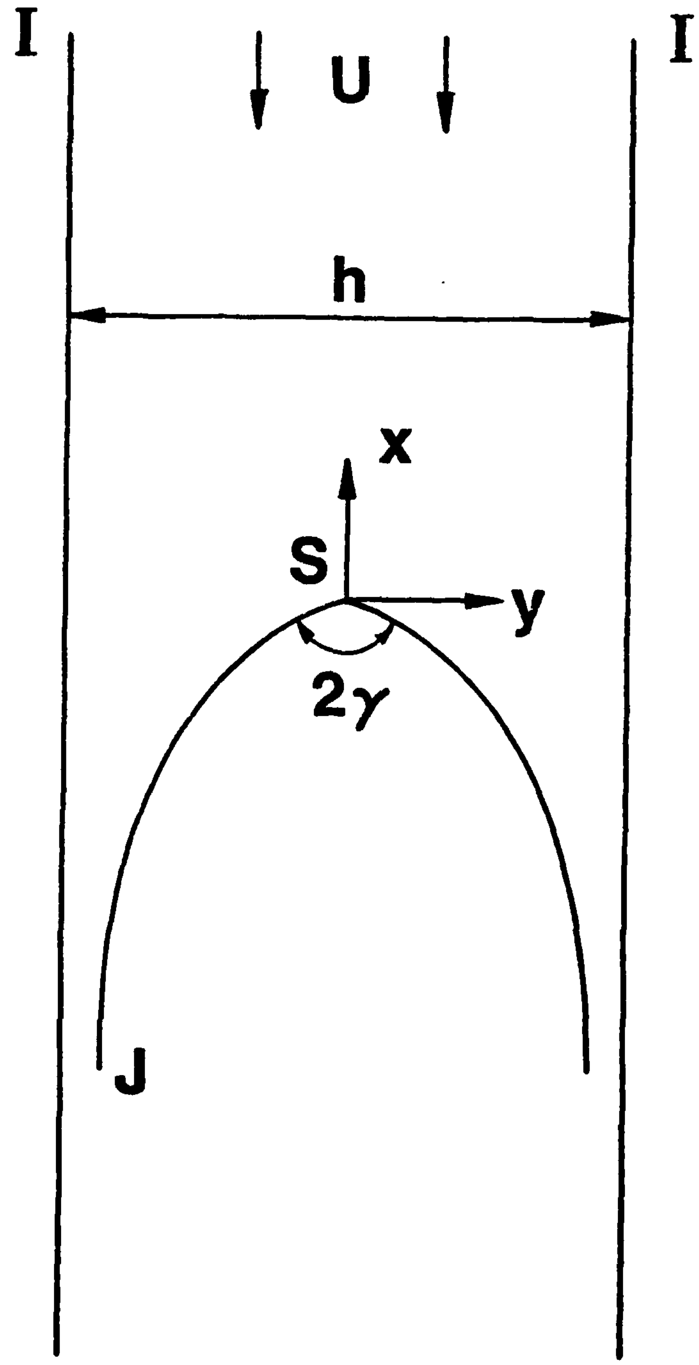

Figure 1: Sketch of the flow and the coordinates.

We define dimensionless variables by choosing h as the unit length and U as the unit velocity. We introduce the potential function φ and the stream function ψ. Without loss of generality we choose $\varphi = 0$ at $x = y = 0$ and $\psi = 0$ on the bubble surface. We denote the complex velocity by $\zeta = u - iv$ and we define the function $\tau - i\theta$ by

$$\zeta = u - iv = e^{\tau - i\theta} . \qquad (2)$$

Following Birkhoff and Carter[7] and Vanden-Broeck[3], we define the new variable t by the relation

$$e^{-\pi f} = \frac{1}{2}\left(t + \frac{1}{t}\right) . \qquad (3)$$

Here $f = \varphi + i\psi$ denotes the complex potential. It can be shown that this transformation maps the flow domain onto the unit circle in the complex t-plane so that the free surface SJ goes into the portion of the circumference in the first quadrant (see reference 3 for details). The complex t-plane is shown in Figure 2.

The flow in the neighborhood of the apex S of the bubble is locally a flow inside an angle $\pi - \gamma$ (see Figure 1). Therefore ζ vanishes like $f^{\gamma/\pi}$ as $f \to 0$. Using (3) we have

$$\zeta \sim (1 - t)^{2\gamma/\pi} \quad \text{as} \quad t \to 1 . \qquad (4)$$

Following the procedure outlined in reference 3 we represent ζ by the expansion

$$\zeta = -[-\ell n\, C(1 + t^2)]^{1/3}(-\ell n\, C)^{-1/3}(1 - t^2)^{2\gamma/\pi}\left[1 + \sum_{n=1}^{\infty} a_n t^{2n}\right] . \qquad (5)$$

Here C is an arbitrary constant between 0 and 0.5. We chose $C = 0.2$. The coefficients a_n have to be determined to satisfy the condition that the pressure is constant on the free surface SJ. We use the notation $t = |t|e^{i\sigma}$ so that points on SJ are given by $t = e^{i\sigma}$, $0 < \sigma < \frac{\pi}{2}$. Vanden-Broeck[3]

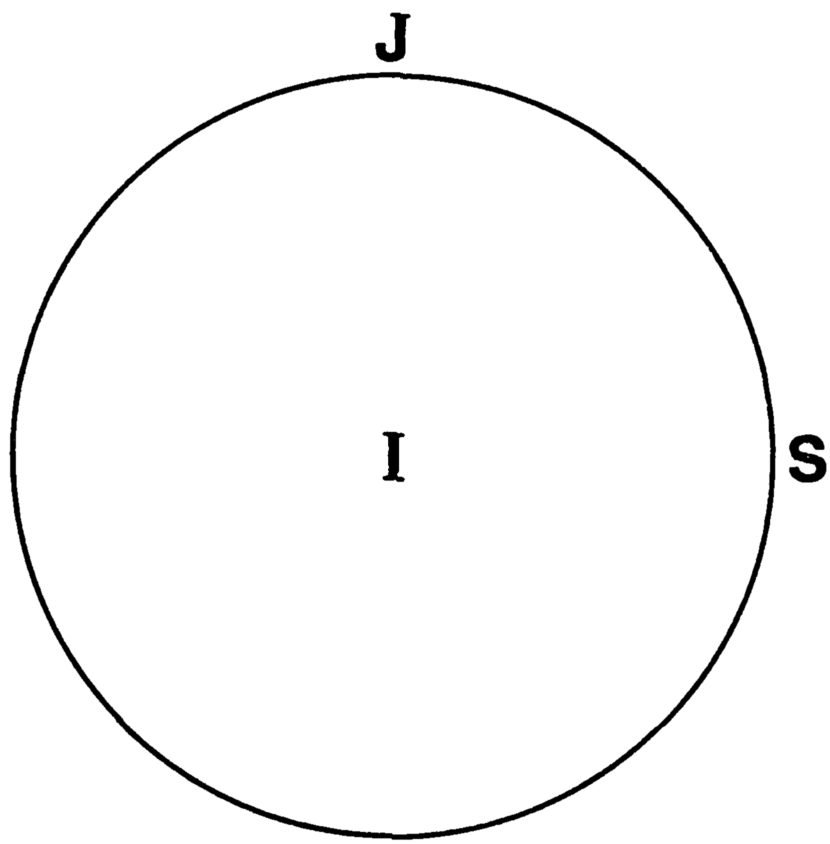

Figure 2: The complex t-plane.

showed that the pressure condition on SJ can be written as

$$\pi \cot \sigma e^{2\tilde{\tau}} \frac{d\tilde{\tau}}{d\sigma} + \frac{1}{F^2} e^{-\tilde{\tau}} \cos \tilde{\theta} = 0 \ . \tag{6}$$

Here $\tilde{\tau}(\sigma)$ and $\tilde{\theta}(\sigma)$ denote the values of τ and θ on the free surface SJ.

For $\gamma = 0$ and $\gamma = \frac{\pi}{2}$, (5) is equivalent to the representations (15), (18) and (19) of reference 3.

In order to find solutions with a 120° angle at the apex, we set $\gamma = \frac{\pi}{3}$ in (5) and determine the coefficients a_n by using the procedure outlined in reference 3. Thus we truncate the infinite series in (5) after $N - 1$ terms. We find the $N - 1$ coefficients a_n and the Froude number F by collocation. Therefore we introduce the N mesh points $\sigma_I = \frac{\pi}{2N}(I - \frac{1}{2})$, $I = 1,\ldots,N$ and satisfy (5) at the N points σ_I. This leads to a system of N nonlinear algebraic equations for the N unknowns a_n, $n = 1,\ldots,N - 1$ and F. We solve this system by Newton's method. Once this system is solved, the shape of the bubble is obtained by numerically integrating the identity

$$\frac{\partial x}{\partial \varphi} + i \frac{\partial y}{\partial \varphi} = e^{-\tau+i\theta} \ . \tag{7}$$

In Table I we present numerical values of F corresponding to different values of N. The value 0.3577 appear to be correct to 4 decimal places. The corresponding profile of the bubble is shown in Figure 3.

As a check on our results, we modified the scheme to allow the Froude number F to be prescribed. Solutions could not be obtained for other values of F.

N	F
10	0.35733
30	0.35763
50	0.35771
70	0.35775

Table I: Values of the Froude number F corresponding to the bubble with a 120° degree angle at the apex for various values of N.

The results in reference 3 results and the present calculations show that

$$\gamma = \frac{\pi}{2}, \qquad 0 < F < F_c, \tag{8}$$

$$\gamma = \frac{\pi}{3}, \qquad F = F_c, \tag{9}$$

$$\gamma = 0, \qquad F_c < F < \infty \tag{10}$$

where $F_c \sim 0.36$.

The results (8)-(10) do not support Garabedian's[2] conjecture that bubbles with a 120° angle at the apex exist for all F between 0.23 and 0.36.

Finally let us mention that Vanden-Broeck[8] solved the problem with surface tension. He found that for each value of the surface tension there exists a countably infinite number of solutions for which $\gamma = \frac{\pi}{2}$. As the surface tension tends to zero, all these solutions approach a unique limiting configuration characterized by $F = F^* \sim 0.23$. The profile corresponding to $F = F^*$ was found to be in good agreement with experimental data.

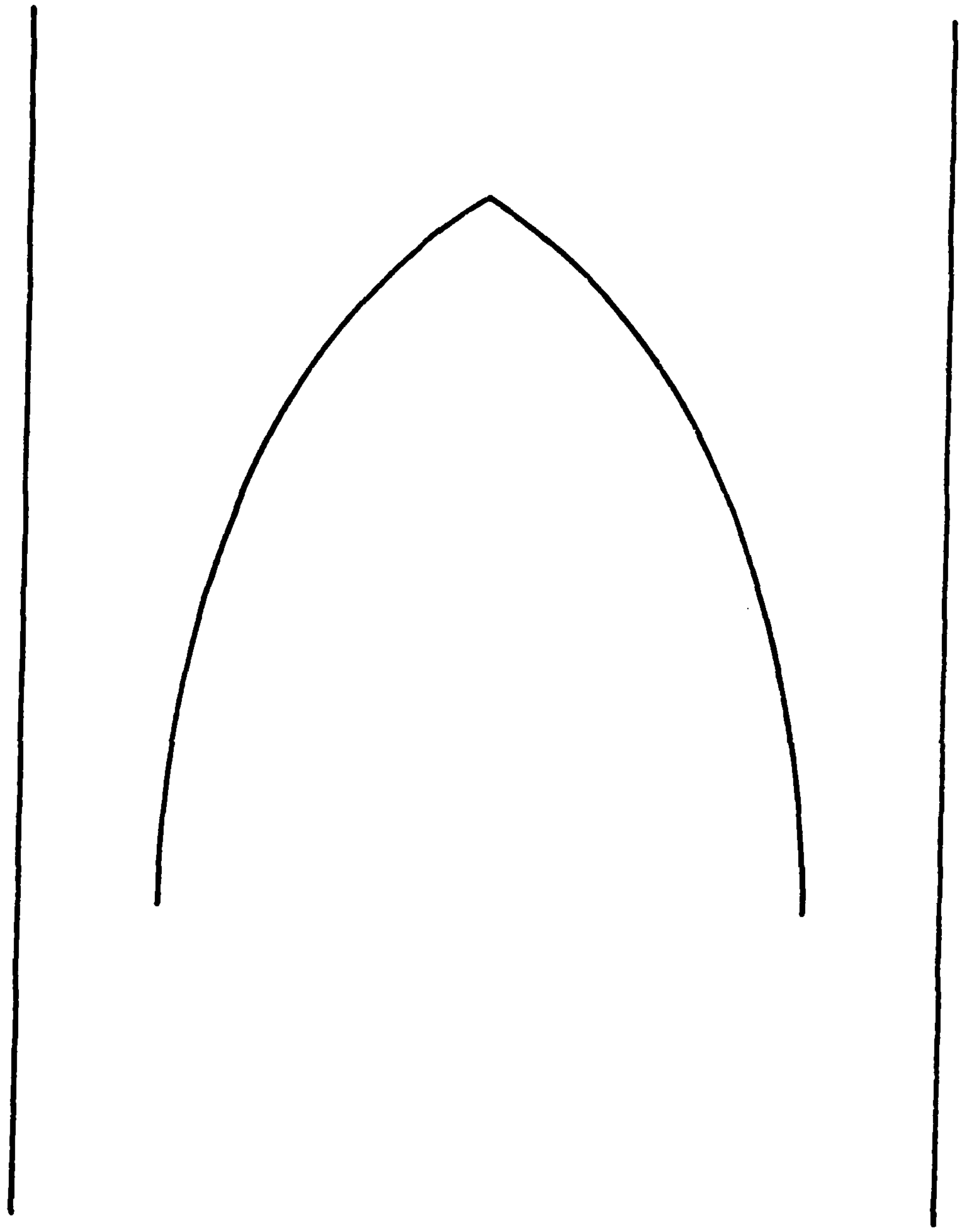

Figure 3: Profile of the bubble with an angle of 120° at the apex.

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