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CONTINUATION OF RESEARCH ON MULTIPARAMETER STOCHASTIC  
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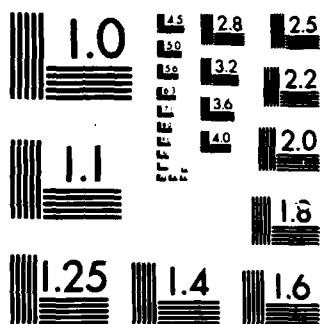
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br><br>To study Markov processes with a multidimensional parameter necessitates the introduction of generalized processes that can be localized to surfaces. A vehicle for dealing with these is "stochastic cochains," processes parameterized by k-dimensional surfaces in n-dimensional surface. The principal result of this project has been the development of a theory of stochastic cochains. |                              |                                                                    |

The focus of this research project is on multiparameter stochastic processes with particular attention being given to Martingale and Markovian properties.

The concept of "Markovian property" for multiparameter processes was first suggested by Lévy [1] some forty years ago. However, the topic is relatively undeveloped, and very few examples of multiparameter Markov processes have been found. Possibly the most important example is the "free Euclidean field," discovered independently by Wong [2] and Nelson [3].

The "free Euclidean field" is a generalized process, not a regular process. Markov property is a "local property." What then does it mean for a generalized process to be Markov? One of the main objectives over the last three years has been to answer this question in a natural and general way. What has emerged is a theory of "stochastic co-chains," which are processes parametrized by  $k$ -dimensional surfaces in  $R^n$  [4]. It turns out that stochastic cochains are geometric objects (independent of coordinates), closed under many, but not all, of the differentio-geometric operation. Viewed as processes parameterized by points in  $R^n$ , stochastic cochains are stochastic differential forms, but not all currents are co-chains. Co-chains are precisely those currents that can be localized to surfaces so as to allow Markov properties to be defined.

From the insight thus provided, we see that the most natural Markov process in  $R^n$  are  $(n-1)$ -cochains. Of these, the simplest example would be Gauss-Markov  $(n-1)$ -cochains that are both isotropic and homogeneous. Characterization of all such cochains in terms of a pair of spectral distributions (solenoidal and irrotational, [5]) is an important open problem.

A multiparameter white noise is best thought of as an  $n$ -cochain in  $R^n$ . Thus, a Wiener process in  $R^n$  is a Gaussian  $n$ -cochain that is isotropic and homogeneous. To use such processes in physical problems, we need to deal with

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2. E. Wong, "Stochastic Differential Forms and Markov Fields," presented at the 1984 ARO Workshop on Signal Tracking, February 13-15, 1984, Fort Belvoir, VA.
3. E. Wong, "Markovian Random Fields," Proceedings of the 23rd IEEE Conference on Decision and Control, vol. 3, pp. 1447-1450, 1984.

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