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ROYAL AIRCRAFT ESTABLISHMENT FARNBOROUGH (ENGLAND)
A M PRIOR ET AL. APR 85 RAE-TR-85046 DRIC-BR-97604

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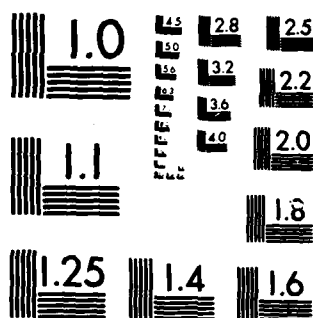
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by

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SUMMARY

The compounding method enables stress intensity factors to be evaluated for complex geometrical configurations using the known results of simpler ancillary configurations. Implementation of the method on a computer system requires that a data file of known results be available, and that a program be written to compound solutions from that data file.

Firstly, a procedure is described to convert available graphical data on stress intensity factors to numerical data and store it in a direct access computer file together with identification parameters. Secondly a computer program is described which uses the data in that file to compound stress intensity factors for other configurations. It is shown that the errors in the results from this program are within acceptable engineering tolerances.

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1 INTRODUCTION

The performance and operational life of engineering structures is often limited by the initiation and subsequent growth of cracks in the materials. Continued use of the structure is often possible, but this must only be undertaken with some knowledge of the effect that the crack is likely to have on the structure. The distribution of stress near the crack tip is one of the factors which determine the crack growth rate and maximum crack length before fast fracture is likely to occur.

The stress intensity factor K_M ($M = I, II, III$) is a parameter that characterises this stress field and is used in many calculations in the analysis of fracture and fatigue. The value of K_M depends on the forces on the structure, the crack length, and also the geometry of the crack and any boundaries that are near it. Solutions have been calculated for many configurations of cracks in sheets near holes, edges, or stiffeners and have been collected together and published¹. These solutions can be used directly to calculate stress intensity factors for simple shapes under simple loadings, but the number of solutions available is limited and more complex practical problems remain unsolved. In order to extend their use to more complex configurations the solutions can be combined, either by superimposing them or using the compounding method^{2,3}.

Although quite simple in principle, compounding can be a tedious and time consuming process because the ancillary solutions appear in graphical form. It would be easier and quicker to calculate stress intensity factors for complex geometries if the ancillary solutions were presented as numerical data. This would also make the results more reliable by eliminating the errors involved with reading data from graphs. A computer file could store such information and make the retrieval and use of the data more efficient in terms of time and cost. The file could be readily updated with new solutions as they became available and so has obvious advantages over a book which would require reprinting.

The first aim of the work reported here was to design and develop a computer data file to store numerical data from stress intensity factor solutions. The second aim was to develop a computer program to use the compounding method to combine solutions in the data file so as to derive stress intensity factors for more complex configurations.

In section 2 of the Report the importance of the stress intensity factor in determining crack behaviour is set out. For cracks in real structures existing analysis needs to be extended. This need is met by the compounding method^{2,3} which is outlined in section 3 along with the defining equations and a summary of its use. Two simple problems are then solved manually illustrating the steps involved in the calculations for cracks near boundaries (section 3.1.1) and cracks at boundaries (section 3.1.2); the latter includes an example of the use of the 'equivalent stress' method⁴.

The problems that arose in section 3 from the use of graphical results for stress intensity factors are described in section 4. These include the errors inherent in reading and interpolating data from graphs and also the lengthy calculations involved. The subsequent need to develop a computer data file in order to alleviate these problems is

then identified; the design of such a file is discussed together with the difficulties encountered and the methods employed to overcome them.

The development of a computer program to use the data file to compound stress intensity factors is described in section 5. The operation of the program is discussed in general terms and a flow diagram is used to indicate the main segments. In the final part of this section (5.3) the results from the program are compared with those from the manual calculations in sections 3.1.1 and 3.1.2. It is shown that the results from the computer program vary by a maximum of 1% from those calculated by hand. Given the tolerances of the compounding method itself the results from the program are considered to be acceptable.

Consideration of the ways in which the operator and computer interact, through questions and answers and explanatory comments, makes up an important part of the development of the program. This aspect of the work including error handling and provision of 'help' comments is discussed in section 6, together with the design of an Operators Manual⁵. The future inclusion in the data file of extra solutions is also discussed, as well as certain limitations in the use of the computer program.

2 STRESS INTENSITY FACTORS

It is important to determine the likely effect of a crack on a structure, both in the short and long term. Knowing this the structure might then continue to be used within certain safe operational limits or may be removed from service completely. One must therefore be able to estimate how fast the crack is likely to grow, if at all, and also to what length it can grow before the structure becomes unsafe. The growth rate of a crack is controlled by the magnitude of the stress field in the region of the crack tip; this distribution is described by the stress intensity factor K_M :

$$K_M = Y\sigma\sqrt{\pi a} \quad , \quad (1)$$

where σ is a generalized stress which depends on the forces applied to the structure, a is the crack half-length and Y is the shape factor which takes into account the shape and orientation of the crack along with effects of any boundaries or other cracks nearby. The subscript M denotes the mode of crack deformation: $M = I$ denotes the opening mode, normally as a result of simple tension; $M = II$ denotes sliding-mode, a characteristic of shear loading; and $M = III$ denotes tearing-mode, resulting from torsional shear stresses. In this Report only opening mode stress intensity factors K_I will be considered.

Solutions are available for two- and three-dimensional problems, which usually include the effects of only one or two boundaries such as holes, edges of plates, stiffeners or changes of thickness in the plate. These solutions are usually presented in graphical form and many have been collected together and published. One example of this is the Compendium of Stress Intensity Factors by Rooke and Cartwright¹.

On the graphs in the Compendium the vertical axis is the stress intensity factor K made non-dimensional by dividing by the stress intensity factor K_0 at the tip of the

crack with no other boundaries present. If the crack is perpendicular to the stress σ applied remote from the crack then $Y = 1$ and

$$K_0 = \sigma\sqrt{\pi a} . \quad (2)$$

However, in the compounding theory described in section 3, K in the absence of any boundaries is referred to as $\bar{K}$ since K_0 has another meaning. The symbol $\bar{K}$ will therefore be used throughout this Report to mean the stress intensity factor at the tip of a crack when there are no other boundaries present.

The horizontal axis is usually a non-dimensional crack length a/b where b is some length obtained from the geometry of the problem. The solutions contained in Ref 1 apply to linear, elastic, isotropic, homogeneous materials. Two typical examples of the Compendium results are shown in Figs 1 and 2.

As was stated earlier, most of the solutions currently available apply to a simple arrangement of a crack with one or two structural boundaries. In practical situations the geometry is often not so simple and many more boundaries may need to be taken into account in order to calculate the stress intensity factor. To solve the problem using standard techniques may be a lengthy and, as a result, expensive process. Therefore to make all the basic solutions available is not enough. Some extra facility is required to extend the use of existing solutions so that more complex problems may be solved. Such a facility was devised by Rooke and Cartwright², and is known as the compounding method. A complete development of compounding is presented in a PhD thesis by Rooke³. As an extension to the method the use of the equivalent stress has been described by Rooke⁴. The general equations of the compounding method have also been derived independently by Panasyuk and others⁶, who used arguments based on dimensional analysis.

3 THE COMPOUNDING METHOD

In this section the general equations of the compounding method are expressed for the two cases of cracks near and at boundaries. In sections 3.1.1 and 3.1.2 examples of each are solved in order to demonstrate the steps involved in using compounding.

The principle of the compounding method is to separate the geometry of a complex problem into a number of simpler configurations for which solutions are known. These ancillary solutions normally involve only one boundary and are presented in graphical form in the Compendium¹ and elsewhere. The results calculated for the ancillary solutions are then compounded together by a form of superposition to produce an approximate value of the stress intensity factor in the complex problem. The compounding process does not take into account the effects of boundary-boundary interaction. This produces an error which can be estimated. A derivation of this error term, K_e , for one specific case is presented in Ref 3. It can be shown that when boundaries are not close together K_e/K is small and K_e can be neglected.

The compounding method can be expressed in general by the following equation:

$$K_r = \bar{K} + \sum_{n=1}^N (K_n - \bar{K}) + K_e, \quad (3)$$

where K_r is the stress intensity factor at the tip of the crack in the complex problem, $\bar{K}$ is the stress intensity factor in the absence of any boundaries (called K_0 in Ref 1), K_n is the stress intensity factor resulting from the n th boundary only, K_e is the error term and N is the total number of boundaries.

This equation can be written, with K_e neglected, as

$$Q_r = 1 + \sum_{n=1}^N (Q_n - 1), \quad (4)$$

where $Q_r = K_r/\bar{K}$, $Q_n = K_n/\bar{K}$.

The normalised parameter Q_n appears as the vertical axis on the graphs in the Compendium¹ (where $K_0 = \bar{K}$), plotted against non-dimensionalised crack length (crack length/structure length). Equation (4) can be used to build up solutions to complex problems involving N boundaries from the solutions for each of the ancillary configurations, up to N in number. The method is applicable to both two-dimensional and three-dimensional problems but its use is concentrated on two-dimensional configurations because so few ancillary solutions exist for three-dimensional problems. Equation (4) is only valid for problems in which the crack does not cross any of the boundaries. In situations where crossing does occur the idea of an equivalent stress has been introduced⁴; this modification can be used for cracks at the edges of holes and for cracks at stiffeners in reinforced structures.

The general equation for the equivalent stress method is

$$Q_r = Q_0 \left[1 + \sum_{n=1}^N (Q_n - 1) \right] + Q_e, \quad (5)$$

where Q_0 is the normalized stress intensity factor ($K_0/\bar{K}$) for a crack at the boundary in the absence of all other boundaries, Q_n is the normalized factor for the n th ancillary solution, and N is the number of ancillary configurations (excluding Q_0). A method of estimating the error term Q_e , for cracks at holes, is described in section 3.1.2. In general Q_e is negligible for cracks at stiffeners and is neglected.

3.1 Examples of the use of compounding (manual)

To illustrate the procedure followed when using the compounding method two simple problems are solved in the following sections. The solutions are developed in a series of steps which are paralleled in the computer program described in section 5. Other examples

of the method appear in an Engineering Sciences Data Unit publication⁷ and RAE Technical Reports^{4,8}. Since the results obtained are in non-dimensionalised form, units need not be specified so long as a consistent system is used.

3.1.1 Cracks near boundaries

Consider a large sheet with a circular hole close to one edge. Between the edge and the hole is a crack, perpendicular to the edge and of length $2a$. This sheet is subjected to a uniaxial tensile stress σ , remote from the crack and in a direction parallel to the edge, as shown in Fig 3a. The stress intensity factor K at crack tip A is required for various lengths of crack.

Step 1

Separate the complex configuration into a number of ancillary geometries for which solutions are available. The Compendium¹ contains two solutions (Q_1 and Q_2 say) which when compounded together will represent the problem shown in Fig 3a. These are for Q_1 (Fig 3b), Case No.1.1.11, which contains K for an internal crack perpendicular to the edge of a sheet subjected to a uniaxial tensile stress acting parallel to the edge; and for Q_2 (Fig 3c), Case No.1.3.5, which contains K for tip A of a crack near a circular hole in a sheet subjected to a uniaxial tensile stress acting perpendicular to the crack.

Step 2

Define the parameters for the ancillary solutions in terms of the dimensions of the complex problem (Fig 3a). The parameters for tip A are, for Case No.1.1.11, $b = 1$; for Case No.1.3.5, $b = 0.5$, $c = 0.952$, $R/c = 0.475$.

Step 3

Estimate the stress intensity factor at tip A for a range of crack lengths, from 0.1 to 0.4 at intervals of 0.05. From the crack length a and dimension b the value of a/b on the X axis can be determined and values of Q can be read from the Y axis for each crack length in the series. Simple linear interpolation is used in Case No.1.3.5 between the curves of $R/c = 0.4$ and $R/c = 0.5$ to obtain results for $R/c = 0.475$. Results for Q_1 and Q_2 for the crack lengths between 0.1 and 0.4 appear in Table 1.

Step 4

Since Q_e will be small for this configuration, the non-dimensionalised stress intensity factor Q_r is obtained from equation (4) with $N = 2$, so that

$$Q_r = Q_1 + Q_2 - 1 \quad (6)$$

This process is repeated for each crack length to produce the set of results presented in Table 1. These results are within 1% of those presented in Fig 3.8 of the thesis by Rooke³.

Table 1

Values of Q_1 , Q_2 and Q_r for a crack near a hole in
a sheet subjected to a uniform tensile stress

a/b	Q_1	Q_2	Q_r
0.10	1.00	1.18	1.18
0.15	1.01	1.17	1.18
0.20	1.01	1.17	1.18
0.25	1.02	1.17	1.19
0.30	1.03	1.18	1.21
0.35	1.04	1.18	1.22
0.40	1.05	1.20	1.25

To calculate the stress intensity factor K for this problem the parameter $\bar{K}$, which depends on the applied stress and the crack length, must be known.

3.1.2 Cracks at boundaries

Consider a very long strip of width b as shown in Fig 4a. In the centre of the strip is a hole of radius R , with two diametrically opposed, radial cracks of length l . The crack line is perpendicular to the axis of the strip. The strip is subjected to a uniform uniaxial tensile stress along its axis, remote from the crack. The stress intensity factor K at one of the crack tips is calculated for various lengths of crack.

Step 1

Separate the complex geometry into a number of simpler geometries for which solutions are available. In this case two ancillary solutions (Q_0 and Q_1 for Fig 4b&c respectively) appear in the Compendium¹ which when compounded together will represent the complex geometry. These are: Case No.1.3.3 which contains Q_0 for one or two cracks at a circular hole in a sheet subjected to a biaxial tensile stress; and Case No.1.1.1 which contains Q_1 for a central crack in a rectangular sheet subjected to a uniform uniaxial tensile stress in a direction perpendicular to the crack line.

Step 2

Determine the parameters for these ancillary solutions in terms of the dimensions of the complex problem. The parameters are: for Case No.1.1.1, $h = \infty$, and $b = 4$; and for Case No.1.3.3, number of cracks = 2, $R = 1$ and $\alpha = 0$, therefore $R/b = 0.25$.

Step 3

Estimate the stress intensity factors for the ancillary configurations for several crack lengths. From inspection of the graphs it is seen that the results change rapidly at small crack lengths; therefore individual crack lengths of 1.04, 1.08, 1.20, 1.40, 1.60, 1.80 and 2.00 were chosen. Since all the other parameters are known the values of Q for each ancillary configuration can be read from the graphs in the Compendium¹. Results for Q_0 and Q_1 for the required crack lengths are presented in Table 2.

Step 4

Obtain Q_r for the complex problem from the results for the ancillary calculations by using equation (5). Values of Q_e are required at this stage. The derivation of Q_e is described in Refs 4 and 8. The basic calculations are as follows: if the values of crack length a and hole radius R are known then the value of Q_e/Δ can be obtained from the graph in Fig 4 of Ref 8. The parameter Δ is given by

$$\Delta = K_t - 3Q_1(R/b) \quad , \quad (7)$$

and is a function of the stress concentration factor K_t of the uncracked configuration and the stress intensity factor Q_1 of the ancillary solution when the crack length is R . Since R/b is known the value of $Q_1(R/b)$ for this particular problem can be determined from Case No.1.1.1¹, and, since the value of K_t can be found from Peterson⁹, Δ can be calculated and Q_e determined. This process is repeated for each crack length. The terms $Q_1(R/b)$ and K_t are constant for a particular geometry and are independent of a .

All the data is now available to compound the solution to the complex problem. Results for Q_r appear in Table 2 and they agree to within 1% with those obtained by Rooke⁴.

Table 2

Values of Q_0 , Q_1 , Q_e and Q_r for a crack at a hole in a long strip subjected to a uniform tensile stress

a	Q_0	Q_1	Q_e	Q_r
1.04	0.63	1.02	0.04	0.68
1.08	0.80	1.02	0.05	0.87
1.20	0.99	1.03	0.06	1.08
1.40	1.06	1.06	0.06	1.18
1.60	1.06	1.10	0.06	1.22
1.80	1.05	1.13	0.05	1.24
2.00	1.04	1.18	0.05	1.27

4 A DATA FILE OF STRESS INTENSITY FACTOR SOLUTIONS

In this section the problems involved with the graphical solutions to stress intensity factors are presented and the subsequent need for a computer data file is identified. The development and final format of the file are discussed in sections 4.2 to 4.4.

4.1 Data file requirement

Collections such as the Compendium¹ go a long way towards making solutions readily available and easy to use but they are not ideal. One problem is that the solutions are presented in graphical form. Although this condenses a large amount of data into a manageable volume, the detailed interpolation and reading of data, required in compounding calculations, can be both tedious and time consuming. Also new solutions are

becoming available all the time, but are not immediately usable because of the time that would be involved in updating and publishing new editions of the Compendium.

There is a need therefore to bring together all the available solutions in a form that is both easier to use and capable of being rapidly updated. A computer data file containing all the available solutions would fulfill this requirement and the design and development of such a system is described below.

It was decided that the file should be developed on a microcomputer since this would make the initial programming work easier than on a more powerful but less accessible main frame, and also because the resulting file could be widely used on any size of computer. Since each item of data was to be accessed independently the file needed to be of the direct access (or random access) type as opposed to the sequential access type. Since it was available, a Hewlett Packard HP 9836 computer was used. The programs were written in the Hewlett Packard BASIC 2.1 language. The essential requirements for this file were that it should be capable of storing the solutions to stress intensity factor problems with all the necessary labels and parameters on any average microcomputer, and that the initial loading and subsequent retrieval of this data should be quick and simple.

4.2 Data file development

Since the Compendium presented data, from many different sources in many different original forms, in a standard way, it was decided to use the Compendium as the basis for the data file. Before the data file can be created it is necessary to convert the Compendium graphical data to numerical data. One way of achieving this is to reduce the curve to a number of discrete points and record the X and Y coordinates of each one. A simple interpolation can then be used to reconstruct the curve between the points when calculations are to be performed. There are a number of disadvantages with this method, the main one being the amount of data that would need to be stored. To record a complex curve with a reasonable degree of accuracy would require between 25 and 50 points, that is 50 to 100 coordinates. If a comprehensive data file were to be produced with perhaps 200 solutions, each with ten curves, it would require 100000 to 200000 numbers. A 5 1/4 inch disk for the HP 9836 could perhaps store 32000 numbers in the required form. Since the aim was to produce a file which could be widely used in stress offices on small 'desk top' computers, this method clearly was not suitable.

To reduce the amount of storage space required, the possibility of curve fitting was investigated with a view to recording some form of equation rather than point data. The method devised reduced the volume of data per curve to a few coefficients and labels, and enabled a single disk to store between three and six times as many curves. The work carried out to develop this method and the final form of the data file is described below.

Many computer manufacturers have produced curve fitting programs. These take the X and Y coordinates of a number of points and then output an equation which when plotted should pass through or close to these points. The programs themselves produce results within seconds, but first the curve must be converted into a number of points and

then the coordinates of each one input to the computer via the keyboard. This is a lengthy process and data read from graphs is prone to errors. Therefore, in order to speed up the input of the point coordinates and reduce the probable errors a graphics tablet was used. A program was written which accepted data from the graphics tablet and used standard curve fitting techniques to produce an equation which represented the original data.

The graphics tablet (Hewlett Packard HP 9111A) has an area of 210 mm (y) by 297 mm (x) which is divided into a number of square units of dimensions 0.024 mm $\times$ 0.024 mm. There are 8740 in the Y direction and 12032 in the X. When a stylus, which is electrically connected to the tablet, is pressed onto the surface the coordinate position of the stylus tip, in terms of the unit squares, is sent to the computer. The action of pressing the stylus and sending the coordinates is known as digitizing.

4.3 Conversion of graphical solution to numerical data

The data could not be taken from the Compendium¹ itself since the pen must be quite close to the tablet in order to digitize a point accurately. Therefore, a photo copy of the required page from the Compendium was used. This is secured to the surface of the tablet with tape, with the axes of the graph in line with the axes marked on the tablet. The curve fitting program is then started. After some initial explanation the operator is asked to type in the values of the X and Y coordinates of the bottom left and top right corners of the graph. Having done this the operator then digitizes these two points and the other two corners with the stylus. The computer program can now scale the surface of the graphics tablet to precisely the same units as the graph. When the operator digitizes any point on the curve the program converts the tablet coordinates into graph coordinates and stores them in an array.

Using this process the graphical solution can be converted to numerical data quite quickly and accurately. The operator is not required to read data from the graph, or to type a large amount of data into the computer. He merely types in four values and then digitizes a number of points (20-30). There are three possible sources of error. The first is that the graph may not be lined up accurately with the axes on the tablet; errors due to this source have been eliminated by including a compensating routine in the digitizing program. The second is that the operator does not digitize the precise point on the curve. A certain amount of care is required here. It is estimated that the digitized point can be consistently within 0.5 mm of the centre of the line, which itself is between 0.5 and 1.0 mm across. On most graphs this represents an error of less than 0.5%. Finally it must be noted that photo-copies are often not exactly the same size as the original. Shrinkage or expansion may occur in both the X and Y directions by different amounts. An expansion of between 1.4% and 1.8% was encountered on one particular copying machine. However, the method of scaling the graphics tablet by digitizing the opposing corners of the graph is not affected by any changes in the size of the paper, so long as the expansions/shrinkages are linear in every direction. Even if the photo-copy had stretched in one axis and shrunk in the other, the results would not be affected

provided that the changes were linear. In digitizing these graphs it was assumed that these changes were linear and so the error was neglected.

Once the curve has been digitized the program uses a least squares method to fit an equation to the point data. This can be a polynomial, logarithmic or exponential function, or some form of power series, the choice being left to the operator. The output from the program consists of a graph displayed on the VDU screen with the point data and fitted curve superimposed on it. The equation of the fitted curve is also displayed. The operator is given the opportunity to inspect the curve and choose another model if the fit is unsatisfactory.

The accuracy of the fit was determined by inspection. For instance, in Fig 5 the data points were plotted as crosses. If the line of the fitted curve did not pass through some part of every cross then the fit was considered unsatisfactory and another model was tried. On recommendation from people who regularly use the compounding method a list of the configurations most often required was drawn up. These were then digitized in the manner described above. It was found that, in general, the best fit was obtained using a polynomial of degree 4 or 5. It proved almost impossible to fit equations to a number of curves, even with modifications to the scales on the axes. In these cases the solutions were split into two or three separate parts and equations were then fitted to these smaller curves. This method, known as partitioning, increased the size of the data file but was considered to be the best way of storing these particular solutions. The output from the polynomial model of the curve fitting program is given as an equation of the form:

$$Y = \sum_{p=0}^P A_p X^p \quad (8)$$

where P is the degree of polynomial, A_p is the coefficient of X^p , Y is the ordinate and X is the abscissa.

In general only six coefficients were needed. These numbers A_0 to A_5 show a significant reduction on the 50 to 100 numbers required by a point data file, with no significant loss of accuracy. Also, since the file is smaller and more compact, it is likely to be easier and quicker to access and retrieve data. To date, some forty different solutions from the Compendium have been reduced to polynomials with this program. The coefficients were then used to create a data file the format of which is described in section 4.4. Also included are some useful solutions which are not in the Compendium, for example those for cracks at loaded holes.

4.4 Data file format

It was decided that the index for the data file should be based on the Compendium¹ for a number of reasons. It had a comprehensive system of labels for the solutions, with figure numbers and curve identifiers, and also many people were already familiar with it as it had been used in stress offices for some time. The file was of the direct (or random) access type, in which each item of data (called a record) is numbered and can be

retrieved independently. Curves with different axes and parameters need to be differentiated in some way and so a type number was used to label curves. Solutions with the same parameters are assigned the same type number, so that a computer program can cope with the differences in data input required for each solution. Some examples of this are illustrated in Table 3. The figure number referred to appears at the bottom of the page in the Compendium.

Table 3

Use of type number to label figure numbers

Figure number	Axes	Data input	Type number
15	$K_I/K_0, a/b$	Tip A or B	1
109-114	$K_I/K_0, a/b$	R/c	3
130, 131	$K_I/K_0, a/h$	$\lambda = 2E_1at/AE_2$	15

The figure number is used to title each ancillary configuration to make it quick and easy to identify the data required. Some limits are also needed to define the range of the curve as it appears in the Compendium¹. This is essential due to the fact that the solutions in the Compendium only exist within the limits of the plotted curve and that the polynomials fitted to the data are only valid over the digitized range. No extrapolation is allowed. A label to identify different curves from the same solution (*ie* with different values of R/c, a/h, etc) is also required. The coefficients of the polynomial are the final pieces of information required for each curve. This complete definition of the solution is known as the data set.

In order to make the file compact but still readable it was designed so as to contain 20 records per data set. The majority of solutions required the data set to contain at least 11 records to store the information so far described. Those solutions which had been partitioned required 12 or 13. This leaves some blank spaces which although not required at present may be needed in the future for more complex curves. The data is stored in a standard pattern so that errors can be more easily located and rectified. The data assigned to each location in the data set is shown in Table 4.

Table 4

Data stored in specific locations in standard data set

Record number	Data stored
1	Figure number
2	Type number
3	Curve identifier
4	X value of 1st split - partitioned curves only
5	X value of 2nd split - partitioned curves only
6	} blank
7	
8	
9	Lower limit of curve
10	Upper limit of curve
11	Coefficient A_0
12	Coefficient A_1
13	Coefficient A_2
14	Coefficient A_3
15	Coefficient A_4
16	Coefficient A_5
17	Coefficient A_6
18	Coefficient A_7
19	Coefficient A_8
20	Coefficient A_9

If, as in the majority of cases, one polynomial has been fitted to the solution over the whole range then the data stored in locations 4 to 8 is zero. If, however, the curve has been partitioned into two, then the data stored in location 4 is not zero. In this case its value gives the position on the X axis of the split between the two sections of the curve. Locations 11 to 20 contain the coefficients of the polynomial fitted to the first part of the curve.

If the curve has been partitioned into three sections then the data stored in location 4 is the X value (X_1) of the first split and that stored in location 5 is the X value (X_2) of the second split. (X_2 is greater than X_1 .) In theory this system can go on, up to the curve being partitioned into six sections. However of the 267 curves that have been digitized to date only 26 were partitioned once and five partitioned twice.

Table 5

Data stored in specific locations in extra
data set for partitioned curves

Record number	Data stored
1	Figure number
2	Type number
3	1E + 15 - label to identify partitioned curve
4	Coefficient A ₀
5	Coefficient A ₁
6	Coefficient A ₂
7	Coefficient A ₃
8	Coefficient A ₄
9	Coefficient A ₅
10	Coefficient A ₆
11	Coefficient A ₇
12	Blank
13	Coefficient A ₀
14	Coefficient A ₁
15	Coefficient A ₂
16	Coefficient A ₃
17	Coefficient A ₄
18	Coefficient A ₅
19	Coefficient A ₆
20	Coefficient A ₇

If a curve has been partitioned then an extra data set (shown in Table 5) follows immediately after the first. This data set is not of the same format since it is an extension to the previous one. The label stored in location 3 identifies it as such. The coefficients of the polynomial fitted to the second part of the curve are stored in locations 4 to 11. If the curve is partitioned into three then the final set of coefficients are stored in locations 13 to 20. If the curve is partitioned again then another extra data set follows immediately. In this way the coefficients for each section of a partitioned curve are stored together, but within the standard pattern of the data file. Also the data file can be kept compact by using extra data sets rather than increasing the size of all the standard ones in order to cope with just a few partitioned curves.

5 A COMPUTER PROGRAM TO COMPOUND STRESS INTENSITY FACTORS

In this section the development of a computer program to compound stress intensity factors is described. The way in which the program is used to obtain stress intensity factors is also presented, together with an analysis of the results and a comparison with the manual technique.

5.1 Program development

As was illustrated in sections 3.1.1 and 3.1.2 with simple examples, the compounding method is an effective, albeit approximate way of obtaining stress intensity factors for complex configurations. It yields results much more quickly than standard analytical techniques by utilising the results from existing solutions which are available in graphical form. The small amount of written work in the example may however be misleading because reading and interpolating data from graphs and doing the calculations by hand still make compounding a somewhat lengthy process. When many more ancillary configurations are involved this time becomes quite significant. The process would be much more efficient if a computer program were able to manipulate the graphical data and carry out compounding calculations.

As a start in meeting this need a program was written to solve one specific problem for which a solution was available. The example chosen was that of a crack near a hole at the edge of a large sheet which is subjected to a uniaxial tensile stress remote from the crack. This is the same problem as that illustrated in section 3.1.1. The data file described in section 4 was used to store data which was obtained from the Compendium¹. The program was written in BASIC on a Hewlett-Packard HP9836 computer.

As the preliminary study was successful the program and data file were then developed further so that the equivalent stress method⁴ could be used and solutions could be obtained from many (40 to date) different configurations from the Compendium.

It must be emphasised that the program is not 'intelligent'. In other words it does not decide which ancillary configurations to use in order to solve a particular problem. The operator follows the same steps as were outlined in sections 3.1.1 and 3.1.2 except that the data acquisition, ancillary calculations and compounding are carried out by the computer. The program is an aid to calculation and, with the correct data, will produce consistent results in a fraction of the time it takes to solve problems by hand.

5.2 Running the program

If the operator were to solve a problem similar to that described in section 3.1.1 the first step would be the same. That is decide which ancillary configurations best describe the complex geometry when compounded together. The program can now be run. The first section is concerned with various aspects of the presentation of the index and results, and whether the equivalent stress method is to be incorporated in the calculations. Help comments are available to explain the running of the program and the consequences of any answer.

The next stage is to choose the ancillary configurations to be compounded. This is done by typing the figure number of the required ancillary solution. The program then checks through an index file to see whether this figure number has been stored in the main data file. If it is available then all the parameters relating to that configuration are requested. The particular curve value can now be determined and the program searches the main file to find the required data set. If the data is not available because the particular combination of parameters is outside the range of the solution in

the Compendium then the operator is asked to try the input again. Once the data set has been found it is transferred from the data file to the memory in the program.

If the curve value requested by the operator is between two curves stored in the data file then both data sets are loaded and the results are interpolated in the calculation routines. The next ancillary configuration is then chosen, all the parameters are input defining its particular geometry and the data set is stored in the computer memory. This process continues until all the ancillary configurations have been stored.

The next step is to decide what crack lengths are to be used in the calculations. The crack lengths can be input in two ways; either as a series of lengths with a set increment between two limits, or as individual crack lengths. Once the operator has chosen which method to use, up to 100 crack lengths can be stored ready for calculation. The operator input is now complete and the program has all the data to calculate the compounded results. Firstly the polynomials are reconstructed from the data set and values of Q are calculated for the given ratios of each ancillary configuration. These are then compounded and, if necessary, the error term Q_e is calculated and added. The results are printed on the screen as Q_r for each crack length. The flow diagram (Fig 6) shows the general pattern of calculation. It can be seen that the computer program follows similar steps to those described in sections 3.1.1 and 3.1.2.

5.3 Analysis of results

Results from the computer program and those from the manual calculations are presented in Tables 6 and 7. In comparing these results it must be remembered that both methods are approximate. Errors appear in reading data from graphs by hand just as they do in approximating an equation to a curve. Therefore the comparison is made in order to determine how much the new method of data analysis differs from the existing one rather than how much it differs from the 'correct answer'.

It can be seen that the computed results differ from those calculated manually by less than 1%. It must be noted that most of the solutions in the Compendium have probable errors of ~1% and that no estimate has been made of the errors arising from the drawing of the graphs. In obtaining stress intensity factors the values of stress, crack length and structural dimensions are to some degree approximate. The results of such calculations are used in calculations of fatigue life and critical crack length which in themselves are approximations. Therefore it is considered that the differences arising from the computer calculations are acceptable when compared to the errors in other methods used in fracture mechanics.

Table 6

Comparison of computed results
with those obtained in
section 3.1.1 by hand

a/b	Manual Q_r	Computed Q_r
0.10	1.18	1.18
0.15	1.18	1.18
0.20	1.18	1.18
0.25	1.19	1.19
0.30	1.21	1.20
0.35	1.22	1.22
0.40	1.25	1.25

Table 7

Comparison of computed results
with those obtained in
section 3.1.2 by hand

a	Manual Q_r	Computed Q_r
1.04	0.68	0.68
1.08	0.87	0.88
1.20	1.08	1.09
1.40	1.18	1.19
1.60	1.22	1.22
1.80	1.24	1.24
2.00	1.27	1.28

6 DISCUSSION

In this section other aspects of the project are discussed, particularly the writing of program code and the requirements of the users manual. The inclusion in the data file of some additions to the solutions contained in the Compendium is also discussed, as well as certain limitations in the use of the computer version of the compounding technique.

6.1 Computer program

Any computer program that is to be used extensively must be easy to use. This means that the parts of the program directly related to the operator, such as comments and requests must be easy to understand and operate. Practical Computer magazine¹¹ explains this more fully and gives ten basic rules to be followed. These are a very useful guideline in assessing the performance of an interactive computer program. In general the qualities exhibited by this type of program ought to be such that the operator should be confident of the meaning of any comment or request displayed on the screen, and that these directions should enable the user to input all the data in the correct format and to get the desired results from the program.

The best way of making a program easy to operate is to have some explanatory text, but not too much, whenever data is to be input or output. A short paragraph of text that is clear in its message will be of great help to an inexperienced operator, so long as it is concise. However, a person who has used the program several times and knows what to do is likely to ignore most of this text. Indeed, it may have a detrimental effect by cluttering up the screen and also increasing run time. So while full explanations are required for first time users, experienced operators may need no more than single word prompts and very basic titles and headings. Ideally the provision of help comments should be infinitely variable between these two extremes and directly related to the experience of the operator. The best way to approach this is by putting the help comments facility entirely in the hands of the operator.

In this program help comments are available on request. The basic program assumes an experienced user and prints short comments at each stage. The help comments can be requested at almost any point in the program. The text appears on the screen and remains until the operator presses <CONTINUE>. The program then returns to precisely the stage where it left off. No data is lost by asking for help comments.

Another important aspect is that the program must be capable of dealing with the occasional typing error as well as with people who do not understand the method of operation. There is a need to enforce the rules of data input in order to make the program foolproof and the results reliable. Any input which could cause unpredictable results later in the program must be declared invalid. It is most important that this is done at the time of input so that the data can be corrected. The fact that an error has occurred may also mean that the operator did not understand what he was supposed to do; therefore a line of explanation describing the cause of the error is also required.

6.2 Operators manual

The operators manual is as much a part of the program as the screen display. It too must be easy to read, clear, concise and comprehensive. It must explain in general terms what needs to be done to operate the program and what sort of output is to be expected. It is especially important to make clear what data is required by the program. The manual written for this program⁵ also contains a question-by-question guide to possible answers and their subsequent effects, by which the operator can quickly gain information about any part of the program. An 'easy reference' section such as this together with a list of possible errors messages and their causes is much more useful than a lengthy description of the program as a whole.

6.3 Additional solutions

There are many solutions for stress intensity factors which have proved useful but are not in the Compendium since they were published after the Compendium. One such set of solutions is that for cracks (one or two) at loaded holes¹¹; some of these have been included in the data file. In order to distinguish these solutions from those in the Compendium (maximum figure number 202), they are numbered from 300 onwards.

6.4 Limitations of the compounding program

Although the method of compounding has been used successfully to obtain stress intensity factors for cracks at loaded holes near boundaries¹², the current computer program cannot be used to solve such problems. When localized loads are in equilibrium with stresses applied remote from the crack, a combination of compounding and superposition techniques is required¹² for the solution. The general formulation of these procedures is not yet available. However, the individual ancillary solutions required can still be obtained from the data file and only the final compounding needs to be done by hand; thus an appreciable time-saving and reduction in labour is still achieved.

The current computer program cannot in general be used to solve problems of edge cracks at straight exterior boundaries of finite configurations. The amount of in-plane bending which occurs is strongly dependent on the relative positions of all the boundaries and may not be adequately represented in ancillary configurations.

Because experience has shown that, provided the crack is not close to any boundaries, the boundary-boundary interaction term (Q_e) is small, it is not included in the simple compounding formula. It is, however, included in the case of cracks growing from the edges of holes.

7 CONCLUSIONS

A method of reducing graphical stress intensity factor solutions was devised whereby a computer program was used to analyse point data relating to the solution and fit a polynomial function to it. The equation of this function was stored in a data file along with other parameters defining the particular solution.

A computer program was written which by using this data file as a source of ancillary solutions satisfactorily solved complex stress intensity factor calculations by the compounding method.

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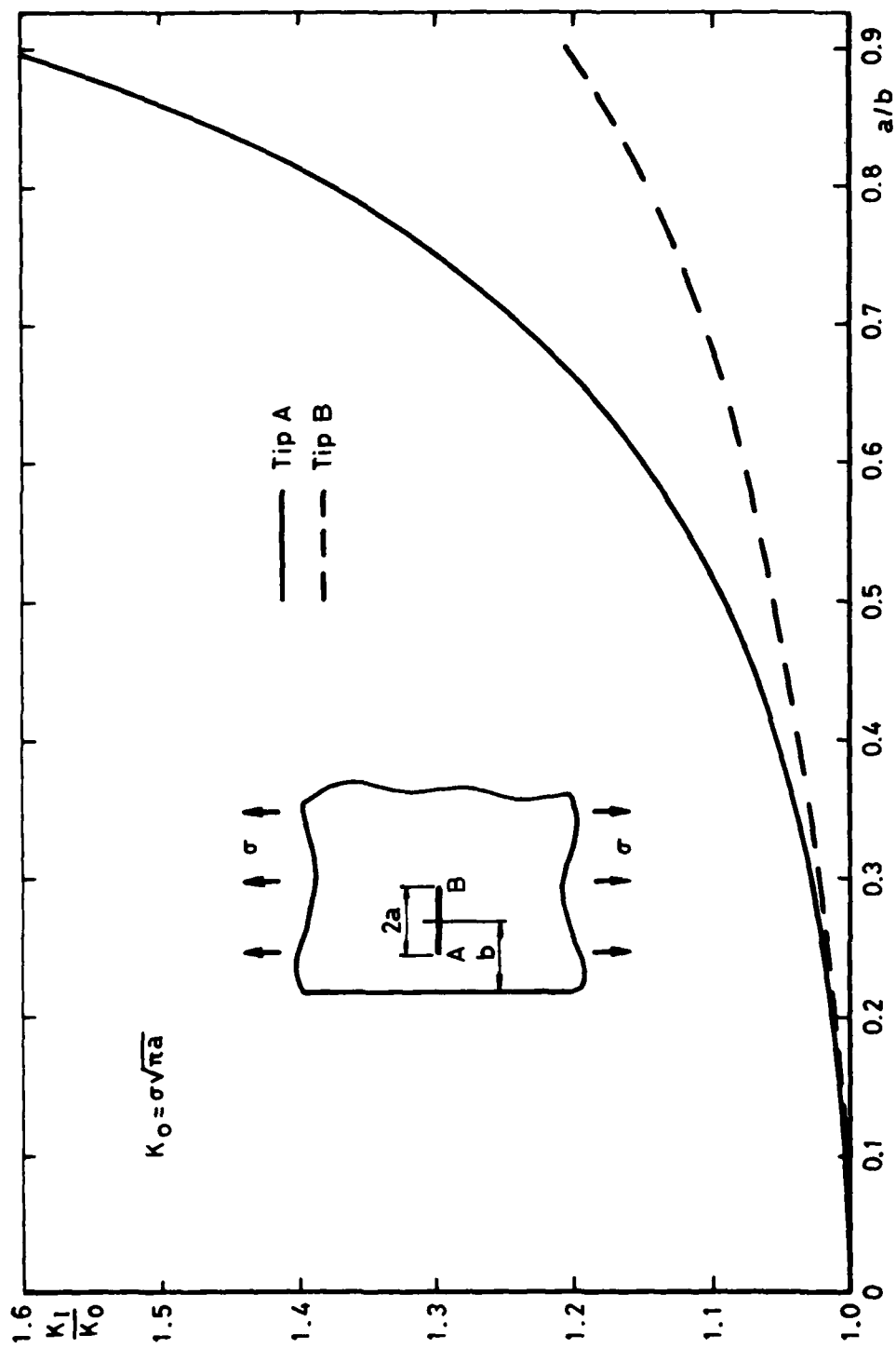


Fig 1 K_I for an internal crack perpendicular to the edge of a sheet subjected to a uniaxial tensile stress

Fig 2

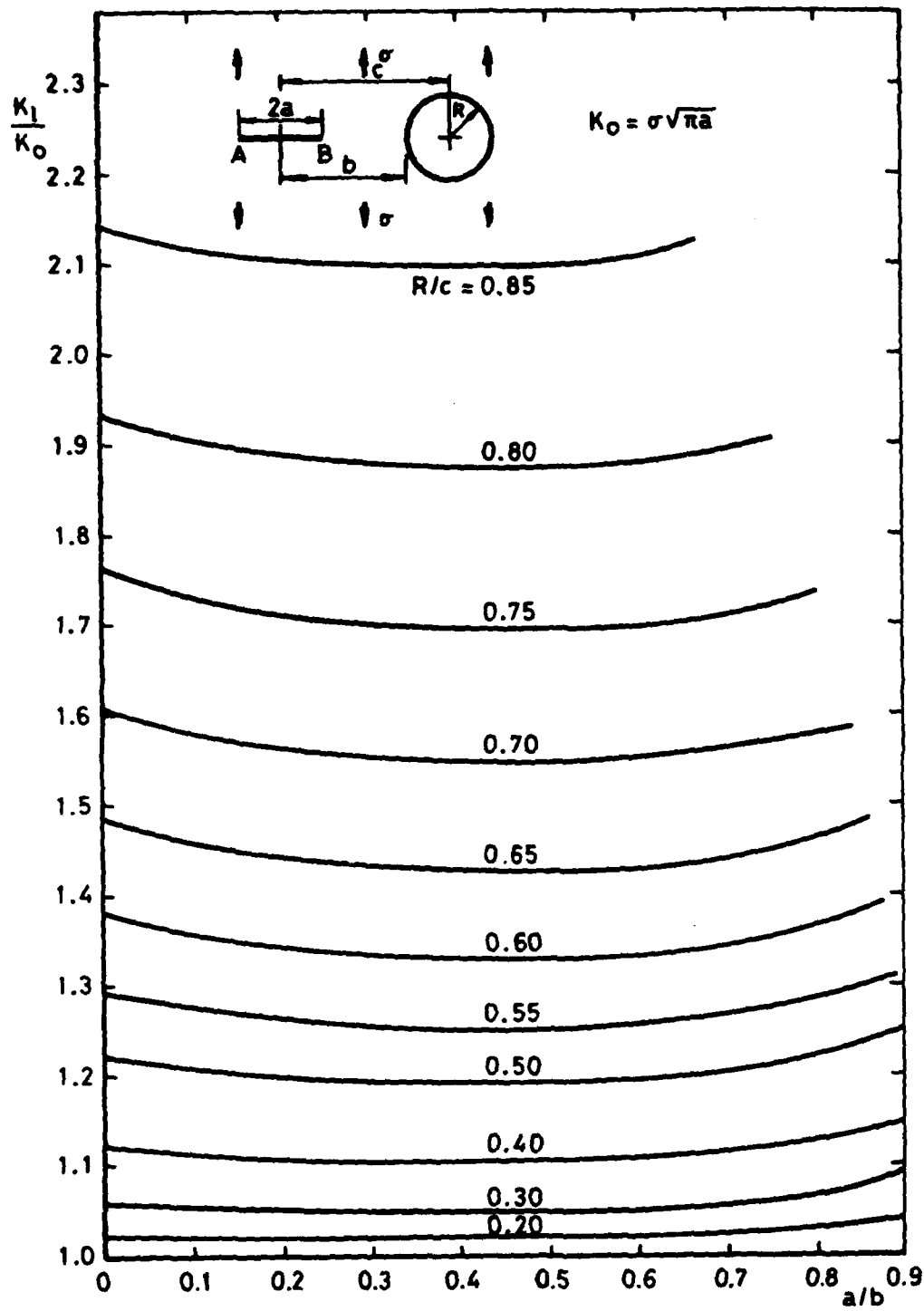


Fig 2 K_I for tip A of a crack near a circular hole in a sheet subjected to a uniform uniaxial tensile stress

Fig 3a-c

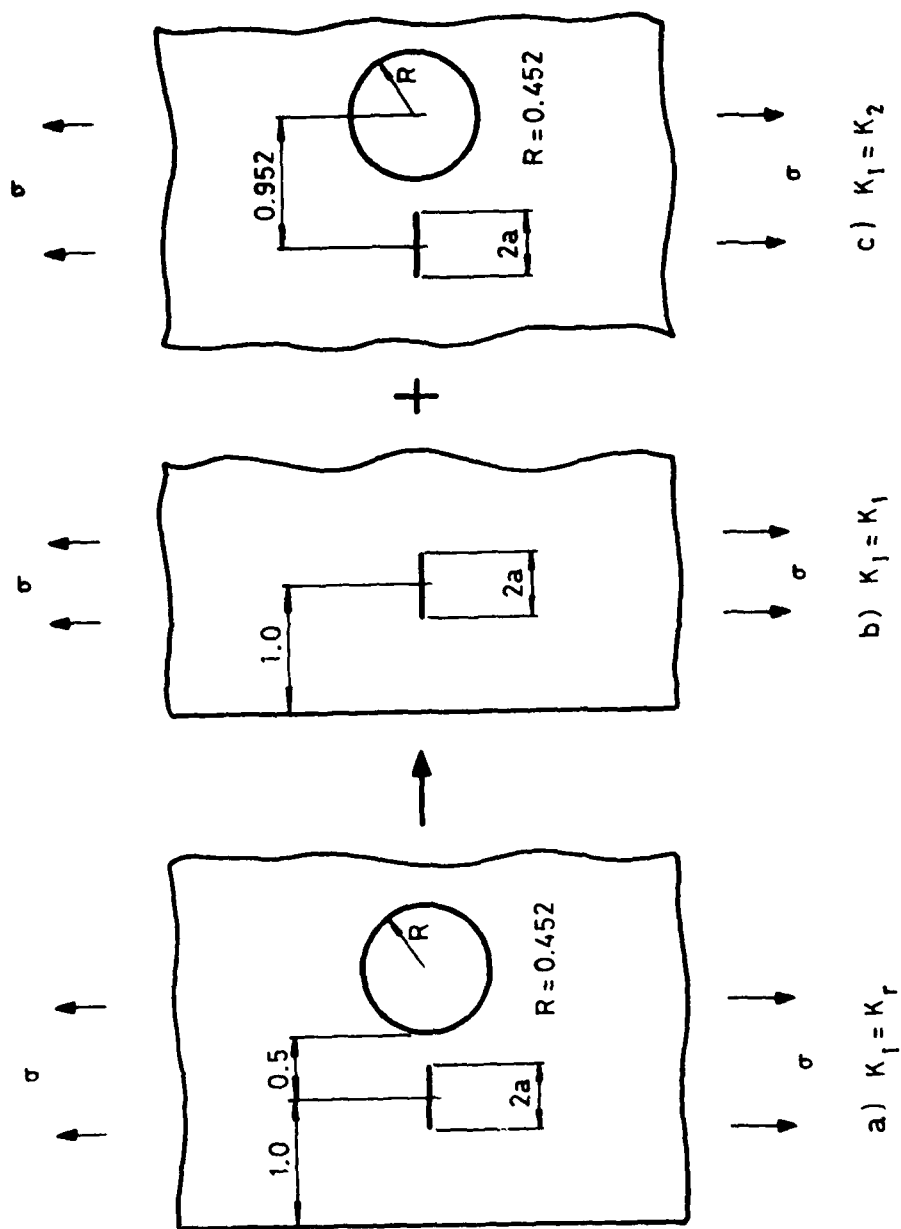


Fig 3a-c Ancillary configurations for a crack near the edge of a large sheet

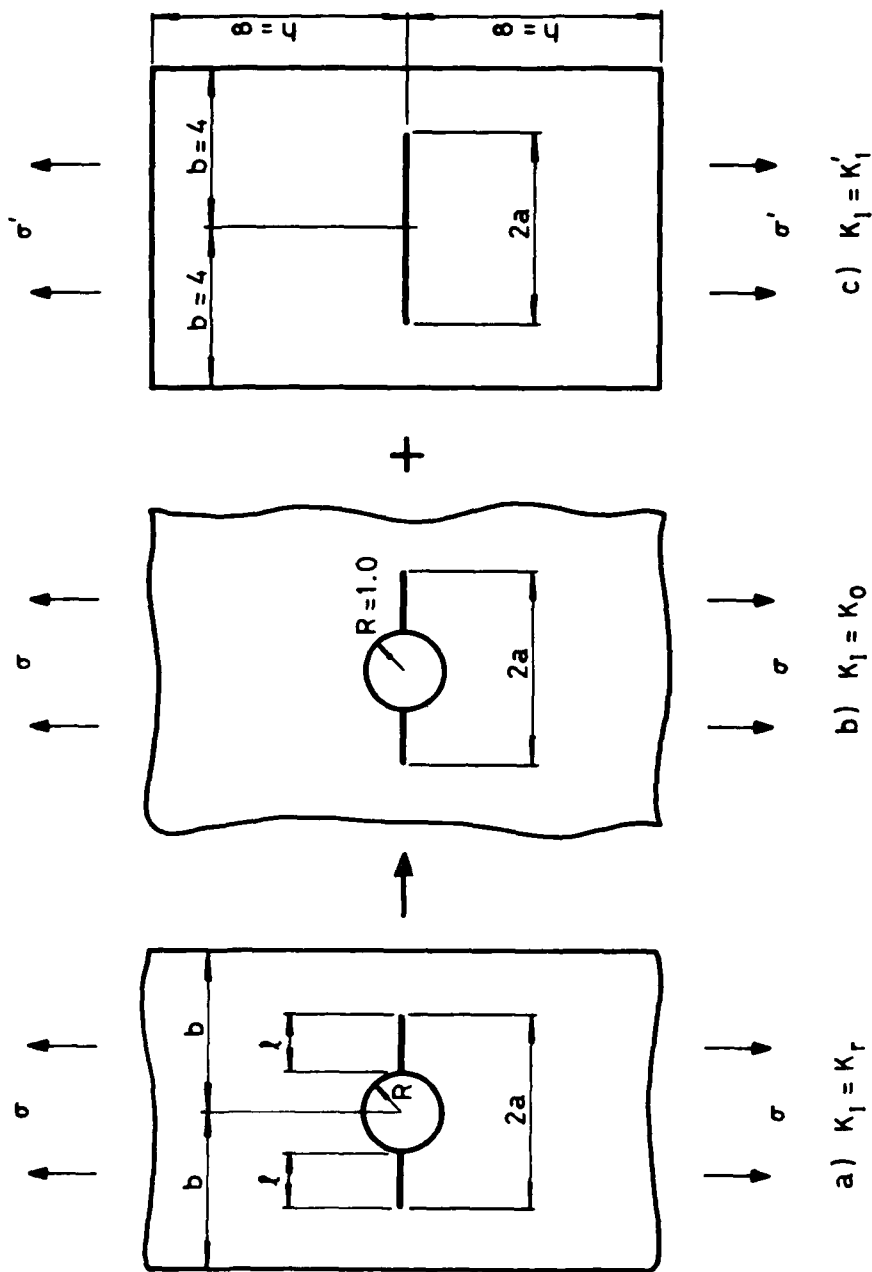


Fig 4a-c Ancillary configurations for a hole in a cracked strip

Fig 5

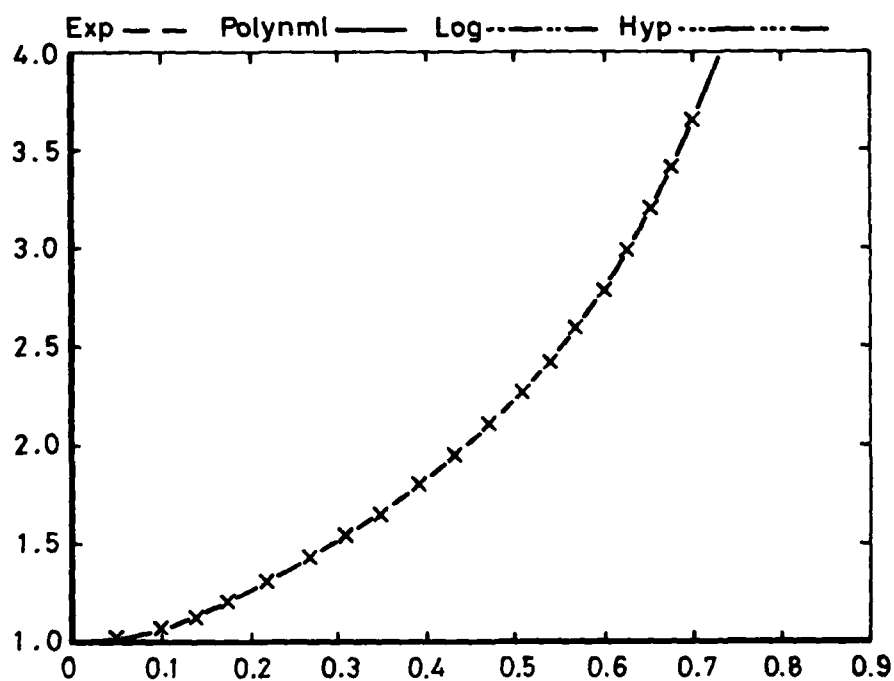


Fig 5 Example of graphics output from curve fitting program showing 5th order polynomial fit

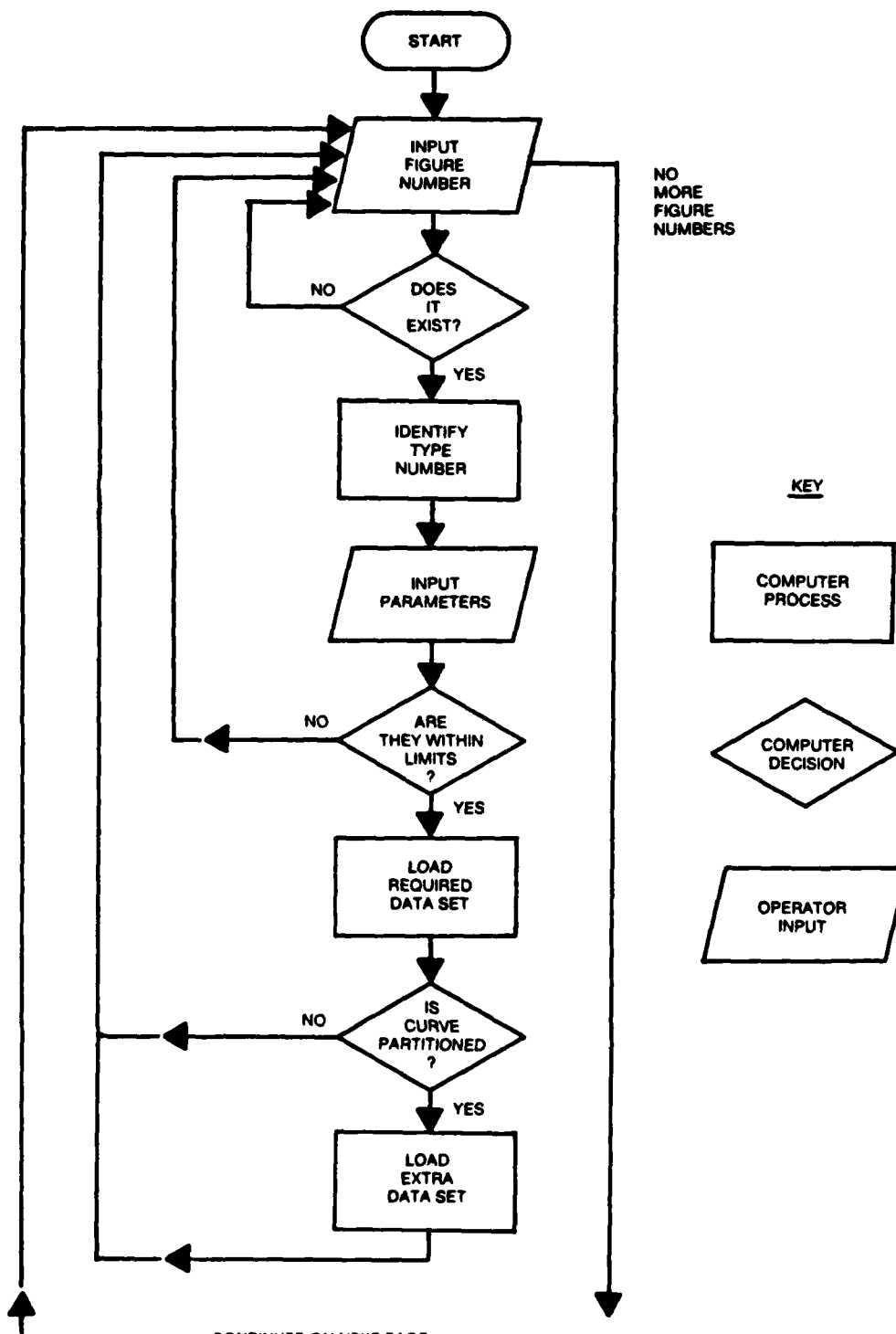


Fig 6 Flow diagram of computer program to compound stress intensity factors

Fig 6 (concl'd)

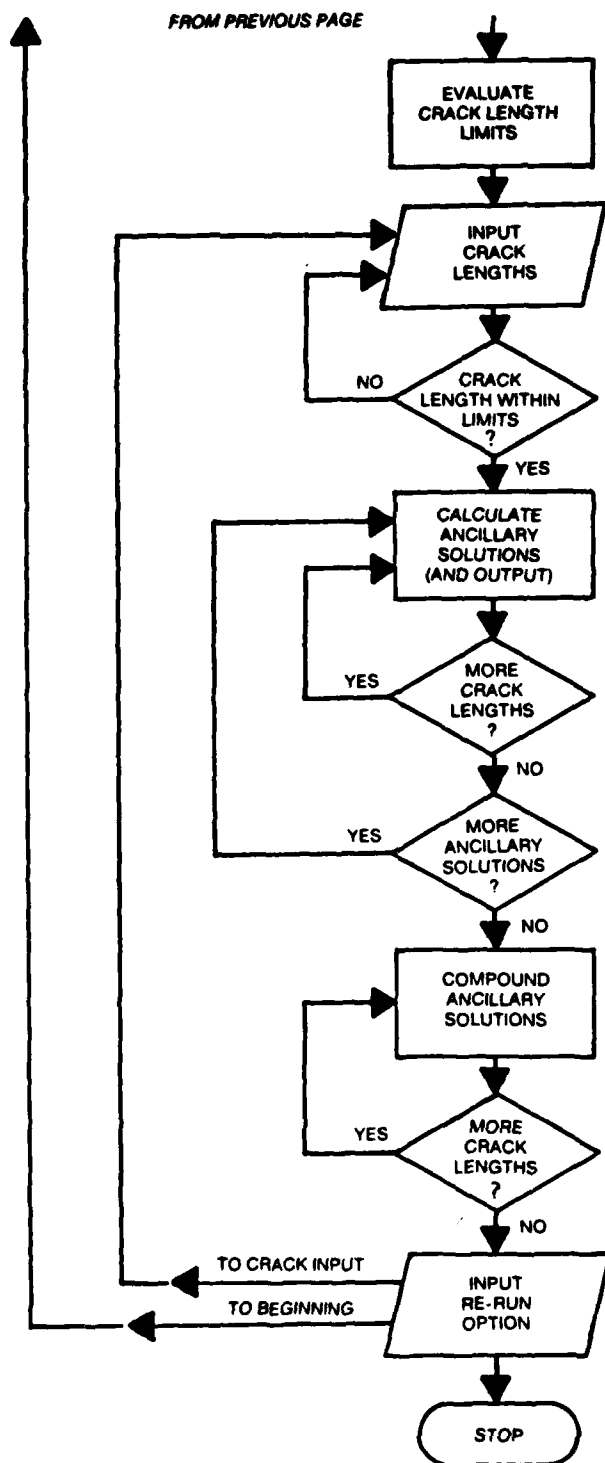


Fig 6 (concl'd) Flow diagram of computer program to compound stress intensity factors

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REPORT DOCUMENTATION PAGE

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17. Abstract The compounding method enables stress intensity factors to be evaluated for complex geometrical configurations using the known results of simpler ancillary configurations. Implementation of the method on a computer system requires that a data file of known results be available, and that a program be written to compound solutions from that data file. Firstly, a procedure is described to convert available graphical data on stress intensity factors to numerical data and store it in a direct access computer file together with identification parameters. Secondly a computer program is described which uses the data in that file to compound stress intensity factors for other configurations. It is shown that the errors in the results from this program are within acceptable engineering tolerances.					

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