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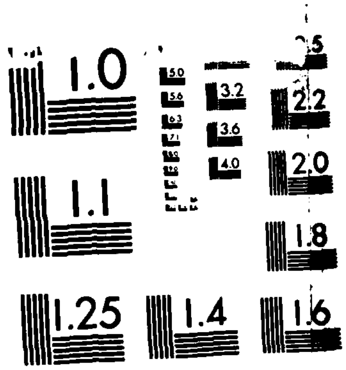

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ON DECENTRALIZED CONTROL
OF A LARGE SPACE STRUCTURE

THESIS

Jonathan B. Sumner
Captain, USAF

AFIT/GA/AA/85D-9

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Astronautical Engineering

Jonathan B. Sumner, B.A.E.

Captain, USAF

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Brian Sumner

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Abstract

Modern optimal control methods are used to develop a multiple-input-multiple-output controller. Focus is made on a three-controller configuration exhibiting high controllability but low observability in the first controller, a median amount of each in the second, and low controllability but high observability in the third. These characteristics are due to the technique used to suppress control and observation spillover among the controllers. A control model for large space structures, which employs full state feedback using deterministic observers, is developed and implemented in a computer simulation. The technique of spillover suppression and the conditions assuring stability of the control system are developed and implemented as well.

The simulation is tailored to address the control of the Draper-2 large flexible space structure model. The model has been used previously for optical pointing (line-of-sight (LOS)) studies. Here, position sensors and point force actuators are used to effect feedback control (regulation) of the damped unforced structural vibrations. The simulation can output both the unsuppressed and suppressed case open-loop and closed-loop eigenvalues and the LOS time response for stability and performance analysis.

With the control problem formulated for modal control, an investigation is made into the effects on time response of assigning three groups of four modes in a permutative fashion

to the three controllers. A fourth residual set of eight modes is carried without spillover suppression to represent the unmodelled modes of a real structure. The groups are assembled based upon a previous investigation's results from applying modal cost analysis for LOS performance. Simple high frequency truncation is also used for comparison.

Controllers based on modal cost analysis alone are found to yield marginal stability and mediocre LOS performance due to little insight into the sensitivity of the residual modes to spillover. However, specific problem modes are readily identified by examining the results of an internal balancing analysis for modal sensitivity. Simple frequency truncation is found to give the best time response here when the modes most contributory to LOS are assigned to the controllers with more controllability. However, the relatively small quantity of modes and the overwhelmingly large relative contribution of the three rigid body modes included may obscure some conclusions. Results indicate more revealing results might be obtained if more modes are added to the model and/or if some of the residual modes are suppressed.

MODAL ASSIGNMENT EFFECTS
ON DECENTRALIZED CONTROL
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I. Introduction

Now that the space shuttle is demonstrating the ability to routinely deliver sizeable payloads to space, the construction of large space structures for commercial manufacturing, scientific research, and military use will soon be reality. There are still practical economic constraints, however, on the amount of material the shuttle can deliver for construction. Efficient structural design then dictates the use of thin, lightweight elements in trusslike frameworks. Although helping eliminate the problem of delivering materials, this introduces a control problem due to the undesirable flexibility inherent in such structures. Often the large space structure is intended for high accuracy pointing of antennas and lasers. Active regulation (control) of structural vibrations is thus required. Unfortunately such structures are lightly damped and have numerous low-frequency vibrational modes, often clumped together, within the bandwidth of practical control systems. Furthermore, there can be hundreds of modes to be concerned with even after techniques (1, 2) have been applied to reduce the number of modes from infinity for a continuous system, to thousands for a finite element structural model, to those the designer concludes to still have significant effect on the stability and time response of the control system. Modern state-space methods of control theory lend themselves well, however, to this concept of a discrete (finite) structural model. The more modes a designer can keep in the model of the system/space structure, the less inaccurate it will be. But the computational burden to an online computer, functioning as the controller, grows also.

Therefore the designer may need to reduce this computational burden by employing multiple controllers. Each would bear only a portion of the computational load, and ideally control only those modes assigned to it, suppressing the effects of other modes. In other words, ideally all modes would be either controlled or suppressed. Practical constraints can still prevent there being enough controllers and/or few enough modes to permit this, however. The remaining modes are called residual modes. The effect of these modes on the system is called observation spillover as they contaminate the sensor (observation) data. In contrast, the control applied to the controlled modes can affect the residual modes. This is called control spillover, and the residual modes may drive the system either unstable or even more stable. Insofar as the suppression of spillover is concerned, however, Calico and Aldridge (3) successfully showed that a transformation matrix could be generated by singular value decomposition of the control and observation matrices which, when applied to the feedback gain matrices, would drive spillover terms to zero, effecting suppression. They employed a mathematical model of a fictitious nontrivial space structure in a simulation whose computer code allowed for the inclusion of either three controllers with a set of residual modes (numbering as few as zero) or four controllers with no residual modes. The code contained an observer to provide full state feedback. Calico and Thyfault (4) used a similar version with altered code to investigate the effect of direct output feedback in place of the observer. In neither case was there time to carry the investigation to examining time response, which could have been quite revealing of the overall performance effects of the various ways in which modes are assigned to controllers.

The thrust of this thesis is to implement time response output for the nontrivial model and investigate the effect on time response of certain modal assignments by fixed groups to any of the three controllers with another fixed group assigned as residuals. The inclusion of residuals provides in a limited sense a truth model for the analysis of stability and performance. There may be significant differences in time response caused by the fact that a controller is known to provide more relative controllability and less observability, or vice versa, for its assigned modes. The investigation will involve initially the fixing of certain baseline parameters to allow a parallel comparison of reasonable results. Then line-of-sight pointing and defocus performance will be generated for comparison among various cases. The performance is actually accomplished by using position sensors to provide modal amplitudes measurement with an observer in place for the unmeasured states, and using point force actuators to execute control. All processes are assumed to be deterministic, as stochastic estimation and control is not within the scope of this investigation. The simulation model is a version of the so-called "Draper-2" space structure model originated at the Charles Stark Draper Laboratories.

The next section ~~will~~ describes the selected model configuration and discuss its finite element representation. Then the modal control and matrix transformation methods will be explained. ~~Afterwards~~ The implementation of the simulation program will be discussed. ~~Finally~~ The last sections will detail the investigation, results, conclusions and recommendations.

II. Selected Model Configuration

The model selected is basically the same as that used by Calico et al in their investigations. It is a fictitious, nontrivial space structure developed by Charles Stark Draper Laboratories for just such control problem studies. Several versions of the model, incorporating only minor changes, have indeed been used extensively throughout the community. The version selected here is that used by Draper Laboratories themselves in a study done for the Defense Advanced Research Projects Agency (DoD) (5). It varies slightly from that version used by Calico et al because of differing masses at some locations. It was selected, however, because the study using it included conversion data to obtain physical, line-of-sight output directly from the generated modal coordinates.

The model provides a realistic flexible space structure simulator for study, and includes a flexible optical support structure in a trusslike framework and an equipment section with solar panels. The optical support structure consists of three trusses: the upper mirror truss containing a convex primary mirror and concave tertiary mirror, the lower mirror truss containing the flat secondary mirror and image focussing plane, and the metering truss to maintain mirror separation. Figure 2-1 depicts the overall structure, and Figure 2-2 the finite element representation. Figure 2-3 gives the model's general structural dimensions. Then Figure 2-4 shows the optical path of a ray of light reflecting from the mirrors.

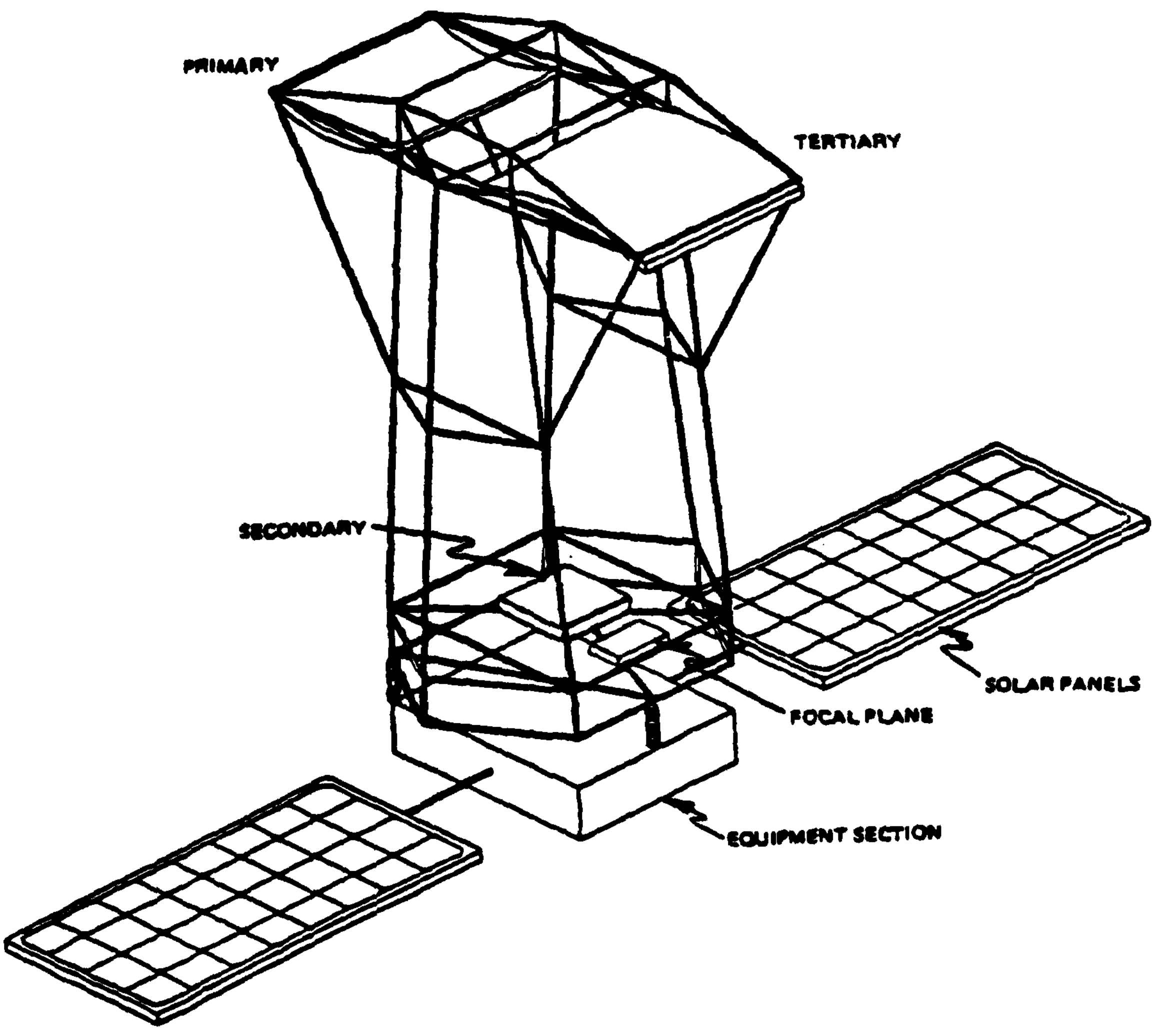
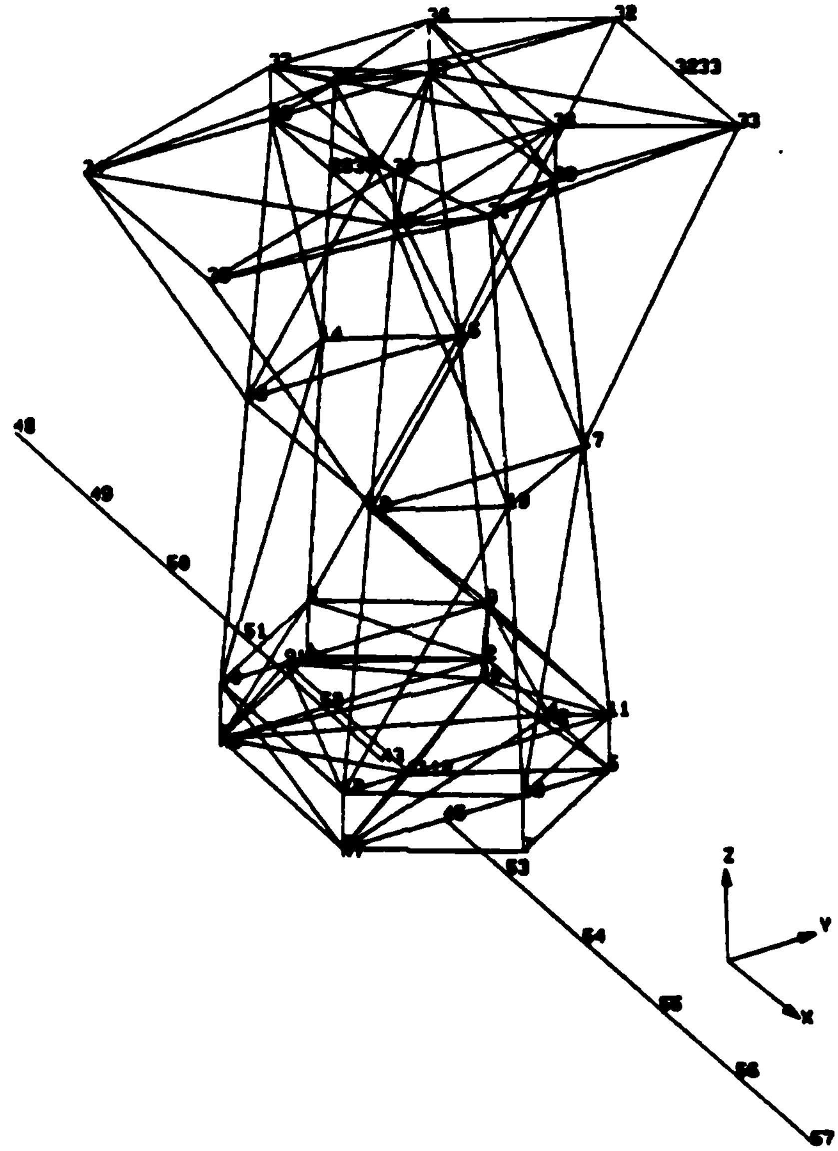


Figure 2-1. Overall Structural Depiction



<u>Figure 2-2.</u> Finite Element Representation

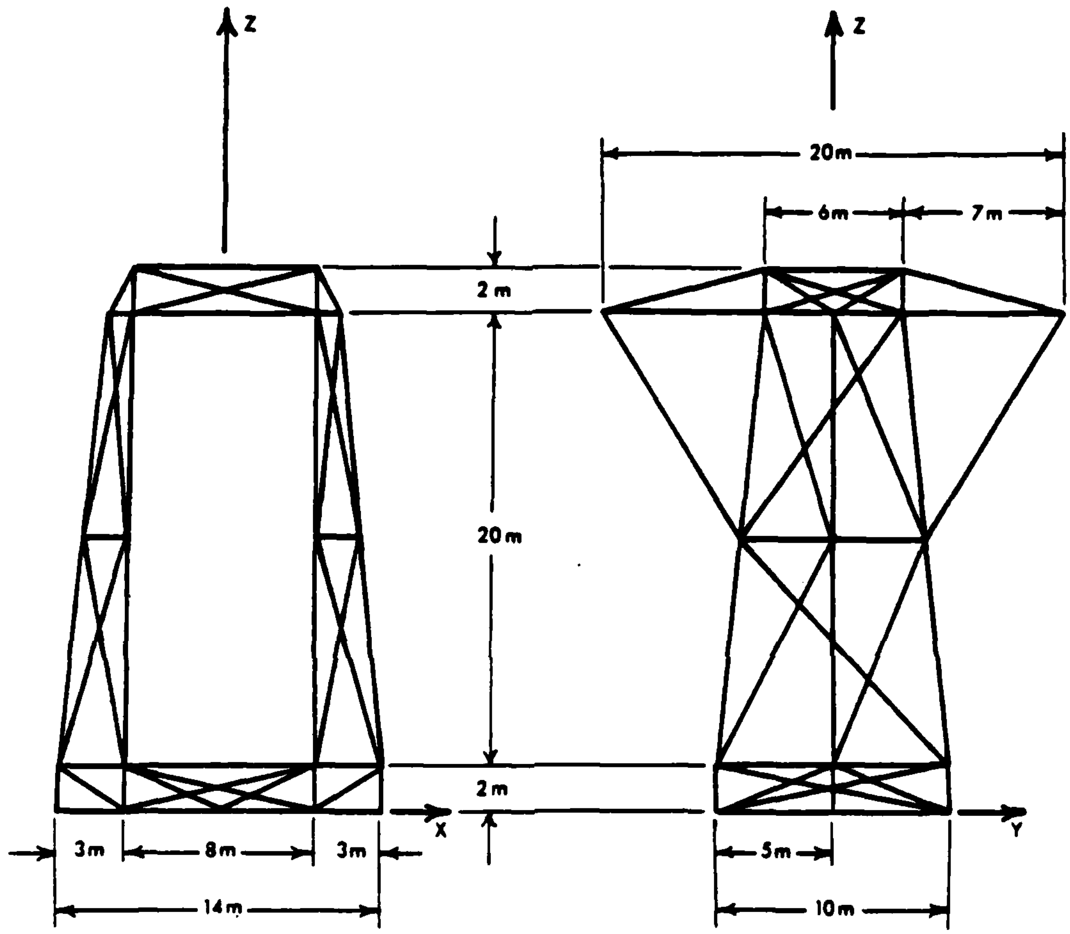


Figure 2-3. Structural Dimensions

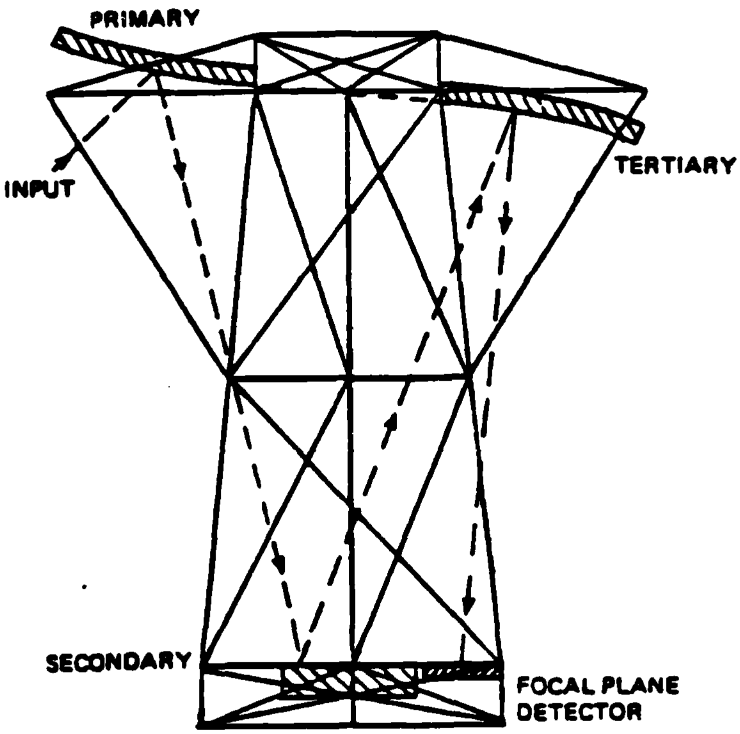


Figure 2-4. Optical Path

Appendix A gives the necessary input cards for a NASTRAN run, and pertinent data for the Draper-2 model. From the "CONM2" input cards it can been seen that the model weighs some 9304 kg, and from Fig 2-3, is 24 m high.

Of interest in this investigation is the calculation of the optical line-of-sight (LOS). For Draper-2 the optical line-of-sight refers to how the ray of light shown in Fig 2-4 deviates in the x-, y-, or z-direction from a nominal "focal" point. The x-y planar deviations can be (and are in this investigation) put into a more meaningful radial deviation, while deviations in the z-direction can be understood better as a measure of defocus of the optical ray. The motions of the structural members, of course, are what causes these deviations, and these motions bear kinematical relationship to each other since the members are connected. Ref (5) provides the basic algorithm for calculating these LOS deviations or errors, while the full expansion of the basic algorithm can be found in Ref (3). Fortunately it is not necessary to employ the fullblown set of equations to calculate the LOS error at some time t in the time history of the motion of the structure. When modal analysis is performed, as in this investigation, a vector of modal coordinates at some time t can be multiplied by some output matrix which effects modal superposition and yields the three quantities of interest in an "LOS vector." This matrix is referred to as Φ_{LOS} and is of row dimension 3 and column dimension n, where n is the number of modes in the model. Table 2-1 provides the matrix in its transposed form for convenient reading. The column labels LOSX and LOSY refer to a normalization imposed on the x- and y-LOS errors, wherein they are divided by a factor of 8.051.

Table 2-1. (5)

Transpose of Matrix Φ_{LOS}

Mode	Freq	LOSX	LOSY	Defocus
1	0.0	0.2705E-04	0.3778E-03	0.9313E-09
2	0.0	-.1580E-04	0.8426E-06	-.1863E-07
3	0.0	0.4927E-03	-.3133E-04	-.9604E-09
4	0.0	0.1166E-03	0.1155E-03	0.4366E-10
5	0.0	-.3502E-04	-.8110E-03	-.2721E-08
6	0.0	-.7835E-03	0.4676E-04	0.6985E-09
7	0.1455	-.2641E-06	0.3263E-03	-.4904E-07
8	0.2632	0.3489E-06	-.2141E-03	-.1099E-05
9	0.3173	-.1505E-05	-.3452E-06	-.5238E-05
10	0.3329	-.2293E-03	0.4407E-06	-.1467E-06
11	0.4432	0.6041E-05	0.1066E-05	0.1559E-04
12	0.5779	0.7359E-03	-.6768E-05	0.1906E-05
13	0.5814	0.2330E-05	0.8731E-03	-.4276E-06
14	1.2238	-.1027E-05	-.2362E-03	-.4831E-07
15	1.3002	0.3759E-05	-.1519E-03	-.7763E-05
16	1.3475	-.1589E-03	-.1413E-05	0.9364E-05
17	1.7209	0.2738E-04	-.2777E-02	-.4955E-04
18	1.8187	0.2590E-05	-.4976E-06	0.2634E-05
19	1.8187	0.1151E-04	-.2364E-05	-.5493E-06
20	1.8892	-.3597E-13	-.3518E-14	0.3030E-14
21	2.3635	0.2237E-04	0.1376E-01	0.9028E-05
22	2.9895	0.1345E-03	0.2724E-05	-.1509E-04
23	3.1795	-.4355E-05	-.2602E-03	0.2500E-05
24	3.3873	0.5573E-06	-.4536E-03	-.4258E-06
25	5.1617	-.1814E-13	-.4398E-15	0.2601E-14
26	5.2603	-.3395E-04	-.7013E-06	0.4747E-05
27	7.8769	0.1038E-14	0.7264E-16	-.9614E-16
28	8.1168	-.4275E-03	-.1701E-02	-.1950E-03
29	8.3600	0.2916E-03	0.4775E-06	-.3638E-05
30	8.5706	0.1868E-01	-.1108E-04	0.1072E-02
31	8.8135	-.3822E-04	-.2919E-06	-.3755E-05
32	8.8135	-.4494E-04	0.7780E-07	0.9546E-06
33	11.3462	0.3762E-05	0.8995E-05	-.5240E-06
34	11.4978	-.1225E-02	0.1870E-02	0.3982E-03
35	12.7258	0.1098E-01	-.5662E-03	-.1593E-01
36	13.5832	0.2097E-04	0.7475E-03	-.6865E-04
37	13.7141	-.7143E-03	-.1011E-01	0.1142E-02
38	14.1604	0.3953E-02	0.5935E-02	-.1084E-02
39	15.6523	-.1414E-01	0.3213E-02	-.4966E-02

Table 2-1. cont'd

Transpose of Matrix Φ_{LOS}

Mode	Freq	LOSX	LOSY	Defocus
40	16.0724	0.1907E-02	0.1044E-01	-.2294E-03
41	16.5248	-.6780E-02	-.2220E-02	0.1413E-01
42	16.7453	0.3853E-02	-.9540E-03	-.1375E-01
43	17.1555	0.1132E-03	-.8405E-02	-.1739E-02
44	17.8283	0.3244E-02	-.1168E-02	0.7174E-02
45	19.0713	-.6439E-03	-.1446E-01	-.1545E-02
46	23.7716	-.2266E-01	0.1669E-01	0.5217E-01
47	24.4140	0.9058E-02	0.3711E-01	-.2293E-01
48	25.9089	0.2688E-03	0.1205E-01	-.3317E-02
49	26.3625	-.1444E-01	-.1042E-01	0.2382E-02
50	26.4292	0.9416E-14	-.6257E-14	-.1485E-14

Note that the first six rows are for the rigid body modes which have zero frequency. The portion of the table used in this investigation included mode numbers 4 through 23, totalling twenty modes.

For this investigation, a set of twenty-one collocated pairs of point force actuators and position displacement sensors were employed at the same locations in the structure as in Ref (3). These locations and the sensor/actuator orientations in direction cosines are given in Table 2-2. There is a sensor and actuator for every motion which affects the LOS deviations. Notice that the orientations are conveniently aligned with the coordinate axes of the model. Strictly speaking, any orientation that is physically possible could be used as long as proper direction cosines are used.

With the structural model described, the system mathematical model and control derivation is at hand.

Table 2-2

Draper-2 Sensor/Actuator Locations and Direction Cosines

Pair	Node	x	y	z
1	9	0	1	0
2	9	0	0	1
3	10	0	0	1
4	11	1	0	0
5	11	0	1	0
6	11	0	0	1
7	12	0	0	1
8	27	1	0	0
9	27	0	1	0
10	27	0	0	1
11	28	0	0	1
12	29	0	1	0
13	29	0	0	1
14	30	0	0	1
15	32	0	0	1
16	33	0	0	1
17	34	1	0	0
18	34	0	1	0
19	34	0	0	1
20	35	0	1	0
21	35	0	0	1

(N.B. These map to only translational components of the modal matrix described in Section III.)

III. System Mathematical Models

Equations of Motion

The motion of a large space structure, such as Draper-2, can be described as that of a typical multiple-degree-of-freedom system with light damping. Here the system will be deterministic, with active control to regulate vibrations. The equations of motion can be expressed in matrix form as

$$M\ddot{\underline{q}} + E\dot{\underline{q}} + K\underline{q} = D\underline{u} \tag{1}$$

where

$M \triangleq$ nxn mass matrix

$E \triangleq$ nxn damping matrix

$K \triangleq$ nxn stiffness matrix

$D \triangleq$ nxm actuator mapping matrix

$\underline{q} \triangleq$ nx1 vector of physical coordinates q

$\underline{u} \triangleq$ mx1 control force input vector

and n is the number of degrees of freedom (corresponding in turn to the number of modes of vibration in the system), while m is the number of actuators employed to effect control.

The matrices M and K may be obtained by finite element analysis of the structure (for example by NASTRAN). The control designer may then decide how many modes must be kept in his model, and how many and what type actuators there should be and where they should be placed.

Proceeding now toward the objective of determining the control $\underline{u}$, transformation is made to state space form.

Solving the undamped, free-vibration equation of motion

$$M\ddot{\underline{q}} + K\underline{q} = \underline{0} \qquad (2)$$

yields the system's natural frequencies ω_k, k=1...n, and the nxn modal matrix Φ. The columns of Φ are the associated eigenvectors determined from $\det| M\lceil -\omega_k{}^2 \rfloor - K | = 0$. The modal matrix is such that

$$\underline{q} = \Phi \underline{\eta} \qquad (3)$$

where $\underline{\eta}$ is an nx1 vector of modal coordinates. The properties of Φ are such that when Eq (3) is directly substituted into Eq (1) which is then premultiplied by Φ^T, the result can be expressed as

$$\ddot{\underline{\eta}} + \begin{bmatrix} \ddots & & \\ & 2\zeta_k\omega_k & \\ & & \ddots \end{bmatrix} \dot{\underline{\eta}} + \begin{bmatrix} \ddots & & \\ & \omega_k{}^2 & \\ & & \ddots \end{bmatrix} \underline{\eta} = \Phi^T D\underline{u} \qquad (4)$$

The second order matrix equation may now be converted to a first order state space expression

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \qquad (5)$$

where

$A \triangleq$ 2nx2n plant matrix

$B \triangleq$ 2nxm control mapping or input matrix

$\underline{x} \triangleq$ 2nx1 state vector

$\underline{u} \triangleq$ mx1 control input vector

These state space matrices have form

$$A \triangleq \left[\begin{array}{c:c} 0 & I \\ \hdashline \diagdown\, -\omega^2 \,\diagdown & \diagdown\, -2\zeta\omega \,\diagdown \end{array}\right] \text{(subscript k implied)}$$

$$B \triangleq \left[\begin{array}{c} 0 \\ \hdashline \Phi^T D \end{array}\right] \qquad (6)$$

$$\underline{x} \triangleq \left[\begin{array}{c} \underline{\eta} \\ \hdashline \underline{\dot{\eta}} \end{array}\right]$$

To obtain feedback of the time dependent motion of the structure, so as to determine the control vector $\underline{u}$, an output equation is introduced.

$$\underline{y} = C_p \underline{q} + C_v \underline{\dot{q}} \qquad (7)$$

where

$C_p \triangleq n_p x n$ displacement matrix

$C_v \triangleq n_v x n$ velocity matrix

$n_p \triangleq$ number of position (displacement) sensors

$n_v \triangleq$ number of position rate (velocity) sensors

In state space form, the output equation becomes

$$\underline{y} = C\underline{x} \tag{8}$$

where

$$C \triangleq [\, C_p\Phi \mid C_v\Phi \,] \tag{9}$$

Control Model

The complete finite element model of the structure is described now by the 2n-dimensional vector $\underline{x}$. Recall that n represents the number of modes in the model, already reduced from infinity for the real structure. Many of the high-frequency modes can usually be discarded as of no consequence in a reduced order model. As previously mentioned, there are techniques available to aid in deciding how many and which modes to keep to maintain some desired accuracy in reproducing the dynamics of the structure. The finalized reduced order model, the control model, may still contain a great number of modes, enough to place practical constraint on an online computer attempting to do realtime computation of the control $\underline{u}$ as well as model the plant A and receive feedback of the output $\underline{y}$. The computational task may be apportioned among N controllers reducing the unwieldy sizes of matrices to multiply. Doing this also means more modes can be controlled overall than by using a single controller. As is the case in this investigation, each of the N controllers would control a subset n_1 of the n modes, where n now represents the actual number of modes used in the control model and subscript i any one controller.

With the finite number of modes selected for control, there remain the rest of the modes characterizing the real structure. Many of these will still have significant effect on stability and performance of the control system, even though they are not included in the controllers. A realistic investigation would include a good number of these "residual" modes in the model. Thus a practical truth model implies a control model containing n modes, of which there are r residual modes and (n-r) controlled modes.

Spillover is suppressed only among the controllers. Again, the residual modes are part of the truth model, and are not suppressed, as their existence is not recognized by the control system.

For each controller, represented by the subscript i, and r for the residual set, Eqs (5), (6), (8), and (9) may be rewritten

$$\dot{\underline{x}}_i = A_i \underline{x}_i + B_i \underline{u} \tag{10}$$

$$A_i = \left[\begin{array}{c:c} 0 & I \\ \hdashline \ddots\ -\omega^2\ \ddots & \ddots\ -2\zeta\omega\ \ddots \end{array}\right], \quad B_i = \begin{bmatrix} 0 \\ \cdots\cdots \\ \Phi^T D_i \end{bmatrix}, \quad \underline{x}_i = \begin{bmatrix} \underline{\eta}_i \\ \cdots \\ \dot{\underline{\eta}}_i \end{bmatrix} \tag{11}$$

(subscript k_i implied)

$$\underline{y} = \sum_{i=1}^{N} C_i \underline{x}_i + C_r \underline{x}_r \tag{12}$$

$$C_i = [\ Cp_i\Phi \mid Cv_i\Phi\ ] \tag{13}$$

Note that the modal components of $\underline{x}_i$ (and $\underline{x}_r$) are simply arbitrary subsets or groupings of the modal states without regard to any particular ordering. In other words, the modes can be assigned in arbitrary groups to the various controllers or to the residual set in the control model.

Interestingly, when only position sensors are used and these are collocated with the actuators in pairs, as in this investigation, the right partition of the C_i matrices is zero. Dropping the subscript i momentarily it is seen that

$$[Cp\Phi] = [\Phi^T D]^T = [D^T\Phi] \tag{14}$$

This is convenient, because now obviously $Cp = D^T$.

These matrix forms and equations can be implemented in a computer simulation for any structure. Only the dimensioning of the matrices would vary based upon the desired number of modes to keep in the model and the applicable number of sensors/actuators.

Modal Control

As previously indicated the type of control in this investigation is full state feedback control, specifically applied to a multiple-controller configuration. The desired control is given by:

$$\underline{u} = \sum_{i=1}^{N} G_i \underline{x}_i \tag{15}$$

where the G_i are the control gain matrices to be determined.

Feedback in the form of Eq (15) assumes complete knowledge of the state variables. This is impossible in a real structure. Here, the system model representing the real structure is, however, completely observable. This means all the states can be reconstructed by use of an observer to "observe" the unmeasured states. (6) The state equations become

$$\dot{\hat{x}}_i = A_i\hat{x}_i + B_i\underline{u} + K_i(\underline{y} - \hat{y}_i) \tag{16}$$

$$\hat{y}_i = C_i\hat{x}_i \tag{17}$$

The $\hat{x}_i$'s are the reconstructed states, the $\hat{y}$'s the estimated outputs, and the K_i's are observer gain matrices.

The objective in selecting the K_i's is to have the reconstruction error

$$\underline{e}_i = \hat{x}_i - \underline{x}_i \tag{18}$$

go asymptotically to zero in steady state. An idea of how this can be done can be obtained by examining the case of a single controller. Here, $\underline{y}$ would just be $\underline{y}_i$. Using this fact and subtracting Eq (16) from Eq (10), then substituting Eq (17) and Eq (8) (subscripted), the observer-error state equation is

$$\dot{\underline{e}}_i = (A_i - K_iC_i)\underline{e}_i \tag{19}$$

The K_i are selected so that the eigenvalues of $(A_i - K_iC_i)$ are stable. Thus the steady state error will go to zero. Notice in Eq (19) that there is no input term. The error is excited only by the initial conditions for the plant, and is thus not affected by control inputs. This is, however, only the case when there is a single controller. With multiple controllers, Eq (16) still applies and there will be spillover due to output $\underline{y}$ from all sensors.

Proceeding, with all the states now available in the vector $\hat{x}_i$, the control is

$$\underline{u}_i = \sum_{i=1}^{N} G_i \hat{x}_i \tag{20}$$

Determination of the control gain matrices as well as the observer gain matrices will now be discussed.

For the development of these gain matrices linear optimal regulator theory is applied. In both cases this involves construction of a quadratic performance index J. For a control gain matrix the index is

$$J \triangleq 1/2 \int_0^{\infty} (\underline{x}_i{}^T Q_i \underline{x}_i + \underline{u}^T R_i \underline{u})\, dt \tag{21}$$

where

$Q \triangleq$ a positive semidefinite square weighting matrix

$R \triangleq$ a positive definite square weighting matrix

J represents a compromise between obtaining minimum deviation of the present state from the desired final value and reaching the desired final value with minimum control input energy.

Thus the objective is to minimize J subject to the state Eq (10). The optimal solution is shown in (6) to be given by

$$G_i = -R_i^{-1}B_i^TS_i \tag{22}$$

where S_i is the solution to the steady state matrix Riccati equation:

$$S_iA_i + A_i^TS_i - S_iB_iR_i^{-1}B_i^TS_i + Q_i = 0 \tag{23}$$

A similar approach is taken to finding the observer gain matrices K_i. To build a similar performance index expression, an equation similar to Eq (19) can be written:

$$\dot{\underline{w}}_i = (A_i - K_iC_i)^T\underline{w}_i \tag{24}$$

The eigenvalues for $(A_i - K_iC_i)^T$ are the same as for those in Eq (19) since only the transpose of the matrix is involved. Eq (24) can be rewritten

$$\dot{\underline{w}}_i = A_i^T\underline{w}_i - C_i^T\underline{r}_i \tag{25}$$

which provides the form of an input

$$\underline{r}_i = K_i^T\underline{w}_i \tag{26}$$

Now, a similar performance index can be written

$$J \triangleq 1/2\int_0^\infty (\underline{w}_i^TQ_{oi}\underline{w}_i + \underline{r}_i^TR_{oi}\underline{r}_i)\ dt \tag{27}$$

where the weighting matrices are not necessarily the same as those for the controller gain matrices.

When Eq (27) is minimized subject to Eq (25) the solution is optimally

$$K_i = +R_{oi}^{-1}C_iP_i \tag{28}$$

where P_i is the solution to the Riccati equation:

$$P_iA_i^T + A_iP_i - P_iC_i^TR_{oi}^{-1}C_iP_i + Q_{oi} = 0 \tag{29}$$

Eq (22) and (28) provide the forms for the control and observer gain matrices. The designer may then determine the weightings "Q" and "R" as desired. It may be desired to improve response settling time, stabilize a control-destabilized system, or build uniformity into the closed-loop system's modal damping ratios.

While each separate controller may be designed such that it is stable, the individual controllers are still coupled when applied simultaneously as in Eq (20). In other words, there is spillover among the controllers. The next subsection will describe how it arises and indicate what is to be done to suppress it and provide stable control in its presence. Section IV will detail the means of suppression.

Multiple Controllers

Repeated here for clarity are the pertinent equations for multiple controllers:

$$\dot{\underline{x}}_i = A_i\underline{x}_i + B_i\underline{u} \tag{10}$$

$$\underline{y} = \sum_{i=1}^{N} C_i\underline{x}_i + C_r\underline{x}_r \tag{12}$$

$$\dot{\hat{x}}_i = A_i\hat{x}_i + B_i\underline{u} + K_i(\underline{y} - \hat{y}_i) \tag{16}$$

$$\hat{y}_i = C_i\hat{x}_i \tag{17}$$

$$\underline{e}_i = \hat{x}_i - \underline{x}_i \tag{18}$$

Subtracting Eq (16) from Eq (10), rearranging, and finally substituting Eqs (12), (17), and (18), a proper expression for the error states associated with multiple controllers follows:

$$\dot{\underline{e}}_i = (A_i - K_i C_i)\underline{e}_i + \sum_{\substack{k=1 \\ k \neq i}}^{r} K_i C_k \underline{x}_k \quad \begin{array}{l} i = 1,\ldots,N \\ k = 1,\ldots,N,r \end{array} \tag{30}$$

Using Eq (15), the state equations for controlled states, with error states included, as well as for the residual states are

$$\dot{\underline{x}}_i = (A_i + B_i G_i)\underline{x}_i + B_i G_i \underline{e}_i + \sum_{\substack{k=1 \\ k \neq i}}^{N} B_k G_i \hat{x}_k \quad i = 1,\ldots,N \tag{31}$$

and

$$\dot{\underline{x}}_r = A_r \underline{x}_r + \sum_{i=1}^{N} B_r G_i \hat{x}_i \tag{31a}$$

Notice there is no residual error state equation because no attempt is made to observe the residual states.

An augmented state vector $\underline{z}$ can now be assembled from the controlled state vectors $\underline{x}_i$ and their associated error states $\underline{e}_i$, wherein Eqs (30) and (31) are combined so that the state and error vectors for individual controllers are kept adjacent:

$$\underline{z} = \{ \underline{x}_1^T, \underline{e}_1^T, \ldots \underline{x}_N^T, \underline{e}_N^T, \underline{x}_r^T \}^T \tag{32}$$

With Eqs (30) and (31) so combined, a single closed-loop matrix state equation can be assembled in terms of $\underline{z}$ as follows:

$$
\dot{\underline{z}} = \begin{bmatrix}
A_1+B_1G_1 & B_1G_1 & .. & B_1G_i & B_1G_i & .. & B_1G_N & B_1G_N & 0 \\
0 & A_1-K_1C_1 & .. & K_1C_i & 0 & .. & K_1C_N & 0 & K_1C_r \\
: & : & & : & : & & : & : & : \\
: & : & & : & : & & : & : & : \\
B_iG_1 & B_iG_1 & .. & A_i+B_iG_i & B_iG_i & .. & B_iG_N & 0 & 0 \\
K_iC_1 & 0 & .. & 0 & A_i-K_iC_i & .. & K_iC_N & 0 & K_iC_r \\
: & : & & : & : & & : & : & : \\
: & : & & : & : & & : & : & : \\
B_NG_1 & B_NG_1 & .. & B_NG_i & B_NG_i & .. & A_N+B_NG_N & B_NG_N & 0 \\
K_NC_1 & 0 & .. & K_NG_i & 0 & .. & 0 & A_N-K_NC_N & K_NC_r \\
B_rG_1 & B_rG_1 & .. & B_rG_i & B_rG_i & .. & B_rG_N & B_rG_N & A_r
\end{bmatrix} \underline{z} \qquad (33)
$$

The entire control model is now contained in this single first order matrix differential equation. The large matrix above has been referred to as the system major matrix. In its present general form it is not easy to see if such a system is stable or not, but this will soon be examined.

Now, for three controllers $N = 3$, and the augmented state vector appears

$$\underline{z} = (\underline{x}_1{}^T, \underline{e}_1{}^T, \underline{x}_2{}^T, \underline{e}_2{}^T, \underline{x}_3{}^T, e_3{}^T, \underline{x}_r{}^T)^T \qquad (34)$$

The components of $\underline{z}$ are then assembled based upon

$$\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1\underline{u} \qquad (35)$$

$$\dot{\underline{x}}_2 = A_2\underline{x}_2 + B_2\underline{u} \qquad (36)$$

$$\dot{\underline{x}}_3 = A_3\underline{x}_3 + B_3\underline{u} \qquad (37)$$

The observer applied is of form

$$\dot{\hat{x}}_i = A_i\hat{x}_i + B_i\underline{u} + K_i(\underline{y} - \hat{y}_i) \quad i = 1,2,3 \tag{38}$$

where

$$\hat{y}_i = C_i\hat{x}_i \quad i = 1,2,3 \tag{39}$$

$$\underline{u} = G_1\hat{x}_1 + G_2\hat{x}_2 + G_3\hat{x}_3 \tag{40}$$

Applying properly chosen K_i such that the individual $\underline{e}_i$ go to zero with time, the expressions for the $\dot{\underline{e}}_i$'s are

$$\dot{\underline{e}}_1 = (A_1-K_1C_1)\underline{e}_1 + K_1C_2\underline{x}_2 + K_1C_3\underline{x}_3 + K_1C_r\underline{x}_r \tag{41}$$

$$\dot{\underline{e}}_2 = (A_2-K_2C_2)\underline{e}_2 + K_2C_1\underline{x}_1 + K_2C_3\underline{x}_3 + K_2C_r\underline{x}_r \tag{42}$$

$$\dot{\underline{e}}_3 = (A_3-K_3C_3)\underline{e}_3 + K_3C_1\underline{x}_1 + K_3C_2\underline{x}_2 + K_3C_r\underline{x}_r \tag{43}$$

Recalling that $\underline{e}_i = \hat{x}_i - \underline{x}_i$, substituting this properly into Eq (40), and then combining Eq (40) properly with Eqs (35), (36), and (37), the remaining components of $\dot{\underline{z}}$ are now obtained:

$$\dot{\underline{x}}_1 = (A_1+B_1G_1)\underline{x}_1 + B_1G_1\underline{e}_1 + B_1G_2\hat{x}_2 + B_1G_3\hat{x}_3 \tag{44}$$

$$\dot{\underline{x}}_2 = (A_2+B_2G_2)\underline{x}_2 + B_2G_2\underline{e}_2 + B_2G_1\hat{x}_1 + B_2G_3\hat{x}_3 \tag{45}$$

$$\dot{\underline{x}}_3 = (A_3+B_3G_3)\underline{x}_3 + B_3G_3\underline{e}_3 + B_3G_1\hat{x}_1 + B_3G_2\hat{x}_2 \tag{46}$$

$$\dot{\underline{x}}_r = A_r\underline{x}_r + B_rG_1\hat{x}_1 + B_rG_2\hat{x}_2 + B_rG_3\hat{x}_3 \tag{47}$$

Equations (41) through (47) can now be put into matrix form

$$\dot{z} = \begin{bmatrix} A_1+B_1G_1 & B_1G_1 & B_1G_2 & B_1G_2 & B_1G_3 & B_1G_3 & 0 \\ 0 & A_1+K_1C_1 & K_1C_2 & 0 & K_1C_3 & 0 & K_1C_r \\ B_2G_1 & B_2G_1 & A_2+B_2G_2 & B_2G_2 & B_2G_3 & B_2G_3 & 0 \\ K_2C_1 & 0 & 0 & A_2-K_2C_2 & K_2C_3 & 0 & K_2C_r \\ B_3G_1 & B_3G_1 & B_3G_2 & B_3G_2 & A_3+B_3G_3 & B_3G_3 & 0 \\ K_3C_1 & 0 & K_3C_2 & 0 & 0 & A_3-K_3C_3 & K_3C_r \\ B_rG_1 & B_rG_1 & B_rG_2 & B_rG_2 & B_rG_3 & B_rG_3 & A_r \end{bmatrix} z \tag{48}$$

It is now readily apparent how spillover can cause overall instability in the system. For example, notice how the control gains G_2 and G_3 make up part of the component state equation for controller no. 1 (row 1). The same type of spillover from the two other control gain terms appears in the component state equations for controller no.'s 2 and 3 (rows 3 and 5). Furthermore all the control gains appear in the component state equation for the residual modes, to which no control was to be applied. If all such terms could be eliminated, spillover would not occur. It is not really necessary, however, to eliminate all such terms.

What is needed is a block triangular form for the major matrix. Since the diagonal terms of the major matrix already have "stable" system eigenvalues by design, it is only necessary that the matrix be block triangular. This is true because the eigenvalues of a block triangular matrix are the same as those of its diagonal blocks. Thus two types of triangularization can be made to eliminate sufficient block terms and effect suppression of spillover. An upper block triangularization for suppression would require the following:

$$B_iG_k = K_iC_k = 0 \quad \begin{array}{l} k = 1,\ldots,N-1 \\ i = k+1,\ldots,N \end{array} \tag{49}$$

A lower block triangularization would require

$$B_iG_k = K_iC_k = 0 \quad \begin{array}{l} i = 1,\ldots,N-1 \\ k = i+1,\ldots,N \end{array} \tag{50}$$

Notice the residual terms are ignored. This means spillover between the residual modes and the controlled modes is not suppressed and the system stability and/or control performance may be affected. Table 3-1 shows which terms to eliminate for upper or lower block triangularization.

Table 3-1

Suppressing Spillover Among Three Controllers

Upper Triangularization	Lower Triangularization
$B_2G_1 = 0$	$B_1G_2 = 0$
$B_3G_1 = 0$	$B_1G_3 = 0$
$B_3G_2 = 0$	$B_2G_3 = 0$
$K_2C_1 = 0$	$K_1C_2 = 0$
$K_3C_1 = 0$	$K_1C_3 = 0$
$K_3C_2 = 0$	$K_2C_3 = 0$

Before discussing the technique of triangularization involving transformation matrices, mention should be made of an alternative way to effect triangularization. Aldridge (3) pointed out that it had been shown previously that by proper placement of sensors/actuators and selective assignment of modes to controllers, the three-controller problem could be reduced to a two-controller problem. This comes about through the changes effected in the control mapping matrix B (for actuation) and the output mapping matrix C (for sensing). With proper modal assignments one controller could be made orthogonal to the other two, thereby eliminating spillover for that controller. No attempt will be made in this investigation to configure the structural model for this. However, such a discussion serves to introduce the next subsection.

The above discussion on sensors/actuators introduces the idea that selection of them impacts the control problem. Earlier it was pointed out that an observer was clearly necessary due to an insufficient quantity of sensors to feed back all the states. The next subsection will detail how, due to the triangularization, there is indeed a minimum number of sensors and actuators required for complete observability and controllability, which are themselves required for the formation of the observer and control gain matrices.

Sensor/Actuator Requirements

Control or observation spillover among the various controllers may be eliminated by constraining the various controller and observer gain matrices. Using the control term B_iG_1 as an example, notice that if the columns of G_1 are constrained to lie in the null space of B_i, then the product is zero. The product could vanish for all values of $i \neq 1$ if G_1 lay in the null space of the matrix B_{2N} defined as follows:

$$
B_{2N} = \begin{bmatrix} B_2 \\ \cdots\cdots \\ : \\ : \\ : \\ \cdots\cdots \\ B_N \end{bmatrix} \tag{51}
$$

The null space of this matrix B_{2N} has dimension P_{2N} defined as

$$
P_{2N} = (n_a - r_{2N}) \tag{52}
$$

where

$n_a \triangleq$ number of actuators

$r_{2N} \triangleq$ rank of $B_{2N} \leq \min(n_2 + n_3 + \ldots + n_N,\ n_a)$

Since the columns of G_1 lie in the null space of B_{2N} of dimension P_{2N}, the number of linearly independent columns of G_1 is also P_{2N}.

As Aldridge pointed out, the number of actuators must exceed the rank of B_{2N} to have $B_{2N}G_1 = 0$. Without this condition being met, no transformation matrix to do this exists. If B_{2N} is full rank, the number of actuators needed will be

$$
n_a > \sum_{i=2}^{N} n_i \tag{53}
$$

whereas if it is rank deficient, then

$$
n_a > r_{2N} \tag{54}
$$

Similarly, the required number of sensors is, for some C_{2N}

with linearly independent columns,

$$n_s > \sum_{i=1}^{N-1} n_i \qquad (55)$$

and for linearly dependent columns,

$$n_s > r_{2N} \quad \text{(here } r_{2N} \text{ is the rank of } C_{2N}\text{)} \qquad (56)$$

The above are for an upper block triangularization. Noting the conditions of Eq (50), the required numbers of actuators and sensors for a lower block triangularization, assuming B_{2N} and C_{2N} are full rank, are respectively

$$n_a > \sum_{i=1}^{N-1} n_i \qquad (57)$$

$$n_s > \sum_{i=2}^{N} n_i \qquad (58)$$

Satisfying Ineq (51) through (58) provides complete controllability and observability of the controlled modes and ensures that spillover among them is suppressed. To give an example, if the finalized control model has three controllers each containing ten modes, and there are no residual modes, at least twenty-one sensors and twenty-one actuators are needed to completely control and observe such a system. This investigation satisfies the requirements on sensors and actuators.

Based upon the above conditions being met, the technique of generating the transformation matrices follows.

IV. Transformation Technique

This section describes how the major matrix can be block triangularized, thereby eliminating the observer and control spillover terms among controllers. The discussion will describe the determination of transformation matrices to eliminate the control and observation spillover terms. These matrices are referred to respectively as T and L.

For a configuration using a single controller, the spillover terms to be eliminated are B_sG and KC_s. The subscript s refers to those modes to be suppressed. The trivial solution that $G = K = 0$ is unacceptable because it means the control and observer feedback loops have effectively been severed by turning off all gain. The system would be without feedback control inputs, and therefore would simply return to the case of a structure regulating itself through natural damping. The objective here is to find a solution (transformation) such that $B_sG = 0$ <u>and</u> $B_cG \neq 0$, as well as $KC_s = 0$ <u>and</u> $KC_c \neq 0$.

Before proceeding, it is worth mentioning here that some attempt could be made to include suppression of some or all of the residual modes. Varhola (7) actually did this for a smaller order model with a single controller. Such suppression will not be addressed in this investigation, however, as it will be assumed that the multiple controllers already have enough to handle in suppressing spillover among themselves. The residual modes are presumably beyond the capacity for control or suppression, and in fact may not even be known in a real structure.

Returning now to discussing multiple controllers, and bearing in mind the conditions cited above, attention will now be focused on control spillover suppression. The B_s matrices can take the form of Eq (51) repeated here:

$$B_{2N} = \begin{bmatrix} B_2 \\ \cdots\cdots \\ \vdots \\ \vdots \\ \vdots \\ \cdots\cdots \\ B_N \end{bmatrix} \tag{51}$$

Recall this form was based upon the condition of Eq (49) for upper block triangularization:

$$B_i G_k = K_i C_k = 0 \qquad \begin{matrix} k = 1,\ldots,N-1 \\ i = 1,\ldots,N \end{matrix} \tag{49}$$

saying that, for example, given N controllers, the columns of G_1 must be made orthogonal to the (N-1) rows of the B_i matrices which were combined into B_{2N} above. This says that the columns of G_1 are constrained to lie in the null space of B_{2N}. This is just an example of the what the objective is. The discussion will continue using the generic term B_s to represent any control mapping matrix (or set) that does not match in index the control gain matrix of interest.

The control transformation matrix T is such that

$$B_s T = 0 \tag{59}$$

It will soon be apparent why, but Eq (59) is introduced here to point out again the conditions for the existence of T. From Eq (59) it is seen that the row dimension of T must match the column dimension of B_s, which is n_a, the number of actuators. The column dimension of T is found by referring to the conditions cited in Section III; namely, its column dimension must be

$$P_{2N} = (n_a - r_{2N}) \tag{52}$$

where it is seen that r_{2N} must be at least as big the number of modes to be suppressed: here n_m. Thus the dimensions of T are n_a by $(n_a - n_m)$, given the conditions cited on linear independence of the rows of B_s.

The application of T will now be illustrated. Consider a system containing controlled and suppressed modes as follows:

$$\dot{x}_c = A_c \underline{x}_c + B_c \underline{u} \tag{60}$$

$$\dot{x}_s = A_s \underline{x}_s + B_s \underline{u} \tag{61}$$

where

$$\underline{u} = G_c \hat{x}_c \tag{62}$$

It is necessary to eliminate the $B_s \underline{u}$ term of Eq 61) so that the system described by Eq 61) no longer admits to any of the control applied to the system of Eq (60). To eliminate this term it is necessary to find a matrix T such that $B_s T = 0$

while $B_cT \neq 0$. Let this transformation matrix T be used to define a new control $\underline{v}$, such that

$$\underline{u} = T\underline{v} \tag{63}$$

Substituting Eq (63) into Eqs (60) and (61), and renaming the term B_cT as B^* while recognizing that the term B_sT is now just zero, results in a transformed system in which spillover has been suppressed:

$$\dot{\underline{x}}_c = A_c\underline{x}_c + B^*\underline{v} \tag{64}$$

$$\dot{\underline{x}}_s = A_s\underline{x}_s \tag{65}$$

In the same manner as the construction was made of the feedback control vector $\underline{u}$, the new control vector is

$$\underline{v} = G^*\hat{x}_c \tag{66}$$

The determination of T will now be discussed.

T is determined through singular value decomposition of the matrix B_s (3), which has dimensions $n_m \times n_a$. The matrix can be decomposed as a product of three matrices:

$$B_s = W \Sigma V^T \tag{67}$$

where

$W \triangleq$ an $n_m x n_m$ orthogonal matrix of left singular vectors

$V \triangleq$ an $n_a x n_a$ orthogonal matrix of right singular vectors

$\Sigma \triangleq$ an $n_m x n_a$ matrix containing the s singular values of B_s in the following form:

$$\Sigma = \begin{bmatrix} S & \vdots & 0 \\ \cdots & \vdots & \cdots \\ 0 & \vdots & 0 \end{bmatrix} \tag{68}$$

where

$$S = \begin{bmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_s \end{bmatrix} \tag{69}$$

The number of non-zero singular values s of B_s equals its rank, and they are all non-negative. If B_s is of full rank, then $s = \min(n_a, n_m)$.

The matrix W is partitioned into

$$W = [\ W_s \mid W_r\ ] \tag{70}$$

where

$W_s \triangleq$ an $n_m x s$ matrix of left singular vectors associated with the non-zero singular values

$W_r \triangleq$ an $n_m x r$ matrix of left singular vectors associated with the zero singular values

and

$$n_m = s + r \tag{71}$$

The V matrix is partitioned in a like manner as

$$V = [\ V_s \mid Vp\ ] \tag{72}$$

where

$V_s \triangleq$ an $n_a$xs matrix of right singular vectors associated with the non-zero singular values

$Vp \triangleq$ a $n_a$xp matrix of right singular vectors associated with the zero singular values

and

$$n_a = s + p \tag{73}$$

Since the V matrix is an orthogonal matrix, see that

$$V_s^T Vp = 0 \tag{74}$$

allowing the following construction:

$$B_s T \triangleq 0 = W_s S V_s^T T \tag{75}$$

Examining Eqs (73) and (74) for similarities, see that

$$T = Vp \tag{76}$$

would satisfy the equality based upon Eq (74). Necessarily $Vp \neq 0$, else the transformation matrix is just a matrix of zeroes--an undesirable result. Now that the transformation matrix T has been determined, it remains to see how the new control gain matrix G* is found.

The G* matrix is found in the same manner as described in Section III in the subsection on <u>Modal Control</u>. It is the optimal solution

$$G_{i*} = -[R_{i*}]^{-1}[B_{i*}]^T S_i \tag{77}$$

in which S_i is the solution to the steady state algebraic matrix Riccati equation:

$$S_iA_i + A_i^TS_i - S_iB_{i*}[R_{i*}]^{-1}[B_{i*}]^TS_i + Q_i = 0 \qquad (78)$$

wherein the subscript i refers again to a particular controller. With the appropriate substitutions, it is now apparent that the form of Eq (64) can be reiterated to show the form of the state equation for any controller i, after suppression:

$$\dot{\underline{x}}_i = A_i\underline{x}_i + B_iT_iG_{i*}\hat{x}_i \qquad (79)$$

Similarly, substituting C_s^T for B_s, K^T for G, and L for T in the above development, results in L equalling Vp. Again, the number of sensors must exceed the number of modes to be suppressed.

This section completes the description of equations used in developing the control model for implementing in a computer simulation. A brief discussion on how the time response is computed is included in the following section, which describes the actual computer program.

V. Computer Simulation Implementation

This section will describe the operation of a FORTRAN coded computer program which implements the equations of Section III. The program is an updated version of that used by Aldridge. Its capabilities have been altered to deliver time response of the LOS deviations mentioned in Section II. As is often the case, a lengthy amount of time in this investigation was devoted to understanding what the program does, how it does it, and what changes could be made to enable it to output LOS time response data. The following discussion will briefly describe how time response is generated.

The theory behind obtaining any form of time response is fairly straightforward. Hence it is included here instead of Section III. Of interest is Eq (33), abbreviated here as

$$\dot{\underline{z}} = [MAJM]\underline{z} \tag{80}$$

where the term MAJM replaces the large block matrix, called the major matrix, shown in Eq (33). This is a commonplace homogeneous first order matrix linear differential equation. The solution of a scalar first order differential equation $\dot{z} = az$ is simply exp(at), wherein t represents some time differential. Analogous is the matrix solution to Eq (80): exp([MAJM]t) . To use some common terms, the state $\underline{z}$ can be found for any time t_f by propagating the state at a previous time t_i forward (or background) through multiplication by the fundamental matrix (8), known also as the state transition matrix, here called [STM], where

$$[STM] = \exp([MAJM](t_f - t_i)) \qquad (81)$$

Thus

$$\underline{\underline{z}}_f = [STM]\underline{\underline{z}}_i \qquad (82)$$

saying that a fixed time differential t can be used to propagate the state incrementally <u>ad infinitum</u>.

Once the state vector can be output at discrete time points, it is only a matter of converting the state at a desired stopping point from modal coordinates to physical coordinates. The desired form of the physical coordinates in this investigation is the LOS vector described in Section II. This version of the program outputs a tabulation of LOSX, LOSY, Defocus, and Radius vs. time with the rest of the printed output, as well as just Radius and Defocus vs. time on another output file for plotting. A description of the program's capabilities now follows.

The capabilities of the program were tailored to the investigation. The user must obtain the appropriate products of the actuator and sensor mapping matrices with the modal matrix beforehand (see Eq (14)). Ref (7) provides the code to read in any modal, sensor mapping, and actuator mapping matrices. The program accepts only modal coordinates for initial conditions, although once again, if reasonable physical initial conditions could be specified, Ref (7) contains code for their input and conversion to modal coordinates.

The Q weighting matrix is assumed to be diagonal and therefore only its diagonal is to be input. A complete matrix could be formed simply enough, but was not of interest in this investigation. Also the control weighting matrix, R, is assumed to be an identity matrix. Extensive revision of the code would be needed to include a non-identity R matrix if it were of interest. These limitations were of little consequence to this investigation.

The program can operate with either three or four controllers specified. If three controllers are specified, then a group of modes can be designated as residuals in place of the group that a fourth controller would have controlled. This group may number as few as zero (no residuals). Thus three controllers with or without residuals or four controllers with no residuals can be run. In either case, the user may also specify other options: (1) time response (with LOS as output), (2) roots (eigenvalues) of the closed-loop (overall) system and the open-loop (individual) systems--controlled, error, and residual states, or (3) both time response and eigenvalue analysis. Moreover, the program makes both an "unsuppressed" and a "suppressed" pass, and any of these three options can be selected for each pass as desired. A sensible choice in the early part of an investigation would be to select "2" for both the unsuppressed and suppressed passes until the user is satisfied with the placement of the closed-loop roots. Later the time response option could be chosen. This is because the subroutine MEXP (9), called only when time response is desired, requires a relatively large amount of central processing time. All options are initiated by input data. A guide illustrating the form and purpose of all the required input data is included in Appendix B.

As the program executes with its specified options, the output contains many statements analogous to prompts in an interactive session. This simply allows the user to follow the execution and recollect input data when reading the output. Much of the input data is restated in the output.

Execution begins with the dimensioning of the numerous indexed variables the program uses as well as initializing of some important parameters used by the various subroutines in properly handling the indexed variables. Thus dimensioning need not be altered anywhere except in the main program. Comment statements in the code provide guidance on specifying these parameters and dimensions before compilation.

As a final note, the program is supported by several facility routines from among the International Mathematical and Statistical Library (IMSL) routines. Their code is not included in the listing in Appendix B.

VI. Investigation

As indicated in Section I, the main objective of the investigation is to see how the assignment of various groups of modes can affect the LOS time response, and see if any general conclusions might be warranted. The foundation for such an investigation is based upon certain facts regarding the basic control model setup.

As set up, this control problem recognizes, as is often the case, that there is a limited number of sensors/actuators that can be practically installed, and the placement of these may not provide all the observability and controllability desired. For a multiple-controller configuration, however, the described block triangularization of the major matrix results in the various controllers commanding varying degrees of controllability and observability. To be sure, there will always be modes which are not directly controlled or observed. A large space structure with many degrees of freedom must be modelled with a greatly reduced number of modes to accommodate the limited capacity and/or speed of an onboard computer acting as the controller. Thus it is of interest to see how the choice of modes to control as well as the assignment of these modes in a multiple control scheme affects system performance.

For this version of the Draper-2 model there are twenty-one pairs of collocated sensors and actuators with fixed placements. Three controllers were used in the control model.

It was assumed each had the computational capacity for four modes. Thus each was assigned four modes. A group of eight modes was considered as residual to represent the unmodelled modes of a real structure. Thus there were twenty modes in total representing a truth model of the structure. With these figures in hand, the relative degrees of controllability and observability of the individual controllers can be evaluated.

Examining Table 3-1 and noting this version of the simulation program performs a lower block triangularization, a pattern is seen as to the amount of suppression of control or observation spillover there must be for a particular controller. To suppress observation spillover from the other eight modes assigned to Controllers 2 and 3, Controller (Ctlr) 1 must use eight of its sensor inputs, leaving it the controller with least observability. Ctlr 2 uses only four of its sensor inputs to suppress the modes of Ctrl 3. It has more observability. In contrast, Ctlr 3 uses eight of its actuators to suppress control spillover from the other eight modes in Ctlrs 1 and 2. It has the least controllability. Once again, Ctlr 2 uses only four actuators in suppression, giving it more controllability than 3. Summarizing, a three-controller configuration with lower block triangularization exhibits one controller (Ctlr 1) commanding the most controllability, one (Ctlr 3) commanding the most observability, and one (Ctlr 2) commanding a median amount of both. Thus the model setup with three controllers allows a straightforward study involving assigning the various groups of modes permutatively to the three controllers. A fourth (residual) group provides more realistic insight into the overall stability and time response, and was thus included. To reiterate the underpinning of this investigation, stability can be regarded a fundamental requirement for any large space structure control system, but time response, in particular LOS pointing here, may be the driving specification.

The investigation began with a search for candidate modal rankings which could be divided into groups for assignment. One way to rank order is simply to put the mode numbers in ascending order (AO). This might be the situation in the early stages of a design study when little is known of the relative effects of each mode on LOS performance. The premise is well known that many of the high frequency modes can be expected to lie outside the control bandwidth, and may be simply truncated (removed from the model). An excellent alternative ranking was made by Varhola (7). It is the so-called modal cost ranking (MC). It ranks the relative effect of a mode on any index of interest against the other modes. Similarly, another scheme called internal balancing (IB) can be used. Varhola generated an MC ranking for LOS performance and an IB ranking for the ability of the sensors/actuators to affect a particular mode. In other words, the second ranking indicates the sensitivity of a mode to control and control spillover, whether it contributes to LOS performance or not.

The AO ranking for the investigation was made by simply taking the three rigid body rotation modes plus the next seventeen in ascending order of frequency. The MC ranking was made by selecting the twenty highest ranking modes among those with a mode number falling within the first twenty so that they would match with the AO ranking. The modes are consistently numbered 1 through 20, with the rigid body translation mode numbers having been dropped. The rank orderings obtained are as follows:

AO: 1,2,3,4 | 5,6,7,8 | 9,10,11,12 | 13,14,15,16,17,18,19,20

MC: 1,2,3,20 | 4,7,10,6 | 9,19,11,12 | 13,18,5,8,15,14,17,16

IB: 1,2,3,7,4,8,5,10,20,6,13,9,12,11,19,18,15,14,17,16

The dividing mark between the groups indicates how the modes were grouped. Notice the first group (Group 1) under both AO and MC analysis contains the three rigid body rotation modes. The fact that they are in the MC Group 1 means they are of top importance to the LOS performance. Notice also no groupings are presented for the IB ranking. This is because this ranking was used only in an auxiliary fashion and was not directly examined in the investigation. However, to illustrate its usefulness, notice that it also lists modes 1, 2, and 3 highest. This is good news for effecting control as it means these are the three modes most sensitive to control.

The AO and MC groups were assigned to various controllers in a permutative fashion. For each ranking there were six possible combinations: 1-2-3, 1-3-2, 2-1-3, 2-3-1, 3-1-2, and 3-2-1, wherein the ordering implies the ordinal number of the controllers. For example, the third combination, 2-1-3, implies that the modes from Group 2 were assigned to Ctlr 1, the modes from Group 1 to Ctlr 2, and the modes from Group 3 to Ctlr 3.

For each run the initial conditions were such that the LOS Radius was unity at time zero. (Note that no units of measure need apply since modal coordinates are used.) All the runs were also set up to cover 20.0 seconds of propagation with a t value of 0.05 seconds. This value of t would definitely meet the requirements on sampling rate to reproduce with accuracy a time history of LOS motion. Some preliminary runs were made to find suitable values for the twenty elements of the Q matrix diagonal. (During this stage of the investigation an error was discovered in the subroutine FORMQ1 algorithm which forms the diagonal Q matrix. The algorithm is now correct.) The objective was to apply a fixed Q matrix such that all combinations run would remain stable and provide meaningful data on time response. After some trial and error, but with much insight gained into how each mode plays into the stability and movement of the closed-loop roots, a final set of Q values was determined. They are given according to ascending mode number as follows: 200 (x3), 500 (x7), 2500, 3000, 500 (x6), 15000 (x2). These values provide for a stable suppressed system for both the AO and MC cases.

With the control model set up, six runs were made for the AO and MC cases. In all twelve runs both time response and eigenvalue analysis were made. It was of interest to see how the closed-loop suppressed roots shifted from their original locations where they had the given damping ratio of 0.005.

But it was more important to see if any pattern might be revealed by the time response plots to warrant general conclusion about the time response performance of a large space structure control-configured as described when various modal assignments can be made. The results of the investigation follow.

VII. Results

Figures 7-1 through 7-12 on the following pages are plots of the closed-loop roots of the system after suppression. For the complex roots only the root with the positive imaginary part is plotted. The controlled modes have roots for both the controlled states and the observer error states. Each residual mode, of course, has only a single root. The roots were obtained by matching them with the identified roots of the individual (open-loop) systems as generated by the program. With the small value of damping ratio of 0.005 before closing the loop, the mode can be identified by recognizing its frequency is approximately equal to the imaginary part of its root.

In all cases the selected values for the Q matrix diagonal resulted not only in a stable system regardless of modal assignment, but also resulted in significant damping improvement of the controlled modes. This can be seen in the spread of the roots away from the constant damping ratio line of 0.005. As a further result of selecting as residual the eight lowest ranking modes under MC analysis and the eight highest frequency modes under AO analysis (frequency truncation), the residual modes remained quite steady. Their damping ratios hardly deviated from the original value of 0.005. In no case did any damping ratio decrease significantly below 0.005 whether for a controlled, error, or residual state. To be noted here for later discussion is the fact

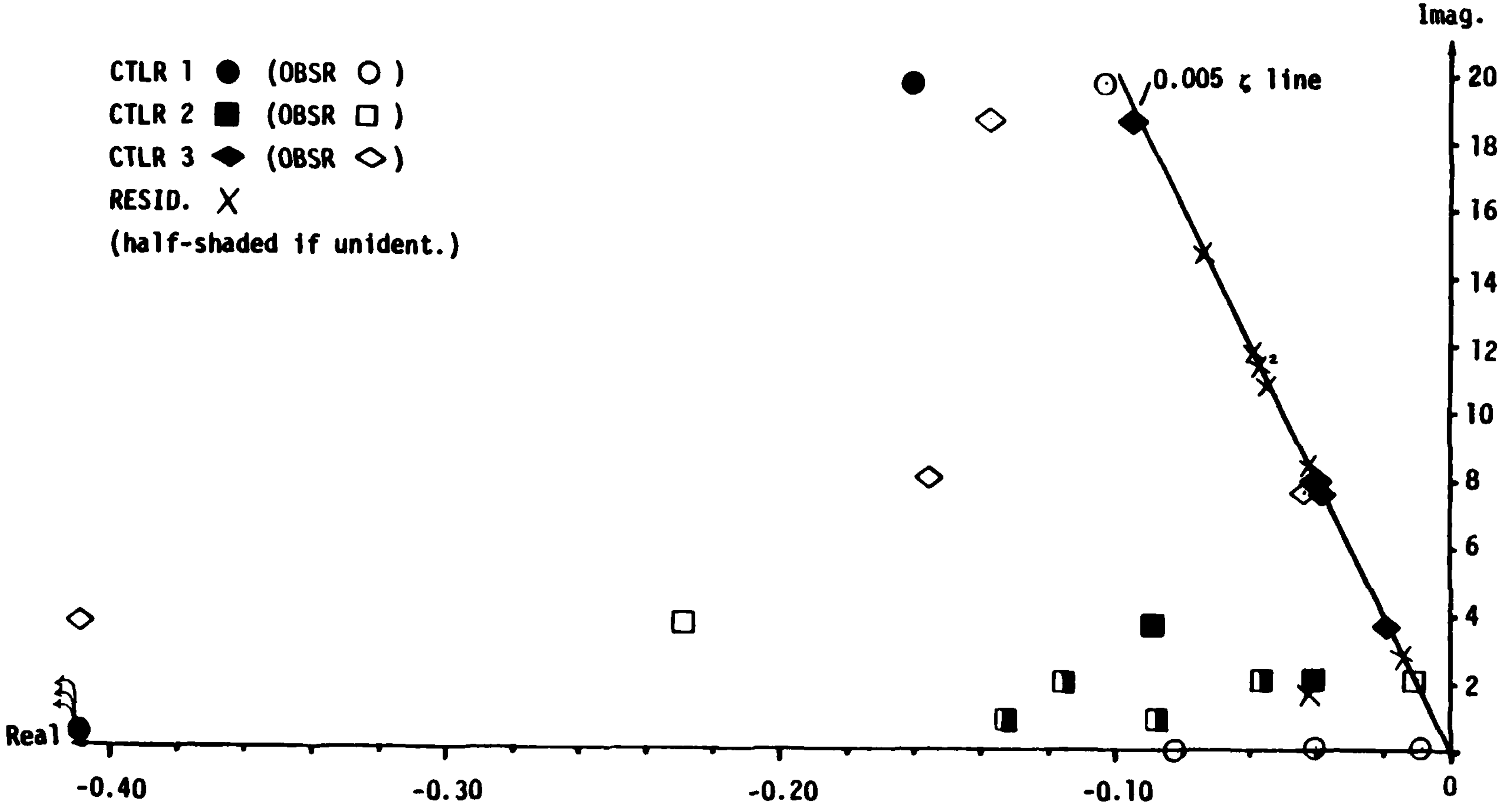


Figure 7-1. MC Comb 1-2-3 Closed-Loop Roots

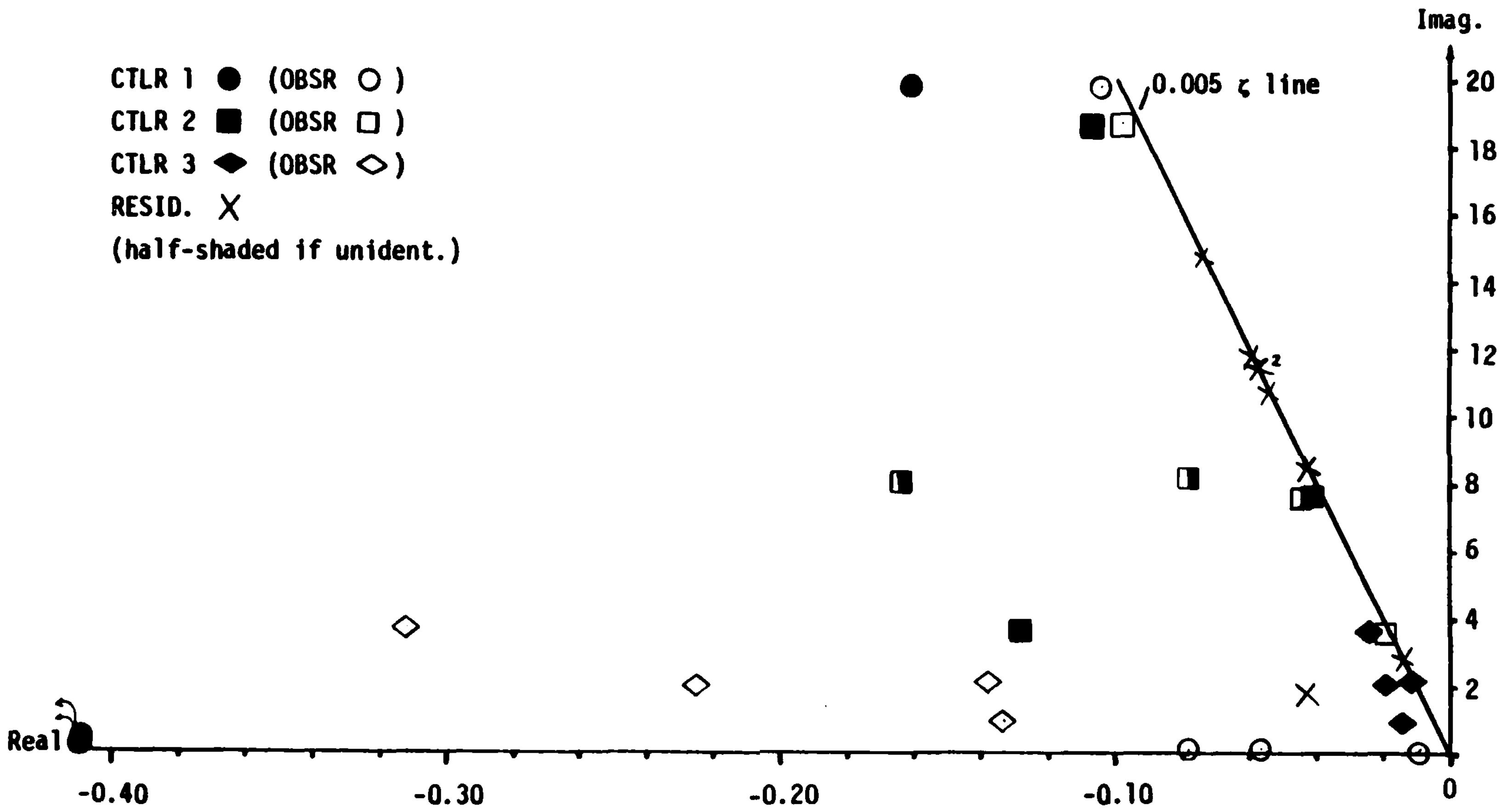


Figure 7-2. MC Comb 1-3-2 Closed-Loop Roots

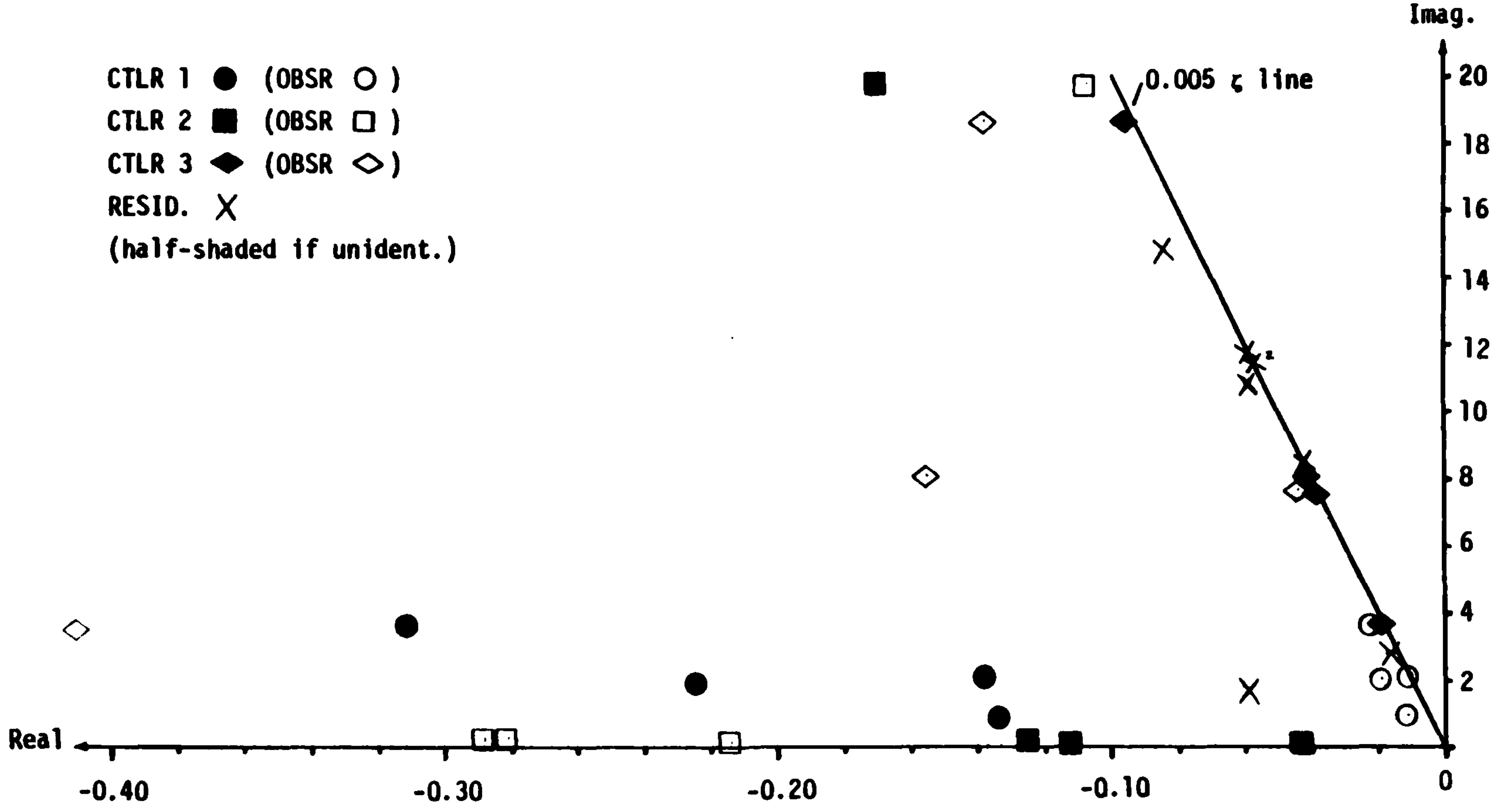


Figure 7-3. MC Comb 2-1-3 Closed-Loop Roots

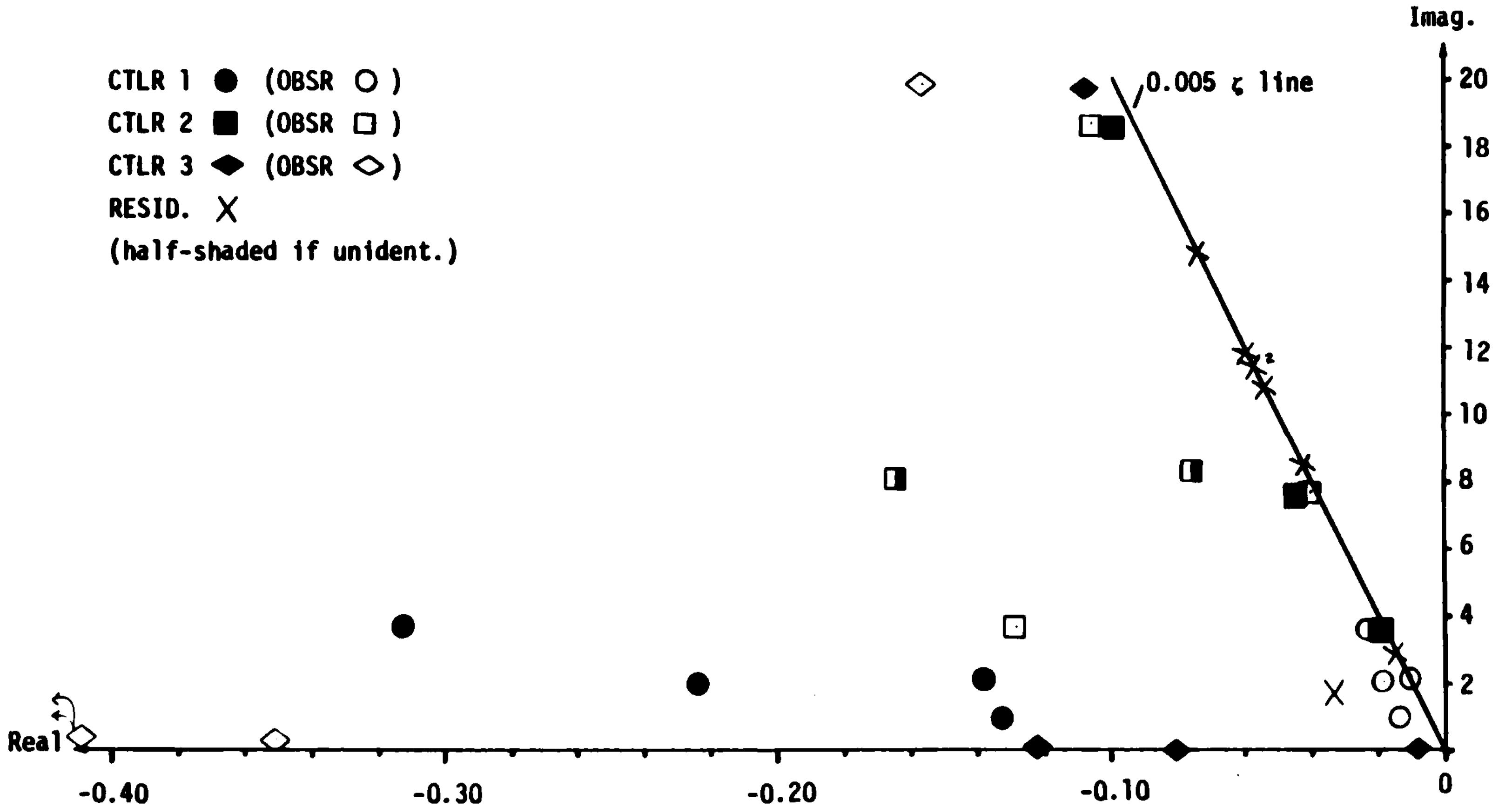


Figure 7-4. MC Comb 2-3-1 Closed-Loop Roots

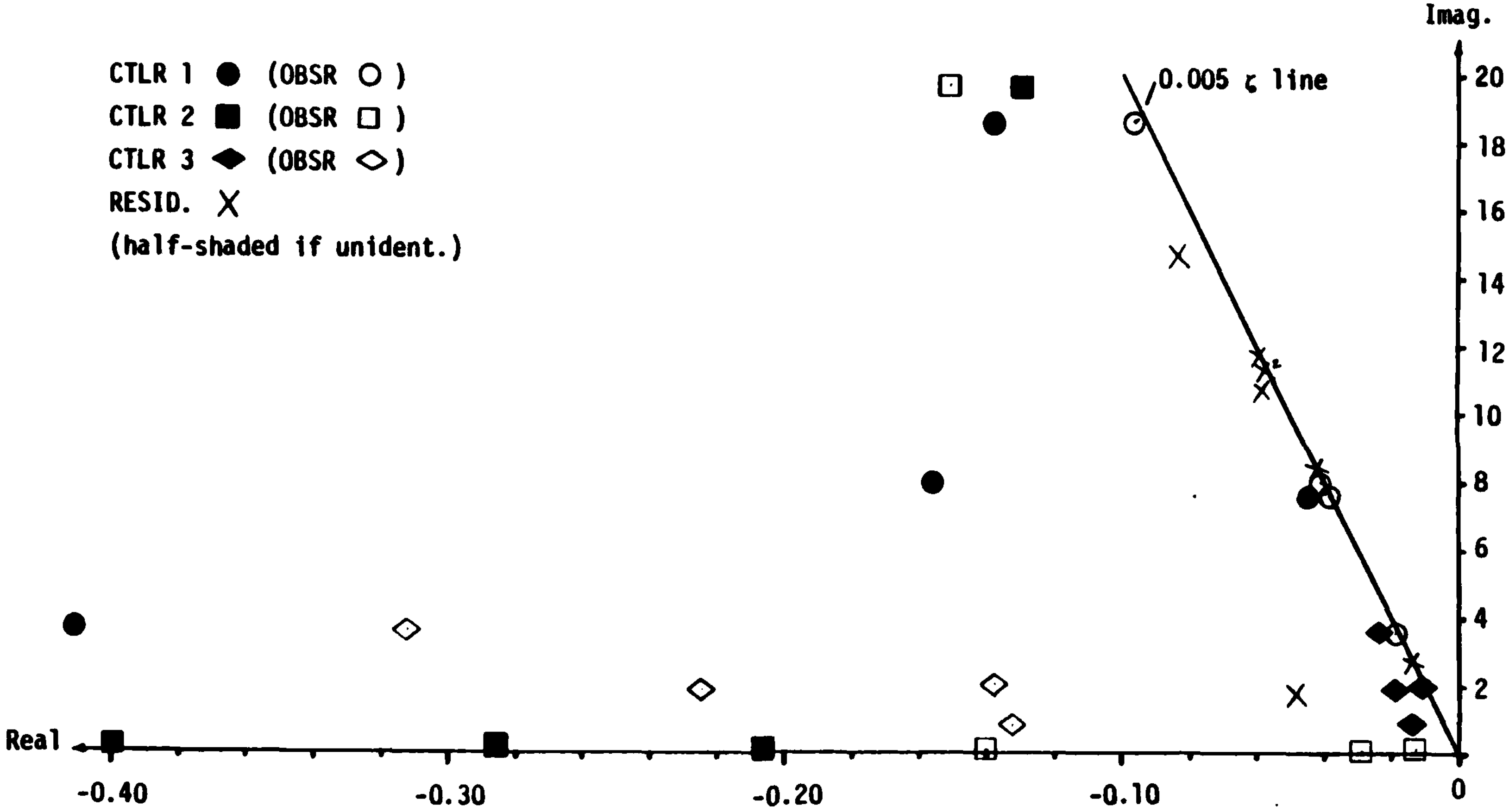


Figure 7-5. MC Comb 3-1-2 Closed-Loop Roots

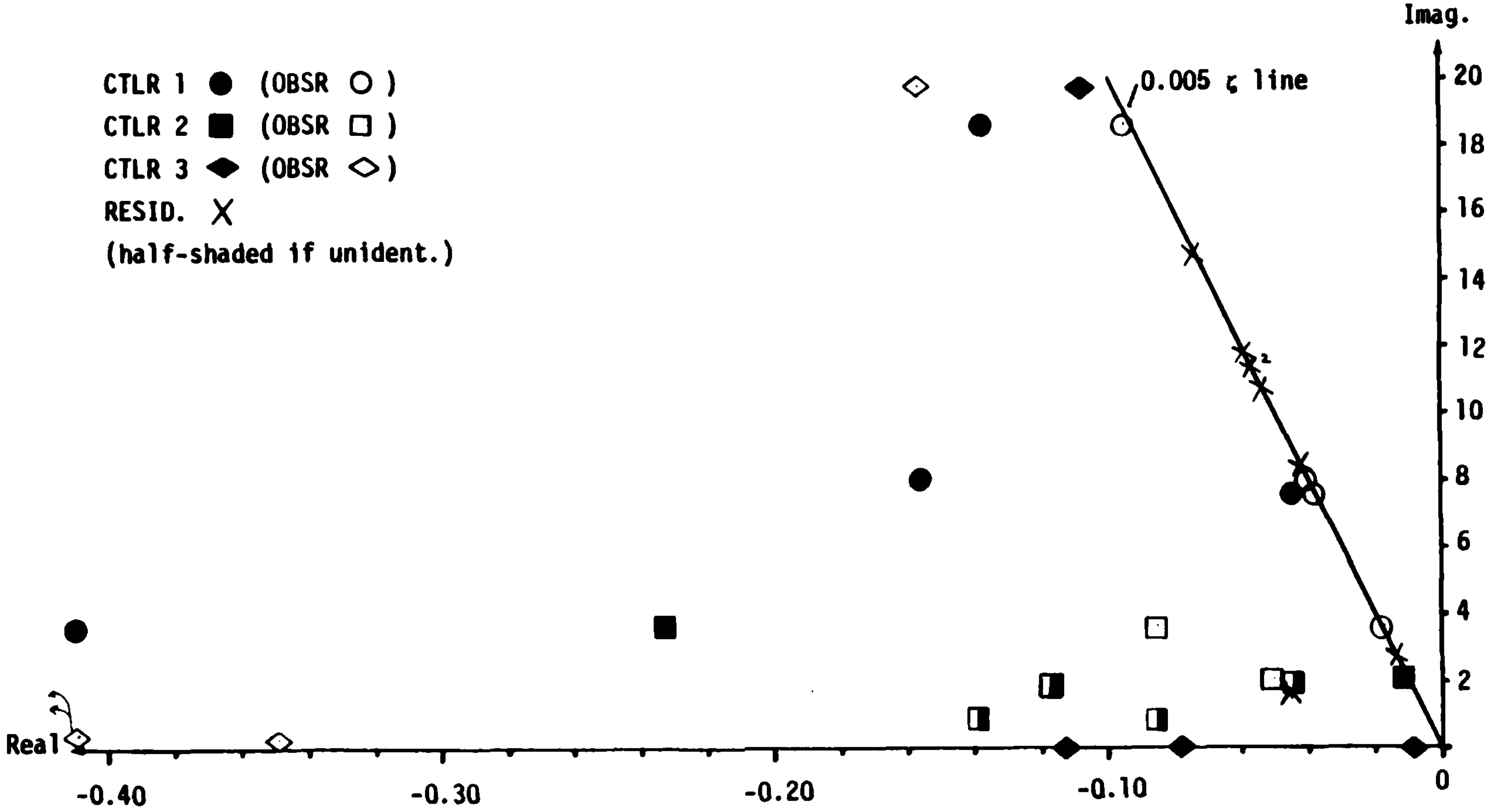


Figure 7-6. MC Comb 3-2-1 Closed-Loop Roots

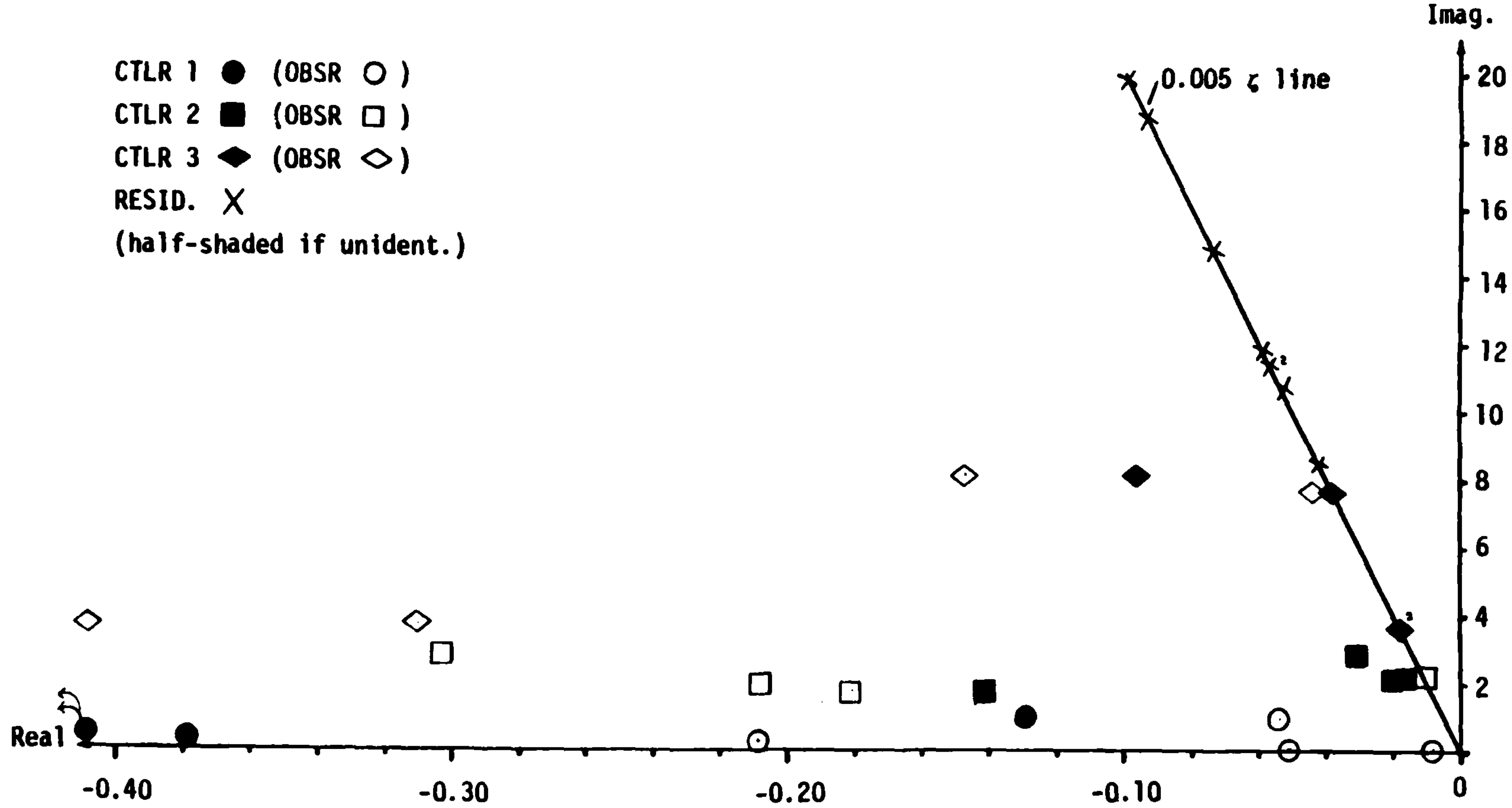


Figure 7-7. AO Comb 1-2-3 Closed-Loop Roots

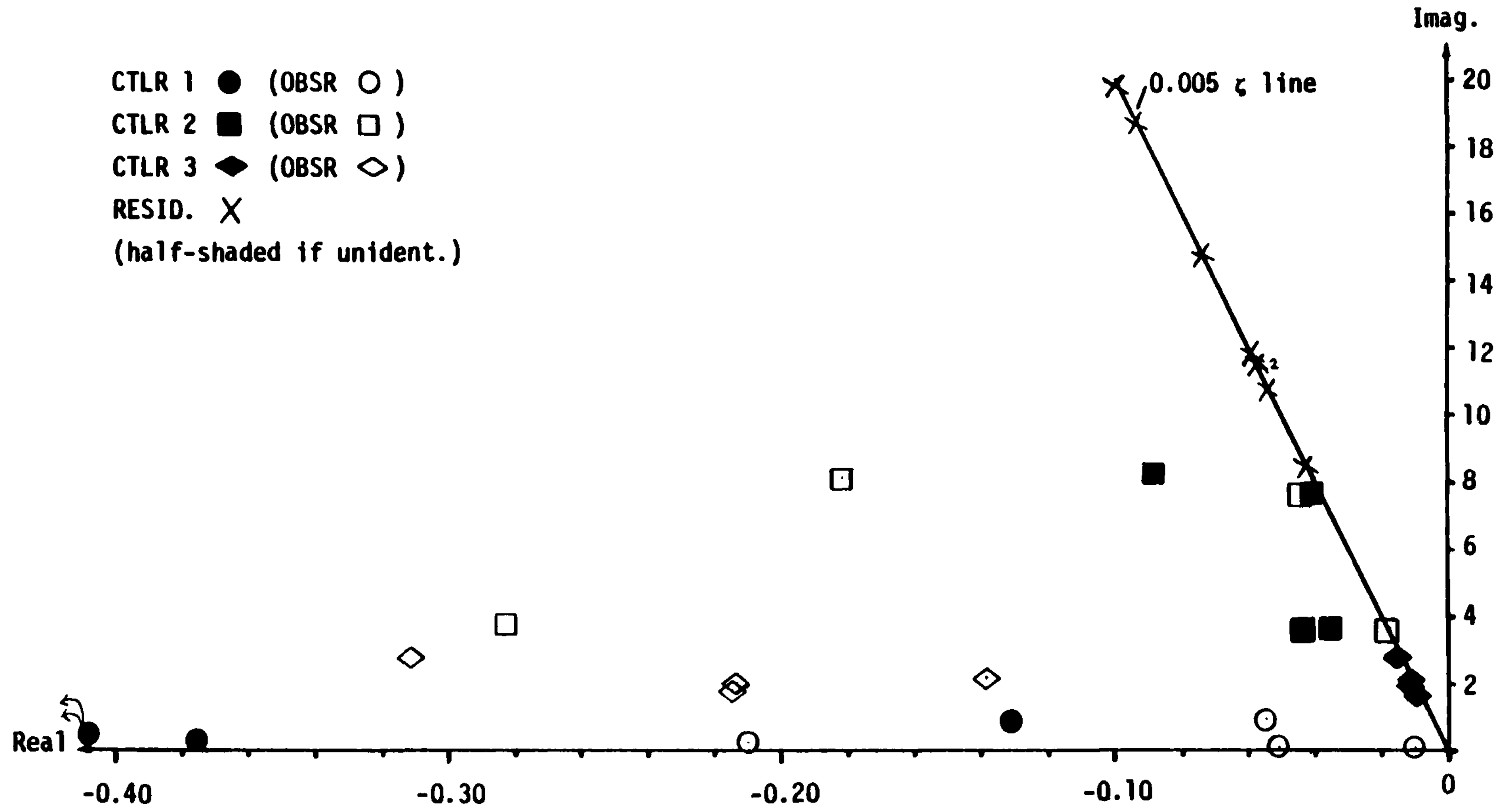


Figure 7-8. AO Comb 1-3-2 Closed-Loop Roots

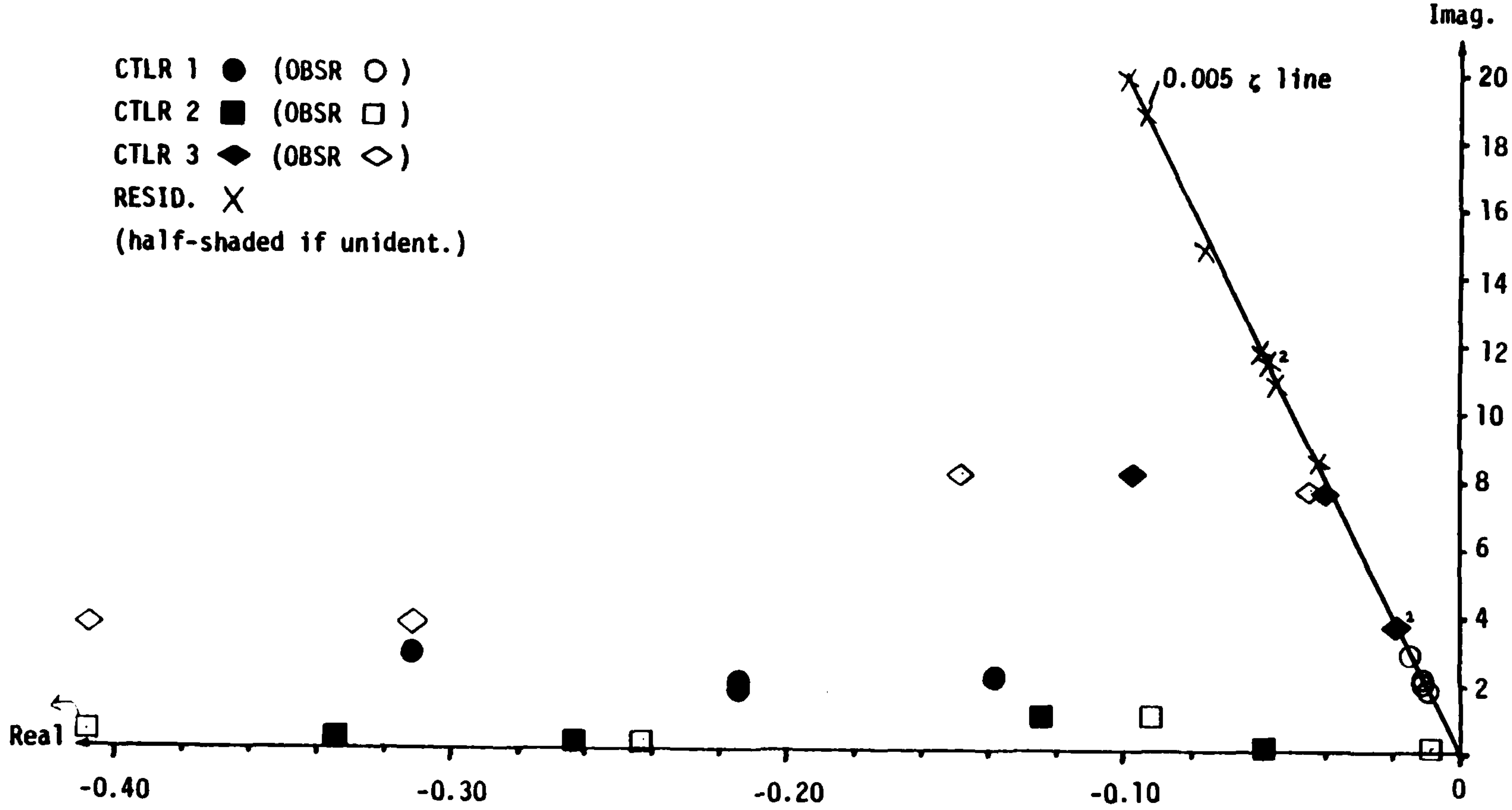


Figure 7-9. AO Comb 2-1-3 Closed-Loop Roots

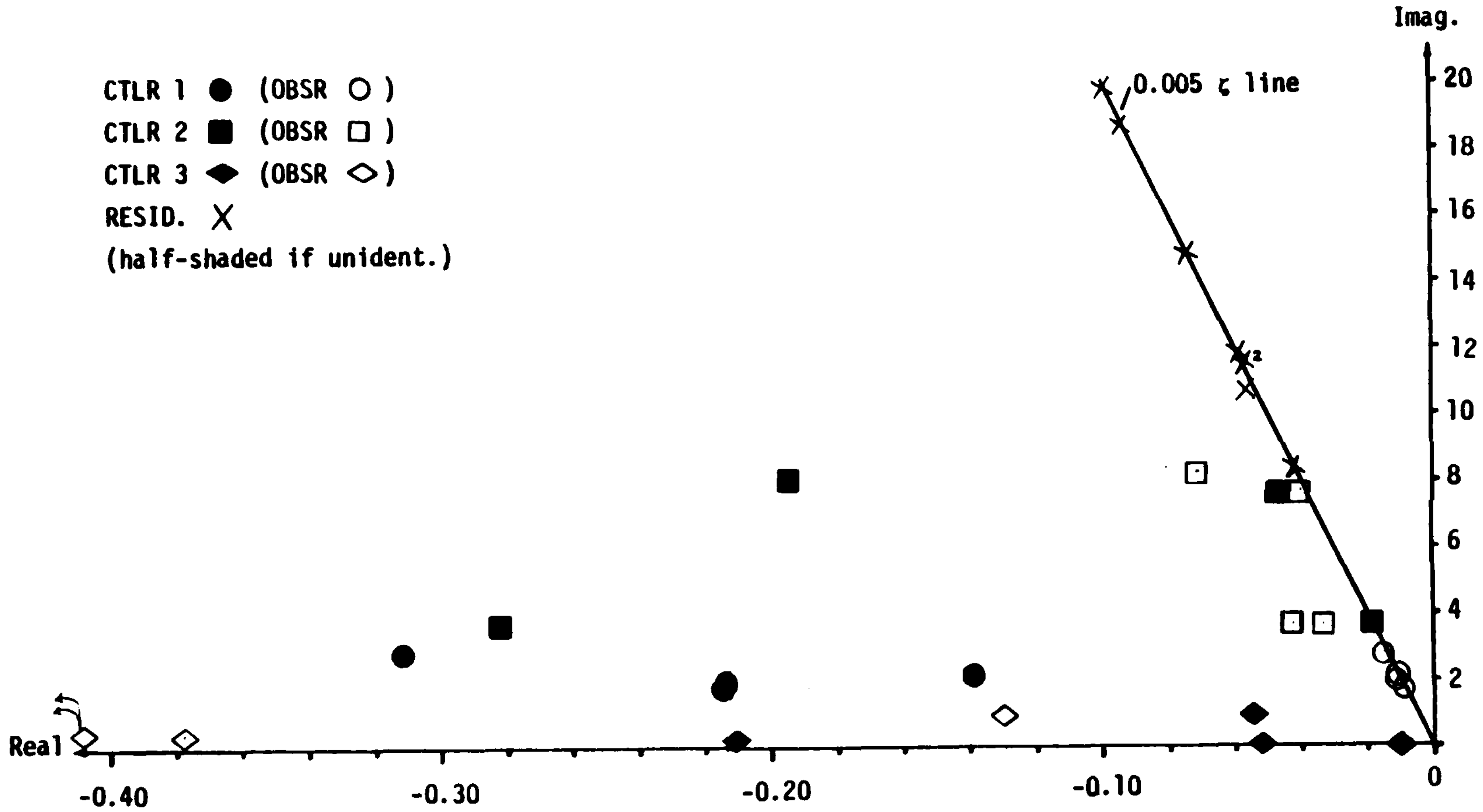


Figure 7-10. AO Comb 2-3-1 Closed-Loop Roots

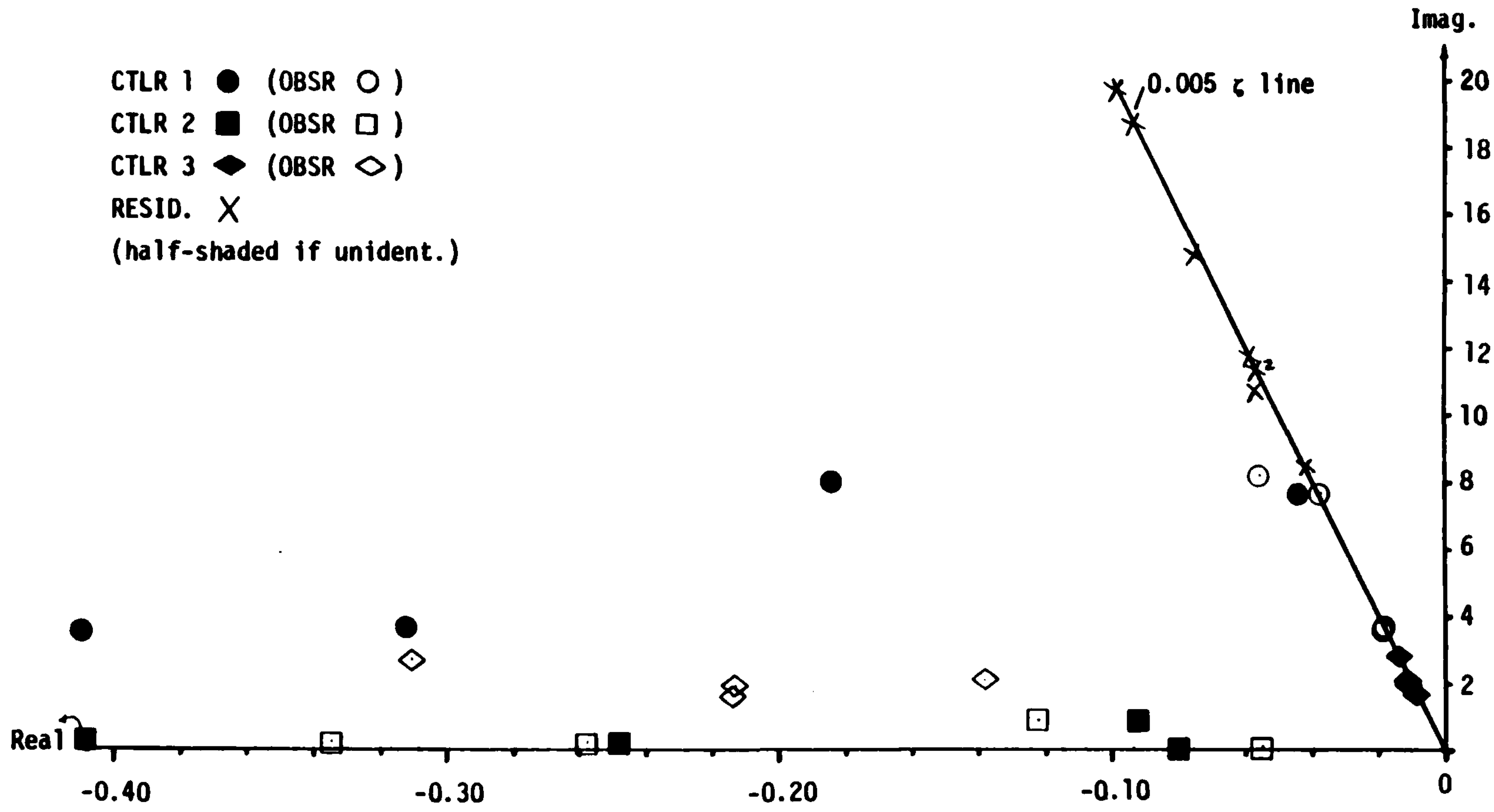
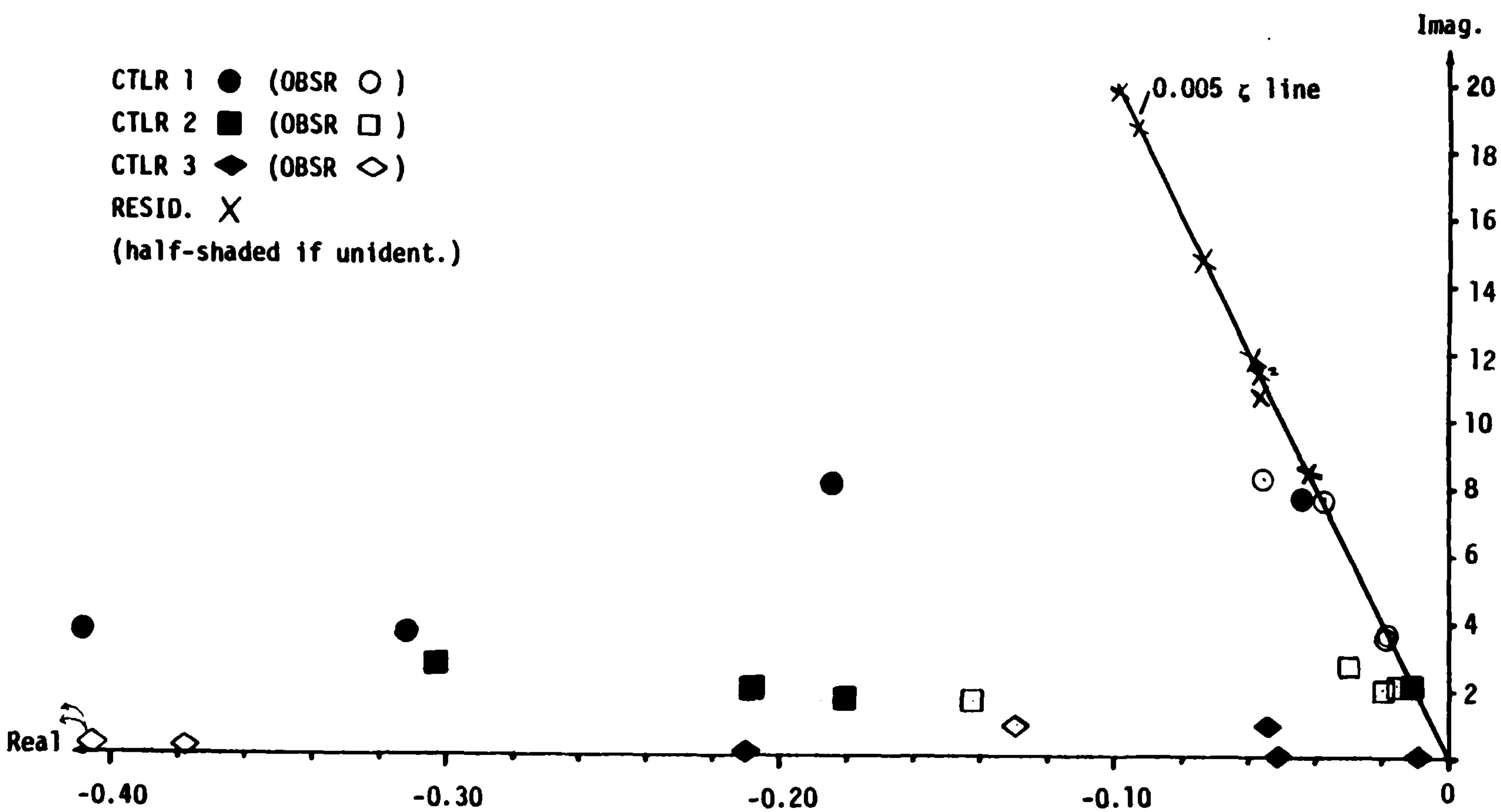


Figure 7-11. AO Comb 3-1-2 Closed-Loop Roots



<u>Figure 7-12.</u> AO Comb 3-2-1 Closed-Loop Roots

that the root for mode 5 (imaginary part of approximately 1.65), which is a residual mode using MC ranking, actually moved left under all six combinations.

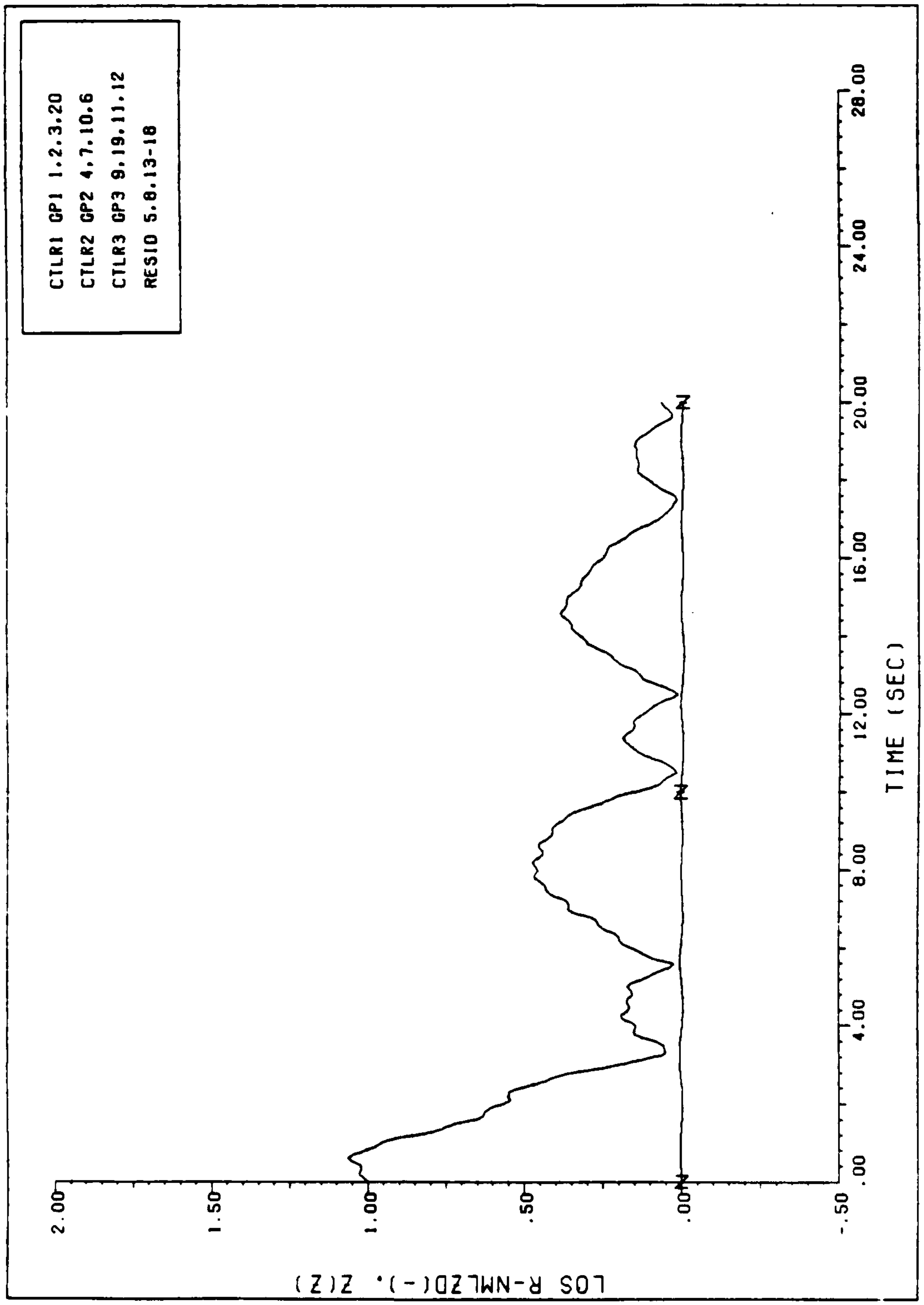
Consistent behavior of the placement of the roots of the of the error states relative to those of the controlled states for a particular controller is exhibited. The controller with most observability, Ctlr 3, consistently achieved greater damping of the error states than the controlled states. On the other hand, Ctlr 1 consistently achieved greater damping of the controlled states due to its greater degree of controllability. Ctlr 2, however, did neither in a consistent fashion. Its roots shifted so drastically after closing the loop that it was difficult to identify or distinguish between the controlled and error states--hence the notation on the figures.

As previously mentioned, under MC analysis mode 5 (residual) was made more stable. In contrast notice that mode 11 (controlled, imaginary part approximately 7.69) did not move away from the original damping ratio despite a relatively large weighting factor of 2500. This was true in both MC and AO analysis. The explanation of these two phenomena is found in the IB ranking. Recall this ranking gives the relative sensitivity to control of an individual mode regardless of its importance to LOS. Despite the fact that mode 5 was of little importance to LOS and was therfore made residual, it was still quite sensitive to control. Thus spillover moved its root significantly. On the other hand, mode 11 may have been ranked slightly higher than mode 5 for its contribution to LOS, but it was relatively insensitive to control. Thus its root did not move much.

With these general comparisons made among the eigenvalue plots, the telling results of the overall system LOS performance can be examined.

Figures 7-13 through 7-24 on the following pages are the time response plots of both the Radius and Defocus. As in previous investigations Defocus was found to be of little consequence to the control problem. Its time dependent amplitude has been miniscule compared to that of the Radius. Therefore it will not be discussed further here since, as mentioned, all cases were set up to be stable and its time response is relatively flat. The Radius' time response, however, was found to be highly dependent on modal assignment. It should be noted that in some cases the Radius was increasing significantly at the end of the time history. Since all cases were previously shown to be stable, this result simply means that a low frequency mode (or modes) was not well controlled in these configurations and would take much longer to damp out.

Comparing MC to AO analysis, the various combinations usually exhibited the same general behavior. When those modes important to control because of their relatively high contribution to LOS were assigned to the controller with most controllability, Ctlr 1, the amplitudes were damped out in a well behaved fashion. However, notice that nearly equivalent time



CASE 1 - M.C. 1-2-3 COMB. (SUPPR'D)

<u>Figure 7-13.</u> MC Comb 1-2-3 Time Response

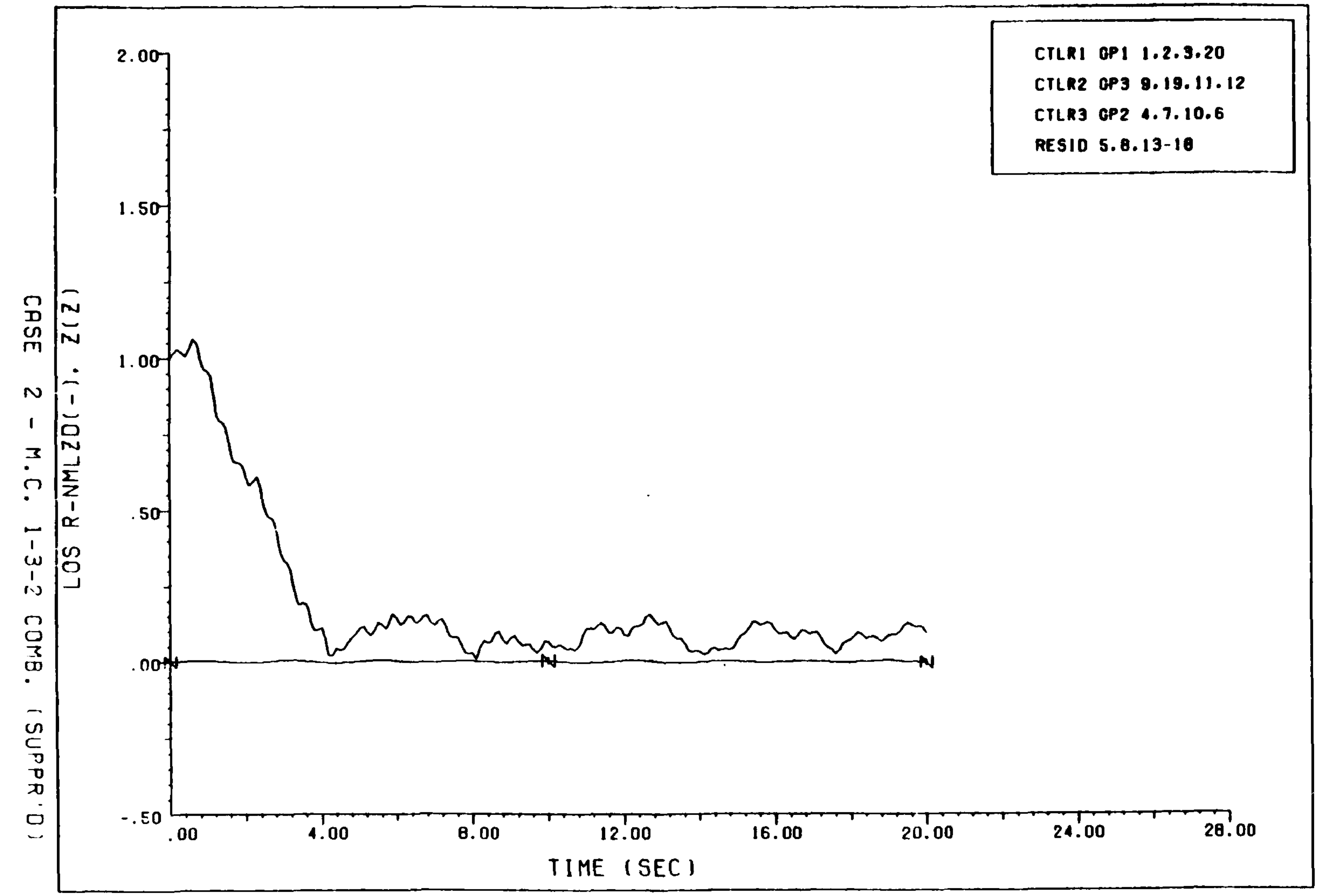


Figure 7-14. MC Comb 1-3-2 Time Response

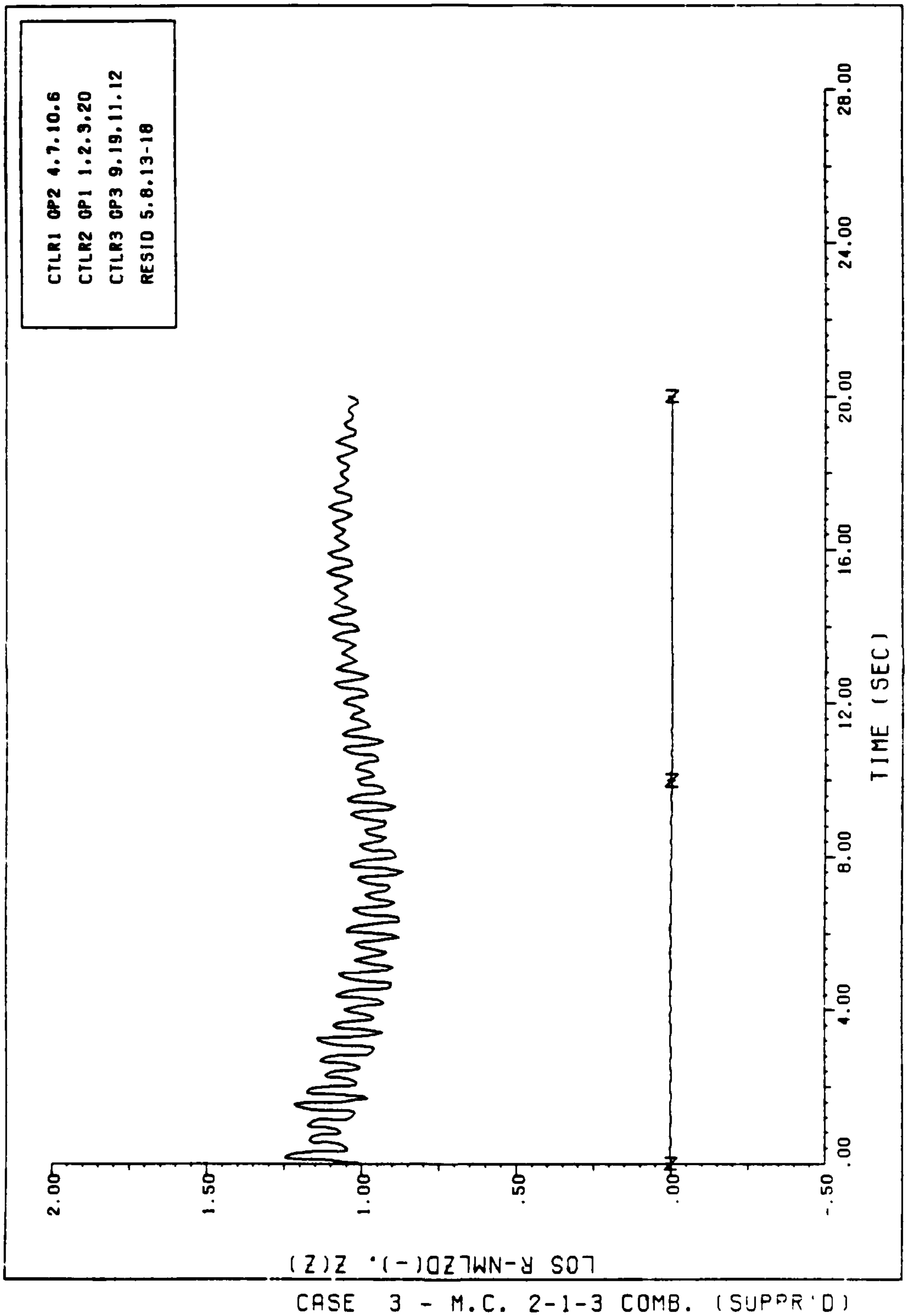


Figure 7-15. MC Comb 2-1-3 Time Response

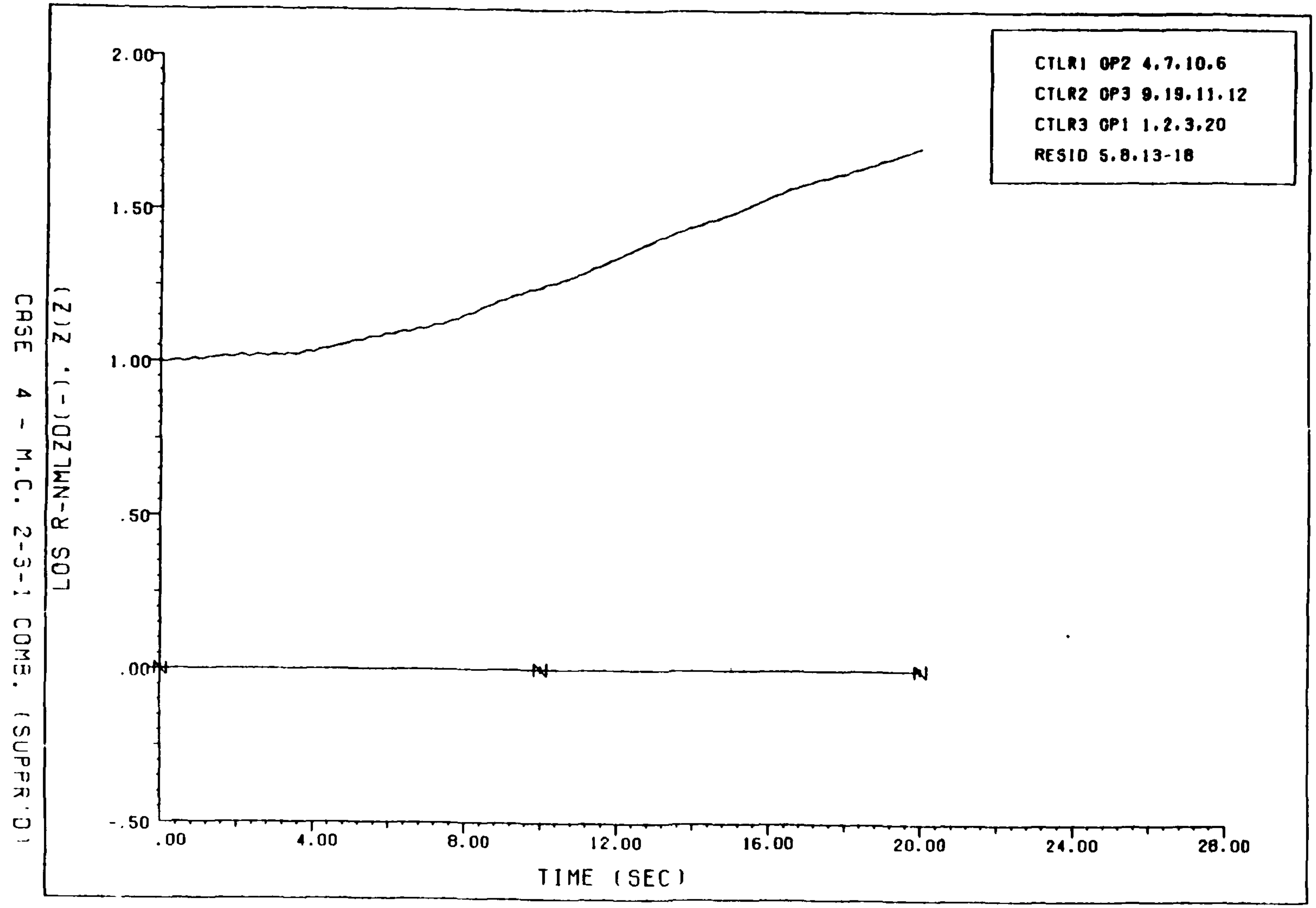
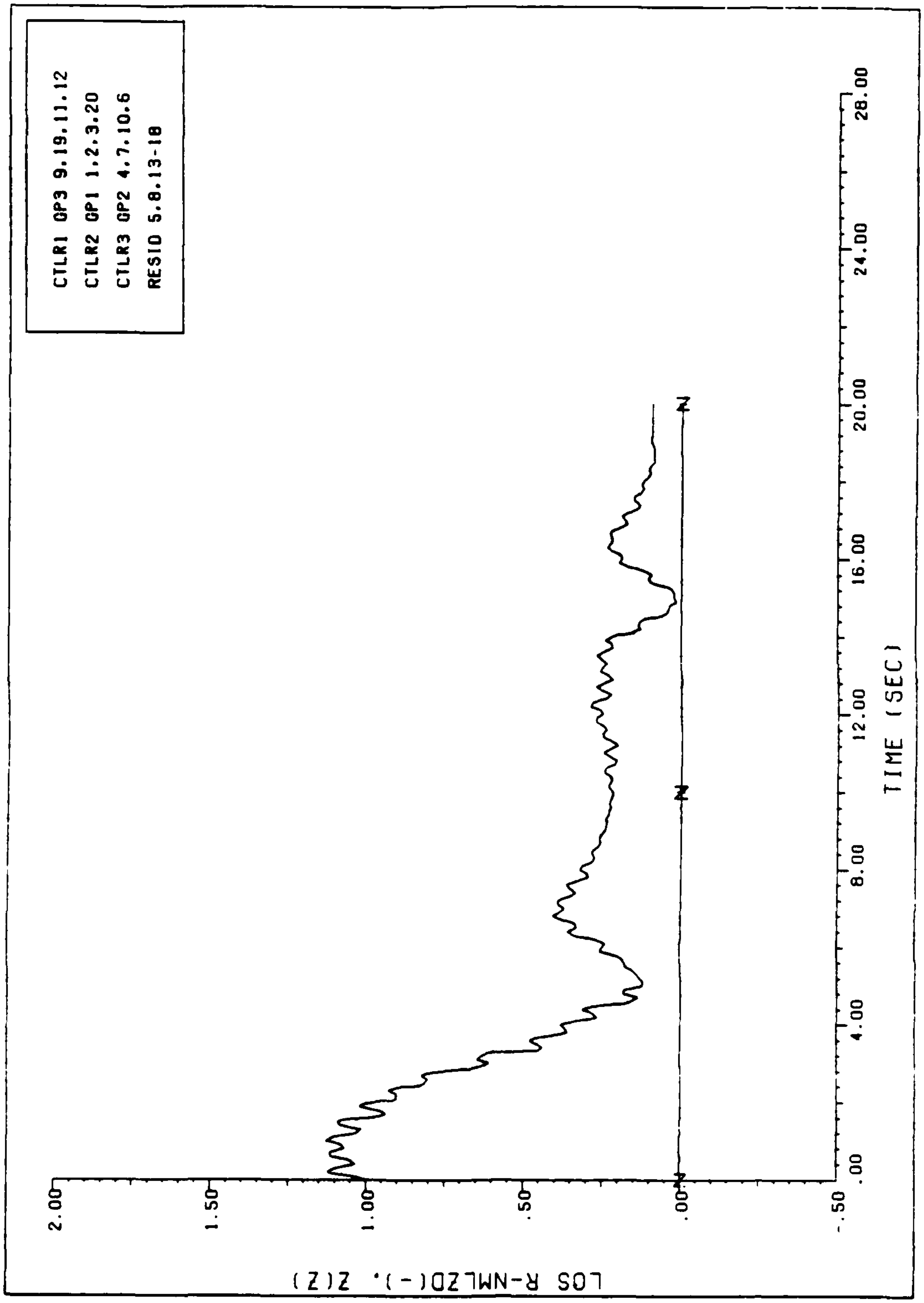


Figure 7-16. MC Comb 2-3-1 Time Response



CASE 5 - M.C. 3-1-2 COMB. (SUPPR'D)

Figure 7-17. MC Comb 3-1-2 Time Response

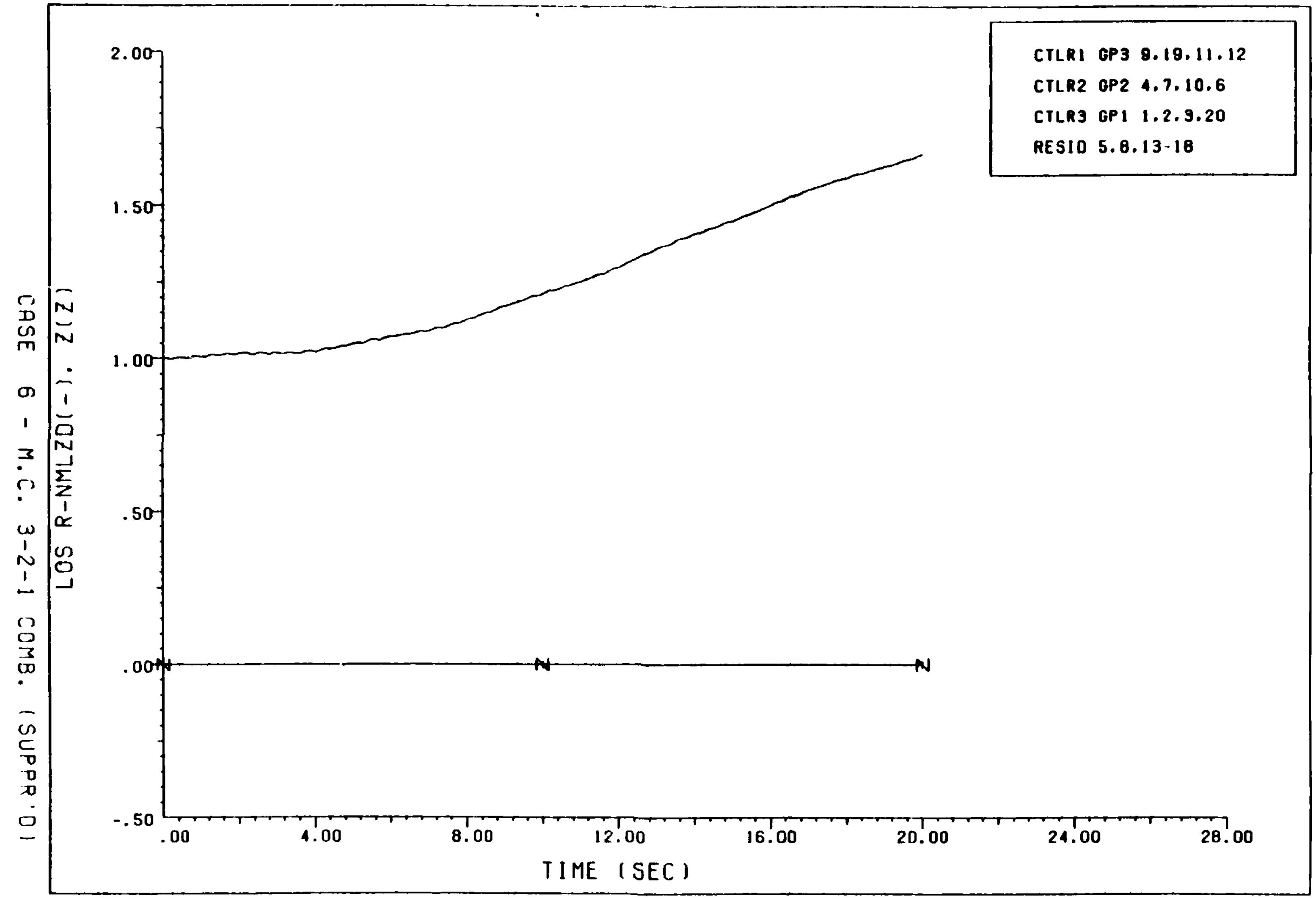


Figure 7-18. MC Comb 3-2-1 Time Response

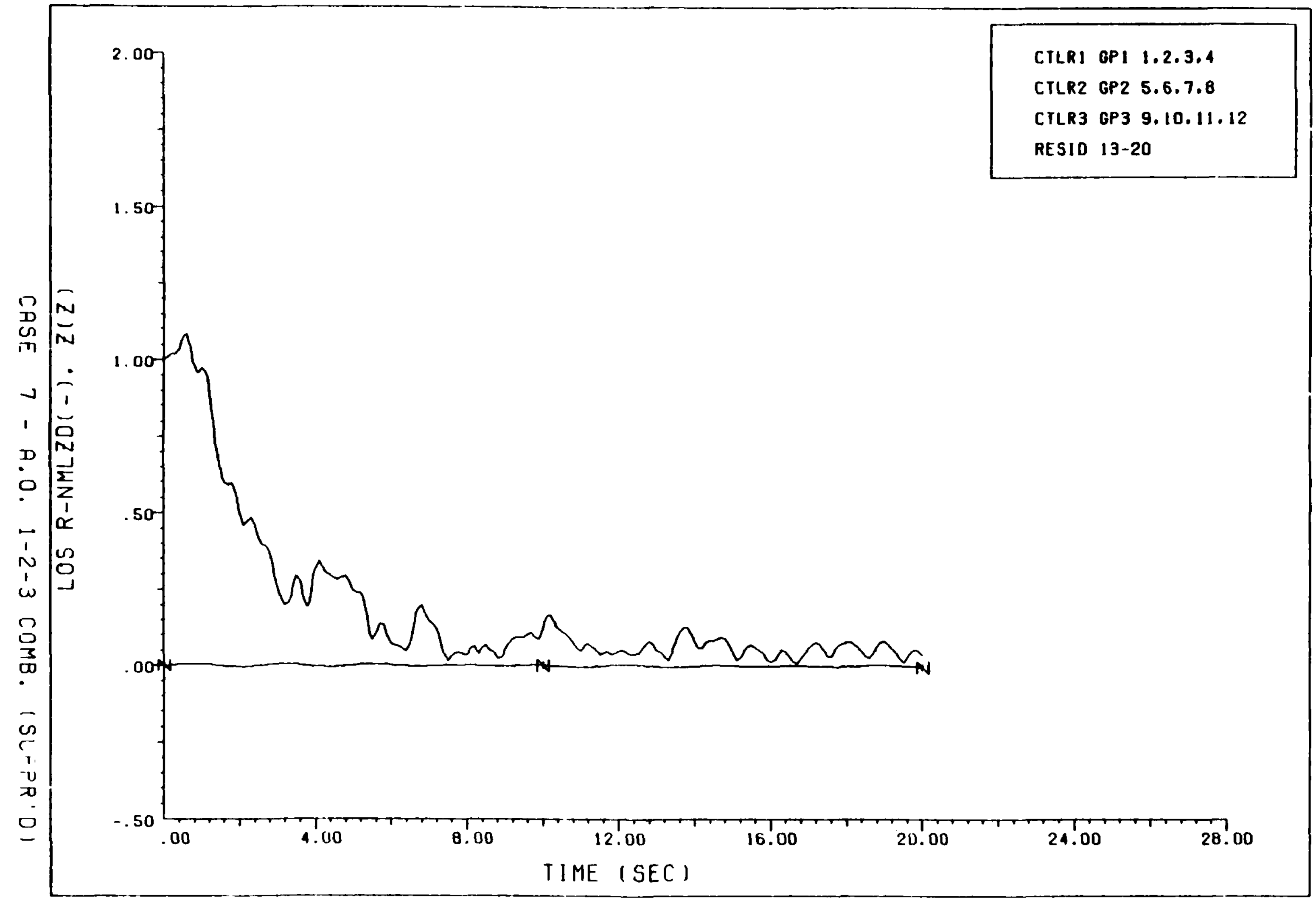


Figure 7-19. AO Comb 1-2-3 Time Response

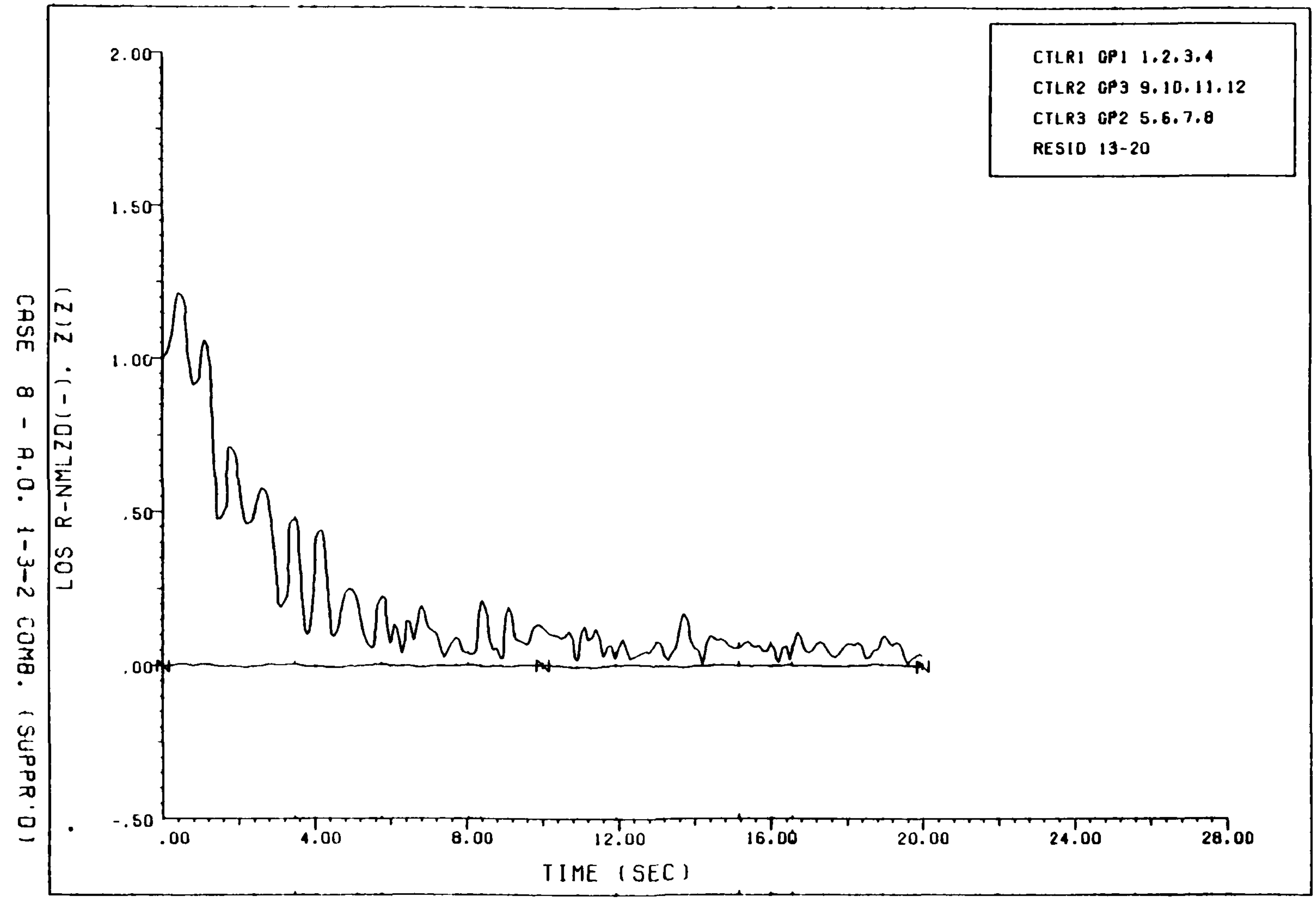


Figure 7-20. AO Comb 1-3-2 Time Response

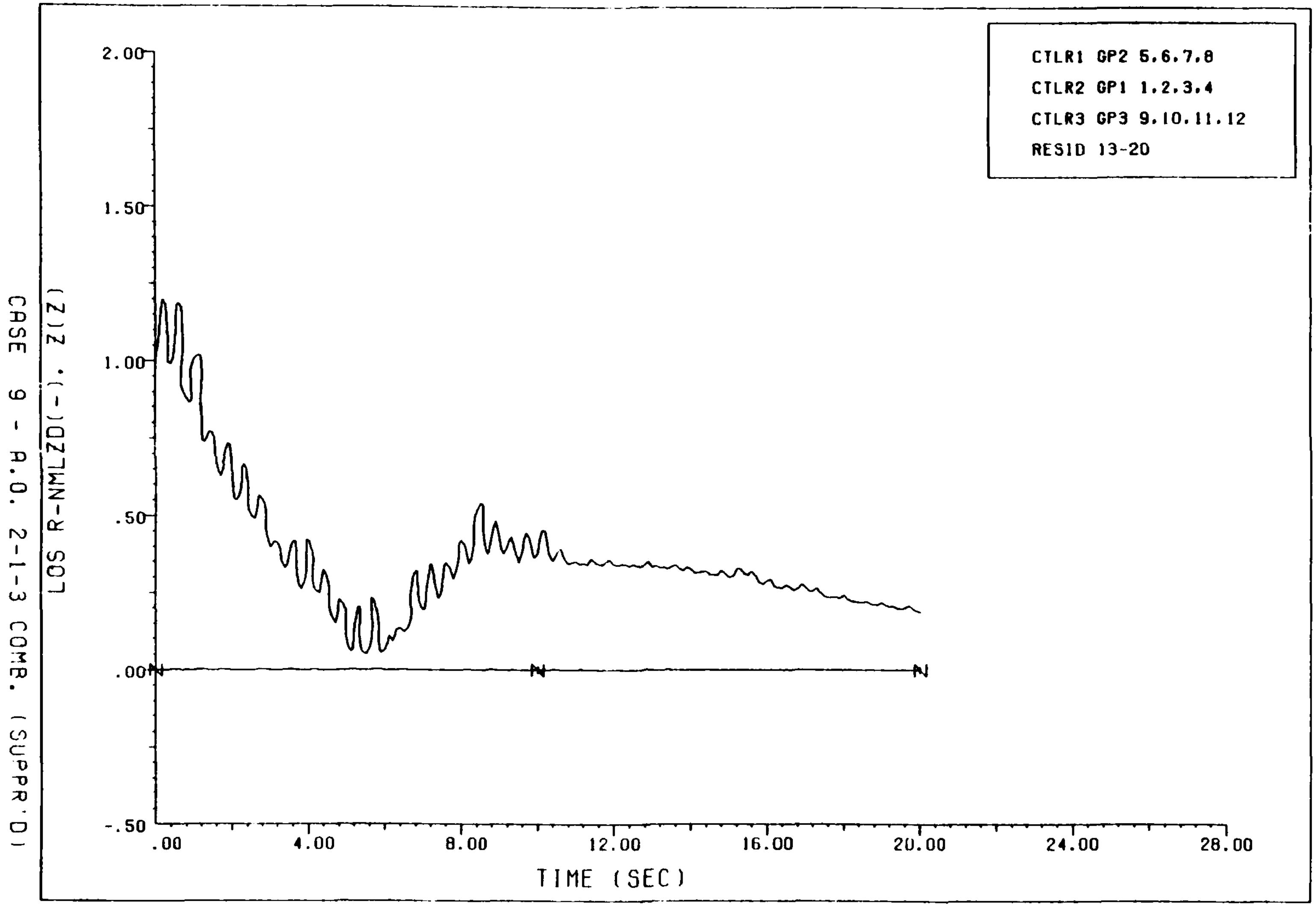


Figure 7-21. AO Comb 2-1-3 Time Response

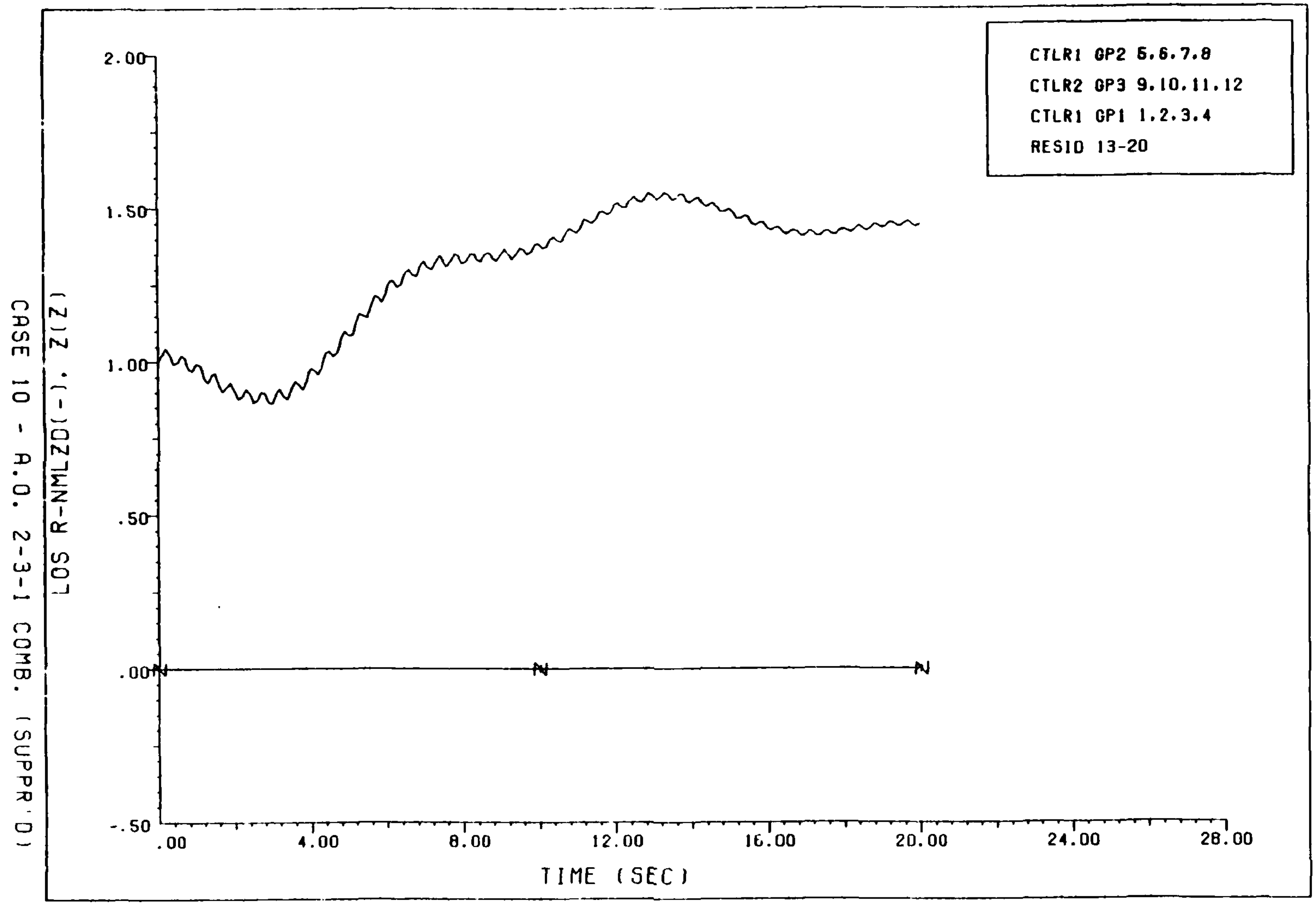


Figure 7-22. AO Comb 2-3-1 Time Response

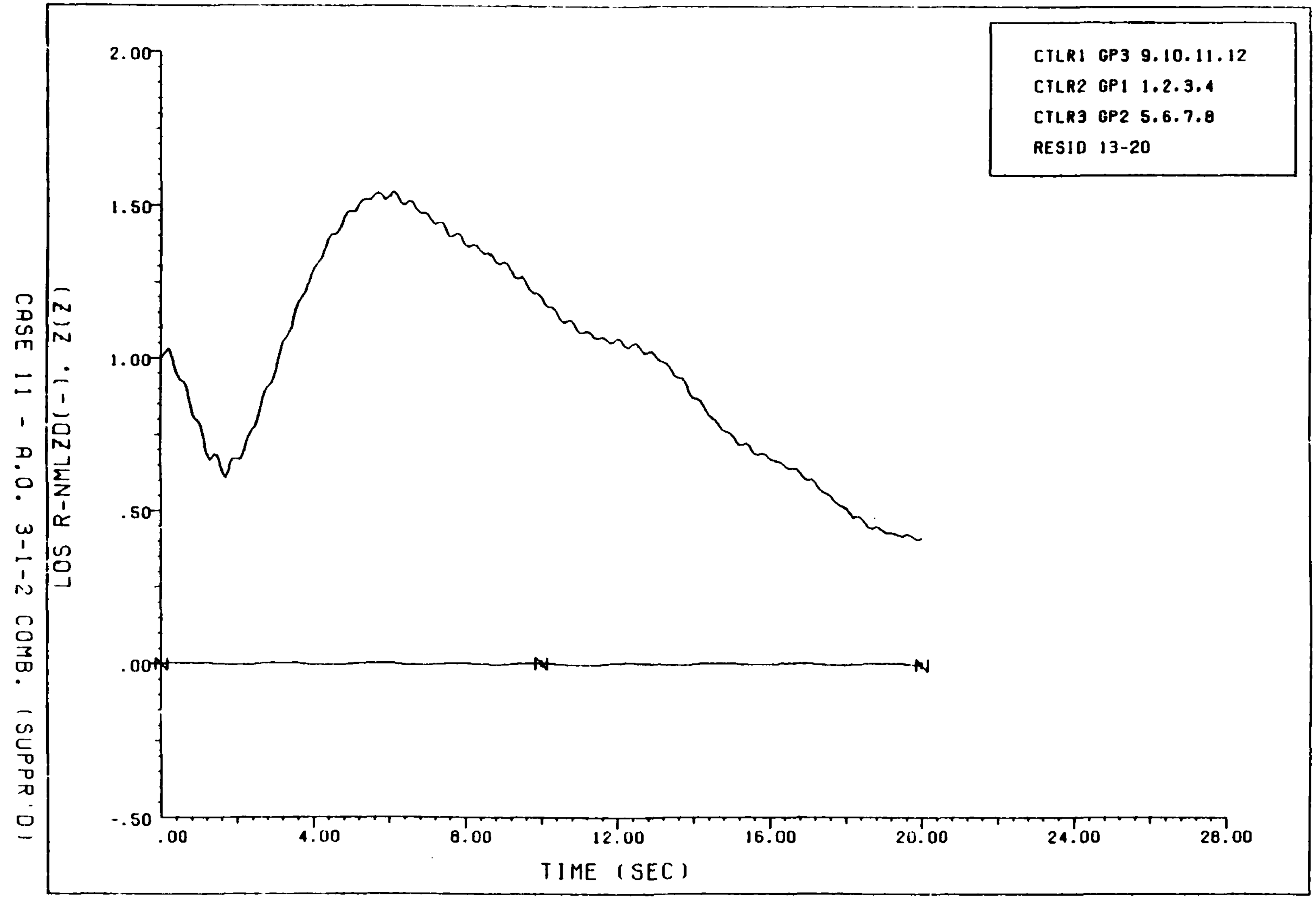


Figure 7-23. AO Comb 3-1-2 Time Response

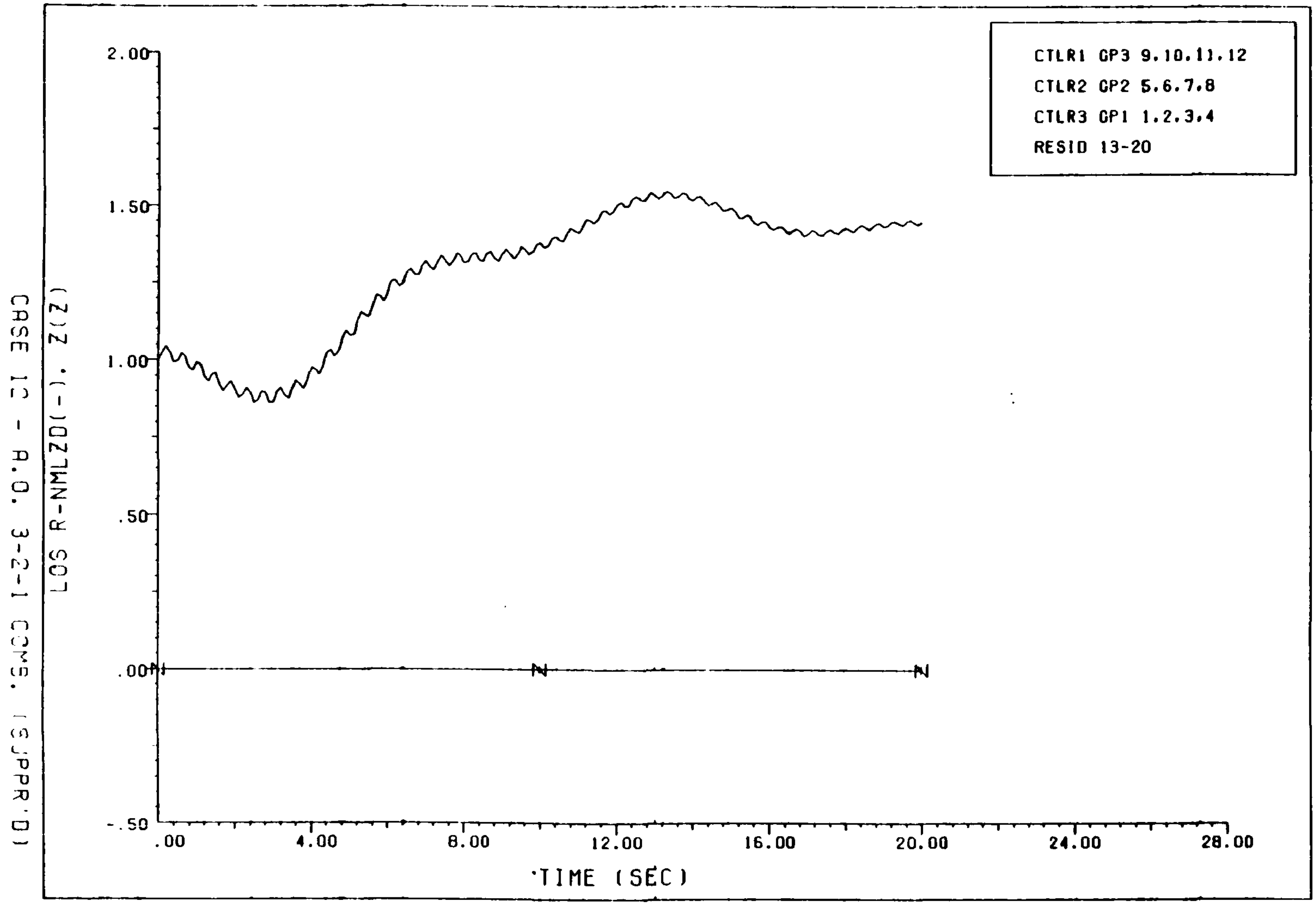


Figure 7-24. AO Comb 3-2-1 Time Response

response was achieved when the modes of Group 3 were assigned to Ctlr 2 as seen in Figs 7-13, 14, 19, and 20. It can be seen that as Group 1 is shifted to Ctlr 2 and then Ctlr 3, the time response deteriorates. If the symbology of the modal combinations is likened to a cardinal number, one might be tempted to conclude that the lower the numerical value of the combination, the better the time response. Combination (Comb) 1-2-3, analogous to the number 123, exhibited very good overall time response, while Comb 3-2-1 exhibited poor behavior. Notice, however, that Comb 3-1-2 (Fig 7-17) exhibited better behavior than combination 2-1-3 (Fig 7-15). Conversely, Figs 7-22 and 24 show Combs 2-3-1 and 3-2-1 had nearly identical time response. (The tabulated values differed only in the third decimal place.) Indeed, the best time response for MC analysis is seen in Fig 7-14 to be Comb 1-3-2 and not 1-2-3.

Though the time response behavior of the various combinations bore similar comparison between the MC and AO analyses, it was not the case that MC analysis resulted in significantly better time response. When the best time response cases (Figs 7-13 and 19) are compared, it is seen simple frequency truncation (AO analysis) has done as well for this model as the effort of making the Modal Cost ranking, if not slightly better. Notice the rankings given in Section VI are quite similar and rank the three rigid body modes highest. The relative value of these modes was so large that Varhola could not scale them onto his ranking charts. In summary of the comparison of AO to MC time response, the two analyses did not exhibit a clear pattern of improvement in time response by combination. In order of improving time response the MC analysis results are as follows: Combs 2-3-1, 3-2-1, 2-1-3, 3-1-2, 1-2-3, 1-3-2. But for AO analysis the ordering is different: 3-2-1, 2-3-1, 3-1-2, 2-1-3, 1-3-2, and 1-2-3. With the above results in mind a discussion of conclusions to be drawn is at hand.

VIII. Conclusions

A number of conclusions can be drawn from the results of this investigation.

From the eigenvalue plots under MC analysis, it is seen that multiple controllers suppressing spillover can effectively control a high frequency mode with little or no adverse effect on lower frequency modes. For example, mode 20 was damped without driving other modes unstable.

However, these same plots show that mode 5, a residual mode, was also affected--by spillover. Though it was damped in this investigation under conditions of these gains, it might just as well have been driven unstable by the spillover. It would be better to be able to predict residual modes will maintain their original damping ratios after suppression so as to not be of concern. Thus it should be concluded that Modal Cost analysis alone is a risky approach to take. It should be accompanied by Internal Balancing analysis to reveal problem areas. These arise when a mode has a high IB ranking but a low enough MC ranking to leave it as a residual mode.

IB analysis can also reveal candidate modes for removal from the controlled groups in exchange for just such problem modes. For example, mode 11, exhibiting little response to control, might well be replaced by mode 5. An advantageously greater range of gain values (Q matrix weighting values) might then be possible because it could be expected that the residual modes would remain "steady." It is now obvious that a better control configuration would have the IB ranking identical to the MC ranking. The designer would accomplish this by moving sensors and actuators to increase or decrease their effect on a particular mode, resulting in changing the IB ranking. An even greater range of gain could result because now the single identical ranking would simultaneously show those modes most contributory to LOS also being those most sensitive to control, and equally important, those contributing little to LOS being relatively insensitive.

In addition to the drawbacks of applying MC analysis alone as mentioned above, another conclusion to be drawn is that MC analysis alone does not necessarily offer significant time response improvement over AO analysis, which is the simplest approach to take. In this investigation, AO Comb 1-2-3, the simplest ordering and assignment possible, resulted in the best time response. It should be noted, however, that the result may have been due to the overwhelming influence of the three rigid body modes on LOS performance.

In summary, for any modal assignment approach taken, unsuppressed residual modes will limit the amount of gain that can be applied to improve time response. Though various analytical rankings can be made and used together to effect significant improvement in time response, this investigation revealed no clear pattern among the analyses to predict behavior based on modal assignment. The results and conclusions do, however, provide good ground for further investigations, as described in the next section.

IX. Recommendations

Several further investigations can stem from this one. As mentioned, mode 5 was found to make little contribution to LOS while being quite sensitive to control spillover. At the same time mode 11 had somewhat opposite characteristics. These two modes are good candidates to lead into an investigation of the effects on time response and stability gain margins for this configuration under mode-swapping conditions when the various cases of modal assignment combination are run. There are other modes, with less pronounced characteristics, which would also make good candidates (e.g., modes 8 and 12).

Some investigations might also be carried out on the effects of new placements and orientations of the sensor/actuator pairs based on the modal rankings of Ref (7). This investigation could include the addition of code to the simulator for disturbance rejection. Moreover, such investigations could include studying the effects of noise in the sensor data.

A similar investigation to this one begs to be run with an increased number of modes. This investigation used for continuity of analysis a twenty-mode truth model to search for general patterns of modal assignment effects. More revealing data may be obtained if the number of modes assigned to each controller could be increased and if simultaneously all significant residual modes could be included in the suppression. Doing this might show that there are some clear patterns to the effects of various modal assignments not revealed in this investigation. As mentioned, the overwhelming value of the rigid body modes in the rankings may have obscured such patterns.

Along these lines the code could be revised to be more flexible in its dimensioning. Presently the program must be recompiled if the quantity of modes changes. This is due to the requirement that certain of the routines (9) have exactly dimensioned arrays, as opposed to the IMSL routines which do not require this. There was not time in this investigation to make these revisions, but they would greatly enhance the flexibility of the program and the speed with which further investigations could be carried out.

Appendix A

Draper-2

NASTRAN and Other Data

E I G E N V A L U E   A N A L Y S I S   S U M M A R Y   (INVERSE POWER METHOD)

NUMBER OF EIGENVALUES EXTRACTED	54
NUMBER OF STARTING POINTS USED	10
NUMBER OF STARTING POINT MOVES	0
NUMBER OF TRIANGULAR DECOMPOSITIONS	61
TOTAL NUMBER OF VECTOR ITERATIONS	673
REASON FOR TERMINATION	7*
LARGEST OFF-DIAGONAL MODAL MASS TERM	.20E-13
MODE PAIR	51
	37
NUMBER OF OFF-DIAGONAL MODAL MASS TERMS FAILING CRITERION	0

(* 1 OR MORE ROOT OUTSIDE FR.RANGE.
SEE NASTRAN U.M. PAGE 3.4-12)

50 MODES AND FREQUENCIES

REAL EIGENVALUES

MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIAN FREQUENCY	CYCLIC FREQUENCY	GENERALIZED MASS	GENERALIZED STIFFNESS
1	35	-2.489640E-05	4.989637E-03	7.941255E-04	1.000000E+00	-2.489640E-05
2	37	-1.357722E-05	3.684727E-03	5.864426E-04	1.000000E+00	-1.357722E-05
3	33	-1.111428E-05	3.333800E-03	5.305920E-04	1.000000E+00	-1.111428E-05
4	31	-9.826475E-06	3.134721E-03	4.989063E-04	1.000000E+00	-9.826475E-06
5	29	-9.587553E-06	3.096377E-03	4.928030E-04	1.000000E+00	-9.587553E-06
6	21	-2.089891E-09	4.571532E-05	7.275820E-06	1.000000E+00	-2.089891E-09
7	26	8.361744E-01	9.144257E-01	1.455354E-01	1.000000E+00	8.361744E-01
8	25	2.735091E+00	1.653811E+00	2.632122E-01	1.000000E+00	2.735091E+00
9	22	3.974426E+00	1.993596E+00	3.172987E-01	1.000000E+00	3.974426E+00
10	27	4.374811E+00	2.091605E+00	3.328892E-01	1.000000E+00	4.374811E+00
11	19	7.754729E+00	2.784731E+00	4.432030E-01	1.000000E+00	7.754729E+00
12	20	1.318287E+01	3.630822E+00	5.778633E-01	1.000000E+00	1.318287E+01
13	23	1.334619E+01	3.653243E+00	5.814317E-01	1.000000E+00	1.334619E+01
14	18	5.912151E+01	7.689061E+00	1.223751E+00	1.000000E+00	5.912151E+01
15	16	6.674240E+01	8.169607E+00	1.300233E+00	1.000000E+00	6.674240E+01
16	14	7.168796E+01	8.466874E+00	1.347545E+00	1.000000E+00	7.168796E+01
17	15	1.169216E+02	1.081303E+01	1.720947E+00	1.000000E+00	1.169216E+02
18	13	1.305813E+02	1.142722E+01	1.818690E+00	1.000000E+00	1.305813E+02
19	11	1.305862E+02	1.142743E+01	1.818732E+00	1.000000E+00	1.305862E+02
20	12	1.408955E+02	1.186994E+01	1.889160E+00	1.000000E+00	1.408955E+02
21	9	2.205301E+02	1.485026E+01	2.363492E+00	1.000000E+00	2.205301E+02
22	8	3.528249E+02	1.878363E+01	2.989508E+00	1.000000E+00	3.528249E+02
23	6	3.982044E+02	1.995506E+01	3.175946E+00	1.000000E+00	3.982044E+02
24	5	4.529573E+02	2.128279E+01	3.387262E+00	1.000000E+00	4.529573E+02
25	1	1.051814E+03	3.243168E+01	5.161662E+00	1.000000E+00	1.051814E+03
26	2	1.092404E+03	3.305153E+01	5.260315E+00	1.000000E+00	1.092404E+03
27	3	2.449482E+03	4.949224E+01	7.876934E+00	1.000000E+00	2.449482E+03
28	4	2.600906E+03	5.099907E+01	8.116755E+00	1.000000E+00	2.600906E+03
29	7	2.759142E+03	5.252754E+01	8.360018E+00	1.000000E+00	2.759142E+03
30	10	2.899910E+03	5.385081E+01	8.570622E+00	1.000000E+00	2.899910E+03
31	17	3.066569E+03	5.537661E+01	8.813461E+00	1.000000E+00	3.066569E+03
32	24	3.066569E+03	5.537661E+01	8.813462E+00	1.000000E+00	3.066569E+03
33	28	5.082329E+03	7.129046E+01	1.134623E+01	1.000000E+00	5.082329E+03
34	30	5.219047E+03	7.224297E+01	1.149783E+01	1.000000E+00	5.219047E+03
35	32	6.393348E+03	7.995842E+01	1.272578E+01	1.000000E+00	6.393348E+03
36	34	7.283877E+03	8.534563E+01	1.358310E+01	1.000000E+00	7.283877E+03
37	38	7.425014E+03	8.616852E+01	1.371415E+01	1.000000E+00	7.425014E+03
38	36	7.916140E+03	8.897269E+01	1.416044E+01	1.000000E+00	7.916140E+03
39	39	9.671943E+03	9.834604E+01	1.565226E+01	1.000000E+00	9.671943E+03
40	41	1.019817E+04	1.009860E+02	1.607242E+01	1.000000E+00	1.019817E+04
41	40	1.078034E+04	1.038284E+02	1.652481E+01	1.000000E+00	1.078034E+04
42	42	1.107000E+04	1.052140E+02	1.674534E+01	1.000000E+00	1.107000E+04
43	43	1.161892E+04	1.077911E+02	1.715548E+01	1.000000E+00	1.161892E+04
44	44	1.254812E+04	1.120184E+02	1.782828E+01	1.000000E+00	1.254812E+04
45	45	1.435881E+04	1.198283E+02	1.907126E+01	1.000000E+00	1.435881E+04
46	46	2.230087E+04	1.493615E+02	2.377163E+01	1.000000E+00	2.230087E+04
47	47	2.353083E+04	1.533976E+02	2.441399E+01	1.000000E+00	2.353083E+04
48	48	2.650074E+04	1.627905E+02	2.590891E+01	1.000000E+00	2.650074E+04
49	50	2.743684E+04	1.656407E+02	2.636253E+01	1.000000E+00	2.743684E+04
50	49	2.757575E+04	1.660595E+02	2.642919E+01	1.000000E+00	2.757575E+04
51	51	2.872906E+04	1.694965E+02	2.697620E+01	1.000000E+00	2.872906E+04
52	52	3.054857E+04	1.747815E+02	2.781734E+01	1.000000E+00	3.054857E+04
53	53	3.076466E+04	1.753986E+02	2.791555E+01	1.000000E+00	3.076466E+04
54	54	3.213600E+04	1.792652E+02	2.853094E+01	1.000000E+00	3.213600E+04

ACOSS6 MODEL EIGENDATA DUPLICATION AUGUST 13, 1985 RELEASE APR. 1984

50 MODES AND FREQUENCIES

S O R T E D   B U L K   D A T A   E C H O

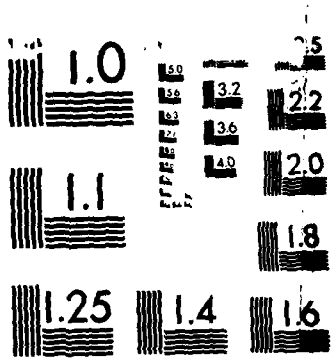
CARD COUNT	. 1 ..	2 ..	3 ..	4 ..	5 ..	6 ..	7 ..	8 ..	9 ..	10 .
1-	BAROR					1.0	.0	.0	1	
2-	CBAR	1	200	1	2					
3-	CBAR	2	200	1	3					
4-	CBAR	3	200	2	3					
5-	CBAR	4	200	2	4	.0	1.0	.0	1	
6-	CBAR	5	200	3	4					
7-	CBAR	6	200	4	5	.0	1.0	.0	1	
8-	CBAR	7	200	4	6					
9-	CBAR	8	200	3	8	.0	1.0	.0	1	
10-	CBAR	9	200	5	6					
11-	CBAR	10	200	5	7					
12-	CBAR	11	200	6	7					
13-	CBAR	12	200	1	8					
14-	CBAR	13	200	2	9					
15-	CBAR	14	200	3	10					
16-	CBAR	15	200	5	11					
17-	CBAR	16	200	6	12					
18-	CBAR	17	200	7	13					
19-	CBAR	18	200	3	8					
20-	CBAR	19	200	2	8					
21-	CBAR	20	200	3	9					
22-	CBAR	21	200	4	9					
23-	CBAR	22	200	4	11					
24-	CBAR	23	200	5	12					
25-	CBAR	24	200	5	13					
26-	CBAR	25	200	6	13					
27-	CBAR	26	200	12	41					
28-	CBAR	27	200	6	41					
29-	CBAR	28	200	10	41					
30-	CBAR	29	200	3	41					
31-	CBAR	30	200	8	9					
32-	CBAR	31	200	8	10					
33-	CBAR	32	200	9	10					
34-	CBAR	33	200	9	40					

50 MODES AND FREQUENCIES

S O R T E D   B U L K   D A T A   E C H O

CARD COUNT	. 1	.. 2	.. 3	.. 4	.. 5	.. 6	.. 7	.. 8	.. 9	.. 10 .
35-	CBAR	34	200	10	40					
36-	CBAR	35	200	11	40					
37-	CBAR	36	200	12	40					
38-	CBAR	37	200	9	11	.0	1.0	.0	1	
39-	CBAR	38	200	10	12	.0	1.0	.0	1	
40-	CBAR	39	200	11	12					
41-	CBAR	40	200	11	13					
42-	CBAR	41	200	12	13					
43-	CBAR	42	300	14	15					
44-	CBAR	43	300	14	16					
45-	CBAR	44	300	16	15					
46-	CBAR	45	300	17	18					
47-	CBAR	46	300	17	19					
48-	CBAR	47	300	18	19					
49-	CBAR	54	300	26	27					
50-	CBAR	55	300	26	28					
51-	CBAR	56	300	27	28					
52-	CBAR	57	300	29	30					
53-	CBAR	58	300	29	31					
54-	CBAR	59	300	30	31					
55-	CBAR	60	300	27	29	.0	1.0	.0	1	
56-	CBAR	61	300	27	30					
57-	CBAR	62	300	28	30	.0	1.0	.0	1	
58-	CBAR	63	300	27	36					
59-	CBAR	64	300	28	37					
60-	CBAR	65	300	30	39					
61-	CBAR	66	300	29	38					
62-	CBAR	67	300	29	36					
63-	CBAR	68	300	27	37					
64-	CBAR	69	300	28	39					
65-	CBAR	70	300	30	38					
66-	CBAR	71	300	36	37					
67-	CBAR	72	300	37	39	.0	1.0	.0	1	
68-	CBAR	73	300	39	38					

AD-A163 977 MODAL ASSIGNMENT EFFECTS ON DECENTRALIZED CONTROL OF A 2/2
LARGE SPACE STRUCTURE(U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AF OH SCHOOL OF ENGI J B SUMNER
UNCLASSIFIED DEC 85 AFIT/GA/AA/85D-9 F/G 22/2 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

50 MODES AND FREQUENCIES

S O R T E D   B U L K   D A T A   E C H O

CARD COUNT	. 1 ..	2 ..	3 ..	4 ..	5 ..	6 ..	7 ..	8 ..	9 ..	10 .
69-	CBAR	74	300	36	38	.0	1.0	.0	1	
70-	CBAR	75	300	37	38					
71-	CBAR	76	400	8	14					
72-	CBAR	77	400	10	14					
73-	CBAR	78	400	10	16					
74-	CBAR	79	400	16	9					
75-	CBAR	80	400	9	15					
76-	CBAR	81	400	11	17					
77-	CBAR	82	400	11	18					
78-	CBAR	83	400	12	18					
79-	CBAR	84	400	12	19					
80-	CBAR	85	400	13	19					
81-	CBAR	86	400	13	17					
82-	CBAR	87	400	14	26					
83-	CBAR	88	400	14	20					
84-	CBAR	89	400	16	20					
85-	CBAR	90	400	16	27					
86-	CBAR	91	400	15	27					
87-	CBAR	92	400	15	26					
88-	CBAR	93	400	17	29					
89-	CBAR	94	400	18	29					
90-	CBAR	95	400	18	30					
91-	CBAR	96	400	19	30					
92-	CBAR	97	400	19	31					
93-	CBAR	98	400	17	31					
94-	CBAR	99	400	15	32					
95-	CBAR	100	400	16	34					
96-	CBAR	101	400	17	33					
97-	CBAR	102	400	18	35					
98-	CBAR	111	400	26	32					
99-	CBAR	112	400	27	32					
100-	CBAR	113	400	27	33					
101-	CBAR	114	400	29	33					
102-	CBAR	115	400	31	33					

50 MODES AND FREQUENCIES

S O R T E D   B U L K   D A T A   E C H O

CARD COUNT	. 1 ..	2 ..	3 ..	4 ..	5 ..	6 ..	7 ..	8 ..	9 ..	10 .
103-	CBAR	116	400	32	33	.0	1.0	.0	1	
104-	CBAR	117	400	26	34					
105-	CBAR	118	400	28	34					
106-	CBAR	119	400	30	34					
107-	CBAR	120	400	30	35					
108-	CBAR	121	400	31	35					
109-	CBAR	122	400	34	35	.0	1.0	.0	1	
110-	CBAR	123	400	32	36					
111-	CBAR	124	400	33	36					
112-	CBAR	125	400	34	37					
113-	CBAR	126	400	35	39					
114-	CBAR	127	300	26	37					
115-	CBAR	128	300	26	36					
116-	CBAR	129	300	31	39					
117-	CBAR	130	300	31	38					
118-	CBAR	131	500	48	49	.0	1.0	.0	1	
119-	CBAR	132	500	49	50	.0	1.0	.0	1	
120-	CBAR	133	500	50	51	.0	1.0	.0	1	
121-	CBAR	134	500	51	52	.0	1.0	.0	1	
122-	CBAR	135	500	52	43	.0	1.0	.0	1	
123-	CBAR	136	500	45	53	.0	1.0	.0	1	
124-	CBAR	137	500	53	54	.0	1.0	.0	1	
125-	CBAR	138	500	54	55	.0	1.0	.0	1	
126-	CBAR	139	500	55	56	.0	1.0	.0	1	
127-	CBAR	140	500	56	57	.0	1.0	.0	1	
128-	CBAR	181	400	8	15					
129-	CELAS2	142	5.79E3	4	1	42	1			
130-	CELAS2	143	5.79E3	4	2	42	2			
131-	CELAS2	144	5.79E3	4	3	42	3			
132-	CELAS2	145	5.79E3	3	1	46	1			
133-	CELAS2	146	5.79E3	3	2	46	2			
134-	CELAS2	147	5.79E3	3	3	46	3			
135-	CELAS2	148	5.79E3	6	1	47	1			
136-	CELAS2	149	5.79E3	6	2	47	2			

50 MODES AND FREQUENCIES

S O R T E D B U L K D A T A E C H O

CARD COUNT	. 1 ..	2 ..	3 ..	4 ..	5 ..	6 ..	7 ..	8 ..	9 ..	10 .
137-	CELAS2	160	5.79E3	6	3	47	3			
138-	CONM2	501	27		375.					
139-	CONM2	502	28		375.					
140-	CONM2	503	29		375.					
141-	CONM2	504	30		375.					
142-	CONM2	505	32		500.					
143-	CONM2	506	33		500.					
144-	CONM2	507	34		250.					
145-	CONM2	508	35		250.					
146-	CONM2	509	9		500.					
147-	CONM2	510	10		500.					
148-	CONM2	511	11		500.					
149-	CONM2	512	12		500.					
150-	CONM2	520	14		8.0					
151-	CONM2	521	15		17.0					
152-	CONM2	522	16		17.0					
153-	CONM2	523	17		17.0					
154-	CONM2	524	18		17.0					
155-	CONM2	525	19		8.0					
156-	CONM2	544	44		3500.					+544
157-	+544	2.10E3		2.10E3			4.20E3			
158-	CONM2	548	48		90.00					+548
159-	+548	270.0								
160-	CONM2	550	50		180.0					+550
161-	+550	540.0								
162-	CONM2	552	52		90.0					+552
163-	+552	270.0								
164-	CONM2	553	53		90.0					+553
165-	+553	270.0								
166-	CONM2	555	55		180.0					+555
167-	+555	540.0								
168-	CONM2	557	57		90.0					+557
169-	+557	270.0								
170-	CRIGD1	141	44	42	43	45	46	47		

50 MODES AND FREQUENCIES

SORTED BULK DATA ECHO

CARD COUNT	. 1 ..	2 ..	3 ..	4 ..	5 ..	6 ..	7 ..	8 ..	9 ..	10 .
171-	EIGR	600	INV	.0	20.0	60	50		1.0 E-03	+10
172-	+10	MASS								
173-	GRID	1		-7.0	.0	.0				
174-	GRID	2		-4.0	5.0	.0				
175-	GRID	3		-4.0	-5.0	.0				
176-	GRID	4		.0	5.0	.0				
177-	GRID	5		4.0	5.0	.0				
178-	GRID	6		4.0	-5.0	.0				
179-	GRID	7		7.0	0	.0				
180-	GRID	8		-7.0	.0	2.0				
181-	GRID	9		-4.0	5.0	2.0				
182-	GRID	10		-4.0	-5.0	2.0				
183-	GRID	11		4.0	5.0	2.0				
184-	GRID	12		4.0	-5.0	2.0				
185-	GRID	13		7.0	.0	2.0				
186-	GRID	14		-6.0	.0	12.				
187-	GRID	15		-4.0	4.0	12.				
188-	GRID	16		-4.0	-4.0	12.				
189-	GRID	17		4.0	4.0	12.				
190-	GRID	18		4.0	-4.0	12.				
191-	GRID	19		6.0	.0	12.0				
192-	GRID	26		-5.0	.0	22.0				
193-	GRID	27		-4.0	3.0	22.0				
194-	GRID	28		-4.0	-3.0	22.0				
195-	GRID	29		4.0	3.0	22.0				
196-	GRID	30		4.0	-3.0	22.0				
197-	GRID	31		5.0	.0	22.0				
198-	GRID	32		-4.0	10.0	22.0				
199-	GRID	33		4.0	10.0	22.0				
200-	GRID	34		-4.0	-10.0	22.0				
201-	GRID	35		4.0	-10.0	22.0				
202-	GRID	36		-4.0	3.0	24.0				
203-	GRID	37		-4.0	-3.0	24.0				
204-	GRID	38		4.0	3.0	24.0				

50 MODES AND FREQUENCIES

S O R T E D   B U L K   D A T A   E C H O

CARD COUNT	. 1 ..	2 ..	3 ..	4 ..	5 ..	6 ..	7 ..	8 ..	9 ..	10 .
205-	GRID	39		4.0	-3.0	24.0				
206-	GRID	40		.0	.0	2.0				
207-	GRID	41		.0	-5.0	1.0				
208-	GRID	42		.0	5.0	.0				
209-	GRID	43		-2.0	.0	.0				
210-	GRID	44		.0	.0	.0				
211-	GRID	45		2.0	.0	.0				
212-	GRID	46		-4.0	-5.0	.0				
213-	GRID	47		4.0	-5.0	.0				
214-	GRID	48		-20.0	.0	.0				
215-	GRID	49		-21.00	.0	.0				
216-	GRID	50		-10.0	.0	.0				
217-	GRID	51		-11.0	.0	.0				
218-	GRID	52		-6.0	.0	.0				
219-	GRID	53		6.0	.0	.0				
220-	GRID	54		11.0	.0	.0				
221-	GRID	55		10.0	.0	.0				
222-	GRID	56		21.00	.0	.0				
223-	GRID	57		20.0	.0	.0				
224-	MAT1	100	1.24E+11		.3					
225-	OMIT1	56	48	50	52	53	55	57		
226-	OMIT1	456	9	10	11	12	14	15	16	
227-	OMIT1	456	17	18	19	27	28	29	30	
228-	OMIT1	456	32	THRU	35					
229-	OMIT1	123456	1	THRU	8					
230-	OMIT1	123456	13	26	31	49	51	54	56	
231-	OMIT1	123456	36	THRU	41					
232-	PARAM	GRDPNT	0							
233-	PBAR	200	100	6.250E-43.095E-63.095E-66.109E-6						
234-	PBAR	300	100	3.133E-41.500E-61.500E-63.110E-6						
235-	PBAR	400	100	3.919E-43.049E-63.049E-66.099E-6						
236-	PBAR	500	100	9.407E-41.874E-51.874E-53.749E-5						
	ENDDATA									

Premapped product of sensor/actuator mapping matrices and modal matrix.

(Each block below is a row in the product matrix. The number of columns equals the number of sensors/actuators. The number of rows equals the number of modes in the model. Here fifty-four modes are given.)

N.B. THE FIRST SIX BLOCKS (ROWS) ARE FOR THE SIX RIGID BODY MODES.

-.315269E-02	.517895E-02	-.138475E-02	-.848555E-03	-.111276E-02
.683988E-02	.274804E-03	-.627424E-02	-.142413E-01	.101214E-01
-.466664E-02	-.142412E-01	-.300683E-02	.386633E-02	.552660E-02
.158803E-02	.426474E-03	-.213272E-02	.272722E-02	-.147629E-02
-.554972E-03				
.462729E-02	-.355801E-03	-.516205E-02	.243005E-02	.811695E-03
-.433856E-02	-.914393E-02	.147711E-01	-.879919E-02	-.193592E-02
-.756388E-02	-.879917E-02	-.115472E-01	-.131719E-02	-.529975E-02
-.818312E-02	.452030E-04	.271952E-02	-.475004E-02	.320011E-02
-.715295E-02				
-.496062E-02	-.201205E-02	-.196797E-02	-.471914E-02	.515882E-02
-.405760E-02	-.401340E-02	-.593003E-02	.524705E-02	-.407961E-02
-.194591E-02	.524705E-02	-.399141E-02	-.200323E-02	-.404874E-02
-.402228E-02	.160550E-02	.991013E-04	-.301276E-02	.946879E-04
-.299068E-02				
-.474338E-02	.754460E-02	.101718E-01	-.157211E-03	-.466108E-02
.332965E-02	.595678E-02	.103287E-01	.594416E-03	.201562E-02
.114861E-01	.594440E-03	.727127E-02	.807005E-02	.385515E-02
.543203E-02	-.105750E-03	-.470226E-02	.675062E-02	-.496501E-02
.806421E-02				
-.160182E-02	-.301424E-02	-.185961E-02	.120376E-01	-.309507E-02
.396111E-03	.155050E-02	.444494E-02	-.786007E-03	-.181398E-03
-.128230E-02	-.786160E-03	.212784E-02	-.278332E-02	.626846E-03
.131966E-02	.111043E-01	-.234845E-02	-.731816E-03	-.246390E-02
-.154559E-03				
-.923363E-02	-.639696E-02	-.281699E-02	-.200268E-02	-.103184E-01
-.667933E-02	-.309936E-02	-.618778E-03	-.315849E-02	-.846932E-02
-.102700E-02	-.315849E-02	-.130938E-02	-.568097E-02	-.596334E-02
-.381536E-02	-.268068E-02	-.977603E-02	-.474816E-02	-.101340E-01
-.295818E-02				
-.675074E-04	.152337E-02	.151056E-02	-.196338E-02	.639319E-04
-.152481E-02	-.151058E-02	.588829E-02	.847191E-04	-.152630E-02
.150731E-02	.838672E-04	-.150657E-02	.152103E-02	-.152188E-02
-.151452E-02	-.188295E-02	-.172713E-05	-.352031E-06	-.131139E-05
.258268E-06				

-.461975E-02	.935707E-03	.791995E-03	-.784817E-02	.461604E-02
-.935799E-03	-.791320E-03	-.102444E-01	.494835E-02	-.104358E-02
.693351E-03	.494942E-02	-.690607E-03	.916870E-03	-.917982E-03
-.806480E-03	-.209309E-02	-.180235E-05	.732430E-06	-.646094E-06
.111913E-05				

.263244E-03	.514215E-02	.517585E-02	-.239311E-05	.263216E-03
.514210E-02	.517585E-02	.334293E-07	.329817E-03	.513579E-02
.519414E-02	.330027E-03	.519422E-02	.515426E-02	.515426E-02
.517286E-02	-.536401E-07	.262004E-03	.515777E-02	.259302E-03
.517541E-02				

-.493235E-02	-.783646E-03	.148636E-02	.108730E-05	-.493470E-02
-.783624E-03	.148747E-02	-.472337E-06	-.389643E-03	-.192450E-02
.262548E-02	-.389543E-03	.262720E-02	-.331138E-03	-.330840E-03
.103421E-02	-.434250E-07	-.493367E-02	.351962E-03	-.515918E-02
.148816E-02				

.279027E-03	.779379E-02	.788086E-02	-.718050E-05	.278830E-03
.779361E-02	.788090E-02	.677495E-07	.449849E-03	.778169E-02
.792822E-02	.450472E-03	.792853E-02	.782726E-02	.782722E-02
.787447E-02	-.189545E-06	.275233E-03	.783367E-02	.268642E-03
.787970E-02				

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-.260616E-02	.454519E-02	-.906189E-05	-.124903E-02	-.623355E-02
.812426E-02	-.124810E-02	.816049E-02	-.121100E-02	-.119006E-02
.312369E-02	.521106E-04	-.155719E-01	.960752E-03	-.162719E-01
.454232E-02				

.380725E-03	.345091E-02	.354388E-02	-.170481E-01	-.479175E-03
-.346788E-02	-.351501E-02	.152819E-02	-.599366E-03	-.344473E-02
.360952E-02	-.600439E-03	-.355857E-02	.347343E-02	-.348078E-02
-.351878E-02	-.175764E-01	-.493670E-04	.269304E-05	-.522509E-04
.140472E-04				

-.606722E-04	.813332E-04	.147687E-03	.485137E-03	.606673E-04
-.814476E-04	-.147368E-03	-.201893E-03	-.623802E-04	-.614378E-04
.182670E-03	-.594099E-04	-.182640E-03	.926131E-04	-.936193E-04
-.136635E-03	.569513E-03	-.223572E-06	-.777539E-07	-.215776E-05
-.895722E-06				

-.104122E-02	-.591641E-03	.410506E-03	-.290310E-02	.104141E-02
.588150E-03	-.407276E-03	.285652E-02	-.123112E-02	.135586E-02
.110052E-02	-.124178E-02	-.110656E-02	-.461495E-03	.467948E-03
-.303508E-03	-.148712E-02	-.945050E-06	-.427879E-05	-.113570E-04
-.493212E-05				

-.130713E-02	.522738E-03	.145587E-02	.126968E-04	-.130948E-02
.522065E-03	.145579E-02	-.151901E-05	.752184E-03	-.486307E-04
.205388E-02	.754158E-03	.205523E-02	.685619E-03	.685142E-03
.131358E-02	-.129328E-06	-.130303E-02	.991604E-03	-.140079E-02
.144910E-02				

-.100571E-01	-.625669E-02	-.142280E-03	-.150481E-01	.100722E-01
.624191E-02	.163093E-03	.954304E-02	-.359604E-02	.111830E-01
.397951E-02	-.368026E-02	-.402034E-02	-.553053E-02	.558021E-02
.872781E-03	-.180601E-02	-.139329E-05	-.250137E-04	-.716567E-04
-.342586E-04				

-.403904E-04	-.836498E-04	-.712110E-04	-.547136E-06	-.391138E-04
-.823301E-04	-.706823E-04	.798046E-06	-.190752E-04	-.922450E-04
-.671748E-04	-.191900E-04	-.672465E-04	-.836282E-04	-.823625E-04
-.754719E-04	-.374604E-06	-.393685E-04	-.763816E-04	-.402739E-04
-.701908E-04				

-.194436E-03	-.102196E-04	.800289E-04	-.752618E-05	-.186437E-03
-.352121E-05	.818227E-04	.442890E-05	-.112822E-04	-.512271E-04
.131915E-03	-.111690E-04	.130320E-03	.621622E-05	.125484E-04
.665097E-04	-.166268E-05	-.190686E-03	.372518E-04	-.197742E-03
.821806E-04				

.790560E-08	-.183191E-07	-.222229E-07	-.387317E-08	-.566197E-08
.230921E-07	.262764E-07	.106553E-08	.367022E-08	.227697E-07
-.268811E-07	.344111E-08	.307527E-07	-.192742E-07	.241328E-07
.272668E-07	-.129133E-07	.112725E-08	.219716E-08	.128728E-08
.207441E-08				

-.359137E-02	.110446E-01	.129350E-01	.207808E-02	.361521E-02
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.153511E-01	-.154157E-02	-.153706E-01	.115545E-01	-.116099E-01
-.133859E-01	.689066E-02	.297970E-05	-.560106E-05	-.684433E-04
-.337446E-04				

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-.706913E-03	-.269481E-03	-.706448E-03	-.215413E-03	-.216107E-03
-.435593E-03	-.483766E-06	.396178E-03	-.313669E-03	.422012E-03
-.379977E-03				

.909885E-03	.716368E-04	-.355725E-03	.689764E-03	-.914633E-03
-.712788E-04	.354204E-03	-.696177E-04	.458267E-04	-.429031E-03
-.696288E-03	.484372E-04	.701136E-03	.261782E-04	-.275099E-04
.325626E-03	-.491253E-03	-.456432E-06	.152662E-05	.101552E-04
.553053E-05				

-.118932E-03	.379430E-03	.391920E-03	-.523880E-04	.119389E-03
-.380480E-03	-.391738E-03	.472439E-04	-.448142E-04	-.391549E-03
.507984E-03	-.420348E-04	-.508923E-03	.396334E-03	-.398301E-03
-.449199E-03	.102954E-03	.270960E-07	-.311648E-06	-.127353E-05
-.436344E-06				

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.589546E-10	.232612E-10	-.883301E-12	.374241E-10	-.196689E-10
.120012E-09	.391314E-10	.119866E-09	.282835E-10	.285883E-10
.674301E-10	.229694E-12	-.511310E-10	.423983E-10	-.517914E-10
.178712E-10				

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.716573E-04	.235031E-04	.723634E-04	.170467E-04	.171069E-04
.405964E-04	.120033E-06	-.307033E-04	.254518E-04	-.310967E-04
.107336E-04				

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-.285738E-08	-.112267E-08	.595835E-10	-.183449E-08	.879504E-09
-.594557E-08	-.191492E-08	-.579802E-08	-.137370E-08	-.141055E-08
-.328203E-08	-.148776E-10	.250274E-08	-.207669E-08	.253572E-08
-.874981E-09				

-.151482E-02	-.788804E-02	-.434479E-02	-.400577E-02	.128025E-02
.733758E-02	.419156E-02	.416830E-02	-.552042E-03	-.212511E-01
-.180890E-01	.250737E-03	.179648E-01	.285214E-02	-.276292E-02
.492848E-02	-.894667E-03	-.119657E-03	-.254803E-03	.390807E-04
.345359E-04				

.471273E-04	.185191E-03	-.165577E-04	.806940E-05	.474279E-04
.193540E-03	-.146923E-04	.124759E-05	-.327388E-04	-.273739E-03
.244052E-03	-.238594E-04	.262235E-03	-.189090E-04	-.230104E-04
.814419E-04	-.519109E-06	.562892E-04	.783117E-04	.768187E-04
-.174910E-04				

-.603767E-02	-.108939E-01	-.150046E-02	.490010E-04	-.597151E-02
-.111844E-01	-.152992E-02	-.190504E-03	.418424E-02	.207360E-01
-.143947E-01	.363771E-02	-.151189E-01	.260540E-02	.285659E-02
-.336751E-02	.431568E-04	-.640327E-02	-.558349E-02	-.779453E-02
-.196951E-02				

-.251762E-04	-.361694E-04	-.400639E-05	.273338E-06	-.256506E-04
-.375830E-04	-.394272E-05	-.676113E-06	.122041E-04	.602995E-04
-.432253E-04	.104030E-04	-.457333E-04	.588253E-05	.668737E-05
-.108560E-04	-.240884E-06	-.266366E-04	-.175571E-04	-.299465E-04
-.402984E-05				

.254078E-05	-.579625E-05	-.653088E-06	.887257E-06	.298614E-05
-.526842E-05	-.556410E-06	.151495E-06	.436827E-06	.256420E-05
-.123658E-04	.739684E-07	-.117262E-04	-.265411E-05	-.274132E-05
-.651099E-05	.217392E-06	.320805E-05	-.238516E-05	.173710E-05
-.833632E-06				

-.556024E-04	-.480965E-04	.122885E-04	-.434313E-04	.586904E-04
.516701E-04	-.152390E-04	-.864821E-05	-.564735E-05	-.121984E-04
-.578764E-05	-.621297E-05	.734617E-05	-.267805E-05	.130638E-05
-.851884E-05	.291648E-04	.260441E-06	-.665818E-06	-.357343E-05
-.403731E-05				

-.537168E-03	.520850E-03	.233605E-02	.501840E-03	.343037E-03
-.311124E-02	.196102E-02	.114088E-01	.158505E-02	.206397E-02
-.955181E-03	.511415E-02	-.725271E-03	.987384E-03	.877781E-03
-.151061E-02	.704593E-03	.204818E-04	.843954E-03	-.237563E-03
.244244E-02				

.226417E-03	-.632991E-02	.160226E-01	.693207E-03	-.403311E-03
-.429029E-02	.161342E-01	-.191693E-02	-.299728E-02	.155110E-02
-.522245E-02	-.397380E-02	-.606902E-02	-.281601E-02	-.256504E-02
-.106591E-01	-.229962E-03	.981327E-03	.852733E-02	-.993008E-06
.187453E-01				

-.637010E-04	-.283002E-03	.107939E-02	.142278E-04	.798854E-04
.255410E-03	-.950240E-03	-.436772E-04	.489813E-04	-.219371E-03
-.410383E-03	.423990E-04	.349771E-03	-.219148E-03	.179056E-03
.733537E-03	.280058E-03	.126739E-04	.334732E-04	-.250635E-04
.464027E-04				

-.119527E-02	-.257750E-02	.141724E-01	.111884E-03	.156016E-02
.161757E-02	-.114949E-01	-.249733E-03	.211875E-02	-.362017E-02
-.463685E-02	.174024E-02	.343649E-02	-.296028E-02	.227946E-02
.104078E-01	.398177E-02	.249900E-03	.557278E-03	-.319956E-03
.109491E-02				

-.736339E-03	.509146E-02	-.267495E-02	-.113467E-02	.217571E-02
-.111604E-01	.107226E-01	.262806E-02	.130227E-01	-.196305E-02
-.540285E-02	.113787E-01	.221183E-02	.188029E-03	-.939477E-03
.279214E-02	-.854186E-03	.652236E-03	.506370E-03	.197179E-03
.405806E-02				

-.195628E-02	.105146E-01	-.107018E-01	.882051E-03	-.149115E-02
.961143E-02	-.110945E-01	.193793E-03	.199362E-02	-.367990E-02
.101007E-01	.320126E-02	.154420E-01	-.257297E-02	-.963129E-03
-.193304E-02	-.266018E-03	-.178052E-06	.232023E-02	.287037E-02
-.825016E-02				

-.164774E-02	.745523E-02	-.105676E-01	.434937E-02	.170984E-02
-.579112E-02	.101516E-01	.248202E-03	.248690E-02	.220379E-02
.159181E-01	.603854E-03	-.155062E-01	-.120381E-05	.274313E-03
-.271237E-03	.360392E-02	.452517E-03	.716848E-03	.566144E-03
-.113763E-04				

.177836E-02	.128502E-02	.877239E-02	.275248E-03	.192555E-02
-.427954E-02	.562841E-02	.676451E-03	-.324161E-02	-.763846E-02
-.165200E-01	-.448668E-03	-.776167E-02	.107322E-01	.392012E-02
.874318E-02	-.133546E-03	.242571E-02	.345981E-02	.164313E-02
.738573E-02				

.240188E-02	-.748250E-02	.756737E-02	.498625E-03	.209599E-02
.291031E-02	.904949E-02	-.548351E-03	-.420382E-03	-.147144E-01
-.897553E-02	-.368983E-02	-.193782E-01	.241906E-02	.920693E-02
.527305E-02	-.135693E-03	.242902E-02	.267954E-02	.930594E-03
.761995E-02				

-.184702E-02	-.154987E-02	-.119169E-01	-.672602E-03	.962223E-03
.430191E-02	.105187E-01	.112567E-03	-.222609E-02	-.124927E-01
.262082E-01	-.198301E-02	-.272186E-01	-.917448E-02	.862136E-02
.655111E-02	-.707292E-03	-.504753E-03	.494954E-03	-.347785E-03
-.582496E-03				

-.552044E-02	.228056E-01	-.208466E-02	-.401412E-03	-.509181E-02
.186123E-01	-.137768E-02	.678153E-04	.528961E-02	-.143599E-02
-.290120E-01	.280986E-02	-.240259E-01	.235477E-02	.105350E-02
-.660742E-02	-.955036E-04	-.640794E-02	.958732E-02	-.466349E-02
-.426498E-03				

-.192568E-02	.209248E-01	-.807090E-02	-.288086E-02	.252983E-02
-.242720E-01	.909374E-02	.109431E-02	-.824936E-02	-.166072E-02
-.129665E-01	-.638068E-02	.146887E-01	-.262354E-02	.313816E-02
.784463E-02	-.531398E-03	.341206E-03	-.636178E-03	.952803E-03
.967373E-03				

-.103436E-02	-.406277E-02	-.566725E-02	-.650086E-03	-.190237E-02
-.828580E-02	-.124772E-01	-.281084E-03	-.588435E-02	-.455617E-02
.252619E-03	.761620E-02	.149419E-02	.148808E-01	.354281E-01
-.550145E-02	.180726E-04	-.175386E-02	-.704243E-02	-.484477E-03
-.863932E-02				

.154460E-02	.835839E-02	.149743E-01	.130651E-02	.345791E-03
-.264185E-02	-.443142E-02	-.611294E-02	-.426798E-02	-.137994E-02
-.184353E-02	.429431E-02	.102076E-02	-.338997E-01	.127035E-01
-.281911E-02	-.554804E-04	.856156E-03	.349911E-02	.164146E-03
.504835E-02				

.142008E-03	.224142E-02	.472764E-02	.689370E-03	.114589E-03
-.378594E-03	-.323057E-02	.296670E-01	.144188E-02	-.349620E-03
.228201E-02	.492052E-02	-.916499E-03	-.698678E-02	.513412E-02
.416381E-03	-.170479E-03	-.901519E-06	.912994E-03	-.140085E-03
.822853E-03				

.124063E-02	.174579E-02	.779892E-02	.701055E-03	.253186E-02
.318422E-02	.780943E-02	.168804E-02	-.380284E-02	-.107403E-02
.127346E-01	-.498741E-03	.757852E-02	.995247E-02	-.305264E-03
-.158982E-01	-.383383E-03	.995313E-03	.384417E-02	-.905353E-03
.648640E-02				

.114790E-07	.160498E-06	.982832E-07	-.449983E-08	.632603E-07
-.876380E-07	.203879E-06	.359133E-07	-.121692E-06	-.303988E-07
.162093E-06	-.480372E-07	.233667E-06	.172712E-06	.133042E-07
-.252357E-06	-.104950E-07	.207673E-07	.690105E-07	-.114697E-07
.128938E-06				

.605729E-02	.219745E-02	.486531E-02	-.416147E-02	-.842852E-02
-.529524E-02	-.124708E-01	-.499496E-02	.685893E-02	.206975E-02
.830728E-02	-.413295E-02	-.111182E-01	.384558E-02	-.821254E-03
.256678E-01	-.281944E-02	-.802565E-03	-.213810E-02	.154408E-02
-.186268E-02				

-.133540E-01	-.118397E-02	.397182E-02	.115957E-01	.168347E-01
-.140901E-03	-.618315E-02	-.477920E-02	.338930E-02	-.469804E-03
-.158237E-03	-.346920E-02	.129432E-02	.160813E-02	.116392E-02
.516107E-02	-.143190E-02	-.290321E-03	-.281857E-03	-.202135E-02
-.209307E-02				

.419823E-02	.300807E-02	.455697E-02	-.272211E-02	-.306943E-02
-.378098E-03	.280769E-02	-.102818E-01	.869922E-02	-.204967E-02
.531960E-02	-.895417E-02	.537789E-02	.165158E-03	.789054E-02
-.144181E-01	-.144233E-04	.212682E-03	.179038E-02	-.361545E-03
.333217E-02				

.100376E-02	.308717E-02	.784317E-02	.956176E-03	.688409E-03
-.546498E-03	-.222561E-02	.365427E-02	-.376257E-02	.979819E-03
.328094E-02	.541218E-02	-.358549E-02	.769268E-03	-.389560E-02
.767619E-02	-.183726E-02	.191140E-03	.146357E-02	-.137414E-03
.295946E-02				

Appendix B

Guide to Input Data

and Program Source Code Listing

Guide to Input Data

Card or Cards ([ ])	Range of Values	Remarks
DEC	3,4	Number of Controllers
NC1,NC2,NC3,NR NACT,NSEN,ZETA	N/A	Numbers of modes asgn'd, Number sensors, actuators, zeta
[PHIA]	N/A	Premapped modal-sensor matrices*
[PHIS]	N/A	Premapped modal-actuator mat.*
[W(I)]	N/A	Modal frequencies* (asc. order)
IC1(I)	N/A	Modes assigned to Ctlr 1
IC2(I)	N/A	Modes assigned to Ctlr 2
IC3(I)	N/A	Modes assigned to Ctlr 3
IR(I)	N/A	Modes assigned to 4th Group
Q	0,1	0≙do not print A,B,C,Q matrices
BB(I)	N/A	Q matrix diagonal elements
SKIP	1,2,3	1≙TR+EA, 2≙EA, 3≙TR **
DUMMY	1,2,3	Conditional SKIP***
[INIT(I,J)]	N/A	Initial conditions state vector*
DT,TMAX,PDT	N/A	TR: t, t_{max}, output counter
[PHIL(J,I)]	N/A	Φ_{LOS} transposed*
DUMMY	1,2,3	Conditional SKIP***

* See the code for how the data should be formatted.

** TR stands for time response; EA for eigenvalue analysis.

*** Covers all possible cases (see code for implied meanings).

Case no.:	(1)	(2)	(3)	(4)
1st pass:	EA	TR(TR+EA)	TR(TR+EA)	TR
2nd pass:	TR(TR+EA)	TR(TR+EA)	EA	EA

```
      PROGRAM ACOSS2(INPUT,OUTPUT,TAPE8,TAPE6=OUTPUT,TAPE7)
C
C     PFN:  CSDLLOWERTIME
C  THIS PROGRAM GENERATES A LOWER TRIANGULAR TRANSFORMATION
C  THE THREE CONTROLLER SOLUTION WILL INCLUDE RESIDUALS.
C
      REAL A1(21,21),A2(21,21),A3(21,21),A4(21,21)
      REAL B1(21,21),B2(21,21),B3(21,21),B4(21,21)
      REAL C1(21,21),C2(21,21),C3(21,21),C4(21,21)
      REAL CTCC1(21,21),CTCC2(21,21),CTCC3(21,21),CTCC4(21,21)
      REAL SAT(21,21),SAT2(21,21),SAT3(21,21),SAT4(21,21)
      REAL AKC(21,21),ACT(21,21),BCG(21,21),KCC(21,21)
      REAL P(21,21),S(21,21)
      REAL QA1(21,21),QA2(21,21),QA3(21,21),QA4(21,21)
      REAL ACG1(21,21),ACG2(21,21),ACG3(21,21),ACG4(21,21)
      REAL ABG1(21,21),ABG2(21,21),ABG3(21,21),ABG4(21,21)
      REAL GAIN1(21,21),GAIN2(21,21),GAIN3(21,21),GAIN4(21,21)
      REAL KT1(21,21),KT2(21,21),KT3(21,21),KT4(21,21)
      REAL KOB1(21,21),KOB2(21,21),KOB3(21,21),KOB4(21,21)
      REAL GAMMA1(21,21),GAMMA2(21,21),GAMMA3(21,21)
      REAL T2(21,21),T3(21,21),T4(21,21),BB(21)
      REAL TRT(21,21),TEN(21,21),CT(21,21),V(21,21)
      REAL RK(21,21),RK1(21,21),RK2(21,21),RK3(21,21)
      REAL RG2(21,21),RG3(21,21),RG4(21,21)
      REAL D(21),W(21),TOL,DT,ABT,TMAX
      REAL CV(3,21),XL(3)
      REAL ZETA,CLOS(3,21),SING(21),XTR(21,21),STOR(21,21)
      REAL PHIA(21,21),PHIS(21,21),MODE(2,21),INIT(4,21)
      REAL PHIL(3,21),RPHIL(3,21)
      REAL MAJM(64,64),STM(64,64),WORK(64,64),XHAT(3,64)
      REAL X0(64),X1(64),XC(64),PETA(21,64),CPHIL(3,64)
      INTEGER N,N2,NC1,NC2,NC3,NC12,NC22,NC32,NR,NR2,NMODE
      INTEGER IC1(21),IC2(21),IC3(21),I,J,K,L,M,KK,LL,MM,PDT
      INTEGER DEC,Q,NACT,NSEN,IR(21),IER,SKIP,NCOL,NCOL1
      INTEGER NDA,NDIM,NDA1,NDIM1,ZZ,E2,E3,E4,P1,P2,P3,AA
      COMPLEX Z(64),W1(21)
      COMMON/MAIN1/NDIM,NDIM1,TEN,X(4096)
      COMMON/MAINA/NDA,NDA1
      COMMON/MAINB/NCOL,NCOL1
      COMMON/MAIN2/STOR
      COMMON/MAIN3/XTR
      COMMON/SAVE/T(100),TS(100)
      COMMON/INOUT/KOUT,TAPE
      COMMON/NUM/IC1,IC2,IC3,IR,NC1,NC2,NC3,NR
C
C
C  INITIALIZATIONS AND COMMENTS ON DIMENSIONING
C
C  NDIM IS THE ACTUAL NUMBER OF MODES (ALSO REP'D BY N) FOR THIS VERSION.
      NDIM = 20
      NDIM1 = NDIM + 1
C  NCOL IS THE MAX NUMBER OF MODES ENVISIONED AS POSSIBLE TO RUN
C  IN ANY CONFIGURATION OF THE 4 CONTROLLERS.  FOR THIS VERSION ASSIGN
```

```
C  NOT LESS THAN 1 NOR MORE THAN 10 MODES TO ANY OF THE 1ST 3 CTLRS.
C  THE A,B,C MATRICES NEED DIM >= 2 * # MODES ASGN'D TO THE INDIV CTLR.
C  SPECIFYING THIS PARAM ALLOWS DIMENSIONING OF THE INDEXED VAR'S
C  TO A STD NUMBER AND AIDS USING THE IMSL ROUTINES.
      NCOL = 21
      NCOL1 = NCOL + 1
C  NDA IS THE MAX NUMBER OF STATES, = 4 * # MODES (ETAS) (= N ETAS +
C  N ETA DOTS + N ERR + N ERR DOTS)   NDA IS BASED ON NDIM, NOT NCOL,
C  BECAUSE IT REPRESENTS 4 * THE ACTUAL NO. OF STATES, AND IS
C  MEANT TO SET UP SPACE FOR 'MM', THE ACTUAL NO. OF STATES.  MM'S
C  VALUE DEPENDS ON WHETHER THERE ARE RESIDUALS (IN PLACE OF A 4TH
C  CONTROLLER) OR NOT (3 OR 4 ACTIVE CONTROLLERS).  IF THE NO.
C  OF RESIDUALS = 0, THEN MM = NDA, ELSE MM = 4 * # CTLD MODES +
C  2 * # RES MODES.  NOTE:  REDUNDANTLY, THIS VERSION CONTAINS
C  THE EQUIVALENT TERMS 'MM' AND 'M'.
C  WARNING:  UNTIL SOMEONE DEVELOPS A SUBSTITUTE FOR SUBROUTINES 'MEXP,'
C  'MMUL,' AND 'MMUL1,' ETC.
C  IT WILL BE NECESSARY TO MAKE NDA AND ITS ASSOCIATED DIMENSIONS IN THE
C  MAIN DIMENSION STATEMENTS EXACTLY THE SAME AS MM,
C  AND RECOMPILE WHENEVER THE VALUE OF 'MM' CHANGES ACCORDING TO NEW
C  CONTROLLER ASSIGNMENTS.  MAJM IS INPUT TO MEXP, A KLEINMAN ROUTINE,
C  WHICH REQUIRES THE MATRIX TO HAVE EXACT DIMENSIONS, IN CONTRAST TO
C  THE IMSL ROUTINES WHICH ALLOW THE USER TO TELL THEM HOW A MATRIX IS
C  ACTUALLY DIMENSIONED RELATIVE TO THE SPACE IT WAS GIVEN IN THE
C  DIMENSION STATEMENT IN THE MAIN.
CTEMP****    NDA MUST EQUAL MM;  THIS VERSION EXPECTS 20 MODES WITH 8 OF
CTEMP****    THEM ASSIGNED TO CTLR # 4 WITH 'DEC' = 3.  THEREFORE, MM=64.
      NDA = 64
      NDA1 = NDA + 1
      KOUT = 6
      TAPE = 9
      Q = 0
      IER = 0
C  ZZ IS THE FLAG DRIVING THE SUPPRESSED AND UNSUPPRESSED PASSES
      ZZ = 0
C  AA IS THE FLAG DRIVING THE SINGLE READ-IN OF INITIAL CONDITIONS
      AA = 0
C
C
      PRINT'(////)'
      PRINT*,'     ********************************************* '
      PRINT*,'     *****                                   ***** '
      PRINT*,'     *****   3 OR 4 C O N T R O L L E R      ***** '
      PRINT*,'     *****    ( O B S E R V E R )            ***** '
      PRINT*,'     *****   L O W E R - R E S I D U A L     ***** '
      PRINT*,'     *****            B L O C K              ***** '
      PRINT*,'     *****          C S D L   I I            ***** '
      PRINT*,'     *****                                   ***** '
      PRINT*,'     ********************************************* '
      PRINT'(//)'
      PRINT*,'  THIS PROGRAM GENERATES A SOLUTION',
     *       ' USING A LOWER TRIANGULAR TRANSFORMATION '
      PRINT'(////)'
```

```
C
C  THIS CODE ALLOWS ASSIGNMENT OF THE MODES TO EITHER
C  ANY OF 3 CTLRS PLUS 1 RESID MODE SET        OR
C  ANY OF 4 CTLRS
C
C
C  INITIAL SELECTION FOR THREE OR FOUR CONTROLLERS
C
C
      PRINT*,' FOR A THREE CONTROLLER RUN, ENTER 3, OR, '
      PRINT*,' FOR A FOUR  CONTROLLER RUN, ENTER 4     >'
      READ(8,*) DEC
C
C  DEC DEFAULT SWITCH
C
      IF (DEC.NE.4) DEC = 3
      PRINT*,' '
      PRINT*,'    THIS IS A ',DEC,' CONTROLLER RUN '
C
C
C  PHI MATRICES AND CONTROLLER ENTRIES
C
C
C
      PRINT'(///)'
      IF (DEC.EQ.3) THEN
      PRINT*,' ENTER NC1,NC2,NC3,NR,NACT,NSEN,ZETA  >'
      ELSE
      PRINT*,' ENTER NC1,NC2,NC3,NC4,NACT,NSEN,ZETA  >'
      ENDIF
      READ(8,*) NC1,NC2,NC3,NR,NACT,NSEN,ZETA
      PRINT*,NC1,NC2,NC3,NR,NACT,NSEN,ZETA
      PRINT*,' '
C
C  NOW N IS CALCULATED TO EQUAL THE NUMBER OF MODES IN THE MODEL (=NDIM)
      N = NC1 + NC2 + NC3 + NR
C
      PRINT*,' ENTER THE ',NACT,' ELEMENTS FOR EACH PHIA '
      PRINT*,' '
      DO 1 I=1,N
      PRINT*,'ENTER PHIA ',I,' >'
      READ(8,*) (PHIA(I,J),J=1,NACT)
      PRINT*,'                 ',(PHIA(I,J),J=1,NACT)
 1    CONTINUE
      PRINT'(//)'
      PRINT*,' ENTER THE ',NSEN,' ELEMENTS FOR EACH PHIS '
      PRINT*,' '
      DO 2 I=1,N
      PRINT*,'ENTER PHIS ',I,' >'
      READ(8,*) (PHIS(I,J),J=1,NSEN)
      PRINT*,'                 ',(PHIS(I,J),J=1,NSEN)
 2    CONTINUE
      PRINT'(//)'
```

```
C
C
C  OMEGAS
C
C
      PRINT*, ' ENTER THE VALUE FOR EACH OMEGA '
      PRINT*,' '
      DO 4 I=1,N
      PRINT*,'ENTER OMEGA ',I,' >'
      READ(8,*) W(I)
      PRINT*,'                  ',W(I)
      D(I) = -2. * ZETA * W(I)
 4    CONTINUE
      PRINT'(/)'
C
C  MODE ASSIGNMENT TO CONTROLLERS
C
C
      PRINT'(///)'
      PRINT*,' THE FOLLOWING MODES ARE ENTERED ACCORDING TO THE '
      PRINT*,' ORDER IN WHICH THEY ARE ENTERED IN THE DATA FILE '
      PRINT*,' AND NOT ACCORDING TO THEIR ACTUAL MODE NUMBER. '
      PRINT'(//)'
      PRINT*,' ENTER THE ',NC1,' CONTROLLER 1 MODES >'
      READ(8,*) (IC1(I),I=1,NC1)
      PRINT*,'     ',(IC1(I),I=1,NC1)
      PRINT*,' '
      PRINT*,' ENTER THE ',NC2,' CONTROLLER 2 MODES >'
      READ(8,*) (IC2(I),I=1,NC2)
      PRINT*,'     ',(IC2(I),I=1,NC2)
      PRINT*,' '
      PRINT*,' ENTER THE ',NC3,' CONTROLLER 3 MODES >'
      READ(8,*) (IC3(I),I=1,NC3)
      PRINT*,'     ',(IC3(I),I=1,NC3)
      PRINT*,' '
      IF (DEC.EQ.3) THEN
      PRINT*,' ENTER THE ',NR,' RESIDUAL MODES >'
      ELSE
      PRINT*,' ENTER THE ',NR,' CONTROLLER 4 MODES >'
      ENDIF
      READ(8,*) (IR(I),I=1,NR)
      PRINT*,'     ',(IR(I),I=1,NR)
      PRINT*,' '
C
C
      NC12 = 2 * NC1
      NC22 = 2 * NC2
      NC32 = 2 * NC3
      N2 = 2 * N
      NR2 = 2 * NR
C  M IS THE ACTUAL NUMBER OF STATES (VALUE DEPENDS ON HAVING RESID MODES)
      IF (DEC.EQ.3) THEN
      M = 2 * NC12 + 2 * NC22 + 2 * NC32 + NR2
```

```
      CALL PRNT(C1,NSEN,NC12)
      PRINT*,' THE CONTROLLER 1 WEIGHTING MATRIX IS '
      CALL PRNT(QA1,NC12,NC12)
      PRINT*,' THE CONTROLLER 2 A MATRIX IS '
      CALL PRNT(A2,NC22,NC22)
      PRINT*,' THE CONTROLLER 2 B MATRIX IS '
      CALL PRNT(B2,NC22,NACT)
      PRINT*,' THE CONTROLLER 2 C MATRIX IS '
      CALL PRNT(C2,NSEN,NC22)
      PRINT*,' THE CONTROLLER 2 WEIGHTING MATRIX IS '
      CALL PRNT(QA2,NC22,NC22)
      PRINT*,' THE CONTROLLER 3 A MATRIX IS '
      CALL PRNT(A3,NC32,NC32)
      PRINT*,' THE CONTROLLER 3 B MATRIX IS '
      CALL PRNT(B3,NC32,NACT)
      PRINT*,' THE CONTROLLER 3 C MATRIX IS '
      CALL PRNT(C3,NSEN,NC32)
      PRINT*,' THE CONTROLLER 3 WEIGHTING MATRIX IS '
      CALL PRNT(QA3,NC32,NC32)
C
      IF (NR.EQ.0) THEN
      PRINT*,'  NO RESIDUAL TERMS '
      GOTO 115
      ENDIF
C
      IF (DEC.EQ.3) THEN
      PRINT*,' THE A RESIDUAL MATRIX IS '
      CALL PRNT(A4,NR2,NR2)
      PRINT*,' THE B RESIDUAL MATRIX IS '
      CALL PRNT(B4,NR2,NACT)
      PRINT*,' THE C RESIDUAL MATRIX IS '
      CALL PRNT(C4,NSEN,NR2)
      ELSE
      PRINT*,' THE CONTROLLER 4 A MATRIX IS '
      CALL PRNT(A4,NR2,NR2)
      PRINT*,' THE CONTROLLER 4 B MATRIX IS '
      CALL PRNT(B4,NR2,NACT)
      PRINT*,' THE CONTROLLER 4 C MATRIX IS '
      CALL PRNT(C4,NSEN,NR2)
      PRINT*,' THE CONTROLLER 4 WEIGHTING MATRIX IS '
      CALL PRNT(QA4,NR2,NR2)
      ENDIF
C
      ENDIF
C
 115  CONTINUE
C
C
C THIS SECTION GENERATES THE RICCATI SOLUTIONS
C AND THE GAIN MATRICES OF EACH CONTROLLER
C
C
      CALL VMULFP(B1,B1,NC12,NACT,NC12,NCOL,NCOL,SAT,NCOL,IER)
```

```
      IF (ZZ.EQ.0) THEN
      CALL VMULFM(C1,C1,NSEN,NC12,NC12,NCOL,NCOL,CTCC1,NCOL,IER)
      ENDIF
  120 CONTINUE
      IER = 0
      TOL = 0.001
      PRINT'(//)'
      CALL MRIC(NC12,A1,SAT,QA1,S,ABG1,TOL,IER)
      IF (ZZ.EQ.0) THEN
      PRINT*,' THE RICCATI SOLUTION OF AC + BCG #1 IS '
      PRINT*,' IER = ',IER
      CALL PRNT(S,NC12,NC12)
      ENDIF
      CALL VMULFM(B1,S,NC12,NACT,NC12,NCOL,NCOL,GAIN1,NCOL,IER)
      PRINT*,' THE G1 GAIN MATRIX IS '
      CALL PRNT(GAIN1,NACT,NC12)
      IER = 0
      TOL = 0.001
      CALL TFR(ACT,A1,NC12,NC12,1,2)
      PRINT'(//)'
      CALL MRIC(NC12,ACT,CTCC1,QA1,P,ACG1,TOL,IER)
      IF (ZZ.EQ.0) THEN
      PRINT*,' THE RICCATI SOLUTION OF AC - KCC #1 IS '
      CALL PRNT(P,NC12,NC12)
      ENDIF
      CALL MMUL(C1,P,NSEN,NC12,NC12,KT1)
      IF (ZZ.EQ.1) THEN
      CALL VMULFP(RK1,GAMMA1,P1,P1,NSEN,NCOL,NCOL,STOR,NCOL,IER)
      CALL MMUL(STOR,KT1,P1,NSEN,NC12,KCC)
      CALL MMUL(GAMMA1,KCC,NSEN,P1,NC12,KT1)
      PRINT*,' THE K1* GAIN MATRIX IS '
      ELSE
      PRINT*,' THE K1 GAIN MATRIX IS '
      ENDIF
      CALL TFR(KOB1,KT1,NSEN,NC12,1,2)
      CALL PRNT(KOB1,NC12,NSEN)
  125 CONTINUE
      IF (ZZ.EQ.0) THEN
      CALL VMULFP(B2,B2,NC22,NACT,NC22,NCOL,NCOL,SAT2,NCOL,IER)
      ENDIF
      IF (ZZ.EQ.0) THEN
      CALL VMULFM(C2,C2,NSEN,NC22,NC22,NCOL,NCOL,CTCC2,NCOL,IER)
      ENDIF
  140 CONTINUE
      IER = 0
      TOL = 0.001
      PRINT'(//)'
      CALL MRIC(NC22,A2,SAT2,QA2,S,ABG2,TOL,IER)
      IF (ZZ.EQ.0) THEN
      PRINT*,' THE RICCATI SOLUTION OF AC + BCG #2 IS '
      PRINT*,' IER = ',IER
      CALL PRNT(S,NC22,NC22)
      ENDIF
```

```
      CALL VMULFM(B2,S,NC22,NACT,NC22,NCOL,NCOL,GAIN2,NCOL,IER)
      IF (ZZ.EQ.1) THEN
      CALL VMULFM(T2,GAIN2,NACT,E2,NC22,NCOL,NCOL,STOR,NCOL,IER)
      CALL MMUL(RG2,STOR,E2,E2,NC22,TEN)
      CALL MMUL(T2,TEN,NACT,E2,NC22,GAIN2)
      PRINT*,' THE G2* GAIN MATRIX IS '
      ELSE
      PRINT*,' THE G2 GAIN MATRIX IS '
      ENDIF
      CALL PRNT(GAIN2,NACT,NC22)
      IER = 0
      TOL = 0.001
      CALL TFR(ACT,A2,NC22,NC22,1,2)
      PRINT'(//)'
      CALL MRIC(NC22,ACT,CTCC2,QA2,P,ACG2,TOL,IER)
      IF (ZZ.EQ.0) THEN
      PRINT*,' THE RICCATI SOLUTION OF AC - KCC #2 IS '
      CALL PRNT(P,NC22,NC22)
      ENDIF
      CALL MMUL(C2,P,NSEN,NC22,NC22,KT2)
      IF (ZZ.EQ.1) THEN
      CALL VMULFP(RK2,GAMMA2,P2,P2,NSEN,NCOL,NCOL,STOR,NCOL,IER)
      CALL MMUL(STOR,KT2,P2,NSEN,NC22,KCC)
      CALL MMUL(GAMMA2,KCC,NSEN,P2,NC22,KT2)
      PRINT*,' THE K2* GAIN MATRIX IS '
      ELSE
      PRINT*,' THE K2 GAIN MATRIX IS '
      ENDIF
      CALL TFR(KOB2,KT2,NSEN,NC22,1,2)
      CALL PRNT(KOB2,NC22,NSEN)
 145  CONTINUE
      IF (ZZ.EQ.0) THEN
      CALL VMULFP(B3,B3,NC32,NACT,NC32,NCOL,NCOL,SAT3,NCOL,IER)
      ENDIF
      IF (ZZ.EQ.0.OR.DEC.EQ.3) THEN
      CALL VMULFM(C3,C3,NSEN,NC32,NC32,NCOL,NCOL,CTCC3,NCOL,IER)
      ENDIF
 150  CONTINUE
      IER = 0
      TOL = 0.001
      PRINT'(//)'
      CALL MRIC(NC32,A3,SAT3,QA3,S,ABG3,TOL,IER)
      IF (ZZ.EQ.0) THEN
      PRINT*,' THE RICCATI SOLUTION OF AC + BCG #3 IS '
      PRINT*,' IER = ',IER
      CALL PRNT(S,NC32,NC32)
      ENDIF
      CALL VMULFM(B3,S,NC32,NACT,NC32,NCOL,NCOL,GAIN3,NCOL,IER)
      IF (ZZ.EQ.1) THEN
      CALL VMULFM(T3,GAIN3,NACT,E3,NC32,NCOL,NCOL,STOR,NCOL,IER)
      CALL MMUL(RG3,STOR,E3,E3,NC32,TEN)
      CALL MMUL(T3,TEN,NACT,E3,NC32,GAIN3)
      PRINT*,' THE G3* GAIN MATRIX IS '
```

```
      ELSE
      PRINT*,' THE G3 GAIN MATRIX IS '
      ENDIF
      CALL PRNT(GAIN3,NACT,NC32)
      IER = 0
      TOL = 0.001
      CALL TFR(ACT,A3,NC32,NC32,1,2)
      PRINT'(//)'
      CALL MRIC(NC32,ACT,CTCC3,QA3,P,ACG3,TOL,IER)
      IF (ZZ.EQ.0) THEN
      PRINT*,' THE RICCATI SOLUTION OF AC - KCC #3 IS '
      CALL PRNT(P,NC32,NC32)
      ENDIF
      CALL MMUL(C3,P,NSEN,NC32,NC32,KT3)
      IF (ZZ.EQ.1.AND.DEC.EQ.4) THEN
      CALL VMULFP(RK3,GAMMA3,P3,P3,NSEN,NCOL,NCOL,STOR,NCOL,IER)
      CALL MMUL(STOR,KT3,P3,NSEN,NC32,KCC)
      CALL MMUL(GAMMA3,KCC,NSEN,P3,NC32,KT3)
      PRINT*,' THE K3* GAIN MATRIX IS '
      ELSE
      PRINT*,' THE K3 GAIN MATRIX IS '
      ENDIF
      CALL TFR(KOB3,KT3,NSEN,NC32,1,2)
      CALL PRNT(KOB3,NC32,NSEN)
155   CONTINUE
      IF (DEC.EQ.4) THEN
      IF (ZZ.EQ.0) THEN
      CALL VMULFP(B4,B4,NR2,NACT,NR2,NCOL,NCOL,SAT4,NCOL,IER)
      ENDIF
      CALL VMULFM(C4,C4,NSEN,NR2,NR2,NCOL,NCOL,CTCC4,NCOL,IER)
160   CONTINUE
      IER = 0
      TOL = 0.001
      CALL MRIC(NR2,A4,SAT4,QA4,S,ABG4,TOL,IER)
      IF (ZZ.EQ.0) THEN
      PRINT*,' THE RICCATI SOLUTION OF AC + BCG #4 IS '
      PRINT*,' IER = ',IER
      CALL PRNT(S,NR2,NR2)
      ENDIF
      CALL VMULFM(B4,S,NR2,NACT,NR2,NCOL,NCOL,GAIN4,NCOL,IER)
      IF (ZZ.EQ.1) THEN
      CALL VMULFM(T4,GAIN4,NACT,E4.NR2,NCOL.NCOL.STOR,NCOL,IER)
      CALL MMUL(RG4,STOR,E4,E4,NR2,TEN)
      CALL MMUL(T4,TEN,NACT,E4,NR2,GAIN4)
      PRINT*,' THE G4* GAIN MATRIX IS '
      ELSE
      PRINT*,' THE G4 GAIN MATRIX IS '
      ENDIF
      CALL PRNT(GAIN4,NACT,NR2)
      IER = 0
      TOL = 0.001
      CALL TFR(ACT,A4,NR2,NR2,1,2)
      CALL MRIC(NR2,ACT,CTCC4,QA4,P,ACG4,TOL,IER)
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```
      CALL MMUL(C4,P,NSEN,NR2,NR2,KT4)
      CALL TFR(KOB4,KT4,NSEN,NR2,1,2)
      IF (ZZ.EQ.0) THEN
      PRINT*,' THE K4 GAIN MATRIX IS '
      ELSE
      PRINT*,' THE K4* GAIN MATRIX IS '
      ENDIF
      CALL PRNT(KOB4,NR2,NSEN)
 165  CONTINUE
      ENDIF
C
C
C  THIS SECTION GENERATES THE BLOCK SEGMENTS
C  OF MAJM AND PUTS THEM INTO THE MAJM MATRIX
C
C  THE THREE CONTROLLER MATRIX WILL CONTAIN
C  RESIDUAL TERMS (SEE DIAGRAM BELOW).
C  THE FOUR CONTROLLER MATRIX DOES NOT IN-
C  CLUDE RESIDUALS (YET).
C
C  THE THREE CONTROLLER MATRIX (MAJM) WITH
C  RESIDUAL TERMS WILL LOOK LIKE:
C
C  ****                                                         ****
C  *                                                               *
C  *  A1+BG1   B1G1    B1G2    B1G2    B1G3    B1G3     0          *
C  *                                                               *
C  *    0     A1-KC1   K1C2     0      K1C3     0      K1CR        *
C  *                                                               *
C  *   B2G1    B2G1   A2+BG2   B2G2    B2G3    B2G3     0          *
C  *                                                               *
C  *   K2C1     0       0     A2-KC2   K2C3     0      K2CR        *
C  *                                                               *
C  *   B3G1    B3G1    B3G2    B3G2   A3+BG3   B3G3     0          *
C  *                                                               *
C  *   K3C1     0      K3C2     0       0     A3-KC3   K3CR        *
C  *                                                               *
C  *   BRG1    BRG1    BRG2    BRG2    BRG3    BRG3     AR         *
C  *                                                               *
C  ****                                                         ****
C
C
      K = 2 * NC12
      KK = K + NC22
      L = 2 * NC22 + K
      LL = L + NC32
      P1 = 2 * NC32 + L
      IF (DEC.EQ.3) THEN
      MM = 2*NC12 + 2*NC22 + 2*NC32 + NR2
      ELSE
      MM = 2*NC12 + 2*NC22 + 2*NC32 + 2*NR2
      P2 = P1 + NR2
      ENDIF
```

```
C
      DO 200 I=1,MM
      DO 200 J=1,MM
 200  MAJM(I,J) = 0.0
      DO 201 I=1,NC12
      DO 201 J=1,NC12
 201  MAJM(I,J) = ABG1(I,J)
      DO 202 I=1,NC22
      DO 202 J=1,NC22
 202  MAJM(I+K,J+K) = ABG2(I,J)
      DO 203 I=1,NC32
      DO 203 J=1,NC32
 203  MAJM(I+L,J+L) = ABG3(I,J)
      CALL TFR(AKC,ACG1,NC12,NC12,1,2)
      DO 204 I=1,NC12
      DO 204 J=1,NC12
 204  MAJM(I+NC12,J+NC12) = AKC(I,J)
      CALL TFR(AKC,ACG2,NC22,NC22,1,2)
      DO 205 I=1,NC22
      DO 205 J=1,NC22
 205  MAJM(I+KK,J+KK) = AKC(I,J)
      CALL TFR(AKC,ACG3,NC32,NC32,1,2)
      DO 206 I=1,NC32
      DO 206 J=1,NC32
 206  MAJM(I+LL,J+LL) = AKC(I,J)
      CALL MMUL(B1,GAIN1,NC12,NACT,NC12,BCG)
      DO 207 I=1,NC12
      DO 207 J=1,NC12
 207  MAJM(I,J+NC12) = BCG(I,J)
      CALL MMUL(B1,GAIN2,NC12,NACT,NC22,BCG)
      DO 208 I=1,NC12
      DO 208 J=1,NC22
      MAJM(I,J+K) = BCG(I,J)
 208  MAJM(I,J+KK) = BCG(I,J)
      CALL MMUL(B1,GAIN3,NC12,NACT,NC32,BCG)
      DO 209 I=1,NC12
      DO 209 J=1,NC32
      MAJM(I,J+L) = BCG(I,J)
 209  MAJM(I,J+LL) = BCG(I,J)
      CALL MMUL(B2,GAIN1,NC22,NACT,NC12,BCG)
      DO 210 I=1,NC22
      DO 210 J=1,NC12
      MAJM(I+K,J) = BCG(I,J)
 210  MAJM(I+K,J+NC12) = BCG(I,J)
      CALL MMUL(B2,GAIN2,NC22,NACT,NC22,BCG)
      DO 211 I=1,NC22
      DO 211 J=1,NC22
 211  MAJM(I+K,J+KK) = BCG(I,J)
      CALL MMUL(B2,GAIN3,NC22,NACT,NC32,BCG)
      DO 212 I=1,NC22
      DO 212 J=1,NC32
      MAJM(I+K,J+L) = BCG(I,J)
 212  MAJM(I+K,J+LL) = BCG(I,J)
```

```
      CALL MMUL(B3,GAIN1,NC32,NACT,NC12,BCG)
      DO 213 I=1,NC32
      DO 213 J=1,NC12
      MAJM(I+L,J) = BCG(I,J)
213   MAJM(I+L,J+NC12) = BCG(I,J)
      CALL MMUL(B3,GAIN2,NC32,NACT,NC22,BCG)
      DO 214 I=1,NC32
      DO 214 J=1,NC22
      MAJM(I+L,J+K) = BCG(I,J)
214   MAJM(I+L,J+KK) = BCG(I,J)
      CALL MMUL(B3,GAIN3,NC32,NACT,NC32,BCG)
      DO 215 I=1,NC32
      DO 215 J=1,NC32
215   MAJM(I+L,J+LL) = BCG(I,J)
      CALL MMUL(KOB1,C2,NC12,NSEN,NC22,KCC)
      DO 216 I=1,NC12
      DO 216 J=1,NC22
216   MAJM(I+NC12,J+K) = KCC(I,J)
      CALL MMUL(KOB1,C3,NC12,NSEN,NC32,KCC)
      DO 217 I= 1,NC12
      DO 217 J= 1,NC32
217   MAJM(I+NC12,J+L) = KCC(I,J)
      CALL MMUL(KOB2,C1,NC22,NSEN,NC12,KCC)
      DO 218 I=1,NC22
      DO 218 J=1,NC12
218   MAJM(I+KK,J) = KCC(I,J)
      CALL MMUL(KOB2,C3,NC22,NSEN,NC32,KCC)
      DO 219 I=1,NC22
      DO 219 J=1,NC32
219   MAJM(I+KK,J+L) = KCC(I,J)
      CALL MMUL(KOB3,C1,NC32,NSEN,NC12,KCC)
      DO 220 I=1,NC32
      DO 220 J=1,NC12
220   MAJM(I+LL,J) = KCC(I,J)
      CALL MMUL(KOB3,C2,NC32,NSEN,NC22,KCC)
      DO 221 I=1,NC32
      DO 221 J=1,NC22
221   MAJM(I+LL,J+K) = KCC(I,J)
      CALL MMUL(B4,GAIN1,NR2,NACT,NC12,BCG)
      DO 222 I=1,NR2
      DO 222 J=1,NC12
      MAJM(I+P1,J) = BCG(I,J)
222   MAJM(I+P1,J+NC12) = BCG(I,J)
      CALL MMUL(B4,GAIN2,NR2,NACT,NC22,BCG)
      DO 223 I=1,NR2
      DO 223 J=1,NC22
      MAJM(I+P1,J+K) = BCG(I,J)
223   MAJM(I+P1,J+KK) = BCG(I,J)
      CALL MMUL(B4,GAIN3,NR2,NACT,NC32,BCG)
      DO 224 I=1,NR2
      DO 224 J=1,NC32
      MAJM(I+P1,J+L) = BCG(I,J)
224   MAJM(I+P1,J+LL) = BCG(I,J)
```

```
      CALL MMUL(KOB1,C4,NC12,NSEN,NR2,KCC)
      DO 225 I=1,NC12
      DO 225 J=1,NR2
 225  MAJM(I+NC12,J+P1) = KCC(I,J)
      CALL MMUL(KOB2,C4,NC22,NSEN,NR2,KCC)
      DO 226 I=1,NC22
      DO 226 J=1,NR2
 226  MAJM(I+KK,J+P1) = KCC(I,J)
      CALL MMUL(KOB3,C4,NC32,NSEN,NR2,KCC)
      DO 227 I=1,NC32
      DO 227 J=1,NR2
 227  MAJM(I+LL,J+P1) = KCC(I,J)
C
C
      IF (DEC.EQ.4) THEN
C
C
      DO 230 I=1,NR2
      DO 230 J=1,NR2
 230  MAJM(I+P1,J+P1) = ABG4(I,J)
      CALL TFR(AKC,ACG4,NR2,NR2,1,2)
      DO 231 I=1,NR2
      DO 231 J=1,NR2
 231  MAJM(I+P2,J+P2) = AKC(I,J)
      CALL MMUL(B1,GAIN4,NC12,NACT,NR2,BCG)
      DO 232 I=1,NC12
      DO 232 J=1,NR2
      MAJM(I,J+P1) = BCG(I,J)
 232  MAJM(I,J+P2) = BCG(I,J)
      CALL MMUL(B2,GAIN4,NC22,NACT,NR2,BCG)
      DO 233 I=1,NC22
      DO 233 J=1,NR2
      MAJM(I+K,J+P1) = BCG(I,J)
 233  MAJM(I+K,J+P2) = BCG(I,J)
      CALL MMUL(B3,GAIN4,NC32,NACT,NR2,BCG)
      DO 234 I=1,NC32
      DO 234 J=1,NR2
      MAJM(I+L,J+P1) = BCG(I,J)
 234  MAJM(I+L,J+P2) = BCG(I,J)
      CALL MMUL(B4,GAIN4,NR2,NACT,NR2,BCG)
      DO 238 I=1,NR2
      DO 238 J=1,NR2
 238  MAJM(I+P1,J+P2) = BCG(I,J)
      CALL MMUL(KOB4,C1,NR2,NSEN,NC12,KCC)
      DO 242 I=1,NR2
      DO 242 J=1,NC12
 242  MAJM(I+P2,J) = KCC(I,J)
      CALL MMUL(KOB4,C2,NR2,NSEN,NC22,KCC)
      DO 243 I=1,NR2
      DO 243 J=1,NC22
 243  MAJM(I+P2,J+K) = KCC(I,J)
      CALL MMUL(KOB4,C3,NR2,NSEN,NC32,KCC)
      DO 244 I=1,NR2
```

```
      DO 244 J=1,NC32
 244  MAJM(I+P2,J+L) = KCC(I,J)
C
      ELSE
C
      CALL FORMA(A4,D,W,NR,NR2,IR)
      DO 250 I=1,NR2
      DO 250 J=1,NR2
 250  MAJM(I+P1,J+P1) = A4(I,J)
      ENDIF
C
C
C  CONSTRUCTION OF THE MAJOR MATRIX 'MAJM' IS NOW COMPLETE
C
C
      IF (DEC.EQ.4) THEN
      PRINT*,' THE FOUR CONTROLLER MAJM IS '
      ELSE
      PRINT*,' THE THREE CONTROLLER MAJM W/RESIDUALS IS '
      ENDIF
      CALL PRNTXL(MAJM,MM,MM)
C
C
C  NEXT, THE TIME RESPONSE SECTION FOLLOWED BY THE E'VALUE ANALYSIS SECTION
C  THE USER HAS THE OPTION OF GETTING BOTH TIME RESP AND E'VALUE ANALYSIS
C  OR 'SKIP'PING EITHER OF THEM.  IF TIME RESP IS SKIPPED, NO IC'S NEEDED.
C
C
      IF (ZZ.EQ.1) THEN
      PRINT*,' THE SUPPRESSED ANALYSIS IS CALCULATED '
      PRINT'(//)'
      ENDIF
      PRINT*,' FOR THE TIME RESPONSE AND THE EIGENVALUE'
     *        ,' ANALYSIS ENTER 1 '
      PRINT*,' FOR ONLY THE EIGENVALUE ANALYSIS ENTER 2       > '
      PRINT*,' FOR ONLY THE TIME RESPONSE ENTER 3      > '
      READ(8,*) SKIP
      IF (SKIP.EQ.1) THEN
      PRINT*,' YOU ASKED FOR BOTH TIME RESP AND EIGENVALUE ANAL'
      PRINT*,' '
      PRINT*,' THE TIME RESPONSE IS FIRST... '
      ENDIF
      IF (SKIP.EQ.2) THEN
      PRINT*,' '
      PRINT*,' YOU CHOSE "2" SO WE WILL FORGET THE TIME RESPONSE '
      GOTO 300
      ENDIF
      IF(SKIP.EQ.3) THEN
      PRINT*,' YOU ASKED FOR THE TIME RESPONSE ONLY'
      PRINT*,' '
      ENDIF
C  INITIAL CONDITIONS
C  THESE ARE READ IN ONLY ONCE FOR EACH JOB (WHEN AA=0.)
```

```
C  REGARDLESS OF THE NUMBER OF CONSECUTIVE RUNS
C
C  REMEMBER --  EVEN THOUGH RESIDUAL MODES DO NOT HAVE E AND
C  E DOT TERMS, THIS READ STATEMENT WILL BE LOOKING FOR THEM,
C  SO INPUT E AND E DOT TERMS FOR THE RESIDUALS ALSO.
C  DON'T WORRY -- SUBROUTINE FORMXO WILL FILTER THEM OUT LATER.
C
C
      IF(AA.EQ.0) THEN
C  READ IN A DUMMY DATA CARD.  ALLOWS STRUCTURING OF INP DATA FILE THE
C  SAME WAY FOR ALL 4 CASES OF DESIRED OUTPUT:  TIME RESP/EIG ANAL VS
C  ONE OR THE OTHER ACCORDING TO SUPPRESSED OR UNSUPPRESSED PASS.
C
C  CASE 1          CASE 2           CASE 3           CASE 4
C
C    EA            TR(TR+EA)        TR(TR+EA)          EA       ZZ = 0
C  TR(TR+EA)       TR(TR+EA)          EA               EA       ZZ = 1
C
C  IMPLIED MEANING OF THE DATA CARDS IN THAT AREA OF THE INPUT DATA
C
C        PASS WITH ZZ = 0
C        ----------------
C        PASS WITH ZZ = 1
C
C  SKIP            SKIP             SKIP             SKIP
C  ----                                              ----
C  SKIP            DUMMY            DUMMY            SKIP
C
C  DATA BLKS       DATA BLKS        DATA BLKS        DATA BLKS(EXTRAN)
C                  ---------        ---------
C  DUMMY(EXTRAN)   SKIP             SKIP             DUMMY (EXTRANEOUS)
C
      IF(ZZ.EQ.0) READ(8,*) DUMMY
      PRINT*,' ENTER THE INITIAL CONDITIONS FOR ',N,' MODES '
      PRINT*,' ROW1=ETA, ROW2=ETA DOT, ROW3=E, ROW4=E DOT '
      DO 90 I=1,4
 90   READ(8,*) (INIT(I,J),J=1,N)
      PRINT'(//)'
      PRINT*,' THE INITIAL CONDITIONS ARE '
      DO 95 I=1,4
   95 PRINT*,(INIT(I,J),J=1,N)
      PRINT'(//)'
      PRINT*,' THE INITIAL STATE VECTOR, Z (XO) IS '
      CALL FORMXO(XO,INIT,DEC)
      CALL PRNT(XO,M,1)
C  READ IN THE TIME PARAMETERS
      READ(8,*) DT,TMAX,PDT
C  READ IN THE PHI-LOS MATRIX, SIMULTANEOUSLY FORMING IT TO (3XN)
      DO 98 I=1,N
   98 READ(8,*) (PHIL(J,I),J=1,3)
C  REFORM THE PHI-LOS MATRIX IAW THE REORDERING OF MODES
C  SIMULTANEOUSLY FORMING THE PRODUCT CPHIL = RPHIL*PETA ONCE AND FOR ALL
      CALL RFMPHIL(PHIL,RPHIL,NC1,NC2,NC3,NR,IC1,IC2,IC3,IR,
```

```
     *PETA,CPHIL,MM)
      AA = 1
      ENDIF
      PRINT*,' THE TIME INCREMENT IS ',DT,' SECONDS '
      TOL = 0.001
      CALL MEXP(MM,MAJM,DT,STM)
      PRINT'(//)'
      PRINT*,' THE SOLUTION IS THE STATE TRANS MATRIX, STM '
      CALL PRNTXL(STM,MM,MM)
C
C
C  STM IS NOW THE SOLUTION TO ZDOT = MAJM * Z
C  WE NOW PROPAGATE THE STATE IN DT STEPS
C
C  FIRST GENERATE THE [C1:C2:C3] MATRIX
C
C
CS      DO 260 I=1,NSEN
CS      DO 260 J=1,NC1
CS 260  V(I,J)=C1(I,J)
CS      DO 270 I=1,NSEN
CS      DO 270 J=1,NC2
CS 270  V(I,J+NC1)=C2(I,J)
CS      DO 280 I=1,NSEN
CS      DO 280 J=1,NC3
CS 280  V(I,J+NC1+NC2)=C3(I,J)
CS      NMODE=NC1+NC2+NC3
CS      IF (NR.GT.0) THEN
CS      DO 290 I=1,NSEN
CS      DO 290 J=1,NR
CS 290  V(I,J+NMODE)=C4(I,J)
CS      NMODE=NMODE+NR
CS      ENDIF
C
CS      PRINT*,' THE C PARTITION MATRIX IS '
CS      CALL PRNT(V,NSEN,NMODE)
C
C
CS      CALL YHAT(CLOS,NSEN,NC1,NC2,NC3,NR,MM,XHAT,PETA,CV,V)
C
      CALL TIMEX(STM,MM,DT,X0,PDT,TMAX,X1,XC,CPHIL,XL)
      IF(SKIP.EQ.3) GOTO 410
C
C
C END OF TIME RESPONSE SECTION
C
C
 300  CONTINUE
C
C
C  EIGENVALUE ANALYSIS SECTION
C
C
```

```
      PRINT'(///)'
      PRINT*,'             OVERALL SYSTEM EIGENVALUES '
C*****NOTICE EIGRF INPUT HERE IS MAJM WHICH HAS ACTUAL DIMENSION 'MM'
C*****SEE COMMENTS ON DIMENSIONING NEAR BEGINNING OF MAIN
      CALL EIGRF(MAJM,MM,NDA,0,Z,TEN,NCOL,WORK,IER)
      PRINT*,' IER = ',IER
      DO 400 I=1,MM
 400  PRINT*,'           ',Z(I)
      PRINT'(//)'
C
      PRINT*,'             EIGENVALUES OF AC + BCG SYSTEM 1 '
      CALL EIGRF(ABG1,NC12,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT*,' IER = ',IER
      DO 401 I=1,NC12
 401  PRINT*,'           ',W1(I)
      PRINT'(//)'
C
      PRINT*,'             EIGENVALUES OF AC - KCC SYSTEM 1 '
      CALL TFR(AKC,ACG1,NC12,NC12,1,2)
      CALL EIGRF(AKC,NC12,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT*,' IER = ',IER
      DO 402 I=1,NC12
 402  PRINT*,'           ',W1(I)
      PRINT'(//)'
C
      PRINT*,'             EIGENVALUES OF AC + BCG SYSTEM 2 '
      CALL EIGRF(ABG2,NC22,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT*,' IER = ',IER
      DO 403 I=1,NC22
 403  PRINT*,'           ',W1(I)
      PRINT'(//)'
C
      PRINT*,'             EIGENVALUES OF AC - KCC SYSTEM 2 '
      CALL TFR(AKC,ACG2,NC22,NC22,1,2)
      CALL EIGRF(AKC,NC22,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT*,' IER = ',IER
      DO 404 I=1,NC22
 404  PRINT*,'           ',W1(I)
      PRINT'(//)'
C
      PRINT*,'             EIGENVALUES OF AC + BCG SYSTEM 3 '
      CALL EIGRF(ABG3,NC32,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT*,' IER = ',IER
      DO 405 I=1,NC32
 405  PRINT*,'           ',W1(I)
      PRINT'(//)'
C
      PRINT*,'             EIGENVALUES OF AC - KCC SYSTEM 3 '
      CALL TFR(AKC,ACG3,NC32,NC32,1,2)
      CALL EIGRF(AKC,NC32,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT*,' IER = ',IER
      DO 406 I=1,NC32
 406  PRINT*,'           ',W1(I)
```

```
C
      PRINT'(//)'
      IF (NR.EQ.0) THEN
      PRINT*,' NO RESIDUAL TERM EIGENVALUES '
      GOTO 410
      ENDIF
      IF (DEC.EQ.4) THEN
C
      PRINT*,'            EIGENVALUES OF AC + BCG SYSTEM 4 '
      CALL EIGRF(ABG4,NR2,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT*,' IER = ',IER
      DO 407 I=1,NR2
  407 PRINT*,'          ',W1(I)
      PRINT'(//)'
C
      PRINT*,'            EIGENVALUES OF AC - KCC SYSTEM 4 '
      CALL TFR(AKC,ACG4,NR2,NR2,1,2)
      CALL EIGRF(AKC,NR2,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT*,' IER = ',IER
      DO 408 I=1,NR2
  408 PRINT*,'          ',W1(I)
C
      ELSE
C
      PRINT*,'            EIGENVALUES OF THE A RESIDUAL MATRIX '
      CALL EIGRF(A4,NR2,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT*,' IER = ',IER
      DO 409 I=1,NR2
  409 PRINT*,'          ',W1(I)
      ENDIF
      PRINT'(///)'
C END OF EIGENVALUE ANALSIS SECTION
  410 CONTINUE
      IF (ZZ.EQ.1) GOTO 600
C
C
C  THIS SECTION FORMS THE TRANSFORMATION MATRICES.
C  TO GET MAJM IN LOWER TRIANGULAR FORM, IT IS
C  NECESSARY TO DRIVE THE B1G2, B1G3, B2G3, K1C2,
C  K1C3, AND K2C3 TERMS TO ZERO (THREE CTLRS).
C
C  WHEN FOUR CONTROLLERS ARE USED, THERE WILL
C  BE ADDITIONAL G4 AND K4 TERMS.     IT WILL
C  THEN BE NECESSARY TO DRIVE THE B(I)G4, AND
C  K(I)C4 TERMS TO ZERO ALSO.
C
C
C  AFTER THE TRANSFORMATION IS COMPLETE,
C  THE THREE CONTROLLER MAJM (WITH RESIDUALS)
C  WILL LOOK LIKE:
C
C  ****                                                     ****
C  *                                                           *
```

```
C  *  A1+BG1    B1G1      0        0        0        0        0      *
C  *                                                                *
C  *    0      A1-KC1     0        0        0        0       K1CR    *
C  *                                                                *
C  *   B2G1     B2G1    A2+BG2    B2G2      0        0        0      *
C  *                                                                *
C  *   K2C1      0        0      A2-KC2     0        0       K2CR    *
C  *                                                                *
C  *   B3G1     B3G1     B3G2     B3G2    A3+BG3    B3G3      0      *
C  *                                                                *
C  *   K3C1      0       K3C2      0        0      A3-KC3    K3CR    *
C  *                                                                *
C  *   BRG1     BRG1     BRG2     BRG2     BRG3     BRG3      AR     *
C  *                                                                *
C  ****                                                          ****
C
C
C  WHERE THE NON-ZERO TERMS INCLUDE THE
C  TRANSFORMATION MATRICES.
C  (THE RESIDUALS ARE NOT ZEROED SINCE THEY
C   ARE NOT ADDRESSED IN THIS PROGRAM)
C
C
C
C  ON WITH THE TRANSFORMATION MATRICES!
C
C FIRST THE OBSERVER GAIN MATRIX, K1
C
C
      CALL TFR(CT,C2,NSEN,NC22,1,2)
      DO 500 I=1,NC2
      DO 500 J=1,NSEN
 500  V(I,J) = CT(I,J)
      CALL TFR(CT,C3,NSEN,NC32,1,2)
      DO 501 I=1,NC3
      DO 501 J=1,NSEN
 501  V(I+NC2,J) = CT(I,J)
      IF (DEC.EQ.4) THEN
      CALL TFR(CT,C4,NSEN,NR2,1,2)
      DO 502 I=1,NR
      DO 502 J=1,NSEN
 502  V(I+NC2+NC3,J) = CT(I,J)
      NRV = NC2 + NC3 + NR
      PRINT*,' V (C2/C3/C4) IS '
      ELSE
      PRINT*,' V (C2/C3) IS '
      NRV = NC2 + NC3
      ENDIF
      CALL PRNT(V,NRV,NSEN)
      CALL LSVDF(V,NCOL,NRV,NSEN,TEN,NCOL,-1,SING,STOR,IER)
      PRINT*,' '
      PRINT*,' LSVDF 1 IER = ',IER
      PRINT'(//)'
```

```
      PRINT*,' V OUT OF LSVDF IS '
      CALL PRNT(V,NSEN,NSEN)
      P1 = NSEN - NRV
      IF (P1.LT.1) THEN
      DO 503 I=1,NSEN
 503  GAMMA1(I,1) = V(I,NSEN)
      P1 = 1
      ELSE
      DO 504 I=1,NSEN
      DO 504 J=1,P1
 504  GAMMA1(I,J) = V(I,J+NRV)
      ENDIF
      PRINT*,' TRANSFORMATION MATRIX GAMMA1 '
      CALL PRNT(GAMMA1,NSEN,P1)
C
C  CHECK TO SEE THAT GAMMA1 IS ORTHOGONAL TO BOTH C2 AND C3
C
C NOTE:   AKC IN THIS SECTION IS JUST A WORK AREA TO TEST
C         THE ORTHOGONALITY OF CT * TR.  IN ALL CASES IT
C         SHOULD BE A BLOCK ZERO MATRIX.
C
      CALL TFR(CT,C2,NSEN,NC22,1,2)
      CALL MMUL(CT,GAMMA1,NC22,NSEN,P1,AKC)
      PRINT*,' C2T * GAMMA1 '
      CALL PRNT(AKC,NC22,P1)
      CALL TFR(CT,C3,NSEN,NC32,1,2)
      CALL MMUL(CT,GAMMA1,NC32,NSEN,P1,AKC)
      PRINT*,' C3T * GAMMA1 '
      CALL PRNT(AKC,NC32,P1)
      IF (DEC.EQ.4) THEN
      CALL TFR(CT,C4,NSEN,NR2,1,2)
      CALL MMUL(CT,GAMMA1,NR2,NSEN,P1,AKC)
      PRINT*,' C4T * GAMMA1 '
      CALL PRNT(AKC,NR2,P1)
      ENDIF
C
      IF (DEC.EQ.4) THEN
      PRINT*,' C234 SINGULAR VALUES '
      ELSE
      PRINT*,' C23 SINGULAR VALUES '
      ENDIF
      CALL PRNT(SING,NRV,1)
      CALL TFR(TRT,GAMMA1,NSEN,P1,1,2)
      CALL MMUL(TRT,GAMMA1,P1,NSEN,P1,RK)
      CALL GMINV(P1,P1,RK,RK1,J,TAPE)
      CALL TFR(CT,C1,NSEN,NC12,1,2)
      CALL MMUL(TRT,C1,P1,NSEN,NC12,AKC)
      CALL MMUL(CT,GAMMA1,NC12,NSEN,P1,KOB1)
      CALL MMUL(KOB1,RK1,NC12,P1,P1,STOR)
      CALL MMUL(STOR,AKC,NC12,P1,NC12,CTCC1)
C
C  THIS CTCC1 WILL BE SUBSTITUTED BACK INTO MRIC
C  SYSTEM 1 TO GET A NEW K1.
```

```
C
C
C  NOW THE OBSERVER GAIN MATRIX, K2
C
C
      CALL TFR(CT,C3,NSEN,NC32,1,2)
      DO 505 I=1,NC3
      DO 505 J=1,NSEN
 505  V(I,J) = CT(I,J)
      IF (DEC.EQ.4) THEN
      CALL TFR(CT,C4,NSEN,NR2,1,2)
      DO 506 I=1,NR
      DO 506 J=1,NSEN
 506  V(I+NC3,J) = CT(I,J)
      NRV = NC3 + NR
      PRINT*,' V (C3/C4) IS '
      ELSE
      NRV = NC3
      PRINT*,' V (C3) IS '
      ENDIF
      CALL PRNT(V,NRV,NSEN)
      CALL LSVDF(V,NCOL,NRV,NSEN,TEN,NCOL,-1,SING,STOR,IER)
      PRINT*,' '
      PRINT*,' LSVDF 2 IER = ',IER
      PRINT'(//)'
      PRINT*,' V OUT OF LSVDF IS '
      CALL PRNT(V,NSEN,NSEN)
      P2 = NSEN - NRV
      IF (P2.LT.1) THEN
      DO 507 I=1,NSEN
 507  GAMMA2(I,1) = V(I,NSEN)
      P2 = 1
      ELSE
      DO 508 I=1,NSEN
      DO 508 J=1,P2
 508  GAMMA2(I,J) = V(I,J+NRV)
      ENDIF
      PRINT*,' TRANSFORMATION MATRIX GAMMA2 '
      CALL PRNT(GAMMA2,NSEN,P2)
C
C  CHECK TO SEE THAT GAMMA2 IS ORTHOGONAL TO C3
C
      CALL TFR(CT,C3,NSEN,NC32,1,2)
      CALL MMUL(CT,GAMMA2,NC32,NSEN,P2,AKC)
      PRINT*,' C3T * GAMMA2 '
      CALL PRNT(AKC,NC32,P2)
      IF (DEC.EQ.4) THEN
      CALL TFR(CT,C4,NSEN,NR2,1,2)
      CALL MMUL(CT,GAMMA2,NR2,NSEN,P2,AKC)
      PRINT*,' C4T * GAMMA2 '
      CALL PRNT(AKC,NR2,P2)
      ENDIF
C
```

```
      IF (DEC.EQ.4) THEN
      PRINT*,' C34 SINGULAR VALUES '
      ELSE
      PRINT*,' C3 SINGULAR VALUES '
      ENDIF
      CALL PRNT(SING,NRV,1)
C
      CALL TFR(TRT,GAMMA2,NSEN,P2,1,2)
      CALL MMUL(TRT,GAMMA2,P2,NSEN,P2,RK)
      CALL GMINV(P2,P2,RK,RK2,J,TAPE)
      CALL TFR(CT,C2,NSEN,NC22,1,2)
      CALL MMUL(TRT,C2,P2,NSEN,NC22,AKC)
      CALL MMUL(CT,GAMMA2,NC22,NSEN,P2,KOB2)
      CALL MMUL(KOB2,RK2,NC22,P2,P2,STOR)
      CALL MMUL(STOR,AKC,NC22,P2,NC22,CTCC2)
C
C
C  CTCC2 WILL BE SUBSTITUTED BACK INTO
C  MRIC-SYSTEM 2 FOR A NEW K2.
C
C
C  NOW THE OBSERVER GAIN MATRIX, K3, WHEN
C  USING FOUR CONTROLLERS
C
C
      IF (DEC.EQ.4) THEN
      CALL TFR(CT,C4,NSEN,NR2,1,2)
      DO 509 I=1,NR
      DO 509 J=1,NSEN
  509 V(I,J) = CT(I,J)
      PRINT*,' V (C4) IS '
      CALL PRNT(V,NR,NSEN)
      CALL LSVDF(V,NCOL,NR,NSEN,TEN,NCOL,-1,SING,STOR,IER)
      PRINT*,' '
      PRINT*,' LSVDF K3 IER = ',IER
      PRINT'(//)'
      PRINT*,' V OUT OF LSVDF IS '
      CALL PRNT(V,NSEN,NSEN)
      P3 = NSEN - NR
      IF (P3.LT.1) THEN
      DO 510 I=1,NSEN
  510 GAMMA3(I,1) = V(I,NSEN)
      P3 = 1
      ELSE
      DO 511 I=1,NSEN
      DO 511 J=1,P3
  511 GAMMA3(I,J) = V(I,J+NR)
      ENDIF
      PRINT*,' TRANSFORMATION MATRIX GAMMA3 '
      CALL PRNT(GAMMA3,NSEN,P3)
C
C  CHECK TO SEE THAT GAMMA3 IS ORTHOGONAL TO C4
C
```

```
      CALL MMUL(CT,GAMMA3,NR2,NSEN,P3,AKC)
      PRINT*,' C4T * GAMMA3 '
      CALL PRNT(AKC,NR2,P3)
C
      PRINT*,' C4 SINGULAR VALUES '
      CALL PRNT(SING,NR,1)
      CALL TFR(TRT,GAMMA3,NSEN,P3,1,2)
      CALL MMUL(TRT,GAMMA3,P3,NSEN,P3,RK)
      CALL GMINV(P3,P3,RK,RK3,J,TAPE)
      CALL TFR(CT,C3,NSEN,NC32,1,2)
      CALL MMUL(TRT,C3,P3,NSEN,NC32,AKC)
      CALL MMUL(CT,GAMMA3,NC32,NSEN,P3,KOB3)
      CALL MMUL(KOB3,RK3,NC32,P3,P3,STOR)
      CALL MMUL(STOR,AKC,NC32,P3,NC32,CTCC3)
      ENDIF
C
C
C  CTCC3 WILL BE SUBSTITUTED BACK INTO
C  MRIC-SYSTEM 3 FOR A NEW K2  (WHEN
C  USING FOUR CONTROLLERS)
C
C
C  NOW THE CONTROLLER GAIN MATRIX, G2
C
C
      DO 512 I=1,NC1
      DO 512 J=1,NACT
  512 V(I,J) = B1(I+NC1,J)
      PRINT*,' V (B1) IS '
      CALL PRNT(V,NC1,NACT)
      CALL LSVDF(V,NCOL,NC1,NACT,TEN,NCOL,-1,SING,STOR,IER)
      PRINT*,' '
      PRINT*,' LSVDF CONTROL 2 IER = ',IER
      PRINT'(//)'
      PRINT*,' V OUT OF LSVDF IS '
      CALL PRNT(V,NACT,NACT)
      E2 = NACT - NC1
      IF (E2.LT.1) THEN
      DO 513 I=1,NACT
  513 T2(I,1) = V(I,NACT)
      E2 = 1
      ELSE
      DO 514 I=1,NACT
      DO 514 J=1,E2
  514 T2(I,J) = V(I,J+NC1)
      ENDIF
      PRINT*,' TRANSFORMATION MATRIX T2 '
      CALL PRNT(T2,NACT,E2)
C
C  CHECK TO SEE THAT T2 IS ORTHOGONAL TO B1
C
C  NOTE:    IN THIS SECTION, BCG IS THE WORK AREA
C           FOR B * T.  IN ALL CASES THESE SHOULD
```

```
C           BE BLOCK ZERO MATRICES.
C
C
      CALL MMUL(B1,T2,NC12,NACT,E2,BCG)
      PRINT*,' B1 * T2 '
      CALL PRNT(BCG,NC12,E2)
C
      PRINT*,' B1 SINGULAR VALUES '
      CALL PRNT(SING,NC1,1)
C
      CALL VMULFM(T2,T2,NACT,E2,E2,NACT,NACT,RK,NACT,IER)
      CALL GMINV(E2,E2,RK,RG2,J,TAPE)
      CALL MMUL(B2,T2,NC22,NACT,E2,KOB2)
      CALL MMUL(KOB2,RG2,NC22,E2,E2,SAT2)
      CALL VMULFP(SAT2,T2,NC22,E2,NACT,NCOL,NACT,KOB2,NCOL,IER)
      CALL VMULFP(KOB2,B2,NC22,NACT,NC22,NCOL,NCOL,SAT2,NCOL,IER)
C
C
C  THIS SAT2 WILL BE SUBSTITUTED BACK INTO MRIC
C  SYSTEM 2 FOR A NEW G2.
C
C
C  NOW THE CONTROLLER GAIN MATRIX, G3
C
C
      DO 515 I=1,NC1
      DO 515 J=1,NACT
 515  V(I,J) = B1(I+NC1,J)
      DO 516 I=1,NC2
      DO 516 J=1,NACT
 516  V(I+NC1,J) = B2(I+NC2,J)
C
C  V IS NOW AN NC1+NC2 BY NACT MATRIX
C
      PRINT*,' V (B1/B2) IS '
      NRV = NC1 + NC2
      CALL PRNT(V,NRV,NACT)
      CALL LSVDF(V,NCOL,NRV,NACT,TEN,NCOL,-1,SING,STOR,IER)
      PRINT*,' '
      PRINT*,' LSVDF CONTROL 3 IER = ',IER
      PRINT'(//)'
      PRINT*,' V OUT OF LSVDF IS '
      CALL PRNT(V,NACT,NACT)
      E3 = NACT - NRV
      IF (E3.LT.1) THEN
      DO 517 I =1,NACT
 517  T3(I,1) = V(I,NACT)
      E3 = 1
      ELSE
      DO 518 I=1,NACT
      DO 518 J=1,E3
 518  T3(I,J) = V(I,J+NRV)
      ENDIF
```

```
      PRINT*,' TRANSFORMATION MATRIX T3 '
      CALL PRNT(T3,NACT,E3)
C
C  CHECK TO SEE THAT T3 IS ORTHOGONAL TO B1 AND B2
C
      CALL MMUL(B1,T3,NC12,NACT,E3,BCG)
      PRINT*,' B1 * T3 '
      CALL PRNT(BCG,NC12,E3)
      CALL MMUL(B2,T3,NC22,NACT,E3,BCG)
      PRINT*,' B2 * T3 '
      CALL PRNT(BCG,NC22,E3)
C
      PRINT*,' B12 SINGULAR VALUES '
      CALL PRNT(SING,NRV,1)
C
      CALL VMULFM(T3,T3,NACT,E3,E3,NACT,NACT,RK,NACT,IER)
      CALL GMINV(E3,E3,RK,RG3,J,TAPE)
      CALL MMUL(B3,T3,NC32,NACT,E3,KOB3)
      CALL MMUL(KOB3,RG3,NC32,E3,E3,SAT3)
      CALL VMULFP(SAT3,T3,NC32,E3,NACT,NCOL,NACT,KOB3,NCOL,IER)
      CALL VMULFP(KOB3,B3,NC32,NACT,NC32,NCOL,NCOL,SAT3,NCOL,IER)
C
C
C  SAT3 WILL BE SUBSTITUTED BACK INTO
C  MRIC-SYSTEM 3 FOR A NEW G3.
C
C
C  NOW THE CONTROLLER GAIN MATRIX, G4,
C  TO BE FOUND WHEN USING FOUR CONTROLLERS
C
C
      IF (DEC.EQ.4) THEN
      DO 519 I=1,NC1
      DO 519 J=1,NACT
 519  V(I,J) = B1(I+NC1,J)
      DO 520 I=1,NC2
      DO 520 J=1,NACT
 520  V(I+NC1,J) = B2(I+NC2,J)
      DO 521 I=1,NC3
      DO 521 J=1,NACT
 521  V(I+NC1+NC2,J) = B3(I+NC3,J)
C
C  V IS NOW A (NC1+NC2+NC3) BY NACT MATRIX
C
      PRINT*,' V(B1/B2/B3) IS '
      NRV = NC1 + NC2 + NC3
      CALL PRNT(V,NRV,NACT)
      CALL LSVDF(V,NCOL,NRV,NACT,TEN,NCOL,-1,SING,STOR,IER)
      PRINT*,' '
      PRINT*,' LSVDF CONTROL IER = ',IER
      PRINT'(//)'
      PRINT*,' V OUT OF LSVDF IS '
      CALL PRNT(V,NACT,NACT)
```

```
      E4 = NACT - NRV
      IF (E4.LT.1) THEN
      DO 522 I=1,NACT
 522  T4(I,1) = V(I,NACT)
      E4 = 1
      ELSE
      DO 523 I=1,NACT
      DO 523 J=1,E4
 523  T4(I,J) = V(I,J+NRV)
      ENDIF
      PRINT*,' TRANSFORMATION MATRIX T4 '
      CALL PRNT(T4,NACT,E4)
C
C  CHECK TO SEE THAT T4 IS ORTHOGONAL TO B1,B2,B3
C
      CALL MMUL(B1,T4,NC12,NACT,E4,BCG)
      PRINT*,' B1 * T4 '
      CALL PRNT(BCG,NC12,E4)
      CALL MMUL(B2,T4,NC22,NACT,E4,BCG)
      PRINT*,' B2 * T4 '
      CALL PRNT(BCG,NC22,E4)
      CALL MMUL(B3,T4,NC32,NACT,E4,BCG)
      PRINT*,' B3 * T4 '
      CALL PRNT(BCG,NC32,E4)
C
      PRINT*,' B123 SINGULAR VALUES '
      CALL PRNT(SING,NRV,1)
C
      CALL VMULFM(T4,T4,NACT,E4,E4,NACT,NACT,RK,NACT,IER)
      CALL GMINV(E4,E4,RK,RG4,J,TAPE)
      CALL MMUL(B4,T4,NR2,NACT,E4,KOB4)
      CALL MMUL(KOB4,RG4,NR2,E4,E4,SAT4)
      CALL VMULFP(SAT4,T4,NR2,E4,NACT,NCOL,NACT,KOB4,NCOL,IER)
      CALL VMULFP(KOB4,B4,NR2,NACT,NR2,NCOL,NCOL,SAT4,NCOL,IER)
      ENDIF
C
C
C  SAT4 WILL BE SUBSTITUTED BACK INTO MRIC-
C  SYSTEM 4 FOR A NEW G4 WHEN USING FOUR
C  CONTROLLERS.
C
C
      ZZ = 1
      GOTO 115
 600  CONTINUE
C
C
C  THE PROBLEM IS NOW COMPLETE
C
C
      PRINT'(///)'
      PRINT*,'  THIS RUN HAS BEEN COMPLETED  '
      PRINT'(///)'
```

```
      END
      SUBROUTINE FORMXO(XO,INIT,DEC)
      COMMON/MAINA/NDA
      COMMON/MAINB/NCOL
C
C
C  FORMXO IS THE SAME FOR THREE OR FOUR
C  CONTROLLERS SINCE THE RESIDUALS BECOME
C  THE FOURTH CONTROLLER.
C
      COMMON/NUM/IC1(21),IC2(21),IC3(21),IR(21),NC1,NC2,NC3,NR
      REAL XO(NDA),INIT(4,NCOL)
      INTEGER M,I,J,K,L
      DO 1 I=1,NC1
      M = IC1(I)
      XO(I) = INIT(1,M)
      XO(I+NC1) = INIT(2,M)
      XO(I+NC1*2) = INIT(3,M)
 1    XO(I+NC1*3) = INIT(4,M)
      J = NC1*4
      DO 2 I=1,NC2
      M = IC2(I)
      XO(I+J) = INIT(1,M)
      XO(I+J+NC2) = INIT(2,M)
      XO(I+J+NC2*2) = INIT(3,M)
 2    XO(I+J+NC2*3) = INIT(4,M)
      K = J + NC2*4
      DO 3 I=1,NC3
      M = IC3(I)
      XO(I+K) = INIT(1,M)
      XO(I+K+NC3) = INIT(2,M)
      XO(I+K+NC3*2) = INIT(3,M)
 3    XO(I+K+NC3*3) = INIT(4,M)
      IF (NR.NE.0) THEN
      L = K + NC3*4
      DO 4 I=1,NR
      M = IR(I)
      XO(I+L) = INIT(1,M)
      XO(I+L+NR) = INIT(2,M)
      IF (DEC.EQ.4) THEN
      XO(I+L+NR*2) = INIT(3,M)
      XO(I+L+NR*3) = INIT(4,M)
      ENDIF
 4    CONTINUE
      ENDIF
      RETURN
      END
      SUBROUTINE FACTOR(N,A,S,MR)
C       A=S'S
      DIMENSION A(1),S(1)
      COMMON/MAINB/ NCOL,NCOL1
      COMMON/INOUT/KOUT
      TOL=1.E-6
```

```
      MR=0
      NN=N*NCOL
      TOL1=0.
      DO 1 I=1,NN,NCOL1
      R=ABS(A(I))
 1    IF (R.GT.TOL1) TOL1=R
      TOL1=TOL1*1.E-12
      II=1
      DO 50 I=1,N
      IM1=I-1
       DO 5 JJ=I,NN,NCOL
 5    S(JJ)=0.
      ID=II+IM1
      R=A(ID)-DOT(IM1,S(II),S(II))
      IF (ABS(R).LT.(TOL*A(ID)+TOL1)) GO TO 50
      IF (R) 15,50,20
 15   MR=-1
      WRITE(KOUT,1000)
 1000 FORMAT(37HOTRIED TO FACTOR AN INDEFINITE MATRIX   )
      RETURN
 20   S(ID)=SQRT(R)
      MR=MR+1
      IF (I.EQ.N) RETURN
      L=II+NCOL
      DO 25 JJ=L,NN,NCOL
      IJ=JJ+IM1
 25   S(IJ)=(A(IJ)-DOT(IM1,S(II),S(JJ)))/S(ID)
 50   II=II+NCOL
      RETURN
      END
      SUBROUTINE FORMA(A,D,W,N,N2,IC)
      COMMON/MAINB/NCOL
      REAL A(NCOL,NCOL),W(NCOL),D(NCOL)
      INTEGER IC(N),I,J,N,M
      DO 1 I=1,N2
      DO 1 J=1,N2
      A(I,J)=0.0
1     CONTINUE
      DO 2 I=1,N
      M= IC(I)
      A((I+N),(I+N))=D(M)
      A(I,(I+N)) = 1.0
      A((I+N),I) = -(W(M)**2)
2     CONTINUE
      RETURN
      END
      SUBROUTINE FORMB(B,PHI,N,N2,NACT,IC)
      COMMON/MAINB/NCOL
      REAL B(NCOL,NCOL),PHI(NCOL,NCOL)
      INTEGER IC(N),NACT,N,M,I,J
      DO 1 I=1,N2
      DO 1 J=1,NACT
      B(I,J) = 0.0
```

```
1     CONTINUE
      DO 2 I=1,N
      M = IC(I)
      DO 2 J=1,NACT
      B((N+I),J) = PHI(M,J)
2     CONTINUE
      RETURN
      END
      SUBROUTINE FORMC(C,PHIS,N,N2,NSEN,IC)
      COMMON/MAINB/NCOL
      REAL C(NCOL,NCOL),PHIS(NCOL,NCOL)
      INTEGER IC(N),M,NSEN,N,N2,I,J
      DO 1 I=1,NSEN
      DO 1 J=1,N2
      C(I,J) = 0.0
1     CONTINUE
      DO 2 I=1,NSEN
      DO 2 J=1,N
      M = IC(J)
      C(I,J) = PHIS(M,I)
2     CONTINUE
      RETURN
      END
      SUBROUTINE FORMQ1(Q,A,N,IC)
      COMMON/MAINB/NCOL
      REAL A(NCOL),Q(NCOL,NCOL)
      INTEGER I,J,K,M,N,N2,IC(NCOL)
      N2 = N * 2
      DO 1 I=1,N2
      DO 1 J=1,N2
      Q(I,J) = 0.0
 1    CONTINUE
      DO 2 I=1,N
      M = IC(I)
      Q(I,I) = A(M)
      Q(I+N,I+N) = Q(I,I)
 2    CONTINUE
      RETURN
      END
      SUBROUTINE GMINV(NR,NC,A,U,MR,MT)
      DIMENSION A(1),U(1)
      COMMON/MAIN1/ NDIM,NDIM1,S(1)
      COMMON/MAINB/NCOL,NCOL1
      COMMON/INOUT/KOUT
      TOL=1.E-12
      MR=NC
      NRM1=NR-1
      TOL1=1.E-20
      JJ=1
      DO 100 J=1,NC
      FAC=DOT(NR,A(JJ),A(JJ))
      JM1=J-1
      JRM=JJ+NRM1
```

```
      JCM=JJ+JM1
      DO 20 I=JJ,JCM
20    U(I)=0.
      U(JCM)=1.0
      IF (J.EQ.1) GO TO 54
      KK=1
      DO 30 K=1,JM1
      IF (S(K).EQ.1.0) GO TO 30
      TEMP=-DOT(NR,A(JJ),A(KK))
      CALL VADD(K,TEMP,U(JJ),U(KK))
30    KK=KK+NCOL
      DO 50 L=1,2
      KK=1
      DO 50 K=1,JM1
      IF (S(K).EQ.0.) GO TO 50
      TEMP=-DOT(NR,A(JJ),A(KK))
      CALL VADD(NR,TEMP,A(JJ),A(KK))
      CALL VADD(K,TEMP,U(JJ),U(KK))
50    KK=KK+NCOL
      TOL1=TOL*FAC
      FAC=DOT(NR,A(JJ),A(JJ))
54    IF (FAC.GT.TOL1) GO TO 70
      DO 55 I=JJ,JRM
55    A(I)=0.
      S(J)=0.
      KK=1
      DO 65 K=1,JM1
      IF (S(K).EQ.0.) GO TO 65
      TEMP=-DOT(K,U(KK),U(JJ))
      CALL VADD(NR,TEMP,A(JJ),A(KK))
65    KK=KK+NCOL
      FAC=DOT(J,U(JJ),U(JJ))
      MR=MR-1
      GO TO 75
70    S(J)=1.0
      KK=1
      DO 72 K=1,JM1
      IF (S(K).EQ.1.) GO TO 72
      TEMP=-DOT(NR,A(JJ),A(KK))
      CALL VADD(K,TEMP,U(JJ),U(KK))
72    KK=KK+NCOL
75    FAC=1./SQRT(FAC)
      DO 80 I=JJ,JRM
80    A(I)=A(I)*FAC
      DO 85 I=JJ,JCM
85    U(I)=U(I)*FAC
100   JJ=JJ+NCOL
      IF (MR.EQ.NR.OR.MR.EQ.NC) GO TO 120
      IF (MT.NE.0) WRITE(KOUT,110)NR,NC,MR
110   FORMAT(I3,1HX,I2,8H M  RANK,I2)
120   NEND=NC*NCOL
      JJ=1
      DO 135 J=1,NC
```

```
      DO 125 I=1,NR
      II=I-J
      S(I)=0.
      DO 125 KK=JJ,NEND,NCOL
  125 S(I)=S(I)+A(II+KK)*U(KK)
      II=J
      DO 130 I=1,NR
      U(II)=S(I)
  130 II=II+NCOL
  135 JJ=JJ+NCOL1
      RETURN
      END
      SUBROUTINE INTEG(N,A,C,S,T)
C     S=INTEGRAL EA*C*EA FROM 0 TO T
C     C IS DESTROYED
      DIMENSION A(1),C(1),S(1)
      COMMON/MAIN1/ NDIM,NDIM1, X(1)
      COMMON/MAINB/NCOL,NCOL1
      COMMON/MAIN2/COEF(100)
      NN=N*NCOL
      NM1=N-1
      IND=0
      ANORM=XNORM(N,A)
      DT=T
    5 IF (ANORM*ABS(DT).LE.0.5) GO TO 10
      DT=DT/2.
      IND=IND+1
      GO TO 5
   10 DO 15 I=1,NN,NCOL
      J=I+NM1
      DO 15 JJ=I,J
   15 S(JJ)=DT*C(JJ)
      T1=DT**2/2.
      DO 25 IT=3,15
      CALL MMUL(A,C,N,N,N,X)
      DO 20 I=1,N
      II=(I-1)*NCOL
      DO 20 JJ=I,NN,NCOL
      II=II+1
      C(JJ)=(X(JJ)+X(II))*T1
   20 S(JJ)=S(JJ)+C(JJ)
   25 T1=DT/FLOAT(IT)
      IF (IND.EQ.0) GO TO 100
      COEF(11)=1.0
      DO 30 I=1,10
      II=11-I
   30 COEF(II)=DT*COEF(II+1)/FLOAT(I)
      II=1
      DO 40 I=1,NN,NCOL
      J=I+NM1
      DO 35 JJ=I,J
   35 X(JJ)=A(JJ)*COEF(1)
      X(II)=X(II)+COEF(2)
```

```
 40     II=II+NCOL1
        DO 55 L=3,11
        CALL MMUL(A,X,N,N,N,C)
        II=1
        T1=COEF(L)
        DO 55 I=1,NN,NCOL
        J=I+NM1
        DO 50 JJ=I,J
 50     X(JJ)=C(JJ)
        X(II)=X(II)+T1
 55     II=II+NCOL1
C       X=EXP(A*DT)
        L=0
 60     L=L+1
        CALL MMUL(X,S,N,N,N,C)
        II=1
        DO 90 I=1,N
        J=II
        IF (I.EQ.1) GO TO 75
        DO 70 JJ=I,II,NCOL
        S(JJ)=S(J)
 70     J=J+1
 75     DO 85 JJ=I,N
        KK=JJ
        DO 80 K=I,NN,NCOL
        S(J)=S(J)+C(K)*X(KK)
 80     KK=KK+NCOL
 85     J=J+NCOL
        DO 87 JJ=I,NN,NCOL
 87     C(JJ)=X(JJ)
 90     II=II+NCOL
        IF (L.EQ.IND) GO TO 100
        CALL MMUL(C,C,N,N,N,X)
        GO TO 60
 100    CONTINUE
        RETURN
        END
        SUBROUTINE MEXP(N,A,T,EA)
        DIMENSION A(1),EA(1),C(84),D(85),E(84)
        COMMON/MAIN1/NDIM,NDIM1,TEN,X(1)
        NN = N**2
        NM1 = N-1
        NP1 = N+1
        IF (N.GT.1) GO TO 5
        EA(1)=EXP(T*A(1))
        RETURN
      5 W=1.0
        DO 10 I=1,NN,N
        IL=I+NM1
        DO 10 J=I,IL
     10 EA(J)=A(J)
        C1=XNORM1(N,A)
        IND=0
```

```
      L=1
      T1=T
   15 IF (ABS(T1*C1).LE.3.0) GO TO 20
      T1=T1/2.
      IND=IND+1
      GO TO 15
   20 C2=0.
      DO 25 I=1,NN,NP1
   25 C2=C2-EA(I)
      C2=C2/FLOAT(L)
      C(L)=C2
      D(L+1)=0.
      II=N+1-L
      E(II)=W
      II=1
      DO 35 I=1,NN,N
      IL=I+NM1
      DO 30 J=I,IL
   30 X(J)=EA(J)
      X(II)=X(II)+C2
   35 II=II+NP1
      IF (L.EQ.N) GO TO 40
      CALL MMUL1(X,A,N,N,N,EA)
      W=W*T1/FLOAT(L)
      L=L+1
      GO TO 20
   40 CONTINUE
C****** CAN CHECK X 0 FOR ACCURACY (63 IS OVERKILL FOR CONVERGENCE)
      J=N+63
      DO 50 L=N,J
      DO 45 K=1,N
      D(K)=(D(K+1)-W*C(K))*T1/FLOAT(L)
   45 E(K)=E(K)+D(K)
   50 W=D(1)
      II=1
      DO 60 I=1,NN,N
      IL=I+NM1
      DO 55 J=I,IL
   55 EA(J)=E(1)*A(J)
      EA(II)=EA(II)+E(2)
   60 II=II+NP1
      IF (N.EQ.2) GO TO 85
      DO 80 L=3,N
      CALL MMUL1(EA,A,N,N,N,X)
      II=1
      C2=E(L)
      DO 75 I=1,NN,N
      IL=I+NM1
      DO 70 J=I,IL
   70 EA(J)=X(J)
      EA(II)=EA(II)+C2
   75 II=II+NP1
   80 CONTINUE
```

```
   85 IF (IND.EQ.0) RETURN
      DO 100 L=1,IND
      DO 90 I=1,NN,N
      IL=I+NM1
      DO 90 J=I,IL
   90 X(J)=EA(J)
  100 CALL MMUL1(X,X,N,N,N,EA)
      RETURN
      END
      SUBROUTINE MLINEQ(N,A,C,X,TOL,IER)
C     SOLVES A'X+XA+C=0
C     A AND X CAN BE IN SAME LOCATION
C     ANSWER RETURNED IN C AND X
      DIMENSION A(1),C(1),X(1)
      COMMON/MAINB/ NCOL, NCOL1
      COMMON/MAIN3/F(1)
      ADV=TOL*1.E-6
      DT=.5
      DT1=0.
      NN=N*NCOL
      DO 5 II=1,NN,NCOL1
 5    DT1=DT1-A(II)
      DT1=DT1/N
      IF (DT1.GT.4.0) DT=DT*4.0/DT1
      II=1
      DO 20 I=1,N
      DO 15 JJ=I,NN,NCOL
 15   X(JJ)=DT*A(JJ)
      X(II)=X(II)-.5
 20   II=II+NCOL1
      CALL GMINV(N,N,X,F,MR,0)
      IER=4
      IF (MR.NE.N) RETURN
      CALL MMUL(C,F,N,N,N,X)
C     INITIALIZATION OF X,F
      I=1
      DO 40 II=1,NN,NCOL
      J=II
      IF (I.EQ.1) GO TO 30
      DO 25 JJ=I,II,NCOL
      C(J)=C(JJ)
 25   J=J+1
 30   ID=J
      DO 35 JJ=II,NN,NCOL
      C(J)=DT*DOT(N,F(II),X(JJ))
 35   J=J+1
      F(ID)=F(ID)+1.0
 40   I=I+1
      DO 90 IT=1,20
      NEZ=0
      CALL MMUL(C,F,N,N,N,X)
      I=1
      II=1
```

```
      J=1
      GO TO 70
60    J=II
      DO 65 JJ=I,II,NCOL
      C(J)=C(JJ)
65    J=J+1
70    ID=J
      DT1=C(J)
      DO 75 JJ=II,NN,NCOL
      C(J)=C(J)+DOT(N,F(II),X(JJ))
75    J=J+1
      J=J-1
      DO 80 JJ=II,J
80    X(JJ)=F(JJ)
      IF (ABS(C(ID)).GT.1.E150) GO TO 95
      IF (ABS(C(ID)-DT1).LT.(ADV+TOL*ABS(C(ID)))) NEZ=NEZ+1
      I=I+1
      II=II+NCOL
      IF (I.LE.N) GO TO 60
      IF (NEZ.EQ.N) GO TO 150
      CALL MMUL(X,X,N,N,N,F)
90    CONTINUE
95    IER=1
      RETURN
150   CONTINUE
      NM1=N-1
      DO 155 I=1,NN,NCOL
      II=I+NM1
      DO 155 JJ=I,II
155   X(JJ)=C(JJ)
      IER=0
      RETURN
      END
      SUBROUTINE MMUL(X,Y,N1,N2,N3,Z)
      COMMON /MAINB/ NCOL
      DIMENSION X(NCOL,1),Y(NCOL,1),Z(NCOL,1)
      DO 3 J=1,N3
      DO 2 I=1,N1
      S=0.
      DO 1 K=1,N2
    1 S=S+X(I,K)*Y(K,J)
    2 Z(I,J)=S
    3 CONTINUE
      END
      SUBROUTINE MMUL1(X,Y,N1,N2,N3,Z)
      REAL X(1),Y(1),Z(1)
      COMMON/MAINA/NDA
      NEND3=NDA*N3
      NEND2=NDA*N2
      DO 1 I=1,N1
      DO 1 J=I,NEND3,NDA
      TM=0.
      K=I
```

```
      KK=J-I
5     KK=KK+1
      TM=TM+X(K)*Y(KK)
      K=K+NDA
      IF (K.LE.NEND2) GOTO 5
1     Z(J)=TM
      RETURN
      END
      SUBROUTINE MRIC(N,A,S,Q,X,Z,TOL,IER)
      DIMENSION A(1),S(1),Q(1),X(1),Z(1)
      COMMON/MAIN1/NDIM, NDIM1, F(1)
      COMMON/MAINB/NCOL,NCOL1
      COMMON/MAIN2/TR(1)
      COMMON/INOUT/KOUT
      ADV=TOL*1.E-6
      NN=N*NCOL
      NM1=N-1
      IND=1
      COUNT=0.
      IF (IER.EQ.1) COUNT=99.
      IF (IER.EQ.1) MR=N
      IF (IER.EQ.1) GO TO 100
      T1=-1.
300   CONTINUE
      IER=0
      COUNT=COUNT+1.
      DO 15 I=1,N
      DO 15 J=I,NN,NCOL
15    X(J)=-S(J)
      CALL INTEG(N,A,X,Z,T1)
      CALL FACTOR(N,Z,X,MR)
      IER=1
      IF (MR.LT.0) GO TO 200
      IER=0
      CALL GMINV(N,N,X,Z,MR,0)
      CALL TFR(TR,Z,N,N,1,2)
      CALL MMUL(Z,TR,N,N,N,X)
      DO 18 II=1,NN,NCOL1
      I=II
      DO 17 J=II,NN,NCOL
      X(J)=(X(J)+X(I))/2.
      X(I)=X(J)
17    I=I+1
18    CONTINUE
100   CONTINUE
      DO 16 I=1,N
16    TR(I)=-1.0
C     A+SX IS STABLE
C     POSSIBLE UNCONTROLLABILITY IF MR.NE.N
C       JIM DILLOW'S REPUTATION PRECEDES HIM
      TOL1=TOL/10.
      MAXIT=40
      DO 40 IT=1,MAXIT
```

```
      IF (IER.EQ.1) GO TO 101
      CALL MMUL(S,X,N,N,N,F)
      CALL MMUL(X,F,N,N,N,Z)
      DO 20 I=1,NN,NCOL
      II=I+NM1
      DO 20 J=I,II
      X(J)=A(J)-F(J)
20    Z(J)=Z(J)+Q(J)
101   CONTINUE
      IER=0
      CALL MLINEQ(N,X,Z,X,TOL1,IER)
      IF (IER.NE.0) GO TO 200
      L=0
      C1=0.0
      II=1
      DO 25 I=1,N
      IF (ABS(X(II)-TR(I)).LT.(ADV+TOL*X(II))) L=L+1
      TR(I)=X(II)
      II=II+NCOL1
25    C1=C1+TR(I)
      IF (ABS(C1).GT.1.E20) GO TO 50
      IF (L.NE.N) GO TO 40
      CALL GMINV(N,N,Z,F,MR,0)
      CALL MMUL(S,X,N,N,N,Z)
      DO 30 I=1,NN,NCOL
      II=I+NM1
      DO 30 J=I,II
30    Z(J)=A(J)-Z(J)
      IF (MR.NE.N) WRITE(KOUT,35)MR
 35   FORMAT(27HORICCATI SOLN IS PSD--RANK ,I3)
      GO TO 65
40    CONTINUE
      WRITE(KOUT,45) MAXIT
 45   FORMAT(27HORICCATI NON-CONVERGENT IN ,I2,11H ITERATIONS)
      GO TO 60
50    WRITE(KOUT,55)IT,T1
 55   FORMAT(30HORICCATI BLOW-UP AT ITERATION ,I2,12H INITIAL T= ,F10.5)
60    IER=1
65    RETURN
200   IF (IND.EQ.2) GO TO 250
      IF (COUNT.GE.10.) RETURN
      T1=T1/(2.**COUNT)
      IND=2
      GO TO 300
250   T1=T1*(2.**COUNT)
      IND=1
      GO TO 300
      END
      SUBROUTINE PRNT(MAT,N,M)
      COMMON/MAINB/NCOL
      REAL MAT(NCOL,NCOL)
      INTEGER N,I,J,K,M
      PRINT*,' '
```

```
      IF (M.GT.12) GOTO 2
      DO 1 I=1,N
      PRINT'(1X,12F10.4)',(MAT(I,J),J=1,M)
1     CONTINUE
      GOTO 10
2     CONTINUE
      IF (M.GT.24) THEN
      CALL PRNTXL(MAT,N,M)
      RETURN
      ENDIF
      DO 3 I=1,N
      PRINT'(1X,12F10.4)',(MAT(I,J),J=1,12)
3     CONTINUE
      PRINT'(//)'
      DO 4 I=1,N
      PRINT'(1X,12F10.4)',(MAT(I,J),J=13,M)
4     CONTINUE
10    PRINT'(///)'
      RETURN
      END
      SUBROUTINE PRNTXL(MAT,N,M)
      COMMON/MAINB/NCOL
      COMMON/MAINA/NDA
      REAL MAT(NDA,NDA)
      INTEGER I,J,K,L,M,N
      PRINT*,' '
      DO 1 L=1,M,12
      K = L + 11
      IF (M-L.LT.11) K = M
      DO 2 I=1,N
      PRINT'(1X,12F10.5)',(MAT(I,J),J=L,K)
2     CONTINUE
      PRINT'(//)'
1     CONTINUE
      PRINT'(///)'
      RETURN
      END
      SUBROUTINE PRNTSM(MAT,N,M)
      COMMON/MAINB/NCOL
      REAL MAT(3,NCOL)
      INTEGER I,J,K,L,M,N
      PRINT*,' '
      DO 1 L=1,M,12
      K = L + 11
      IF (M-L.LT.11) K = M
      DO 2 I=1,N
      PRINT'(1X,12F10.5)',(MAT(I,J),J=L.K)
2     CONTINUE
      PRINT'(//)'
1     CONTINUE
      PRINT'(///)'
      RETURN
      END
```

```
      SUBROUTINE TFR(X,A,N,M,K,I)
C
C        I - 1  GIVES  X = A
C            2  GIVES  X = A'
C            3  GIVES  X = A AS A VECTOR
C            4  GIVES  A = X   WHERE X WAS A VECTOR
C
      DIMENSION X(1),A(1)
      COMMON/MAINB/NCOL
      JS=(K-1)*NCOL*M
      JEND=M*NCOL
      GO TO (10,30,50,70),I
 10   DO 20 II=1,N
      DO 20 JJ=II,JEND,NCOL
 20   X(JJ)=A(JJ+JS)
      RETURN
 30   DO 40 II=1,N
      KK=(II-1)*NCOL
      DO 40 JJ=1,M
      LL=(JJ-1)*NCOL+II
 40   X(KK+JJ)=A(LL+JS)
      RETURN
 50   KK=0
      DO 60 II=1,JEND,NCOL
      LL=II+N-1
      DO 60 JJ=II,LL
      KK=KK+1
 60   X(KK)=A(JJ+JS)
      RETURN
 70   KK=M*N+1
      DO 80 II=1,M
      LL=(M-II)*NCOL+1
      DO 80 IJ=1,N
      KK=KK-1
      JJ=LL+N-IJ
 80   A(JJ+JS)=X(KK)
      RETURN
      END
      FUNCTION DOT(NR,A,B)
      DIMENSION A(1),B(1)
      DOT=0.
      DO 1 I=1,NR
 1    DOT=DOT+A(I)*B(I)
      RETURN
      END
      SUBROUTINE VADD(N,C1,A,B)
      DIMENSION A(1),B(1)
      DO 1 I=1,N
 1    A(I)=A(I)+C1*B(I)
      RETURN
      END
      FUNCTION XNORM1(N,A)
C     COMPUTES AN APPROXIMATION TO NORM OF A-- NOT A BOUND
```

```
      DIMENSION A(N*N)
      NN=N*N
      NP1 = N+1
      C1=0.
      TR=A(1)
      IF (N.EQ.1) GO TO 20
      I=2
      DO 10 II=NP1,NN,N
      J=II
      DO 5 JJ=I,II,N
      C1=C1+ABS(A(J)*A(JJ))
 5    J=J+1
      TR=TR+A(J)
 10   I=I+1
      TR=TR/FLOAT(N)
      DO 15 II=1,NN,NP1
 15   C1=C1+(A(II)-TR)**2
 20   XNORM1=ABS(TR)+SQRT(C1)
      RETURN
      END
      FUNCTION XNORM(N,A)
C  COMPUTES AN APPROXIMATION TO NORM OF A -- NOT A BOUND
      DIMENSION A(1)
      COMMON/MAINB/NCOL,NCOL1
      NN=N*NCOL
      C1=0.
      TR=A(1)
      IF (N.EQ.1) GO TO 20
      I=2
      DO 10 II=NCOL1,NN,NCOL
      J=II
      DO 5 JJ=I,II,NCOL
      C1=C1+ABS(A(J)*A(JJ))
 5    J=J+1
      TR=TR+A(J)
 10   I=I+1
      TR=TR/FLOAT(N)
      DO 15 II=1,NN,NCOL1
 15   C1=C1+(A(II)-TR)**2
 20   XNORM=ABS(TR)+SQRT(C1)
      RETURN
      END
      SUBROUTINE RFMPHIL(PHIL,RPHIL,NC1,NC2,NC3,NR,IC1,IC2,IC3,IR,
     *PETA,CPHIL,MM)
      COMMON/MAINA/NDA
      COMMON/MAINB/NCOL
      REAL PHIL(3,NCOL),RPHIL(3,NCOL),CPHIL(3,NDA),PETA(NCOL,NDA)
      INTEGER I,J,K,KK,L,M,MM,NC1,NC2,NC3,NR,NMODE
      INTEGER IC1(NCOL),IC2(NCOL),IC3(NCOL),IR(NCOL)
      NMODE=NC1+NC2+NC3+NR
C
C  FIRST REFORM PHILOS IAW MODE NUMBER REORDERING
C
```

```
      L=0
      DO 1 I=1,NC1
      M = IC1(I)
      DO 1 J=1,3
      RPHIL(J,I+L) = PHIL(J,M)
    1 CONTINUE
      L = L + NC1
      DO 2 I=1,NC2
      M = IC2(I)
      DO 2 J=1,3
      RPHIL(J,I+L) = PHIL(J,M)
    2 CONTINUE
      L = L + NC2
      DO 3 I=1,NC3
      M = IC3(I)
      DO 3 J=1,3
      RPHIL(J,I+L) = PHIL(J,M)
    3 CONTINUE
      IF (NR.GT.0) THEN
      L = L + NC3
      DO 4 I=1,NR
      M = IR(I)
      DO 4 J=1,3
      RPHIL(J,I+L) = PHIL(J,M)
    4 CONTINUE
      ENDIF
C
C  NOW FORM P(ETA) TO PICK THE ETAS OUT OF THE STATE VECTOR
C
      DO 60 I=1,NMODE
      DO 60 J=1,MM
 60   PETA(I,J)=0.0
      DO 61 I=1,NC1
      DO 61 J=1,NC1
      IF (I.EQ.J) THEN
      PETA(I,J)=1.0
      ENDIF
 61   CONTINUE
      K=NC1
      KK=4*NC1
      DO 62 I=1,NC2
      DO 62 J=1,NC2
      IF (I.EQ.J) THEN
      PETA(I+K,J+KK)=1.0
      ENDIF
 62   CONTINUE
      K=K+NC2
      KK=KK+4*NC2
      DO 63 I=1,NC3
      DO 63 J=1,NC3
      IF (I.EQ.J) THEN
      PETA(I+K,J+KK)=1.0
      ENDIF
```

```
 63   CONTINUE
      IF (NR.EQ.0) GOTO 65
      K=K+NC3
      KK=KK+4*NC3
      DO 64 I=1,NR
      DO 64 J=1,NR
      IF (I.EQ.J) THEN
      PETA(I+K,J+KK)=1.0
      ENDIF
 64   CONTINUE
 65   CONTINUE
C      PRINT*,' THE MATRIX P(ETA) IS '
C      PRINT*,' ****TRANSPOSED**** '
C      PRINT'(//)'
C      DO 66 J=1,MM
C   66 PRINT'(1X,20F4.0)',(PETA(I,J), I=1,20)
C
C  NOW GET CPHIL = RPHIL*PETA
C
      CALL VMULFF(RPHIL,PETA,3,NMODE,MM,3,NCOL,CPHIL,3,IER)
      RETURN
      END
      SUBROUTINE TIMEX(STM,MM,DT,XO,PDT,TMAX,X1,XC,CPHIL,XL)
      COMMON/MAINA/NDA
      REAL DT,TMAX,ABT,STM(NDA,NDA),XO(NDA),X1(NDA),XC(NDA)
      REAL CPHIL(3,NDA),XL(3)
      INTEGER I,J,MM,PDT
C
C THIS ROUTINE PROPAGATES THE STATE VECTOR IN TIME AND OUTPUTS LOS DATA
C TO BOTH THE MAIN PRINTED OUTPUT FILE AND TO A PLOT FILE.  IT GIVES
C LOS RADIUS AND DEFOCUS VS TIME TO THE PLOT FILE, BUT X,Y,Z,R AND T
C TO THE PRINT FILE.
C
      ABT=0.
C MAKE A COPY OF THE I.C. VECTOR AND USE THE COPY.
      DO 10 I=1,MM
      XC(I) = XO(I)
   10 CONTINUE
C GET THE LINE-OF-SIGHT DATA OUTPUT BY
C WRITING IT TO THE PRINTED OUTPUT,
C  AND TO TAPE7.
      WRITE(6,'(//)')
      WRITE(6,*)'   TIME       LOSX         LOSY       DEFOCUS      RADIUS'
      WRITE(6,'(/)')
C CHECK THAT MAX TIME, 'TMAX', IS NOT REACHED AND CONTINUE TO
C PROPAGATE THE STATE VECTOR AND GET LOS EVERY 'PDT' TIME INCREMENTS
C FIRST OUTPUT THE INITIAL CONDITION
      CALL VMULFF(CPHIL,XC,3,MM,1,3,NDA,XL,3,IER)
      RAD = ((XL(1)**2) + (XL(2)**2))**0.5
      WRITE(6,1000)ABT,XL(1),XL(2),XL(3),RAD
      WRITE(7,1001)ABT,RAD,XL(3)
C NOW BEGIN PROPAGATION
   20 DO 40 I=1,PDT
```

```
      CALL VMULFF(STM,XC,MM,MM,1,NDA,NDA,X1,NDA,IER)
      DO 50 J=1,MM
      XC(J) = X1(J)
   50 CONTINUE
   40 CONTINUE
C INCREMENT THE ABSOLUTE TIME AND CALCULATE LOS FOR OUTPUT
      ABT=ABT+DT*PDT
      CALL VMULFF(CPHIL,XC,3,MM,1,3,NDA,XL,3,IER)
      RAD = ((XL(1)**2) + (XL(2)**2))**0.5
      WRITE(6,1000)ABT,XL(1),XL(2),XL(3),RAD
 1000 FORMAT(1X,1F7.3,4E12.4)
      WRITE(7,1001)ABT,RAD,XL(3)
 1001 FORMAT(2X,1F7.3,2E14.6)
      IF(ABT.LE.TMAX) GOTO 20
   60 CONTINUE
      RETURN
      END
```

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Title: MODAL ASSIGNMENT EFFECTS ON DECENTRALIZED CONTROL OF A LARGE SPACE STRUCTURE

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Modern optimal control methods are used to develop a multiple-input-multiple-output controller. Focus is made on a three-controller configuration exhibiting high controllability but low observability in the first controller, a median amount of each in the second, and low controllability but high observability in the third. These characteristics are due to the technique used to suppress control and observation spillover among the controllers. A control model for large space structures, which employs full state feedback using deterministic observers, is developed and implemented in a computer simulation. The technique of spillover suppression and the conditions assuring stability of the control system are developed and implemented as well.

The simulation is tailored to address the control of the Draper-2 large flexible space structure model. The model has been used previously for optical pointing (line-of-sight (LOS)) studies. Here, position sensors and point force actuators are used to effect feedback control (regulation) of the damped unforced structural vibrations. The simulation can output both the unsuppressed and suppressed case open-loop and closed-loop eigenvalues and the LOS time response for stability and performance analysis.

With the control problem formulated for modal control, an investigation is made into the effects on time response of assigning three groups of four modes in a permutative fashion to the three controllers. A fourth residual set of eight modes is carried without spillover suppression to represent the unmodelled modes of a real structure. The groups are assembled based upon a previous investigation's results from applying modal cost analysis for LOS performance. Simple high frequency truncation is also used for comparison.

Controllers based on modal cost analysis alone are found to yield marginal stability and mediocre LOS performance due to little insight into the sensitivity of the residual modes to spillover. However, specific problem modes are readily identified by examining the results of an internal balancing analysis for modal sensitivity. Simple frequency truncation is found to give the best time response here when the modes most contributory to LOS are assigned to the controllers with more controllability. However, the relatively small quantity of modes and the overwhelmingly large relative contribution of the three rigid body modes included may obscure some conclusions. Results indicate more revealing results might be obtained if more modes are added to the model and/or if some of the residual modes are suppressed.

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