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SIMULATION OF CHAFF
CLOUD SIGNATURE
THESIS

Richard P. Fray
Captain, USAF
AFIT/GE/ENG/85D-16

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SIMULATION OF CHAFF
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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Electrical Engineering

Richard P. Fray, B.S.
Captain, USAF

December 1985

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Preface

The purpose of this study was to develop a computer algorithm capable of generating correlated random processes for simulating time-varying samples of chaff cloud radar cross sections. The need for this model arises in realistic, dynamic, electronic warfare training scenarios. The model may also be used to evaluate radar systems and their effectiveness against chaff.

The algorithm was developed on a VAX II/785 computer using a 4.2 BSD version of the UNIX operating system, the FORTRAN 77 programming language and The International Mathematical and Statistical Library (IMSL) of subroutines.

I developed an interest in electronic warfare and chaff modeling through some of my classes at AFIT and was fortunate to find a topic that matches my interests and an advisor who was enthusiastic about supporting it.

Richard P. Fray



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ABSTRACT

In this thesis, time-varying radar cross sections of chaff clouds are generated for use in radar/ECM computer simulations, under the assumption that scattering from chaff clouds is a wide sense stationary random process. For a jointly gaussian random process, second order statistics are developed from correlated samples of radar cross sections. Applicable statistical tests are performed to validate a set of generated samples. The goodness of fit of the samples compared to the prespecified density function is determined along with a comparison of the correlation coefficients from the generated set to the desired correlation coefficients. Rayleigh and Weibull statistics are also derived from gaussian variables to present an alternative way of describing the probability distributions of chaff cloud cross sections. Topics for further study are suggested.

SIMULATION OF CHAFF

CLOUD SIGNATURE

I. Introduction

Simulations of radar, electronic countermeasure (ECM), or electronic countercountermeasure (ECCM) systems involving chaff have been of interest to the military community for years in the evaluation of such systems and in electronic warfare scenarios used in training pilots, radar operators and electronic warfare officers.

The term chaff denotes a confusion type of radar ECM which employs a large number (hundreds of thousands) of resonant dipoles packaged into small cartridges (16:1). The individual chaff dipoles are manufactured in a variety of ways and from different materials including glass fibers dipped in molten silver or aluminum, but the standard material used in the production of chaff is aluminum foil cut into thin strips (5:18-14).

Individual chaff dipoles have bandwidths corresponding to about 10% of their resonant frequency, so a single cartridge of chaff usually contains several different lengths that will resonate over a wide band of frequencies. When the tubes are dispensed into the air from an aircraft, helicopter, or as a mortar round, the cartridges burst and create a "radar smoke screen" that confuses the enemy radar (5:18-16).

The radar echo produced is the result of the return from a large number of individual dipoles (5:18-10). The echo can be used to saturate a radar coverage area, break

lock on a tracking radar, or create false targets (5:18-17). Corridors of chaff can also be formed to provide friendly aircraft with areas to fly through without being detected or tracked by enemy radar.

Background

Various attempts have been made to simulate chaff clouds realistically using the various parameters associated with the clouds and their probability distributions. Studies have been made to analyze the distributions of dipole angle orientation (21), radar cross section (1, 2), and drift velocity (3). The existing methods of synthetically generating radar reflections from chaff clouds are either deterministic or at best restricted to the first order statistics which merely indicate the magnitudes of the fluctuations of the cross sections about some mean value. The deterministic methods predict the exact outcome in any given situation and often require some very involved calculations. Due to limited computer memory and time, only a few hundred dipoles can usually be analyzed in representing one chaff cloud (14:1).

The restriction to first order statistics is not very satisfactory since in a dynamic situation one has to know not only the magnitudes but also the rates of the fluctuations. On the other hand, statistical methods are not concerned with any particular situation. They predict by a probabilistic method what might happen under a given set of conditions. There is no restriction on the number of dipoles a cloud can consist of. In fact, the larger the number, the more accurate the representation will be (14:1).

To determine the rates of the fluctuations, second order statistics of the chaff cloud radar cross sections are needed. These statistics give the probability of jointly finding two values of a random variable at different times.

Problem Statement

The radar return of a chaff cloud at one instant of time will be correlated to some extent with the return at a previous time. For small time differences, the correlation will be high, and for large ones, the correlation will be low (5:18-8). To model this, the correlation function will be specified as an exponential function of the time difference. The purpose of this thesis is to develop an algorithm to generate pseudorandom samples of radar cross sections, with these correlation coefficients, for a representative chaff cloud. For a given autocorrelation function, second order statistics will be generated.

Scope

This model will remain a fairly simplified one in the respect that it will deal only with the radar cross sections of a chaff cloud. The aerodynamic and electromagnetic qualities, drift velocities, and birdnesting effects (clumping together of several chaff elements) of individual dipoles, and the transient effects of cloud blooming immediately following dispensation of a chaff cartridge will not be addressed.

Assumptions

The primary assumption made is that radar backscattering from chaff clouds is a wide sense stationary random process. This implies that the statistics of one dipole or cloud will not change from one time instant to another, and that the autocorrelation is dependent only on the time difference at which the dipole or cloud is evaluated. It also implies that the average or mean value of the statistics is independent of time. This is justified due to the fact that chaff has a very high aerodynamic drag and therefore will not alter its position very much between radar returns.

Each elemental dipole will be assumed to have random phase and amplitude for two reasons. The orientation of the dipole may change due to rotation of the dipole, and the distance between the radar and the dipole center may change. This implies the cloud will

also have random phase and amplitude. Finally, it is assumed the separation between dipoles is great enough to prevent any mutual coupling or interaction between adjacent dipoles. The effect of the random phases and amplitudes merely suggests that the model will generate a radar cross section that is uniform throughout a chaff cloud.

General Approach

The model entails generating a set of correlated pseudorandom samples from a gaussian probability density function. Two exponential correlation functions, as shown in equations 1 and 2, will be specified for each set of samples.

$$\rho(\tau) = \exp [- |\tau|] \quad (1)$$

$$\rho(\tau) = \exp (- \tau^2) \quad (2)$$

Where τ corresponds to the pulse repetition interval of a radar system and represents the time between consecutive samples of the radar cross section. Several sets of data will be generated with different quantities of cloud samples, for both correlation functions, to form a basis of comparison and validation for the model. The number of samples will be limited to at most 100 because in using pulse integration to obtain additional data, rarely are more than 100 pulses integrated.

For each set of samples, a chi-squared goodness of fit test will be performed to see if the generated samples do indeed fit the theoretical distribution they were originally generated from. Then a comparison will be made between the autocorrelation coefficients of the generated samples and the originally specified exponential correlation functions. Finally, Rayleigh and Weibull samples will be derived from the gaussian samples.

Sequence of Presentation

First the existing theories of representing chaff cloud returns, methods developed for generating correlated random variables, and the derivations of Rayleigh and Weibull statistics from gaussian variables are discussed in chapter 2.

Then the algorithms for generating the correlated random samples is flowcharted, developed, and discussed in chapter 3.

Chapter 4 presents validation of the model through applicable statistical testing of the generated sets of data.

Finally, conclusions are made and recommendations for further study suggested in chapter 5.

A review of the applicable probability theory and a computer listing of the algorithm are presented as appendices.

II. Existing Theory

In this chapter, existing theories developed for generating correlated random variables and some methods used to represent chaff clouds are reviewed. Also the derivations of Rayleigh and Weibull statistics from gaussian variables are shown.

Chaff Models

One model reported in the literature was developed at the Georgia Institute of Technology in 1977 (4). A preliminary chaff radar cross section (RCS) model, developed to estimate the RCS of deployed chaff, was improved and extended to compute the mean RCS of deployed chaff bundles and their variations about the mean value known as scintillations (4:1). The Georgia Tech study proposed two methods to simulate the problem of chaff pattern backscatter scintillation, the Rayleigh distributed draw method, and the dynamic time series model (4:3). Each model's major features will be discussed briefly.

Backscatter power from a chaff cloud is not deterministic. Rather, it varies around some mean value. It can be thought of as an analog process being digitally sampled by a pulsed radar system (4:10). The Rayleigh draw model entailed inputting parameters such as the RCS mean, the period of the highest scintillation rate, and the observation time desired. A draw was then made from a Rayleigh distributed random number generator and processed through an interpolation formula. The interpolation formula allowed the user to determine the RCS at any instant of time between samples or at any rate slower than the sampling rate. The resulting output was an RCS chaff pattern generated as a function of time.

Amplitude fluctuations have been shown to fit a power Rayleigh distribution (15), so this method is realistic for most chaff simulations. Other distributions may be used in

different applications with satisfactory results. To ensure accurate sampling, draws from the Rayleigh generator must be made at the Nyquist rate or higher, corresponding to the highest scintillation rate. But care must be taken because inputting a rate higher than the maximum scintillation rate would introduce errors due to the inconsistency of specifying a maximum and then exceeding it.

A problem may also arise with the validity of the generator draws if the statistics of the RCS change with time. During the bloom period this may present a serious problem, but for first order models, if the mean RCS was assumed to vary slowly with respect to the rate at which a new RCS value is generated, then the results would be considered valid (4:14-18). No testing was done to compute the correlation of the RCS from pulse to pulse, so no comparison could be made between the correlation of the generated variables and the desired correlation.

A more mathematically based method developed in this Georgia Tech report was the dynamic time series model. A time series is a sequence of successive observations or simulations of a stochastic or random process. The observations are usually equally spaced and could represent the RCS return of a chaff cloud from consecutive radar pulse returns (4:19). The time series models basically use previous samples to determine future samples. This model is more rigorously developed but could require longer computer processing time than the Rayleigh draw method (4:34).

Another chaff model, also developed at Georgia Tech, was the Chaff Theoretical/Analytical Characterization and Validation Program (7). This model incorporated the aerodynamic and electromagnetic behavior of chaff dipoles and modeled the propagation of received signals through a radar system (7:1).

This model was a very detailed one, taking into consideration dipoles of different lengths, and weighting chaff returns according to a dipole's angular location within the antenna radiation pattern (7:1-2). The approach was to develop a quasi-deterministic

description of the cloud. Further assumptions made concerning the dipole density function resulted in integrals that could not be evaluated, not even numerically. These integrations were too time consuming on a computer and that made the model unworkable.

They suggested future efforts be devoted to making approximations of the volume integral and empirical descriptions of the dipole density variations (7:4). Also better experimental data was needed to help predict and validate the model.

Other studies exist taking different approaches including Wickliff's and Garbacz' (20), which included mutual coupling among dipoles, and Schiff's (18), which modeled chaff decoy placement in relation to antiship missile defense problems of the Navy.

Methods for Generating Correlated Random Numbers

Several methods were reported in the literature for generating correlated random number sequences. Some of the algorithms were strictly designed to generate gaussian variables while others could be used to generate any probability distribution. One method, developed by Nawathe and Rao, used the theory of optimal linear prediction to generate cross correlated random number sequences (11).

The technique first required generation of two random number sequences, for example $\underline{X}$ and $\underline{Z}$, each having the desired probability distribution. The vector $\underline{Z}$ was used as a parent population for generating a correlated vector $\underline{Y}$. Then for each element of the vector $\underline{X}$, $x(i)$ ($\underline{X} = [x(1) \dots x(n)]$), a calculation of the best linear predictor of $\underline{Y}$ was made. For every $x(i)$, a search was made through $\underline{Z}$ to find a sample which was close to $x(i)$, closeness defined as a specified mean square error interval. If no match was found, a sample was picked from $\underline{Z}$ which had the lowest mean square error, away from $x(i)$, and the algorithm assigned this sample to the corresponding $y(i)$ (11:98).

This method is simple and straight forward and requires only knowledge of the marginal densities of $\underline{X}$ and $\underline{Z}$. Using this method, several sets of data were tested with

different desired correlation coefficients and probability densities. The estimate of the coefficient of correlation between X and Y was defined (11:97) as

$$\rho = (1/nr) \sum_{j=1}^{nr} \{ [x(j) - \mu_x] [y(j) - \mu_y] / \sigma_x \sigma_y \} \quad (3)$$

A table of the calculated correlation coefficients showed the difference from the desired coefficients increased as ρ approached zero, and was on the order of only 1 to 2 percent for a correlation of $\rho=.8$ (11:99). This confirmed the statement in the paper that results tended to improve with increasing magnitude of the correlation coefficient and sample size. The model was further validated through derivations of the expressions for the expectation and variance of the correlation coefficients (11:96).

This technique proved satisfactory for three typical distributions, normal, exponential, and uniform, and thus appears applicable to any continuous distribution. Although it is only an approximation for nonnormal distributions, the results of the Monte Carlo simulation here are valid (11:101).

Another method, developed by Li and Hammond, involved transforming a vector of independent gaussian random variables into a vector of gaussian variables with a specified predistorted correlation coefficient matrix (8). First, a vector of independent gaussian random variables was generated. Then this vector was multiplied by a linear transformation matrix which resulted in correlated gaussian variables possessing the predistorted correlation coefficients. Finally, a nonlinear transformation was performed to create a vector of correlated random variables with the desired univariate probability density and correlation coefficients.

The difficulty in this technique lies in finding the nonlinear transformation. For the desired probability density function, a set of transformations involving inverse cumulative density functions and the error function are required (8:558). The calculation of the predistorted correlation coefficients also requires complicated integral evaluations.

However, data produced by the algorithm passed all statistical tests and generated correlation coefficients with less than .1% error as compared to the desired coefficients (8:559). Therefore, this method proved itself mathematically sound though it may be too complex for the task at hand.

The technique chosen for the chaff cloud signature model was developed by Scheuer and Stoller (17) and a similar study done by Moonan (10). This method proved the most straight forward one found in the literature and was well suited for machine computation.

The technique first required the generation of a vector of independent normal random variables with zero mean and variance of one. The resulting vector will also have a zero mean and variance-covariance matrix specified by the user (17:278). To obtain the correlated vector, the independent vector is multiplied by a linear transformation matrix, created from the covariance matrix. The formulas used to generate the transformation matrix and the correlated variables, will be presented in chapter 3 along with a detailed explanation of the entire algorithm (17:279).

Rayleigh and Weibull Derivations

The purpose of this section is merely to present a method of obtaining Rayleigh and Weibull distributed random variables from independent gaussian variables. These densities are often used to define the distributions of chaff cloud radar returns, and are therefore pertinent to this study.

First, two independent identically distributed gaussian random variables, X and Y , each with zero mean, and identical variances s , are generated. Then, X and Y are each squared and added together to form the variable Z .

$$Z = X^2 + Y^2 \quad (4)$$

Finally, the variable Z is raised to the power k .

$$\underline{W} = Z^k \quad (5)$$

Now, by specifying k , the variable $\underline{W}$ can be transformed into one of several different distributions. For example, if $k=1$, $\underline{W}$ becomes exponential, if $k=1/2$, $\underline{W}$ becomes Rayleigh, and if $k=1/3$, $\underline{W}$ becomes Weibull although the exponential and Rayleigh distributions are both just special cases of the Weibull distribution. The derivation of these results follows.

The moment generating function was used to transform the gaussian variables and is shown in equation 6 for $Z=X^2$ (13:115).

$$\begin{aligned} M_z(t) &= E[\exp(tz)] \\ &= E[\exp(tx^2)] \\ &= \int_{-\infty}^{\infty} f(x) \exp(tx^2) dx \end{aligned} \quad (6)$$

After making some substitutions and evaluating the expected value integral, the result is simplified to equation 7.

$$M_z(t) = [1/(1 - 2st)]^{1/2} \quad (7)$$

This moment generating function represents a gamma function with parameters $a=1/2s$ and $r=1/2$ (9:195). If the substitution of $s=1$ is made, then $\underline{Z}$ has a chi-squared distribution with one degree of freedom. This simplification can be justified by assuming the generated gaussian variables have a variance of one, which was intended when generating the gaussian variables. Since $\underline{X}$ and $\underline{Y}$ are identically distributed, $\underline{Z}=\underline{Y}^2$ can also be represented by a moment generating function of a chi-squared distribution with one degree of freedom. Meyer proved that a sum of n chi-squared distributed random variables each with one degree of freedom will result in another chi-squared variable with n degrees

of freedom (9:201). It can further be shown that this distribution is exponential with parameter $a=1/2$.

$$f_z(z) = (1/2) \exp(-z/2) \quad (8)$$

Finally, by transforming $Z=X^2+Y^2$ into $W=Z^k$ using the Jacobian (13:95), equation 9 results.

$$\begin{aligned} f_w(w) &= f_z(z) |dz/dw| \\ &= f_z(w^{1/k}) |dz/dw| \\ &= (1/2) \exp[-(w^{1/k})/2] | (1/k) w^{(1-k)/k} | \\ &= (1/2k) w^{(1-k)/k} \exp[-(w^{1/k})/2], \quad w, k > 0 \end{aligned} \quad (9)$$

The Weibull density has the general expression

$$f_w(w) = abw^{b-1} \exp(-aw^b) \quad (10)$$

If the parameters are set to $b=1/k$ and $a=1/2$, equation 9 results. Then by substituting different values in for k , the densities mentioned previously will result.

III. New Model Development

In this chapter, the algorithm will be discussed in detail to help the reader adapt it to his own application. The algorithm was written in the Fortran 77 programming language on a VAX 11/785 computer. A flowchart of the algorithm is shown in figure 1.

First, several parameters have to be specified according to the user's desires. This model assumed a generic pulsed radar system for receiving the cross section returns, so a pulse repetition frequency (PRF) was specified. τ was used in the program to designate the time interval between radar pulses and is equal to the reciprocal of the PRF (19:2-3). In this model, the value for τ was chosen as .001 corresponding to a PRF of 1000 Hz. This is a reasonable value for a low to medium PRF radar and corresponds to an unambiguous range of 100 nautical miles (19:2). Also, the number of consecutive pulse returns desired, nr , was specified. Three trials of nr equal to 20, 50, and 100 were used to form a basis of comparison.

The first step in the algorithm required generation of a vector of independent gaussian random numbers, called $\underline{Y}$, with zero mean and variance one (17:278). Throughout this algorithm, several subroutines from the International Mathematical and Statistical Library (IMSL) of subroutines were utilized (6). These subroutines are also written in the Fortran programming language. The subroutine GGNML was called twice to generate two vectors of length nr random numbers. One was used to generate the correlated sequence and the second was used in conjunction with the first to create the Rayleigh and Weibull variables discussed in chapter two.

The next step involved selecting the desired variance-covariance matrix. Two expressions given by equations 1 and 2 were used to represent the correlation between time samples. These exponential functions slowly decrease with small time difference and

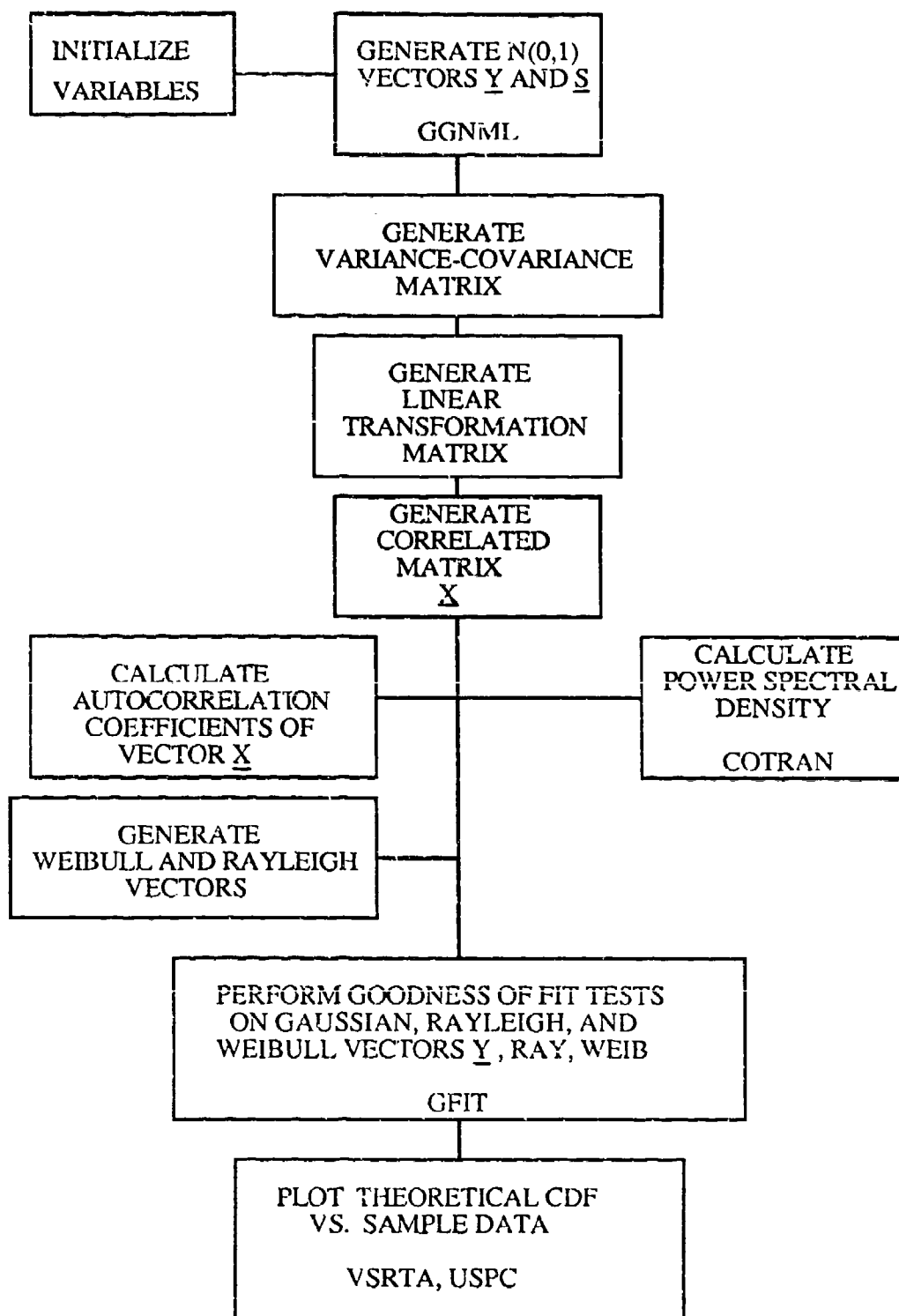


Figure 1. Flowchart of Algorithm

therefore model a high correlation very well.

The section of the program that computes the correlated vector $\underline{X}$ is repeated twice with a different variance-covariance matrix each time. The two matrices are generated by using the two exponential correlation functions described in chapter one. This will further provide a comparison of different correlations often used and hopefully help the user in selecting the appropriate form.

A transformation matrix was needed to change the independent vector into a correlated sequence. The method developed by Scheuer and Stoller (17), as presented briefly in chapter two, was used. A lower diagonal transformation matrix $\underline{C}$ is calculated from the variance - covariance matrix $\underline{CC}$ using the following formulas.

$$C(i,1) = CC(i,1)/[CC(1,1)]^{1/2} \quad 1 < i \leq nr \quad (11)$$

$$C(i,i) = [CC(i,i) - \sum_{k=1}^{i-1} C(i,k)^2]^{1/2} \quad 1 < i \leq nr \quad (12)$$

$$C(i,j) = \{CC(i,j) - \sum_{k=1}^{j-1} [C(i,k) * C(j,k)]\} / C(j,j) \quad 1 < j < i \leq nr \quad (13)$$

$$C(i,j) = 0 \quad i < j \leq nr \quad (14)$$

Finally, to generate the correlated vector $\underline{X}$, the transformation matrix $\underline{C}$ is multiplied by the independent vector $\underline{Y}$ as shown in equation 15 (17:279).

$$X(i) = \sum_{j=1}^i C(i,j) * Y(j) \quad i = 1,2,\dots,nr \quad (15)$$

The next section of the program calculated the autocorrelation of the vector $\underline{X}$. This was done to determine the degree of correlation between the samples. The autocorrelation was computed for nr time lags (the entire sequence) through use of equation 16 (22:69).

$$R_x(i) = (1/nr) \sum_{j=1}^{nr-i} X(j) X(j+i) \quad i = 1, 2, \dots, nr \quad (16)$$

The power spectral density (PSD), one of the second order statistics, was also calculated from the correlated sequence. The purpose of this computation was to obtain an approximate bandwidth of the fluctuations of the radar cross sections. The cosine transform technique was used because it was simple to adapt to machine computation and it gives the closest result to the fast fourier transform (FFT) without the complexity (12:137-139). Subroutine COTRAN performed the computations to produce a normalized magnitude of the power spectrum. The equations used in the cosine transform are presented below.

$$\text{spectr}(k \Delta f) = \tau \sum_{i=1}^{nr-i} x(i) \cos(2\pi ik/nr) \quad k = 0, 1, \dots, nr/2 \quad (17)$$

$$\Delta f = 1/(nr * \tau) \quad (18)$$

Next, the Rayleigh and Weibull variables were generated. The derivations of these variables were discussed in chapter two.

At this point, the model has generated a set of data and performed its transformations and calculations. The last major section of the program dealt with testing the variables for validation purposes. The IMSL subroutine GFIT was called three times to test the goodness of fit of the three generated vectors y, ray, and weib to their theoretical gaussian, Rayleigh, and Weibull distributions. A graph of each sample vector is also plotted vs. the theoretical cumulative distribution function with 95 percent confidence bands using IMSL subroutine USPC. The IMSL subroutine VSRTA merely sorts the vector into ascending order for graphing purposes.

The program also converts the uncorrelated vector $\underline{Y}$ and the correlated vector $\underline{X}$ into vectors with means of 40. This was done to demonstrate a chaff cloud simulating a decoy target with an average RCS of 40 square meters.

IV. Model Validation

The purpose of this chapter is to generate several sets of data and validate the model through statistical testing and by making observations on the data.

First, the program generated independent gaussian vectors with means of zero and variances of one. Three tests were performed for each value of *nr* equal to 20, 50, and 100. Each test used a different seeding number to drive the IMSL random number generators. Table I shows the results of these tests.

TABLE I
Comparison of Means and Variances

nr	test	dseed	mean	variance
20	1	999348	.0000000	.961984
	2	60259	.0000000	.860671
	3	565553	.0000000	1.034006
50	1	39814055	.0000000	.967838
	2	24681055	.0000000	.887949
	3	872257	.0000000	1.206430
100	1	7866543	.0000000	1.242569
	2	629895	.0000000	1.055229
	3	2833745	.0000001	1.073428

As expected, the means are all virtually zero due to the fact that the program subtracts out the mean of the initially generated random numbers. However, the variances fluctuate around one but tend to approach one as *nr* increases. This follows from basic probability theory that as the number of samples increases, the mean tends to approach a constant number. The variance also tends to fluctuate less dramatically as the number of samples increases. So the model behaves well in producing vectors of gaussian random numbers with zero mean and variances of one.

Next, the model created the correlated variables. To show the effect of the correlation, two graphs were generated to compare the independent variables with the correlation, two graphs were generated to compare the independent variables with the

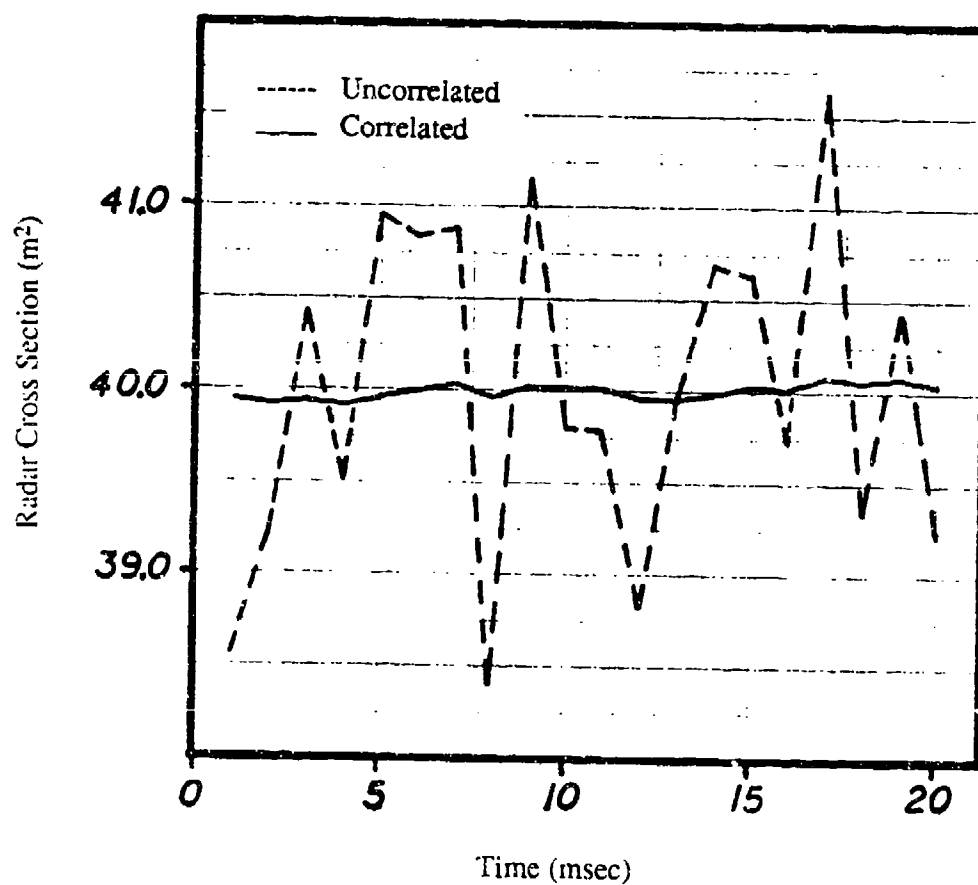


Figure 2. Correlated vs. Unrelated Vectors For nr=20

correlated variables for nr equal to 20 and 50. Figures 2 and 3 show these relationships. A similar effect can be shown for nr equal to 100.

The effect of the correlation is very evident and thus confirms that the model is indeed smoothing out the fluctuations in a dramatic way.

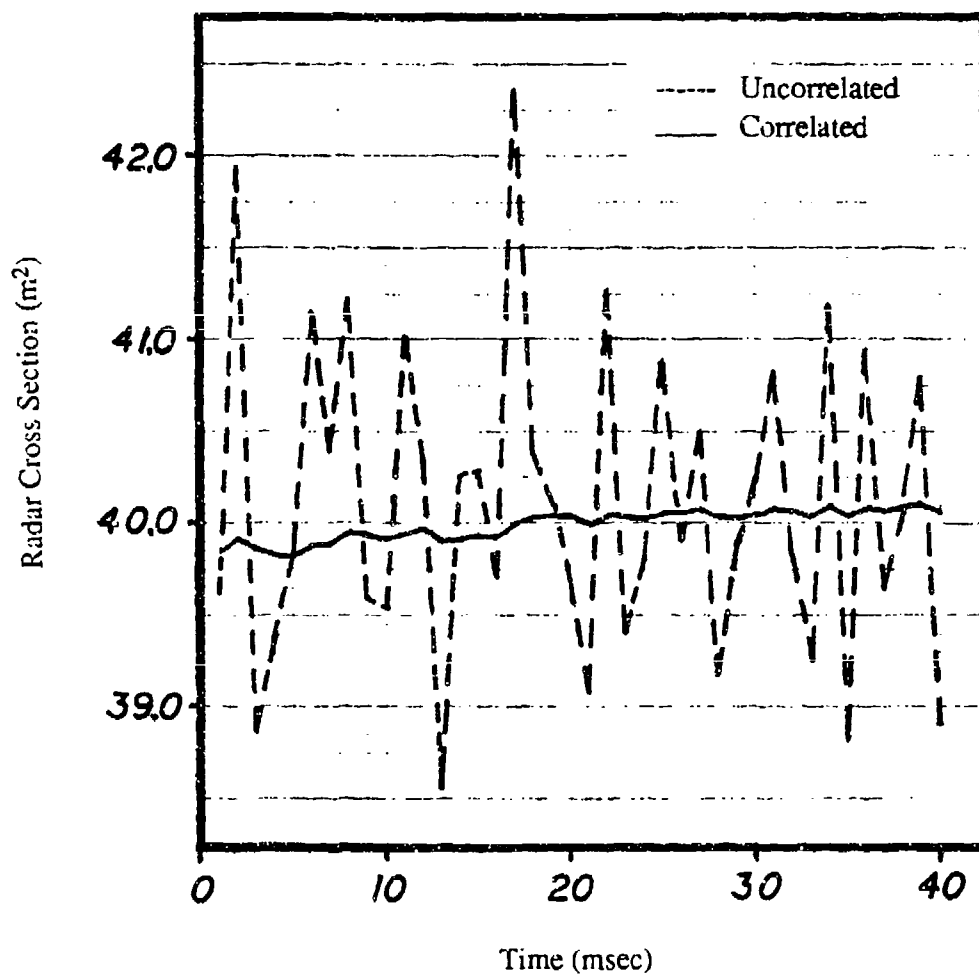


Figure 3. Correlated vs. Uncorrelated Vectors For $nr=50$

The next section of the program computed the autocorrelation of the correlated samples to provide a numerical validation of the data graphically represented in figures 2 and 3. Two graphs were produced for n_r equal to 20 and 100 in figures 4 and 5.

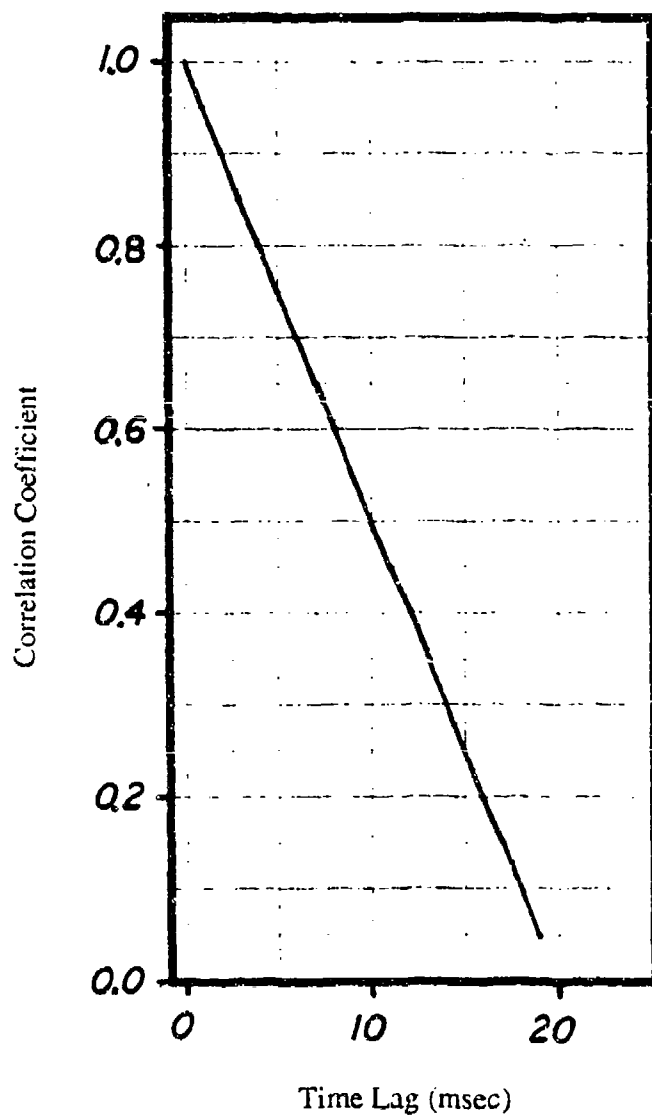


Figure 4. Autocorrelation vs. Time Lag for $n_r=20$

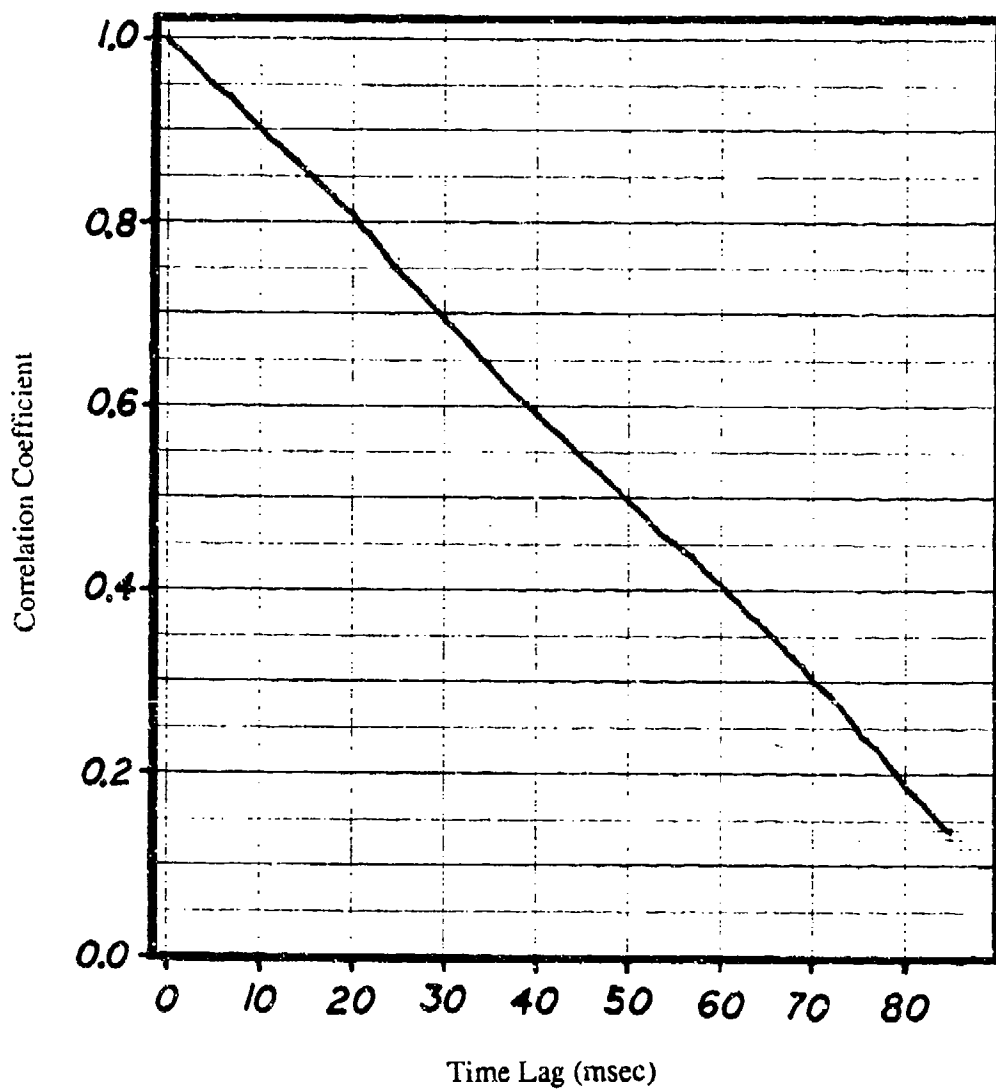


Figure 5. Autocorrelation vs. Time Lag for $n_r=100$

A few quick calculations can show that using equation 1 to compute the variance-covariance matrix will generate numbers in the range of 1 to .90574 for time lags of 0 to .099 seconds. This corresponds to 99 time intervals using a 1000 Hz PRF radar as

specified in this model. Also, the numbers generated by equation 2 will decrease from 1 to .999901. Both sets of numbers decrease in a nearly linear fashion. This may seem wrong at first since both functions are exponential. But it is easy to show that using only .099 time units is a very small portion of the exponential curve and this portion near zero closely approximates a linear function. Thus, the autocorrelation curves in figures 4 and 5 accurately portray the true correlation that was specified in the beginning.

The graphs also show that the vectors exhibit a correlation time, the time required for the autocorrelation to decrease to zero, that is related to the number of samples used. For example, 19 msec for 20 samples, 49 msec for 50 samples, etc. This is due to the fact that a limited number of samples was "windowed" out of an infinitely long random process. A larger number of samples would result in a correlation time that was independent of the sample length as expected. These values are still reasonable though taking into consideration the dynamics of a chaff cloud and the random nature of the dipole orientation, drift velocity, and phase variations. Although these parameters were not directly modeled into the simulation, the results show a realistic behavior in the relation of one cross section to another.

Furthermore, a comparison was made between the autocorrelation functions generated by equations 1 and 2. The function generated with equation 2 was slightly greater than the first, but the difference was at most 3-4% and represented no significant advantages over the equation 1 function.

The power spectral density (PSD) was computed by the program next. Figures 6, 7, and 8 show the PSD for n_r equal to 20, 50, and 100 respectively.

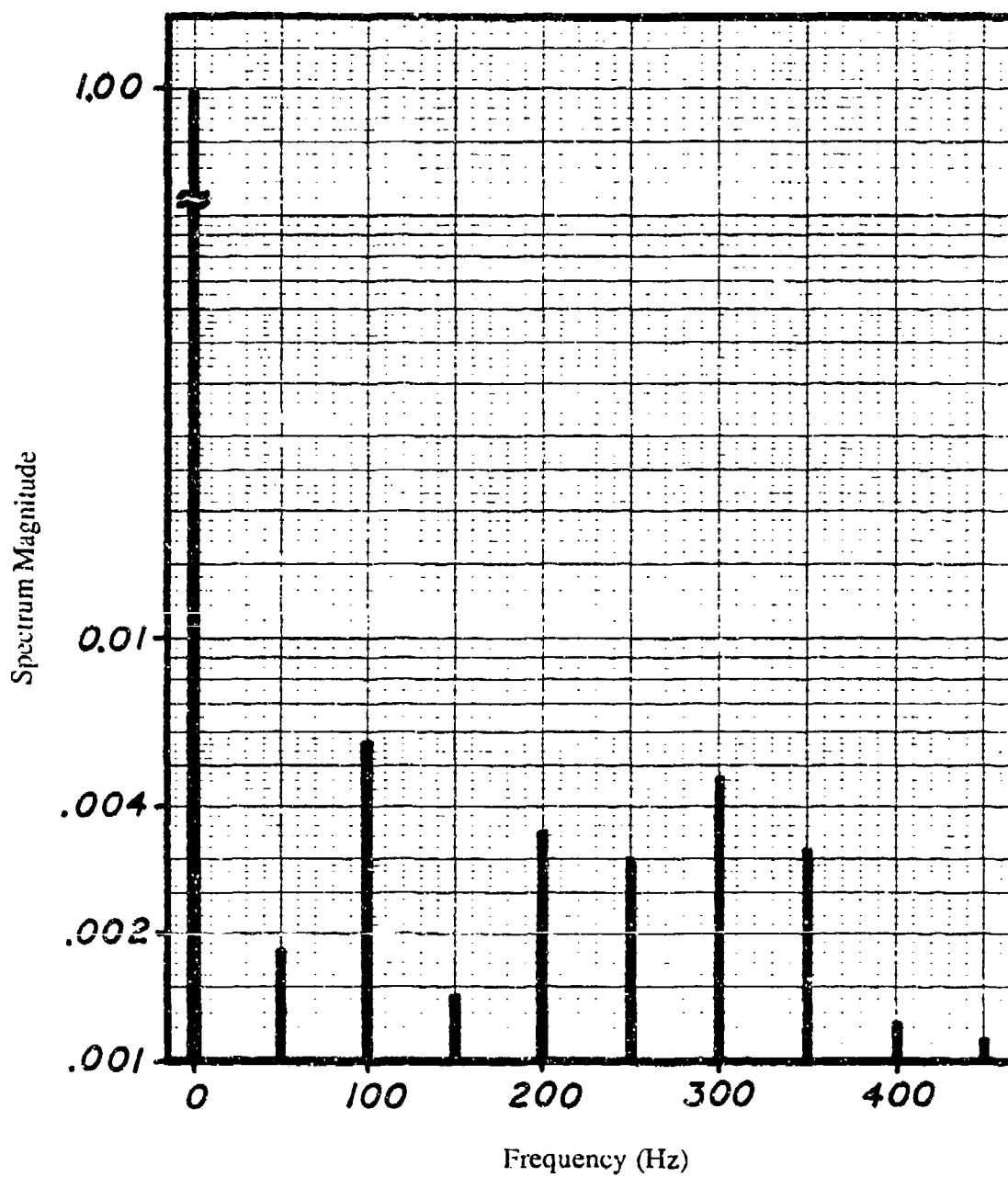


Figure 6. Power Spectral Density For $nr=20$

Again it can be seen that as n_r increases, the graphs tend to display a better representation. In this model, the power should be concentrated around zero because the PSD represented the degree of fluctuations that the mean value undergoes. After correlation of the random variables, the fluctuations should be very small, and the graphs

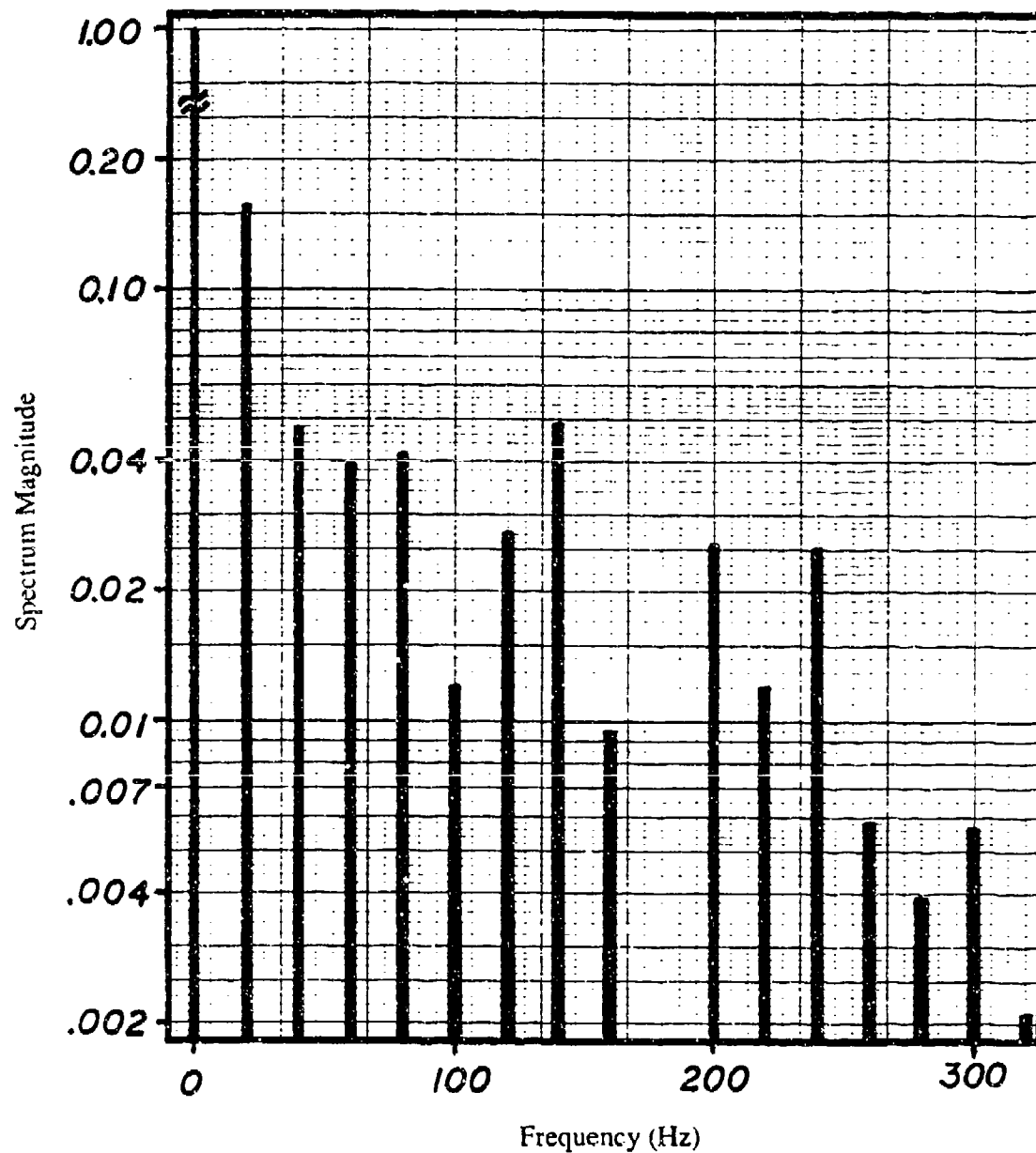


Figure 7. Power Spectral Density For $n_r=50$

show that they are. A comparison was also made concerning the power spectral densities generated by equations 1 and 2. A quick computation can show that the values generated by equation 2 were 3 decibels lower than the equation 1 values. This is not surprising

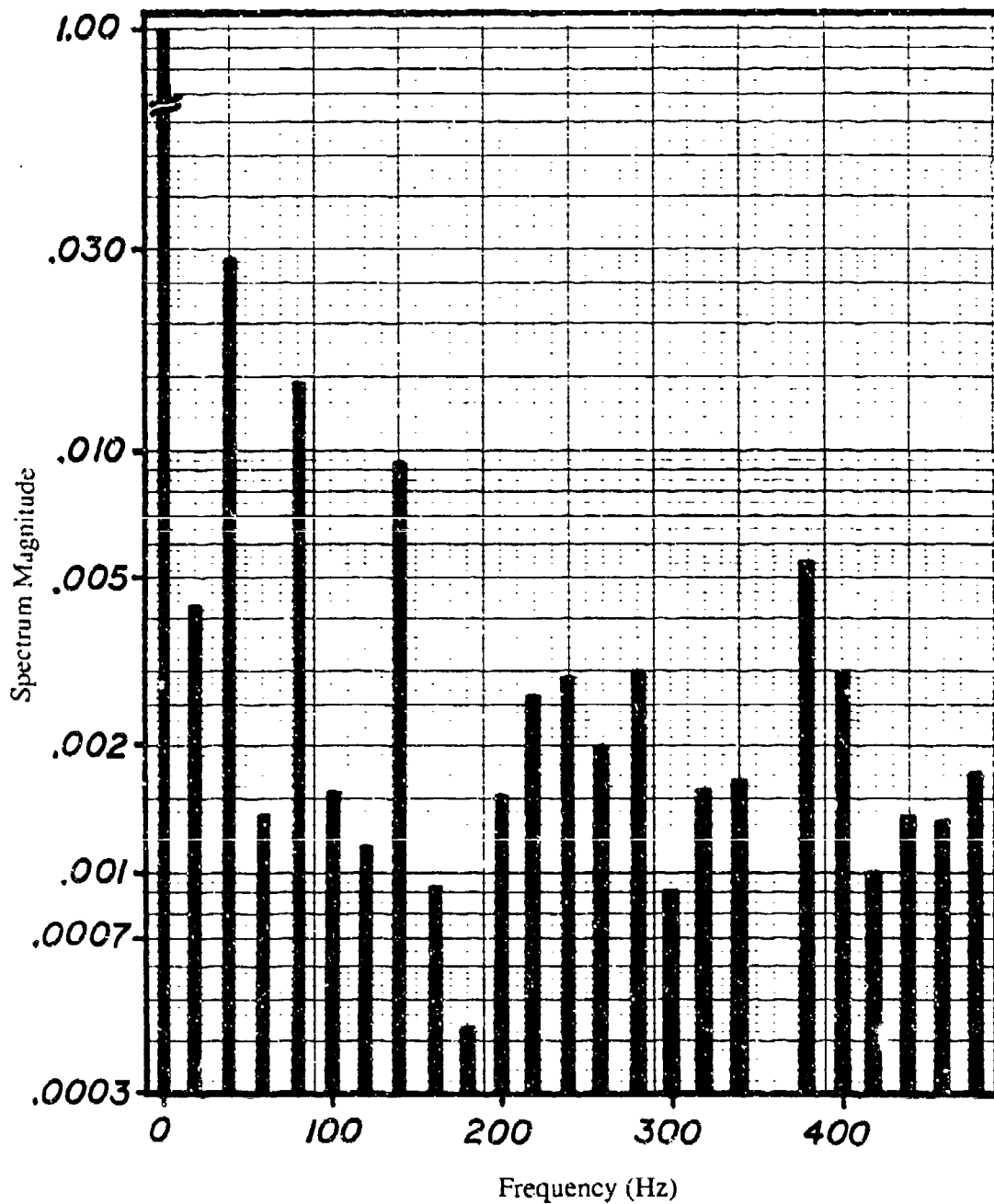


Figure 8. Power Spectral Density For nr=100

since equation 2 is just the square of equation 1. On a normalized magnitude plot, the values are less than or equal to 1 which would result in a 3 decibel loss when the value is squared instead of a gain, as might be expected. Again no significant advantages appear from using equation 2.

The last section of the algorithm computed the goodness of fit of the generated sample gaussian, Rayleigh, and Weibull random variables. Three tests were again performed with different seeding numbers for each nr equal to 20, 50, and 100. Table II displays these results.

TABLE II
Comparison of Goodness of Fit Tests

nr	dseed 1	dseed 2	normal	Weibull	Rayleigh
20	56347856	45678	.6594	.7047	.0786
	49	65478	.3618	.4493	.1116
	2938445	864120	.3080	.8495	.1991
50	765449	8237126	.0077	.0186	.0719
	434819	23569	.1935	.0905	.0992
	76239	49876123	.0591	.0160	.0008
100	99267	8839212	.3286	.0970	.1847
	234598723	102	.1197	.1374	.0215
	8092	5454	.5724	.2214	.4481

One problem arose when computing these values. It is easily seen that some values are quite good (approaching 0), implying high accuracy in the generated variables. But others were computed around 0.5 and even higher. It appeared that the chi-squared statistic was highly dependent on the seeding value. Also, the number of equiprobable tally cells influenced the chi-squared statistic. Care should be taken in specifying this parameter. It was observed that using the same number of tally cells for the Weibull and Rayleigh tests resulted in identical chi-squared statistics. In summary, this did not appear to be a dependable test because the statistic varied so much with the input parameters.

An alternative test performed was graphing a sample vector vs. its theoretical cumulative distribution function. This was done with the IMSL subroutine USPC. This subroutine plotted the sample values, theoretical values, and a two-sided 95% confidence band around the sample values. Every data set generated produced a graph that was well behaved inside the confidence bands. These graphs are presented in figures 9, 10, and 11. This test was deemed much more stable and suitable for confirming the distribution of the generated variables.

This concludes the intended validation of the model with the applicable tests and comparisons. The final chapter will draw some conclusions from these tables and figures and make some suggestions for improving the model.

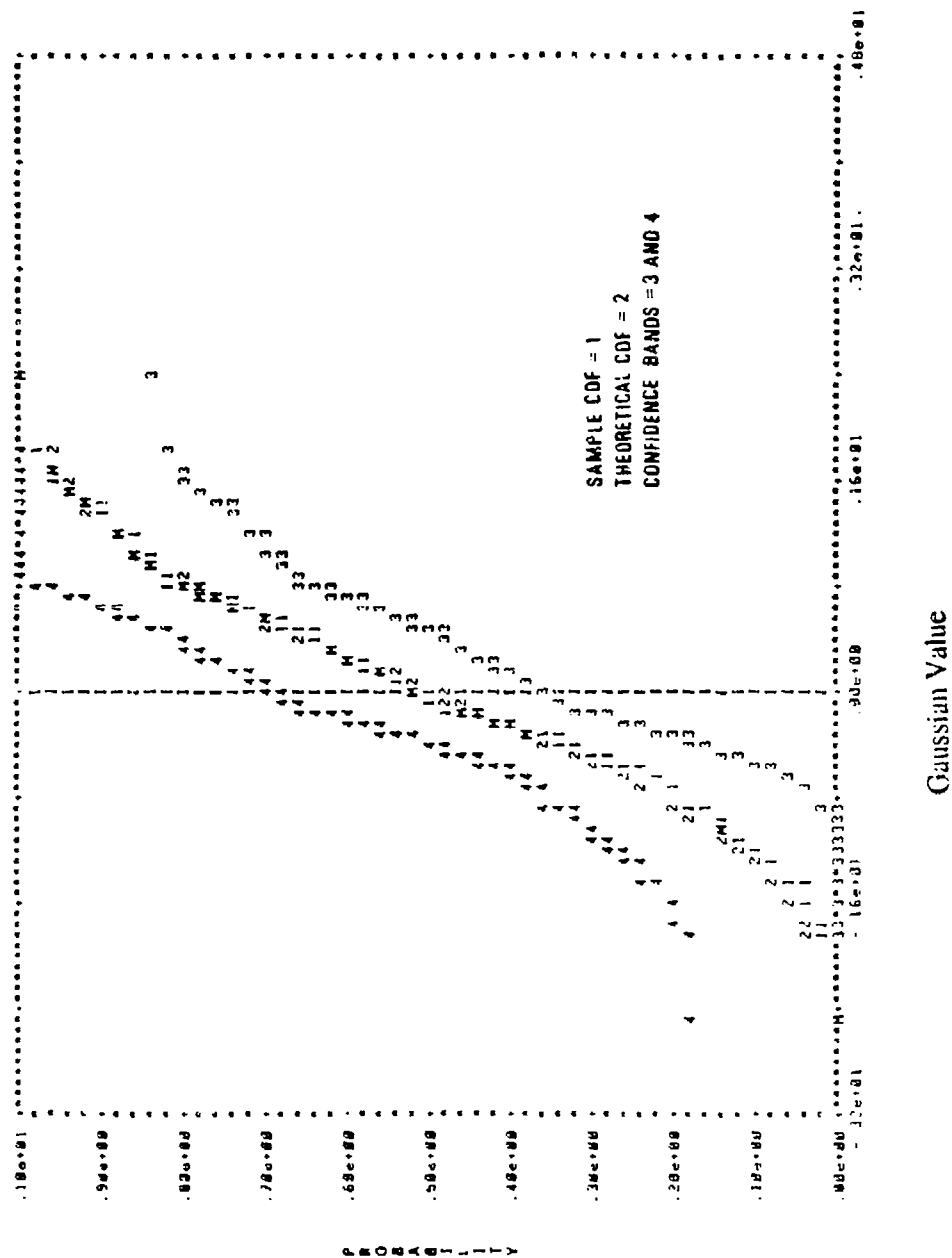


Figure 9. Sample Gaussian Vector vs. Theoretical Distribution for $n=100$

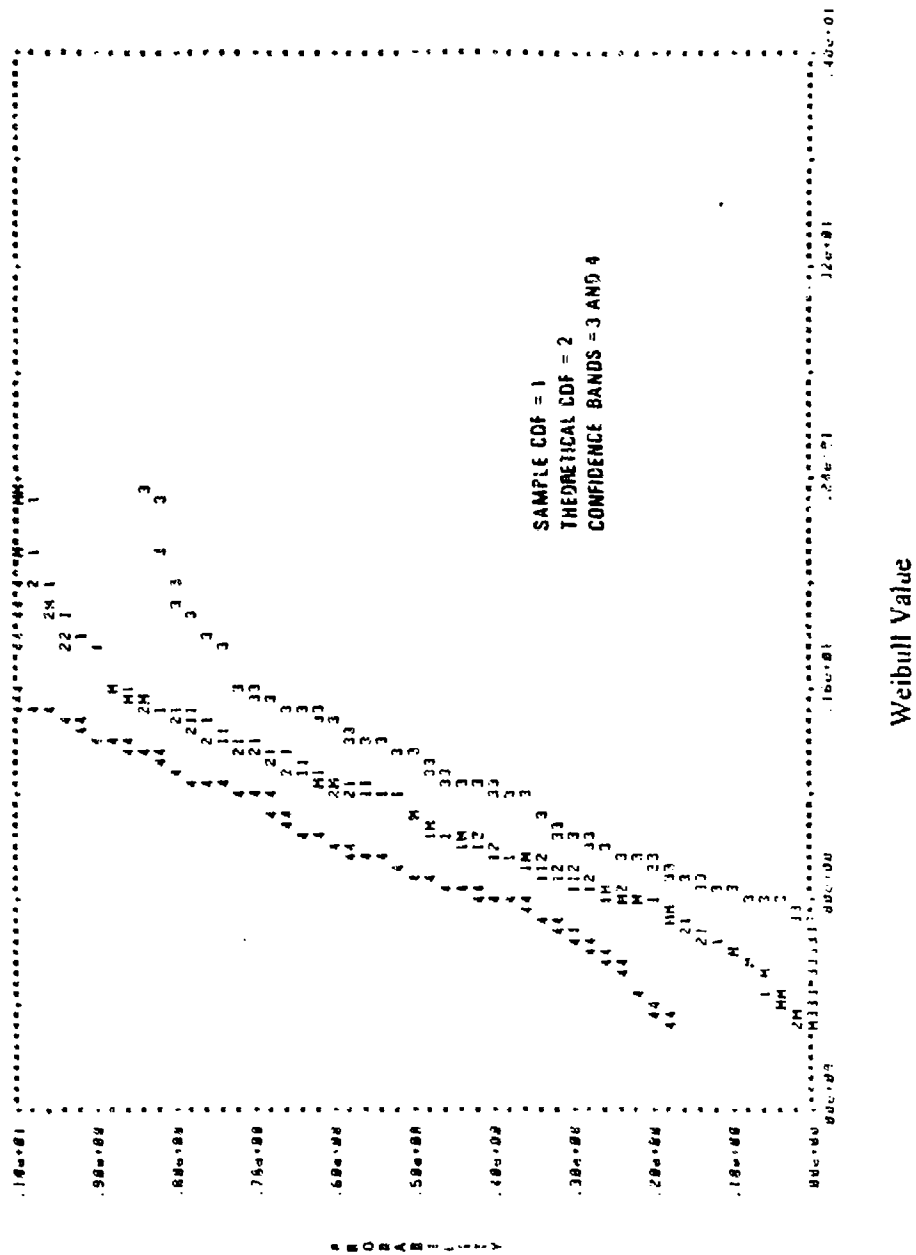


Figure 10 Sample Weibull Vector vs. Theoretical Distribution For $n=100$

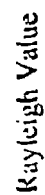


Figure 11. Sample Rayleigh Vector vs. Theoretical Distribution For $nr=100$

V. Conclusions and Recommendations

Conclusions

The purpose of this study was to develop a computer algorithm capable of generating correlated gaussian random sequences. These sequences could be used to represent consecutive chaff cloud radar cross sections in electronic warfare training scenarios. Several sets of data were generated by the model and subjected to various tests and observations.

The model proved to be well behaved in all areas tested except the numerical goodness of fit tests. Several inconsistencies were noted involving the relationship between the input parameters to the IMSL subroutine GFIT and the resulting chi-squared statistics. It was concluded that this test is not adequate for this model because the statistic varied so dramatically with the input seeding numbers and the number of tally cells specified. This problem was offset by using a graphical method of determining the goodness of fit. The data produced by the model was declared satisfactory by falling within the 95% confidence bands on all data generated.

The model realistically generated the independent gaussian variables required and the corresponding Rayleigh and Weibull variables that were derived from the gaussian numbers all proved to be well behaved. The large fluctuations present in the independent variables were transformed into smoothly varying ones just as the model was designed to do.

Furthermore, the autocorrelation and power spectral density properties followed from the predicted theories. The autocorrelation should have been linear, and the PSD should have been concentrated around zero. It was noted as n increased, these computations became steadier and more well behaved.

In summary, this model accomplished all of its initial objectives satisfactorily. Several improvements can be made and there are areas which could be studied more thoroughly as discussed in the next section.

Recommendations

Further study should be done into the problems discussed concerning the goodness of fit tests. An analysis might be performed to determine the optimum number of tally categories that would result in the most consistent chi-squared statistic results.

A study could be done to see if an upper or lower limit exists on the PRF that may inject some inconsistencies into the model.

Determine if there is a more efficient or more accurate algorithm available for computing the power spectral density.

Consider what additional tests the model could be exposed to that might simulate a training environment and determine if the model is realistic in simulating real world dynamics.

The model could be used as suggested earlier to test other radar systems and their effectiveness against chaff, or it could be used to evaluate a model of a radar system.

Introduce the blooming effects of a dispensed chaff cartridge into the model.

Appendix A - Probability Theory

In this appendix, a brief review of probability and statistical theory will be presented. This section is included to make the entire paper self contained.

A random process, stochastic process, or time series is a random variable that is a function of time (14:3). A random variable X and its value at time t_k may be denoted as

$$X_k = X(t_k) \quad (A-1)$$

Because of its random nature, it is meaningless to talk about the value of a random variable obtained at a certain time or values observed during a particular time interval. Instead, the idea of probability provides precise definitions of certain distributions and averages which can be predicted and observed with some level of confidence (14:3). The first and second order probability distributions can be defined as

$$F(x,t) = \text{Prob} [X(t) \leq x] = \int_{-\infty}^x f(v) dv \quad (A-2)$$

$$F(x_1, x_2; t_1, t_2) = \text{Prob} [X(t_1) \leq x_1, X(t_2) \leq x_2]$$

$$= \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} f(u, z) dz du \quad (A-3)$$

The probability density function (pdf) $f(x)$ and the cumulative distribution function (cdf) $F(x)$ are related more clearly in equations A-4 and A-5.

$$F_x(a) = \text{Prob} (-\infty < x \leq a) = \int_{-\infty}^a f(x) dx \quad (A-4)$$

$$f(x) = d/dx [F(x)] \quad (A-5)$$

Oftentimes, the pdf of a random variable X is known and the pdf of a function of x , for example $y=f(x)$ is desired. If the inverse function $x=f^{-1}(y)$ is single valued, then

$$f_y(y) = f_x(x)/|dy/dx| \quad (A-6)$$

The formula holds for multiple valued functions (13:95) such as

$$\begin{aligned} x_1 &= f_1^{-1}(y), \quad x_2 = f_2^{-1}(y), \dots \\ f_y(y) &= f_x(x_1)/|dy/dx_1| + f_x(x_2)/|dy/dx_2| + \dots \end{aligned} \quad (A-7)$$

Random variables are usually classified as stationary or nonstationary depending on whether the statistics of the variables are independent of or dependent on the independent variable time. Since this paper deals strictly with wide sense stationary (WSS) variables, only that type will be described here. A variable that is WSS possesses a first order pdf that is independent of time (a constant), and its second order pdf, known as the autocorrelation function, is dependent only on the time difference $\tau=t_2-t_1$ (14:4).

The expected value or mean of a random variable X is defined as follows.

$$E[X] = m_x = \int_{-\infty}^{\infty} x f(x) dx \quad (A-8)$$

The variance and the standard deviation of the random variable X are defined as

$$\text{VAR}(X) = s^2 = E(X^2) - E^2(X) \quad (A-9)$$

$$\text{SD}(X) = [\text{VAR}(X)]^{1/2} \quad (A-10)$$

where $E[X]$ is commonly referred to as the DC component, $E(X^2)$ is the mean square value, and $\text{SD}(X)$ is the AC component if X represents a voltage (14:5-6).

One of the most important second order quantities of interest is the autocorrelation function $R(\tau)$, which is defined as

$$R(\tau) = E(X_1 X_2) = E[x(t) x(t+\tau)] \quad (A-11)$$

It may be noted that for $\tau=0$, $R(\tau)$ is equal to the mean square value, and as τ approaches infinity, $R(\tau)$ approaches the DC value squared (14:6).

The power spectrum or power spectral density (PSD) is another second order pdf that is of importance. It is the fourier transform of the autocorrelation function and provides information about where the energy in a function is concentrated (13:265). It is defined as follows.

$$X(f) = \int_{-\infty}^{\infty} R(\tau) \exp(-j2\pi f\tau) d\tau \quad (A-12)$$

$$X(f) = \tau \sum_{i=0}^{nr-1} x(i) \exp(-j 2\pi f \tau) \quad -\infty < f < \infty \quad (A-13)$$

It can easily be shown that since $R(-\tau)=R^*(\tau)$, the power spectrum of a real or complex valued process is a real function of f . Other transforms are used to provide information about the PSD, including the cosine transform. The cosine transform uses this real value property to simplify the representation. It deals directly with the discrete sample values of the random variable and does not require transformation of the autocorrelation function (12:139).

The moment generating function (MGF), as presented in chapter 2, is useful in transforming a random variable from one distribution to another. For a variable $Z=X^2$, the MGF is defined as

$$M_z(t) = E[\exp(tz)] = E[\exp(tx^2)]$$

$$= \int_{-\infty}^{\infty} \exp(tx^2) f(x) dx \quad (A-14)$$

$$M_z(t) = (1/\sqrt{2\pi} \sigma) \int_{-\infty}^{\infty} \exp(tx^2) \exp[-(x-\mu)^2 / 2\sigma^2] dx \quad (A-15)$$

for a gaussian random variable X with mean μ and variance σ^2 .

After expanding the exponential, completing the square and making some substitutions,

$$M_z(t) = (1 - 2\sigma^2 t)^{-1/2} = [(1/2\sigma^2) / (1/2\sigma^2 - t)]^{1/2} \quad (A-16)$$

If $\sigma^2=1$, then X^2 has a chi-squared distribution with one degree of freedom.

Meyer proved that the sum of n chi-squared variables each with 1 degree of freedom is a chi-squared variable with n degrees of freedom (9:201). It can also be shown that this variable is exponential with parameter $\alpha=1/2$. Below are moment generating functions of some well known distributions (9).

$$\text{Uniform on } [a,b] \quad M_z(t) = [\exp(bt) - \exp(at)] / (b-a)t \quad (A-17)$$

$$\text{Binomial } (n,p) \quad M_z(t) = [p \exp(t) + (1-p)]^n \quad (A-18)$$

$$\text{Exponential } (\alpha) \quad M_z(t) = \alpha/(\alpha-t) \quad t < \alpha \quad (A-19)$$

$$\text{Gaussian } (\mu, \sigma^2) \quad M_z(t) = \exp[\mu t + \sigma^2 t^2/2] \quad (A-20)$$

Appendix B

Program Listing

```

real ac(100),c(100,100),cells(5),comp(5),cs,freq(51),q
+,q1,q2,ray(100),rho(100),s(100),smean,spe(51),spectr(5
+1),sum,sum1,svar,s(100),vc(100,100),w(400),weib(100),
+x(100),xmean,xvar,x1(100),y(100),ymean,yvar,y1(100),z,
+z1,z2,zmean
integer ic,ldf,ier,ip,k,nr,nl2,n95
double precision dseed
external norm,weib1,ray1
data cells,freq,rho,spectr,x/307*0.0/
?c=1
ldf=0
ip=1
nl2=2
n95=95
sum=0.0
sum1=0.0

```

tau=.001

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15

```

      y1(im)=y(im)+zmean
      sum=sum+y(im)
16      continue
      ymean=sum/nr

*      vector y1 now contains variables
*      with a mean=zmean

*      generate a second vector of independent
*      random variables for use in generating
*      the weibull and rayleigh variables

      dseed=283153
      write(6,18)dseed
18      format(' dseed 2 =',f12.1)
      call ggnml(dseed,nr,s)
      sum=0.0
      sum1=0.0
      do 20 id=1,nr
      sum=sum+s(id)
20      continue
      smean=sum/nr
      do 25 if=1,nr
      sum1=sum1+(s(if)-smean)**2
25      continue
      svar=sum1/nr
      sum=0.0
      do 26 in=1,nr
      s(in)=s(in)-smean
      si(in)=s(in)+zmean
      sum=sum+s(in)
26      continue
      smean=sum/nr

*      vector si now contains variables
*      with a mean=zmean

*      generate the desired variance-covariance matrix

      do 32 i=1,nr
      do 30 j=1,i
      if (iz.eq.2) then
      vc(i,j)=exp(-(abs(i-j))*tau**2)
      else
      vc(i,j)=exp(-(abs(i-j))*tau)
      endif
30      continue
32      continue

*      generate transformation matrix C

      do 40 ik=1,nr
      c(ik,1)=vc(ik,i)
40      continue
      do 48 im=2,nr
      do 46 jm=2,im
      if (im.eq.jm) then
      sum=0.0
      do 42 km=1,im-1
      sum=sum+(c(im,km))**2
42      continue
      c(im,im)=sqrt(vc(im,im)-sum)
      else
      sum1=0.0
      do 44 kn=1,jm-1
      sum1=sum1+c(im,kn)*c(jm,kn)

```

```

44      continue
      c(im,jm)=(vc(im,jm)-sum1)/c(jm,jm)
end if
46      continue
48      continue

*      generate correlated vector X

do 52 i1=1,nr
do 50 jk=1,11
x(i1)=x(i1)+(c(i1,jk))*y(jk)
50      continue
52      continue

*      calculate mean of correlated vector X

sum=0.0
do 60 ia=1,nr
sum=sum+x(ia)
60      continue
xmean=sum/nr

*      compute variance of correlated vector X

sum1=0.0
do 70 id=1,nr
sum1=sum1+(x(id)-xmean)**2
70      continue
xvar=sum1/nr

*      translate X into a random variable
*      with a mean=zmean

do 72 i4=1,nr
x1(i4)=x(i4)-xmean+zmean
72      continue

*      calculate the autocorrelation
*      of the correlated vector X

do 82 i=1,nr
sum=0.0
do 80 j=i,nr
jj=j-i+1
sum=sum+x(j)*x(jj)
80      continue
ac(i)=sum/nr
82      continue
write(6,84)xmean,xvar
84      format("      xmean=",f10.7,"      xvar=",f10.7)
do 88 k1=1,nr
rho(k1)=ac(k1)/ac(1)

*      print out correlated vector and
*      autocorrelation values

write(6,86)k1,x(k1),k1,x1(k1),k1-1,rho(k1)
86      format(" x(",i3,")=",f9.5," x1(",i3,")=",f9.5," rho("
+,i3,")=",f9.5)
88      continue

*      compute the power spectral density
*      of correlated vector X

call cotran(nr,tau,freq,spectr,x,spe)

```

```

*      print out autocorrelation values

      do 92 i=1,nr/2+1
      write(6,98)i-1,freq(i),i-1,spe(i)
98      format(" freq(",i3,")=",f9.3," Hz  spe(",i3,")="
      +,f18.7)
92      continue

      if (iz.eq.2) then
      goto 178

*      recompute parameters with different
*      variance-covariance matrix

      else

*      generate weibull and rayleigh variables

      do 182 i=1,nr
      z=(y(i))**2+(s(i))**2
      q1=1.8/3.8
      z1=z**q1
      weib(i)=z1
      q2=8.5
      z2=z**q2
      ray(i)=z2

*      print out independent normal vectors y and s,
*      print out their corresponding mean=zmean vectors,
*      and print out weibull and rayleigh vectors

      write(6,188)i,y(i),i,y1(i),i,s(i),i,weib(i),i,ray(i)
188      format("y(",i3,")=",f9.5," y1(",i3,")=",f9.5," s(",
      +i3,")=",f9.5," weib(",i3,")=",f9.5," ray(",i3,")=",
      +f9.5)
182      continue
      write(6,184)ymean,yvar,smean,svar
184      format("ymean=",f18.7," yvar=",f18.7," smean=",
      +f18.7," svar=",f18.7)

*      perform goodness of fit test on gaussian variables

      k=4
      call gfit(norm,k,y,nr,cells,comp,cs,ldf,q,ier)
      write(6,114)
114      format("goodness of fit test for gaussian variables")
      write(6,116)/cells(1),cells(2),cells(3),cells(4),
      +cells(5)
116      format("cells=(,f5.1,,f5.1,,f5.1,,f5.1,,f5.1,,f5.1)")
      write(6,118)cs
118      format("chi-squared statistic for gaussian variables
      +=",f9.5)
      write(6,120)ldf
120      format("gaussian degrees of freedom =",i2)
      write(6,122)q
122      format("probability of chi-squared statistic exceedin
      +g gaussian cs if null hypothesis is true =",f18.7)

*      graph normal vector vs. theoretical cdf

      call vsrta(y,nr)
      call uspc(norm,y,nr,n12,n95,'p,ic,w)

*      perform goodness of fit test on weibull variables

```

```

      idf=0
      k=4
      do 124 i=1,6
      cells(i)=0.0
124      continue
      call gfit(weib1,k,weib,nr,cells,comp,cs,idf,q,ier)
      write(6,130)cells(1),cells(2),cells(3),cells(4),
+cells(5)
130      format("cells=(,f5.1,.,,f5.1,.,,f5.1,.,,
+f5.1,.,,f5.1,.)")
      write(6,132)cs
132      format("chi-squared statistic for weibull variables =
+ ",f9.5)
      write(6,134)idf
134      format("weibull degrees of freedom = ",i2)
      write(6,136)q
136      format("probability of chi-squared statistic exceedin
+g weibull cs if null hypothesis is true = ",f10.7)

*      graph weibull vector vs. theoretical cdf

      call vsrta(weib,nr)
      call uspc(weib1,weib,nr,n12,n95,ip,ic,w)

*      perform goodness of fit on rayleigh variables

      idf=0
      k=5
      do 138 i=1,6
      cells(i)=0.0
138      continue
      call gfit(ray1,k,ray,nr,cells,comp,cs,idf,q,ier)
      write(6,144)cells(1),cells(2),cells(3),cells(4),
+cells(5)
144      format("cells=(,f5.1,.,,f5.1,.,,f5.1,.,,
+f5.1,.,,f5.1,.)")
      write(6,146)cs
146      format("chi-squared statistic for rayleigh variables
+ ",f9.5)
      write(6,148)idf
148      format("rayleigh degrees of freedom = ",i2)
      write(6,150)q
150      format("probability of chi-squared statistic exceedin
+g rayleigh cs if null hypothesis is true = ",f10.7)

*      graph rayleigh vector vs. theoretical cdf

      call vsrta(ray,nr)
      call uspc(ray1,ray,nr,n12,n95,ip,ic,w)

      endif
170      continue
      end

*****
*      subroutine cotran
*      computation of cosine transform (PSD) of samples
*
*****

      subroutine cotran(nr,tau,freq,spectr,x,spe)
      real freq(51),pi,spe(51),spectr(51),tau,temp,thet,
+x(100),yy,zz
      integer i,ka,nr
      pi=3.141592654

```

```

do 205 ka=1,nr/2+1
  kb=ka-1
  temp=0.0
  yy=kb/(nr*tau)
  freq(ka)=yy+.0001
do 200 i=1,nr
  thet=2*pi*i*kb/nr
  zz=cos(thet)
  temp=temp+zz*cc(i)
200 continue

*   spectr is the raw computed power spectrum
  spectr(ka)=temp*tau

*   spe is the normalized magnitude of the power spectrum
  spe(ka)=abs(spectr(ka)/spectr(1))
205 continue
return
end

*****
*
*   subroutine norm
*   gaussian cumulative distribution function
*
*****

  subroutine norm(x,p1)
  p1=.5*erfc(-x*.70710668)
  return
end

*****
*
*   subroutine weib1
*   weibull cumulative distribution function
*
*****

  subroutine weib1(x,y1)
  y1=1-exp(-(x**3)/2)
  return
end

*****
*
*   subroutine ray1
*   rayleigh cumulative distribution function
*
*****

  subroutine ray1(x,y2)
  y2=1-exp(-(x**2)/2)
  return
end

```

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See → In this thesis, time-varying radar cross sections of chaff clouds are generated for use in radar/ECM computer simulations, under the assumption that scattering from chaff clouds is a wide sense stationary random process. For a jointly gaussian random process, second order statistics are developed from correlated samples of radar cross sections. Applicable statistical tests are performed to validate a set of generated samples. The goodness of fit of the samples compared to the prespecified density function is determined along with a comparison of the correlation coefficients from the generated set to the desired correlation coefficients. Rayleigh and Weibull statistics are also derived from gaussian variables to present an alternative way of describing the probability distributions of chaff cloud cross sections. Topics for further study are suggested. *Key word: —*

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