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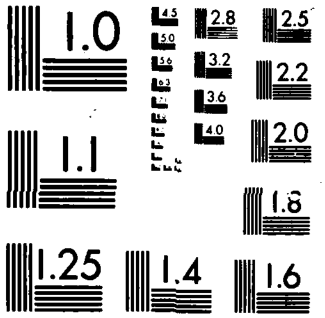
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Mechanics and Materials Center
TEXAS A&M UNIVERSITY
College Station, Texas

AD-A164 121

RESEARCH ON DAMAGE MODELS FOR CONTINUOUS FIBER COMPOSITES

Annual Technical Report

by

D. H. Allen
W. E. Haisler
and
C. E. Harris

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Continuous fiber composite laminate are known to undergo a substantial amount of complex load-induced damage which can adversely affect component performance [1]. Therefore, it is desirable to develop new models capable of accounting for the effect of damage on materials properties.

This report documents research completed during the first year of a three year effort under AFOSR grant no. AFOSR-84-0067 and originally detailed under Texas A&M Research Foundation proposal no. RF-84-34 and dated October 1983. The objective of this research is to develop an accurate damage model for predicting strength and stiffness of continuous fiber composite media subjected to fatigue or monotonic loading and to verify this model with experimental results obtained from composite specimens of selected geometry and makeup to be described herein.

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AEROSPACE ENGINEERING DEPARTMENT
TEXAS A&M UNIVERSITY

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Air Force Office of Scientific Research
Office of Aerospace Research
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1. INTRODUCTION

1.1 Summary

Continuous fiber composite laminates are known to undergo a substantial amount of complex load-induced damage which can adversely affect component performance [1]. Therefore, it is desirable to develop new models capable of accounting for the effect of damage on materials properties.

This report documents research completed during the first year of a three year effort under AFOSR grant no. AFOSR-84-0067 and originally detailed under Texas A&M Research Foundation proposal no. RF-84-34 and dated October 1983. The objective of this research is to develop an accurate damage model for predicting strength and stiffness of continuous fiber composite media subjected to fatigue or monotonic loading and to verify this model with experimental results obtained from composite specimens of selected geometry and makeup to be described herein.

1.2 Statement of Work

The following is a brief summary of work to be performed under the present grant:

1) develop constitutive equations relating stresses to strains and damage internal state variables (ISV) which may be used in a stress gradient field;

2) develop ISV growth laws as a function of load history for matrix cracking, interlaminar fracture, etc.;

3) develop finite element algorithms capable of evaluating ply properties in damaged components.

4) perform experiments on components with selected stacking sequences in order to verify the model.

2. RESEARCH COMPLETED TO DATE

2.1 Summary of Completed Research

The following research has been completed during the first year:

1) the general damage-dependent stress-strain relations have been revised and completed;

2) an ISV growth law has been constructed for matrix cracking;

3) laminate equations have been constructed for matrix cracking;

4) the relation between the damage ISV and surface area of cracks has been established analytically;

5) all finite element programs are complete; and

6) the model has been compared to both experimental and finite element results for glass/epoxy laminates.

In addition, the following research is partially completed;

1) experimental characterization of ply and laminate properties in a variety of graphite/epoxy AS4/3502 laminates;

2) non-destructive evaluation of damage in various graphite/epoxy laminates; and

3) correlation of model to experiment for degraded stiffness due to damage in graphite/epoxy composites.

The following sections further detail the results summarized above.

2.2 Development of the General Model

Thermodynamic and symmetry constraints have been utilized to construct stress-strain relations of the form:

$$\sigma_{L_{ij}} = \sigma^{R}_{L_{ij}} + C'_{L_{ijkl}}\left(\epsilon_{L_{kl}} - \epsilon^{T}_{L_{kl}}\right) \quad , \qquad (1)$$

where $\sigma_{L_{ij}}$ is the stress tensor, $\epsilon_{L_{kl}}$ is the strain tensor, $\sigma^{R}_{L_{ij}}$ is the damage dependent residual stress, $\epsilon^{T}_{L_{kl}}$ is the damage dependent thermal strain, and $C'_{L_{ijkl}}$ is the damage dependent modulus tensor, given by

$$C'_{L_{ijkl}} \equiv C_{L_{ijkl}} + M^{\eta}_{ijklmn}\,\alpha^{\eta}_{L_{mn}} \quad . , \qquad (2)$$

where $C_{L_{ijkl}}$ and M^{η}_{ijklmn} are material constants and $\alpha^{\eta}_{L_{mn}}$ are second-order tensor-valued damage dependent internal state variables.

A kinematically based description of the internal state variable has been constructed of the form

$$\alpha^{\eta}_{L_{ij}} \equiv \frac{1}{V_L}\int_{S^{\eta}_{2}} u^{c}_{i}\, n^{c}_{j}\, dS = \frac{1}{V_L}\int_{S^{\eta}_{2}} \alpha_{ij}\, dS \quad , \qquad (3)$$

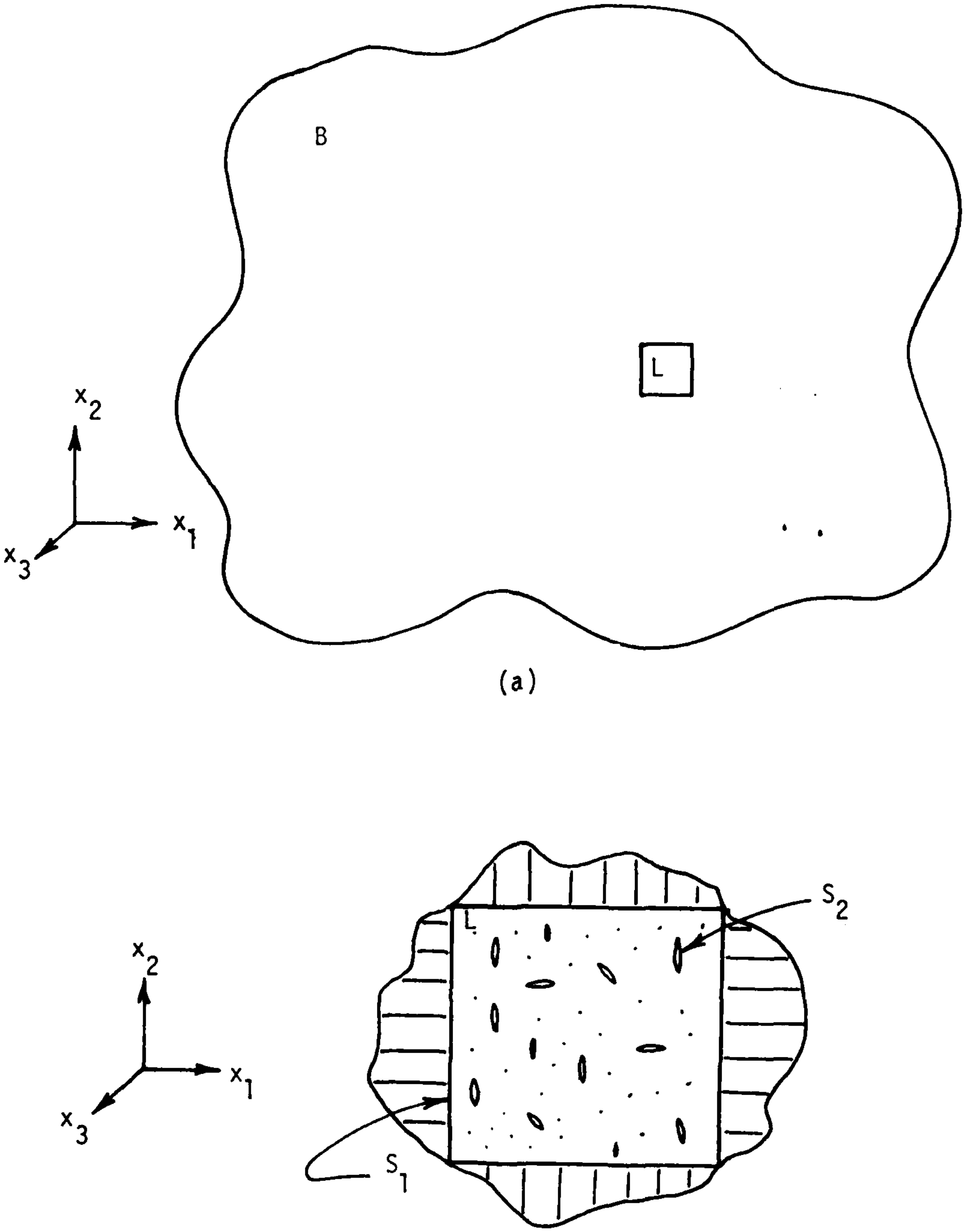
where u^{c}_{i} are the components of the crack-opening displacement, n^{c}_{i} are the components of the crack unit normal, and S_2 is the total surface area of cracks in a given volume element, V_L, as shown in Fig. 1.

An ISV growth law has been constructed for matrix cracking, as given by

$$\dot{\alpha}^{1}_{L_{22}} = K \left(\frac{\epsilon_{n} - \epsilon_{n_{min}}}{\epsilon_{n_{max}} - \epsilon_{n_{min}}}\right)^{n_1} \frac{d\epsilon_n}{dt} \quad \text{if} \quad \epsilon_{n_{min}} < \epsilon_n < \epsilon_{n_{max}} \text{ , and} \qquad (4)$$

$$\dot{\alpha}^{1}_{L_{22}} = 0 \quad \text{if } \epsilon_n < \epsilon_{n_{min}} \text{ or } \epsilon_n > \epsilon_{n_{max}},$$

where the parameters are as described in Appendix 6.2.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L
x_2
x_1
x_3
L
S_2
x_2
x_1
x_3
S_1

(a)

(b)

Fig. 1. A Body with Damage (a) General Body, (b) Local Element.

The results outlined in this section are described in detail in Appendix 6.1.

2.3 Application to Matrix Cracking

The model has been applied to composite laminates with a single damage mode consisting of matrix cracks, as shown in Fig. 2. This was accomplished by imposing symmetry constraints, reducing to single index notation, and constructing laminate equations. It was found that for matrix cracks which lay in a plane normal to the fiber (x_1) direction,

$$\alpha^1_{L_{ij}} = |\alpha^1_L| \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \qquad (5)$$

in local ply coordinates, where $\alpha^1_{L_{ij}}$ is the damage tensor for a typical ply due to matrix cracking.

An examination of the dependence of free energy, u_L, on damage led to the conclusion that the magnitude of damage $|\alpha^1_L|$ depends only on the surface area of cracks S_2. Furthermore, since the free energy depends explicitly on damage, it follows that the relation between $|\alpha^1_L|$ and S_2 could be determined from

$$|\alpha^1_L| = \frac{\int_{S_2(o)}^{S_2(t)} G_L(S_2)ds}{\rho_c V_c f^1_{22}} = g'(S_2) , \qquad (6)$$

where G_L is the energy release rate.

The energy release rate was then calculated as a function of S_2 by utilizing a finite element simulation of a $[0,90]_s$ laminate.

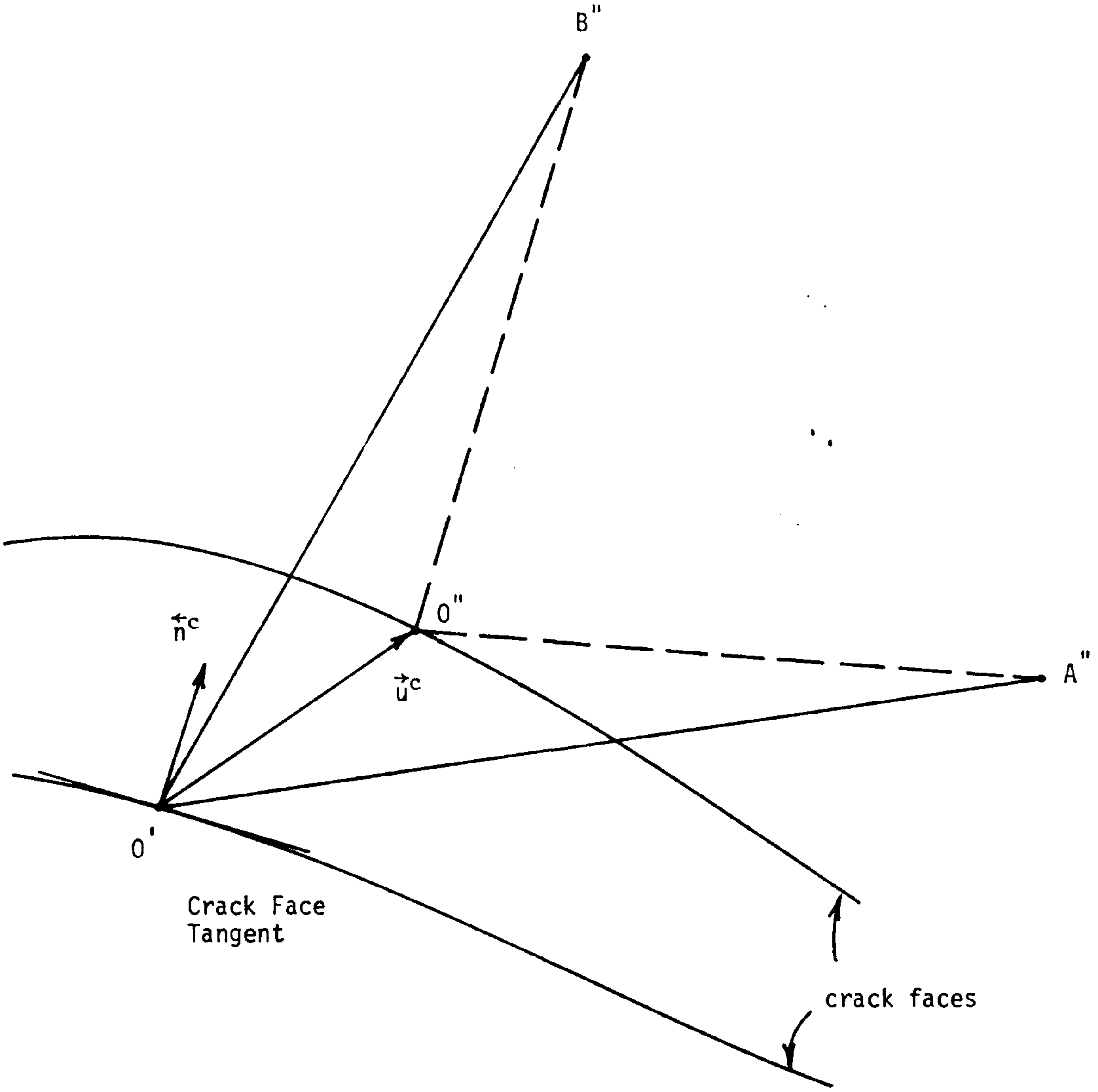
B"
O"
$\vec{n}^c$
$\vec{u}^c$
A"
O'
Crack Face Tangent
crack faces

Fig. 2. Description of the Internal State at Point O'.

Using the procedure briefly described above, it is possible to relate all stiffness components to cracked surface area for composite laminates with matrix cracks. It is emphasized that the model predicts all stiffness components for any layup using material property input which is stacking sequence independent.

Figs. 3 & 4 show predictions of the model for axial and other stiffness components of $[0,90_3]_s$ E-glass epoxy laminates. Fig. 3 indicates that the model predicts somewhat higher stiffness than experimental results. Although experimental results are not available for other components, Fig. 4 shows excellent agreement between FEM results and the model for ν_{xz}. Details of the results outlined in this section are contained in Appendix 6.2.

The following section will detail attempts to obtain more useful experimental results.

2.4 Experimental Research Activities

The purpose of the experimental research is to study and document the progression of damage in a systematic manner that will fully support the development of the constitutive model. The authors have determined via an extensive literature survey, partially documented in reference 1, that the experimental data necessary to characterize and verify the model does not exist in the current literature. The comprehensive experimental data base will provide two fundamental types of essential information which are not currently available in the open literature. First, the progression-of-damage study will establish the phenomenological characterization of the internal state variables and will provide the essential ingredients for formulating the associated

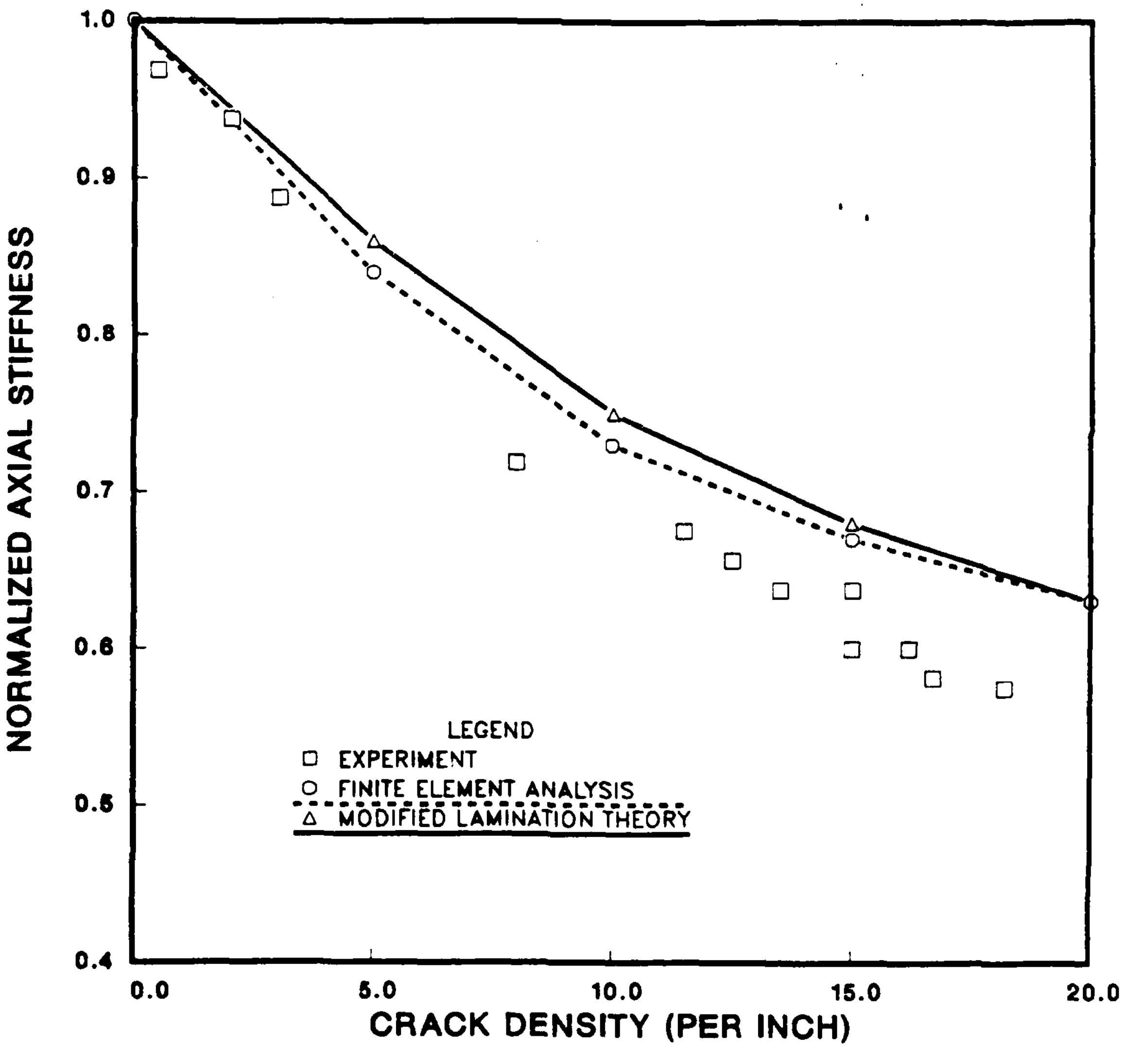

Fig. 3. Normalized Axial Stiffness Predictions for a $[0,90_3]_s$ E-Glass Epoxy Laminate.

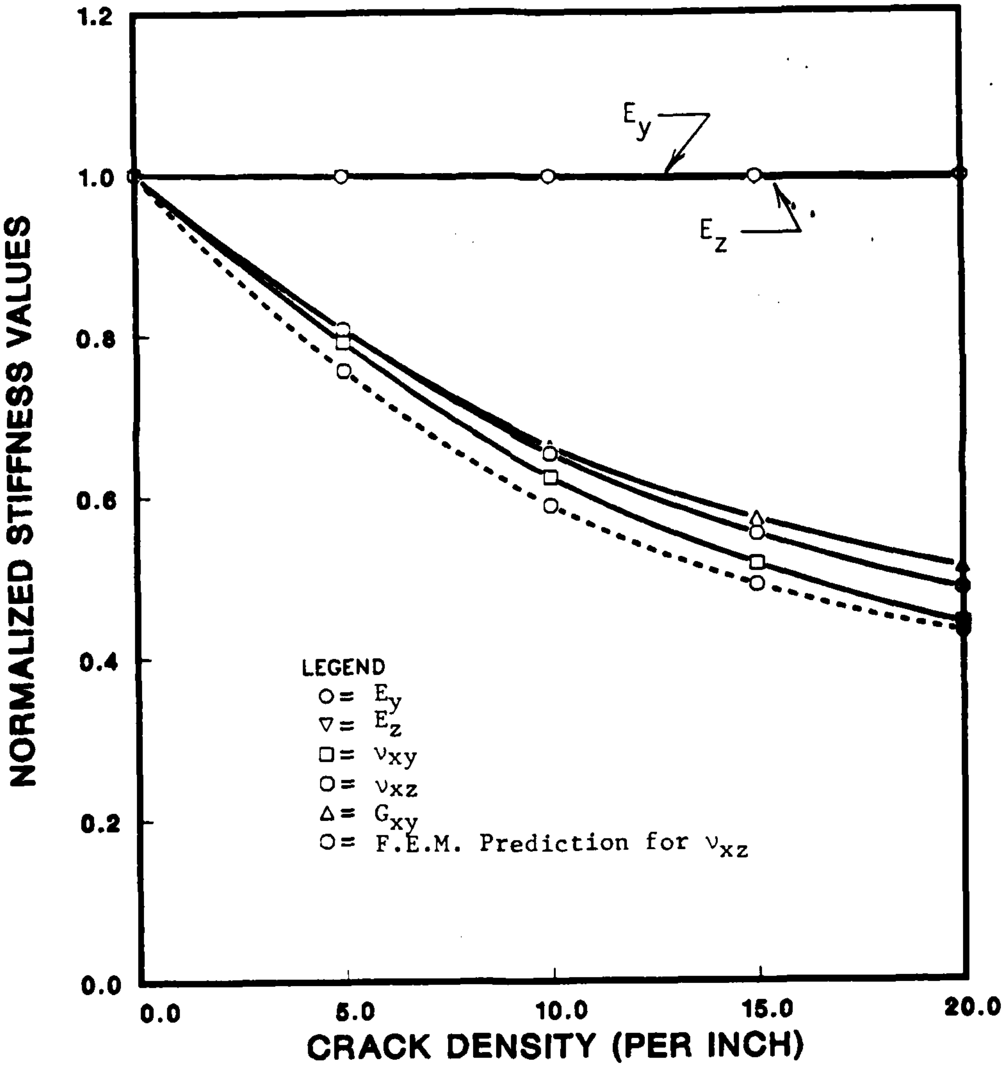

Fig. 4. Effective Orthotropic Property Predictions for $[0,90_3]_s$ E-Glass Epoxy Laminate.

damage growth laws. Second, the experimental data base will provide direct measurements of the effective (damage-degraded) moduli. These direct measurements will be compared to the constitutive model predictions. This is an essential task for both the development and "fine-tuning" of the model as well as establishing the validity and accuracy of the model. The following sections describe the results of the three primary tasks that have been completed to date. The first task was the development of a materials characterization laboratory. The second task was the formulation of a test matrix, laminate panel preparation and specimen coupon fabrication. Finally, the study of matrix crack damage has commenced.

2.4.1 Materials Characterization Laboratory

The research objectives of this grant necessitated the establishement of a materials characterization laboratory. The effort was supported jointly by Texas A&M University and by an equipment grant from AFOSR. The established laboratory meets the dual requirements of measuring the mechanical properties of materials and nondestructively studying load-induced microstructural damage. The primary laboratory facilities consist of a 20-kip Instron testing machine for monotonic loading conditions and a 110-kip MTS 880 testing system (partially funded by AFOSR Equipment Grant-84-0257) for both monotonic and cyclic loading conditions. Support equipment include computer controlled data acquisition and data reduction systems, as well as redundant real-time analog signal recording systems. (The MTS 880 testing system also includes an environmental chamber for cryogenic or elevated temperature testing in a controlled humidity environment.) The primary nondestructive evaluation (NDE) facility is a portable x-ray radiography system for in-situ x-ray examinations. A second NDE facility is an ultrasonic spectroscopy laboratory. This technology is being developed by Dr. V.K. Kinra as part of a "sister" AFOSR Grant. Additional NDE facilities, already in place at Texas A&M include edge-replication, ultrasonic C-scan, optical microscopes and a scanning electron microscope. All of the above equipment is now in place and functional along with fully trained personnel.

2.4.2 EXPERIMENTAL TEST PROGRAM

Laminate panels are being prepared from pre-impregnated graphite/epoxy tape, AS4/3502, by the Mechanics and Materials Laboratory at Texas A&M University. The curing process was developed from the procedure recommended by the pre-preg tape vendor. Each 12" x 12" panel yields ten 1" x 11" tensile test coupons. The matrix of test coupons is given in Table 1. The anticipated specimen requirements necessitated the fabrication of 2 panels for each laminate stacking sequence. While panel fabrication is still an ongoing activity, all Type I and II panels have been fabricated. The cured materials are being stored in a dessicant chamber.

The number of specimens in each test category listed in Table 1 were selected primarily to insure adequate replicate test data. The specified load histories for the four fatigue tests are considered to be tentative. The exact cyclic load histories will be selected at a later date. For this reason, it is desirable to have a sufficient number of specimens available should the research warrant a redirection. It should be noted that there are two spare specimens in addition to the eighteen specimens listed in the test matrix for each laminate. The

TABLE 1 TEST MATRIX

A. LAMINA MATERIAL CHARACTERIZATION

$[0]_8$	5
$[90]_8$	5
$[\pm 45]_{2S}$	5

B. LAMINATE CHARACTERIZATION & DAMAGE GROWTH LAWS

LAMINATE TYPE	QUASI - STATIC		FATIGUE			
	LAMINATE CHARACTERIZATION	MONOTONIC DAMAGE GRW LAWS	R=10 HIGH	R=10 LOW	R≠10	COMPR
I. $[0/90/0]_S$	2	4	3	3	3	3
$[90/0]_S$	2	4	3	3	3	3
$[0/90]_S$	2	4	3	3	3	3
$[0/90]_{2S}$	2	4	3	3	3	3
$[0_2/90_2]_S$	2	4	3	3	3	3
$[0/90_2]_S$	2	4	3	3	3	3
$[0/90_3]_S$	2	4	3	3	3	3
$[0/90_4]_S$	2	4	3	3	3	3
II. $[90/45/0/-45]_S$	2	4	3	3	3	3
$[90/\pm 45/0]_S$	2	4	3	3	3	3
III. $[0/\pm 45]_S$	2	4	3	3	3	3
$[45/0/-45]_S$	2	4	3	3	3	3
IV. $[30_2/-30_2]_S$	2	4	3	3	3	3
$[(\pm 30)_2]_S$	2	4	3	3	3	3

total number of specimens (20 per laminate) will allow for some destructive examinations of the microstructural damage without diminishing the overall data base.

The laminates listed in Table 1 were selected for specific reasons. The Type I laminates were selected to study matrix cracks. The primary mode of microstructural damage in cross-ply laminates is matrix cracks in the 90° plies [2,3] In addition, the free edge interlaminar stresses are minimal and should not affect the microstructural damage that develops prior to fracture. Therefore, the cross-ply laminates are ideal for isolating the single damage mode of matrix cracking. The specific cross-ply stacking sequences were selected so that both the thicknesses of the 90° ply layers and the 0° ply constraint layers are varied.

The quasi-isotropic laminates, Type II, were selected because of their practical significance and in order to study adjacent ply matrix crack interaction. The microstructural damage should be free of edge effects in both stacking sequences because they both have compressive interlaminar normal stresses at the free edge [4]. Also, the + 45° and -45° plies are adjacent in one laminate and separated in the other. This will provide information concerning the effect of adjacent ply constraint on matrix cracking and crack interaction. Finally, these two laminates will be useful for the study of localized internal delaminations which accompany the intersection of adjacent ply matrix cracks.

The Type III laminates were chosen to isolate and study matrix crack damage in (+) and (-) θ plies in the absence of matrix cracks in 90° plies. The two laminate stacking sequences are consistent with the

Type II laminates, excluding the 90° plies, and both have minimal interlaminar normal stress at the straight free edges.

The Type IV laminates were selected to study the influence of free edge effects [2,5]. The failure of the $[30_2/-30_2]_s$ laminate is dominated by the interlaminar shear stress, τ_{xz}. Free edge delaminations open at the +30/-30 interface. On the other hand, delaminations do not form at the free edges of the $[(\pm30)_2]_s$ laminate. The significance of free edge effects is illustrated by the fact that the strength of the $[(\pm30)_2]_s$ laminate is much higher than that of the $[30_2/-30_2]_s$ laminate. It is anticipated that free edge delaminations can only be addressed as a boundary value problem since the delamination surfaces form external boundaries. However, this damage mode is included herein because it represents a realistic damage mode for many practical structural geometries.

2.4.3 Experimental Results

The first phase of the research has the objective of establishing the internal state variable and the associated damage growth law for the isolated damage mode of matrix cracks. Type I cross-ply laminates have been selected for this purpose. This initial phase of experimental research is being confined to the quasi-static, monotonic loading condition. The general expeimental procedure is briefly outlined below:

1. Establish the basic lamina properties for input to all analyses.
2. Determine the undamaged moduli and the load-to-failure versus displacement characteristics of the laminate.
3. Using the results of 2 as a guide, study and document the progression of damage as follows:

(a) At appropriate load increments, perform nondestructive examinations.

(b) Determine the tangent moduli and secant moduli. Record all extensometer versus load data.

(c) At a few specially selected damage states, perform destructive examinations.

The basic lamina properties are listed in Table 2. In each case, the mean value and standard deviation are listed. From the limited comparisons available, it can be seen that the recently measured lamina properties are in good agreement with the specified values supplied by the pre-preg tape vendor.

The experimental investigation of matrix cracks in the 90° plies of cross-ply laminates has commenced. A typical progression-of-damage pattern in the $[0/90_3]_s$ laminate is shown in Fig. 5. Also, Fig. 6 shows an associated x-ray radiograph of the apparent saturation damage state in the $[0/90_3]_s$ laminate. The edge replicas shown in Fig. 5 were taken of the same specimen at increasing applied stress levels. The increasing number of cracks and the apparent saturation spacing at 67.1 ksi are consistent with well known results for transverse matrix cracking. It is obvious from Fig. 5 that all of the matrix cracks are not straight cracks through the thickness of the 90° layer. The initial cracks that form are straight. However, as cracks begins to fill in the spaces between the initial cracks, some cracks have distinctive parabolic profiles. This can clearly be seen in the edge replica taken at 56.8 ksi. The curved cracks have greater surface area than straight cracks and unit normals that are not aligned with the x_1-axis direction as is the case with straight cracks. Therefore, the presence of the

TABLE 2 MEASURED LAMINA PROPERTIES

	MEAN VALUE	STANDARD DEVIATION	VENDOR SUPPLIED PROPERTIES
E_{11}	21.5×10^6 psi	2.0%	21.5×10^6 psi
E_{22}	1.39×10^6 psi	2.1%	
G_{12}	0.694×10^6 psi	-	
ν_{12}	0.31	3.7%	
F_{tu_1}	326.0 ksi	3.5%	310.0 ksi
ε_{tu_1}	0.0144	4.6%	

Notes:

1. F_{tu_1} = tensile strength of 0° unidirectional laminate.
2. ε_{tu_1} = tensile failure strain of 0° unidirectional laminate.

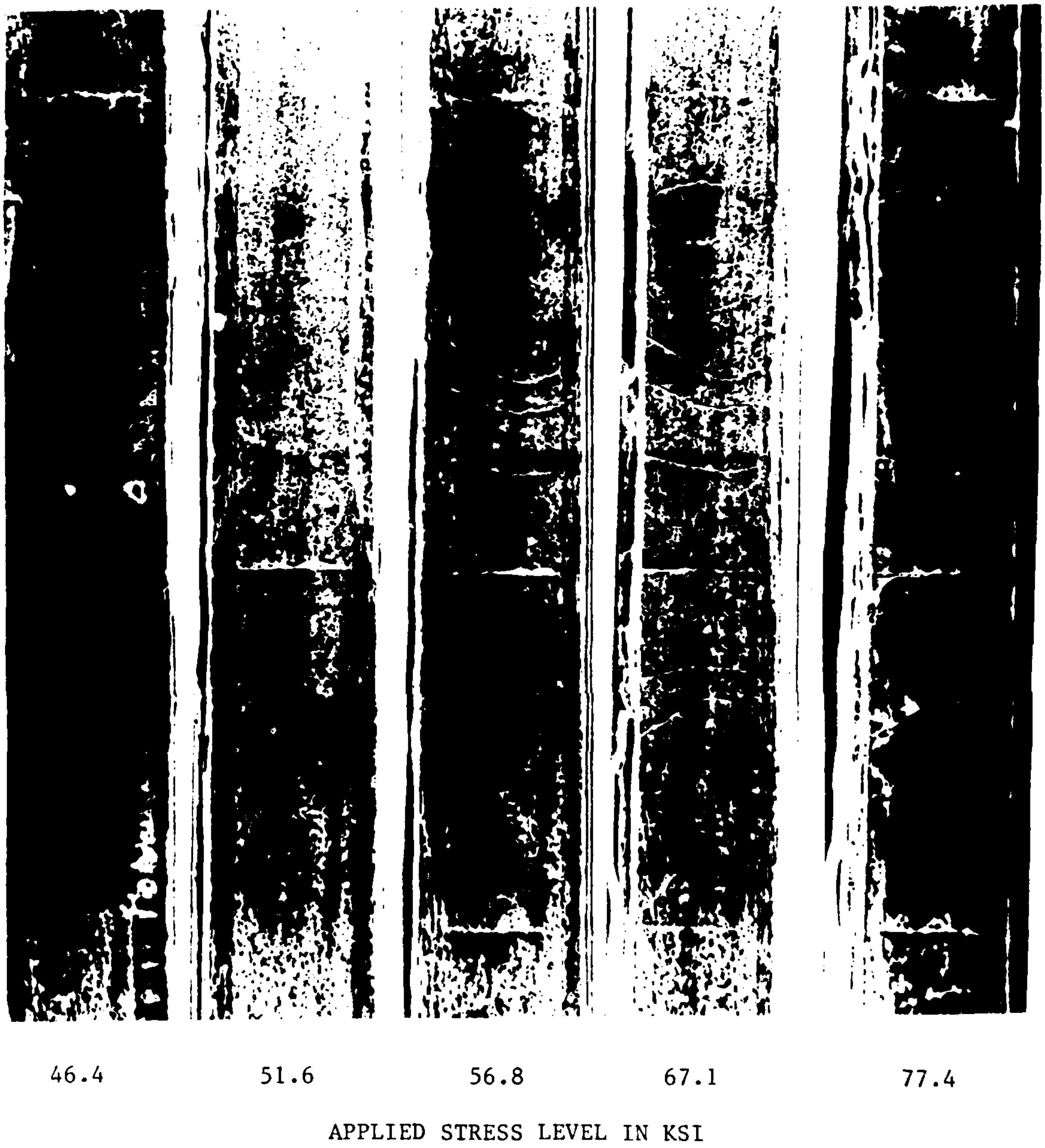
46.4
51.6
56.8
67.1
77.4
APPLIED STRESS LEVEL IN KSI

Fig. 5 Edge Replicas at Increasing Stress Levels of a Specimen from the $[0/90_3]_s$ Laminate.

Fig. 6 X-Ray Radiograph of the Apparent Saturation Crack Pattern in the $[0/90_3]_s$ Laminate.

curved cracks may have a significant influence on the formulation of the internal state variable for matrix cracking. The initial indication is that the profile of the curved cracks is greatly influenced by the thickness of the 90° layer and to some extent by the thickness of the adjacent 0° plies. This is probably related to the previously identified shear lag effect on matrix crack formation. This is currently the focus of the experimental research effort.

2.5 Conclusion

A general model has been developed for predicting the relation between stresses and strains in composites with load-induced damage. This model is capable of predicting structural component response in a stress gradient field. Although the general form of the model is fairly complex, it has been shown herein that for the case of matrix cracking the model is only slightly more cumbersome that standard laminate analysis. Furthermore, the model has been compared with some success to experimental results for $[0,90_3]_s$ E-glass epoxy laminates.

Current efforts deal with verification of the model for AS4/3502 graphite-epoxy laminates, with primary emphasis on matrix cracking. Since necessary experimental results are not available in the literature, a comprehensive experimental program has been initiated under the current grant. Initial results indicate that the model is accurate for graphite-epoxy systems.

Activities in the second year of research will concentrate on three important issues: 1) effect of matrix cracking on reduction of all stiffness components as a function of stacking sequence in cross-ply laminates; 2) refinement of the ISV growth law for matrix cracking; and 3) initial studies of the effects of interlaminar delamination on stiffness loss.

The authors are quite encouraged by the current results and believe that they provide ample justification for continuing the research effort.

2.6 References

[1] Groves, S.E., and Allen, D.H., "A Survey of Damage in Continuous Fiber Composites," Texas A&M University Mechanics and Materials Center, MM-5023-84-6, March, 1984.

[2] Herakovich, C.T., Nagarkar, A., and O'Brien, D.A., "Failure Analysis of Composite Laminates with Free Edges," Modern Developments in Comp. Mat. and Struc. Edited by J.R. Vinson, ASME, 345 East 47th, St., NY, NY, 10017, 1979, pp. 53-66.

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[5] Herakovich, C.T., "Influence of Layer Thickness on the Strength of Angle-Ply Laminates," J. of Comp. Mat., Vol. 16, May 1982, pp. 216-227.

3. PUBLICATIONS LIST

3.1. To Be Submitted for Publication

1. "A Damage Model for Continuous Fiber Composites, Part I-Theoretical Development," by D.H. Allen, S.E. Groves, and R.A. Schapery (Appendix 6.1), to be submitted to Mechanics of Materials.

2. "A Damage Model for Continuous Fiber Composites, Part II - Model Applications," by S.E. Groves, D.H. Allen, C.E. Harris, R.A. Schapery, (Appendix 6.2), to be submitted to Mechanics of Materials.

3.2. In Preparation

1. "Modelling of Stiffness Reduction Due to Matrix Cracks in Graphite/Epoxy Laminates," by C.E. Harris, R.G. Norvell, D.H. Allen, and S.E. Groves.

2. "Experimental/Analytical Correlation of Damage Accumulation in Laminated Composites," R.G. Norvell, C.E. Harris, D.H. Allen, and I. Georgiou.

3. "Damage Growth Laws for Matrix Cracks in Graphite/Epoxy Laminates," by D.H. Allen, S.E. Groves, I. Georgiou, and C.E. Harris.

4. PROFESSIONAL PERSONNEL INFORMATION

4.1 Faculty Research Assignments

1. Dr. D.H. Allen (Co-principal Investigator)- overall program coordination; development of stiffness relationships; construction of ISV growth laws; mechanical testing.
2. Dr. W.E. Haisler (Outgoing Co-principal Investigator)- finite element modelling.
3. Dr. C.E. Harris (Incoming Co-Principal Investigator)- overall experimental coordination; mechanical testing; nondestructive evaluation; stiffness modelling.

4.2 Additional Staff

1. Mr. S.E. Groves (Lecturer, Research Assistant, and Ph.D. Candidate)- ISV growth laws; laminate analysis; finite element modelling; mechanical testing.
2. Mr. R.G. Norvell (Research Assistant and M.S. Candidate)- mechanical testing; nondestructive evaluation.
3. Mr. I. Georgiou (Research Assistant and M.S. Candidate)- mechanical testing; nondestructive evaluation; ISV growth laws.
4. Mr. B. Harbert (Lab Technician)- experimental lab support.
5. Mr. C. Fredericksen (Lab Technician)- experimental lab support.
6. Mrs. T. Marquez (Secretary)- secretarial support.

5. INTERACTIONS

5.1 Papers Presented

5.1.1 Presented Under This Grant

1. D.H. Allen, "A Damage Model for Continuous Fiber Composites," Tenth Annual Composites Review, Dayton, October, 1984.

2. D.H. Allen, "A Damage Model for Continuous Fiber Composites," Society of Engineering Science 21st Annual Meeting, Blacksburg, VA, October, 1984.

5.1.2 Related Research Presentations

1. D.H. Allen, "On the Use of Internal State Variables in Thermoviscoplastic Constitutive Equations," 2nd Symposium on Nonlinear Constitutive Relations for High Temperature Applications, Clevelend, June, 1984.

2. W.E. Haisler, "Numerical Considerations in the Development and Implementation of Constitutive Models," 2nd Symposium on Nonlinear Constitutive Relations for High Temperature Applications, Cleveland, June, 1984.

3. C.E. Harris, "Comparison of the Fracture Behavior of Thick Laminated Composites Utilizing the Compact Tension, Three-Point Bend and Center-Cracked Tension Specimens," presented at the Seventeenth Symposium on Fracture Mechanics sponsored by ASTM, Albany, NY, August 1984.

4. C.E. Harris, "A Damage Tolerant Design Parameter for Graphite/Epoxy Laminated Composites," presented at the 21st Annual Meeting of the Society of Engineering Science, Blacksburg, VA, October 15-17, 1984.

5. C.E. Harris, "The Fracture of Thick Graphite/Epoxy Laminates with Part-Through Surface Flaws," presented at the Symposium on Composite Materials: Fatigue and Fracture, Dallas, TX, October 1984.

6. C.E. Harris, "Unique Aspects of Conducting Strength Tests of Thick Composites" invited paper presented at the fall meeting of the Society for Experimental Mechanics, Milwaukee, WI., November 1984..

5.1.3 To Be Presented

1. D.H. Allen, "Modelling of Stiffness Reduction Due to Matrix Cracks in Graphite/Epoxy Laminates," Society of Engineering Science 22nd Annual Meeting, State College, PA, October, 1985.

2. C.E. Harris, "Experimental/Analytical Correlation of Damage Accumulation in Laminated Composites," Society of Engineering Science 22nd Annual Meeting, State College, PA, October, 1985.

5.2 Research Related Travel and Consultative Functions

1. D.H. Allen visited Hercules Corp., Magna, UT, to discuss composite materials, April 1984.

2. D.H. Allen visited Martin-Marietta, Denver, to discuss composites in large space structures and to see the PACOSS experiment, April 1984.

3. D.H. Allen and W.E. Haisler attended the 2nd AFOSR Forum on Large Space Structures, Washington, D.C., June 1984.

4. D.H. Allen and W.E. Haisler attended the 2nd Symposium on Nonlinear Constitutive Equations for Elevated Temperature Applications, Cleveland, June 1984.

5. D.H. Allen attended NASA Shuttle Launch 41-C to see the launch of LDEF, which contains one of Dr. Allen's experiments, April 1984.

6. W.E. Haisler attended the AFFDL Vibration Damping Conference, Long Beach, CA, April 1984.

7. W.E. Haisler attended the AIAA SDM Conference, Palm Springs, CA, May 1984.

8. D.H. Allen consulted with engineers at the General Electric Aircraft Engine Business Group on viscoplasticity in metals at elevated temperature, Cincinnati, April and November, 1984.

9. D.H. Allen visited Drs. G. Sendeckyj and D. Paul at AFML to discuss composites, Dayton, November, 1984.

10. D.H. Allen, and C.E. Harris attended the ASTM Conference in Dallas, November, 1984.

6. APPENDIX

INTERIM TECHNICAL REPORTS

APPENDIX 6.1

Mechanics and Materials Center
TEXAS A&M UNIVERSITY
College Station, Texas

A DAMAGE MODEL FOR CONTINUOUS FIBER COMPOSITES

PART I: Theoretical Development

by

D. H. Allen

S. E. Groves

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and

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Texas A&M University

College Station, Texas 77843

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August 1984
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A continuum mechanics approach is utilized herein to develop a model for predicting the thermomechanical constitution of continuous fiber composites subjected to both monotonic and cyclic fatigue loading. In this model the damage is characterized by a set of second order tensor valued internal state variables representing locally averaged measures of specific damage states such as matrix cracks, fiber-matrix debonding, interlaminar cracking, or any other damage state. Globally averaged history dependent constitutive equations are posed utilizing constraints imposed from thermodynamics with internal state variables as well as fracture mechanics.

In Part I the thermodynamics with internal state variables is constructed and it is shown that suitable definitions of the locally averaged field variables will lead to equivalent thermodynamic constraints on a scale assumed to be large compared to the scale of the damage. Based on this result the Helmholtz free energy is then expanded in a

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In Part I the thermodynamics with internal state variables is constructed and it is shown that suitable definitions of the locally averaged field variables will lead to equivalent thermodynamic constraints on a scale assumed to be large compared to the scale of the damage. Based on this result the Helmholtz free energy is then expanded in a Taylor series expansion in terms of strain, temperature, and damage to obtain the stress-strain relation for composites with internal state

variables representing damage. Finally, an internal state variable growth law is proposed for matrix cracking.

In Part II the resulting three dimensional tensor equations are simplified using material symmetry constraints and are written in engineering notation. The resulting constitutive model is then cast into laminate equations and an example problem is solved.

It is concluded that although the model requires further development and extensive experimental verificatiion it may be a useful tool in characterizing the thermomechanical constitutive behavior of continuous fiber composites with damage.

INTRODUCTION

Ultimate failure of continuous fiber composite structural components is preceded by a sequence of microstructural and macrostructural events such as microvoid growth, matrix cracking, fiber-matrix debonding, interlaminar cracking, edge delamination, and fiber fracture which are all loosely termed damage. Considerable experimental research has been performed in the last decade detailing the growth of damage under both monotonic and cyclic loading conditions [1-7]. The significance of this damage lies in the fact that numerous global material properties such as stiffness and residual strength may be substantially altered during the life of the component, as shown in Fig. 1 [8]. It has been found that the first phase of fatigue is typified by development of a characteristic damage state (CDS) [9] which is composed primarily of matrix cracking in off-axis plies. During the second phase of damage development the CDS contributes to fiber-matrix

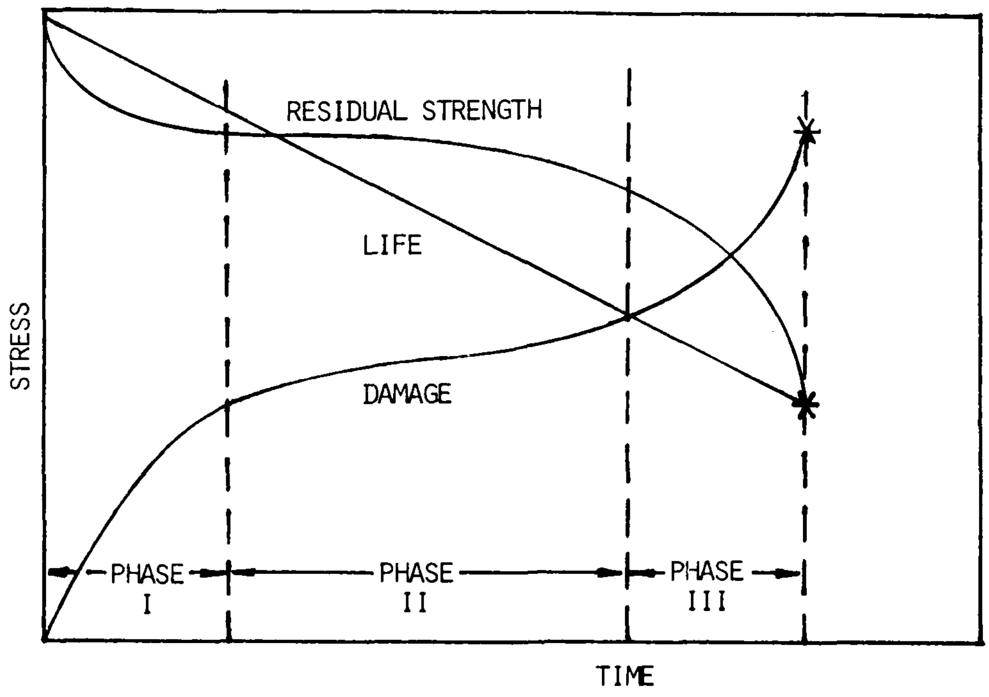

Fig. 1. Damage Accumulation in a Continuous Fiber Composite Subjected to Monotonic Load or Strain Controlled Cyclic Fatigue (from ref. 8).

debonding, delamination, and fiber microbuckling. These phenomena in turn contribute to a tertiary damage phase in which edge delamination and fiber fracture lead to ultimate failure of the specimen [6].

Analytical modeling of the damage state appears to be only recently studied. The earliest attempts fall under the general heading of laminate analysis, in which various empirical schemes have been developed to discount ply properties in the presence of damage [10-12]. Axial stiffness reduction and stress distribution in the CDS have also been predicted using a one-dimensional shear lag concept [5]. Several researchers have obtained solutions for effective moduli of elastic bodies with distributed cracks [13-17]. In the case where cracks are either randomly distributed or oriented the effect of total crack surface area is found to cause a first order effect on the stiffness [13-16].

Fracture based concepts have recently been utilized to model damage development [18-21]. Although the first of these studies [18] contains a general theory which may be applied to fibrous composites, it has so far only been utilized for quasi-isotropic random particulate composites such as solid rocket propellant, [9] and as such has not been applied to continuous fiber composites. The theory in the latter two [20,21] has not been utilized to predict reduction of off-axis stiffness components. Kachanov's modulus reduction technique [22] has also been utilized for fibrous composites [23] and although promising results were obtained, the model was constructed in uniaxial form only.

A complex interactive experiment and analysis model (called a mechanistic model) has been proposed [8]. The approach used therein is fundamentally quite different from that developed in this paper.

Furthermore, the mechanistic model requires numerous experimental results for each geometric layup in order to determine which damage mode predominates.

Finally, extended forms of Miner's rule [24] have been proposed [18,25]. However, they are based on simplified microphysical models at this time.

The concept of damage as an internal state variable [26] has been previously utilized in continuum mechanics/thermodynamics based theories for crystalline and/or brittle materials [27-34], as well as for nonlinear viscoelastic materials [18]. A study has been made of the effect of vector-valued damage parameters on various compliance terms [35], and this methodology is currently undergoing further development [36, 37].

The research reviewed above indicates that although important progress has been made in characterizing damage in fibrous composites, substantial and continued research is warranted before several issues can be resolved.

In this paper an attempt will be made to assemble many of the concepts embodied in the research efforts mentioned briefly above and to utilize these concepts to develop a model for damage in continuous fiber composites which is rigorously based in continuum mechanics/thermodynamics and is generic with regard to material type, load spectrum, and specimen geometry.

CHARACTERIZATION OF DAMAGE AS A SET OF INTERNAL STATE VARIABLES

Consider an initially unloaded and undamaged continuous fiber composite structural component as shown in Fig. 2a, where undamaged is defined here to mean that the body may be considered to be continuous (without voids) on a scale several orders of magnitude smaller than the smallest external dimension of the component. Although voids may exist in the initial state, their total surface area is assumed to be small compared to the external surface area of the component. Under this assumption the body is assumed to be simply connected and we call the initial bounding surface the external boundary. Although the component is undamaged, there may exist local heterogeneity due to such causes as *processing inhomogeneities and second phase materials including fibers* and matrix tougheners. In addition, the body may be subjected to some residual stress state due to prior loading, cool down, etc.

Now suppose that the component is subjected to some traction or deformation history, as shown in Fig. 2b. The specimen will undergo a thermodynamic process which will in general be in some measure irreversible. This irreversibility is introduced by the occurrence of such phenomena as material inelasticity (even in the absence of damage) fracture (both micro- and macroscale), friction (due to rubbing of fractured surfaces), and chemical change. While all of these phenomena can and do commonly occur in continuous fiber composites, in the present research it will be assumed that fracture is the only irreversible phenomenon of significance. Thus, all fracture events will be termed damage. Due to these fracture events, the body will necessarily become

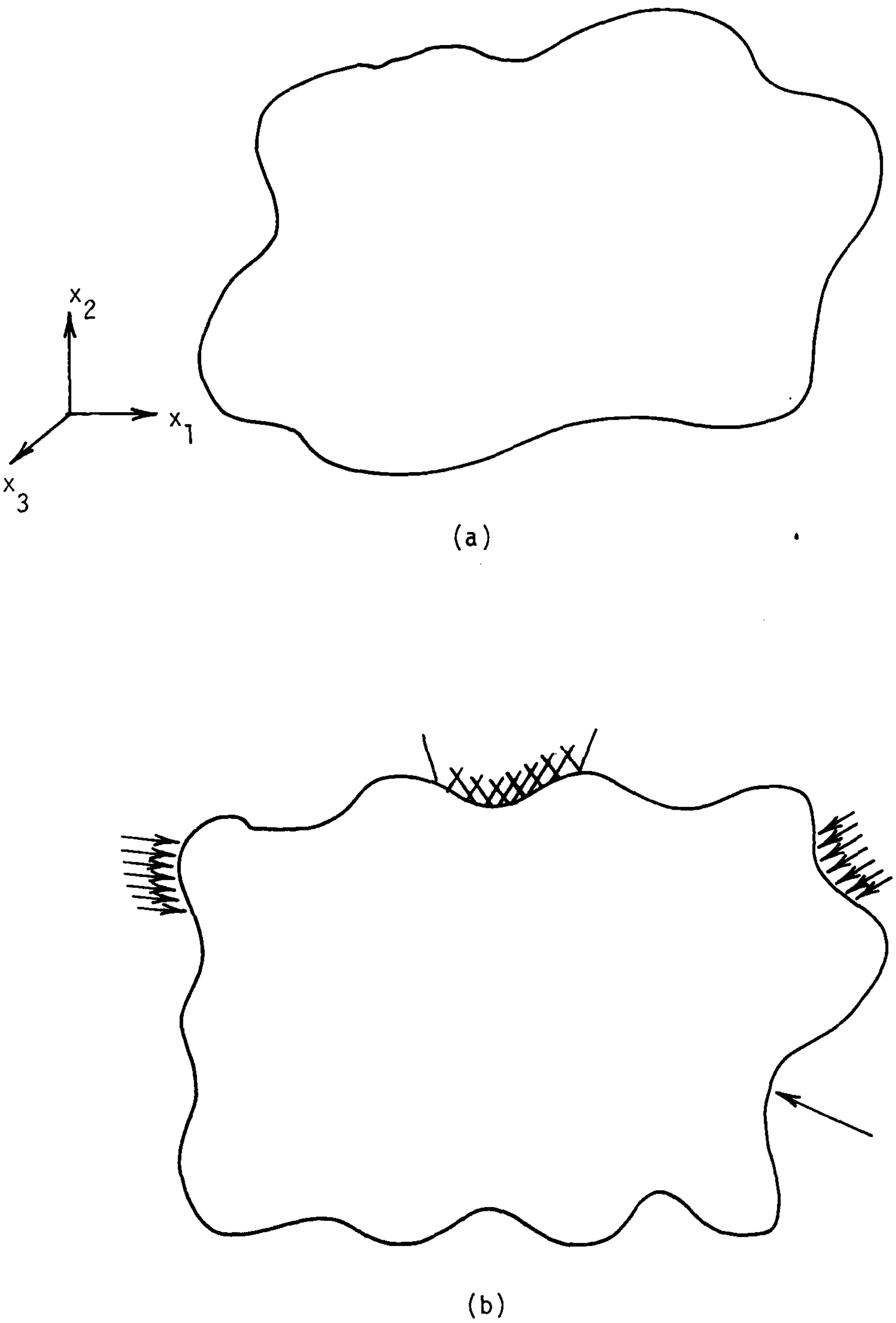

Fig. 2. Fibrous Composite Structural Component in

a) Undamaged State, and

b) With Applied Tractions.

multiply connected, and all newly created surfaces not intersecting the external boundary will be termed internal boundaries. Because of the above assumptions the model will be limited to polymeric matrix composites at temperatures well below the glass transition temperature T_g, where viscoelasticity in matrix materials is small. Metal matrix composites will be excluded due to complex post-yield behavior of the matrix.

While fracture classically involves changes in the boundary conditions governing a complex field problem, it is hypothesized that one may neglect boundary condition changes caused by creation and alteration of both internal and external surfaces created during fracture as long as the resulting damage in the specimen is statistically homogeneous on a scale which is small compared to the scale of the body of interest. This implies that the total newly created surface area (which includes internal surfaces) may be of the same order of magnitude as the original external surface area. Under the condition of small scale statistical homogeneity all continuum based conservation laws are assumed to be valid on a global scale in the sense that all changes in the continuum problem resulting from internal damage are reflected only through alterations in constitutive behavior. Typical microstructural events which qualify as damage are therefore matrix cracking in lamina, fiber/matrix debonding, and localized interlaminar delamination. Large scale changes in the external surface such as edge delaminations, however, are treated as boundary effects which must be reflected in conservation laws via changes in the external boundary conditions rather than in constitutive equations [39].

Although the damage process actually involves the conversion of strain energy to surface energy, the fact that the damage is reflected in the local constitutive equations requires that it be treated as a set of energy dissipative local state variables which are not discernible on the external boundary. Therefore, since the damage can be determined only through a precise knowledge of the entire history of observable inputs, it is characterized as a set of internal state variables. This concept will be further developed in the next section.

THERMODYNAMICS OF MEDIA WITH DAMAGE

We now proceed to construct a concise model of the continuum with damage. To do this, consider once again the structural component, denoted B in Fig. 3a. The body B is assumed to be of the scale of some appropriate boundary value problem of interest. Now consider some local element labelled L and with external surfaces S_1 arbitrarily chosen normal to a set of Cartesian coordinate axes (x_1, x_2, x_3), as shown in Fig. 3b. The element L is extracted from B and the newly created surfaces are subjected to appropriate boundary conditions so that the element response is identical to that when it is in B. Internal surfaces caused by fracture are labelled S_2 such that the intersection of S_1 and S_2 is a null set and $S = S_1 + S_2$. Furthermore, the volume of the element is defined to be V_L, which includes the volume of voids V_v. The scale of L is chosen so that its dimensions are small compared to the dimensions of B, but at the same time, the dimensions of L are large enough to guarantee statistical homogeneity of the material

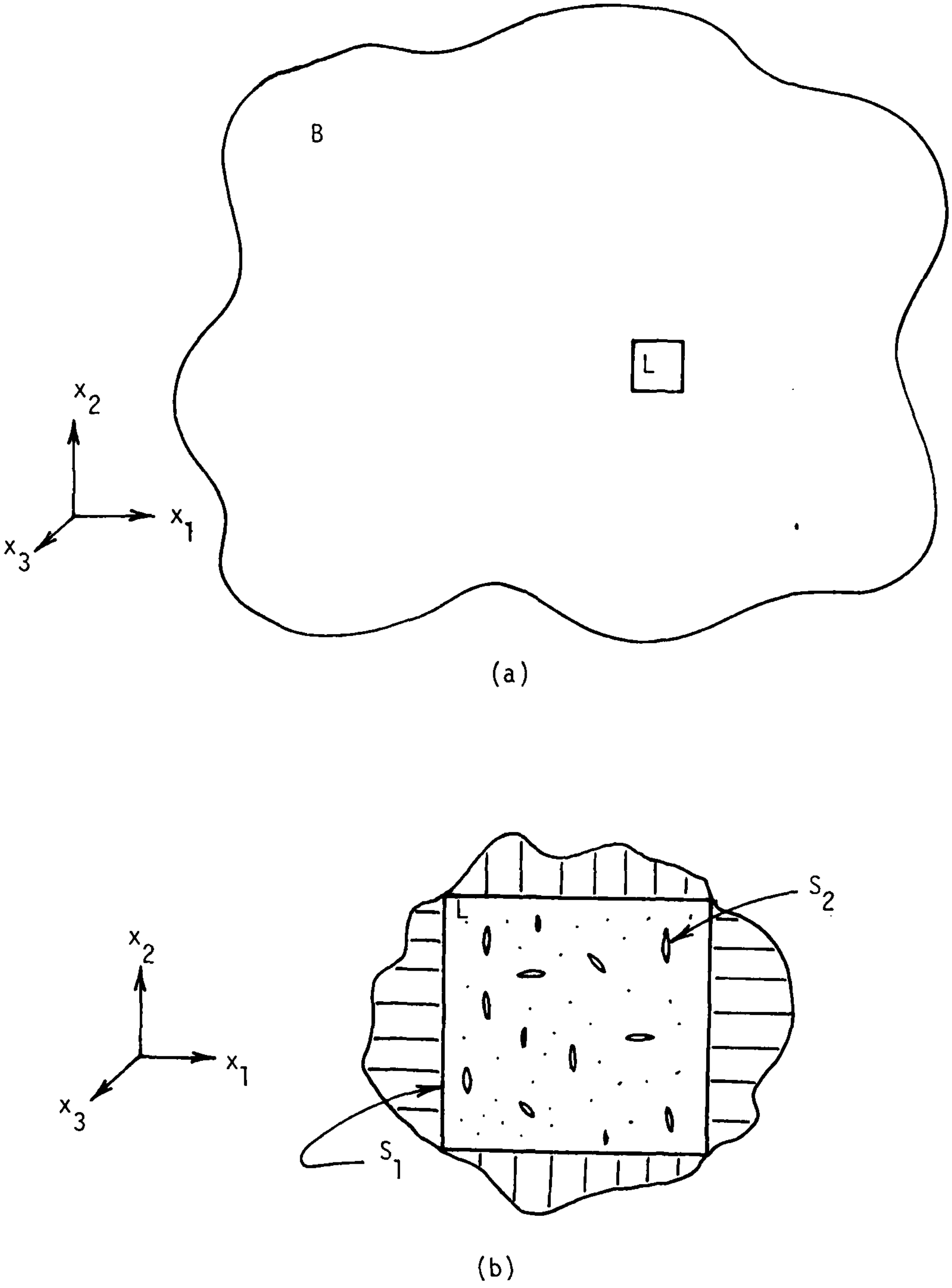
B
L
x_2
x_1
x_3
(a)
L
S_2
S_1
x_2
x_1
x_3
(b)

Fig. 3. A Body with Damage (a) General Body,
(b) Local Element.

inhomogeneities and defects in L even though the total surface area of defects may be of the same order of magnitude as S_1 [38]. Suppose further that in the absence of defects or at constant damage state the material behavior is linear thermoelastic, thus specifically excluding the effects of crack face rubbing.

The following notation is adopted. Quantities without capitalized subscripts denote pointwise quantities. Those with subscripts L denote quantities which are averaged over the local element L. Finally, the subscript E denotes linear thermoelastic properties.

Review of Thermodynamic Constraints on Linear Thermoelastic Media

Under the above conditions the pointwise Helmholtz free energy per unit mass h of the undamaged linear elastic medium may be expressed as a second order expansion in terms of strain ε_{ij} and temperature change ΔT as follows [40]:

$$h \equiv u - Ts = h_E(\varepsilon_{ij}, T) =$$

$$A + B_{ij}\varepsilon_{ij} + 1/2\, C_{ijkl}\varepsilon_{ij}\varepsilon_{kl} + D\Delta T + E_{ij}\varepsilon_{ij}\Delta T + 1/2\, F\Delta T^2 \qquad (1)$$

where u and s are the internal energy and entropy per unit mass, respectively, and A, B_{ij}, C_{ijkl}, D, E_{ij} and F are material parameters which are independent of strain and temperature and $\Delta T \equiv T - T_R$, where T_R is the reference temperature at which no deformation is observed at zero load. In addition, we assume here that all motions produce small deformations so that ε_{ij} is the infinitesimal strain tensor. Furthermore, inertial effects are assumed to be negligible.

Pointwise conservation laws appropriate to the body are as follows:

1) conservation of linear momentum

$$\sigma_{ji,j}=0 \qquad ; \qquad (2)$$

where σ_{ij} is the work conjugate stress tensor to the strain tensor ε_{ij} and body forces are assumed to be negligible;

2) *conservation of angular momentum* (assuming body moments may be neglected)

$$\sigma_{ij} = \sigma_{ji} \qquad ; \qquad (3)$$

3) balance of energy

$$\rho\dot{u} - \sigma_{ij}\dot{\varepsilon}_{ij} + q_{j,j} = \rho r \qquad ; \qquad (4)$$

where ρ is the mass density, q_j are the components of the heat flux vector, and r is the heat source term; in addition, dots denote time differentiation and, $_{,j} \equiv \partial/\partial x_j$;

4) the second law of thermodynamics

$$\rho\dot{s} - \frac{\rho r}{T} + \left(\frac{q_j}{T}\right)_{,j} \geq 0 \qquad , \qquad (5)$$

Furthermore

$$\varepsilon_{ij} \equiv \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad , \qquad (6)$$

The above set of equations may be cast with appropriate boundary conditions so that constraints imposed by the second law of thermodynamics will result in [40]

$$s = s_E = \frac{-\partial h_E}{\partial T} = -D - E_{ij}\varepsilon_{ij} - F\Delta T \qquad , \qquad (7)$$

$$\sigma_{ij} = \sigma_{E_{ij}} = \rho\,\frac{\partial h_E}{\partial \varepsilon_{ij}} = \rho\left(B_{ij} + C_{ijk\ell}\varepsilon_{k\ell} + E_{ij}\,\Delta T\right) \qquad , \qquad (8)$$

where ρB_{ij} are components of residual stresses at the reference temperature at which $T = 0$,

$$q_i = -k_{ij}g_j + 0\,(|g_j|^2) \quad , \tag{9}$$

where

$$g_j \equiv T_{,j} \quad . \tag{10}$$

Thermodynamic Constraints with Local Damage

Now consider the local element shown in Fig. 3b with traction boundary conditions on the external surface S_1. In addition, the interior of L is assumed to be composed entirely of linear elastic material and voids. It is our aim to construct locally averaged field equations which are similar in form to the pointwise field equations discussed above. In performing this averaging process the pointwise Helmholtz free energy described in equation (1) will undergo a natural modification to include the energy conversion due to crack formation.

In order to construct locally averaged equations first define the locally averaged stress tensor:

$$\sigma_{L_{ij}} \equiv \frac{1}{V_L} \int_{V_L} \sigma_{ij} dV \tag{11}$$

Integrating the balance of energy (4) over the local volume and dividing through by the local volume results in

$$\frac{1}{V_L}\int_{V_L}\rho\dot{u}dV - \frac{1}{V_L}\int_{V_L}\sigma_{ij}\dot{\varepsilon}_{ij}dV + \frac{1}{V_L}\int_{V_L}q_{j,j}dV = \frac{1}{V_L}\int_{V_L}\rho r dV \quad . \tag{12}$$

In order to simplify the above define the locally by averaged density ρ_L such that

$$\rho_L \equiv \frac{1}{V_L}\int_{V_L}\rho dV \quad . \tag{13}$$

Furthermore, define the local internal energy rate per unit mass $\dot{u}_L$ such that

$$\dot{u}_L \equiv \frac{1}{\rho_L V_L}\int_{V_L}\rho\dot{u}dV \quad , \tag{14}$$

which can be constructed as long as $\rho_L \neq 0$.

Now consider the second term in equation (12). Recall that since σ_{ij} is a symmetric tensor

$$\sigma_{ij}\dot{\varepsilon}_{ij} = \frac{1}{2}\ \sigma_{ij}\ (\dot{u}_{i,j} + \dot{u}_{j,i}) = \sigma_{ij}\dot{u}_{i,j} \quad . \tag{15}$$

Thus, using the divergence theorem gives

$$\frac{1}{V_L}\int_{V_L}\sigma_{ij}\dot{\varepsilon}_{ij}dV = \frac{1}{V_L}\int_{V_L-V_v}\sigma_{ij}\dot{u}_{i,j}dV = \frac{1}{V_L}\int_{S_1}\sigma_{ij}\dot{u}_i n_j dS + \frac{1}{V_L}\int_{S_2}\sigma_{ij}\dot{u}_i n_j dS \quad , \tag{16}$$

where it is assumed that the stress tensor is negligible in the void volume V_v, and n_j are the components of the unit outer normal vector to the surface $S = S_1 + S_2$.

Now define

$$\dot{u}_L^c \equiv - \frac{1}{\rho_L V_L} \int_{S_2} \sigma_{ij} \dot{u}_i n_j dS \quad , \qquad (17)$$

which is the time rate of change of surface energy release per unit local volume due to cracking in L. Further, define

$$\varepsilon_{L_{ij}} \equiv \frac{1}{V_L} \int_{S_1} u_i n_j dS \quad , \qquad (18)$$

so that under the assumption that all displacements are infinitesinal and for the case of either spacially uniform surface tractions or applied displacements on the local element external surface S_1

$$\sigma_{L_{ij}} \dot{\varepsilon}_{L_{ij}} = \frac{1}{V_L} \int_{S_1} \sigma_{ij} \dot{u}_i n_j dS \qquad (19)$$

Thus, equation (16) becomes

$$\frac{1}{V_L} \int_{V_L} \sigma_{ij} \dot{\varepsilon}_{ij} dV = \rho_L \dot{u}_L^c + \sigma_{L_{ij}} \dot{\varepsilon}_{L_{ij}} \quad . \qquad (20)$$

Define also

$$q_{L_{j,j}} \equiv \frac{1}{V_L} \int_{V_L} q_{j,j} dV \quad , \tag{21}$$

and

$$r_L \equiv \frac{1}{\rho_L V_L} \int_{V_L} \rho r dV \quad . \tag{22}$$

Substituting equations (14), (20), (21), and (22) into equation (12) yields the following locally averaged balance of energy:

$$\rho_L \dot{u}_L + \rho_L \dot{u}_L^c - \sigma_{L_{ij}} \dot{\varepsilon}_{L_{ij}} + q_{L_{j,j}} = \rho_L r_L \quad . \tag{23}$$

We now define the effective internal energy u_L' such that

$$u_L' \equiv u_L + u_L^c \quad . \tag{24}$$

Substitution of (24) into (23) results in

$$\rho_L \dot{u}_L' - \sigma_{L_{ij}} \dot{\varepsilon}_{L_{ij}} + q_{L_{j,j}} = \rho_L r_L \quad , \tag{25}$$

which can be seen to be equivalent in form to energy balance law (4).

In order to construct a similar statement for entropy production inequality (5), first multiply through by T and then integrate over the local volume V_L and divide by this quantity to obtain

$$\frac{1}{V_L}\int_{V_L} \rho \dot{s} T dV - \frac{1}{V_L}\int_{V_L} \rho r dV + \frac{1}{V_L}\int_{V_L} T\left(\frac{q_j}{T}\right)_{,j} dV \geq 0 \quad . \tag{26}$$

Now define

$$T_L \equiv \frac{1}{V_L}\int_{V_L} T dV \quad , \tag{27}$$

and

$$\dot{s}_L \equiv \frac{1}{\rho_L T_L V_L}\int_{V_L} \rho \dot{s} T dV \quad , \tag{28}$$

so that substitution of definitions (22), (27) and (28) into (26) will result in

$$\rho_L \dot{s}_L T_L - \rho_L r_L + \frac{1}{V_L}\int_{V_L} T\left(\frac{q_j}{T}\right)_{,j} dV \geq 0 \quad . \tag{29}$$

Now note that the last term in (29) may be written as follows using the product rule:

$$\frac{1}{V_L}\int_{V_L} T\left(\frac{q_j}{T}\right)_{,j} dV = \frac{1}{V_L}\int_{V_L} q_{j,j} dV - \frac{1}{V_L}\int_{V_L} \frac{q_j , g_j}{T} dV \quad . \tag{30}$$

Thus, define

$$q_{L_j} T_{L,j} \equiv \frac{T_L}{V_L} \int_{V_L} \frac{q_j T_{,j}}{T} \, dV \quad . \tag{31}$$

Therefore, substituting definitions (21) and (31) into (30) and this result into (29) gives, after dividing through by T_L:

$$\rho_L \dot{s}_L - \frac{\rho_L r_L}{T_L} + \left(\frac{q_{L_j}}{T_L}\right)_{,j} \geq 0 \quad . \tag{32}$$

We now assume that the local volume is small enough compared to B that the standard procedure may be utilized to obtain the linear conservation of momentum equations [38]

$$\sigma_{L_{ji,j}} = 0 \quad , \tag{33}$$

similar to pointwise equations (2), and the conservation of angular momentum may also be obtained

$$\sigma_{L_{ji}} = \sigma_{L_{ij}} \quad , \tag{34}$$

similar to equations (3). Thus, it is assumed that no body moments are introduced via material inhomogeneity or other sources.

Locally averaged field equations (25), (32),(33) and (34) have now been constructed which are similar to pointwise field equations (2) through (5). On the basis of this similarity we now define the locally averaged Helmoltz free energy [18, 38]:

$$h_L \equiv u'_L - T_L s_L = u_L - T_L s_L + u^c_L = h_{E_L} + u^c_L \quad , \tag{35}$$

where it can be seen from definitions (14) and (28) that h_{E_L} is the locally averaged elastic Helmholtz free energy for which residual damage is assumed to be small , given by equation (1) to be

$$h_{E_L} \equiv \frac{1}{V_L} \int_{V_L} (A + B_{ij}\varepsilon_{ij} + \frac{1}{2} C_{ijk\ell}\varepsilon_{ij}\varepsilon_{k\ell} + D\Delta T + E_{ij}\varepsilon_{ij}\Delta T + \frac{1}{2} F\Delta T^2)\, dV, \tag{36}$$

Utilizing definitions (18) and (27) we may further define

$$A_L \equiv \frac{1}{V_L} \int_{V_L} A dV \quad , \tag{37}$$

$$B_{L_{ij}}\, \varepsilon_{L_{ij}} \equiv \frac{1}{V_L} \int_{V_L} B_{ij}\, \varepsilon_{ij}\, dV \quad , \quad \text{no sum on } i,j, \tag{38}$$

$$C_{L_{ijkl}}\, \varepsilon_{L_{ij}}\, \varepsilon_{L_{kl}} \equiv \frac{1}{V_L} \int_{V_L} C_{ijkl}\, \varepsilon_{ij}\, \varepsilon_{kl}\, dV \quad , \quad \text{no sum on } i,\ j,\ k,\ l, \tag{39}$$

$$D_L \equiv \frac{1}{V_L T_L} \int_{V_L} D \Delta T dV \quad , \tag{40}$$

$$E_{L_{ij}} \; \varepsilon_{L_{ij}} \equiv \frac{1}{V_L T_L} \int_{V_L} E_{ij} \; \varepsilon_{ij} \; TdV \quad , \qquad \text{and sum on } i,j, \text{ and} \tag{41}$$

$$F_L \equiv \frac{1}{V_L T_L{}^2} \int_{V_L} FT^2 dV \quad . \tag{42}$$

Note that when the actual strain state ε_{ij} is spatially invariable and $S_2 = 0$, the above equations result in $B_{L_{ij}} = B_{ij}$, $C_{L_{ijkl}} = C_{ijkl}$ and $E_{L_{ij}} = E_{ij}$. Substituting definitions (37) through (42) into local elastic free energy equation (36) results in

$$h_{E_L} = A_L + B_{L_{ij}} \varepsilon_{L_{ij}} + \frac{1}{2} C_{L_{ijkl}} \varepsilon_{L_{ij}} \varepsilon_{L_{kl}} + D_L T_L + E_{L_{ij}} \varepsilon_{L_{ij}} T_L + \frac{1}{2} F_L T_L{}^2 \quad . \tag{43}$$

The similarity between the pointwise and local field equations leads to the conclusion that

$$s_L = \frac{-\partial h_L}{\partial T} \tag{44}$$

$$\sigma_{L_{ij}} = \rho_L \frac{\partial h_L}{\partial \varepsilon_{L_{ij}}} = \rho_L \left(B_{L_{ij}} + C_{L_{ijk\ell}} \varepsilon_{L_{k\ell}} + E_{L_{ij}} T_L\right) + \rho_L \frac{\partial u_L^c}{\partial \varepsilon_{L_{ij}}} \quad , \tag{45}$$

and

$$q_{L_i} = -k_{L_{ij}} g_{L_j} + O\left(|g_{L_j}|^2\right) \quad , \tag{46}$$

where

$$k_{L_{ij}} g_{L_{ij}} \equiv \frac{1}{V_L} \int_{V_L} k_{ij} g_j dV \quad , \qquad \text{no sum on } i,j, \tag{47}$$

and

$$g_{L_j} \equiv T_{L,j} \quad . \tag{48}$$

Note the similarity between equations (7) through (10) and (44) through (48), respectively.

Equations (45) will serve as the basis for thermomechanical stress-strain relations in damaged fibrous composites. All damage will be reflected through the local energy due to cracking u_L^c. This term will be modelled with internal state variables characterizing the various damage modes.

Description of the Internal State

In order to describe the internal state, we first consider the kinematics of a typical point 0 with neighboring points A and B, as shown in Fig. 4. Before deformation lines OA and OB are orthogonal, as shown in (a). After deformation we imagine that lines joining O',A', and B' are as shown in (b), and just at the instant that deformation is completed, a crack forms normal to the plane of AOB through point O', as shown in (c). Furthermore, point O' becomes two material points O' and O" on opposite crack faces and points A' and B' deform further to points A" and B". It is assumed that all displacements, including crack opening, are infinitesimal, so that an observer at an appropriate

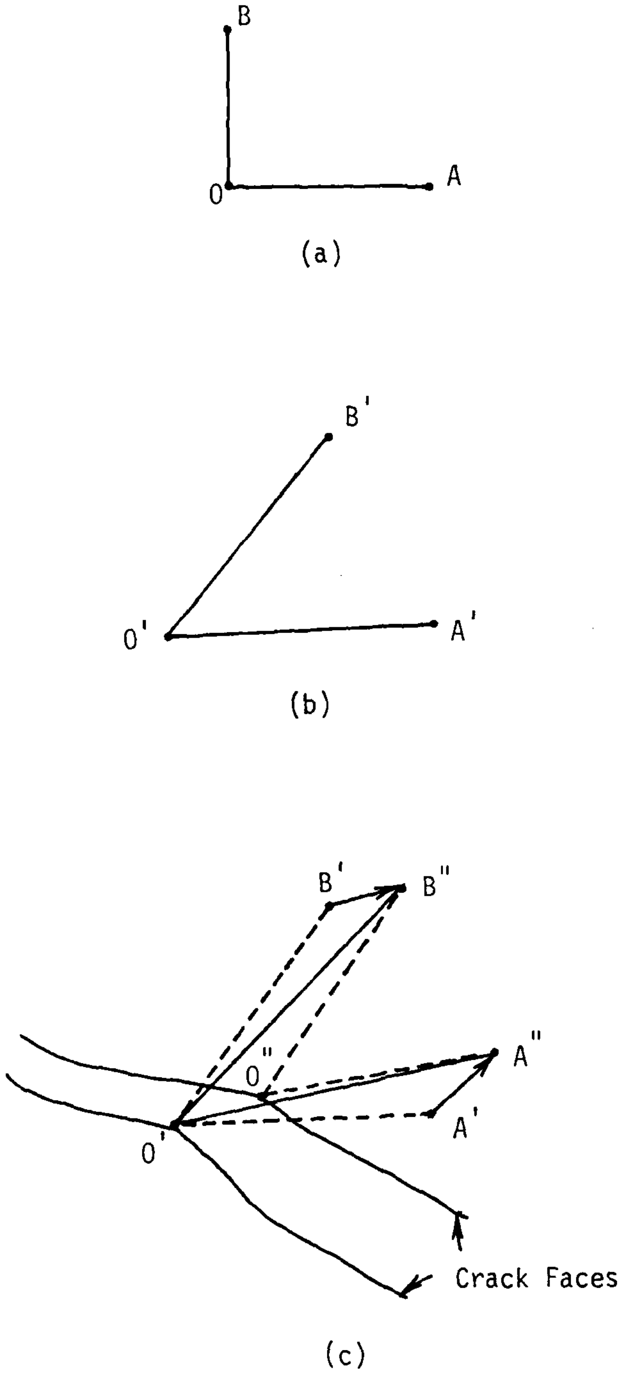
B
0
A
(a)
B'
0'
A'
(b)
B'
B"
0"
A"
A'
0'
Crack Faces
(c)

Fig. 4. Kinematics of the Damage Process

a) Point "0" Prior to Deformation,

b) Point "0" after Deformation and Prior to Fracture Process,

c) Point "0" after Fracture.

observation distance from point O "sees" only the deformation A" O' B". The strain associated with this deformation is appropriately called an observable state variable. However, the strain of interest is associated with A"O"B". Therefore, it is essential to construct an internal state variable which will relate these two strain descriptions. We therefore construct the vectors $\vec{u}^c$ connecting O' and O" and $\vec{n}^c$ describing the normal to the crack face at O', as shown in Fig. 5. It should be noted that $\vec{u}^c$ can be used to construct a pseudo-strain representing the difference in rotation and extension of lines A"O'B" and A"O"B".

Now recall that the energy released during cracking is given by equation (17). Since the body is elastic, we assume that this process is reversible and that tractions T_i^c can be applied at point O' which will close the crack:

$$\dot{u}_L^c = \frac{1}{\rho_L V_L} \int_{S_2} T_i^c \ \dot{u}_i^c \ dS \quad . \tag{49}$$

Using Cauchy's formula the above can be placed in a form similar to (17):

$$\dot{u}_L^c = \frac{1}{\rho_L V_L} \int_{S_2} \sigma_{ij}^c \ \dot{u}_i^c \ n_j^c \ dS \quad , \tag{50}$$

where the superscripts denote quantities associated with the actual crack geometry. Although these quantities do not necessarily coincide with the terms in the integrand of (17), their surface integration will

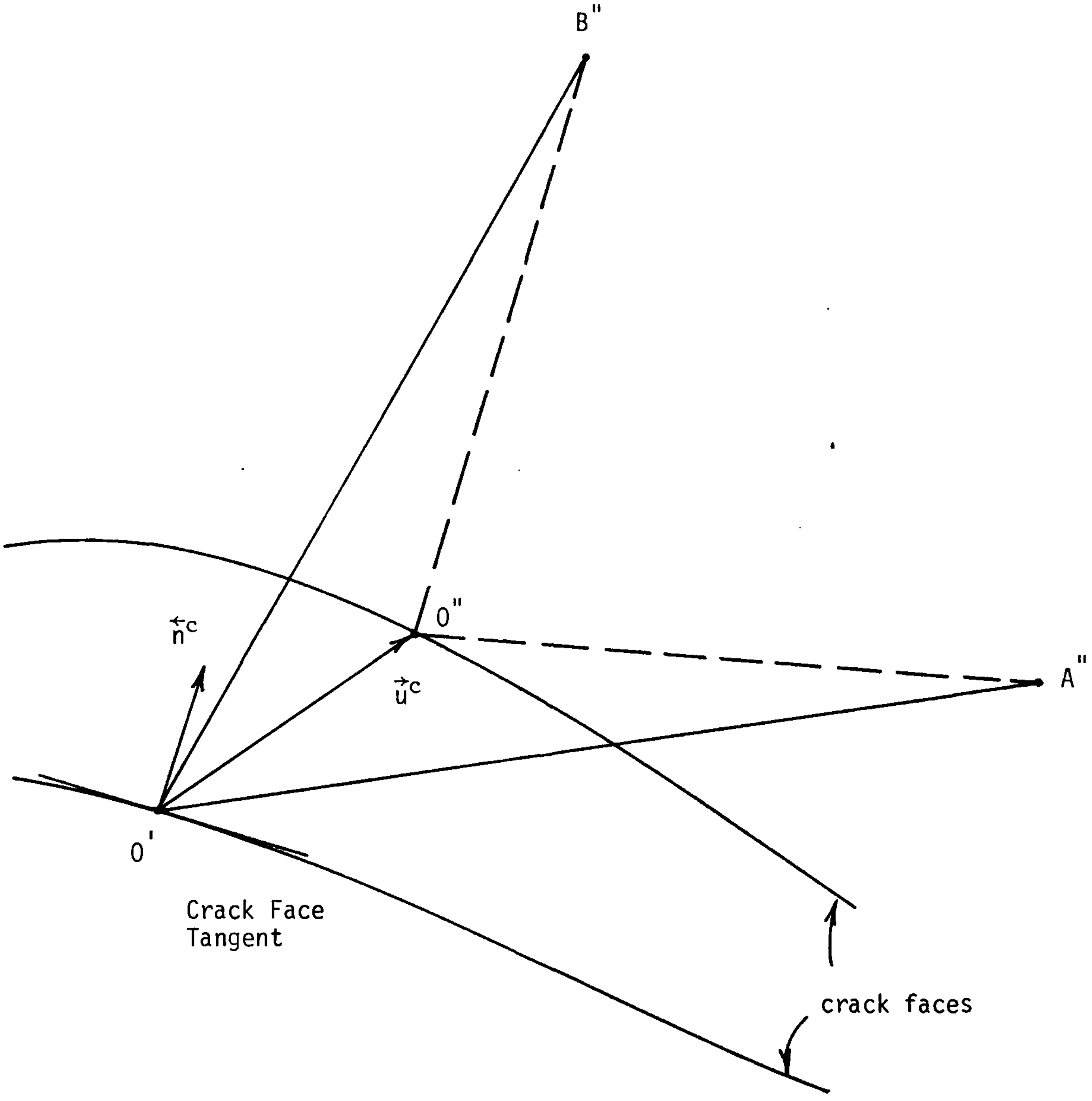
B"
O"
$\vec{n}^c$
$\vec{u}^c$
A"
O'
Crack Face
Tangent
crack faces

Fig. 5. Description of the Internal State at Point O'.

result in precisely the same energy release rate $\dot{u}_L^c$ due to the reversibility of the process.

Guided by the fact that $\vec{u}^c$ and $\vec{n}^c$ describe the kinematics of the cracking process at point 0, we now define the following second order tensor valued internal state variable:

$$\alpha_{ij} \equiv u_i^c \; n_j^c \quad . \tag{51}$$

Note that when the crack normal is time independent the above may be time differentiated to give

$$\dot{u}_L^c = \frac{1}{\rho_L \, V_L} \int_{S_2} \sigma_{ij}^c \; \dot{\alpha}_{ij} \; dS \quad . \tag{52}$$

Note that the components of $\vec{u}^c$ and $\vec{n}^c$ can be recovered from (51) by using single row and column multiplication of α_{ij}:

$$u_i^2 = u_i^c n_j \; u_i^c n_j \qquad \text{(no sum on i),} \tag{53}$$

$$n_i^2 = u_i^c n_j \; u_i^c n_j \qquad \text{(no sum on j).} \tag{54}$$

Therefore, the normal and shear modes of crack displacement can be recovered from α_{ij}.

Now suppose that $\dot{u}_L^c$ is subdivided into integrals over a finite number of internal surface areas S_2^n of fundamentally different nature:

$$\dot{u}_L^c = \sum_{n=1}^{N} \frac{1}{\rho_L \, V_L} \int_{S_2^n} \sigma_{ij}^c \; \dot{\alpha}_{ij}^n \; dS \quad , \tag{55}$$

such that

$$\sum_{\eta=1}^{N} S_2^{\eta} = S_2 \quad , \tag{56}$$

and the integer N represents the number of damage modes, to be described below.

Now define the locally averaged internal state variable α_{ij}^{η} for the η th damage mode as follows:

$$\alpha_{L_{ij}}^{\eta} \equiv \frac{1}{V_L} \int_{S_2^{\eta}} u_i^c n_j^c \, dS = \frac{1}{V_L} \int_{S_2^{\eta}} \alpha_{ij} \, dS \quad . \tag{57}$$

Furthermore, if we define the average crack closure stress $\sigma_{L_{ij}}^{c\eta}$ for the η th damage mode such that

$$\sigma_{L_{ij}}^{c\eta} \dot{\alpha}_{ij}^{\eta} \equiv \frac{1}{\rho_L V_L} \int_{S_2^{\eta}} \sigma_{ij}^{c\eta} \dot{\alpha}_{ij}^{\eta} \, dS \quad \text{, no sum on } i, j, \eta, \tag{58}$$

it follows that, from (55), (57) and (58)

$$\dot{u}^c_L = \sigma^{c\eta}_{L_{ij}} \dot{\alpha}^{\eta}_{L_{ij}} \quad , \tag{59}$$

where we have assumed that repeated indices η imply summation over the range N. It is clear from the above discussion that the value of N must be sufficiently large to recover the essential physics of the damage process. In a mathematical sense, this implies that, whereas the mapping from α_{ij} to $\alpha^{\eta}_{L_{ij}}$ is unique, the inverse should also be true in an approximate sense. Note also that both u^c_i and n^c_j in equations (57) will be affected by crack interaction in the local volume.

Now consider equation (59) in further detail. The kinetic quantities $\sigma^{c\eta}_{L_{ij}}$ may be interpreted as generalized forces which are energy conjugates to the kinematic strain-like internal state variables $\alpha^{\eta}_{L_{ij}}$.

We infer from this that there exists a constitutive relation between these variables of the form

$$\sigma^{c\eta}_{L_{ij}} = \sigma^{c\eta}_{L_{ij}} (\varepsilon_{L_{kl}}, T_L, \alpha^{\mu}_{L_{kl}}) \quad . \tag{60}$$

Therefore, substituting (60) into (59) and integrating in time will give

$$u^c_L (t_1) = \int_{-\infty}^{t_1} \dot{u}^c_L (t)dt = u^c_L (\varepsilon_{L_{k1}} (t_1), T_L (t_1), \alpha^{\mu}_{L_{k1}} (t_1)) \quad , \tag{61}$$

It is now proposed that u^c_L be expanded in a Taylor series which is second order in each of the arguments in equation (61) as follows:

$$\begin{aligned} u^c_L = \rho_L \Big(& G^\eta_{ij}\, \alpha^\eta_{L_{ij}} + H^\eta_{ij}\, \alpha^\eta_{L_{ij}}\, \Delta T_L + I^\eta{}_{ijkl}\, \varepsilon_{L_{ij}}\, \alpha^\eta_{L_{kl}} + J^{\eta\zeta}_{ijkl}\, \alpha^\zeta_{L_{ij}}\, \alpha^\zeta_{L_{kl}}\, \Delta T_L \\ & + L^{\eta\zeta}_{ijklmn}\, \varepsilon_{L_{ij}}\, \alpha^\eta_{L_{kl}}\, \alpha^\zeta_{L_{mn}} + \frac{1}{2} M^\eta_{ijklmn}\, \varepsilon_{L_{ij}}\, \varepsilon_{L_{kl}}\, \alpha^\eta_{L_{mn}} + N^\eta_{ijkl}\, \varepsilon_{L_{ij}}\, \alpha^\eta_{L_{kl}}\, \Delta T_L \\ & + P^\eta_{ij}\, \alpha^\eta_{L_{ij}}\, \Delta T^2_L + \frac{1}{2} Q^{\eta\zeta}_{ijklmnpq}\, \varepsilon_{L_{ij}}\, \varepsilon_{L_{kl}}\, \alpha^\eta_{L_{mn}}\, \alpha^\zeta_{L_{pq}} \\ & + R^{\eta\zeta}_{ijklmn}\, \varepsilon_{L_{ij}}\, \alpha^\eta_{kl}\, \alpha^\eta_{L_{mn}}\, \Delta T_L + S^{\eta\zeta}_{ijkl}\, \alpha^\eta_{L_{ij}}\, \alpha^\zeta_{L_{kl}}\, \Delta T^2_L \\ & + T^{\eta\zeta}_{ijklmn}\, \varepsilon_{L_{ij}}\, \alpha^\eta_{L_{kl}}\, \alpha^\zeta_{L_{mn}}\, \Delta T^2_L + \frac{1}{2} U^{\eta\zeta}_{ijklmn}\, \varepsilon_{L_{ij}}\, \alpha^\eta_{L_{kl}}\, \alpha^\zeta_{L_{mn}} \cdot \Delta T_L \\ & + V^{\eta\zeta}_{ijklmnpq}\, \varepsilon_{L_{ij}}\, \varepsilon_{L_{kl}}\, \alpha^\eta_{L_{mn}}\, \alpha^\zeta_{L_{pq}}\, \Delta T^2_L \Big) \quad , \qquad (62) \end{aligned}$$

where all terms are at least linear in $\alpha^\eta_{L_{ij}}$ due to the fact that u^c_L depends explicitly on damage, and $\Delta T_L \equiv T_L - T_R$. Thus, substituting (43) and (62) into equations (45) and neglecting higher order terms yields:

$$\begin{aligned} \sigma_{L_{ij}} = {} & B_{L_{ij}} + E_{L_{ij}}\, \Delta T_L + C_{L_{ijkl}}\, \varepsilon_{L_{kl}} + I^\eta_{ijkl}\, \alpha^\eta_{L_{kl}} \\ & + M^\eta_{ijklmn}\, \varepsilon_{L_{kl}}\, \alpha^\eta_{L_{mn}} + N^\eta_{ijkl}\, \alpha^\eta_{L_{kl}}\, \Delta T_L \quad . \qquad (63) \end{aligned}$$

Equations (63) may be written in the following convenient form

$$\sigma_{L_{ij}} = \sigma^R_{L_{ij}} + C'_{L_{ijkl}} \left(\varepsilon_{L_{kl}} - \varepsilon^T_{L_{kl}} \right) \quad , \qquad (64)$$

where

$$\sigma^R_{L_{ij}} = B_{L_{ij}} + I^{\eta}_{ijkl}\,\alpha^{\eta}_{L_{kl}} \quad , \tag{65}$$

is the residual stress in the absence of strain and temperature change, which may be induced by damage;

$$C'_{L_{ijkl}} \equiv C_{L_{ijkl}} + M^{\eta}_{ijklmn}\,\alpha^{\eta}_{L_{mn}} \quad , \tag{66}$$

is called the effective modulus tensor, which is degraded due to damage; and

$$\varepsilon^T_{L_{kl}} \equiv C'_{L_{ijkl}} \left(E_{L_{ij}} - N^{\eta}_{ijkl}\,\alpha^{\eta}_{kl}\,\Delta T_L \right) \quad , \tag{67}$$

is called the thermal strain, which also may be affected by damage. Note that the only second order terms are due to damage induced stiffness reduction and thermal strains.

Equations (64) through (67) are the completed description of the stress-strain relationship. Note that these equations reduce to the standard linear thermoelastic equations in the absence of damage ($\alpha^{\eta}_{L_{ij}}$ =0). Furthermore, it is notable that these equations reduce to Kachanov's model [22] in uniaxial form.

Damage Growth Laws

The model is completed with the construction of the damage growth laws, which may be described in the following differential equation form:

$$\dot{\alpha}^{\eta}_{L_{ij}} = \Omega^{\eta}_{ij} (\varepsilon_{L_{kl}}, T_L, \alpha^{\mu}_{L_{kl}}) \tag{68}$$

or equivalently, when Ω^{η}_{ij} are analytic in time,

$$\alpha^{\eta}_{L_{ij}}(t_1) = \int_{-\infty}^{t_1} \Omega^{\eta}_{ij}(\varepsilon_{L_{kl}}(t), T_L(t), \alpha^{\mu}_{L_{kl}}(t))dt \quad , \tag{69}$$

where the dot denotes time differentiation. Although the above equations are called "growth" laws they have the more general capability to model such phenomena as healing.

The precise nature of equations (69) is determinable only through a concise experimental program coupled with an understanding of the micromechanics of the medium. Indeed, these growth laws constitute the single most complex link in the model development.

In this section an example of a first generation growth law will be constructed for predicting damage up to the CDS in continuous fiber composites. Experimental evidence suggests that matrix cracks predominate prior to development of the CDS [4-6]. Guided by this observation, a single damage tensor is considered herein: $\alpha^{1}_{L_{ij}}$ representing matrix cracking.

In order to completely define equations (69), it is necessary to construct indicators of both the magnitude and direction of the damage tensor. In this first generation model it is assumed that the direction of the damage tensor is known a priori and does not vary as the damage state changes. Specifically, in a typical laminate, it is assumed that, in accordance with equation (57), the locally averaged resultants of $\vec{u}^c$ and $\vec{n}^c$ are normal to the fiber direction in each ply, as shown in Fig. 6. Thus, for example, in a 0° ply $\alpha^1_{L_{22}} \neq 0$, and all other components are zero, whereas in a 90° ply, $\alpha^1_{L_{11}} \neq 0$ and all other components are zero in global coordinates.

Under the above assumptions, the magnitude of the damage tensor is the sole repository for history dependence in each ply. Experimental evidence indicates that for matrix cracking in randomly oriented particulate composites [41] and matrix cracks in fibrous composites [20,21] the growth of damage surface area is related to the energy release rate G by

$$\frac{da}{dN} \propto G^n \quad , \tag{70}$$

where N represents the number of cycles in a fatigue test, and n is some material parameter. Guided by these results, a similar law is constructed here. To accomplish this, first multiply both sides of (70) by dN/dt to obtain for the 90° ply, for example,

$$\frac{d\alpha^1_{L_{22}}}{dt} \frac{dN}{dt} = \dot{\alpha}^1_{L_{22}} = k\, G^n \frac{dN}{dt} \quad . \tag{71}$$

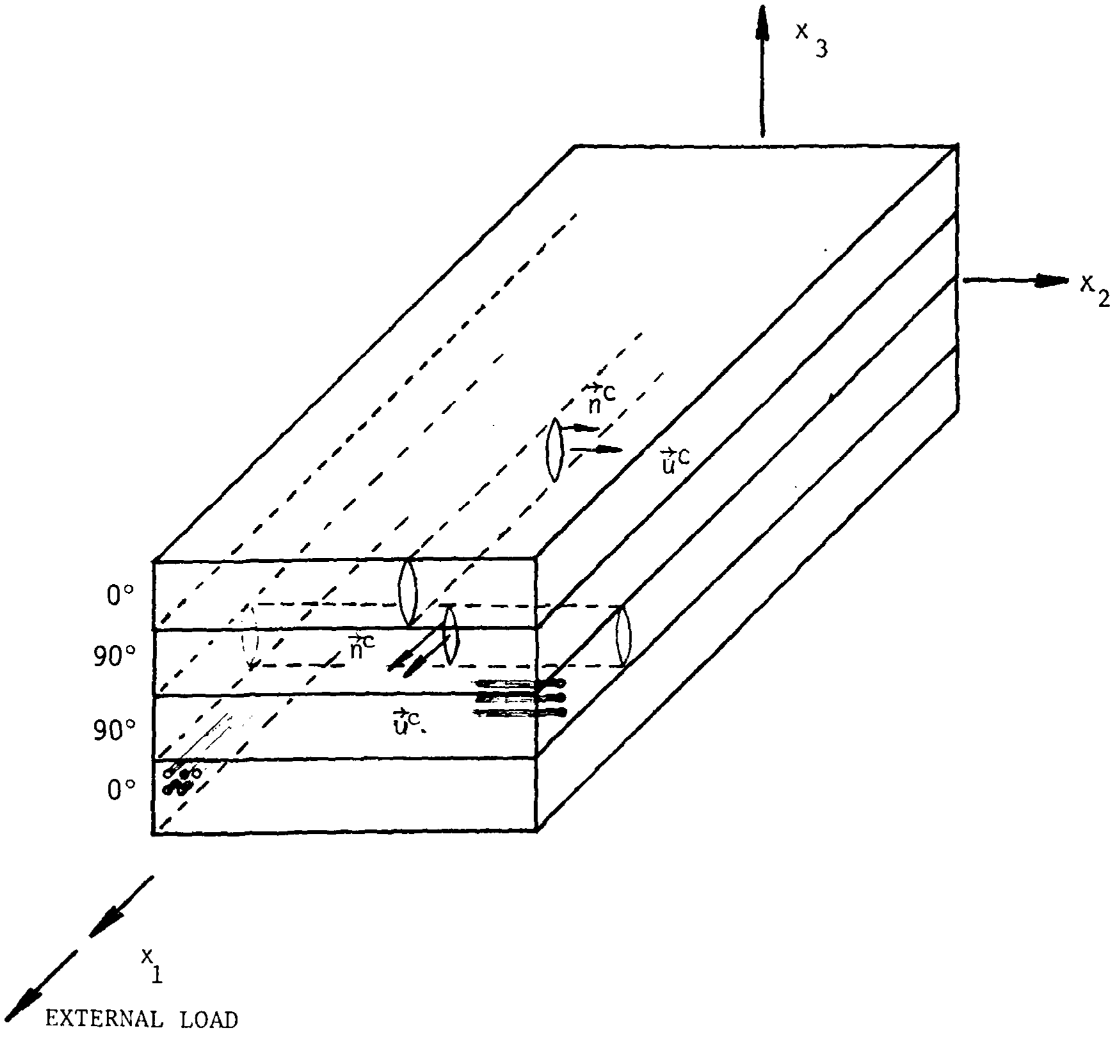
x_3
x_2
$\vec{n}^c$
$\vec{u}^c$
0°
90°
$\vec{n}^c$
90°
$\vec{u}^c$
0°
x_1
EXTERNAL LOAD

Fig. 6. Assumed Damage Vector Directions in a $[0,90_s]$ Laminate.

Assuming that the energy release rate is essentially mode I and therefore depends on the maximum normal strain, the damage growth law for transverse cracking is thus hypothesized to be of the form

$$\dot{\alpha}^{1}_{L_{22}} = K \left(\frac{\varepsilon_n - \varepsilon_{n_{min}}}{\varepsilon_{n_{max}} - \varepsilon_{n_{min}}} \right)^{n_1} \frac{d\varepsilon_n}{dt} \quad \text{if} \quad \varepsilon_{n_{min}} < \varepsilon_n < \varepsilon_{n_{max}} \text{, and}$$

$$\dot{\alpha}^{1}_{L_{22}} = 0 \quad \text{if } \varepsilon_n < \varepsilon_{n_{min}} \quad \text{or} \quad \varepsilon_n > \varepsilon_{n_{max}}, \tag{72}$$

where ε_n is the local normal strain component which is normal to the fibers. Furthermore, $\varepsilon_{n_{min}}$ is the value of ε_n at which transverse cracking initiates, and $\varepsilon_{n_{max}}$ is the corresponding value at which transverse cracking saturates. K and n are experimentally determined material parameters which may depend on the initial damage state or on history dependent damage such as matrix microvoids. The use of ε_n presupposes that the fracture mode is predominantly mode I in nature, which may not be the case in some complex layups. In these cases, mode II and mode III terms may be required. Note that all components of $\alpha_{L_{ij}}$ are zero except $\alpha_{L_{22}}$, which is nonzero in the local coordinate system wherein the fibers are aligned parallel to the x_1 axis.

The time derivative of ε_n reflects the time independent nature of the damage mode, and actually represents a departure from the form of equations (69) in that explicit strain rate dependence is now reflected in the growth law. However, damage growth law (73) need not satisfy the principal of equipresence [42] since it is not an equation of state [43].

Futhermore, a study of thermodynamic constraints with linear strain rate independence in equations (69) will indicate that equations (44) through (46) remain valid [44,45].

Fig. 7 shows a typical growth history for a specimen subjected to monotonically increasing deformation u(L).

Equation (72) completes the description of the damage model. Integration of these equations in time will lead to current values of the damage tensor which is input to constitutive equations (64) through (67). It should be pointed out, however, that these equations may be extremely nonlinear and as such must in many cases be integrated numerically with stiff integration schemes [48].

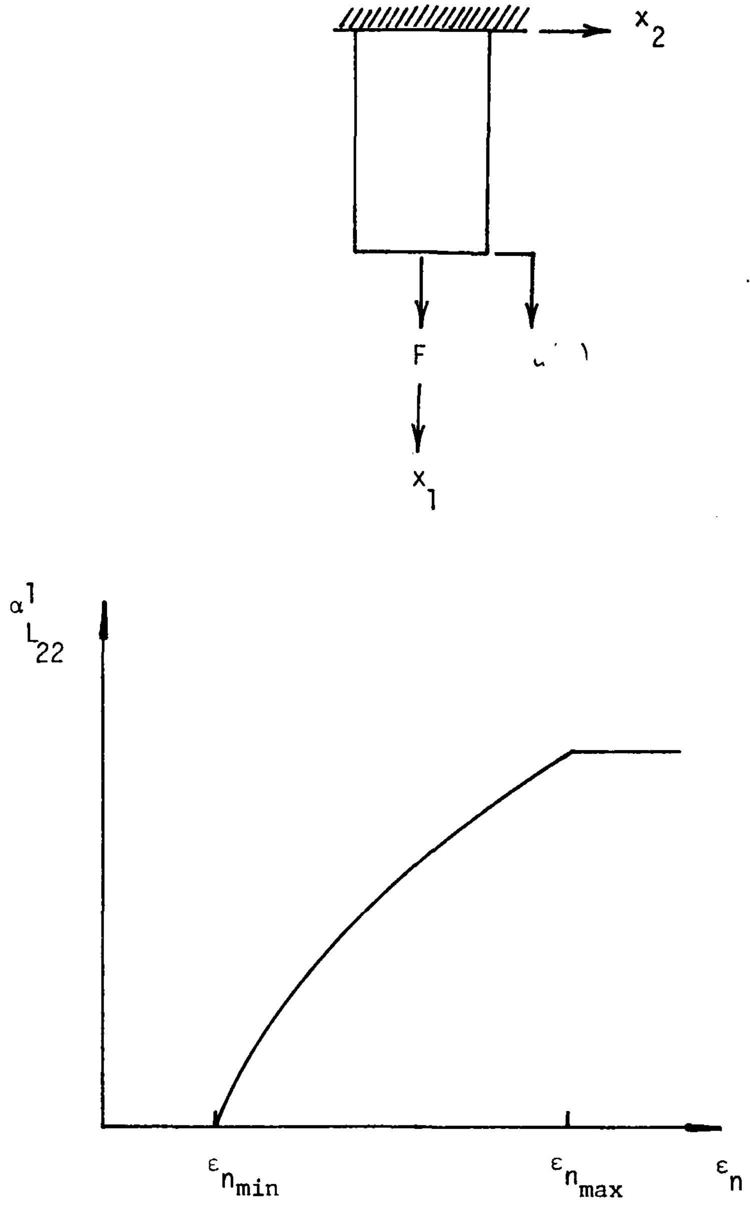

Fig. 7. Typical Growth of Damage in a Specimen with Matrix Cracks.

CONCLUSION

Stres-strain relations have been developed herein which account for various forms of damage in continuous fiber composites. Furthermore, a damage growth law has been proposed for matrix cracking in fibrous composites. The model developed herein is thus a complete description necessary to characterize the thermomechanical constitution of a fibrous composite (excluding failure). However, the actual use of this model is complicated by the requirement for numerous experimentally determined quanitites, as well as the necessity to determine locally based observable state variables by analytic methods. The construction of these parameters constitutes an entire separate research effort which is considered in Part II.

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APPENDIX 6.2

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A DAMAGE MODEL FOR CONTINUOS FIBER COMPOSITES

PART II: Model Applications

by

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A continuum mechanics approach is utilized herein to develop a model for predicting the thermomechanical constitution of continuous fiber composites subjected to both monotonic and cyclic fatigue loading. In this model the damage is characterized by a set of second order tensor valued internal state variables representing locally averaged measures of specific damage states such as matrix cracks, fiber-matrix debonding, interlaminar cracking, or any other damage state. Locally averaged history dependent constitutive equations are posed utilizing constraints imposed by thermodynamics with internal state variables.

In part I the thermodynamic with internal state variables was constructed. It was shown that suitable definitions of the locally averaged field variables led to equivalent thermodynamic constraints on a scale assumed to be large compared to the scale of the damage. Based on this result the Helmholtz free energy was expanded in a Taylor series in terms of strain, temperature, and damage to obtain the stress-strain relations for composites with internal state variables representing damage. Finally, an internal state

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variable growth law was proposed for matrix cracking.

In this paper, the three dimensional tensor equations from Part I [1] are simplified using symmetry constraints. After introducing engineering notation and expressing the constitutive equations in the standard laminate coordinate system, a specified constitutive model is developed for the case of matrix cracks only. The potential of the model to predict degraded or effective stiffness moduli is demonstrated by solving the problem of transverse matrix cracks in the 90° layer of $[0/90]_s$ and $[0/90_3]s$ laminates.

To solve the example problems, the undamaged modulii are determined from experimental data. The damage tensor is determined analytically from a finite element analysis assuming a variety of matrix crack spacings in the 90° layers. The internal state variable for transverse matrix cracking is related to the strain energy release rate due to cracking by utilizing linear elastic fracture mechanics. The values of effective (damage degraded) stiffness predicted by the constitutive model are in close agreement with both experimental and finite element results. The close agreement obtained in these example problems, while limited to transverse matrix cracks only, demonstrates the potential of the constitutive model to predict degraded stiffness.

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A continuum mechanics approach is utilized herein to develop a model for predicting the thermomechanical constitution of continuous fiber composites subjected to both monotonic and cyclic fatigue loading. In this model the damage is characterized by a set of second order tensor valued internal state variables representing locally averaged measures of specific damage states such as matrix cracks, fiber-matrix debonding, interlaminar cracking, or any other damage state. Locally averaged history dependent constitutive equations are posed utilizing constraints imposed by thermodynamics with internal state variables.

In Part I the thermodynamics with internal state variables was constructed. It was shown that suitable definitions of the locally averaged field variables led to equivalent thermodynamic constraints on a scale assumed to be large compared to the scale of the damage. Based on this result the Helmholtz free energy was expanded in a Taylor series in terms of strain, temperature, and damage to obtain the stress-strain relations for composites with internal state variables representing damage. Finally, an internal state variable growth law was proposed for matrix cracking.

In this paper, the three dimensional tensor equations from Part I [1] are simplified using symmetry constraints. After introducing engineering notation and expressing the constitutive equations in the standard laminate coordinate system, a specialized constitutive model is developed for the case of matrix cracks only. The potential of the model to predict degraded or effective stiffness moduli is demonstrated by solving the problem of transverse matrix cracks in the 90° layer of $[0/90]_s$ and $[0/90_3]_s$ laminates.

To solve the example problems, the undamaged moduli are determined from experimental data. The damage tensor is determined analytically from a finite element analysis assuming a variety of matrix crack spacings in the 90° layers. The internal state variable for transverse matrix cracking is related to the strain energy release rate due to cracking by utilizing linear elastic fracture mechanics. The values of effective (damage degraded) stiffnesses predicted by the constitutive model are in close agreement with both experimental and finite element results. The close agreement obtained in these example problems, while limited to transverse matrix cracks only, demonstrates the potential of the constitutive model to predict degraded stiffnesses.

INTRODUCTION

In Part I [1] it was hypothesized that damage can be modeled in continuous fiber composites by a set of internal state variables which represent locally averaged measures of matrix cracking, interlaminar delamination, and other damage mechanisms on a scale which is assumed to be small compared to the scale of the boundary value problem of interest. Continuum mechanics with internal state variables [1] was then utilized to construct stress-strain relations in which all components of the degraded modulus tensor can be determined for a given damage state.

The purpose of this paper (Part II) is to demonstrate how the model may be utilized to predict the stiffness of laminates which are subjected to known damage states. This procedure is illustrated via specific examples in which there is a single damage mode consisting of transverse matrix cracking. It is shown that single lamina properties in the presence of damage can be utilized as given properties to obtain favorable comparisons to both experimental and finite element results for specific laminates.

The model application is accomplished by first imposing symmetry constraints, performing the laminate integration, and finally reducing to generalized plane stress.

SIMPLIFICATION OF THE MODEL

We now consider the stress-strain relation described in equations (64) through (67) of Part I (see Appendix A). For the examples to be considered herein, it is assumed that all residual stress components are zero ($\sigma^R_{L_{ij}}=0$), and that there are no thermal transients ($\Delta T_L=0$).

Reduction to Single-Index Notation

By incorporating the symmetry of the stress and strain tensors, the quadratic dependence of the Helmholtz free energy on strain, and the

Voigt single index notation, [2] the constitutive equations reduce to (see appendix A)

$$\sigma_{L_{ij}} = C'_{L_{ij}} \varepsilon_{L_j} \qquad (1)$$

where

$$C'_{L_{ij}} = C_{L_{ij}} + M^{\eta}_{ijmn} \alpha^{\eta}_{mn} \qquad (2)$$

In equations (1) and (2) the subscript L represents quantities which are locally averaged at the lamina level. The subscripts i and j range from one to six, the subscripts m and n ranges from 1 to 3, and the superscript η ranges from 1 to N, the number of damage modes.

At this point no further reductions can be made to the number of unknown constants in equation (2) without specifying the specific damage modes and material symmetry.

Application to Matrix Cracking

As discussed in the introduction, the potential of the constitutive model will be demonstrated by considering the case of matrix cracking in continuous fiber composites. An example of this damage state is shown in Fig. 1. In order to construct the proposed constitutive model for this system we first examine the response of a single ply subjected to transverse matrix cracking as shown in Fig. 2. Assuming that the crack geometry is symmetric about normals to each of the ply coordinates, the internal state variable associated with matrix cracking in local ply coordinates is represented by

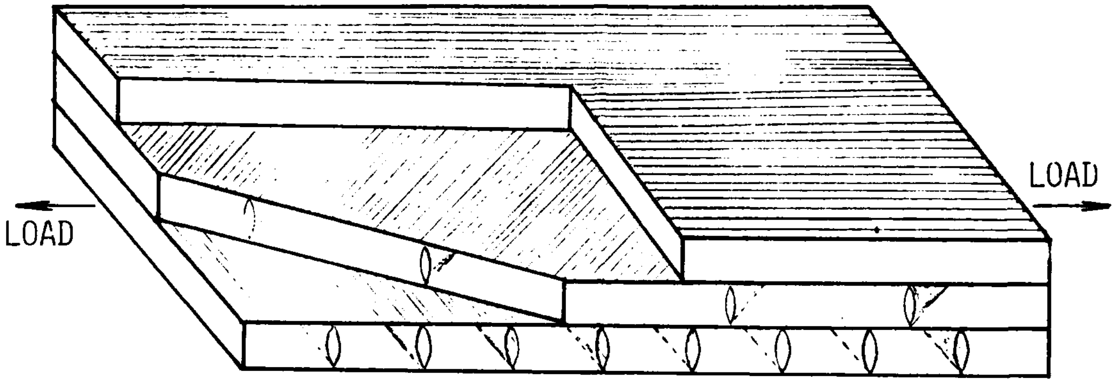


Fig. 1. Matrix Cracking in a Laminated Continuous Fiber Composite.

$$\alpha^1_{L_{ij}} = |\alpha^1_L| \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad . \qquad (3)$$

This implies that the damage in a single ply is defined by the direction of vectors $\vec{n}^c$ and $\vec{u}^c$ and the magnitude $|\alpha^1_L|$ (see Fig. 2). It will be shown in a later section that the magnitude of $\alpha^1_{L_{ij}}$ is related to the surface area of matrix cracks per unit volume in a ply.

Note that a second order tensor representation of the internal state variable is insufficient if the crack orientation within the ply is time dependent. In this case a higher order tensor is required. However, since the crack is matrix dominated and constrained by fibers, rotation is assumed to be negligible and the second order tensorial representation is considered adequate in this example.

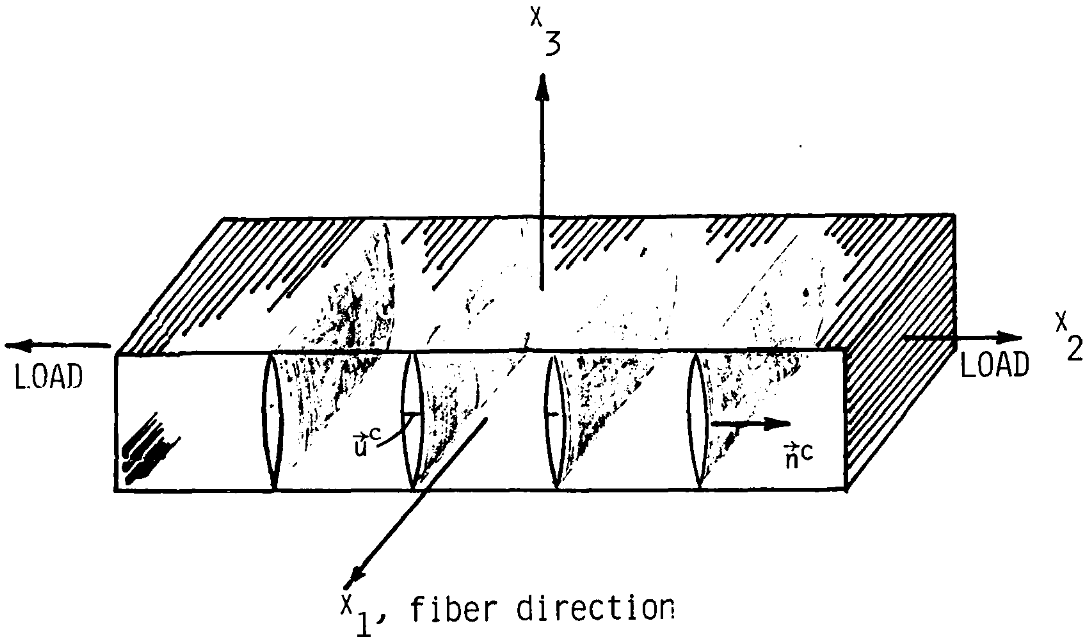
For the single damage mode of matrix cracking described in Fig. 1, equation (2) reduces to

$$C'_{L_{ij}} = C_{L_{ij}} + M^1_{ijmm}\, \alpha^1_{mn} \quad , \qquad (4)$$

where $\alpha^1_{L_{mn}}$ represents matrix cracking and the above properties are constructed in ply coordinates. Since the only non-zero component of $\alpha^1_{L_{mn}}$ is $\alpha^1_{L_{22}}$; equation (4) reduces to

$$C'_{L_{ij}} = C_{L_{ij}} + M^1_{ij22}\, \alpha^1_{22} \quad , \qquad (5)$$

where M^1_{ij22} represent the non-zero components of M^1_{ijkl} associated with matrix cracking. At this point M^1_{ij22} has been reduced to 21 unknown coefficients in equation (5).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x_2
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x_1, fiber direction

Fig. 2. Transverse Matrix Cracking in a Single Ply.

Material symmetries may now be utilized to further simplify the constitutive equations. The material in question is assumed to be initially transversely isotropic in the undamaged state on the local scale, where the plane of isotropy is the $x_2 - x_3$ plane shown in figure 2. In the undamaged state the effective modulus tensor $C'_{L_{ijkl}}$ is equivalent to the elastic transversely isotropic ply properties given by [3]

$$\begin{bmatrix} \sigma_{L_1} \\ \sigma_{L_2} \\ \sigma_{L_3} \\ \sigma_{L_4} \\ \sigma_{L_5} \\ \sigma_{L_6} \end{bmatrix} = \begin{bmatrix} C_{L_{11}} & C_{L_{12}} & C_{L_{12}} & 0 & 0 & 0 \\ C_{L_{12}} & C_{L_{22}} & C_{L_{23}} & 0 & 0 & 0 \\ C_{L_{12}} & C_{L_{23}} & C_{L_{22}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{L_1} \\ \varepsilon_{L_2} \\ \varepsilon_{L_3} \\ \varepsilon_{L_4} \\ \varepsilon_{L_5} \\ \varepsilon_{L_6} \end{bmatrix} \tag{6}$$

It can be shown using irreducible integrity bases [4,5,8] that with the inclusion of the damage tensor for transverse cracking the effective modulus tensor becomes orthotropic. Therefore, the damage tensor $M^1_{L_{ij22}}$ is an orthotropic tensor containing 9 unknown constants. Thus, the complete constitutive equation (5) (assuming the damage growth law $\alpha^1_{L_{22}}$ is known) requires the determination of independent material constants for the undamaged modulus tensor $C_{L_{ij}}$, and 9 independent constants for the damage tensor, $M^1_{L_{ij22}}$.

For relatively thin laminates it is useful to apply the conditions of generalized plane stress where the out-of-plane shear stresses σ_4 and σ_5 are neglected. Applying these conditions to equation (1) and using matrix notation results in

$$\begin{Bmatrix}\sigma_{L_1}\\ \sigma_{L_2}\\ \sigma_{L_3}\\ \sigma_{L_6}\end{Bmatrix}=\left[\begin{bmatrix}C_{L_{11}} & C_{L_{12}} & C_{L_{12}} & 0\\ C_{L_{12}} & C_{L_{22}} & C_{L_{23}} & 0\\ C_{L_{12}} & C_{L_{23}} & C_{L_{22}} & 0\\ 0 & 0 & 0 & C_{L_{66}}\end{bmatrix}-\alpha^1_{L_{22}}\begin{bmatrix}M^1_{1122} & M^1_{1222} & M^1_{1322} & 0\\ M^1_{1222} & M^1_{2222} & M^1_{2322} & 0\\ M^1_{1322} & M^1_{2322} & M^1_{3322} & 0\\ 0 & 0 & 0 & M^1_{6622}\end{bmatrix}\right]\begin{Bmatrix}\varepsilon_{L_1}\\ \varepsilon_{L_2}\\ \varepsilon_{L_3}\\ \varepsilon_{L_6}\end{Bmatrix} \qquad (7)$$

where for generalized plane stress conditions there are 12 unknown material constants in equation (7).

Laminate Equations

To utilize single lamina equations to characterize the response of multilayered laminates, it is necessary to globally average the local ply constitutive equations. This is accomplished herein by imposing the Kirchhoff hypothesis for thin plates. This procedure is very similar to the method used in Jones [3]. However, generalized plane strain conditions are imposed rather than plane strain because this is consistent with the stress state in equation (7) (A detailed description of the global averaging is given in appendix B). The resulting equations are as follows:

$$\{N\} = [A]\,\{\varepsilon^o\} \qquad (8)$$

or

$$\{\varepsilon^o\} = [A^{-1}]\,\{N\} \quad , \qquad (9)$$

where

$$A_{ij} = \sum_{k=1}^{n} (C'_{L_{ij}})_k \, (t_k) \qquad (10)$$

and $\{\varepsilon_o\}$ contains the laminate midplane strains where k specifies the ply and t_k is the ply thickness.

DEPENDENCE OF THE FREE ENERGY ON CRACK SURFACE AREA

It was previously shown in Part 1 [1] that the free energy, u_L^c was first order in the internal state variables, $\alpha^\eta_{L_{ij}}$. Therefore, in general

$$U_L^c = f_{ij}^\eta \left(\varepsilon_{L_{kl}}, T_L\right) \cdot \alpha^\eta_{L_{ij}} \quad , \qquad (11)$$

where f_{ij}^η is a tensor associated with each specific mode of damage, η. Therefore, for the "fixed grip" condition ($\varepsilon_{L_{ij}}$ = constants) and constant temperature conditions, the free energy is a linear function of the internal state variables.

The free energy, u_L^c, depends on the energy released during cracking as well as the total surface area of cracks in the local volume. For an elastic material undergoing stable self-similar crack growth, the available energy is related to the energy required for crack extension by [6,7]

$$\dot{U}_L^c = \frac{1}{\rho_L v_L} G_L \dot{S}_2 \quad , \qquad (12)$$

where G_L is the local volume averaged strain energy release rate during crack extension, and S_2 is the total surface area of matrix cracks. Also, it has been previously shown [1] that the local energy is related to the total energy by

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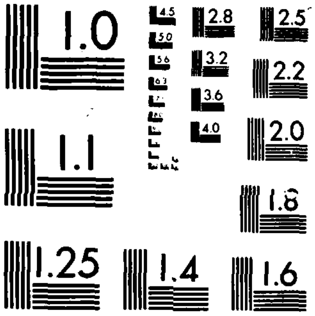
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$$u_L^c(t) = \frac{U_L^c(t)}{\rho_L v_L} \quad . \tag{13}$$

The total energy released during the fracture process is obtained by substituting equation (13) into equation (12) and integrating in time. This results in

$$U_L^c(t) \equiv \int_o^t G_L \dot{S}_2 \, dt' \quad , \tag{14}$$

where t is the time of interest, t' is a dummy variable of integration, and S_2 is a time dependent geometric quantity. It should be noted that G_L may be time dependent if, for example, cracks propagate through heterogeneous zones such as resin rich regions during the damage process.

Since the body is assumed to be elastic at a constant damage state, the strain energy release rate, G_L, may be assumed to be rate independent. Therefore, equation (14) becomes

$$U_L^c(t) \equiv \int_{S_2(o)}^{S_2(t)} G_L(S_2) ds = g(S_2) \quad , \tag{15}$$

where it is apparent that the total energy is a function of surface area only.

It therefore follows from equations (11), (13) and (15) that for the case of transverse matrix cracking

$$U_L^c = \frac{1}{\rho_L v_L} g(S_2) = f_{22}^1(\varepsilon_{L_{kl}}, T_L) \cdot \alpha_{L_{22}}^1 \; . \tag{16}$$

Thus, finally

$$\alpha_{L_{22}}^1 = \frac{g(S_2)}{\rho_c V_c f_{22}^1} = g'(S_2) \quad . \tag{17}$$

The precise nature of the function g' is determined either analytically (using a finite element solution) or experimentally.

From equations (17) and (15), it is seen that the internal state variable depends on the strain energy release rate, G_L. For the "fixed grip" condition, the strain energy release rate is given by [7]

$$G_L = -\frac{1}{B}\left(\frac{dU_T}{da}\right) \quad , \qquad (18)$$

where B is the crack depth, and U_T is the elastic strain energy. For a uniaxial stress state, the elastic strain energy is defined as

$$U_T \equiv \int_V \left[\int_0^{\varepsilon_{11}} \sigma_{11}\, d\varepsilon_{11}\right] dV \quad , \qquad (19)$$

where σ_{11} and ε_{11} are the uniaxial stress and strain states, respectively, and V is the volume of interest. For a linear elastic material, the term in the brackets of equation (19) can be integrated and equation (19) becomes

$$U_T = \int \left[\frac{1}{2}\, E_{11}\, \varepsilon_{11}^2\right] dV \qquad (20)$$

where E_{11} is the laminate stiffness (A_{11}/t) in the x-axis direction. For the uniaxial fixed grip condition, ε_{11} does not vary over the volume. Therefore, equation (20) becomes

$$U_T = \frac{1}{2}\, V\, E_{11}\, \varepsilon_{11}^2 \quad . \qquad (21)$$

Finally, substituting equation (21) into equation (18) and recalling that ε_{11} is constant for the "fixed grip" condition, the following expression for strain energy release rate is obtained:

$$G_L = -\frac{1}{2B}\, V\varepsilon_{11}^2\, \frac{\partial E_{11}}{\partial a} \qquad (22)$$

This result implies that only the laminate modulus, E_{11} changes with the formation of new crack surfaces.

The above result may be considered intuitively obvious because E_{11} is defined as the load in the x_1-direction divided by the associated displacement in the x_1-direction. Since the displacement is held constant, the load will decrease as crack extension occurs and the structure becomes more compliant. This can be shown to be analogous to [7]

$$G_L = \frac{P^2}{2B}\frac{\partial C}{\partial A} \quad , \tag{23}$$

where C is the specimen compliance ($1/E_{11}$).

As an example, consider a $[0/90]_s$ laminate with matrix cracking in the 90° plies only. The average stiffness is given by

$$E_{11} = (E_0 + E_{90})\ /2 \quad , \tag{24}$$

where E_0 and E_{90} are the moduli in the global x_1 direction (0° fiber orientation) of the 0° and 90° plies, respectively. Assuming that no cracks form in the outer 0° plies, equation (22) becomes

$$G_L = -\frac{1}{4B} V\varepsilon^2 \frac{\partial E_{90}}{\partial a} \quad . \tag{25}$$

This result implies that only changes in the modulus of the constrained 90° plies, where matrix cracks are assumed to occur, effect the value of the strain energy release rate for increasing crack formation.

Determination of Damage Modulus Tensor

The material constraints used in M^1_{ij22} can be determined using equation (5)

$$M^1_{ij22} = \frac{C'_{L_{ij}} - C_{L_{ij}}}{\alpha^1_{L_{22}}} \tag{26}$$

provided that $C_{L_{ij}}$, $C'_{L_{ij}}$, and $\alpha^{1}_{L_{22}}$ are known. In reality it is $C'_{L_{ij}}$ that one seeks to predict; however, it is possible to analytically or experimentally determine $C'_{L_{ij}}$ for prescribed values of $\alpha^{1}_{L_{22}}$. The terms in $C_{L_{ij}}$ are the orthotropic material properties determined experimentally from tests conducted on undamaged composite lamina.

In order to determine the terms in $C'_{L_{ij}}$, it is necessary to determine the effective orthotropic properties of a single ply for a given value of $\alpha^{1}_{L_{22}}$. The effective orthotropic properties due to matrix cracking are determined by modeling the cracks as internal boundaries and solving the resultant boundary value problem using a finite element continuum model. Alternatively, $C'_{L_{ij}}$ may be determined experimentally where methods exist to determine appropriate stiffness components.

RESULTS

This section presents the results obtained from the finite element continuum model which was used to predict the damage modulus tensor and the globally averaged laminate equations relating stress and strain. These results demonstrate the ability of the model to predict all components of the effective stiffness due to matrix cracking for a $[0,90_3]_s$ laminate. The experimental data used in this analysis, shown in Table 1, were obtained from reference 9.

The first series of tests were conducted to determine the strain energy release rate and the damage modulus tensor M'_{ij22} for $[0,90]_s$ and $[90,90]_s$ laminates using the finite element continuum model with various

Table 1. Undamaged ply properties for E-glass epoxy [9]

$E_1 = 6.05 \times 10^6$ PSI

$E_2 = 1.89 \times 10^6$ PSI

$E_3 = 1.89 \times 10^6$ PSI

$\nu_{12} = \nu_{13} = \nu_{23} = 0.30$

$G_{12} = G_{13} = 0.63 \times 10^6$ PSI

$G_{23} = 0.7269 \times 10^6$ PSI

cured ply thickness = 0.008 inches

crack densities in the center 90° ply. The finite element mesh used for the analysis is shown in Fig. 3. The effect of different constraining layers on the strain energy release rates calculated from the finite element results are shown in Fig. 4. Also shown in Fig. 4 is the theoretical result for repeated cracks in an isotropic infinite domain [10]. This result is intended only to verify the cubic nature of the strain energy release rate calculated from the finite element model. For the current state of model development, only the $[0,90]_s$ relation for the energy release rate was used. It should be noted that, in calculating the energy release rate, the derivative of the change in stiffness with respect to crack length was approximated with a first order finite difference relation.

The next step was to determine an approximate relation between the strain energy release rate and the total crack length. A least squares cubic fit of the curve for the $[0,90]_s$ laminate, shown in Fig. 4 was obtained and is given as follows:

$$G(a) = 3.88690 - 0.280162\ a - 44.11385\ a^2 + 69.22121\ a^3, \tag{27}$$

where the nonlinear terms are believed to be caused by boundary effects and crack interaction. Substituting equation (27) into equation (15) results in an expression for the internal state variable as follows:

$$\alpha_L = 3.8869a - 0.140081\ a^2 - 14.70462\ a^3 + 17.305303\ a^4 \quad . \tag{28}$$

The values for the damage modulus tensor M'_{ij22} can now be determined from the finite element results for the $[0,90]_s$ laminate. In

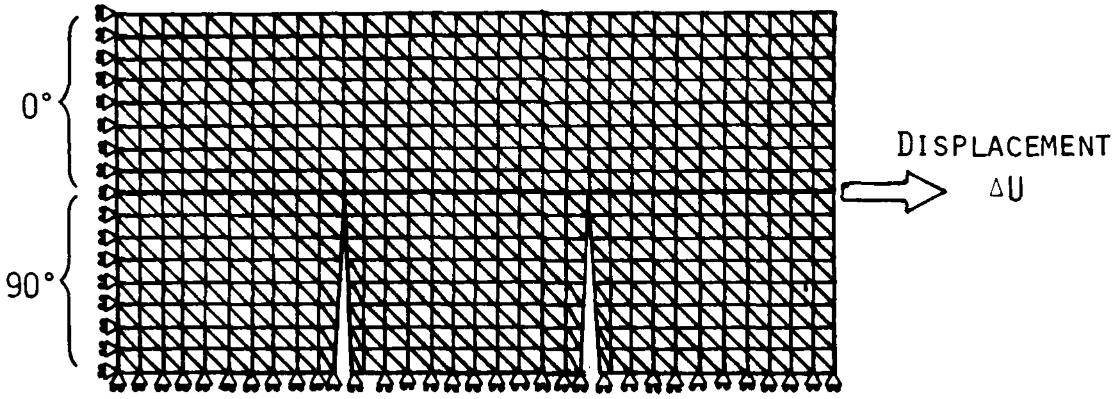
0°
90°
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Fig. 3. Finite Element Mesh for a $[0,90]_s$

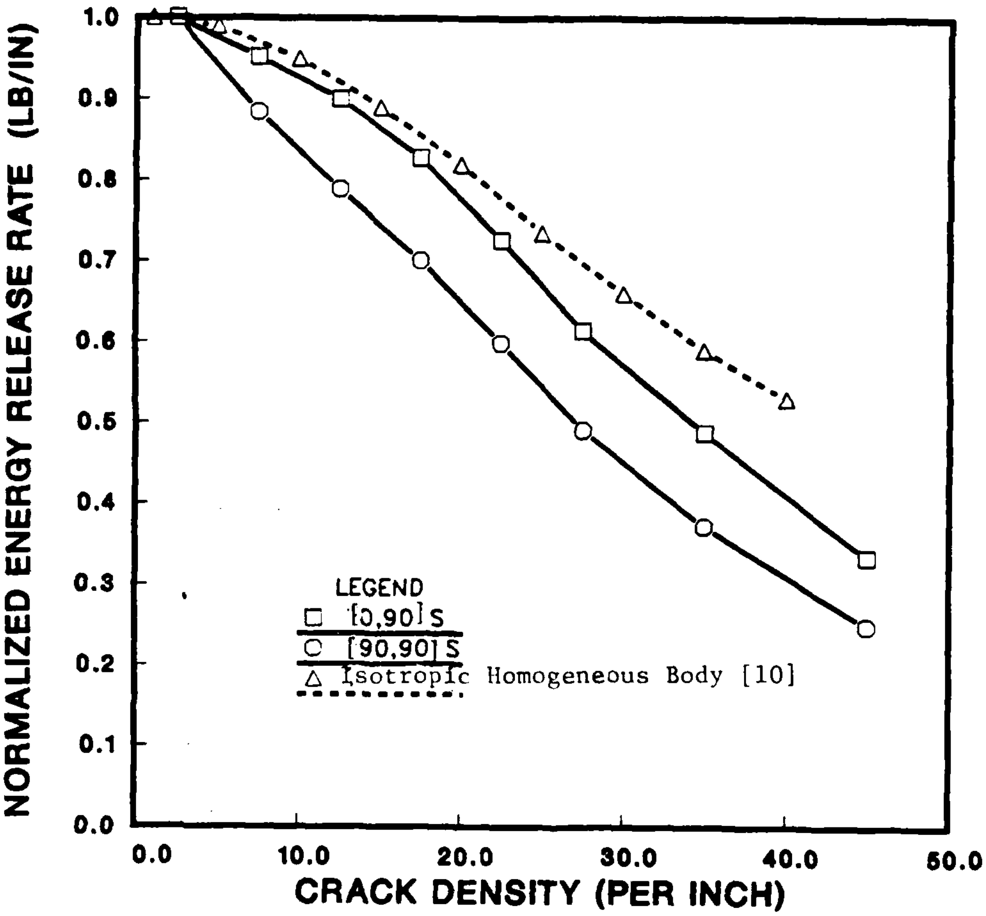


Fig. 4. Strain Energy Release Rate for E-Glass Epoxy.

order to evaluate the constants in M'_{ij22}, a particular crack density is selected and the finite element results for this case are utilized in equation (25). These results are then used as follows: 1) compare damage model predictions for the axial stiffness loss in the $[0,90]_s$ laminate to the finite element results, and 2) compare the damage model predictions for the axial stiffness loss in a $[0,90_3]_s$ laminate to both finite element predictions and experimental results.

The results for case one are shown in Fig. 5. It can be seen that the damage model underpredicts the stiffness loss. This result is expected since laminate analysis does not account for interlaminar shearing which would effectively reduce the axial stiffness. Furthermore, the strain energy release rate predicted by the finite element continuum model is conservative due to the inherent "over stiffness" problems associated with the finite element model, especially since constant strain triangular elements were incorporated. This same argument will hold true when the finite element model results are compared to experimental data.

The normalized axial stiffness results for case 2 ($[0,90_3]_s$ laminate) are shown in Fig. 6. As discussed previously, the results predicted from the damage model as well as finite elements underestimate the stiffness loss in the laminate. Nevertheless the good comparison of the damage model to the experimental results can only be encouraging since the only experimental data used were the undamaged stiffness properties. The inclusion of experimental data for the damaged stiffness properties and a more refined energy release rate prediction can only help decrease the current differences between model predictions and experimental results. A phenomenological explanation for the

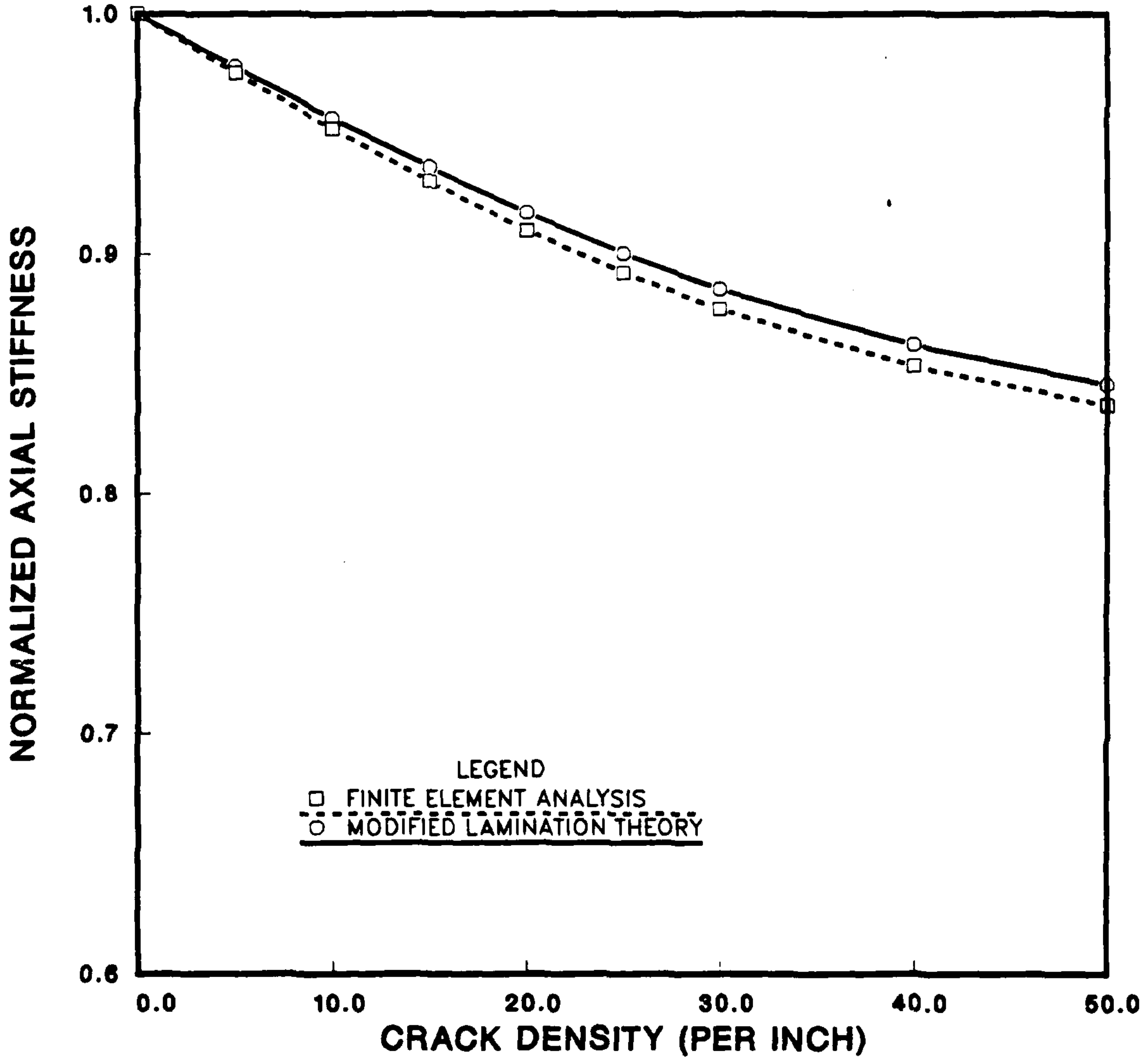


Fig. 5. Normalized Axial Stiffness Predictions for a $[0,90]_s$ E-Glass Epoxy Laminate.

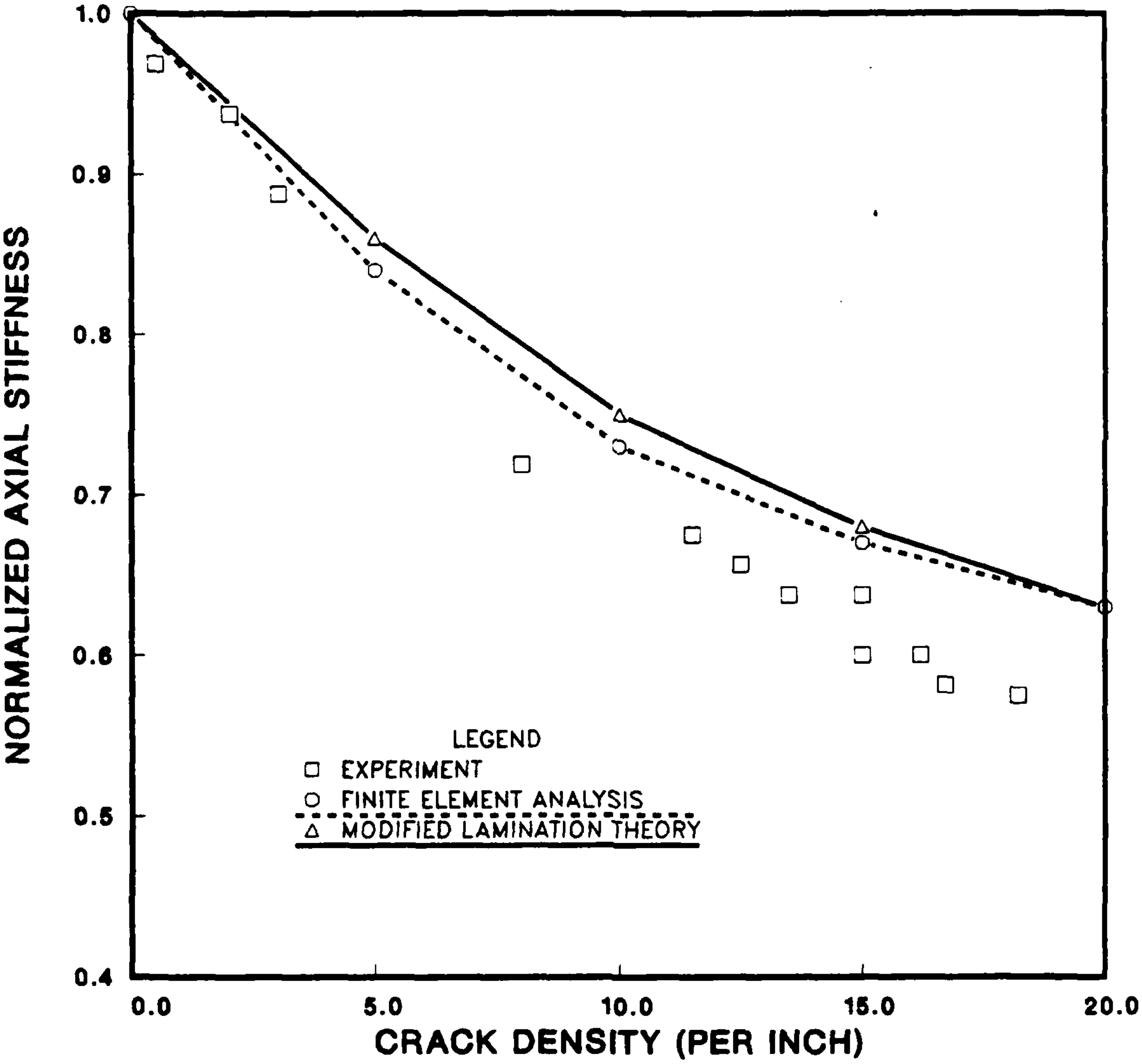


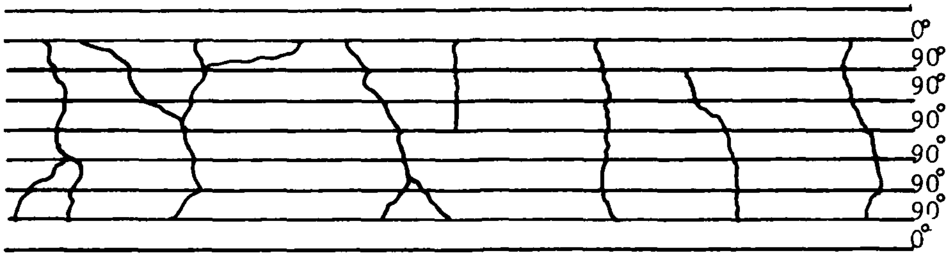
Fig. 6. Normalized Axial Stiffness Predictions for a $[0,90_3]_s$ E-Glass Epoxy Laminate.

underpredicted stiffness loss can be seen. If one examines a schematic diagram of a typical saturation crack pattern for the $[0,90_3]_s$ laminate [9,11] as shown in Fig. 7a. It is observed that a network of branched or curved cracks exist in addition to a more uniform spacing of transverse cracks. This suggests that the surface area of transverse matrix cracks may actually be greater than that assumed by the damage model as shown in Fig. 7b. Since the internal state variable depends directly on the surface area of cracks, one concludes that by allowing for the increased surface area due to branching or curvature then the model results would be in better agreement with the experimental data. This suggests that it may be preferrable to obtain all material parameters from experimental tests rather than finite element results.

In Fig. 8 the remaining components of the effective modulus tensor are predicted using the laminate equations. It can be seen from the figure that the moduli in the y and z directions do not vary significantly for transverse matrix cracking. However, it can be seen that the Poisson's ratios ν_{xy} and ν_{xz}, as well as the shear modulus G_{xy}, vary significantly. Also shown in Fig. 8 are results obtained from the finite element continuum model for ν_{xz}. The good agreement demonstrated here supports the ability of the model to predict all components of the effective modulus tensor.

CONCLUSION

In part I constitutive relations were developed to account for various forms of damage in continuous fiber composites by utilizing the concept of internal state variables. These constitutive equations have been simplified to a usable form by imposing symmetry constraints. The


0°
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90°
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90°
90°
0°

Fig. 7a. Typical Saturation Crack Patterns Observed in $[0,90_3]_s$ E-Class Laminate [9,10].

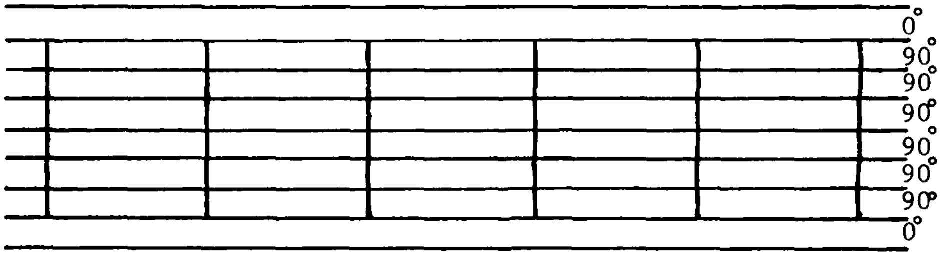
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Fig. 7b. Assumed Saturation Crack Pattern in $[0,90_3]_s$ Laminate for Damage Model.

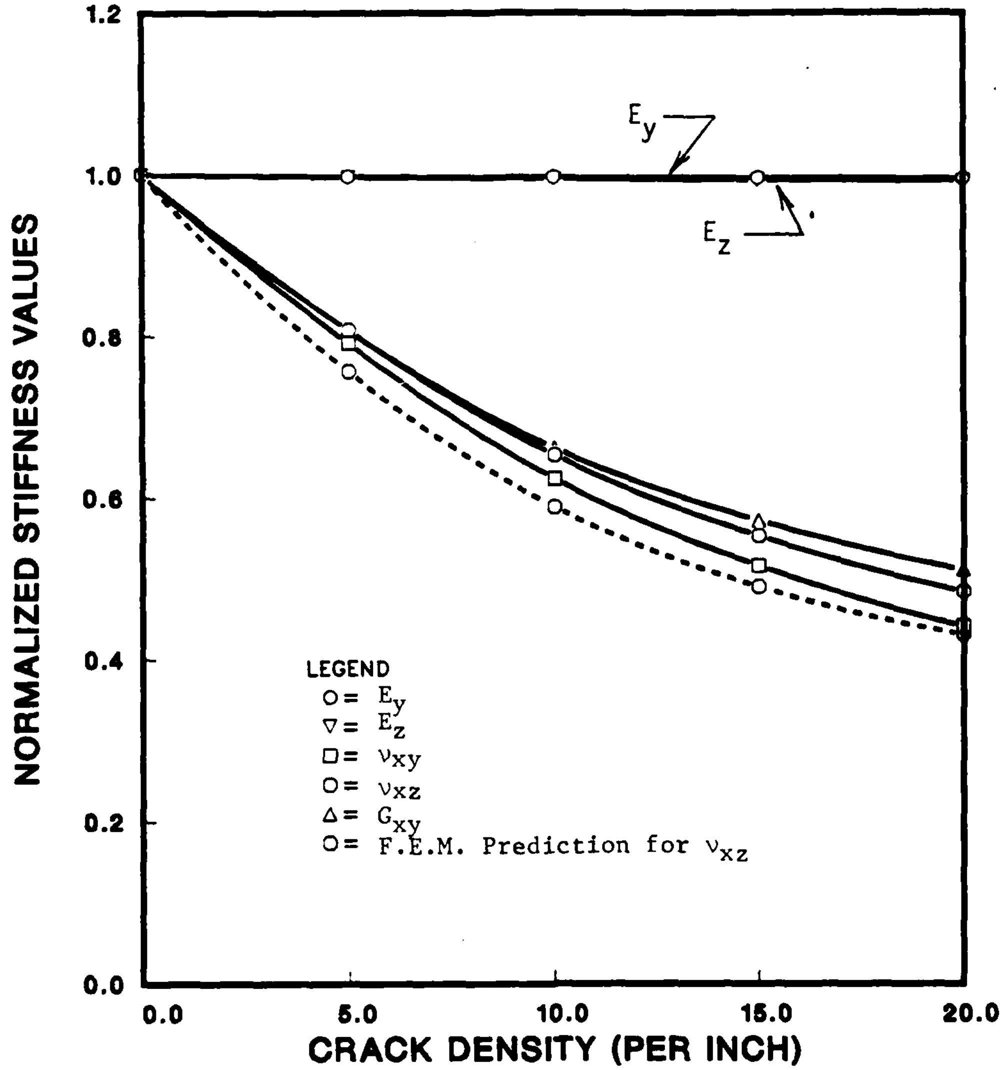

Fig. 8. Effective Orthotropic Property Predictions for $[0,90_3]_s$ E-Glass Epoxy Laminate.

potential of the resulting constitutive model was demonstrated by considering the case of matrix cracking only. Results were obtained for $[0,90]_s$ and $[0,90_3]_s$ laminates with matrix cracking in the 90° plies. Although some of the material constants used to characterize the local ply constitutive equations were determined analytically, model results compare favorable to experimental and finite element analyses for specified values of crack densities. It is anticipated that when the material constants used in the model are determined experimentally, the results should be notably better due to the inability of the finite element results to account for unusual crack geometry. Finally, the model was shown to be capable of predicting all components of the damage induced effective modulus tensor. However, the accuracy of this capability remains to be verified through experimental correlation.

The objective of this research has been threefold: 1) to develop a model for predicting the constitutive behavior of a laminate given the mode and extent of damage, 2) to develop growth laws for each mode and the extent of damage, and 3) to incorporate both 1) and 2) into a usable form. Although the model has been demonstrated for given states of matrix cracking, parts 2) and 3) require further research and are indeed the most difficult link in the model development since they will require extensive correlation with experiment as well as finite element analysis.

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Appendix A: Application of Symmetry Constraints

The damage-dependent constitutive model (equations (64) through (67) of Part I [1]) is defined as follows:

$$\sigma_{L_{ij}} = \sigma^{R}_{L_{ij}} + C'_{L_{ijkl}} \left(\varepsilon_{L_{kl}} - \varepsilon^{T}_{kl}\right) \tag{1a}$$

where $\sigma_{L_{ij}}$ ≡ local stress tensor

$\varepsilon_{L_{kl}}$ ≡ local strain tensor

$$\sigma_{L_{ij}} \equiv \rho_L \left(B_{L_{ij}} + I^{\eta}_{ijkl}\, \alpha^{\eta}_{L_{kl}}\right) \tag{2a}$$

≡ Residual stress in the absence of strain and temperature change

$$C'_{L_{ijkl}} \equiv \rho_L \left(C_{L_{ijkl}} + M^{\eta}_{ijklmn}\, \alpha^{\eta}_{L_{mn}}\right) \tag{3a}$$

≡ effective modulus tensor

$C_{L_{ijkl}}$ ≡ undamaged modulus tensor

$$\varepsilon^{T}_{kl} \equiv C'^{-1}_{L_{ijkl}}\, \rho_L \left(E_{L_{ij}} - N^{\eta}_{ijkl}\, \alpha^{\eta}_{kl}\, \Delta T_L\right) \tag{4a}$$

≡ thermal strain tensor

$\alpha^{\eta}_{L_{kl}}$ ≡ internal state variable for damage modes

$\Delta T_L \equiv$ change in temperature

and $B_{L_{ij}}$, I^{η}_{ijkl}, M^{η}_{ijklmn}, $E_{L_{ij}}$ & N^{η}_{ijkl} are tensorial material constants as required by the Taylor Series expansion.

For demonstration purposes, the residual stress terms are neglected and isothermal behavior is imposed on equations (1a) through (4a), yielding

$$\sigma_{L_{ij}} = C'_{L_{ijkl}} \varepsilon_{L_{kl}} \quad , \tag{5a}$$

where the effective modulus tensor is given by

$$C'_{L_{ij}} = \rho_L \left(C_{L_{ijkl}} + M^{\eta}_{ijklmn} \alpha^{\eta}_{L_{mn}} \right). \tag{6a}$$

Note that M^{η}_{ijklmn} is a sixth order tensor with 729 coefficients for each value of η. It is assumed here that the constitutive equations given by (5a) are statistically homogeneous. Therefore, the conditions of stress and strain symmetry can be applied to equations (5a) and (6a) to obtain

$$C'_{L_{ijkl}} = C'_{L_{jikl}} \ , \quad C'_{L_{ijkl}} = C'_{L_{ijlk}} \quad . \tag{7a}$$

Using equations (6a) and (7a) it can be shown that

$$M^{\eta}_{ijklmn} = M^{\eta}_{jiklmn}, \ M^{\eta}_{ijklmn} = M^{\eta}_{ijlkmn} \quad . \tag{8a}$$

Using the Helmholtz free energy and the quadratic dependence of the effective modulus tensor on strain, it can be shown that

$$C'_{L_{ijkl}} = C'_{L_{klij}} \quad . \tag{9a}$$

Substitution of equation (6a) into (9a) will result in

$$M^{\eta}_{ijklmn} = M^{\eta}_{klijmn} \qquad (10a)$$

The above symmetry constraints reduce the number of coefficients to 189 terms in equation (10a) for each value of η. It is most convenient at this point to reindex the constitutive tensors using the Voigt notation [2] where

$$\begin{aligned} \sigma_1 &= \sigma_{11} & \sigma_4 &= \sigma_{23} = \sigma_{32} \\ \sigma_2 &= \sigma_{22} & \sigma_5 &= \sigma_{13} = \sigma_{31} \\ \sigma_3 &= \sigma_{33} & \sigma_6 &= \sigma_{12} = \sigma_{21} \end{aligned} \qquad (11a)$$

and

$$\begin{aligned} \varepsilon_1 &= \varepsilon_{11} & \varepsilon_4 &= 2\,\varepsilon_{23} = 2\,\varepsilon_{32} \\ \varepsilon_2 &= \varepsilon_{22} & \varepsilon_5 &= 2\,\varepsilon_{13} = 2\,\varepsilon_{31} \\ \varepsilon_3 &= \varepsilon_{33} & \varepsilon_6 &= 2\,\varepsilon_{12} = 2\,\varepsilon_{21} \end{aligned} \qquad (12a)$$

Using the contracted notation, equations (5a) and (6a) can be written as

$$\sigma_{L_i} = C'_{L_{ij}}\,\varepsilon_j \quad , \qquad (13a)$$

where i and j range from 1 to 6 and

$$C'_{L_{ij}} = C_{L_{ij}} + M^{\eta}_{ijmn}\,\alpha^{\eta}_{mn} \quad . \qquad (14a)$$

Note that m and n range from 1 to 3 and η ranges from 1 to N, where N is the number of damage modes.

Appendix B

The values of generalized plane strain are given by

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \varepsilon_z^o \\ \varepsilon_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \\ \kappa_{xy} \end{Bmatrix} , \qquad (1b)$$

where the superscript "o" denotes the midsurface strains and the κ matrix denotes the midsurface curvatures. Under the condition of generalized plane strain there is no warping allowed out-of-plane, which implies, that $\kappa_z = 0$. Thus the generalized plane strain equation becomes

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \varepsilon_z^o \\ \varepsilon_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ 0 \\ \kappa_{xy} \end{Bmatrix} . \qquad (2b)$$

It is now assumed that no moments or curvatures are imposed and that all laminates studied are symmetric. Therefore, in order to determine the resultant forces, it is necessary only to integrate the given stress state as specified in equation (7) over the laminate thickness to obtain

$$\begin{Bmatrix} N_x \\ N_y \\ N_z \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} {}^{o}{}_{x} \\ {}^{o}{}_{y} \\ {}^{o}{}_{z} \\ {}^{o}{}_{xy} \end{Bmatrix} dz \quad , \qquad (3b)$$

where t is the total thickness of the laminate.

Substituting equations (13a) and (2b) into (3b) for the case where there are no rotations results in

$$\{N\} = \int_{-t/2}^{t/2} [\bar{C}'_L] \{\varepsilon^o\} \, dz \quad , \qquad (4b)$$

where {N} denotes the force resultants, $[\bar{C}'_L]$ is the transformed effective stiffness matrix, and $\{\varepsilon^o\}$ represent the mid-surface strains. Note that since transverse cracks go completely through the thickness of the cracked plies the stiffness is assumed to be spacially constant through the thickness. Therefore, equation (4b) can be written as

$$\{N\} = \sum_{k=1}^{n} [\bar{C}'_L]_k \, (Z_k - Z_{k-1}) \, \{\varepsilon^o\} \ , \qquad (5b)$$

where k specifies the ply and $Z_k - Z_{k-1}$ is the thickness of each ply. One can define

$$A_{ij} = \sum_{k=1}^{n} (\bar{C}_{L_{ij}})_k \, (Z_k - Z_{k-1}) \quad , \qquad (6b)$$

where A_{ij} represents the globally averaged stiffness matrix. Thus the globally averaged constitutive equations become

$$\{N\} = [A]\,\{\varepsilon^o\}. \tag{7b}$$

Experimental testing is often conducted on uniaxial testing machines in which the applied force resultants are input and the strains are experimentally determined output. Therefore, at times, it is more convenient to express the strains in terms of the applied force resultants as follows,

$$\{E^o\} = [A]^{-1}\,\{N\} \quad . \tag{8b}$$

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