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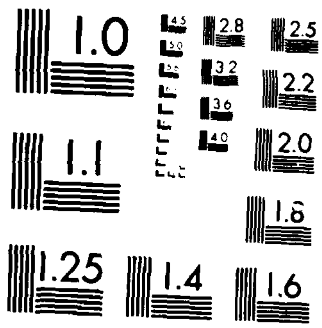

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During this research period, the use of certain nonconforming schemes for solving viscous incompressible flow problems was put on a rigorous foundation, and the resulting theory was used to construct new convergent schemes. The investigator was able to show that a certain nonconforming quadratic element actually has the same accuracy as a well known nonconforming cubic. This is of great practical significance because the costs of impelmenting the quadratic elements are far less than those for cubics. The above work is reported in "Analysis of nonconforming stream function and pressure finite element spaces for the Navier-Stokes equations", submitted to Mathematics of Computations.

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1. Nonconforming finite element methods for Navier-Stokes equations

The standard weak problem of viscous incompressible flow, expressed in terms of the stream function is

$$\int_\Omega \nu \Delta\Psi\Delta\Phi + \Delta\Psi(\xi_y\Phi_x - \xi_x\Phi_y) = \int_\Omega f \,\vec{\mathrm{curl}}\, \Phi \qquad \forall\Phi \in H_0^2(\Omega)$$

The appearance of second order derivatives necessitates the use of time functions with continuous first derivatives in this formulation. Unfortunately, these are excessively difficult to construct and work with, and are usually avoided in favor of nonconforming methods. The problems with the latter are (i) They fail to work as expected in many cases and (ii) There was almost no rigorous theory to justify the cases which do yield valid numerical schemes.

In view of the unquestioned greater efficiency of those nonconforming schemes which actually do work, it seemed essential to provide rigorous justifications for their use, and also to use whatever theory resulted for the construction of new convergent schemes. This project has been completed successfully. It was necessary to introduce a new weak form, but one which reduces to the standard one given above in the conforming case. Then, a detailed analysis using some new tools developed for the purpose enabled us to produce several interesting and significant results. For example, we were able to show that a certain nonconforming quadratic element actually has the same accuracy as a well known nonconforming cubic. This is of great practical significance, because the costs of implementing the quadratic element are far less than those for cubics. It is quite surprising that a quadratic space can ever have enough smoothness to yield a convergent scheme, but we have verified our conclusions in numerous computations of physical interest. The explanation involves the nonconforming behavior in an essential way. In addition to this work, several new elements were introduced and analysed, and error estimates for them produced in several norms. Further, our work from the previous grant period on optimal pressure recovery was extended to the nonconforming case.

The above work is reported in

> Analysis of nonconforming stream function and pressure finite element spaces for the Navier-Stokes equations by M. E. Cayco and R. A. Nicolaides (submitted to Mathematics of Computation)

This work is also elaborated in Ms. Cayco's thesis, successfully defended in August 1985, entitled

> Finite element methods for the stream function formulation of the stationary Navier-Stokes equations.

2. Improved methods for parallel computer implementation of Substructuring.

One of the basic parallelizable algorithms for solving discretized elliptic partial differential equations is that of substructuring. The basic idea is to break up the physical domain into "substructures", ie, smaller domains, and then to solve the resulting local equations in parallel. The local calculations cannot be done completely without reference to each other, because the solution within a given subdomain is dependent on the solutions of the other subdomains. However, within a given subdomain, a reasonable amount of the computation is, in fact, local to that subdomain. This represents the parallel part of the computation. Following it, a relatively small computation is done (serially) to provide the linking data for joining the subdomain solutions together to create the solution of the original problem.

Expressed in matrix terms, the coefficient matrix is written- by suitable ordering of the equations and variables - into the form

$$\begin{bmatrix} A_{11} & & & A_{1m} \\ & A_{22} & & A_{2m} \\ & & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mm} \end{bmatrix}$$

and Gauss elimination or some other algorithm is used. Inversion of the diagonal blocks is done in parallel, so then the elimination process becomes trivial.

The description given above necessarily requires the invertibility of the diagonal blocks A_{ii} $i = 1,2,\ldots,m$. For the common second order elliptic cases, this invertibility is guaranteed by the ellipticity of the equation, so no difficulties arise. For the Navier-Stokes equations in the incompressible case however the local matrices A_{ii} are singular, a situation which is inherited from the global problem where, it will be recalled, the singularity is removed by specifying one linear functional, of the pressure p, e.g. $\int_{\Omega} p d\Omega = 0$. We cannot specify such functional within each subdomain because the global equation system would then be overdetermined.

For a long time this difficulty was brought to preclude the use of the substructuring algorithm for flow problems. However, a solution has been found, which does not even require the diagonal blocks to be square matrices. It is based on the use of the generalized inverses of the diagonal blocks. These are computed in the usual way by performing eliminations. The new method is just as convienent as the standard method, and has been employed to compute numerous flow simulations. It is described fully with numerous applications in the report

> <u>On substructuring algorithms and solution techniques for the numerical analysis of partial differential equations</u>
> by M. D. Gunzburger and R. A. Nicolaides
> submitted to International Journal for Numerical Methods in Engineering (will also appear in M. E. Rose 65th birthday volume)

3. New developments in particle and point vortex methods

We have begun a substantial investigation into the application of particle and point vortex methods for flow probiems. (Please see "Proposed Work 1986-1987" for a description of the usefulness and importance of these methods).

Although some of the basic concepts of particle methods are quite old, a lot of interest is being shown in developing a wide ranging numerical methodology for applying them to serious technological problems beyond the range of the usual finite difference and finite element methods. For incompressible flows, until recently, all that existed were a single low order of accuracy algorithm for inviscid calculations, even without boundaries. A way for handling viscosity and boundaries was added to this algorithm by Chorin in 1973. In the early 1980's, the first rigorous convergence proofs for the point vortex method in the inviscid case (Eulers equations) were produced by Dushane, Hald, and Beale and Majda. Very recently, Raviart and co-workers have refined these proofs and placed them in a more general setting.

Our initial contributions have been on the algorithmic side. Specifically, the goal is to reduce the cost of computation (particle methods-at least low order ones-are rather expensive) by constructing methods which use more information, associated with many fewer particles. To our knowledge, the methods we have proposed are the first higher order particle methods. Several classes of higher order methods have been defined as set out in the attached preliminary report. A great deal of work must now be done to find the most efficient methods and apply them to new problems. A highly preliminary report on this work entitled

<u>Construction of higher point vortex and particle methods</u> is enclosed.

[Modified version will be submitted to Journal of Computational Physics]