

DEVELOPMENT OF COMPUTER PROGRAMS
AND RELATED SOFTWARE FOR
THE DETECTION AND ANALYSIS OF P12 PULSATIONS
RECORDED AT GROUND BASED STATIONS

J. C. Samson

Institute of Earth and Planetary Physics
Department of Physics
University of Alberta
Edmonton, Alberta, Canada
T6G-2J1

Final Report
2 March 1982 - 2 September 1985

2 September 1985

Approved for public release; distribution unlimited

AIR FORCE GEOPHYSICS LABORATORY
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HANSCOM AFB, MASSACHUSETTS 01731

DTIC
ELECTE
MAR 14 1986
E

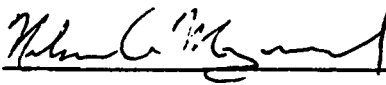
DTIC FILE COPY

86 3 14 009

AD-A165 224

"This technical report has been reviewed and is approved for publication"


HOWARD J. SINGER
Contract Manager


NELSON C. MAYNARD
PHG Branch Chief

FOR THE COMMANDER


RITA C. SAGALYN
PH DIVISION DIRECTOR

This report has been reviewed by the ESD Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS).

Qualified requestors may obtain additional copies from the Defense Technical Information Center. All others should apply to the National Technical Information Service.

If your address has changed, or if you wish to be removed from the mailing list, or if the addressee is no longer employed by your organization, please notify AFGL/DAA, Hanscom AFB, MA 01731. This will assist us in maintaining a current mailing list.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

AD-A165 224

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; Distribution unlimited.		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			4. PERFORMING ORGANIZATION REPORT NUMBER(S)		
5. MONITORING ORGANIZATION REPORT NUMBER(S) AFGL-TR-85-0146			6a. NAME OF PERFORMING ORGANIZATION Institute of Earth and Planetary Physics		
6b. OFFICE SYMBOL (If applicable)			7a. NAME OF MONITORING ORGANIZATION Air Force Geophysics Laboratory		
6c. ADDRESS (City, State and ZIP Code) Dept. of Physics University of Alberta Edmonton, Alberta, Canada T6G 2J1			7b. ADDRESS (City, State and ZIP Code) Hanscom AFB Massachusetts 01731		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION			8b. OFFICE SYMBOL (If applicable)		
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F19628-82-K-0032			8c. ADDRESS (City, State and ZIP Code)		
10. SOURCE OF FUNDING NOS.			11. TITLE (Include Security Classification) Computer Programs & Related Software for (over)		
PROGRAM ELEMENT NO. 62101F			PROJECT NO. 7601		
TASK NO. 08			WORK UNIT NO. AR		
12. PERSONAL AUTHOR(S) J.C. Samson					
13a. TYPE OF REPORT FINAL REPORT		13b. TIME COVERED FROM 3/2/82 TO 9/2/85		14. DATE OF REPORT (Yr., Mo., Day) 1985 July 2	
15. PAGE COUNT 34					
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB. GR.	geomagnetic pulsations, detection, polarized waves		
19. ABSTRACT (Continue on reverse if necessary and identify by block number)					
The major objectives of the study were to develop algorithms for the estimation of the polarization states of Pi 2 geomagnetic pulsations, to code these algorithms in FORTRAN, and to develop models for the prediction of the polarization states of Pi 2's. The Pi 2 models are based on the magnetic fields associated with transient field-aligned currents, on geomagnetic field lines threading the auroral ionosphere. These models have proven to be extremely successful in predicting the polarization states of high and mid latitude Pi 2's. The computer programs are based on algorithms for detecting pure states, and estimating the parameter of pure states or polarized waves in multiple time series. All the software for these algorithms has been written in FORTRAN, and the programs have been used with considerable success in the parameterization of the polarization states of Pi 2's. <i>Continued on reverse side</i>					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS ☐			21. ABSTRACT SECURITY CLASSIFICATION Unclassified		
22a. NAME OF RESPONSIBLE INDIVIDUAL Howard J. Singer			22b. TELEPHONE NUMBER (Include Area Code) (617)861-5757		22c. OFFICE SYMBOL AFGL/PHG

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

11. (Cont'd)

the Detection and Analysis of P1 2 or PI 2 Pulsations Recorded at
Ground Based Stations

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

TABLE OF CONTENTS

1.	Introduction.....	2
1.1	General introduction.....	2
1.2	An introduction to Pi2s.....	4
2.	Work completed.....	6
2.1	Pi2 experimental studies.....	6
2.2	Pi2 models.....	8
2.3	Multivariate time series studies.....	10
2.4	Computer programs and documentation.....	13
3.	Publications.....	15
3.1	Publications in refereed journals.....	15
3.2	Abstracts of presentations at conferences.....	15
3.3	Other publications.....	16
4.	References.....	17
5.	Figures.....	21

[illegible]

1. INTRODUCTION

1.1 General Introduction

The research program for Contract Number F19628-82-K-0032 had both scientific and technical objectives. The scientific objectives included the measurement of the polarizations and wave characteristics of Pi2s. These data were then to be used to develop a model of the magnetic fields and currents associated with Pi2s. This model was to be used in evaluating methods for detecting Pi2s with ground based magnetometer arrays. The technical objectives included the design of algorithms for the estimation of the wave and polarization characteristics of Pi2s, the programming of these algorithms, and the writing of documentation to accompany the computer programs

The work under this contract falls in four specific areas including:

A. Pi2 Experimental Studies.

These studies used data from ground based magnetometer arrays, including the AFGL (Air Force Geophysics Laboratory), the Alberta (University of Alberta), and the IMS (International Magnetosphere Study) arrays. The studies were designed to measure the polarizations, phase velocities, group velocities, frequencies, and wave envelopes of the Pi2s.

B. Pi2 Models

These models are based on field aligned and ionospheric currents and were used to model the magnetic fields of Pi2s at ground level.

C. Multivariate Time Series Studies

These studies used the methods of multivariate analysis to develop analytic forms for the estimation of the parameters of waves, and in particular Pi2s.

D. Computer Programs and Documentation

The computer programs are the FORTRAN codes needed to implement the various time series algorithms on a computer. The documentation gives a brief introduction and the technical details needed to run the programs.

1.2 An introduction to Pi2s

Pi2 pulsations are oscillations of the geomagnetic field which occur in conjunction with intensifications of the magnetospheric currents associated with polar magnetic substorms. The Pi2s are seen as quasi sinusoidal oscillations on ground based magnetometers. The oscillations typically have frequencies of about 10 mHz (0.01 Hz), and amplitudes of up to several nT at mid latitudes and tens of nT at high latitudes. A pulsation train can last for one cycle or more.

These pulsations are probably the magnetic signatures of transient changes in the ionospheric and field aligned currents (fac) associated with the onset of the substorm expansive phase. Observations which support this possibility, are those showing the association of Pi 2's with the initial brightening of auroral arcs and precipitation of electrons at the beginning of the substorm expansive phase [Troitskaya and Gul'elmi, 1967; Pytte and Trefall, 1972; Samson 1982], and the close correlation of the polarization - pattern of Pi 2's with the magnetic fields from the substorm current "wedge" [Rostoker, 1967; Bjornsson et al., 1971; Baransky et al., 1980; Lester et al., 1983, 1984; Samson and Harrold, 1983]. (See McPherron et al. [1973] for a description of the substorm current "wedge".)

Samson and Harrold [1983] showed that all the various ground based observations of the polarizations could be fitted to a

Final Report, September 2, 1985

consistent pattern, with counterclockwise (cc) polarizations at mid-latitudes, and a more complicated pattern near the region of the onset of substorm fac (see Fig. 7 in Samson and Harrold). Studies conducted under this contract [Samson, 1985] have shown that the observed pattern of polarizations is compatible with westward moving fac and ionospheric current systems which are associated with a brightening arc.

The midlatitude wave-numbers and phase velocities of Pi2's [Mier-Jedrzejowicz and Southwood, 1979, 1981; Baransky et al., 1980; Lester et al., 1983, 1984] indicate that most Pi2's show westward propagation, with phase velocities near 2 degrees longitude /sec (longitudinal wavelengths near 200 degrees, and mean frequencies near 0.01 Hz) at least near the longitudinal center of the substorm current wedge.

2. WORK COMPLETED

2.1 A. Pi2 experimental studies

The experimental work used data from the AFGL, Alberta, and IMS arrays (Fig.1) to study the polarizations, phase velocities, group velocities, and characteristic time constants of Pi2s.

The Pi2s appear to be correlated with the brightening of auroral arcs [Samson, 1985]. Often this arc is equatorward of the Harang Discontinuity. The Pi2s show their maximum intensities near the brightened arc and the substorm enhanced auroral electrojets (Fig.2). Although the polarization patterns of individual events can be quite complicated, these patterns do appear to be consistent from event to event (Fig. 3, see also Fig. 7 in Samson and Harrold [1983]). Near the longitudinal center of the substorm current wedge, the polarizations range from counterclockwise (cc) (viewed down on the H-D plane) at midlatitudes, to clockwise (cw) just equatorward of the intensity maximum, to cc just poleward of the intensity maximum. Our model studies have shown that these polarizations are compatible with a westward moving region of transient fac and ionospheric current associated with a brightening arc [Samson, 1985].

Analyses of data from the midlatitude AFGL stations have yielded excellent measurements of the characteristic rise and fall times, frequencies, phase velocities, and group velocities of Pi2s [Samson et al., 1985]. These measurements were based on a

Final Report, September 2, 1985

polarized wavelet model of the vector time series of the Pi2 (see the discussion below or Samson et al.[1985].). Measured group delays over the AFGL array were often less than 5-10 sec, indicating that Pi2s should give good timings of substorm onsets. The rise and fall times for the envelopes of the Pi2 pulsation trains were typically 60-100 sec.(Fig. 4). These short time intervals suggest that the Pi2s are not associated with hydromagnetic resonances. However, the rise and fall times are quite compatible with transient reflections of fac over a brightening arc [Samson, 1985].

Near the center of the substorm current wedge (see Samson and Harrold [1983] for a description of substorm centered coordinates), most midlatitude Pi2s have westward phase velocities with typical values of 1 to 4 degrees longitude/ sec. (Fig.5). There are some indications of eastward propagation more than 30° east of the center (see also Lester et al. [1984]).

2.2 B. Pi2 models

Due in part to the inherent difficulties in a self consistent theoretical description of transient hydromagnetic processes in the magnetosphere, we chose an heuristic model for Pi2s. This model was based on fac and ionospheric currents compatible with reflection of an Alfvén wave from a conducting strip in the auroral ionosphere. This conducting strip is oriented east-west, and is associated with a brightening arc. The model, shown in Fig. 6, has fac at the edge of the conducting strip, and ionospheric currents closing the circuit. The model is compatible with an incident Alfvén wave with an equatorward directed electric field. To maintain a curl free electric field, this configuration requires the longitudinal scale size of the Pi2 to be much larger than the latitudinal scale size. Observations indicate that this should be the case since longitudinal wavelengths of Pi2s are often near 180° (see Samson et al. [1985] and references therein).

More details on the formation of fac due to reflections from a nonuniform ionosphere can be found in Samson [1985]. Southwood and Hughes [1985] have also considered a Pi2 model which takes into account reflections from a nonuniform ionosphere. Unfortunately their model assumes an infinite latitudinal extent, and longitudinal wavelengths which are comparable to the latitudinal width of the enhanced conductivity band associated with an arc. These assumptions are not compatible with the observations mentioned above.

Final Report, September 2, 1985

To run the model simulations, we have used a computer program which computes the magnetic fields associated with the fac and ionospheric currents in the model depicted in Fig. 6. A dipolar geometry is used for the fac. We assumed westward propagation, and consequently, the current system must move westward at a velocity appropriate for the measured phase velocities. The test for the validity of the model was based on the prediction of the observed polarization pattern of the Pi2s. The predicted pattern is shown in Fig. 7 and compares favorably with the pattern in Fig. 3 and in Fig. 7 of Samson and Harrold [1983].

2.3 C. Multivariate time series

Methods of multivariate time series analysis were required in order to determine the characteristic polarization patterns of Pi2s, to measure the group and phase velocities, and to develop data-adaptive filters to enhance Pi2 waveforms in noisy data. We have also made some preliminary progress in using multivariate detection theory for the detection of Pi2 wavetrains. Almost all our theoretical work has been based on the concept of pure states in multivariate processes [Samson, 1983a]. Numerous tests have shown that Pi2s fit this model very well [Samson, 1985; Samson et al., 1985]. We have also developed theoretical expressions for reducing sample biases in the estimators of the pure states, based on the work of Samson [1983b]. These revised estimators have been incorporated into the FORTRAN programs written under this contract.

The specific model which we have found suitable for Pi2s has the mathematical form

$$\underline{w}(x,t) = 1/2[a(x,t)p(x)\exp(-i2\pi f_0 t) + c.c.] + \underline{e}(x,t) \quad [1]$$

where $\underline{w}(x,t)$ is the vector wavelet of the Pi2 at position x and time t , $a(x,t)$ is the amplitude, $p(x)$ is the complex polarization vector [Samson, 1983a], f_0 is the frequency, and $\underline{e}(x,t)$ is the noise vector [Samson et al., 1985]. The estimators of the

Final Report, September 2, 1985

amplitude, $a(t)$, phase, $\phi(t)$, and wave-frequency, f_0 , are respectively

$$a_j(t) = (r_j^2(t) + q_j^2(t))^{1/2}, \quad [2]$$

$$\phi_j(t) = (t_2 - t_1 + 1)^{-1} \sum_{t=t_1}^{t_2} \tan^{-1}[q_j(t)/r_j(t)], \quad [3]$$

$$f_0 = (t_2 - t_1 + 1)^{-1} \sum_{t=t_1}^{t_2} f(t), \quad [4]$$

and

$$2\pi f(t) = a_j^{-2}(t)[r_j(t)q_j(t+1) - q_j(t)r_j(t+1)] \quad [5]$$

(any j)

where

$$\underline{r}(t) = N^{-1} \sum_{k=0}^{N/2} c(k)b(k)\underline{p} \exp(2\pi ikt/N) + c.c., \quad [6]$$

$$\underline{q}(t) = N^{-1} \sum_{k=0}^{N/2} c(k)b(k)\underline{p} \exp(2\pi ikt/N) + c.c., \quad [7]$$

$$\underline{p} = \underline{E}^{-1/2} \underline{u}, \quad [8]$$

$$b(k) = \underline{p}'[\underline{E}^{-1} - \underline{S}^{-1}]\underline{z}(k)/(\underline{p}'\underline{E}^{-1}\underline{p}), \quad [9]$$

$$\underline{z}(k) = \sum_{t=0}^{N-1} \underline{d}(t) \exp(-2\pi ikt/N), \quad [10]$$

and

$$\underline{S} = \sum_{k=0}^{N/2} c(k) \underline{z}(k) \underline{z}^t(k).$$

[11]

In the above equations, $\underline{E}$ ($\underline{E}=\underline{I}_n$) is the noise spectral matrix, $\underline{u}$ is the eigenvector corresponding to the largest eigenvalue of $\underline{E}^{-1/2} \underline{S} \underline{E}^{-1/2}$, and $c(k)$ is the spectral window. The above estimators were used by Samson et al. [1985] in their analysis of Pi2s. An illustration of the use of the above estimators in the analysis of mid latitude Pi2s is given in Fig. 8,9. Figure 8 shows Pi2 pulsations recorded by the AFGL magnetometers, after application of a 3 mHz highpass filter. Figure 9 shows the estimated envelopes $a(t)$, and the frequency $f(t)$.

2.4 D. Computer programs and documentation

Under the terms of this contract, four programs have been written in FORTRAN, documented, and sent to AFGL. These programs are POLFILT1, POLFILT2, QUADFILT, and LHFILT. All of these programs were designed to aid in the estimation of the parameters of propagating waves, and in particular Pi2s.

POLFILT1 consists of a main program and subroutines for the adaptive filtering of multichannel time series. The filters extract the polarized or pure state by modulating the spectral representation with a function based on the degree of polarization of the multichannel data [Samson, 1983a]. These filters also have an adjustment for lagged arrivals in the wavelets, whereas the more sophisticated POLFILT2 does not. POLFILT1 has a number of biases in its estimators and is designed largely to act as a rapid, inexpensive filter for the initial processing of the data. For more details read the manual DOCUMENTATION FOR POLFILT1.

POLFILT2 is a more ambitious program, and uses a number of optimal estimators to determine the parameters of pure states or polarized waves. This program can be used to filter data to extract the time series of the pure states, and to estimate the parameters of polarized waves (eg. the parameters of the polarization ellipse). Because it is a much more sophisticated program, POLFILT2 tends to be much more expensive to run, and requires a much larger number of input parameters than does POLFILT1. Further information on POLFILT2 can be found in the manual DOCUMENTATION FOR POLFILT2.

Final Report, September 2, 1985

QUADFILT is a FORTRAN program for estimating the amplitudes, frequencies and phases of polarized wave packets (see equations [1] to [10]). This program uses a quadrature filter in the frequency domain to determine the Hilbert transform of the multichannel data [Goodman, 1960] (see Equation [7]). This program has been used extensively in the estimation of the phase and group velocities of Pi2s [Samson et al., 1985]. For more details on this program please read DOCUMENTATION FOR QUADFILT.

The fourth program written for AFGL is LHFILT. This program is a filter to low pass, high pass, or band pass selected time sequences. We have used this filter in detrending our data before using POLFILT1, POLFILT2, or QUADFILT. Details on this filter are given in DOCUMENTATION FOR LHFILT.

3. PUBLICATIONS

3.1 Publications in refereed journals

Samson, J. C., Harrold, B. G., and Yeung K. L., Characteristic time constants and velocities of midlatitude Pi2's, J. Geophys Res., 90, 3448-3456, 1985.

Samson, J. C., Large scale studies of Pi2's associated with auroral breakups, J. Geophys., in press, 1985.

3.2 Abstracts of presentations at conferences

Samson, J. C., Estimators of the parameters of pure states and polarized waves in multichannel, ULF magnetic field data, Chapman Conference on Waves in Magnetospheric Plasmas, Hawaii, Feb. 1983

Samson, J. C., Field aligned currents associated with pulsations at high latitudes, Chapman Conference on Magnetospheric Currents, Virginia, April, 1983.

Final Report, September 2, 1985

Samson, J. C., Field aligned currents associated with Pi2s near the westward travelling surge, Chapman Conference on Magnetospheric Currents, Virginia, April, 1983.

Samson, J. C., Large scale, ground based studies of the polarization states of Pi2s, IAGA, Hamburg, August, 1984.

3.3 Other publications

Documentation for POLFILT1

Documentation for POLFILT2

Documentation for QUADFILT

Documentation for LHFILT

REFERENCES

Baransky, L.N., V.A. Troitskaya, I.V. Sterlikova, M.B

Gokhberg, N.A. Ivanov, I.P. Khartchenko, J.W. Munch, and

K. Wilhelm,

The analysis of simultaneous observations of nighttime Pi
pulsations on an East-West profile, J. Geophys., 48, 1, 1980.

Bjornsson, A., O. Hillebrand, and H. Voelker,

First observational results of geomagnetic pi2 and pc5 pulsations
on a north-south profile through Europe,

Z. Geophys., 37, 1031-1042, 1971.

Goodman, N. R., Measuring amplitude and phase, J. Franklin Inst.,
270, 437-450, 1960.

Lester, M., W.J. Hughes, and H.J. Singer,

Polarization patterns of Pi2 magnetic pulsations and the substorm
current wedge, J. Geophys. Res., 88, 7958-7966, 1983.

Lester, M. W.J. Hughes, and H.J. Singer,

Longitudinal structure in Pi2 pulsations and the substorm current
wedge, J. Geophys. Res., 89, 5489-5494, 1984.

Final Report, September 2, 1985

McPherron, R.L., C.T. Russell, and M.P. Aubry,
Satellite studies of magnetospheric substorms on August 15, 1968,
9, Phenomenological model for substorms, J. Geophys. Res., 78,
3131-3149, 1973.

Mier-Jedrzejowicz, W.A.C., and D.J. Southwood,
The East-West structure of pulsation activity in the 8-20 mHz
band, Planet. Space Sci., 27, 617-630, 1979.

Pytte, T., and H. Trefall,
Auroral-zone electron precipitation events observed before and at
the onset of negative magnetic bays, J. Atmos. Terr. Phys., 34,
315-337, 1972.

Rostoker, G.,
The polarization characteristics of Pi2 micropulsations and their
relation to the determination of possible source mechanisms for
the production of nighttime impulsive micropulsation activity,
Can. J. Phys., 45, 1319-1335, 1967.

Samson, J.C.,
Pi2 pulsations: High latitude results, Planet. Space Sci., 30,
1239-1247, 1982.

Samson, J. C.,
Pure states, polarized waves, and principal components in the
spectra of multiple geophysical time series, Geophys. J. Roy
Astr. Soc., 72, 647-664, 1983a.

Final Report, September 2, 1985

Samson, J. C.,

The reduction of sample bias in polarization estimators for multichannel geophysical data with anisotropic noise, Geophys. J. Roy. Astron. Soc., 75, 289-308, 1983b.

Samson, J.C., and B.G. Harrold,

Maps of the polarizations of high latitude Pi2's, J. Geophys. Res., 88, 5736-5744, 1983.

Samson, J.C., and G. Rostoker,

Polarization characteristics of Pi2 pulsations and implications for their source mechanisms, 2, Influence of the westward travelling surge, Planet. Space Sci., 31, 435-458, 1983.

Samson, J.C., Harrold, B. G., and Yeung, K. L., Characteristic time constants and velocities of midlatitude Pi2s, J. Geophys. Res., 90, 3448-3456, 1985.

Samson, J.C.,

Large scale studies of Pi2's associated with auroral breakups, J. Geophys., in press, 1985.

Southwood, D.J., and W.J. Hughes,

Concerning the structure of Pi2 pulsations, J. Geophys. Res., 90, 386-392, 1985.

Final Report, September 2, 1985

Troitskaya, V.A., and A.V. Gulelmi,

Geomagnetic micropulsations and diagnostics of the magnetosphere,
Space Sci. Rev., 7, 689-768, 1967.

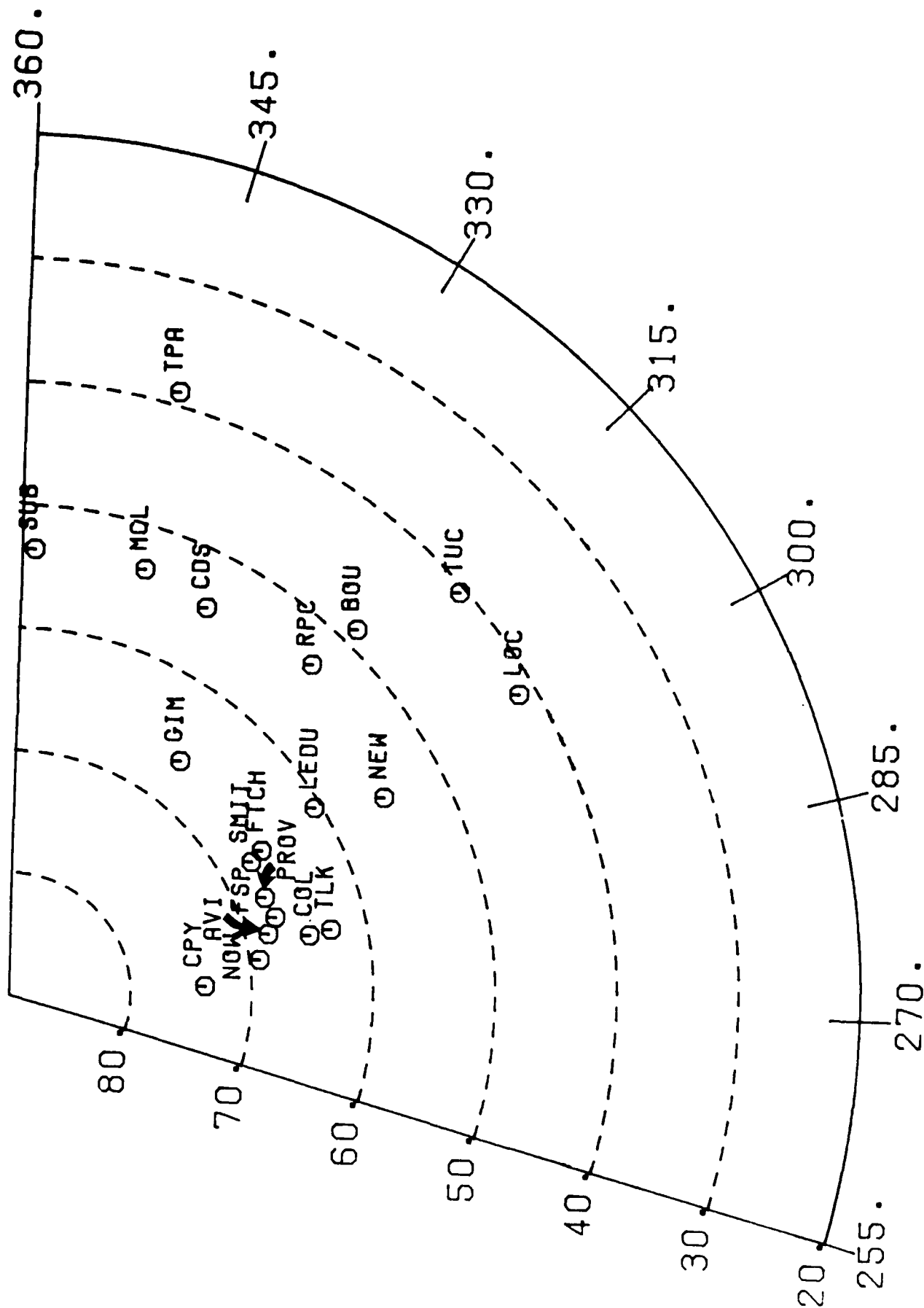


Figure 1. Stations in the AFGL array (NEW, RPC, CDS, MCL, SUB, LOC, TPA), the Alberta array (PROV, HAYR, SMT, UAN, FTCH, MCMU, LEDU), and the IMS array (remainder). Coordinates are centered dipole.

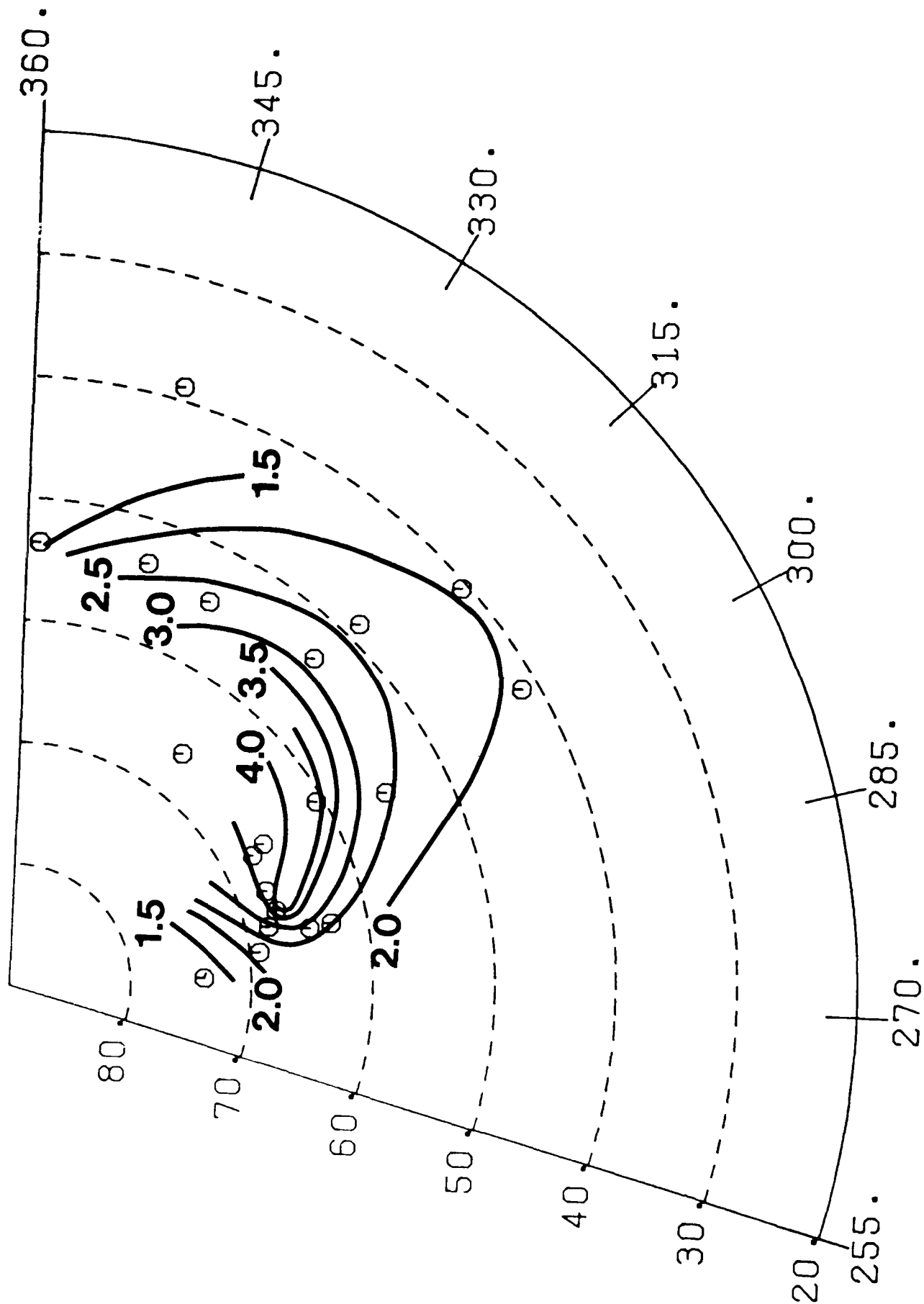


Figure 2. Log-intensity contours of a Pi2 event occurring on Day 298, 1979 (after Samson [1985]). The intensity is the sum of the power in the H, D, and Z coordinates. Intensity is measured in nT^2 .

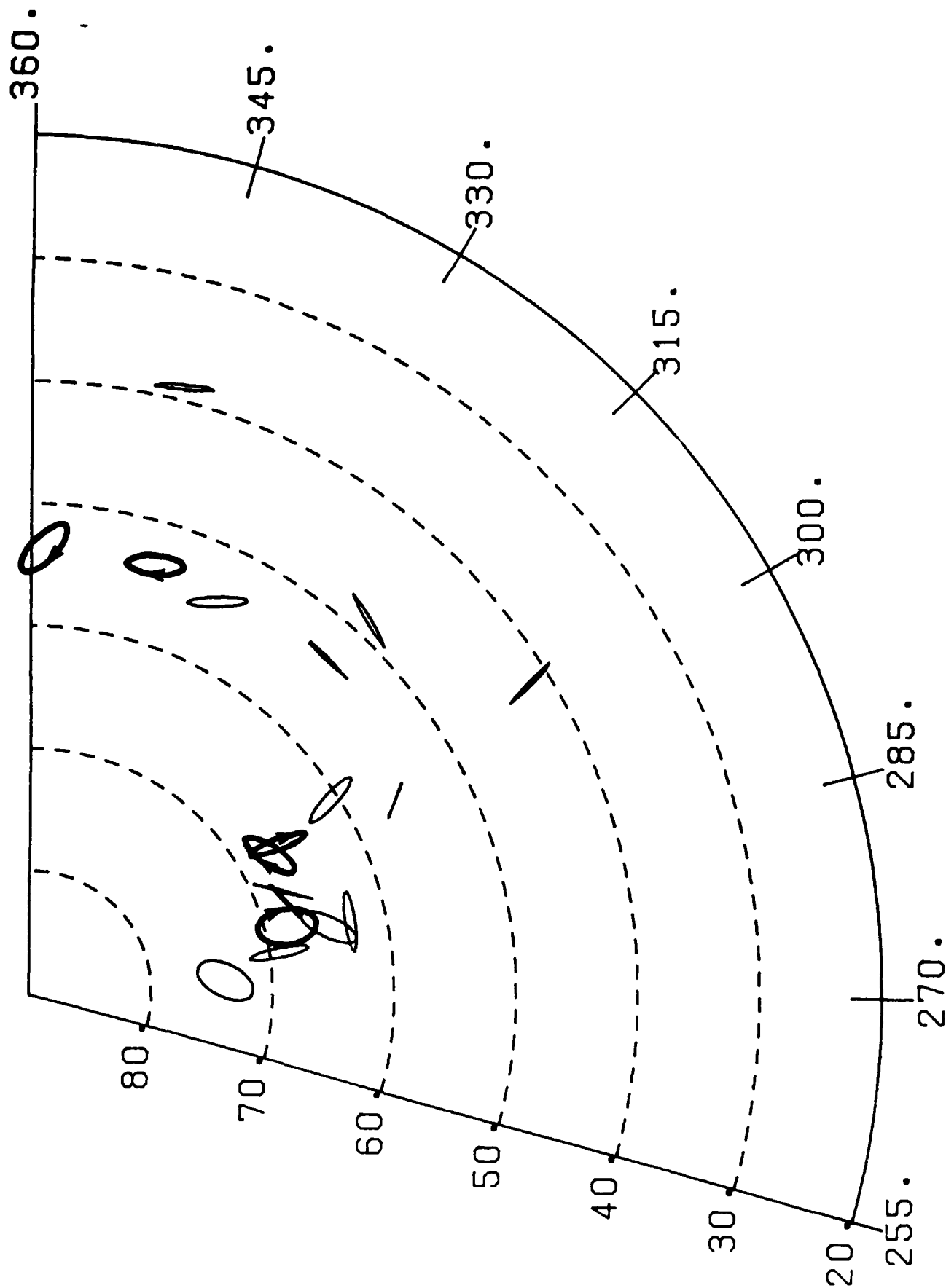


Figure 3. Polarization in the H-D plane for the event occurring on Day 298, 1979. Clockwise polarization is depicted by a dark ellipse with an arrow. The others are counterclockwise.

RISE TIME

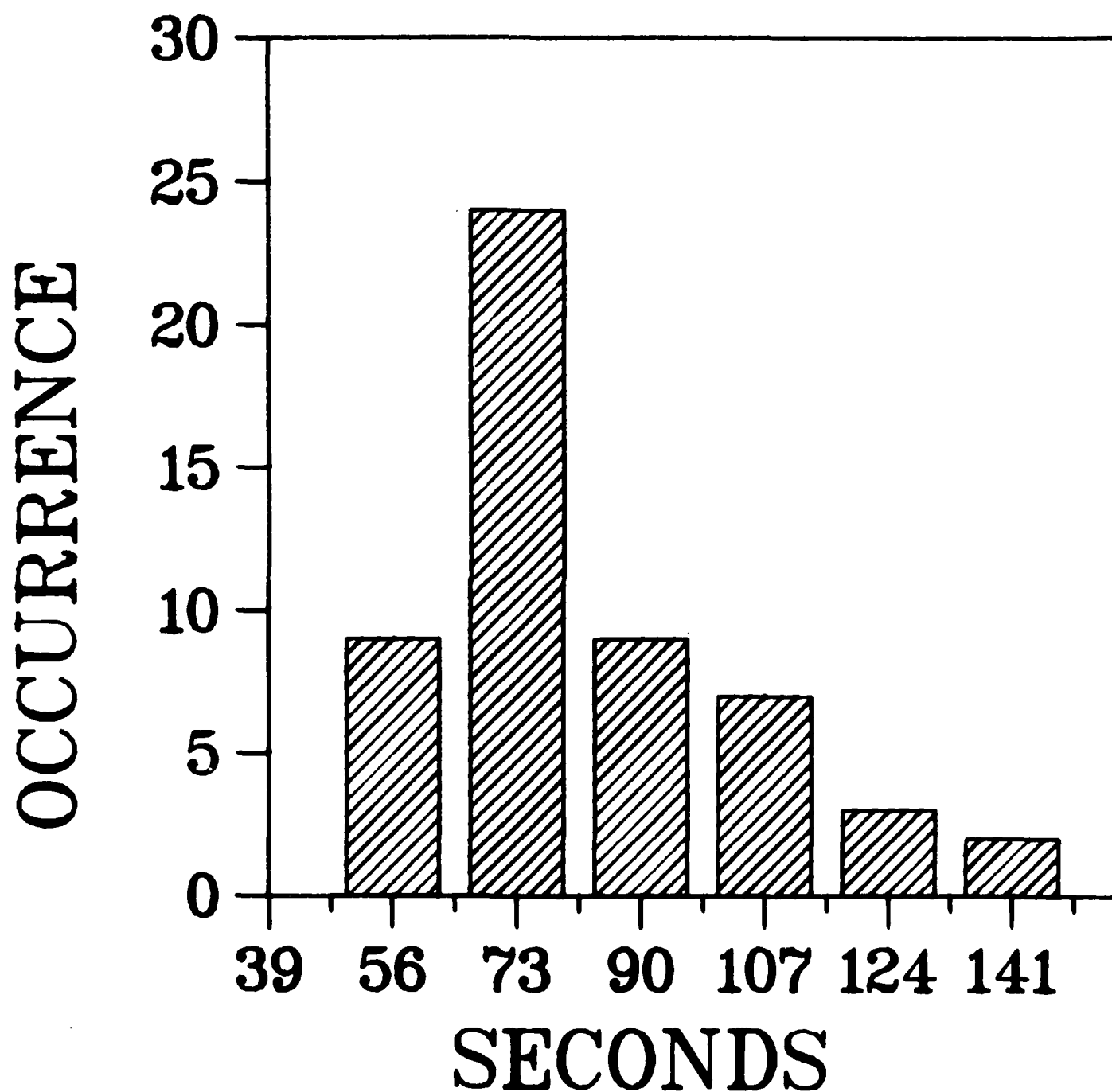


Figure 4. Rise times for mid-latitude Pi2s (after Samson et al. [1985]). The rise time is from the $1/e$ maximum to the maximum.

PK PHASE VELOCITY

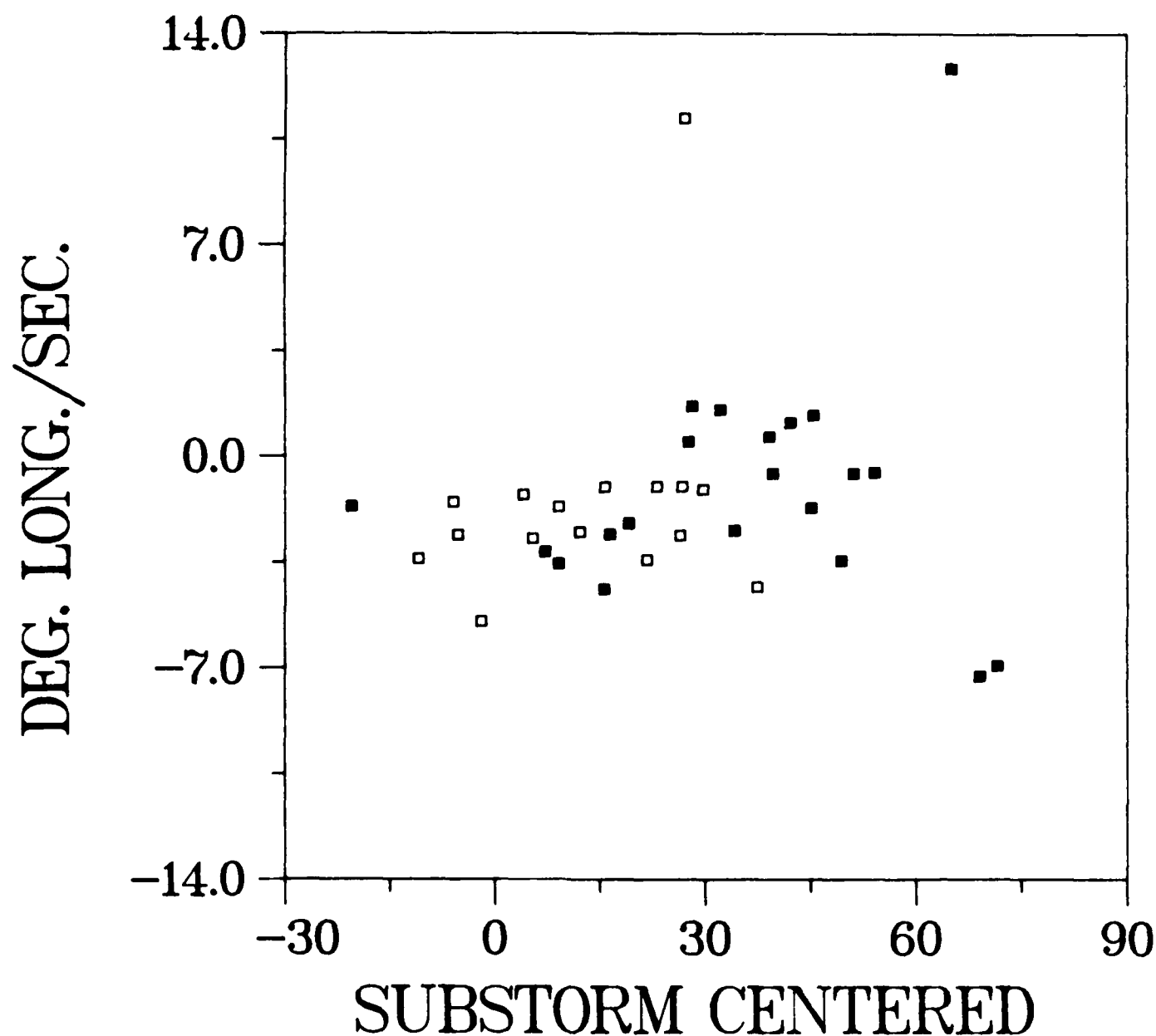


Figure 5. Phase velocities of mid-latitude Pi2s (H-component) (after Samson et al. [1985]). Negative phase velocities indicate westward propagation.

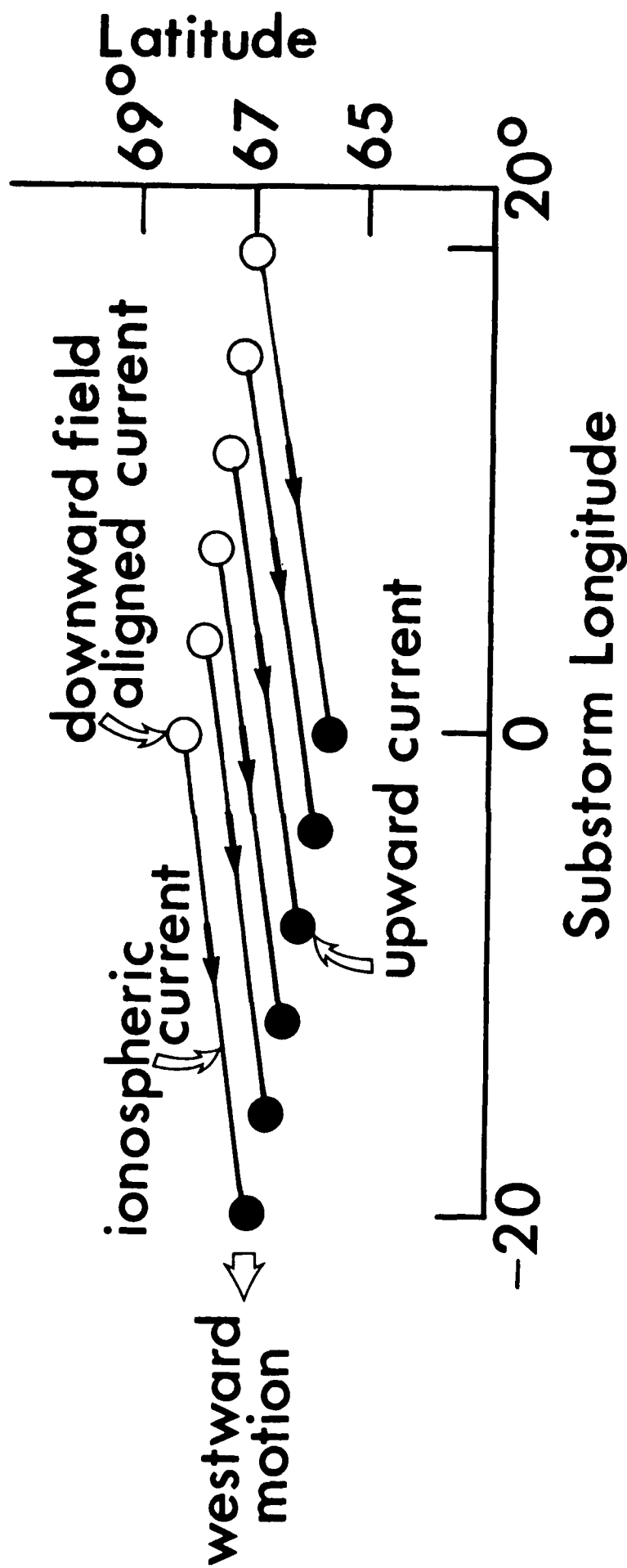


Figure 6. A field aligned and ionospheric current model for Pi2s (after Samson [1985]).

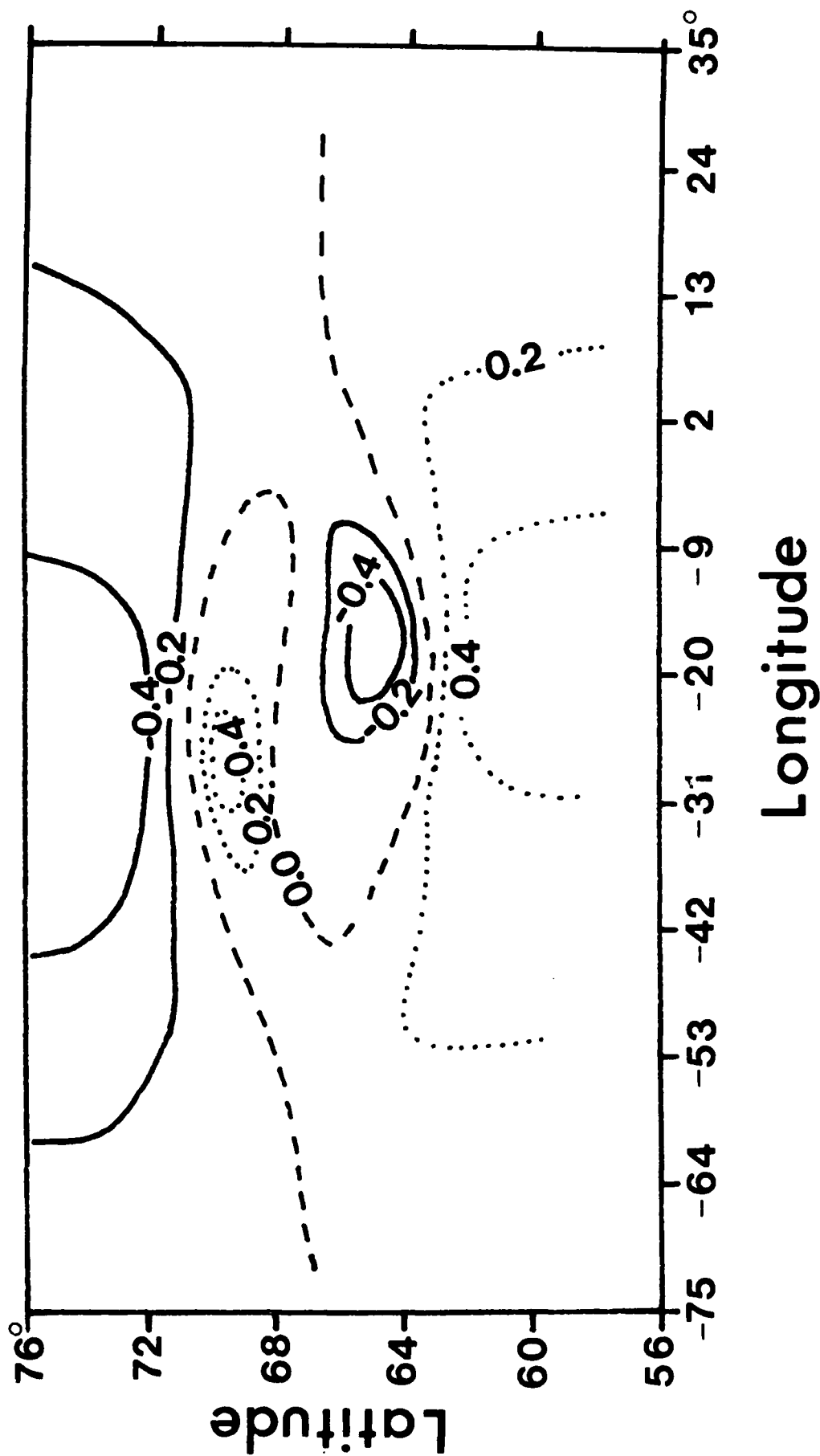


Figure 7. The predicted polarizations in the H-D plane using the model in Figure 6. The values indicate the ratio of the minor axis to the major axis of the polarization ellipse. Negative values indicate cw polarization.

Filter 1 H Component

Filter 1 D Component.

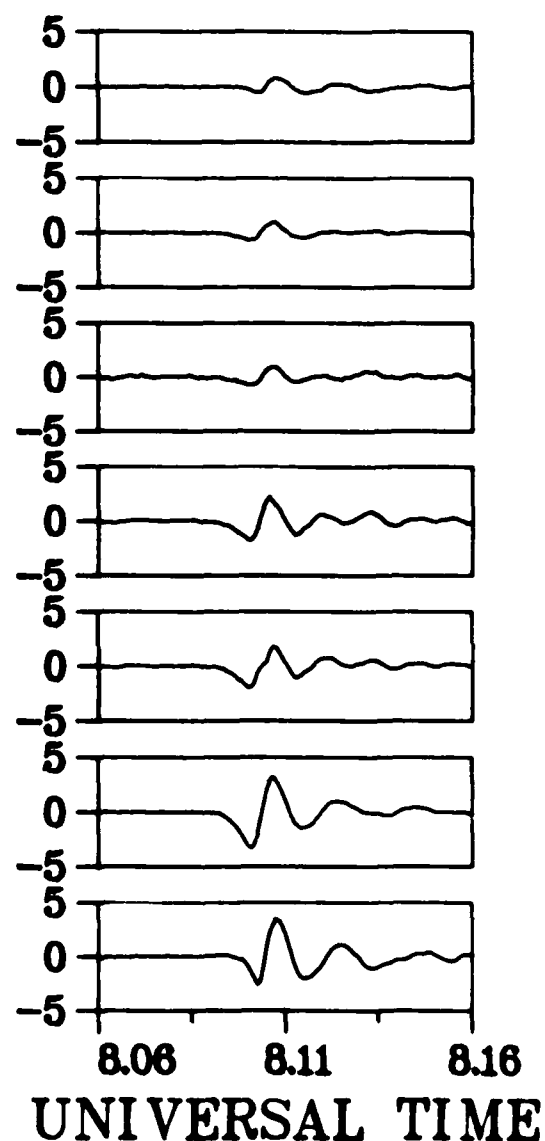
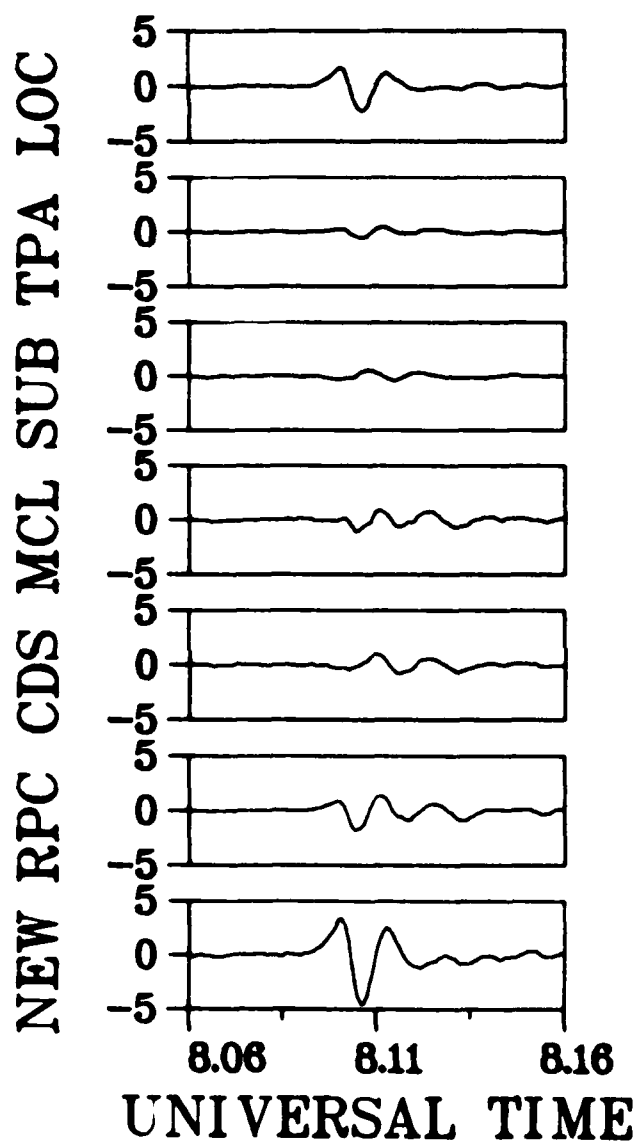


Figure 8. A Pi2 recorded at the AFGL array. The time series have been detrended with LHFILT, and a highpass cutoff of 3mHz.

Envelope H Component

Frequency H Component

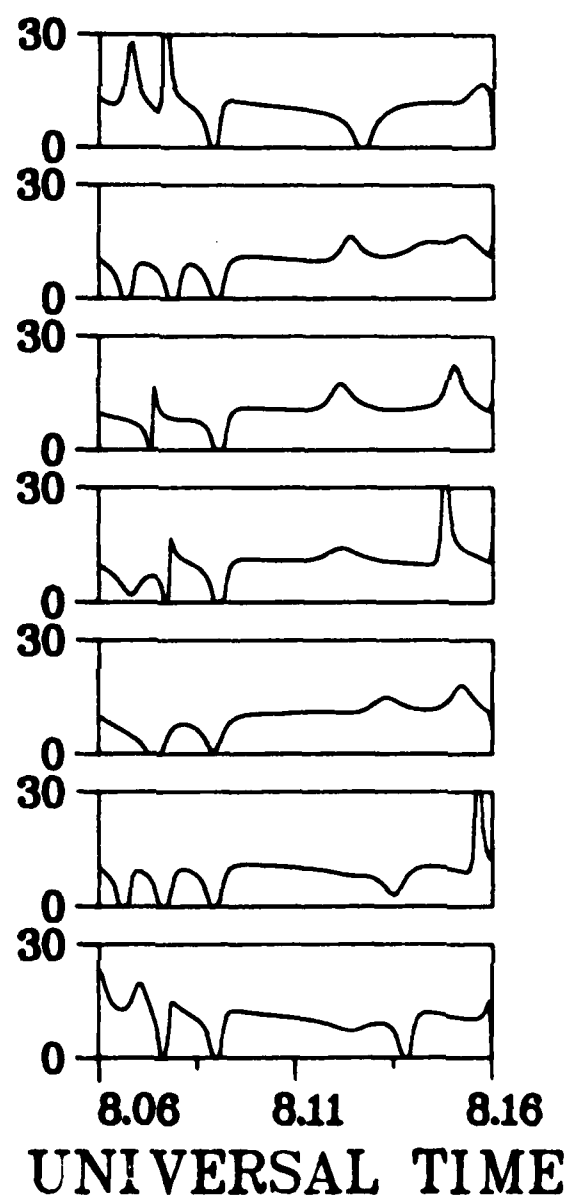
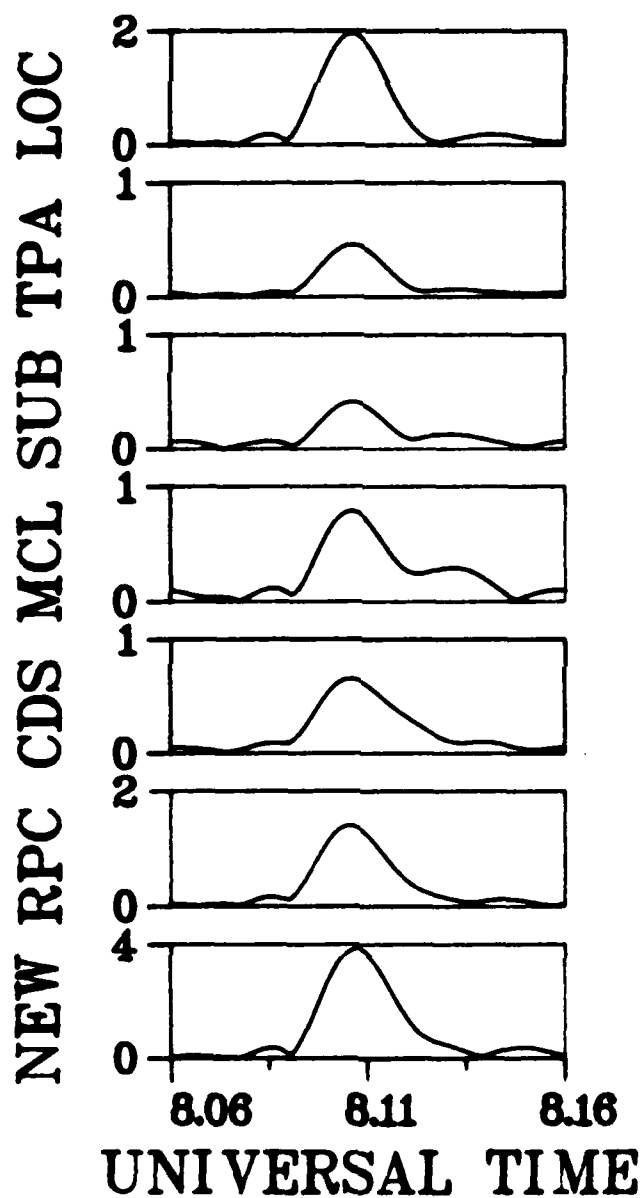


Figure 9. The estimated envelope, $a(t)$ (Equation [2]), and frequency, $f(t)$ (Equation [5]) for the P12 in Figure 8.