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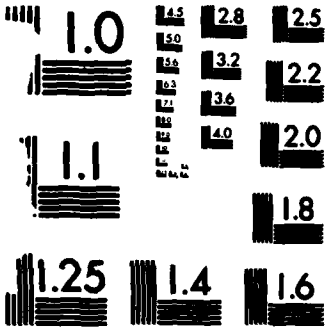

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August 1985

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THE DE BROGLIE RELATIONS: LORENTZ INVARIANCE AND PHOTONS

Ronald G. Newburgh

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A simple covariant derivation of the de Broglie relations breaks down when one considers propagation of light in an anisotropic medium. This breakdown results in two distasteful conclusions. The first is that the de Broglie relations are not completely universal and their covariance is restricted. The second is that wave-particle duality is not universal. In this paper a resolution of these problems is suggested, a resolution which is an extrapolation of ideas of de Broglie and Andrade e Silva. The relations are generalized for propagation in anisotropic media by defining a new phase wave with wave vector ℓ in the direction of energy transport and distinct from the conventional electromagnetic wave. The introduction of ℓ follows the method by which de Broglie associated a wave with an electron. The magnitude of ℓ is $\omega u/c^2$ where ω is the angular frequency and u the velocity of energy transport. The physical meaning of ℓ and the covariance of physical laws in the presence of matter are discussed. An experiment is described which

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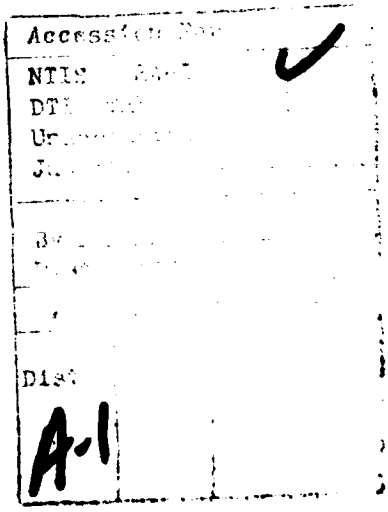
would test the existence of the phase wave associated with the new wave vector ℓ. This experiment is essentially a Young two-slit experiment carried out within an anisotropic crystal.

Preface

It would be difficult for me to exaggerate my debt to Dr. Hüseyin Yilmaz of Hamamatsu TV—Japan. In many hours of talk we have examined these ideas and, with his help and insight, I have been able to go far beyond the original notions.



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The de Broglie Relations: Lorentz Invariance and Photons

1. INTRODUCTION

The de Broglie relations which apply to both matter and light connect energy and frequency as well as momentum and wave vector. As the mathematical embodiment of wave-particle duality, they are a cornerstone of modern physics. As an expression of this duality, they unite disparate quantities once regarded as being completely distinct but today accepted as being intimately connected. Less widely appreciated is the covariant nature of the relations. This covariance is demonstrable through a remarkably simple derivation. The simplicity and lucidity of the derivation are such that far reaching implications of the de Broglie relations become evident.

In this paper we first present the derivation and then examine the implications. These are three in number. The first is the interpretation of Planck's constant as a Lorentz invariant on equal footing with the vacuum velocity of light.

The second implication is related to the breakdown of the de Broglie relations for light propagating in anisotropic media. As commonly written, the relations are valid only for propagation in vacuo or in isotropic media. However, the covariant derivation of the relations in vacuo gives clues that provide a way to generalize the relations for anisotropic media. This generalization preserves covariance by the introduction of a new wave and new wave vector distinct from the electromagnetic wave and wave vector propagating in vacuo.

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The third implication is really a consequence of the first two. These suggest that a broadened statement of the principle of covariance may be valid, a statement that includes the form of physical laws in the presence of matter. We will examine this as a possibility, at least for those laws describing the propagation of light.

2. A COVARIANT DERIVATION OF THE DE BROGLIE RELATIONS

Let us first state the de Broglie relations. Perhaps it is better to refer to them as the de Broglie - Planck relations, since they apply to both matter and light. They may be written as

$$E = \hbar\omega \ , \tag{1}$$

and

$$\underset{\sim}{p} = \hbar\underset{\sim}{k} \ . \tag{2}$$

In Eqs. (1) and (2) E is the energy, $\underset{\sim}{p}$ the 3-momentum, ω the circular frequency, $\underset{\sim}{k}$ the 3-wave vector, and $\hbar$ Planck's constant divided by 2π. The relations are peculiar in that the quantities on the left, energy and momentum, are characteristic of particles, while those on the right, frequency and wave vector, are descriptors of waves. The relations themselves are valid for both matter and electromagnetic radiation. This peculiarity is the mathematical expression of wave-particle duality that is the basis of so much twentieth century physics.

From the theory of special relativity we know that we can define a 4-momentum as

$$\underset{\sim}{P} = (\underset{\sim}{p}, \ iE/c) \tag{3}$$

and a 4-wave vector as

$$\underset{\sim}{K} = (\underset{\sim}{k}, \ i\omega/c) \tag{4}$$

The constant c is the velocity of light in vacuo. We can therefore rewrite Eqs. (1) and (2) in the succinct form

$$\underset{\sim}{P} = \hbar\underset{\sim}{K} \tag{5}$$

By using 4-vectors and making but two assumptions, we can derive Eqs. (1) and (2) quite simply.[1,2]

Landau and Lifshitz[3] have stated an important property of 4-vectors. If two 4-vectors are parallel in one inertial frame, the ratio of corresponding components is a Lorentz invariant in all inertial frames. (Obviously the two vectors remain parallel in all inertial frames.) We now introduce our two assumptions. The first is that wave-particle duality does indeed exist, that matter and light have both wave and particle aspects. The second is an assumption of parallelism, namely that the directions of energy transport and wave propagation are identical. The energy transport is characterized by the 4-momentum $\underset{\sim}{P}$, the wave propagation by the 4-wave vector $\underset{\sim}{K}$. Our assumption states that $\underset{\sim}{P}$ and $\underset{\sim}{K}$ are parallel. Applying the Landau and Lifshitz rule, we take ratios of corresponding components of $\underset{\sim}{P}$ and $\underset{\sim}{K}$. These ratios are invariant and therefore equal a constant value. This value, $\hbar$, can be determined by experiment. Writing out the values, we have

$$p_1/k_1 = p_2/k_2 = p_3/k_3 = \hbar = E/\omega \quad , \tag{6}$$

q.e.d.

This derivation is quite general and equally valid for particles of nonzero rest mass and electromagnetic radiation. It differs from those of Planck, Einstein, and de Broglie. Planck postulated Eq. (1) for material oscillators in the walls comprising a black body cavity. At no time, though, before Einstein's photoelectric paper[4] did Planck apply it to the electromagnetic radiation within the cavity. This was done first by Einstein who, by asserting the validity of Eq. (1) for radiation and thus explaining the photoelectric effect, indeed revealed the quantum nature of light. De Broglie revised the argument nearly twenty years later when he postulated the wave nature of electrons. His doctoral thesis,[5] as summarized by Ruark and Urey,[6] used Einstein's mass energy relation

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1. Newburgh, R. G. (1980) The de Broglie relations viewed as Lorentz invariants, Lettere al Nuovo Cimento 29:195.
2. Newburgh, R. G. (1981) Optics in four dimensions - 1980, M. A. Machado and L. M. Narducci, Eds., AIP Conference Proceedings 65:131-137.
3. Landau, L. D., and Lifshitz, E. M. (1962) The Classical Theory of Fields, Rev. 2nd Edition, Section 10, p. 31, Pergamon Press, Reading, Mass.
4. Einstein, A. (1905) Ueber einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt, Ann. Physik 17:132.
5. de Broglie, L. (1924) Recherches sur la Theorie des Quanta, PhD Thesis, University of Paris, Paris, France.
6. Ruark, A. B., and Urey, H. C. (1930) Atoms, Molecules and Quanta, McGraw-Hill Publishing Co., New York, pp. 516-522.

$$E = mc^2 \tag{7}$$

and Planck's quantum condition, Eq. (1), to associate a frequency with the electron. This frequency, ω_o, calculated from the electron's rest energy, is that of the wave co-moving with the electron, and is therefore defined in the electron's rest frame. However, an observer, with respect to whom the electron moves, will see a frequency ω which differs from ω_o because of the Lorentz transformation of frequency from one inertial frame to another. De Broglie identified this frequency with the laboratory frequency of the periodic motion of the electron within the atom. He then argued that the phase of this motion at a given position of the electron must agree with the phase of the accompanying wave at this point. From this argument and assuming Fermat's principle, he showed that the possible paths of the electron may be interpreted as rays of the accompanying phase waves. In de Broglie's words (as translated by Ruark and Urey[6]) "the motion can be stable only if the phase wave is tuned with the length of the path".

De Broglie's treatment of the electron may be summarized quite simply. Motion of a particle is equivalent to energy transport by virtue of the Einstein mass-energy relation. He then postulated a plane wave accompanying the energy transport, the frequency of which derives from the Planck quantum condition, Eq. (1). He made the argument consistent by showing that the possible electron paths are indeed rays of the postulated set of phase waves. This has been a leit motif in de Broglie's thought. His interpretation of the particle and wave is the basis of his disagreement with the Copenhagen interpretation of quantum mechanics. As he stated in a paper of 1968 written with Andrade e Silva,[7] "we have shown that the 'mean' path of the particle is determined according to the shape of the wave by a certain 'guidance' law."

Before publication of de Broglie's ideas, Dirac[8] had followed a line of thought very similar to the one we have used to derive Eqs. (1) and (2). He was concerned solely with the Bohr frequency condition for radiation emitted or absorbed in atomic transitions,

$$\delta E = \hbar\omega \ . \tag{8}$$

Here δE is the loss (gain) of energy of an atom which emits (absorbs) a photon of frequency ω. Dirac, too, concluded that the direction of energy transport

7. de Broglie, L., and Andrade e Silva, J. (1968) Interpretation of a recent experiment on interference of photon beams, Phys. Rev. 172:1284.

8. Dirac, P. A. M. (1924) Note on the Doppler principle and Bohr's frequency condition, Proc. Cambridge Phil. Soc. 22:432.

(really that of the energy-momentum 4-vector) and that of light propagation (taken in vacuo) must be identical for Eq. (8) to hold in all inertial frames.

From this derivation one can appreciate the deceptive simplicity of Eqs. (1) and (2). Wave-particle duality and energy-wave parallelism are the only assumptions used. Note that the assumption of duality is indifferent to the Copenhagen or causal statistical interpretation of quantum mechanics. That Eqs. (1) and (2) are fundamental relations of far reaching importance is built into the structure of twentieth century physics. Not one of the four variables in the two equations is invariant under a Lorentz transformation. Yet if the two assumptions of duality and parallelism hold, the ratio of energy and frequency as well as that of momentum and wave vector are invariant in all inertial frames. Moreover the Lorentz invariant is identified with Planck's constant. Two of the basic relations of quantum mechanics have thus acquired significant meaning within the framework of the special theory. It is clear that this derivation welds relativity and quantum theory firmly together.

3. PROPAGATION IN VACUO OR ISOTROPIC MEDIA

There is a logical difficulty in discussing light propagation in vacuo or in an isotropic medium before considering anisotropic media. This is because the directions of the energy transport and wave vector coincide in space. However, discussing anisotropic media first carries with it a difficulty which is the mirror image of the first. The problem derives, I believe, from a century of development of the wave theory of light. By the time Einstein proposed the light quantum, the electromagnetic wave was so firmly established that relatively little attention was given to the meaning of the photon. With matter waves the situation was different. When de Broglie introduced matter waves, the quantum particle was hardly two years old (if we date it from the Bohr-Rutherford atom). Working out the implications of quantisation for wave or particle (for matter with nonzero rest mass) was a far more equitable development.

In light propagation in vacuo or isotropic media, we deal with an electromagnetic wave from which the Poynting vector derives. The energy propagates in the same direction We do not consider a wave equation other than that given by Maxwell's equations (with relativistic correction, if necessary). Nor do we spend any great amount of time on the description of the photon. At this point it is worth quoting a paragraph from de Broglie and Andrade e Silva.[7]

> "For us, according to classical concepts, a particle is a very small object which is constantly localized in space, and a wave is a physical process which is propagated in space in the course of time according to

a given equation of propagation. This wave, we call the wave v, must be clearly distinguished from the statistical wave ψ, which is arbitrarily normalized in current quantum mechanics. This wave v has a very low amplitude and does not carry energy, at least not in a noticeable manner. The particle is a very small zone of highly concentrated energy incorporated in the wave, in which it constitutes a sort of generally mobile singularity. By reason of this incorporation of the particle in the wave, the particle possesses an internal vibration which, as it moves, remains constantly in phase with the vibration of its wave. In our former papers we have shown that the 'mean' path of the particle is determined according to the shape of the wave by a certain 'guidance law'."

This is a most suggestive passage, if we accept their interpretation. The immediate question it raises is whether or not the electromagnetic wave can be identified with the wave called v by them. The question does not seem important for vacuum or an isotropic medium, since wave and energy transport are collinear. In describing propagation in an anisotropic medium, we shall find that the question is hardly trivial.

4. PROPAGATION IN ANISOTROPIC MEDIA: BREAKDOWN AND REESTABLISHMENT OF LORENTZ INVARIANCE

The foundation of the argument of this paper has been the parallelism of wave propagation and energy transport. From the derivation in Section 2, it is clear that this parallelism is a necessary condition for the Lorentz invariance of the de Broglie relations. Satisfying this condition for particles is not a problem, since de Broglie built it into his very definition of matter waves. The wave is a phase wave defined as accompanying the particle and, of necessity, parallel with it. The situation becomes more complicated, though, when we deal with light. For propagation in free space as well as in isotropic media, parallelism between propagation direction and energy transport holds; therefore Eqs. (1) and (2) are valid in all inertial frames. We speak of a light quantum or photon and the electromagnetic wave. Applying de Broglie's terminology, we have looked on the electromagnetic wave for almost sixty years as the phase wave that accompanies or guides the photon. Yet we have not really defined wave or photon in the process. In many ways the result is a curious pastiche of classical electromagnetics and early quantum physics. It certainly does not embody the clearly developed relations of wave mechanics. However, as long as the parallelism holds, there seems no need of further concepts; indeed, adding them would violate the principle of William of Occam's razor.

Complications arise when light propagates in an anisotropic medium. In such a medium, of which many crystals and plasmas are examples, the energy transport is not parallel to the wave normal or vector. Born and Wolf[9] discuss this phenomenon at length. The momentum $\underset{\sim}{p}$ of the radiation is not parallel to the wave vector $\underset{\sim}{k}$. Therefore Eq. (2) is no longer valid. If Eq. (2) is not true, we are forced to conclude that the de Broglie relations are not universal and, equally disturbing, that their covariance is restricted. The second problem is that wave and particle no longer coincide. *If we associate the light quantum or* particle with energy transport (for energy is a particle concept), we must conclude that the light quantum is not accompanied by a phase wave. The classical wave of the electromagnetic field seems inadequate to play the role demanded. Therefore, wave-particle duality is no longer universally true.

This is now our problem. The de Broglie relations are not Lorentz invariant; wave-particle duality is not universal. These are the inescapable conclusions to be drawn from the facts of propagation in anisotropic media. Both are patently uncomfortable propositions, at odds with the ideas underlying modern physics since the early nineteen twenties. In this paper, I wish to suggest a way to resolve the dilemma, a way that will preserve both Lorentz invariance and wave-particle duality. *The proposed resolution results from examining the problem in* the light of the ideas of de Broglie and Andrade e Silva[7] and insisting on the necessity *of general covariance of the relations.*

Given the fact of energy transport, we associate a light quantum or photon with energy. The photon has energy E and moves with velocity $\underset{\sim}{u}$, the group velocity of the electromagnetic wave. We seek a phase wave, distinct from the electromagnetic wave described by k, to accompany or guide the photon. To describe this wave we introduce a new propagation 3-vector $\underset{\sim}{\ell}$, parallel to the momentum and different from $\underset{\sim}{k}$. (The details of the introduction of $\underset{\sim}{\ell}$ and the associated 4-vector L are given in Appendix A.) This new wave vector $\underset{\sim}{\ell}$ is the descriptor of a phase wave propagated in the direction of energy transport with phase velocity w (equal to c^2/u). This wave is not the same as the electromagnetic waves describing the electromagnetic field, waves which are functions of $\underset{\sim}{k}$ and ω, rather than $\underset{\sim}{\ell}$ and ω. As de Broglie and Andrade e Silva wrote, we consider the particle to be a very small zone of highly concentrated energy incorporated in and enveloped by the wave. The wave itself acts as a pilot or guide for the particle.

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9. Born, M., and Wolf, E. (1964) Principles of Optics, 2nd Rev. Edition, 1969, Chap. 14, pp. 665-718, Pergamon Press, Reading, Mass.

At this point we have a propagation theory comprising three distinct parts: a light quantum, an electromagnetic wave, and a phase wave associated with the quantum and distinct from the electromagnetic wave. The quantum has energy E which moves with velocity u, which we can call a group velocity. The phase wave is characterized by a wave vector $\underset{\sim}{\ell}$ and a phase velocity w, such that the product of u and w is always c^2,

$$u\,w = c^2 \quad . \tag{9}$$

The electromagnetic wave is characterized by a wave vector $\underset{\sim}{k}$ and velocity v. (For a pictorial description of the various velocities and wave vectors, see Figure 1.)

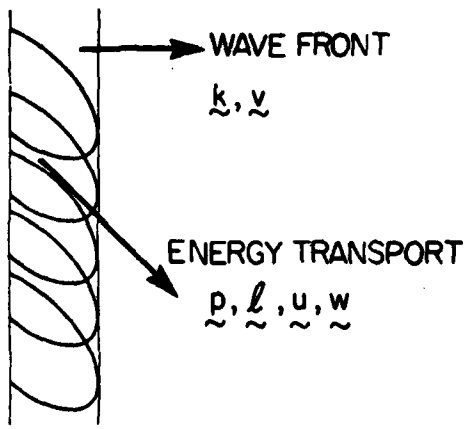

Figure 1. Propagation in an Anisotropic Medium. The phase velocity of the electromagnetic wave is v, the velocity of energy transport is $\underset{\sim}{u}$, and the phase velocity introduced in this paper is w. The wave vector of the electromagnetic wave is $\underset{\sim}{k}$. The photon momentum is $\underset{\sim}{p}$. The new wave vector introduced in this paper is $\underset{\sim}{\ell}$

For propagation in vacuo, the two waves are clearly degenerate. The two wave vectors, $\underset{\sim}{\ell}$ and $\underset{\sim}{k}$, are parallel, and all three velocities, u, w, and v, are equal to c. Introduction of the phase wave ℓ would be redundant since it would be identical with the electromagnetic wave. For propagation in an anisotropic medium, the difference between the two waves is clear; only through the phase wave

can we preserve covariance. For propagation through a material isotropic medium, the situation is less clear. The two waves do coincide in space, so that there exists a spatial degeneracy. However the phase velocity w of the phase wave is always $\geq c$, whereas the phase velocity of the electromagnetic wave is almost always $< c$. (It can sometimes exceed c for x-rays and propagation in plasmas.) This, of course, implies that the phase velocity w of the phase wave has an associated index of refraction in solids which differs from that associated with v. For this index we use the ray index of refraction N_R defined in crystal optics as

$$N_R = c/u \quad . \tag{10}$$

(See, for example, Born and Wolf.[9]) Since u, the energy velocity, can never exceed c, N_R is always ≥ 1. Therefore, the phase velocity w of the wave function may be written as

$$w = \omega/\ell = c^2/u = N_R c \geq c \quad . \tag{11}$$

These results are not surprising when one considers that only the group or energy velocity (not the phase velocity) must obey the Einstein velocity addition theorem. Therefore it is the energy velocity to which we look when we seek to generalize our relations. Our measurements, after all, always require energy transfer from wave or particle to the detector. Ko and Chuang[10] have made this point in discussing flux in a moving dispersive dielectric.

The final point is that it is the index associated with ℓ, rather than k, that we must use in Fermat's theorem to determine the ray in an anisotropic medium. We apply the theorem in the form

$$\int_{P_1}^{P_2} N_R \, ds \quad , \tag{12}$$

and determine the path which makes the integral an extremum.

10. Ko, H. C., and Chuang, C. W. (1977) On the energy flux and Poynting vector in a moving dispersive dielectric, Radio Science 12:337.

5. EXPERIMENT TO VERIFY THE PHASE WAVE

If the concept of the phase wave is to have meaning, one should be able to test it by experiment. As long as the phase wave (ℓ-wave) and electromagnetic wave (k-wave) coincide in space, as discussed in Section 3, the phase wave is an unnecessary complication. However, when propagating in anisotropic materials, they are physically separated in both velocity and direction. This fact suggests an experiment to test the reality of the phase wave.

Given a wave, any wave, physicists have nearly always turned to phenomena of interference and diffraction for existence proofs. For example, the wave aspect of particles with nonzero rest mass was first shown by crystal diffraction of the phase wave. All subsequent work with matter has shown that the phase waves are responsible for the wave phenomena. This suggests that we reconsider the interference of light. Is the quantal phase wave rather than the classical electromagnetic wave the true cause of wave phenomena of light? My contention is that the phase wave is indeed the cause. For example, consider interference in a Young two-slit experiment evidenced by the darkening of a photographic film. The darkening occurs when photons arrive at the film and interact with the silver halide grains. The photons are the energy carriers. The interference occurs with waves whose direction is coincident with that of the energy transport.

Let us apply this reasoning to the anisotropic crystal and carry out a two-slit experiment in such a medium. The arrangement of slits and matter is shown in Figure 2. The crystal is cut into three pieces. A single slit is cemented between I and II at a position normal to the phase wave. A double slit, parallel to the single slit, is cemented between II and III. The arrangement is such that the electromagnetic wave cannot enter pieces II and III. One may even absorb it by using suitable slit material. Film placed on the rear side of III will record the arrival of light energy.

My prediction is that this experiment will exhibit interference, interference originating in the passage of the ℓ-phase wave through the double slit. The evidence of the interference is the blackening of the silver halide grains in the film caused by the arrival of light energy. According to our hypothesis, it is the ℓ-phase wave, not the k-electromagnetic wave, that accompanies energy transport in anisotropic media. Indeed the k-wave travels in a different direction. Therefore the appearance of interference under these conditions would verify the independent existence of the ℓ-phase wave. Moreover it would confirm de Broglie's view of the phase wave as a pilot or guiding wave. Demonstration of physical reality for the light-phase wave would establish even closer correspondence between light and matter, since it would mean their wave natures have the same form.

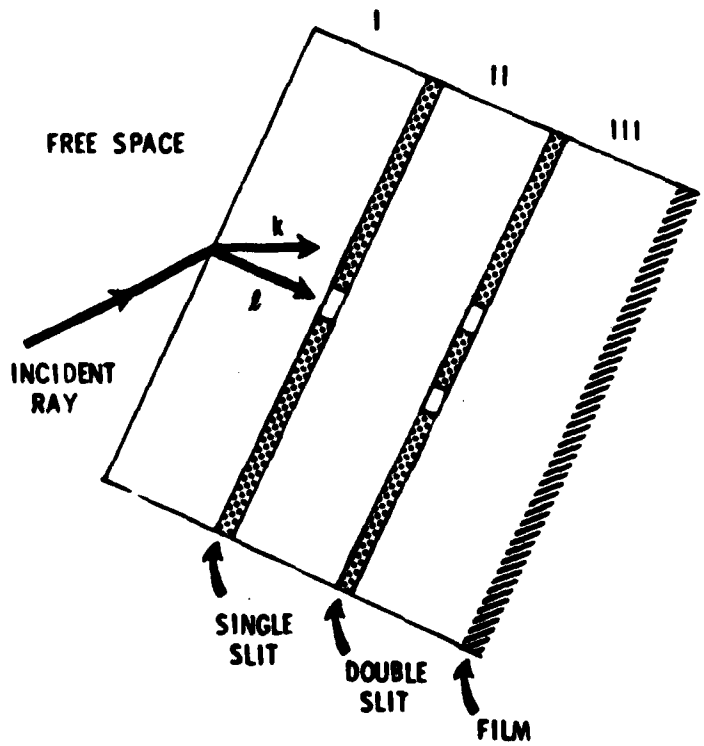

Figure 2. Proposed Young Experiment to Verify the ℓ-Phase Wave. The anisotropic crystal is divided into three parts, with a simple slit between I and II and a double slit between II and III

6. CONCLUSIONS

We started with a remarkably simple derivation of de Broglie's relations. The derivation underlined their covariant nature. When light propagates in anisotropic media, it s clear that the relations, as we know them, break down. This forces us to the conclusion that the covariance of the relations is restricted in the presence of matter. However, de Broglie's method of obtaining matter waves suggests the existence of a light wave associated with the light quantum and distinct from the elec romagnetic wave. For propagation in vacuo, the two waves are completely degen rate; for propagation in a material isotropic medium, they coincide spatially but travel with different phase velocities. The result of introducing this new light or phase wave is the reestablishment of the universality of de Broglie's relations and their covariance. Most important is the assertion of

the covariance of these laws in the presence of matter. Following Møller,[11] we can write a law of nature as

$$F\left(A, B, \ldots; \frac{\delta A}{\delta x_i}, \frac{\delta B}{\delta x_k}, \ldots\right) = 0 \quad , \tag{13}$$

where A, B, ... are a set of physical quantities measured in an inertial frame Σ. In a frame Σ', the law would be expressed as

$$F\left(A', B', \ldots; \frac{\delta A'}{\delta x_i'}, \frac{\delta B'}{\delta x_k'}, \ldots\right) = 0 \quad . \tag{14}$$

From the results of this paper, A, B, ... include not only so-called field quantities but those quantities which represent matter at the point of observation, not separated at a distance. As such our covariant view of nature is enlarged.

Our insistence on a wave accompanying the photon that, in anisotropic matter, is clearly different from the classical electromagnetic wave, is of great importance. Its close relation to energy transport is particularly satisfying, since it is only from the evidence of energy arriving at our detectors that we make our far-reaching inference concerning wave phenomena. To demonstrate the existence of this phase wave, we have proposed an experiment to show phase wave interference. A positive result from this experiment would also tend to buttress de Broglie's hypothesis of the pilot or guiding wave for transport of a quantum.

There is, of course, an element of speculation in these ideas. Yet they have an appealing simplicity to them that is very powerful as well; they enable us to preserve both covariance and wave-particle duality. Questions remain. It should be possible to find a closer relation between this new wave and the classical electromagnetic wave. As of now, this relation is still obscure. Equally important, what is the wave equation to which this new wave is the solution? In other words, what is the quantum wave equation for the photon? To answer these questions goes beyond the scope of this paper. Our purpose was to state the fundamental problems exposed by propagation in anisotropic crystals and, following de Broglie and Andrade e Silva, suggest a possible solution. It now remains to develop the implications contained in this solution.

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Appendix A

Physical Meaning of the Phase Wave

In order to preserve the covariance of the de Broglie relations, we assume that a propagation 3-vector $\underset{\sim}{\ell}$ does exist, parallel to the 3-momentum and different from $\underset{\sim}{k}$ in both magnitude and direction. The defining equation for $\underset{\sim}{\ell}$ is

$$\underset{\sim}{p} = \hbar \underset{\sim}{\ell} \quad . \tag{A1}$$

Taking $i\omega/c$ as the temporal component, we form from ℓ a new 4-vector L, equivalent to $(\ell, i\omega/c)$. This new 4-vector may be written in terms of L as

$$\underset{\sim}{P} = \hbar \underset{\sim}{L} \quad . \tag{A2}$$

Before dealing with the physical meaning of L and ℓ, let us examine their transformation properties. If we consider two inertial frames Σ and Σ' such that Σ' moves with relative velocity V along the x-axis with respect to Σ, the standard Lorentz transformations give the relations for $\underset{\sim}{p}$, E, $\underset{\sim}{\ell}$, and ω in the two frames. These transformations are

$$\begin{aligned} p'_x &= \gamma(p_x - VE/c^2) \\ p'_y &= p_y \\ p'_z &= p_z \\ E' &= \gamma(E - p_x V) \end{aligned} \tag{A3}$$

and

$$\begin{aligned} \ell'_x &= \gamma(\ell_x - V\omega/c^2) \\ \ell'_y &= \ell_y \\ \ell'_z &= \ell_z \\ \omega' &= \gamma(\omega - \ell_x V) \quad , \end{aligned} \tag{A4}$$

where γ equals $(1 - V^2/c^2)^{-1/2}$ and c is the velocity of light in vacuo.

To determine the physical significance of $\underset{\sim}{\ell}$, let us introduce a quantity w defined as a vector in the direction of $\underset{\sim}{\ell}$ with the magnitude $\omega/|\ell|$. Its dimensions are those of a velocity. There is a second velocity v that is the velocity of energy transport and in the direction of $\underset{\sim}{p}$; therefore, by definition, it is also in the direction of $\underset{\sim}{\ell}$. We have done precisely what de Broglie did in treating an electron. To a particle of velocity u moving in a given direction, he ascribed a plane wave propagating in the same direction with phase velocity c^2/u, which we call w. We conclude that the vector $\underset{\sim}{\ell}$ defined in Eq. (9) is the wave vector for a plane wave with phase velocity w (equal to c^2/u) propagating in the direction of energy transport. This wave is not the same as the electromagnetic waves describing the electric and magnetic field vectors, waves which are functions of $\underset{\sim}{k}$ and ω, rather than $\underset{\sim}{\ell}$ and ω. This new wave is one we associate directly with the light quantum. It is the guiding wave in the sense of de Broglie and Andrade e Silva.[7]

Appendix B

Relation of Photon Momentum to the Minkowski and Abraham Formulations

It is quite obvious that propagation in any medium is far more complicated than that in free space. Though not immediately the concern of this paper, the disagreement between the Minkowski and Abraham formulations for the electromagnetic momentum in a *medium is related.* Peierls[B1, B2] has examined the problem in depth. His general conclusion is that Abraham's expression gives the part of the momentum associated with the electromagnetic field but excludes that part carried by matter. The Minkowski expression, according to Peierls, gives a quantity known as pseudo-momentum that is useful in describing the invariance of physical laws under displacement of all physical parameters from one point of the medium to another. The momentum associated with the photon, and thus used in the defining equation for $\underline{L}$, is that given by the Abraham expression.

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