AD-A165 515 FINAL SUMMARY REPORT FOR CONTRACT NUMBER N00014-83-K-0624 FOR THE PERIOD (U) VIRGINIA UNIV CHARLOTTESVILLE DEPT OF SYSTEMS ENGINEERING C M HARRIS 1/1

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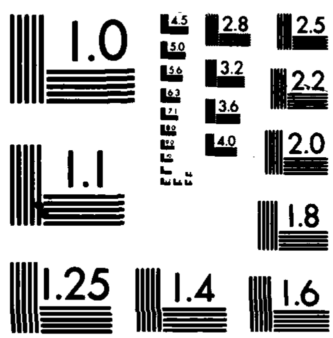

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FINAL SUMMARY REPORT

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for the period September 1, 1983 - September 30, 1985

Submitted by:

Carl M. Harris

January 1986

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1. GENERAL COMMENTS AND SUMMARY OF ACCOMPLISHMENTS

1.1:

It has often been noted in the literature of applied probability (for example, see Gross and Harris, 1985) that a lack of appropriate solution procedures frequently prevents the application of major theoretical advances. In the study of complex large-scale systems, analysis aimed at obtaining exact answers becomes especially ineffective. Typically, the opportunity to study real systems depends upon the ready access to either closed-form answers or computer-based numerical routines. There has indeed been some important progress made in implementation but they are only the initial steps.

Various numerical procedures and approximation techniques have, ~~in fact,~~ been used over the years for deriving information on the major performance measures of stochastic systems. But, at this stage, the major point is that it is now realized that the ready availability of inexpensive and friendly computing is making it a lot easier to bring the theory much more into the mainstream of real-problem solution methods.

We have had a particular interest in providing an assessment of the impact that recent advances in numerical procedures and their computer implementation may have on the effective application of probability methods in the modeling of dynamic, large-scale systems.

Of special concern are three key classes of applications, namely, the operational modeling of large maintained systems of complex engineering equipments; stochastic models of network flows with potential queueing before processing; and special types of inventory models with random re-order leadtimes and replenishment processes. All of these three modeling environments require the wide-spread use of stochastic processes of a fairly general nature.

Queueing models are common in these problem areas, but their effective incorporation into the study of maintenance, for example, has not been very well exploited to date. The modeling of preventive maintenance and its interaction with inventory stocks and generalized logistics flows leads to very complex numerical problems. This becomes even truer when the systems are allowed to be large and possibly time varying. Further complications include non-constant and multiple service providers, non-Markovian probability structures, and large state-space problems, for example.

It is in the area of queueing where we have probably seen the most progress to date on the use of numerical approximations. We make special note here of the recent work of Neuts (major portions of which are documented in his 1981 book and the referenced 1975 paper). He has coined the phrase "computational probability" to describe these directions of research, defined as the study of stochastic models with special attention to algorithmic feasibilty over a realistic range of parameter values. Professor Neuts is able to avoid transform solutions and complex variable methods for prime classes of problems by the creative use of iterative (computer-based) matrix methods. But, it should be clear that such methods are still very heavy on computations. The obstacles to reasonable solution using Neuts-type arguments are often formidable, though the effective use of contemporary computer technology has made them quite attractive for models of dynamic systems under uncertainty. However, the very nature of the underlying complexity of such models will often diminish their usefulness, particularly since it may be impossible to estimate population parameters in any practical way. A major direction of our research has been to find related computationally feasible methods, for which statistical inference procedures are easier to develop.

The numerical analysis of queueing systems carried out by Neuts, using phase-type distributions, can be identified as an important approximating technique. The incorporation of these distributions can be viewed as a system of approximation, in that they may be used for approximating interarrival and service times. Our generalized hyperexponential (GH) distributions are an important relative of the phase types (see Harris and Sykes, 1985), and their use in single-server queueing systems is well documented in Harris (1985).

The distributions are linear combinations of exponential CDFs with mixing parameters (positive and negative) that sum to unity. The denseness of the class GH with respect to the class of all CDFs defined on $(0,\infty)$ has been established in Botta and Harris (1985) by showing that a GH distribution can be found that is as close as desired, with respect to a suitably defined metric, to a given CDF. The metric induces the usual topology of weak convergence so that, equivalently, there exists a sequence (G_n) of GH CDFs that converges weakly to any CDF. The result follows from a similar well-known result for weak convergence of Erlang mixtures. Various set inclusion relations have also been obtained relating the GH distributions to other commonly used clases of approximating distributions including generalized Erlang, mixed generalized Erlang, those with reciprocal polynomial Laplace transforms, those with rational Laplace transforms, and phase-type distributions.

The nonlinear optimization routine we have developed for the likelihood estimation of the GH parameters is described in Harris and Sykes (1985). Earlier papers by Kaylan and Harris (1981) and Mandelbaum and Harris (1982) provide an interesting history of the evolution of the algorithm and its original application to mixtures of exponential and Weibull densities.

Validation is also an integral part of approximation procedues.

It is needed to support the applicability of a technique and the reliability of results. An applied scientist has constantly to evaluate the trade-off between the ease of application of a particular technique and the accuracy of the ensuing results. Experimentation is the most common validation technique for approximations, and a very common form of experimentation is digital simulation. Most authors have estimated approximation accuracy by the relative size of the percentage difference between simulated and approximate results. Very few of them have resorted to a statistical analysis of simulated results and provide information such as confidence bounds on their estimates. Clearly wider use of key statistical techniques, for point and interval estimation, should be made when simulation is preferred. We note then that some of our recent research has been directed at the development of optimal simulation sampling designs for such computer-based experiments (Harris, 1985b).

Probability researchers have been widely critized for studying systems that are not relevant to the real world. However, it seems that this criticism is largely due to the complexity of available results in the literature rather than due to the system themselves. If one looks at some of the applied areas of queueing, one finds more complex systems than those in the general research and applied probability literature. The distinction is in the nature of analysis. The results found in the applied area literature are applicable, though approximate. A large percentage of results found in the general literature is less useful, though rigorous. Therefore if we want the major methods of applied probability to be important problem-solving tools, then approximation techniques should be put to increasing use whenever necessary and possible. The trend over the past decade or so has been in this direction, and there is every reason to believe that this pattern is going to continue further in the coming years, bringing more reliability and applicability for the techniques used.

Additional project support was provided by visiting Research Fellow Martin Krakowski. His work emphasized some specific aspects of modeling complex problems in optimal system design and reliability under uncertainty.

Krakowski's work on the relevation and related transforms fits very naturally into the general theme of the project. The relevation transform generates an entropy-like expression which appears to be useful in predicting changes in longevity for systems affected by aging or life-improving treatments. His work had a special focus on approximations and numerical problems, to match up with the overall problem and application thrust of the rest of the project.

1.2:

As noted, the use of linear (not necessarily convex) combinations of exponential distributions as approximants permits the relatively easy solution of many problems. We have since learned that such densities are in fact dense in the class of all probability distributions and that some of our previous work on parameter estimation for Weibull mixtures can be adjusted for the nonconvex combination of negative exponential distribution. Two papers were prepared on the problem in project year 1. The first one documented a Jacobi-like nonlinear optimization algorithm for the maximum-likelihood estimation of parameters of generalized mixed exponential distributions. The second manuscript provided an important application of this comprehensive family of probability functions to a specific class of problems frequently encountered in maintenance, queueing and network systems.

The MLE algorithm documented in Harris and Sykes obtains one answer from among a potentially large set of local solutions. This work details the development and testing of a practical procedure for the MLE of the parameters of generalized exponential mixtures. The algorithm offered for the problem guarantees that the result will provide a legitimate probability function of the prescribed type. Extensive testing has been performed and results are very favorable: convergence is rapid and the use of computer resources rather limited.

A revision version of the original project technical report number UVA/525393/SE84/102 has been accepted for publication in the Naval Research Logistics Quarterly and should be appearing around the middle of 1986.

In the second of these papers (Harris, 1985), an effective way was presented for the estimation of the stationery waiting time distribution of the general single-server queueing model GI/G/1. The approach relies on obtaining a generalized exponential mixture as an approximation for the distribution of the service times. This is done using the Harris/Sykes nonlinear optimization algorithm as discussed above. The method is very well suited for obtaining the delay distribution beginning from raw interarrival and service - time data.

In project year 2, two additional papers were prepared on this general subject. The first of these (by Botta and Harris) addressed the denseness of GH distributions in the class of all CDFs, in addition to providing some special characterizations of the GHs. The second 1985 paper was concerned with the construction of optimium statistic simulation experimental designs.

In Botta and Harris, the denseness of the class GH with respect to the class of all CDFs defined on $[0,\infty)$ is established by showing that a GH distribution can be found that was as close as desired, with respect to a suitably defined metric, to a given CDF.

The metric induces the usual topology of weak convergence so that, equivalently, there exists a sequence (G_n) of GH CDFs that converges weakly to any CDF. The result follows from a similar well-known result for weak convergence of Erlang mixtures. Various set inclusion relations were also obtained relating the GH distributions to other commonly used classes of approximating distributions including generalized Erlang (GE), mixed generalized Erland (MGE), those with reciprocal polynomial Laplace transforms (K_n), those with rational Laplace transforms (R_n), and phase-type (PH) distributions. A brief survey of the history and use of approximating distributions in queueing theory was also included.

In the second paper, by Harris, there is a concern for the construction of optimum model-sampling simulation experimental designs. The accurate estimation of the distribution of any random variable by computer is almost always difficult, especially when reasonable accuracy is desired for its tail probabilities. If, in addition, each element of the computer generated pseudo-random sequence is itself the result of a stochastic limit or possibly a functional of a continuous-time process, it becomes most complicated to assess the final statistics.

In this paper, we focused on the estimation of tail probabilities for the distribution function of the maximum on the unit interval of a continuous-time Wiener process approximated as a multi-variate normal of increasing dimension. We critiqued recent approaches to sample-size determination for such distribution-sampling problems and built on insights from probability theory to find more reasonable run sizes.

1.2.1 The following manuscripts were completed:

Harris, C.M., and Sykes, E., "Maximum Likelihood Estimation for Generalized Mixed Exponential Distributions," Report No. UVA/525393/SE/84/102, accepted for publication in Naval Research Logistics Quarterly, expected to appear mid-1986.

Harris, C.M., "A Note on Mixed Expoential Approximations for GI/G/1 Waiting Times," Report No. UVA/525393SE/84/101, published in Computers and Operations Research, Volume 12, No. 3, pp. 285-289, 1985.

Botta, R.F. and Harris, C.M., "Characterizations of Generalized Hyperexponential Distributions," Report No. UVA/525393/SE85|107, accepted for publication in Queueing Systems: Theory and Applications, expected to appear mid - 1986.

Harris, C.M., "Comments on Estimating Quantiles for Gaussian Functionals by Simulation," Technical Report No. UVA|525393|SE85|109, currently pending review with Journal of Statistical Computation and Simulation.

1.2.2 The following talks were given:

Harris, C.M., "Likelihood Estimation of the Parameters of Generalized Exponential Mixtures," Fall 1983 Meeting of ORSA/TIMS, Orlando, Florida, 7 November 1983.

Harris, C.M., "Likelihood Estimation of the Parameters of Generalized Exponential Mixtures," Department of Computer Systems and Production Management, University of Toledo, 26 January 1984.

Harris, C.M., "Mixtures of Discrete Distributions," Department of Computer Systems and Production Management, University of Toledo, 27 January 1984.

Harris, C.M., "Mixed Exponential Approximations for GI/G/1 Waiting Times," Operations Research and Systems Analysis Curriculum, University of North Carolina at Chapel Hill, 5 April 1984.

Harris, C.M., "Mixed Exponential Approximations for GI/G/1 Waiting Times," Operations Research Seminar, University of Delaware, 27 April 1984.

Harris, C.M., "The Modeling and Application of Mixtures of Exponential Distributions," to Department of Mathematical Sciences, The College of William and Mary, Williamsburg, Virginia, 26 October 1984.

Harris, C.M., "The Modeling and Application of Mixtures of Exponential Distributions," to Department of Mathematical Sciences, The Johns Hopkins University, Baltimore, Maryland, 7 December 1984.

Harris, C.M., "The Modeling and Application of Mixtures of Exponential Distributions," to jointly sponsored seminar of the Departments of Industrial Engineering and Mathematical Sciences, Clemson University, Clemson, South Carolina, 20 February 1985.

Harris, C.M., and Sykes, E.A., "Likelihood Estimation for Generalized Mixed Exponential Distributions," Spring 1985 Meeting of ORSA/TIMS, Boston, Massachusetts, invited session sponsored by ORSA/TIMS Applied Probability Group, 29 April 1985.

1.3:

As a result of our early research in year 1, we decided to move up some work proposed for Task C. Of particular interest, there was the unit on the solution of multi-dimensional difference equations. In the proposal we noted that this problem was closely related to some of those in Task A in the sense that the usual solution to a stochastic model with a multivariate state space was presented as a transform. So we have worked on developing a strategy to convert multi-dimensional problems down to much simpler ones.

The primary application for us of multi-dimensional difference equations has been the steady-state solution of networks of Markovian queues. A very important problem that has been of special concern is the solution of queues with feedback and the estimation of their retrial parameter.

1.3.1 The following manuscript was completed:

Harris, C.M., and Hoffman, K.L., "Estimation of a Caller Retrial Rate in Communication Networks with Feedback," accepted for publication in the European Journal of Operational Research, to appear in August 1986.

1.3.2 The following talk was given:

Harris, C.M., on "Queueing Analysis of Communication Networks with Overflow and Retrial," George Mason University Engineering Seminar, 28 November 1983.

1.4:

Throughout the contracted period Martin Krakowski's work has been concrned with the joint use of ergodic variables and of omni-transforms (both terms are ad hoc designations) to the problems of stationary random processes. The essence of the conservation method as applied to stochastic models consists of the selection of one or more random variables each of which has an expected change of zero over any arbitrary interval (which can be taken to be finite, very long or infinitesimal); we refer to such a variable as an ergodic variable. The omni-transform of a random variable Y (in particular Y can be an ergodic variable) is the expectation of an arbitrary (though well enough behaved to permit differentiation or any other needed formal operations) function of Y. Simple as the notion of the omni-transform is, its consequence and wide reach of applications are surprising; complex theorems are at times replaced by short and incisive arguments. The omni-transform is applicable to models other than queueing, e.g. to some models treated by Feller in his Introduction to Probability Theory, Volume 1. The two notions, ergodic variables and omni-transform, go well together because if Y is an ergodic variable, then so is an arbitrary function of Y. The conservation approach to single-server queues, in particular to the M/G/1 queueing model, combines these two notions. So does the treatment of the renewal model, which has a bearing on some aspects of the G/G/1 model.

1.4.1 The following manuscripts were completed:

Krakowski, Martin, "Omni Transforms: Applications to Renewal Theory," Report No. UVA/5253/SE84/103.

Krakowski, M., "The Omni-Transform in Single-Channel Queues," Technical Report No. UVA/525393/SE85/106, accepted for publication in the _Annals of Operations Research_, expected to appear in late 1986.

1.4.2 The following talks were given:

Krakowski, Martin, On "Quick and Clean Methods in Queueing Theory," Department of Systems Engineering, University of Virginia, 17 February 1984.

Krakowski, Martin, "The Omni-Trasform in Single-Channel Queues," invited talk at ORSA/TIMS Conference on Statistical and Computational Problems in Probability Modeling, Williamsburg, VA 9 January 1985.

2. REFERENCES

Botta, R. F. and C. M. Harris, "Characteristics of Generalized Hyperexponential Distributions," ONR Project Report No. UVA/525393/SE85/107, Department of Systems Engineering, Univ. of Virginia, Charlottesville (a portion to appear in Queueing Systems).

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Harris, C. M. (1985a). "Mixed Exponential Approximations for GI/Gl Waiting Times," Computers and Operations Research 12 285-289.

Harris, C. M. (1985b). "Determination of Optimal Simulation Run Sizes: A Brownian Motion Example," ONR Project Report No. UVA/525/393/SE85/105, Department of Systems Enginering, Univ. of Virginia, Charlottesville (pending review with Journal of Statistical Computation and Simulation).

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Krakowski, M. (1985). "The Omni Transform and Its Application to Renewal Theory," ONR Project Report No. UVA/525393/SE84/103, Department of Systems Engineering, Univ. of Virginia, Charlottesville (to appear in Annals of OR).

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