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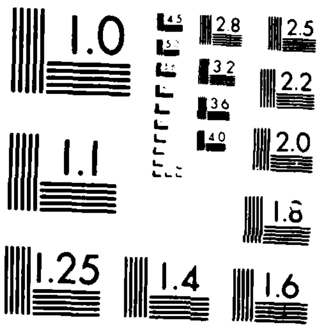

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

SOME NAVIGATION AND ALMANAC ALGORITHMS

REX H. SHUDDE

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This report presents a detailed discussion of the use of the general spherical triangle as a means of eliminating the many special cases of quadrant determination that plague most navigation computations and steal memory from computer programs. Also included are two programs, NAVALGOR and NAVEPHM which are written for the Radio Shack TRS-80 Model 4 Computer.

NAVALGOR implements three sets of computational procedures: direct solution algorithms for speroid and spherical earth models, inverse solution algorithms for spheroid and spherical earth models, and rhumb line approximations to great circle routes.

NAVEPHM implements a procedure for the determination of the position of the Sun, Moon and navigational planets that reproduce "The Nautical Almanac" and "The Air Almanac" to within 0.2'.

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CONTENTS

I. INTRODUCTION.

This report presents some navigation and almanac algorithms with implementation programs that are written in the Radio Shack TRS-80 MODEL 4 BASIC language. This language can be customised for the Navy adopted Sharp PC-1500A or other BASIC language computers.

Because the PC-1500A has limited memory, many of the classical navigation formulas from spherical trigonometry have been rewritten in a form which minimises or eliminates the need to determine special cases—especially those of quadrant. Details are contained in Section II.

The navigation algorithms are detailed in the BASIC program NAVALGOR, presented in Section III. Programs for the determination of the position of the Sun, Moon, Venus, Mars, Jupiter and Saturn are contained in the BASIC program NAVEPHM which is presented in Section IV. These programs reproduce the tables in *The Nautical Almanac* and *The Air Almanac* to within 0.2′.

II. THE GENERAL SPHERICAL TRIANGLE AND COORDINATE TRANSFORMATIONS.

A. Background. Many of the algorithms and procedures used for navigational computations were designed for use with tables of sines, cosines and tangents. To save space and eliminate repetition, these tables usually contained only values for the first quadrant. To generate values for other quadrants, many cumbersome rules were adopted. These rules—contained in classical references such as Bowditch [Ref. 1]—are promulgated in modern computer programs, causing needlessly additional programming effort. This additional effort can largely be eliminated by using the concept of the general spherical triangle which is described in the next section.

B. Theory. The formulas for the general spherical triangle expounded on by Wm. Chauvenet [Ref. 3] in his book, *A Treatise on Plane and Spherical Trigonometry*, which was first published in 1850. One usually thinks of a spherical triangle as looking something like the object depicted in Figure 1, where the arc length of each leg is less than 180°. Chauvenet's general triangle does not have the 180° limitation and so objects similar to the one shown in Figure 2 are also considered to be spherical triangles. In Figure 2, the crosspoint P is 180° from the vertex of angle A. In labeling spherical triangles it is important that an

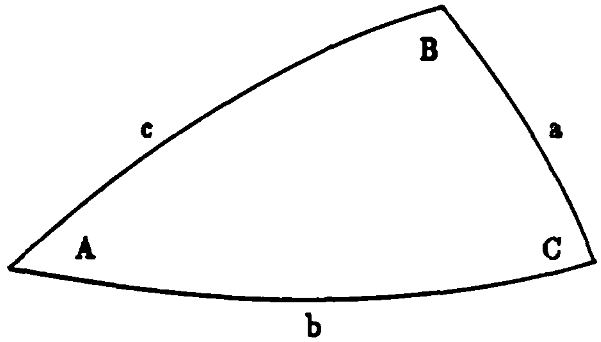

Figure 1. A Spherical Triangle

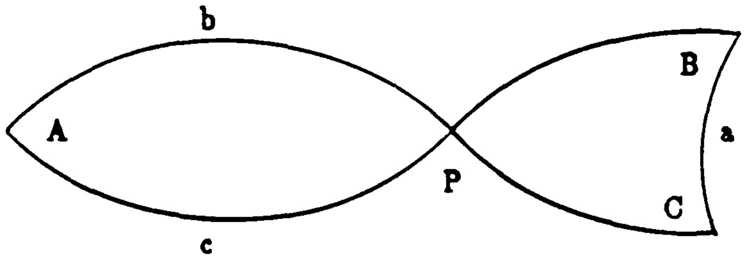

Figure 2. A General Spherical Triangle

angle be labeled with a capital letter and that the opposite side be labeled with the same, but lower case letter.

Depending upon the quantities to be determined, either the set of equations

$$\begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c \cos A, \\ \sin a \cos B &= \cos b \sin c - \sin b \cos c \cos A, \\ \sin a \sin B &= \sin b \sin A, \end{aligned} \tag{1}$$

or the polar form of Equs. (1)

$$\begin{aligned} \cos A &= -\cos B \cos C + \sin B \sin C \cos a, \\ \sin A \cos b &= \cos B \sin C + \sin B \cos C \cos a, \\ \sin A \sin b &= \sin B \sin a, \end{aligned} \tag{2}$$

may be used.

Chauvenet stated that, "It will be found that *all the six cases of the general triangle admit two solutions, but that they all become determinate, when, in addition to the other data, the sign of the sine or cosine of one of the parts is given.*"

Formulas for spherical coordinates should be developed so that quantities such as latitude, declination and altitude, which have values in the range $-90°$ to $+90°$, are determined by arcsine or arctangent formulas; so that quantities such as longitude, azimuth, Greenwich hour angle and local hour angle, which have values in the range $-180°$ to $+180°$ or $0°$ to $360°$, are determined by both a sine and a cosine formula used in conjunction with a quadrant determining arctangent function (the qatn function); and so that a quantity such as distance, which has a spherical arc value in the range of $0°$ to $180°$ (assuming the user wishes the shortest distance), is determined by an arccosine formula.

Illustration 1. Consider the "inverse" problem of spherical geometry, which is: Given the latitude, ϕ, and longitude, λ, of two points $P_1(\phi_1, \lambda_1)$ and $P_2(\phi_2, \lambda_2)$, determine the distance d, the forward azimuth α_{12} and the backward azimuth α_{21}. The solutions will be derived using the convention that eastern longitudes are positive and western longitudes are negative. Also, northern latitudes are positive and southern latitudes are negative. The geometry is illustrated in Figure 3.

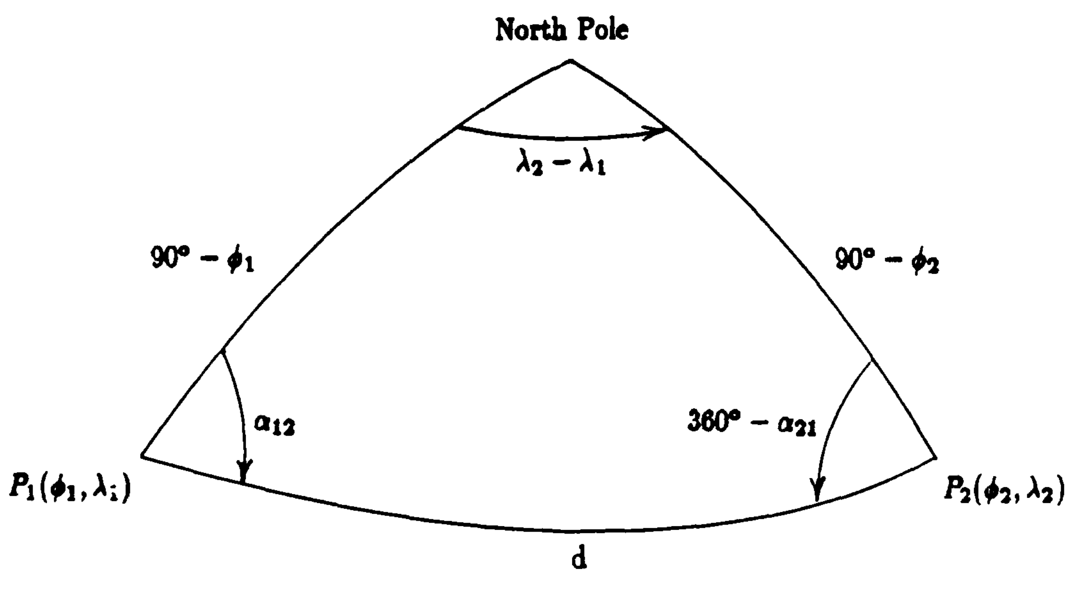

Figure 3. Geometry of the Navigation Triangle.

Solve for d and α_{12}. Label the angles and legs so that $A = \lambda_2 - \lambda_1$, $B = \alpha_{12}$, $C = 360° - \alpha_{21}$, $a = d$, $b = 90° - \phi_2$ and $c = 90° - \phi_1$. Substituting these into Equs. (1) and simplifying the differences of angles, we obtain

$$\begin{aligned}\cos d &= \sin\phi_2 \sin\phi_1 + \cos\phi_2 \cos\phi_1 \cos(\lambda_2 - \lambda_1),\\ \sin d \cos\alpha_{12} &= \sin\phi_2 \cos\phi_1 - \cos\phi_2 \sin\phi_1 \cos(\lambda_2 - \lambda_1),\\ \sin d \sin\alpha_{12} &= \cos\phi_2 \sin(\lambda_2 - \lambda_1).\end{aligned}$$

The first equation can be used to solve for d. Using the principle angle of the arccosine, d lies in the range 0° to 180°. If the non–principle angle is used, d lies in the range 180° to 360°, we can either travel the shortest great circle distance from P_1 to P_2 or we can travel the great circle route which goes around the backside of the earth—the former solution is assumed to be preferred. Hence, d is restricted to lie between 0° and 180°. Multiply d by 60 to get the distance in nautical miles.

Chauvenet states that this system of equations has two solutions, but has only one solution if the sign of the sine or cosine of one of the parts is known. Since d lies between 0° and 180°, the sign of $\sin d$ is known, thus the system has only one solution. Further, and most important, the sign of $\sin d$ is positive for all d between 0° and 180° and so there are no special cases.

The first equation is used to compute d:

$$d = \arccos[\sin\phi_2 \sin\phi_1 + \cos\phi_2 \cos\phi_1 \cos(\lambda_2 - \lambda_1)]. \tag{3}$$

The second and third equations are used to compute α_{12}. A natural tendency would be to solve either the second equation or the third equation, but doing so loses the information concerning the quadrant of α_{12}. Use the second equation to determine $\cos\alpha_{12}$ and the third equation to determine $\sin\alpha_{12}$. With both the sine and the cosine known, the quadrant is uniquely determined using the quadrant determining arctangent function. The solution is

$$\alpha_{12} = \text{qatn}(\sin\alpha_{12}, \cos\alpha_{12})$$

as long as d is not 0° or 360°. Even though the chance of division by zero with the occurrence of a value for d of exactly 0° or exactly 360° is very remote, it can be eliminated entirely. Since the sign of $\sin d$ is positive or zero in both the second and third equations, it effectively cancels out using the qatn function. The azimuth α_{12} is found solving

$$\alpha_{12} = \text{qatn}[\cos\phi_2 \sin(\lambda_2 - \lambda_1), \sin\phi_2 \cos\phi_1 - \cos\phi_2 \sin\phi_1 \cos(\lambda_2 - \lambda_1)]. \tag{4}$$

To find the back azimuth, label the angles and legs so that $A = \lambda_2 - \lambda_1$, $B = 360^\circ - \alpha_{21}$, $C = \alpha_{12}$, $a = d$, $b = 90^\circ - \phi_1$ and $c = 90^\circ - \phi_2$. Substitute these into Equs. (1) and simplify to obtain

$$\alpha_{21} = \mathrm{qatn}[-\cos\phi_1 \sin(\lambda_1 - \lambda_2), \sin\phi_1 \cos\phi_2 - \cos\phi_1 \sin\phi_2 \cos(\lambda_1 - \lambda_2)]. \qquad (5)$$

Illustration 2. Consider the "direct" problem of spherical geometry, which is: Given the latitude, ϕ, and longitude, λ, of a point $P_1(\phi_1, \lambda_1)$, the distance d and the forward azimuth α_{12} to a point $P_2(\phi_2, \lambda_2)$, determine ϕ_2, λ_2 and the backward azimuth α_{21}.

Solve for ϕ_2 and λ_2. Relabel the angles and legs so that $A = \alpha_{12}$, $B = \lambda_1 - \lambda_2$, $C = 360^\circ - \alpha_{12}$, $a = 90^\circ - \phi_2$, $b = d$ and $c = 90^\circ - \phi_1$. Substituting these into Equs. (1) and simplifying the differences of angles, we obtain

$$\begin{aligned} \sin\phi_2 &= \cos d \sin\phi_1 + \sin d \cos\phi_1 \cos\alpha_{12}, \\ \cos\phi_2 \cos(\lambda_2 - \lambda_1) &= \cos d \cos\phi_1 - \sin d \sin\phi_1 \cos\alpha_{12}, \\ \cos\phi_2 \sin(\lambda_2 - \lambda_1) &= \sin d \sin\alpha_{12}. \end{aligned}$$

The first equation can be used to solve for ϕ_2. Using the principle angle of the arcsine, ϕ_2 lies in the range -90° to 90°, which is correct for a latitude. Since ϕ_2 lies between -90° and $+90^\circ$, the sign of $\cos\phi_2$ is known, thus the system has only one solution. Further, and again most important, the sign of $\cos\phi_2$ is positive for all ϕ_2 between -90° and $+90^\circ$ and so there are no special cases.

The first equation is used to compute ϕ_2:

$$\phi_2 = \arcsin(\cos d \sin\phi_1 + \sin d \cos\phi_1 \cos\alpha_{12}). \qquad (6)$$

The second and third equations determine $\cos(\lambda_2 - \lambda_1)$ and $\sin(\lambda_2 - \lambda_1)$. As before, with both the sine and the cosine known, the quadrant is uniquely determined using the qatn function. Since $\cos\phi_2$ is always positive, it can be eliminated in both equations. Then

$$\lambda_2 = \lambda_1 + \mathrm{qatn}(\sin d \sin\alpha_{12}, \cos d \cos\phi_1 - \sin d \sin\phi_1 \cos\alpha_{12}). \qquad (7)$$

The back azimuth can be found by relabeling the angles and legs so that $A = \alpha_{21}$, $B = 360^\circ - \alpha_{21}$, $C = \lambda_1 - \lambda_2$, $a = 90^\circ - \phi_2$, $b = 90^\circ - \phi_1$ and $c = d$. Substituting these into Equs. (1) and simplifying the differences of angles, we obtain

$$\alpha_{21} = \mathrm{qatn}(-\cos\phi_1 \sin\alpha_{12}, \sin\phi_1 \sin d - \cos\phi_1 \cos d \cos\alpha_{12}). \qquad (8)$$

This second formula for the back azimuth is used in the spirit that the unknown parameters should be determined as a function of the known parameters rather than determining the second unknown parameter as a function of the previously evaluated first unknown parameter. That is, any one of the unknowns can be found as a function of the knowns without having to first find one of the other unknowns.

Illustration 3. Although derived from different principles, the equations of coordinate transformation obey the rule of Chauvenet. For example, the equations which transform local hour angle and declination to altitude and azimuth are

$$\begin{aligned} \cos a \sin A &= -\cos\delta \sin h, \\ \cos a \cos A &= \sin\delta\cos\phi - \cos\delta\sin\phi\cos h, \\ \sin a &= \sin\delta\sin\phi + \cos\delta\cos\phi\cos h, \end{aligned} \tag{9}$$

where a = altitude, A = azimuth, δ = declination, h= local hour angle and ϕ = latitude [Ref. 4, pg. 26].

The third equation is used to solve for altitude

$$a = \arcsin(\sin\delta\sin\phi + \cos\delta\cos\phi\cos h). \tag{10}$$

The altitude lies between $-90°$ and $+90°$, and so is correctly determined by the arcsin function. Since $\cos a$ is always positive it can be eliminated between the first two equations when using the qatn function. The local hour angle, which lies in the range 0° to 360°, is determined by

$$A = \text{qatn}(-\cos\delta\sin h, \sin\delta\cos\phi - \cos\delta\sin\phi\cos h). \tag{11}$$

C. Simplification of Standard Works. Using the general triangle and the qatn function, the simplifications listed below can be made in Volume II of Bowditch [Ref. 1]. The various sections are preceded by the section symbol § and equation numbers from Bowditch within those sections are prefixed with the letter B. Unprefixed equation numbers are those contained in this document.

§706. Solving for altitude.—Using the conventions that northern latitudes and declinations are positive and that southern latitudes and declinations are negative, the altitude of a star can be computed directly from Equ. B(2a) or Equ. B(2b). Equ. B(2a) is equivalent to Equ. (10). All of the special cases are thus eliminated.

§707. Solving for azimuth.—Instead of using Equ. B(4b) or Equ. B(5b), use Equ. (11). See §708 below.

§708. **Time azimuth.**—Bowditch uses meridian angles which can be labeled either east or west. Since altitude/azimuth is a left-handed coordinate system, the convention is that west meridian angles are positive and east meridian angles are negative.

Example 1.—The latitude of the observer is 30°25.′0N; the declination of the celestial body is 22°06.′2N; the local hour angle is 39°54.′7W. Using Equ. (11),

$$
\begin{aligned}
A &= \text{qatn}(-\cos 22°06.'2 \sin 39°54.'7, \sin 22°06.'2 \cos 30°25.'0 \\
&\qquad - \cos 22°06.'2 \sin 30°25.'0 \cos 39°54.'7) \\
&= \text{qatn}[-(0.92651)(0.64161), (0.37628)(0.86237) \\
&\qquad - (0.92651)(0.50628)(0.76703)] \\
&= \text{qatn}(-0.59445, 0.32449 - 0.35980) \\
&= \text{qatn}(-0.59445, -0.03531) \\
&= -93.°39913 \\
&= 266.°60087 \\
A &= 266°36.'1.
\end{aligned}
$$

Example 2.—The latitude of the observer is 30°25.′0S; the declination of the celestial body is 22°06.′2N; the local hour angle is 39°54.′7E. Using Equ. (11),

$$
\begin{aligned}
A &= \text{qatn}[-\cos 22°06.'2 \sin(-39°54.'7), \sin 22°06.'2 \cos(-30°25.'0) \\
&\qquad - \cos 22°06.'2 \sin(-30°25.'0) \cos(-39°54.'7)] \\
&= \text{qatn}[-(0.92651)(-0.64161), (0.37628)(0.86237) \\
&\qquad - (0.92651)(-0.50628)(0.76703)] \\
&= \text{qatn}[0.59445, 0.32449 - (-0.35980)] \\
&= \text{qatn}(0.59445, 0.68429) \\
&= 40.°98141 \\
A &= 40°58.'9.
\end{aligned}
$$

D. Spheroid Earth and Great Circle Formulas.

1. Spheroid Earth Formulas. The spheroid earth algorithms are those of Thomas [Ref. 5]. Some changes by Shudde [Ref. 6] make quadrant determination automatic by use of the qatn function. East longitudes and south latitudes are negative. The earth's equatorial radius is denoted by a_e and the earth's flattening factor is denoted by f.

a. Inverse Solution. Input variables are the latitude ϕ_1 and longitude λ_1 of P_1, the latitude ϕ_2 and longitude λ_2 of P_2, ϕ_2 and λ_2. Output variables are distance, (S/a_e), forward asimuth from P_1 to P_2, α_{12}, and back asimuth from P_2 to P_1, α_{21}. All angular input and output is in radians. Compute:

$$
\begin{aligned}
\theta_i &= \tan^{-1}[(1-f)\tan\phi_i], \text{ for } i = 1,2,\\
\Delta\lambda &= \lambda_2 - \lambda_1,\ \theta_m = (\theta_1+\theta_2)/2,\ \Delta\theta_m = (\theta_2-\theta_1)/2,\\
H &= \cos^2\Delta\theta_m - \sin^2\theta_m,\ L = \sin^2\Delta\theta_m + H\sin^2(\Delta\lambda/2),\\
d &= 2\sin^{-1}(L^{1/2}),\ U = 2\sin^2\theta_m\cos^2\Delta\theta_m/(1-L),\\
V &= 2\sin^2\Delta\theta_m\cos^2\theta_m/L,\ X = U+V,\ Y = U-V,\\
T &= d/\sin d,\ D = 4T^2,\ E = 2\cos d,\ A = DE,\\
C &= T-(A-E)/2,\ n_1 = X(A+CX),\\
B &= 2D,\ n_2 = Y(B+EY),\ n_3 = DXY,\\
\delta_1 d &= f(TX-Y)/4,\ \delta_2 d = f^2(n_1-n_2+n_3)/64,\\
S/a_e &= (T-\delta_1 d+\delta_2 d)\sin d,\ M = 32T-(20T-A)X-(B+4)Y,\\
F &= 2Y-E(4-X),\ G = fT/2+f^2M/64,\ Q = -(FG\tan\Delta\lambda)/4,\\
\Delta\lambda'_m &= (\Delta\lambda+Q)/2,\\
t_1 &= \text{qatn}(-\sin\Delta\theta_m\cos\Delta\lambda'_m, \cos\theta_m\sin\Delta\lambda'_m),\\
t_2 &= \text{qatn}(\cos\Delta\theta_m\cos\Delta\lambda'_m, \sin\theta_m\sin\Delta\lambda'_m),\\
\alpha_{12} &= t_1+t_2 \text{ and } \alpha_{21} = t_1-t_2.
\end{aligned}
\tag{12}
$$

b. Direct Solution. Input variables are the latitude ϕ_1 and the longitude λ_1 of P_1, the forward asimuth α_{12} from P_1 to P_2, and the distance (S/a_e) from P_1 to P_2. The output variables are the latitude ϕ_2 and longitude λ_2 of P_2, and the backward asimuth α_{21} from P_2 to P_1. All angular input and output is in radians. Compute:

$$
\begin{aligned}
\theta_1 &= \tan^{-1}[(1-f)\tan\phi_1],\\
M &= \cos\theta_1\sin\alpha_{12},\ c_1 = fM,\ c_2 = f(1-M^2)/4,\\
D &= (1-c_2)(1-c_2-c_1M),\ P = c_2(1+c_1M/2)/D,\\
N &= \cos\theta_1\cos\alpha_{12},\ \sigma_1 = \text{qatn}(N, \sin\theta_1),\\
d &= (S/a_e)/D,\ u = 2(\sigma_1-d),\ W = 1-2P\cos u,\\
V &= \cos(u+d),\ X = c_2^2\sin d\cos d(2V^2-1),\\
Y &= 2PVW\sin d,\ \Delta\sigma = d+X-Y,
\end{aligned}
\tag{13}
$$

$$K = [M^2 + (N\cos\Delta\sigma - \sin\theta_1 \sin\Delta\sigma)^2]^{1/2},$$
$$\tan\theta_2 = (\sin\theta_1 \cos\Delta\sigma + N\sin\Delta\sigma)/K,$$
$$\phi_2 = \tan^{-1}[(\tan\theta_2)/(1-f)],$$
$$\Delta\eta = \text{qatn}(\sin\Delta\sigma \sin\alpha_{12}, \cos\theta_1 \cos\Delta\sigma - \sin\theta_1 \sin\Delta\sigma \cos\alpha_{12}),$$
$$H = c_1(1-c_2)\Delta\sigma - c_1 c_2 \sin\Delta\sigma \cos(2\sigma_1 - \Delta\sigma),$$
$$\lambda_2 = \lambda_1 + \Delta\eta - H,$$
$$\alpha_{21} = \text{qatn}[-M, -(N\cos\Delta\sigma - \sin\theta_1 \sin\Delta\sigma)].$$

2. Spherical Earth Formulas. The inverse solution [Equs. (3), (4) and (5)] and direct solution [Equs. (6), (7) and (8)] formulas were developed in Section B of this report. They are summarised here for convenience.

a. Inverse Solution. Input variables are the latitude ϕ_1 and longitude λ_1 of P_1, the latitude ϕ_2 and longitude λ_2 of P_2, ϕ_2 and λ_2. Output variables are distance, d, forward azimuth from P_1 to P_2, α_{12}, and back azimuth from P_2 to P_1, α_{21}. All angular input and output is in radians. Compute:

$$d = \arccos[\sin\phi_2 \sin\phi_1 + \cos\phi_2 \cos\phi_1 \cos(\lambda_2 - \lambda_1)],$$
$$\alpha_{12} = \text{qatn}(\cos\phi_2 \sin(\lambda_2 - \lambda_1), \sin\phi_2 \cos\phi_1 - \cos\phi_2 \sin\phi_1 \cos(\lambda_2 - \lambda_1)), \text{ and} \qquad (14)$$
$$\alpha_{21} = \text{qatn}(-\cos\phi_1 \sin(\lambda_1 - \lambda_2), \sin\phi_1 \cos\phi_2 - \cos\phi_1 \sin\phi_2 \cos(\lambda_1 - \lambda_2)).$$

b. Direct Solution. Input variables are the latitude ϕ_1 and the longitude λ_1 of P_1, the forward azimuth α_{12} from P_1 to P_2, and the distance d from P_1 to P_2. The output variables are the latitude ϕ_2 and longitude λ_2 of P_2, and the backward azimuth α_{21} from P_2 to P_1. All angular input and output is in radians. Compute:

$$\phi_2 = \arcsin(\cos d \sin\phi_1 + \sin d \cos\phi_1 \cos\alpha_{12}),$$
$$\lambda_2 = \lambda_1 + \text{qatn}(\sin d \sin\alpha_{12}, \cos d \cos\phi_1 - \sin d \sin\phi_1 \cos\alpha_{12}), \text{ and} \qquad (15)$$
$$\alpha_{21} = \text{qatn}(-\cos\phi_1 \sin\alpha_{12}, \sin\phi_1 \sin d - \cos\phi_1 \cos d \cos\alpha_{12}).$$

c. Given Longitude, Find Latitude. Input variables are the latitude ϕ_1 and longitude λ_1 of P_1, the forward azimuth α_{12} from P_1 to P_2, and the longitude λ of some point P on the great circle joining P_1 and P_2. The output variable is ϕ, the latitude of P. All angular input and output is in radians. Using Equs. (2), label the angles and legs so that $A = 360° - \alpha_{21}$, $B = \alpha_{12}$, $C = \lambda - \lambda_1$, $a = 90° - \phi_1$, $b = 90° - \phi$ and $c = d$.

Substituting these into Equs. (2) and simplifying the differences of angles, we obtain

$$\cos\alpha_{21} = -\cos\alpha_{12}\cos(\lambda - \lambda_1) + \sin\alpha_{12}\sin(\lambda - \lambda_1)\sin\phi_1,$$
$$\sin\alpha_{21}\sin\phi = \cos\alpha_{12}\sin(\lambda - \lambda_1) + \sin\alpha_{12}\cos(\lambda - \lambda_1)\sin\phi_1, \text{ and}$$
$$\sin\alpha_{21}\cos\phi = \sin\alpha_{12}\cos\phi_1.$$

Since the only quantity of interest is ϕ, and since latitude lies in the range of $-90°$ to $+90°$, it suffices to divide the second equation by the third equation to obtain an expression for $\tan\phi$. We obtain

$$\phi = \tan^{-1}\left[\frac{\cos\alpha_{12}\sin(\lambda - \lambda_1) + \sin\alpha_{12}\cos(\lambda - \lambda_1)\sin\phi_1}{\sin\alpha_{12}\cos\phi_1}\right]. \quad (16)$$

For almost all applications, Equ. (16) will suffice. It is, however, possible for the denominator of Equ. (16) to become zero in certain circumstances. We now face a situation in which special cases must be examined.

One way to handle the dilemma is to first evaluate the denominator and the numerator. If the value of the denominator is zero, then $\phi = 90°$ if the numerator is positive or $\phi = -90°$ if the numerator is negative. If the denominator is not zero, then use Equ. (16).

A second way to handle the dilemma is to compute

$$\phi = \text{qatn}[\cos\alpha_{12}\sin(\lambda - \lambda_1) + \sin\alpha_{12}\cos(\lambda - \lambda_1)\sin\phi_1, \sin\alpha_{12}\cos\phi_1], \quad (17)$$

but this leads to another problem because ϕ may now be in the range of $-180°$ to $+180°$. To correct the output to a proper latitude, subtract 180° if $\phi > 90°$ or add 180° if $\phi < -90°$.

The question of which method to use is a matter of personal choice. Generally, a simple one line logic function can handle all of the special cases.

d. Find the Vertex. The vertex of a great circle route is the most northerly or/and the most southerly point on the great circle. Bowditch [Ref. 1, §1016, Equs. B25-B30] gives formulas for first, computing the latitude of the vertex, and second, computing the longitude of the vertex as a function of the latitude of the vertex. The Bowditch latitude formula, from a straight forward application of the Law of Sines, is $\phi_v = \cos^{-1}(\sin\alpha_{12}\cos\phi_1)$, where ϕ_v is the latitude of the vertex and ϕ_1 and α_{12} have the same meaning as in the previous section. The principle angle of the arccosine lies between 0° and 180° whereas latitudes lie between $-90°$ and $+90°$. Bowditch illustrates only cases in which ϕ_v is between 0° and +90° and, unfortunately, makes no mention of how to handle cases when the arccosine is greater than 90°.

There are several alternate methods of developing equations for the vertices. One method is presented here and an alternate method is presented at the end of §e. Input variables are the latitude ϕ_1 and longitude λ_1 of P_1, and the forward asimuth α_{12} from P_1 to P_2. Let the latitude and longitude of some arbitrary point P on the great circle from P_1 to P_2 be denoted by ϕ and λ, respectively. All angular input and output is in radians. Label the angles and legs so that $A = \lambda - \lambda_1$, $B = \alpha_{12}$, $C = 360° - \alpha_{21}$, $a = d$, $b = 90° - \phi$ and $c = 90° - \phi_1$. Substituting these into Equs. (1) and simplifying the differences of angles, we obtain

$$\begin{aligned}\cos d &= \sin\phi\sin\phi_1 + \cos\phi\cos\phi_1\cos(\lambda-\lambda_1),\\ \sin d\cos\alpha_{12} &= \sin\phi\cos\phi_1 - \cos\phi\sin\phi_1\cos(\lambda-\lambda_1), \text{ and}\\ \sin d\sin\alpha_{12} &= \cos\phi\sin(\lambda-\lambda_1).\end{aligned}$$

Divide the third equation by the second equation to obtain

$$\tan\alpha_{12} = \frac{\cos\phi\sin(\lambda-\lambda_1)}{\sin\phi\cos\phi_1 - \cos\phi\sin\phi_1\cos(\lambda-\lambda_1)}.$$

Then rearrange the equation in the form

$$\tan\alpha_{12}[\sin\phi\cos\phi_1 - \cos\phi\sin\phi_1\cos(\lambda-\lambda_1)] = \cos\phi\sin(\lambda-\lambda_1). \tag{18}$$

Next, we wish to find what value of ϕ is an extremum of λ. To do this we must first compute $\partial\phi/\partial\lambda$ in Equ. (18):

$$\begin{aligned}\tan\alpha_{12}&\left[\cos\phi\cos\phi_1\frac{\partial\phi}{\partial\lambda} + \sin\phi\sin\phi_1\cos(\lambda-\lambda_1)\frac{\partial\phi}{\partial\lambda} + \cos\phi\sin\phi_1\sin(\lambda-\lambda_1)\right]\\ &= -\sin\phi\sin(\lambda-\lambda_1)\frac{\partial\phi}{\partial\lambda} + \cos\phi\cos(\lambda-\lambda_1).\end{aligned}$$

Then we must set

$$\left.\frac{\partial\phi}{\partial\lambda}\right|_{\substack{\phi=\phi_v\\ \lambda=\lambda_v}} = 0.$$

Thus,

$$\tan\alpha_{12}\cos\phi_v\sin\phi_1\sin(\lambda_v-\lambda_1) = \cos\phi_v\cos(\lambda_v-\lambda_1),$$

which rearranges to

$$\tan(\lambda_v-\lambda_1) = \frac{1}{\sin\phi_1\tan\alpha_{12}}$$

or

$$\lambda_v = \lambda_1 + \tan^{-1}\left[\frac{1}{\sin\phi_1\tan\alpha_{12}}\right]. \tag{19}$$

Once λ_v is known, ϕ_v can be found using one of the methods in §c such as Equ. (17). Since $|\lambda_v - \lambda_1| \leq 90°$, it follows that ϕ_v must be in the same hemisphere as ϕ_1. Thus, if ϕ_1 is in the northern (southern) hemisphere, then ϕ_v must be the northern (southern) vertex.

e. Given Latitude, Find Longitude. Input variables are the latitude ϕ_1 and longitude λ_1 of P_1, the forward azimuth α_{12} from P_1 to P_2 and some latitude ϕ. It is convenient but not necessary to know the latitude of the vertex ϕ_v of the great circle from P_1 to P_2. We wish to find the longitude(s) on the great circle from P_1 to P_2 which have latitude ϕ. There are several possible outcomes: (1) if $|\phi| > |\phi_v|$, there is no solution; (2) if $\phi = \phi_v$, there is one solution and that is $\lambda = \lambda_v$; and (3) if $|\phi| < |\phi_v|$, there are two longitudes which satisfy the given conditions.

Rewrite Equ. (16) in the form

$$\cos\alpha_{12}\cos\phi\sin(\lambda-\lambda_1) + \sin\alpha_{12}\sin\phi_1\cos\phi\cos(\lambda-\lambda_1) = \sin\alpha_{12}\cos\phi_1\sin\phi,$$

or

$$S\sin(\lambda-\lambda_1) + C\cos(\lambda-\lambda_1) = K, \tag{20}$$

where $S \equiv \cos\alpha_{12}\cos\phi$, $C \equiv \sin\alpha_{12}\sin\phi_1\cos\phi$, and $K \equiv \sin\alpha_{12}\cos\phi_1\sin\phi$. Define $\rho > 0$ and η as the solutions to the equations

$$C = \rho\cos\eta \quad \text{and} \quad S = \rho\sin\eta. \tag{21}$$

Solving, we find that

$$\rho = +\sqrt{S^2 + C^2} \quad \text{and}$$
$$\eta = \text{qatn}(S, C)$$

Substituting Equs. (21) into Equ. (20) and rearranging, we find

$$\cos[\lambda - (\lambda_1 + \eta)] = K/\rho.$$

So that

$$\lambda - (\lambda_1 + \eta) = \pm\cos^{-1}(K/\rho),$$

or

$$\lambda = (\lambda_1 + \eta) \pm \cos^{-1}(K/\rho). \tag{22}$$

Recall that a knowledge of ϕ_v is not known to determine the number of solutions. (1) if $|K/\rho| > 1$, there are no solutions; (2) if $|K/\rho| = 1$, there is one solution; and (3) if $|K/\rho| < 1$, there are two solutions.

In the previous section it was mentioned that there was yet another way to determine the position of the vertex. To do this, for fixed ϕ, define $f(\lambda)$ in terms of Equ. (20) as

$$f(\lambda) = S\sin(\lambda - \lambda_1) + C\cos(\lambda - \lambda_1) - K.$$

Then $f'(\lambda)$, the derivative of $f(\lambda)$ with respect to λ is

$$f'(\lambda) = S\cos(\lambda - \lambda_1) - C\sin(\lambda - \lambda_1)$$

Define λ^* so that $f'(\lambda^*) = 0$. Thus

$$S\cos(\lambda^* - \lambda_1) - C\sin(\lambda^* - \lambda_1) = 0,$$

or

$$\frac{\sin(\lambda^* - \lambda_1)}{\cos(\lambda^* - \lambda_1)} = \frac{S}{C},$$

or

$$\lambda^* - \lambda_1 = \text{qatn}(S, C),$$

so that

$$\lambda^* = \lambda_1 + \text{qatn}(S, C) = \lambda_1 + \eta.$$

Hence $\lambda^* = \lambda_1 + \eta$ is an extremum of Equ. (20), or $\lambda_v = \lambda^*$. Note that

$$\begin{aligned} f''(\lambda^*) &= -S\sin(\lambda^* - \lambda_1) - C\cos(\lambda^* - \lambda_1) \\ &= -S\sin\eta - C\cos\eta \\ &= -\frac{S^2}{\rho} - \frac{C^2}{\rho} \\ &= -\frac{S^2 + C^2}{\rho} \\ &= -\rho < 0. \end{aligned}$$

Thus $\lambda_v = \lambda^*$ is the northern vertex.

E. Rhumb Line (Mercator) Formulas.

1. Spheroid Earth Formulas. Bowditch [Ref. 1, Explanation of Table 5, Meridional Parts] gives the formula for the computation of meridional parts as

$$\begin{aligned} M &= a_e \ln\tan\left(45^\circ + \frac{\phi}{2}\right) - a_e\left(e^2\sin\phi + \frac{e^4}{3}\sin^3\phi + \frac{e^6}{5}\sin^5\phi + \ldots\right) \\ &= a_e \ln\tan\left(45^\circ + \frac{\phi}{2}\right) - a_e e\left(e\sin\phi + \frac{e^3}{3}\sin^3\phi + \frac{e^5}{5}\sin^5\phi + \cdots\right) \end{aligned}$$

where M is the number of meridional parts between the equator and the given latitude ϕ, a_e is the equatorial radius of the earth, and $e = \sqrt{(2f - f^2)}$ is the eccentricity of the earth, where $f = 1/298.26$ is the WGS 1972 earth flattening factor.

Using the identity [Ref. 7, #769]

$$\log\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \cdots\right], \quad [x^2 < 1]$$

it is possible to rewrite the formula for M as

$$\begin{aligned} M &= a_e \ln\tan\left(45^\circ + \frac{\phi}{2}\right) - \frac{a_e e}{2}\ln\left(\frac{1+e\sin\phi}{1-e\sin\phi}\right) \\ &= a_e\left[\ln\tan\left(45^\circ + \frac{\phi}{2}\right) - \ln\left(\frac{1+e\sin\phi}{1-e\sin\phi}\right)^{e/2}\right] \\ &= a_e \ln\frac{\tan\left(45^\circ + \frac{\phi}{2}\right)}{\left(\frac{1+e\sin\phi}{1-e\sin\phi}\right)^{e/2}} \end{aligned}$$

If we denote the course (forward asimuth) by α_{12}, then [Bowditch, Vol. II, §1001 and §1013] $\Delta\lambda = \lambda_2 - \lambda_1 = m\tan\alpha_{12}$ where $m = M(\phi_2) - M(\phi_1)$, so that

$$\tan\alpha_{12} = \frac{(\lambda_2 - \lambda_1)/a_e}{\ln\frac{\tan\left(45^\circ + \frac{\phi_2}{2}\right)}{\left(\frac{1+e\sin\phi_2}{1-e\sin\phi_2}\right)^{e/2}} - \ln\frac{\tan\left(45^\circ + \frac{\phi_1}{2}\right)}{\left(\frac{1+e\sin\phi_1}{1-e\sin\phi_1}\right)^{e/2}}} \tag{23}$$

This equation is also given by Shufeld and Newcomer [Ref. 8, pp 81–84]. It is the spheroid earth rhumb line equation.

a. Inverse Solution. Input variables are the latitude ϕ_1 and longitude λ_1 of P_1, the latitude ϕ_2 and longitude λ_2 and P_2, ϕ_2 and λ_2. Output variables are distance, d, forward asimuth α_{12} from P_1 to P_2, and back asimuth α_{21} from P_2 to P_1. All angular input and output is in radians. In Equ. (23), express the angles in radians, then to obtain α_{12} in the proper quadrant, rewrite Equ. (23) in the form

$$\alpha_{12} = \text{qatn}\left[\lambda_2 - \lambda_1, \ln\frac{\tan\left(\frac{\pi}{2} + \frac{\phi_2}{2}\right)}{\left(\frac{1+e\sin\phi_2}{1-e\sin\phi_2}\right)^{e/2}} - \ln\frac{\tan\left(\frac{\pi}{2} + \frac{\phi_1}{2}\right)}{\left(\frac{1+e\sin\phi_1}{1-e\sin\phi_1}\right)^{e/2}}\right] \tag{24}$$

The distance is

$$d = \begin{cases} a_e \dfrac{\phi_2 - \phi_1}{\cos \alpha_{12}}, & \text{if } \cos \alpha_{12} \neq 0, \\ a_e |\lambda_2 - \lambda_1| \cos \phi_1, & \text{otherwise.} \end{cases} \tag{25}$$

In the WGS 1972 earth model, $a_e = 3443.9174$ nautical miles. Also, $\alpha_{21} = \alpha_{12} + \pi$.

b. Direct Solution. Input variables are the latitude ϕ_1 and the longitude λ_1 of P_1, the forward azimuth α_{12} from P_1 to P_2, and the distance d from P_1 to P_2. The output variables are the latitude ϕ_2 and longitude λ_2 of P_2, and the backward azimuth α_{21} from P_2 to P_1. All angular input and output is in radians. In the direct rhumb line solution for the spheroid earth, if $\cos \alpha_{12} = 0$, then

$$\begin{aligned} \lambda_2 &= \lambda_1 + \frac{d/a_e}{\cos \phi_1}, \text{ and} \\ \phi_2 &= \phi_1, \end{aligned} \tag{26a}$$

otherwise, if $\cos \alpha_{12} \neq 0$,

$$\begin{aligned} \lambda_2 &= \lambda_1 + \tan \alpha_{12} \left[\frac{\ln \tan \left(\frac{\pi}{2} + \frac{\phi_2}{2} \right)}{\left(\frac{1 + e \sin \phi_2}{1 - e \sin \phi_2} \right)^{e/2}} - \frac{\ln \tan \left(\frac{\pi}{2} + \frac{\phi_1}{2} \right)}{\left(\frac{1 + e \sin \phi_1}{1 - e \sin \phi_1} \right)^{e/2}} \right], \text{ and} \\ \phi_2 &= \phi_1 + (d/a_e) \cos \alpha_{12}. \end{aligned} \tag{26b}$$

Also, $\alpha_{21} = \alpha_{12} + \pi$.

2. Spherical Earth Formulas. The spherical earth formulas are obtained by setting $e = 0$ in Equations (24) and (26b), and replacing a_e by the standard approximation of 60 nautical miles per degree of arc on the earth's surface.

a. Inverse Solution. Input variables are the latitude ϕ_1 and longitude λ_1 of P_1, the latitude ϕ_2 and longitude λ_2 and P_2, ϕ_2 and λ_2. Output variables are distance, d, forward azimuth from P_1 to P_2, α_{12}, and back azimuth from P_2 to P_1, α_{21}. All angular input and output is in radians.

The spherical earth rhumb line course is obtained by setting $e = 0$ in Equ. (24). The result is

$$\alpha_{12} = \text{qatn} \left[\lambda_2 - \lambda_1, \ln \tan \left(\frac{\pi}{2} + \frac{\phi_2}{2} \right) - \ln \tan \left(\frac{\pi}{2} + \frac{\phi_1}{2} \right) \right] \tag{27}$$

For the spherical earth model, the distance d in nautical miles is

$$d = \begin{cases} 60 \left(\dfrac{180}{\pi} \right) \dfrac{\phi_2 - \phi_1}{\cos \alpha_{12}}, & \text{if } \cos \alpha_{12} \neq 0, \\ 60 \left(\dfrac{180}{\pi} \right) |\lambda_2 - \lambda_1| \cos \phi_1, & \text{otherwise.} \end{cases} \tag{28}$$

Also, $\alpha_{21} = \alpha_{12} + \pi$.

b. Direct Solution. Input variables are the latitude ϕ_1 and the longitude λ_1 of P_1, the forward asimuth α_{12} from P_1 to P_2, and the distance d from P_1 to P_2. The output variables are the latitude ϕ_2 and longitude λ_2 of P_2, and the backward asimuth α_{21} from P_2 to P_1. All angular input and output is in radians.

If $\cos\alpha_{12} = 0$, then

$$\lambda_2 = \lambda_1 + \left(\frac{\pi}{180}\right)\left(\frac{d}{60}\right)\frac{1}{\cos\phi_1}, \text{ and} \qquad (29a)$$
$$\phi_2 = \phi_1,$$

otherwise, if $\cos\alpha_{12} \neq 0$,

$$\lambda_2 = \lambda_1 + \tan\alpha_{12}\left[\ln\tan\left(\frac{\pi}{2} + \frac{\phi_2}{2}\right) - \ln\tan\left(\frac{\pi}{2} + \frac{\phi_1}{2}\right)\right], \text{ and} \qquad (29b)$$
$$\phi_2 = \phi_1 + \left(\frac{\pi}{180}\right)\left(\frac{d}{60}\right)\cos\alpha_{12}.$$

Also, $\alpha_{21} = \alpha_{12} + \pi$.

III. NAVALGOR: Navigation Algorithm Program.

A. Introduction. NAVALGOR implements three sets of computational procedures: (1) Direct Solution Algorithms, (2) Inverse Solution Algorithms, and (3) Rhumb Line Approximations to Great Circle Routes.

The "Direct Solution Algorithms" compute the latitude and longitude of a position P_2 and the backward azimuth from P_2 to P_1 given the latitude and longitude of a position P_1, the forward azimuth from P_1 to P_2 and the distance from P_1 to P_2. The "Inverse Solution Algorithms" compute the distance from position P_1 to position P_2, the forward azimuth from P_1 to P_2, and the backward azimuth from P_2 to P_1 given the latitude and longitude of positions P_1 and P_2. For comparision, the direct and inverse computations are made using four procedures: (1) the spheroid earth model, (2) the spherical earth model, and rhumb line approximations using both (3) the spheroid earth and (4) the spherical earth models.

The spheroid earth model should be considered the standard of comparison for all other models. Computations performed using an IBM 3033 and double precision FORTRAN to compare the inverse solution algorithm [Equs. (12)] to the ACIC long lines (50 to 6000 miles) [Ref. 9] demonstrated that the maximum average error of the inverse solution algorithm was -0.005 meters with a standard deviation of 0.014 meters (unpublished result).

The spherical earth model is included because of its popularity and simplicity. For many procedures the spherical earth model is adequate, and it certainly is more compact for use in a small computer than the spheroid earth model.

The "Rhumb Line Approximations to Great Circle Routes" procedure may be used to find piecewise constant course (rhumb line) approximations to great circle routes for any given increment in longitude. In addition, if the vertex of a course is, for example, too far to the north, a limiting latitude may be input to restrict the rhumb line approximation to go no further north than that limiting latitude. The spheroid earth rhumb line equations are used in this section of the program.

In the sample problems and in the program listing, the east minus and south minus convention is used.

B. Sample Problems—NAVALGOR. The output shown in these sample problems was computed using Microsoft BASICA/D (double precision) on an IBM PC.

Master Menu. The master menu for NAVALGOR is shown below.

```
NAVIGATION ALGORITHM DEMO

1) DIRECT SOLUTION
2) INVERSE SOLUTION
3) RHUMB LINE APPROXIMATIONS
   TO GREAT CIRCLE ROUTES

4) QUIT
```

Option (1) selects the "Direct Solution Algorithm", option (2) selects the "Inverse Solution Algorithm", option (3) selects the "Rhumb Line Approximations to Great Circle Routes" algorithm, and option (4) returns the user to the operating system.

Problem 1. Suppose you are at San Francisco (latitude 37°47′ north and longitude 122°25′ west), that your initial course is 260° and that you travel a distance of 4000 n. mi. What is your final position? Select Option 1 from the master menu.

```
DIRECT SOLUTION

1st LATITUDE    DD.MMSS (-S)  ? 37.47
1st LONGITUDE DDD.MMSS (-E)   ? 122.25
INITIAL COURSE  DDD.MMSS      ? 260
DISTANCE (n. mi.)             ? 4000

SPHEROID EARTH DIRECT SOLUTION
    2nd LATITUDE        6°38.7′
    2nd LONGITUDE   -172°08.8′
    BACK AZIMUTH       51°40.7′

SPHERICAL EARTH DIRECT SOLUTION
    2nd LATITUDE        6°41.9′
    2nd LONGITUDE   -172°00.7′
    BACK AZIMUTH       51°35.9′

RHUMB LINE SOLUTIONS
    SPHERICAL        26°12.4′   -159°56.0′
    SPHEROID         26°12.4′   -160°18.4′

PRESS ANY KEY TO CONTINUE
```

Problem 2. Suppose you are at San Francisco (latitude 37°47′ north and longitude 122°25′ west) and that your destination is Sydney, Australia (latitude 33°51′ south and longitude 151°13′ east). How far do you travel, what is your initial course, and what is the backward asimuth from Sydney to San Fransisco? Select Option 2 from the master menu.

```
                    INVERSE SOLUTION

               1st LATITUDE    DD.MMSS (-S) ? 37.47
               1st LONGITUDE DDD.MMSS (-E) ? 122.25
               2nd LATITUDE    DD.MMSS (-S) ? -33.51
               2nd LONGITUDE DDD.MMSS (-E) ? -151.13

          SPHEROID EARTH INVERSE SOLUTION
              DISTANCE          6443.52 n.mi.
              FORWARD COURSE    240°29.3′
              BACK COURSE        55°55.7′

          SPHERICAL INVERSE SOLUTION
              DISTANCE          6446.3 n.mi.
              FORWARD COURSE    240°18.9′
              BACK COURSE        55°45.9′

          RHUMB LINE SOLUTIONS: SPHERE      SPHEROID
              DISTANCE          6464.49     6485.71 n.mi.
              COURSE            228°19.7′   228°29.7′

               PRESS ANY KEY TO CONTINUE
```

Problem 3. Suppose you are at latitude 37° north, longitude 76° west and that your destination is latitude 45° north, longitude 1° west. Compute the initial great circle course and distance, the latitude and longitude of the vertex, and a rhumb line approximation to the course traveling at most 7° degrees of longitude on each leg of the course. In addition, no portion of the course is to be more northerly than 47°. Select Option 3 from the master menu.

1st Screen:

```
                    RHUMB LINE APPROXIMATIONS

          INITIAL LATITUDE   DD.MMSS (-S) ? 37
          INITIAL LONGITUDE DDD.MMSS (-E) ? 76
          FINAL LATITUDE     DD.MMSS (-S) ? 45
          FINAL LONGITUDE   DDD.MMSS (-E) ? 1

            GREAT CIRCLE SOLUTION
               INITIAL COURSE   58°21.2′
               DISTANCE        3307.8 n.mi.

            RHUMB LINE (MERCATOR) SOLUTION
               COURSE           81°58.2′
               DISTANCE        3435.9 n.mi.

            VERTEX: LATITUDE    48°19.8′
                    LONGITUDE   28°07.3′

          PRESS ANY KEY TO CONTINUE
```

In this first screen, the great circle solution and a single rhumb line solution are computed and displayed. In addition, the location of the nearest vertex is displayed. The vertex may or may not lie between the initial and final positions. If the vertex is not between the initial and final positions then any limiting latitude which may be input on the next screen prompt will be ignored. If the vertex is between the initial and final positions, a limiting latitude will be ignored unless it is lies between the latitude of the vertex and the latitude of the position which is closest to the latitude of the vertex.

2nd Screen:

```
                 RHUMB LINE APPROXIMATIONS

          INPUT THE MAXIMUM NUMBER OF DEGREES
            OF LONGITUDE ON EACH RHUMB LINE LEG ? 7

          LIMITING LATITUDE DD.MMSS (-S)
            (0 TO OMIT) ? 47
          LIMITING LONGITUDES:
              10°45.8′        45°28.8′

          PRESS ANY KEY TO CONTINUE
```

In the second screen a prompt appeared for the maximum longitude between each rhumb line leg—a value of 7° was input. Also input was a limiting latitude of 47°, which lies between the vertex latitude of 48°19.8′ and 45°, the latitude of the final position. Limiting longitudes of 10°45.8′ and 45°28.8′ are computed and displayed. The latitude of a great circle route between these limiting longitudes will be more northerly than 47°, consequently any rhumb line approximation will be due east or due west at a latitude of 47° between these longitudes.

3rd Screen:

RHUMB LINE APPROXIMATION TO GREAT CIRCLE COURSE

LAT	LONG	GREAT CIRCLE COURSE	GREAT CIRCLE DIST	RHUMB LINE COURSE	RHUMB LINE DIST
37°00.0′	76°00.0′	56	0	58	0
39°27.9′	71°00.0′	59	278	62	279
42°18.8′	64°00.0′	64	639	66	641
44°32.0′	57°00.0′	69	971	71	974
46°11.7′	50°00.0′	74	1283	76	1287
47°00.0′	45°28.8′	77	1475	90	1480
47°00.0′	10°45.8′	103	2884	105	2900
45°56.3′	5°00.0′	107	3130	108	3148
45°00.0′	1°00.0′	110	3308	108	3326

1) NEW APPROXIMATION OR 2) NEW PROBLEM?

Screen 3 displays the rhumb line approximation for a maximum of 7° of longitude between course changes. The latitude and longitude of the initial and final positions and of the positions at which course changes are made are given in columns 1 and 2, respectively. Column 3 shows the great circle heading at each position, while Column 4 shows the cumulative great circle distance from the initial position. Similarly, Column 5 shows the rhumb line course to be followed between each pair of positions and Column 6 shows the cumulative rhumb line distance from the initial position. Note that at 47° north and 45°28.8′ west (the first limiting latitude position) the rhumb line course changes to due east until 47° north and 10°45.8′ west in reached. Also note that the rhumb line approximation, including the 'detour' at the limiting latitude, is only 18 n. mi. longer than the great circle route. The third screen ends with two options: Option 1 returns to Screen 2 and prompts for new inputs while Option 2 returns to the master menu.

C. Program Listing—NAVALGOR.

```
10 REM "NAVIGATION ALGORITHMS" R.H. SHUDDE, 03-12-85.  REV. 09-14-85 1600
13 REM "NAVALGOR/BAS"
30 DEFDBL A-Y
40 P4=ATN(1):P2=P4+P4:PI=P2+P2:TP=PI+PI:RD=PI/180:EP=1E-33
50 FL=1/298.26:AE=6378135/1852:EC=SQR(FL*(2-FL))
60 DEFFNM(X)=X-MO*INT(X/MO):REM X MOD MO FUNCTION.
70 DEFFNL(X)=X-TP*INT((X+PI)/TP):REM LONGITUDE ADJUST (-PI,PI)
80 DEFFNR(X)=INT(X*MO+.5)/MO:REM ROUNDING FUNCTION.
90 DEFFNG(X)=X+PI*SGN(X)*(ABS(X)>P2):REM LATITUDE ADJUST (-PI/2,PI/2)
100 DEFFNC(X)=ATN(SQR(1-X*X)/(X-EP*(X=0#)))-PI*(X<0#):REM ARCCOS
110 DEFFNS(X)=ATN(X/(SQR(1-X*X)-EP*(ABS(X)=1))):REM ARCSIN
115 GOTO 2000
120 REM QATN (-PI,PI) FUNCTION:
130 A=ATN(Y/(X-EP*(X=0#)))-PI*(X<0#)*(SGN(Y)-(Y=0#)):RETURN
140 REM QATN (0,TWOPI) FUNCTION:
150 A=ATN(Y/(X-EP*(X=0#)))-PI*(X<0#)+TP*(X>=0#)*(Y<0#):RETURN
170 REM
200 REM DIRECT SOLUTION, SPHEROID EARTH. ALL ANGLES MUST BE IN RADIANS.
210 REM INPUT: LATITUDE G1, LONGITUDE L1, FORWARD AZIMUTH A1 AND
220 REM DISTANCE DD=(S/AE) TO A POINT P2.  NOTE: DD HAS RADIAN UNITS.
230 REM OUTPUT: LATITUDE G2, LONGITUDE L2 AND BACKWARD AZIMUTH A2.
240 S9=SIN(A1):C9=COS(A1):TA=ATN((1-FL)*TAN(G1))
250 S8=SIN(TA):C8=COS(TA):M=-S9*C8:C1=FL*M
260 C2=FL*(1-M*M)/4:D=(1-C2)*(1-C2-C1*M)
270 P=C2*(1+.5*C1*M)/D:N=C8*C9
280 Y=N:X=S8:GOSUB 130:SG=A:SD=DD/D:U=2*(SG-SD)
290 W=1-2*P*COS(U):V=COS(U+SD):X=C2*C2*SIN(SD)*COS(SD*(2*V*V-1))
300 Y=2*P*V*W*SIN(SD):DQ=SD+X-Y:SL=SIN(DQ):CL=COS(DQ)
310 K=SQR(M*M+(N*CL-S8*SL)^2):G2=ATN((S8*CL+N*SL)/K/(1-FL))
320 Y=-SL*S9:X=C8*CL-S8*SL*C9:GOSUB 130:DJ=A
330 H=C1*(1-C2)*DQ-C1*C2*SL*COS(SG+SG-DQ)
340 L2=FNL(L1+DJ-H):Y=M:X=-(N*CL-S8*SL):GOSUB 150:A2=A
350 RETURN
360 REM
400 REM INVERSE SOLUTION, SPHEROID EARTH. ALL ANGLES MUST BE IN RADIANS.
410 REM INPUT: LATITUDES G1 & G2, AND LONGITUDES L1 & L2.
420 REM OUTPUT: DISTANCE DD=(S/AE), FORWARD AZIMUTH A1, AND BACKWARD
430 REM AZIMUTH A2.  NOTE: DD HAS RADIAN UNITS.
440 TA=ATN((1-FL)*TAN(G1)):TB=ATN((1-FL)*TAN(G2))
450 DM=L2-L1:DT=(TB-TA)/2:TM=(TA+TB)/2
460 SH=SIN(DT):CH=COS(DT):SM=SIN(TM):CM=COS(TM)
470 H=CH*CH-SM*SM:L=SH*SH+H*(SIN(DM/2))^2
480 SD=2*FNS(SQR(L)):U=2*SM*SM*CH*CH/((1-L)-(EP*(L=1)))
490 V=2*SH*SH*CM*CM/L:X=U+V:Y=U-V:T=SD/SIN(SD)
500 D=4*T*T:E=2*COS(SD):A=D*E:C=T-(A-E)/2:N1=X*(A+C*X)
510 B=D+D:N2=Y*(B+E*Y):N3=D*X*Y:D4=FL*FL*(N1-N2+N3)/64
520 D3=FL*(T*X-Y)/4:DD=(T-D3+D4)*SIN(SD)
530 M=32*T-(20*T-A)*X-(B+4)*Y:F=Y+Y-E*(4-X)
540 G=FL*(T/2+FL*M/64):Q=-(F*G*TAN(DM))/4
550 DW=(DM+Q)/2:SW=SIN(DW):CW=COS(DW)
```

```
560 Y=SH*CW:X=CM*SW:GOSUB 130:T1=A
570 Y=-CH*CW:X=SM*SW:GOSUB 130:T2=A
580 MO=TP:A1=FNM(T1+T2):A2=FNM(T1-T2):RETURN
590 REM
600 REM DIRECT SOLUTION, SPHERICAL EARTH. ALL ANGLES MUST BE IN RADIANS.
610 REM INPUT: LATITUDE G1, LONGITUDE L1, FORWARD AZIMUTH A1 AND
620 REM DISTANCE DD TO A POINT P2.  NOTE: DD IS IN RADIANS.
630 REM OUTPUT: LATITUDE G2, LONGITUDE L2 AND BACKWARD AZIMUTH A2.
640 S9=SIN(A1):C9=COS(A1):S3=SIN(G1):C3=COS(G1)
650 SD=SIN(DD):CD=COS(DD)
660 Y=SD*S9:X=C3*CD-S3*C9*SD:GOSUB 130:L2=FNL(L1-A)
670 Y=-S9*C3:X=SD*S3-CD*C9*C3:GOSUB 150:A2=A
680 G2=FNS(S3*CD+C3*SD*C9):RETURN
690 REM
700 REM INVERSE SOLUTION, SPHERICAL EARTH. ALL ANGLES MUST BE IN RADIANS.
710 REM INPUT: LATITUDES G1 & G2, AND LONGITUDES L1 & L2.
720 REM OUTPUT: DISTANCE DD TO A POINT P2.  (NOTE: 0 <= DD <= PI RADIANS).
730 REM FORWARD AZIMUTH A1, AND BACKWARD AZIMUTH A2.
740 S3=SIN(G1):C3=COS(G1):S4=SIN(G2):C4=COS(G2)
750 DL=L1-L2:SU=SIN(DL):CU=COS(DL)
760 DD=FNC(S3*S4+C3*C4*CU)
770 Y=SU*C4:X=C3*S4-S3*C4*CU:GOSUB 150:A1=A
780 Y=-SU*C3:X=C4*S3-S4*C3*CU:GOSUB 150:A2=A:RETURN
790 REM
800 REM INVERSE SOLUTION, RHUMB LINE FOR SPHERICAL & SPHEROIDAL EARTH.
810 REM INPUT: LATITUDES G1 & G2, AND LONGITUDES L1 & L2.
820 REM OUTPUT: DISTANCES DA & DB, AND AZIMUTHS ZA & ZB.
830 GA=TAN(P4+G1/2):GB=TAN(P4+G2/2):DL=FNL(L1-L2):DK=ABS(DL/RD)
840 GA=GA-EP*(GA=0):GB=GB-EP*(GB=0)
850 DG=(G2-G1)/RD:Y=DL:X=LOG(GB)-LOG(GA):GOSUB 150:ZA=A
860 CZ=COS(ZA):DA=60*DK*COS(G1):IF ABS(CZ)>.0001THEN DA=60*DG/CZ
870 E1=EC*SIN(G1):E2=EC*SIN(G2):ED=EC/2
880 Y=DL:X=LOG(GB/((1+E2)/(1-E2))^ED)-LOG(GA/((1+E1)/(1-E1))^ED)
885 GOSUB 150:ZB=A
890 CZ=COS(ZB):DB=60*DK*COS(G1):IF ABS(CZ)>.0001THEN DB=60*DG/CZ
900 RETURN
910 REM
1000 REM DIRECT SOLUTION, RHUMB LINE FOR SPHERICAL & SPHEROIDAL EARTH.
1010 REM INPUT: LATITUDE G1, LONGITUDE L1, FORWARD AZIMUTH A1 AND
1020 REM DISTANCE DD TO A POINT P2.  NOTE: DD IS IN RADIANS.
1030 REM OUTPUT: LATITUDE G2 AND LONGITUDE LR & LS
1040 C6=COS(A1)
1050 IF C6=0THEN LR=L1+DD/COS(G1):LS=LR:G2=G1:RETURN
1060 G2=G1+DD*C6
1070 GA=TAN(P4+G1/2):GB=TAN(P4+G2/2):T6=TAN(A1)
1080 LR=L1-T6*(LOG(GB)-LOG(GA))
1090 E1=EC*SIN(G1):E2=EC*SIN(G2):ED=EC/2
1100 LS=L1-T6*(LOG(GB/((1+E2)/(1-E2))^ED)-LOG(GA/((1+E1)/(1-E1))^ED))
1110 RETURN
1120 REM
1200 REM DECIMAL TO DDD MM.F
1210 V$=" ":IF X<0THEN V$="-":X=-X
1220 X=X+1/1200:Y=INT(X):V$=V$+RIGHT$("  "+STR$(Y),3)+"°"
```

```
1230  X=600*(X-Y):Y=INT(X):X$=STR$(1000+Y)
1240  V$=V$+MID$(X$,3,2)+"."+RIGHT$(X$,1)+"'":RETURN
1250  REM
1260  REM DDD.MMSS TO DECIMAL
1270  IX=0:FOR Z=1TO LEN(V$):C$=MID$(V$,Z,1):IF C$="."THEN IX=Z
1280  NEXT :IF IX=OTHEN X=VAL(V$):RETURN
1290  X=VAL(LEFT$(V$,IX)):SN=1:IF X<OTHEN SN=-SN:X=-X
1300  V$=V$+"0000":Y=VAL(MID$(V$,IX+1,2)):Z=VAL(MID$(V$,IX+3,2))
1310  X=SN*((Z/60+Y)/60+X):RETURN
1320  REM
2000  CLS :PRINT SPC(15);"NAVIGATION ALGORITHM DEMO
2010  PRINT :PRINT :PRINT
2020  PRINT SPC(15);"1) DIRECT SOLUTION
2030  PRINT SPC(15);"2) INVERSE SOLUTION
2040  PRINT SPC(15);"3) RHUMB LINE APPROXIMATIONS"
2050  PRINT SPC(15);" TO GREAT CIRCLE ROUTES
2060  PRINT :PRINT SPC(15);"4) QUIT
2070  GOSUB 6010:C=VAL(C$):ON CGOSUB 3000,3500,4000,5500
2080  GOTO 2000
2090  REM
3000  CLS :PRINT SPC(15);"DIRECT SOLUTION":PRINT :PRINT
3010  PRINT SPC(10);:PRINT "1ST LATITUDE DD.MMSS (-S) ";
3020  INPUT V$:GOSUB 1270:G1=X*RD
3030  PRINT SPC(10);:PRINT "1ST LONGITUDE DDD.MMSS (-E) ";
3040  INPUT V$:GOSUB 1270:L1=X*RD
3050  PRINT SPC(10);:PRINT "INITIAL COURSE DDD.MMSS ";
3060  INPUT V$:GOSUB 1270:A1=X*RD
3070  PRINT SPC(10);"DISTANCE (N.MI.) ";:INPUT D
3080  D1=D*RD/60:DD=D/AE
3090  GOSUB 240:PRINT :PRINT SPC(8);"SPHEROID EARTH DIRECT SOLUTION
3100  MO=100:GOSUB 3210
3110  DD=D1:GOSUB 640:PRINT :PRINT SPC(8);"SPHERICAL EARTH DIRECT SOLUTION
3120  GOSUB 3210
3130  GOSUB 1040:PRINT :PRINT SPC(8);"RHUMB LINE SOLUTIONS";
3135  PRINT SPC(4);"LAT";SPC(14);"LONG
3140  PRINT SPC(12);"SPHERICAL ";
3150  X=FNG(G2):X=X/RD:GOSUB 1210:VS$=V$:PRINT V$;SPC(8);
3160  X=FNL(LR):X=X/RD:GOSUB 1210:PRINT V$
3170  PRINT SPC(12);"SPHEROID ";VS$;SPC(8);
3180  X=FNL(LS):X=X/RD:GOSUB 1210:PRINT V$
3190  GOSUB 6000:GOTO 2000
3200  REM
3210  PRINT SPC(12);"2ND LATITUDE ";:X=G2/RD:GOSUB 1210:PRINT V$
3220  PRINT SPC(12);"2ND LONGITUDE ";:X=L2/RD:GOSUB 1210:PRINT V$
3230  PRINT SPC(12);"BACK AZIMUTH ";:X=A2/RD:GOSUB 1210:PRINT V$:RETURN
3240  REM
3500  CLS :PRINT SPC(15);"INVERSE SOLUTION":PRINT :PRINT
3510  PRINT SPC(10);:PRINT "1ST LATITUDE DD.MMSS (-S) ";
3520  INPUT V$:GOSUB 1270:G1=X
3530  PRINT SPC(10);:PRINT "1ST LONGITUDE DDD.MMSS (-E) ";
3540  INPUT V$:GOSUB 1270:L1=X
3550  PRINT SPC(10);:PRINT "2ND LATITUDE DD.MMSS (-S) ";
3560  INPUT V$:GOSUB 1270:G2=X
```

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3570 PRINT SPC(10);:PRINT "2ND LONGITUDE DDD.MMSS (-E) ";
3580 INPUT V$:GOSUB 1270:L2=X
3590 G1=G1*RD:G2=G2*RD:L1=L1*RD:L2=L2*RD
3600 GOSUB 440:PRINT :PRINT SPC(8);"SPHEROID EARTH INVERSE SOLUTION
3610 MO=100:I%=1:GOSUB 3700
3620 DD=D1:GOSUB 740:PRINT :PRINT SPC(8);"SPHERICAL INVERSE SOLUTION
3630 I%=2:GOSUB 3700:GOSUB 830
3640 PRINT :PRINT SPC(8);"RHUMB LINE SOLUTIONS: SPHERE SPHEROID
3650 PRINT SPC(12);"DISTANCE ";FNR(DA);SPC(8);FNR(DB);" N.MI.
3660 PRINT SPC(12);"COURSE ";:X=ZA/RD:GOSUB 1210:PRINT V$;SPC(6);
3670 X=ZB/RD:GOSUB 1210:PRINT V$
3680 GOSUB 6000:GOTO 2000
3690 REM
3700 PRINT SPC(12);"DISTANCE ";
3710 IF I%=1THEN PRINT FNR(DD*AE);" N.MI.
3720 IF I%=2THEN PRINT FNR(60*DD/RD);" N.MI.
3730 PRINT SPC(12);"FORWARD COURSE ";:X=A1/RD:GOSUB 1210:PRINT V$
3740 PRINT SPC(12);"BACK COURSE ";:X=A2/RD:GOSUB 1210:PRINT V$:RETURN
3750 REM
4000 CLS :PRINT SPC(15);"RHUMB LINE APPROXIMATIONS":PRINT :PRINT
4010 PRINT SPC(10);:PRINT "INITIAL LATITUDE DD.MMSS (-S) ";
4020 INPUT V$:GOSUB 1270:G1=X*RD:G6=G1
4030 PRINT SPC(10);:PRINT "INITIAL LONGITUDE DDD.MMSS (-E) ";
4040 INPUT V$:GOSUB 1270:L1=X*RD:L6=L1
4050 PRINT SPC(10);:PRINT "FINAL LATITUDE DD.MMSS (-S) ";
4060 INPUT V$:GOSUB 1270:G2=X*RD:G7=G2
4070 PRINT SPC(10);:PRINT "FINAL LONGITUDE DDD.MMSS (-E) ";
4080 INPUT V$:GOSUB 1270:L2=X*RD:L7=L2
4090 GOSUB 740:MO=10:PRINT :PRINT SPC(10);"GREAT CIRCLE SOLUTION
4100 PRINT SPC(15);"INITIAL COURSE ";:AZ=A1:X=A1/RD:GOSUB 1210:PRINT V$
4110 PRINT SPC(15);"DISTANCE ";FNR(60*DD/RD);" N.MI.
4120 GOSUB 830:PRINT :PRINT SPC(12);"RHUMB LINE (MERCATOR) SOLUTION
4130 PRINT SPC(15);"COURSE ";:X=ZB/RD:GOSUB 1210:PRINT V$
4140 PRINT SPC(15);"DISTANCE ";FNR(DB);" N.MI.
4150 GOSUB 4610
4160 PRINT :PRINT SPC(12);"VERTEX: LATITUDE ";:X=GV/RD:GOSUB 1210:PRINT V$
4170 PRINT SPC(20);"LONGITUDE ";:X=LV/RD:GOSUB 1210:PRINT V$
4180 GOSUB 6000
4190 CLS :PRINT SPC(15);"RHUMB LINE APPROXIMATIONS":PRINT
4200 PRINT :PRINT SPC(10);"INPUT THE MAXIMUM NUMBER OF DEGREES
4210 PRINT SPC(10);" OF LONGITUDE ON EACH RHUMB LINE LEG ";
4220 INPUT V$:GOSUB 1270:IN=X*RD
4230 GS=0:RS=0:IF IN=0THEN IN=TP
4240 DL=FNL(L7-L6):IF DL=0THEN F=1
4250 DV=FNL(LV-L6):IF DL<>0THEN F=DV/DL
4260 L%=0:IF F<=0OR F>=1THEN L%=1:GOTO 4370
4270 PRINT :PRINT SPC(10);"LIMITING LATITUDE DD.MMSS (-S)
4280 PRINT SPC(10);" (0 TO OMIT) ";:INPUT V$:GOSUB 1270:LL=X*RD
4290 L8=0:L9=0:G1=G6:G3=G1:L1=L6:LA1=L1:PH=LL:G2=G7:L2=L7
4300 IF LL=0THEN L%=1:GOTO 4370
4310 Q1=ABS(PV-G6):Q2=ABS(PV-G7):A=Q1:IF Q2<A THEN A=Q2
4320 A=A*SGN(PV):B=PV-LL:IF A=0 THEN G=5:GOTO 4340
4330 G=B/A
```

```
4340 IF G<=0 OR G>1 THEN L%=1:PRINT SPC(10);"CANT USE. - IGNORED.":GOTO 4370
4350 GOSUB 4810:PRINT SPC(10);"LIMITING LONGITUDES:
4360 X=L8/RD:GOSUB 1210:PRINT SPC(13);V$;SPC(10):X=L9/RD:GOSUB 1210:PRINT V$
4370 GOSUB 6000:GOSUB 5220
4380 D8=FNL(L8-L1):D9=FNL(L9-L1):IF ABS(D9)<ABS(D8)THEN T=L9:L9=L8:L8=T
4390 M%=1:LX=L6:PX=G6:IF L%=1THEN 4450
4400 LY=L8:PY=LL:GOSUB 4910
4410 L1=L8:G1=LL:L2=L9:G2=LL:GOSUB 740:GOSUB 830
4420 GOSUB 5110:GS=GS+DD:RS=RS+DB
4430 LX=L9:PX=LL:L1=LX:G1=PX
4440 L2=L7:G2=G7
4450 M%=0:LY=L7:PY=G7:GOSUB 4910
4460 REM
4470 PRINT :PRINT SPC(10);"1) NEW APPROXIMATION OR 2) NEW PROBLEM?
4480 GOSUB 6010:C=VAL(C$):ON CGOSUB 4190,2000
4490 GOTO 4480
4500 REM
4600 REM COMPUTE VERTEX
4610 SA=SIN(AZ):CA=COS(AZ):SP=SIN(G1):CP=COS(G1)
4620 IF SA=0THEN GV=P2:LV=L6:RETURN
4630 LV=L1-ATN(1/SP/TAN(AZ)):LA=L1:LV=FNL(LV):LM=LV:GOSUB 4720
4640 GV=PH:RETURN
4650 REM
4700 REM FIND LATITUDE GIVEN LONGITUDE
4710 SA=SIN(AZ):CA=COS(AZ):SP=SIN(PA):CP=COS(PA)
4720 DL=LA-LM:Y=SP*COS(DL)*SA+SIN(DL)*CA:X=CP*SA:GOSUB 130:PH=A
4730 PH=FNG(PH):RETURN
4740 REM
4800 REM FIND LONGITUDE GIVEN LATITUDE
4810 K%=0:IF ABS(PH)=>ABS(GV)THEN K%=1:RETURN
4820 SA=SIN(AZ):CA=COS(AZ):SP=SIN(G3):CP=COS(G3)
4830 K=CP*SA*TAN(PH):C=SP*SA:S=CA:RO=SQR(S*S+C*C):Y=S:X=C:GOSUB 130:NU=A
4840 CC=FNC(K/RO):DD=LA1-NU:L8=FNL(DD-CC):L9=FNL(DD+CC):RETURN
4850 REM
4900 REM RHUMB APPROXIMATIONS SUBROUTINE. INPUT: LX, PX, LY & PY.
4910 DL=FNL(LY-LX):IC=ABS(IN):AG=ABS(DL)/IC
4920 IF IC/RD<1THEN NP=INT(AG+.5):MO=.1
4930 IF IC/RD>=1THEN NP=INT(AG):MO=1
4940 IF NP<1THEN IC=DL:L2=LY:GOTO 4970
4950 IC=SGN(DL)*IC
4960 L2=FNR((LX+(DL-(NP-1)*IC)/2)/RD+.5*SGN(DL))*RD
4970 L1=LX:G1=PX:LM=L2:GOSUB 4720:G2=PH
4980 GOSUB 740:GOSUB 830:GOSUB 5110:GS=GS+DD:RS=RS+DB
4990 IF NP<1THEN 5060
5000 NC=1:IF NP=1THEN 5040
5010 L1=L2:G1=G2:L2=FNL(L1+IC):LM=L2:GOSUB 4720:G2=PH
5020 GOSUB 740:GOSUB 830:GOSUB 5110:GS=GS+DD:RS=RS+DB
5030 NC=NC+1:IF NC<NPTHEN 5010
5040 L1=L2:G1=G2:L2=LY:G2=PY
5050 GOSUB 740:GOSUB 830:GOSUB 5110:GS=GS+DD:RS=RS+DB:IF M%=1THEN RETURN
5060 L1=LY:G1=PY:MO=TP:A1=FNM(A2+PI):GOSUB 5110:RETURN
5070 REM
5100 REM PRINT LINE OF OUTPUT
```

```
5110  MO=1:X=G1/RD:GOSUB 1210:PRINT V$;SPC(2);
5120  X=L1/RD:GOSUB 1210:PRINT V$;
5130  X=FNR(A1/RD):GOSUB 5200:PRINT V$;SPC(4);
5140  X=FNR(60*GS/RD):GOSUB 5200:PRINT V$;SPC(4);
5150  X=FNR(ZB/RD):GOSUB 5200:PRINT V$;SPC(3);
5160  X=FNR(RS):GOSUB 5200:PRINT V$
5170  NL=NL+1:IF NL=20THEN GOSUB 6000:GOSUB 5220
5180  RETURN
5190  REM
5200  V$="  "+STR$(X):V$=RIGHT$(V$,7):RETURN
5210  REM
5220  CLS :PRINT SPC(10);"RHUMB LINE APPROXIMATION TO GREAT CIRCLE COURSE":
      PRINT
5230  PRINT " GREAT GREAT RHUMB RHUMB
5240  PRINT " CIRCLE CIRCLE LINE LINE
5250  PRINT " LAT LONG COURSE DIST COURSE DIST
5260  NL=5:RETURN
5270  REM
5500  CLS :END
5510  REM
6000  PRINT :PRINT SPC(10);"PRESS ANY KEY TO CONTINUE
6010  FOR I!=1TO 9:C$=INKEY$ :NEXT
6020  C$=INKEY$ :IF C$=""THEN 6020
6030  RETURN
```

D. Program Annotation—NAVALGOR.

Line(s)	Usage
40	PI4 = $\pi/4$. PI2 = $\pi/2$. PI = π. TP = 2π. RD = $\pi/180$ is the degree-to-radian conversion.
50	FL is the earth's flattening factor. AE is the earth's equatorial radius. (Both of these constants are from the WGS–72 earth model.) ECC is the earth's eccentricity.
60	FNM is the x mod M0 function.
70	FNL adjusts longitude to lie between $-\pi$ and π.
80	FNR rounds x to the nearest M0*th*.
90	FNG adjusts latitudes to lie between $-\pi/2$ and $\pi/2$.
100	FNACS is the arccosine function. In the Sharp PC–1500A, function calls may be replaced inline by the ACS function.
110	FNASN is the arcsine function. In the Sharp PC–1500A, function calls may be replaced inline by the ASN function.
130	FNATN2 is the qatn function which returns a principle value between $-\pi$ and π.
140	FNATNP is a qatn function which returns a principle value between 0 and 2π.
160	GOTO 2000 to commence execution.
240-350	Computation of the *spheroid* earth direct solution. G1 = ϕ_1, L1 = λ_1, A1 = α_{12} and distance DD are input. G2 = ϕ_2, L2 = λ_2 and A2 = α_{21} are output.
440-580	Computation of the *spheroid* earth inverse solution. G1 = ϕ_1, L1 = λ_1, G2 = ϕ_2 and L2 = λ_2 are input. A1 = α_{12}, A2 = α_{21} and distance DD are output.
640-680	Computation of the *spherical* earth direct solution. G1 = ϕ_1, L1 = λ_1, A1 = α_{12} and distance DD are input. G2 = ϕ_2, L2 = λ_2 and A2 = α_{21} are output.
740-780	Computation of the *spherical* earth inverse solution. G1 = ϕ_1, L1 = λ_1, G2 = ϕ_2 and L2 = λ_2 are input. A1 = α_{12}, A2 = α_{21} and distance DD are output.
830-900	Computation of the rhumb line inverse solution. G1 = ϕ_1, L1 = λ_1, G2 = ϕ_2 and L2 = λ_2 are input. Spherical earth distance DA and azimuth ZA as well as spheroid earth distance DB and azimuth ZB are output.
1040-1100	Computation of the rhumb line direct solution. G1 = ϕ_1, L1 = λ_1, A1 = α_{12} and distance DD are input. G2 = ϕ_2, spherical earth longitude LR and spheroid earth longitude LS are output.
1200-1240	Convert decimal degrees to degrees, minutes and tenths of minute format.
1270-1310	Convert packed degrees, minutes and seconds format (DDD.MMSS) to decimal degrees.
2000-2060	Master menu option display.
2070	Transfer to selected menu item.
3000-3230	Direct solution option.
3080	D is the distance (input) in nautical miles, D1 is the spherical earth distance in radians and DD is the distance in earth equatorial radius units.
3090	Compute spheroid earth direct solution. Print heading.
3100	Print spheroid earth direct solution.

3110 Compute spherical earth direct solution. Print heading.
3120 Print spherical earth direct solution.
3130 Compute rhumb line solution. Print heading.
3140-3160 Print spherical earth rhumb line latitude and longitude.
3170-3180 Print spheroid earth rhumb line latitude and longitude.
3190 Continue prompt. Go to master menu.
3210-3230 Output subroutine for option 1.
3500-3740 Inverse solution option.
3600 Compute spheroid earth inverse solution. Print heading.
3610 Print spheroid earth inverse solution.
3620 Compute spherical earth inverse solution. Print heading.
3630 Print spherical earth inverse solution.
3640 Heading for rhumb line solutions.
3650 Print spherical and spheroid earth rhumb line distances.
3660-3670 Print spherical and spheroid earth rhumb line courses.
3680 Continue prompt. Go to master menu.
3700-3740 Output subroutine for option 2.
4000-5260 Rhumb line approximation to great circle option.
4000-4080 Input prompts.
4090-4110 Compute great circle solution and print initial course and distance.
4120-4140 Compute rhumb line solution and print course and distance.
4150-4170 Compute and print the latitude and longitude of the vertex of the great circle route.
4180 Display continue prompt.
4190-4220 Prompt for the longitude increment of the rhumb line approximation.
4230 GS and RS are the cumulative great circle and rhumb line distances traveled on each leg, respectively. Initialize them to zero. IN is the longitude increment in radians. If input as zero (no increments requested), it is set to 2π.
4240-4260 Determine if the vertex lies between the origin and destination. If not, go to 4370. Otherwise proceed.
4270-4280 If the vertex is on the great circle route, prompt for a limiting latitude LL.
4290-4340 Set limiting longitudes L8 and L9 equal to zero. Determine if the limiting latitude cuts the great circle course. If it does not, inform the user that the limiting latitude is ignored. L% is zero if the limiting latitude is to be used, otherwise it is one.
4350-4360 If the limiting latitude cuts the great circle course, compute and display the longitudes L8 and L9 at which the limiting latitude cuts the great circle.
4370 Display continue prompt, then print heading.
4380 Determine which limiting longitude is closest to the initial position. L8 becomes the closest, L9 the farthest.
4390 LX and PX are the initial great circle longitude and latitude. If the limiting latitude is not to be used, go to 4450.

4400 LY and PY are the longitude and latitude of the final point on the first segment of the great circle course. GOSUB 4910 to compute the rhumb line approximation up to the first limiting longitude.

4410 Compute the great circle and rhumb line distances from the limiting latitude at the first limiting longitude to the second limiting longitude.

4420 Print one line of output (for the limiting longitudes). Update the distance counters.

4430-4440 Reset LX, PX, LY and PY for the section of the great circle and rhumb line following the limiting latitude section of the course.

4450 GOSUB 4910 to compute the final section of a course following a limiting latitude leg or compute the entire course for cases in which the limiting latitude is not used.

4470-4490 Prompt for either a rework of the rhumb line approximation with new parameters or for a new problem.

4610-4640 Compute the latitude GV an longitude LV of a great circle vertex.

4710-4730 For a given longitude, compute the corresponding latitude of on a great circle.

4810-4840 For a given latitude, compute the corresponding longitudes L8 and L9 on a given gr .t circle. K% is set to one if there is no solution, otherwise K% is zero.

4910-5060 This is the subroutine which selects the longitude endpoints of the rhumb line approximation to a great circle between a starting position with latitude PX and longitude LX and a final position with latitude PY and longitude LY. It then computes and prints the heading and cumulative distance along each leg.

4910-4950 Using the increment size, IN and IC, compute the number of interior points, NP, on the rhumb line approximation exclusive of the initial and final points (the number of interior legs is NP − 1. Redefine IC if it is longer than the difference of the initial and final longitudes.

4960-4980 Find the heading and distance on the initial leg. Print out first line of output.

4990 If there are no internal legs, go to 5060.

5000 Initialize the counter IC. If there is only one leg remaining, go to 5040.

5010-5030 Loop to compute print each internal leg.

5040-5050 Compute the headings and distances for the final leg. If M% is one, return to compute the limiting latitude leg.

5060 Print the final summary line of output.

5110-5200 Subroutine to print one line of output. NL counts the number of lines of output. If 20 lines are output, a new screen is started.

5220-5260 Subroutine to print screen heading.

5500 Return to BASIC.

6000-6030 "PRESS ANY KEY TO CONTINUE" subroutine.

IV. NAVEPHM: Almanac and Ephemeris Program.

A. Introduction. The basis for NAVEPHM are the equations of VanFlandern and Pulkkinen (VFP) [Ref. 10]. These equations can be used to compute the mean heliocentric positions of the sun and planets and the mean geocentric position of the moon for the mean ecliptic and equinox of date. The authors claim their formulas to be of low-precision (1′) and valid for any epoch within 300 years of the present. When corrected for nutation and aberration the accuracy of their formulas, at least for the sun, moon and navigational planets, appears to be much better, *i.e.* 0.2′, at least.

The formulas of VFP are used to compute heliocentric spherical ecliptic coordinates for any specified ephemeris time, ET. These coordinates are longitude λ, latitude β and radius vector r. These coordinates must be converted to rectangular coordinates x, y, and z using the standard transformation

$$\begin{aligned} x &= r\cos\beta\cos\lambda, \\ y &= r\cos\beta\sin\lambda, \text{ and} \\ z &= r\sin\beta. \end{aligned} \tag{30}$$

To obtain the geocentric x, y, and z coordinates of the planets, subtract the x, y, and z coordinates of the sun from the x, y, and z coordinates of the planets, respectively.

The Julian day number, required in many calculations, is obtained using the equation on page B2 of Ref. 2. The ephemeris time is obtained from the universal time, UT, from the equation ET = UT + ΔT. The factor ΔT is obtained by astronomical observation only. The formula used here, $\Delta T = 81.94T - 15$, was obtained by least squares from the 1980.5 through 1985.5 values given on pages B5 and K9 of *The Astronomical Almanac 1985* [Ref. 11], where T is the number of Julian centuries elapsed from 1900 January 0, 12^h ET. Although a plot of ΔT as a function of time is linear for 1980.5 through 1985.5, this should be checked with each new edition of *The Astronomical Almanac*. An accurate value of ΔT affects only the computation of the moon's position. Errors of as much as 9.6^s in ΔT will affect the computation the moon's position by at most 0.1′, consequently it will not be necessary to change FMPL distribution tapes yearly.

The heliocentric in-plane velocity components, $\dot{x}_\omega$ and $\dot{y}_\omega$ of the planets, required for the aberration correction, can be computed from the formulas [Ref. 12, pg. 85]:

$$\begin{aligned} \dot{x}_\omega &= -ae\dot{E}\sin E, \text{ and} \\ \dot{y}_\omega &= a\dot{E}\sqrt{1-e^2}\cos E, \end{aligned} \tag{31}$$

where E is the eccentric anomaly, $\dot{E}$ is the time derivative of E, a is the semimajor axis and e is the eccentricity. Unfortunately, to find E one must solve Kepler's equation iteratively, which is a slow process. With an error of no more than 2% for the navigational planets, one can use the mean anomaly M in place of E in Equs. (31). The in-plane velocity components are thus approximated by

$$\begin{aligned} \dot{x}_\omega &= -ae\dot{E}\sin M, \text{ and} \\ \dot{y}_\omega &= a\dot{E}\sqrt{1-e^2}\cos M. \end{aligned} \tag{32}$$

From Equ. (3.76) of Ref. 12, it can be shown that

$$\dot{E} = \frac{k}{r}\left(\frac{\mu}{a}\right)^{1/2},$$

where μ is the reduced mass and k is the gravitational constant. The heliocentric ecliptic velocity components, $\dot{x}$, $\dot{y}$ and $\dot{z}$, can then be obtained by the transformation

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_\omega \\ \dot{y}_\omega \\ 0 \end{bmatrix},$$

where Ω is the longitude of the ascending node, i is the orbital inclination and ω is the argument of perihelion. The equations for computing a, e, M, ω, i and Ω were obtained from Escobal [Ref. 13, pp. 8–9]. The geocentric $\dot{x}$, $\dot{y}$ and $\dot{z}$ coordinates of the planets are obtained by subtracting the $\dot{x}$, $\dot{y}$ and $\dot{z}$ coordinates of the sun from the heliocentric $\dot{x}$, $\dot{y}$ and $\dot{z}$ coordinates of the planets. Since the aberration of the moon is negligible, its velocity components are not computed.

Once the mean geocentric rectangular positions (x_m, y_m and z_m) and velocities ($\dot{x}_m$, $\dot{y}_m$ and $\dot{z}_m$) have been obtained, the longitude and latitude for the mean ecliptic and equinox of date can be obtained by inverting Equs. (30), so that

$$\begin{aligned} \lambda_m &= \mathrm{qatn}(y_m, x_m), \text{ and} \\ \beta_m &= \sin^{-1}\frac{z_m}{r} = \tan^{-1}\frac{z_m}{\sqrt{x_m^2 + y_m^2}}. \end{aligned} \tag{33}$$

Next the nutation of longitude $\Delta\psi$ and obliquity $\Delta\epsilon$ are computed [Ref. 4, §2C]. The geocentric rectangular positions (x_t, y_t and z_t) for the true ecliptic and equinox of date can be obtained from the transformation

$$\begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \cos\Delta\psi & -\sin\Delta\psi & 0 \\ \sin\Delta\psi & \cos\Delta\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix}.$$

The same transformation is used for the velocity components. The longitude and latitude for the true ecliptic and equinox of date can be obtained by substituting the true positions into Equs. (33).

The aberration or light-time correction converts true positions into apparent positions. The x-coordinate transformation is

$$x_{ae} = x_t - \frac{r\dot{x}_t}{c},$$

where r is the geocentric distance and c is the velocity of light. Similar transformations are used for the y- and z-coordinates. The apparent longitude and latitude for the true ecliptic and equinox of date can be obtained by substituting the apparent positions into Equs. (33).

The apparent geocentric rectangular positions (x_{aq}, y_{aq} and z_{aq}) for the true equator and equinox of date are obtained from the transformation

$$\begin{bmatrix} x_{aq} \\ y_{aq} \\ z_{aq} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Delta\epsilon & -\sin\Delta\epsilon \\ 0 & \sin\Delta\epsilon & \cos\Delta\epsilon \end{bmatrix} \begin{bmatrix} x_{ae} \\ y_{ae} \\ z_{ae} \end{bmatrix}.$$

The apparent right ascension α and declination δ for the true equator and equinox of date can be computed from

$$\begin{aligned} \alpha &= \mathrm{qatn}(y_{aq}, x_{aq}), \text{ and} \\ \delta &= \sin^{-1}\frac{z_{aq}}{r} = \tan^{-1}\frac{z_{aq}}{\sqrt{x_{aq}^2 + y_{aq}^2}}. \end{aligned} \tag{34}$$

The Greenwich mean sidereal time, Greenwich apparent sidereal time, Greenwich hour angle and local hour angle are computed from formulas given on pages B3 and B4 of Ref. 2. The altitude and azimuth are computed from Equs. (10) and (11), respectively. The equation for refraction is Equ. (3), page B15 of Ref. 2. The equations for planetary magnitude are given on pg. 315 of Ref. 4 with the correction for Saturn's rings given on pages 362 to 365. Formulas for the table on page 365 were obtained using least squares. Equations for the semidiameter of the sun and moon are found on page B16 of Ref. 2. The lunar parallax in altitude formula is on page B16 of Ref. 2 and the lunar phase formula is on page 311 of Ref. 4. The lunar age approximation developed by the author is within ±1 day.

B. Sample Problem. Calculate the position of the sun, moon and navigational planets for 24 May 1984 at $17^h 58^m 45^s$ zulu time at latitude 38°35′24″ north and longitude 76°18′00″ west. Assume that the temperature and pressure are the default values of 10°C. and 1010 mb., respectively. The output shown in this sample problem was computed using Microsoft BASICA/D (double precision) on an IBM PC. Several intermediate results, such as the true position, and apparent positions for the true equator and for the true equinox of date are printed only as a debugging aid for those wishing to reproduce the results on a computer other than those for which NAVEPHM is available.

Input Screen:

```
YEAR (4 DIGITS)        ? 1984
MONTH NUMBER           ? 5
DAY OF THE MONTH       ? 24
ZULU TIME (HH.MMSS)    ? 17.5845
LAT (DD.MMSS)  (+N/-S)? 38.3524
LON (DDD.MMSS) (+W/-E)? 76.18
TEMP (DEG CELSIUS)     ? 10
PRESSURE (MILLIBARS)   ? 1010

PRESS C TO CONTINUE
```

The input screen is used for input only—there are no options from which to choose. The temperature and pressure are used only for the atmospheric refraction computation.

1st Output Screen:

```
SUN

TRUE POS, TRUE ECL & EQNX OF DATE (UT):
 63.73236355163271  0

APP POS, TRUE ECL & EQNX OF DATE (UT):
 63.72674538386791  0

APP POS, TRUE EQU & EQNX OF DATE (UT):
 4.11446999613666  20.89927908414652

GREENWICH HOUR ANGLE & DECL (UT):
  90°29.0'     20°54.0'

ALTITUDE =  68.50534784507299        68°30.3'
AZIMUTH  =  218.6611413863376       218°39.7'
SD = 15.8'
REFRACTION =  .4'

PRESS C TO CONTINUE
```

The true and apparent positions for the true ecliptic and equinox of date printed are the ecliptic longitude and latitude, respectively, in decimal degrees. The apparent position for the true equator and equinox of date is the right ascension in decimal hours and the declination in decimal minutes. The Greenwich hour angle and declination are given in degrees, minutes, and tenths of minute notation. The altitude and asimuth are given in both decimal degrees and degrees, minutes, and tenths of minute notation. SD is the sun's semi-diameter. The refraction is for the computed altitude of the sun.

2nd Output Screen:

```
MOON

TRUE POS, TRUE ECL & EQNX OF DATE (UT):
 355.6933313549804 -4.991896504201657

APP POS, TRUE ECL & EQNX OF DATE (UT):
 355.6933313549804 -4.991896504201657

APP POS, TRUE EQU & EQNX OF DATE (UT):
 23.86924533529027 -6.291911880374428

GREENWICH HOUR ANGLE & DECL (UT):
 154°09.7'    - 6°17.5'

ALTITUDE =  5.451726462048884         5°27.1'
AZIMUTH  =  257.4665215802323       257°28.0'
PHASE .31    AGE = 24  DAYS
SD = 14.8'   SD AUG = 14.8'
P IN A =  54'
REFRACTION =  10.2'

PRESS C TO CONTINUE
```

The output for the moon is similar to that for the sun. In addition, the moon's phase, approximate age, augmented semi-diameter (topocentric semi- diameter) and parallax in altitude P IN A are output. The true position and the apparent position for the true ecliptic and equinox of date for the moon are identical because no aberration correction is made for the moon (see Section A).

3rd Output Screen:

```
VENUS

TRUE POS, TRUE ECL & EQNX OF DATE (UT):
 57.72453552418941 -.6528498164408445

APP POS, TRUE ECL & EQNX OF DATE (UT):
 57.71230545494549 -.6532096744515719

APP POS, TRUE EQU & EQNX OF DATE (UT):
 3.706665485856183  19.01586622158487

GREENWICH HOUR ANGLE & DECL (UT):
 96°36.0'     19°01.0'

ALTITUDE =  63.67703088674409        63°40.6'
AZIMUTH  =  227.7063425077327       227°42.4'
MAGNITUDE  = -3.4
REFRACTION =  .5'

PRESS C TO CONTINUE
```

The output for Venus is similar to that for the sun except that the apparent magnitude is output instead of the semi-diameter.

4th Output Screen:

```
MARS

TRUE POS, TRUE ECL & EQNX OF DATE (UT):
 226.1549997906556 -.7543527638263081

APP POS, TRUE ECL & EQNX OF DATE (UT):
 226.1561434601825 -.7542105991415879

APP POS, TRUE EQU & EQNX OF DATE (UT):
 14.89744688277315 -17.39602620686829

GREENWICH HOUR ANGLE & DECL (UT):
 288°44.3'    - 17°23.8'

ALTITUDE = -54.68434644133615       - 54°41.1'
AZIMUTH  =  62.30703743152624         62°18.4'
MAGNITUDE  = -1.6
REFRACTION = -.7'

PRESS C TO CONTINUE
```

The output for Mars is similar to that for Venus. Note that the altitude of Mars is negative, that is, Mars is below the horizon. The leads to the anomolous computation of a negative value for the refraction. When a planet is below the horizon, the value computed for refraction is meaningless.

5th Output Screen:

```
JUPITER

TRUE POS, TRUE ECL & EQNX OF DATE (UT):
 282.0081844949666  .1412184643412081

APP POS, TRUE ECL & EQNX OF DATE (UT):
 282.0100408579675  .1412662061186586

APP POS, TRUE EQU & EQNX OF DATE (UT):
 18.86941225466823 -22.75882790181082

GREENWICH HOUR ANGLE & DECL (UT):
 229°09.5'    - 22°45.5'

ALTITUDE = -61.97025716907849       - 61°58.2'
AZIMUTH  = 296.4711031451715         296°28.3'
MAGNITUDE  = -2.1
REFRACTION = -.5'

PRESS C TO CONTINUE
```

The output for Jupiter is similar to that for Venus. See the comments for Mars regarding a negative refraction value.

6th Output Screen:

```
SATURN

TRUE POS, TRUE ECL & EQNX OF DATE (UT):
 221.5236884913894  2.580801361264901

APP POS, TRUE ECL & EQNX OF DATE (UT):
 221.5271566155949  2.58091611295858

APP POS, TRUE EQU & EQNX OF DATE (UT):
 14.66067525585222 -12.83630953737889

GREENWICH HOUR ANGLE & DECL (UT):
 292°17.4'    - 12°50.2'

ALTITUDE = -49.04279937791162        - 49°02.6'
AZIMUTH  =  60.93733251413396          60°56.2'
MAGNITUDE  =  .4
REFRACTION = -.8'

PRESS C TO CONTINUE
```

The output for Saturn is similar to that for Venus. See the comments for Mars regarding a negative refraction value.

C. Program Listing—NAVEPHM

```
10  REM VANFLANDERN & PULKKINEN PLANET EPHEMERIS. 07-20-85.
    REV 08-27-85 @ 0800
12  REM "ASTRO EPHEM V"
13  REM "NAVEPHM.BAS"
20  DEFDBLA-M,O-Y:DIM A(25),B$(5)
30  P2=2*ATN(1):PI=P2+P2:TP=PI+PI:RD=PI/180:SD=RD/3600
40  EK=.01720209895:MB$="1010":DC$="10":DN$=CHR$(26)
50  DEFFNA(X)=(X-INT(X))*TP:DEFFNB(X)=X-TP*INT(X/TP):
    DEFFNR(X)=INT(X*MO+.5)/MO
60  DEFFNM(X)=X-MO*INT(X/MO)
70  B$(0)="SUN":B$(1)="MOON":B$(2)="VENUS":B$(3)="MARS"
80  B$(4)="JUPITER":B$(5)="SATURN"
90  GOTO 2090
100 :
110 REM 2-ARG ATAN FCTN. RADIANS.
120 A2=ATN(Y/(X-1E-09*(X=0)))-PI*(X<0)+TP*(X>0)*(Y<0):RETURN
130 :
140 REM ARCSIN & ARCCOS FCTN'S
150 AS=ATN(X/(SQR(1-X*X)-1E-09*(ABS(X)=1))):RETURN
160 AC=ATN(SQR(1-X*X)/(X-1E-09*(X=0)))-PI*(X<0):RETURN
170 :
180 REM HELIO SPHER-TO-RECT + MOTION COMPUTATION
190 R=R/100000:CS=R*COS(B)
200 XH=CS*COS(L):YH=CS*SIN(L):ZH=R*SIN(B)
210 IF N=1THEN RETURN
220 MU=1:IF N<>0THEN MU=1+1/RM(N)
230 FA=EK*SQR(AM(N)*MU)/R
240 X=-FA*SIN(M(N)):Y=FA*SQR(1-EC(N)^2)*COS(M(N)):Z=0
250 A=-AP(N):GOSUB 280:A=-IN(N):GOSUB 290:A=-AN(N):GOSUB 280
260 UH=X:VH=Y:WH=Z:RETURN
270 :
280 CA=COS(A):SA=SIN(A):T=X*CA+Y*SA:Y=Y*CA-X*SA:X=T:RETURN :REM Z-AXIS ROT
290 CA=COS(A):SA=SIN(A):T=Y*CA+Z*SA:Z=Z*CA-Y*SA:Y=T:RETURN :REM X-AXIS ROT
300 :
310 REM RECT-TO-SPHER
320 GOSUB 120
330 L=A2/RD:R2=X*X+Y*Y:B=ATN(Z/SQR(R2))/RD:R=SQR(R2+Z*Z):RETURN
340 :
350 REM DECIMAL TO DDD MM.F
360 V$=" ":IF X<0THEN V$="-":X=-X
370 X=X+1/1200:Y=INT(X):V$=V$+RIGHT$("  "+STR$(Y),3)+"°"
380 X=600*(X-Y):Y=INT(X):X$=STR$(1000+Y)
390 V$=V$+MID$(X$,3,2)+"."+RIGHT$(X$,1):RETURN
400 :
410 REM DDD.MMSS TO DECIMAL
420 IX=0:FOR Z=1TO LEN(V$):C$=MID$(V$,Z,1):IF C$="."THEN IX=Z
430 NEXT :IF IX=0THEN X=VAL(V$):RETURN
440 X=VAL(LEFT$(V$,IX)):SN=1:IF X<0THEN SN=-SN:X=-X
450 V$=V$+"0000":Y=VAL(MID$(V$,IX+1,2)):Z=VAL(MID$(V$,IX+3,2))
460 X=SN*((Z/60+Y)/60+X):RETURN
```

```
470  :
480  REM FUNDAMENTAL ARGUMENTS
490  A(1)=.606434+.03660110129*TS:A(2)=.374897+.03629164709*TS
500  A(3)=.259091+.0367481952*TS:A(6)=0:A(7)=.779072+.00273790931*TS
510  A(8)=.993126+.0027377785*TS
520  A(12)=.505498+4.450468670000003E-03*TS:A(13)=.140023+.00445036173*TS
530  A(14)=.292498+.00445040017*TS:A(15)=.987353+.00145575328*TS
540  A(16)=.053856+.00145561327*TS:A(17)=.849694+.00145569465*TS
550  A(18)=.089608+.00023080893*TS:A(19)=.056531+.00023080893*TS
560  A(20)=.814794+.00023080893*TS:A(21)=.133295+.00009294371*TS
570  A(22)=.882987+.00009294371*TS:A(23)=.821218+.00009294371*TS
580  A(25)=.400589+3.269438000000001E-05*TS
590  A(4)=A(1)-A(7):A(5)=A(1)-A(3)
600  FOR Z=1TO 25:A(Z)=FNA(A(Z)):NEXT Z:RETURN
610  :
620  PRINT B$(0):REM SUN(0)
630  X=A(8):Y=A(13):Z=A(19)
640  L=(6910-17*TJ)*SIN(X)+72*SIN(2*X)-7*COS(X-Z)+6*SIN(A(1)-A(7))
650  L=L+5*SIN(4*X-8*A(16)+3*Z)-5*COS(2*(X-Y))-4*SIN(X-Y)
660  L=L+4*COS(4*X-8*A(16)+3*Z)+3*SIN(2*(X-Y))-3*SIN(Z)-3*SIN(2*(X-Z))
670  L=L*SD+A(7):B=0
680  R=100014-1675*COS(X)-14*COS(2*X)
690  AM(0)=1.00000023:EC(0)=.016709114:M(0)=A(8)
700  AP(0)=((4.52778E-04*TJ+1.719175)*TJ+101.2208333)*RD:
     IN(0)=0:AN(0)=180*RD
710  RM(0)=1/329390
720  GOSUB 190:RR=R:XS=XH:YS=YH:ZS=ZH:US=UH::VS=VH:WS=WH:RETURN
730  PRINT B$(1):REM MOON(1)
740  W=A(2):X=A(3):Y=A(4):Z=A(8)
750  L=22640*SIN(W)-4586*SIN(W-2*Y)+2370*SIN(2*Y)+769*SIN(2*W)
760  L=L-668*SIN(Z)-412*SIN(2*X)-212*SIN(2*(W-Y))-206*SIN(W-2*Y+Z)
770  L=L+192*SIN(W+2*Y)+165*SIN(2*Y-Z)+148*SIN(W-Z)-125*SIN(Y)
780  L=L-110*SIN(W+Z)-55*SIN(2*(X-Y))-45*SIN(W+2*X)+40*SIN(W-2*X)
790  L=L-38*SIN(W-4*Y)+36*SIN(3*W)-31*SIN(2*W-4*Y)+28*SIN(W-2*Y-Z)
800  L=L-24*SIN(2*Y+Z)+19*SIN(W-Y)+18*SIN(Y+Z)+15*SIN(W+2*Y-Z)
810  L=L+14*SIN(2*(W+Y))+14*SIN(4*Y)-13*SIN(3*W-2*Y)
820  AG=W+16*A(7)-18*A(12):L=L-11*SIN(AG)+9*COS(AG)+4*TJ*(COS(AG)+SIN(AG))
830  L=L+10*SIN(2*W-Z)+9*SIN(W-2*X-2*Y)-9*SIN(2*W-2*Y+Z)
840  L=L-8*SIN(W+Y)+8*SIN(2*Y-2*Z)-8*SIN(2*W+Z)-7*SIN(2*Z)
850  L=L-7*SIN(W-2*Y+2*Z)+7*SIN(A(5))-6*SIN(W-2*X+2*Y)
860  L=L-6*SIN(2*X+2*Y)-4*SIN(W-4*Y+Z)-4*SIN(2*W+2*X)
870  L=L+3*(SIN(W-3*Y)-SIN(W+2*Y+Z)-SIN(2*W-4*Y+Z)+SIN(W-2*Z)
     +SIN(W-2*Y-2*Z))
880  L=L+2*(SIN(W+4*Y)-SIN(2*W-2*Y-Z)-SIN(2*X-2*Y+Z))
890  L=L+2*(SIN(4*W)+SIN(4*Y-Z)+SIN(2*W-Y))
900  B=18461*SIN(X)+1010*SIN(W+X)+1000*SIN(W-X)-624*SIN(X-2*Y)
910  B=B-199*SIN(W-X-2*Y)-167*SIN(W+X-2*Y)+117*SIN(X+2*Y)
920  B=B+62*SIN(2*W+X)+33*SIN(W-X+2*Y)+32*SIN(2*W-X)-30*SIN(X-2*Y+Z)
930  B=B-16*SIN(2*W+X-2*Y)+15*SIN(W+X+2*Y)+12*SIN(X-2*Y-Z)
940  B=B-9*SIN(W-X-2*Y+Z)-8*SIN(X+A(5))+8*SIN(X+2*Y-Z)
950  B=B+7*(-SIN(W+X-2*Y+Z)+SIN(W+X-Z)-SIN(W+X-4*Y))
960  B=B+6*(-SIN(X+Z)-SIN(3*X)+SIN(W-X-Z))
970  B=B+5*(-SIN(X+Y)-SIN(W+X+Z)-SIN(W-X+Z)+SIN(X-Z)+SIN(X-Y))
```

```
980  B=B+4*(SIN(3*W+X)-SIN(X-4*Y))+3*(-SIN(W-X-4*Y)+SIN(W-3*X))
990  B=B+2*(-SIN(2*W-X-4*Y)-SIN(3*X-2*Y)+SIN(2*W-X+2*Y))
1000 B=B+2*(SIN(W-X+2*Y-Z)+SIN(2*W-X-2*Y)+SIN(3*W-X))
1010 R=6036298-327746*COS(W)-57994*COS(W-2*Y)-46357*COS(2*Y)
1020 R=R-8904*COS(2*W)+3865*COS(2*W-2*Y)-3237*COS(2*Y-Z)
1030 R=R-2688*COS(W+2*Y)-2358*COS(W-2*Y+Z)-2030*COS(W-Z)
1040 R=R+1719*COS(Y)+1671*COS(W+Z)+1247*COS(W-2*X)+704*COS(Z)
1050 R=R+529*COS(2*Y+Z)-524*COS(W-4*Y)+398*COS(W-2*Y-Z)-366*COS(3*W)
1060 R=R-295*COS(2*W-4*Y)-263*COS(Y+Z)+249*COS(3*W-2*Y)-221*COS(W+2*Y-Z)
1070 R=R+185*COS(2*X-2*Y)-161*COS(2*Y-2*Z)+147*COS(W+2*X-2*Y)-142*COS(4*Y)
1080 R=R+139*COS(2*W-2*Y+Z)-118*COS(W-4*Y+Z)-116*COS(2*W+2*Y)
1090 R=R-110*COS(2*W-Z)
1100 L=L*SD+A(1):B=B*SD:R=R/23454.8:GOSUB 190:RP=R:RETURN
1110 PRINT B$(2):REM VENUS(2)
1120 X=A(8):Y=A(13):Z=A(14)
1130 L=(2814-20*TJ)*SIN(Y)-181*SIN(2*Z)+12*SIN(2*Y)-10*COS(2*(X-Y))
1140 L=L+7*COS(3*(X-Y))
1150 B=12215*SIN(Z)+83*(SIN(Y+Z)+SIN(Y-Z))
1160 R=72335-493*COS(Y)
1170 L=L*SD+A(12):B=B*SD
1180 AM(2)=.723331600000001:EC(2)=6.77304100000001E-03:M(2)=A(13)
1190 AP(2)=((-.001386389*TJ+.50818611)*TJ+54.3841861)*RD:IN(2)=3.3946*RD
1200 AN(2)=((.00041*TJ+.89985)*TJ+75.7796472200001)*RD:RM(2)=408523.5
1210 GOSUB 190:RP=R:RETURN
1220 PRINT B$(3):REM MARS(3)
1230 X=A(8):Y=A(16):Z=A(17):Q=A(19)
1240 L=(38451+37*TJ)*SIN(Y)+(2238+4*TJ)*SIN(2*Y)+181*SIN(3*Y)-52*SIN(2*Z)
1250 L=L-22*COS(Y-2*Q)-19*SIN(Y-Q)+17*COS(Y-Q)+17*SIN(4*Y)-16*COS(2*(Y-Q))
1260 L=L+13*COS(X-2*Y)-10*SIN(Y-2*Z)-10*SIN(Y+2*Z)+7*COS(X-Y)-7*COS(2*X-3*Y)
1270 L=L-5*SIN(A(13)-3*Y)-5*SIN(X-Y)-5*SIN(X-2*Y)-4*COS(2*X-4*Y)+4*COS(Q)
1280 L=L+3*COS(A(13)-3*Y)+3*SIN(2*(Y-Q))
1290 B=6603*SIN(Z)+622*SIN(Y-Z)+615*SIN(Y+Z)+64*SIN(2*Y+Z)
1300 R=153031-14170*COS(Y)-660*COS(2*Y)-47*COS(3*Y)
1310 L=L*SD+A(15):B=B*SD
1320 AM(3)=1.52368839:EC(3)=.0934048870000002:M(3)=A(16)
1330 AP(3)=((1.3125E-04*TJ+1.06976667)*TJ+285.431761000001)*RD:
     IN(3)=1.8497*RD
1340 AN(3)=((-1.389E-06*TJ+.770991670000001)*TJ+48.78644167)*RD:
     RM(3)=3098710
1350 GOSUB 190:RP=R:RETURN
1360 PRINT B$(4):REM JUPITER(4)
1370 X=A(18):Y=A(19):Z=A(22)
1380 L=19934*SIN(Y)+5023*TJ+2511+1093*COS(2*Y-5*Z)+601*SIN(2*Y)
1390 L=L-479*SIN(2*Y-5*Z)-185*SIN(2*Y-2*Z)+137*SIN(3*Y-5*Z)-131*SIN(Y-2*Z)
1400 L=L+79*COS(Y-Z)-76*COS(2*Y-2*Z)-74*TJ*COS(Y)+68*TJ*SIN(Y)
     +66*COS(2*Y-3*Z)
1410 L=L+63*COS(3*Y-5*Z)+53*COS(Y-5*Z)+49*SIN(2*Y-3*Z)-43*TJ*SIN(2*Y-5*Z)
1420 L=L-37*COS(Y)+25*SIN(2*X)+25*SIN(3*Y)-23*SIN(Y-5*Z)-19*TJ*COS(2*Y-5*Z)
1430 L=L+17*COS(2*Y-4*Z)+17*COS(3*Y-3*Z)-14*SIN(Y-Z)-13*SIN(3*Y-4*Z)
     -9*COS(2*X)
1440 L=L+9*COS(Z)-9*SIN(Z)-9*SIN(3*Y-2*Z)+9*SIN(4*Y-5*Z)+9
     *SIN(2*Y-6*Z+3*A(25))
1450 L=L-8*COS(4*Y-10*Z)+7*COS(3*Y-4*Z)-7*COS(Y-3*Z)-7*SIN(4*Y-10*Z)
```

```
1460 L=L-7*SIN(Y-3*Z)+6*COS(4*Y-5*Z)-6*SIN(3*Y-3*Z)+5*COS(2*Z)
     -4*SIN(4*Y-4*Z)
1470 L=L-4*COS(3*Z)+4*COS(2*Y-Z)-4*COS(3*Y-2*Z)-4*TJ*COS(2*Y)+3*TJ*SIN(2*Y)
1480 L=L+3*COS(5*Z)+3*COS(5*Y-10*Z)+3*SIN(2*Z)-2*SIN(2*X-Y)+2*SIN(2*X+Y)
1490 L=L-2*TJ*SIN(3*Y-5*Z)-2*TJ*SIN(Y-5*Z)
1500 B=-4692*COS(Y)+259*SIN(Y)+227-227*COS(2*Y)+30*TJ*SIN(Y)+21*TJ*COS(Y)
1510 B=B+16*SIN(3*Y-5*Z)-13*SIN(Y-5*Z)-12*COS(3*Y)+12*SIN(2*Y)
1520 B=B+7*COS(3*Y-5*Z)-5*COS(Y-5*Z)
1530 R=520883-25122*COS(Y)-604*COS(2*Y)+260*COS(2*(Y-Z))-170*COS(3*Y-5*Z)
1540 R=R-106*SIN(2*(Y-Z))-91*TJ*SIN(Y)-84*TJ*COS(Y)+69*SIN(2*Y-3*Z)
1550 R=R-67*SIN(Y-5*Z)+66*SIN(3*Y-5*Z)+63*SIN(Y-Z)-51*COS(2*Y-3*Z)
1560 R=R-46*SIN(Y)-29*COS(Y-5*Z)+27*COS(Y-2*Z)-22*COS(3*Y)-21*SIN(2*Y-5*Z)
1570 L=L*SD+X:B=B*SD
1580 AM(4)=5.202561:EC(4)=.0484942519:M(4)=A(19)
1590 AP(4)=((7.0405E-04*TJ+.5994316700000001)*TJ+273.2775417)*RD:
     IN(4)=1.3031*RD
1600 AN(4)=((3.52222E-04*TJ+1.01053)*TJ+99.44338611)*RD:RM(4)=1047.355
1610 GOSUB 190:RP=R:RETURN
1620 PRINT B$(5):REM SATURN(5)
1630 W=A(19):X=A(21):Y=A(22):Z=A(25)
1640 L=23045*SIN(Y)+5014*TJ-2689*COS(2*W-5*Y)+2507+1177*SIN(2*W-5*Y)
1650 L=L-826*COS(2*W-4*Y)+802*SIN(2*Y)+425*SIN(W-2*Y)-229*TJ*COS(Y)
1660 L=L-153*COS(2*W-6*Y)-142*TJ*SIN(Y)-114*COS(Y)+101*TJ*SIN(2*W-5*Y)
1670 L=L-70*COS(2*X)+67*SIN(2*X)+66*SIN(2*W-6*Y)+60*TJ*COS(2*W-5*Y)
1680 L=L+41*SIN(W-3*Y)+39*SIN(3*Y)+31*SIN(W-Y)+31*SIN(2*(W-Y))
1690 L=L-29*COS(2*W-3*Y)-28*SIN(2*W-6*Y+3*Z)+28*COS(W-3*Y)
1700 L=L+22*TJ*SIN(2*W-4*Y)-22*SIN(Y-3*Z)+20*SIN(2*W-3*Y)
1710 L=L+20*COS(4*W-10*Y)+19*COS(2*Y-3*Z)+19*SIN(4*W-10*Y)
1720 L=L-17*TJ*COS(2*Y)-16*COS(Y-3*Z)-12*SIN(2*W-4*Y)+12*COS(W)
1730 L=L-12*SIN(2*(Y-Z))-11*TJ*SIN(2*Y)-11*COS(2*W-7*Y)
1740 L=L+10*SIN(2*Y-3*Z)+10*COS(2*(W-Y))+9*SIN(4*W-9*Y)
1750 L=L-8*SIN(Y-2*Z)-8*COS(2*X+Y)+8*COS(2*X-Y)+8*COS(Y-Z)
1760 L=L-8*SIN(2*X-Y)+7*SIN(2*X+Y)-7*COS(W-2*Y)-7*COS(2*Y)
1770 L=L-6*TJ*SIN(4*W-10*Y)+6*TJ*COS(4*W-10*Y)+6*TJ*SIN(2*W-6*Y)
1780 L=L-5*SIN(3*W-7*Y)-5*COS(3*(W-Y))-5*COS(2*(Y-Z))+5*SIN(3*W-4*Y)
1790 L=L+5*SIN(2*W-7*Y)+4*SIN(3*(W-Y))+4*SIN(3*W-5*Y)
1800 L=L+4*TJ*COS(W-2*Y)+3*TJ*COS(2*W-4*Y)+3*COS(2*W-6*Y+3*Z)
1810 L=L-3*TJ*SIN(2*X)+3*TJ*COS(2*W-6*Y)-3*TJ*COS(2*X)
1820 L=L+3*COS(3*W-7*Y)+3*COS(4*W-9*Y)+3*SIN(3*W-6*Y)
1830 L=L+3*SIN(2*W-Y)+3*SIN(W-4*Y)+2*COS(3*(Y-Z))
1840 L=L+2*TJ*SIN(W-2*Y)+2*SIN(4*Y)-2*COS(3*W-4*Y)
1850 L=L-2*COS(2*W-Y)-2*SIN(2*W-7*Y+3*Z)+2*COS(W-4*Y)+2*COS(4*W-11*Y)
1860 L=L-2*SIN(Y-Z)
1870 B=8297*SIN(Y)-3346*COS(Y)+462*SIN(2*Y)-189*COS(2*Y)+185
1880 B=B+79*TJ*COS(Y)-71*COS(2*W-4*Y)+46*SIN(2*W-6*Y)
1890 B=B-45*COS(2*W-6*Y)+29*SIN(3*Y)-20*COS(2*W-3*Y)
1900 B=B+18*TJ*SIN(Y)-14*COS(2*W-5*Y)-11*COS(3*Y)-10*TJ
1910 B=B+9*SIN(W-3*Y)+8*SIN(W-Y)-6*SIN(2*W-3*Y)+5*SIN(2*W-7*Y)
1920 B=B-5*COS(2*W-7*Y)+4*SIN(2*W-5*Y)-4*TJ*SIN(2*Y)
1930 B=B-3*COS(W-Y)+3*COS(W-3*Y)+3*TJ*SIN(2*W-4*Y)
1940 B=B+3*SIN(W-2*Y)+2*SIN(4*Y)-2*COS(2*(W-Y))
1950 R=955774-53252*COS(Y)-1878*SIN(2*W-4*Y)-1482*COS(2*Y)
1960 R=R+817*SIN(W-Y)-539*COS(W-2*Y)-524*TJ*SIN(Y)
```

```
1970  R=R+349*SIN(2*W-5*Y)+347*SIN(2*W-6*Y)+328*TJ*COS(Y)
1980  R=R-225*SIN(Y)+149*COS(2*W-6*Y)-126*COS(2*(W-Y))
1990  R=R+104*COS(W-Y)+101*COS(2*W-5*Y)+98*COS(W-3*Y)
2000  R=R-73*COS(2*W-3*Y)-62*COS(3*Y)+42*SIN(2*Y-3*Z)
2010  R=R+41*SIN(2*(W-Y))-40*SIN(W-3*Y)+40*COS(2*W-4*Y)
2020  R=R-28*TJ-23*SIN(W)+20*SIN(2*W-7*Y)
2030  L=L+SD+X:B=B+SD:L9=L:B9=B
2040  AN(5)=9.55474700000001:EC(5)=.05554609270000001:M(5)=A(22)
2050  AP(5)=((9.78542E-04*TJ+1.08522069)*TJ+338.3077722)*RD:IN(5)=2.4886*RD
2060  AN(5)=((-1.52181E-04*TJ+.8731951400000001)*TJ+112.7903889)*RD:
      RM(5)=3498.5
2070  GOSUB 190:RP=R:RETURN
2080  :
2090  CLS :INPUT "YEAR(4 DIGITS) ";K
2110  INPUT "MONTH NUMBER ";M
2130  INPUT "DAY OF THE MONTH ";I
2150  INPUT "ZULU TIME (HH.MMSS) ";V$:GOSUB 420:UT=X
2170  INPUT "LAT (DD.MMSS) (+N/-S) ";V$:GOSUB 420:LT=X
2190  INPUT "LON (DDD.MMSS) (+W/-E) ";V$:GOSUB 420:LG=X
2210  INPUT "TEMP (DEG CELSIUS) ";DC
2230  INPUT "PRESSURE (MILLIBARS) ";MB
2240  GOSUB 2720
2250  JD=367*K-INT(7*(K+INT((M+9)/12))/4)+INT(275*M/9)+I+1721013.5
2260  CLS
2270  TJ=(JD-2415020)/36525:DT=81.94+TJ-15:TS=JD-2451545+DT/3600/24
2280  TO=TS/36525:TS=TS+UT/24:TJ=TS/36525+1
2290  TM$="(UT):":IF DT=0THEN TM$="(TDT):"
2300  GOSUB 490
2310  GM=(2.58622E-05*TO+2400.051336)*TO+6.697374560000001+1.002737909*UT
2320  MO=24:GM=FNM(GM):GS=GM-.00029*SIN(A(5))
2330  DL=(-17.23-.02*TJ)*SIN(A(5))-1.27*SIN(2*A(7))+.21*SIN(2*A(5))
2340  DL=DL-.2*SIN(2*A(1)):DL=DL/3600*RD:REM DL=NUTATION OF LONGITUDE
2350  OE=((.00181*TJ-.0059)*TJ-46.845)*TJ+84428.26+9.21*COS(A(5))
2360  OE=(OE+.552*COS(2*A(7)))*SD
2370  FOR N=0TO 5:ON N+1GOSUB 620,730,1110,1220,1360,1620
2380  IF N=0OR N=1THEN X=XH:Y=YH:Z=ZH:IF N=1THEN U=0:V=0:W=0:GOTO 2410
2390  IF N=0THEN U=UH:V=VH:W=WH:XS=X:YS=Y:ZS=Z:US=U:VS=V:WS=W:GOTO 2410
2400  X=XS+XH:Y=YS+YH:Z=ZS+ZH:U=US+UH:V=VS+VH:W=WS+WH
2410  A=-DL:GOSUB 280
2420  PRINT DN$;"TRUE POS, TRUE ECL & EQNX OF DATE ";TM$:GOSUB 320:PRINT L;B
2430  GD=R:L1=L:B1=B:IF N=0THEN LO=L
2440  C=173.142:RC=R/C:REM VELOCITY OF LIGHT
2450  X=X-RC*U:Y=Y-RC*V:Z=Z-RC*W
2460  PRINT :PRINT "APP POS, TRUE ECL & EQNX OF DATE ";TM$:
      GOSUB 320:PRINT L;B
2470  A=-OE:GOSUB 290:GOSUB 320:LP=L:BP=B:IF N=0THEN LS=L:BS=B
2480  L=L/15
2490  PRINT :PRINT "APP POS, TRUE EQU & EQNX OF DATE ";TM$:PRINT L;B
2500  MO=360:GH=FNM(15*(GS-L))
2510  PRINT :PRINT "GREENWICH HOUR ANGLE & DECL ";TM$
2520  X=GH:GOSUB 360:L$=V$:X=B:GOSUB 360:PRINT L$+"'";SPC(3); V$+"'"
2530  LH=(GH-LG)*RD:CL=COS(LH):SL=SIN(LH):BD=B*RD:SB=SIN(BD):CB=COS(BD)
```

```
2540  LD=LT*RD:CX=COS(LD):SX=SIN(LD):X=SX*SB+CX*CB*CL:GOSUB 150:
      A9=AS:AL=AS/RD
2550  Y=-CB*SL:X=SB*CX-CB*SX*CL:GOSUB 120:AZ=A2/RD
2560  PRINT :PRINT "ALTITUDE = ";AL;:X=AL:GOSUB 360:PRINT SPC(5);V$+"'"
2565  PRINT "AZIMUTH = ";AZ;:X=AZ:GOSUB 360:PRINT SPC(5);V$+"'"
2570  ON NGOSUB 2780,2780,2780,2860,2870
2580  IF N=1THEN PRINT "PHASE";K;" AGE =";AG;" DAYS
2590  IF N>1THEN MO=10:PRINT "MAGNITUDE = ";FNR(MG)
2600  IF N=0THEN S=15.994/R:MO=10:PRINT "SD =";STR$(FNR(S))+"'"
2610  IF N<>1THEN 2660
2620  D=23454.8*R:S=936.75/D:MO=10:PRINT "SD =";STR$(FNR(S))+"'";
2630  S=S*(1+SIN(A9)/D):PRINT " SD AUG =";STR$(FNR(S))+"'"
2640  X=COS(A9)/D:GOSUB 150:PA=AS/RD*60
2650  PRINT "P IN A = ";STR$(FNR(PA))+"'"
2660  RF=1/TAN((AL+7.31/(AL+44.4))*RD)
2670  RF=RF*((MB-80)/930)/(1+.00008*(RF+39)*(DC-10))
2680  RF=RF-.06*SIN((14.7*RF+13)*RD):MO=10:RF=FNR(RF)
2690  PRINT "REFRACTION = ";STR$(RF)+"'"
2700  GOSUB 2720:CLS :NEXT N:GOTO 2090
2710  :
2720  PRINT :PRINT "PRESS C TO CONTINUE"
2730  C$=INKEY$ :IF C$=""THEN 2730
2740  IF C$="C"THEN RETURN
2760  GOTO 2730
2770  :
2780  CD=SIN(BS*RD)*SIN(BP*RD)+COS(BS*RD)*COS(BP*RD)*COS((LS-LP)*RD)
2790  X=CD:GOSUB 160:Y=RR*SIN(AC):X=GD-RR*CD:GOSUB 120:PA=ABS(A2/RD)
2800  ON NGOTO 2810,2840,2850
2810  K=(1+COS(A2))/2:SN=SIN((L0-L1)*RD):X=SQR(K):GOSUB 150
2820  AG=29.53*AS/PI:IF SN>0THEN AG=29.53-AG
2830  MO=100:K=FNR(K):MO=1:AG=FNR(AG):RETURN
2840  MG=(PA*PA*4.247E-07+.01322)*PA+2.1715*LOG(GD*RP)-4:RETURN
2850  MG=.01486*PA+2.1715*LOG(GD*RP)-1.3:RETURN
2860  MG=2.1715*LOG(GD*RP)-8.93:RETURN
2870  B0=(168.1176+1.394091*TJ)*RD:II=(28.0743-.0127991*TJ)*RD
2880  NN=((.242202*TJ+3.98599)*TJ+126.3629)*RD
2890  JJ=((.0171656*TJ-.4549142)*TJ+6.912935)*RD
2900  S0=(-.239992*TJ-2.731279)*TJ+42.92039
2910  CB=COS(B9):Y=SIN(II)*SIN(B9)+COS(II)*CB*SIN(L9-B0)
2920  X=CB*COS(L9-B0):GOSUB 120:UP=A2/RD
2930  D=BP*RD:CB=COS(D):SB=SIN(D):SJ=SIN(JJ):CJ=COS(JJ)
2940  AG=LP*RD-NN:SN=SIN(AG)
2950  Y=SJ*SB+CJ*CB*SN:X=CB*COS(AG):GOSUB 120:UU=A2/RD
2970  BB=SJ*CB*SN-CJ*SB
2980  MG=2.1715*LOG(GD*RP)-8.68+.044*ABS(UP+S0-UU)-2.6*ABS(BB)+1.25*BB*BB
2990  RETURN
```

D. Program Annotation—NAVEPHEM

Line(s)	Usage
30	P2 = $\pi/2$, PI = π, TP = 2π, RD = degree–to–radian conversion.
40	EK = Sun's gravitational constant. MB$ = atmospheric pressure in millibar's (1010 is the default value), DC$ = temperature in degrees Celsius (10 is the default value).
50	FNA converts angles in revolutions to degrees between 0 and 2π, FNR rounds to the nearest M0.
60	FNM is the X modulus M0 function.
70-80	B$ is an array of planet names.
90	GOTO 2090 to commence execution.
120	Two argument arctangent function with output interval of 0 to 2π. The −1E–9 must be replaced by +1E–9 in the PC–1500A.
150	The arcsine function can be replaced inline by the ASN function in the PC–1500A.
160	The arccosine function can be replaced inline by the ACS function in the PC–1500A.
190-200	Unscale the radius vector by a factor of 1E5. Convert spherical coordinates to rectangular coordinates.
210	Return if the body is the moon.
220-260	MU is the reduced mass of the body. Compute the approximate heliocentric velocity vector of the body.
280	Z–axis rotation.
290	X–axis rotation.
320-330	Convert rectangular coordinates to spherical coordinates. L = longitude, B = latitude and R = distance.
360-390	Convert decimal degrees to degrees, minutes and tenth minute notation.
420-460	Convert DDD.MMSS or HH.MMSS format to decimal.
490-600	Compute the fundamental planetary arguments [Ref. 10].
620-720	Compute the geocentric spherical position and velocity of the Sun. Convert to geocentric rectangular.
720-1100	Compute the geocentric spherical and rectangular coordinates of the Moon.
1110-1210	Compute the heliocentric spherical position and velocity of Venus. Convert to heliocentric rectangular.
1220-1350	Compute the heliocentric spherical position and velocity of Mars. Convert to heliocentric rectangular.
1360-1610	Compute the heliocentric spherical position and velocity of Jupiter. Convert to heliocentric rectangular.
1620-2070	Compute the heliocentric spherical position and velocity of Saturn. Convert to heliocentric rectangular. Saturn's longitude and latitude are saved as L9 and B9 on line 2030.
2090-2240	Input prompts should be designed so that default values or previously entered values are displayed. Pressing the return key will input a displayed value. If

any value is changed, then the *entire* value must be keyed in. This feature had to be removed on the TRS–80 Model–4 version since it was difficult to implement.

2250 Computation of the Julian Day Number.

2270 TJ = number of Julian centuries from noon, January 0, 1900 to midnight of input date. DT = ΔT correction factor. [NOTE: This equation should be examined for accuracy yearly—see text]. TS = number of Julian days from noon, January 1, 2000 to 0000 hours Universal Time of the input date.

2280 T0 = number of Julian centuries from noon, January 1, 2000 to 0000 hours Universal Time (UT) of the input date. TS = number of Julian days from noon, January 1, 2000 to the input date and time. TJ = number of Julian centuries from noon, January 0, 1900 to the input date and time.

2290 If the ΔT correction is set to zero, then the positions are referenced to TDT (terrestial dynamic time) rather than to UT.

2300 Compute fundamental arguments.

2310 GM = Greenwich Mean Sidereal Time.

2320 GS = Greenwich Apparent Sidereal Time.

2330-2340 DL = nutation of longitude.

2350-2360 OE = obliquity of the ecliptic corrected for nutation.

2370 FOR loop to cycle through Sun, Moon and planets. N = body number. The loop ends on line 2700.

2380 X, Y and Z are the geocentric coordinates of the Sun or the Moon. Set the velocity components of the Moon equal to zero.

2390 Save the position (XS, YS & ZS) and velocity (US, VS & WS) components of the Sun.

2400 Compute the geocentric position and velocity components of the Nth planet.

2410 Correct position for nutation.

2420 Display true position for the true ecliptic and equinox of date.

2430 Save true distance GD, longitude L1, latitude B1 and solar longitude L0.

2440-2450 Correct position for aberration (light time).

2460 Display apparent position for the true ecliptic and equinox of date.

2470 Convert ecliptic coordinates to equatorial coordinate. Save the right ascension LP and declination BP of the body. For the Sun, save right ascension and declination as LS and BS.

2480 Convert right ascension from degrees to hours.

2490 Display apparent position for the true equator and equinox of date.

2500 GH = Greenwich Hour angle.

2510-2520 Display Greenwich Hour angle and declination.

2530-2565 Compute and display the altitude and azimuth. The altitude is saved as A9 on line 2540.

2570 Subroutine call for N = 1 through 5.

2580 Display the Moon's phase and age.

2590 Display the planet's magnitude.

2600 Compute and display the Sun's semidiameter [Ref. 2, pg. B16].

2610 Transfer if the body is not the Moon.
2620 Compute and display the Moon's semidiameter [Ref. 2, pg. B16].
2630 Compute and display the Moon's augmented semidiameter [Ref. 2, pg. B16].
2640-2650 Compute and display the Moon's parallax in altitude [Ref. 2, pg. B16].
2660-2690 Compute and display the refraction correction at the body's altitude [Ref. 2, pp. B14–B15].
2700 Continue prompt. End of FOR–NEXT loop on body number N.
2720-2760 Continue prompt query.
2780-2790 Compute the phase angle PA. CD is the cosine of the elongation [Ref. 4, pg. 312].
2800 Transfer for the Moon, Venus or Mars.
2810 Compute the Moon's phase angle K [Ref. 4, pg. 311].
2820 AG is an approximation to the Moon's age developed by the author.
2830 Round the Moon's age to the nearest day and return.
2840 MG is the magnitude of Venus [Ref. 4, pg. 314].
2850 MG is the magnitude of Mars [Ref. 4, pg. 314].
2860 MG is the magnitude of Jupiter [Ref. 4, pg. 314].
2870-2980 Compute the magnitude MG of Saturn. This computation is complicated by the varying aspect of Saturn's rings. Details follow.
2870-2900 Computation of the ring's ascending node BO, inclination II, right ascension NN, inclination to the mean equator JJ and arc SO from NN to BO. These equations result from curve fitting the table in Reference 4, page 365.
2910-2920 Computation of UP = U′ [Ref. 4, pg. 364].
2930-2970 Computation of UU = U and BB = sin B [Ref. 4, pg. 345].
2980 MG is the magnitude of Saturn [Ref. 4, pg. 314].

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APPENDIX: The QATN Function

This routine is the standard arctangent function corrected for quadrant. The quadrant arctangent function is occasionally implemented as the ATAN2 function, the ANGLE function or the Rectangular-to-Polar function.

Entering variables are the x- and y-coordinates, X and Y. The exiting variable is the angle Θ, where $-\pi < \Theta \leq \pi$. Use of the quadrant arctangent function is denoted by $\Theta = \text{qatn}(Y, X)$.

1. If $X \neq 0$, go to step 4.
2. Set $\Theta = (\pi/2) * \text{sgn}(Y)$.
3. Go to step 8.
4. Set $\Theta = \arctan(Y/X)$.
5. If $X > 0$, go to step 8.
6. Set $\Theta = \Theta + \pi * \text{sgn}(Y)$.
7. If $Y = 0$, set $\Theta = \pi$.
8. Return.

Note:

If $Y > 0$ then $\text{sgn}(Y) = +1$.
If $Y = 0$ then $\text{sgn}(Y) = 0$.
If $Y < 0$ then $\text{sgn}(Y) = -1$.

Users of Microsoft BASIC can simplfy the qatn function significantly by using the code given below. To return an angle of Θ (designated by A in the code) in the range of $(-\pi, \pi)$, use:

```
PI = 4*ATN(1):  TP = PI + PI: EPS = 1E-33
A = ATN(Y/(X-EPS*(X=0))) - PI*(X<0)*(SGN(Y) - (Y=0))
```

To return a value of A in the range of $(0, 2\pi)$, use:

```
PI = 4*ATN(1):  TP = PI + PI: EPS = 1E-33
A = ATN(Y/(X-EPS*(X=0))) - PI*(X<0) + TP*(X >= 0)*(Y<0)
```

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