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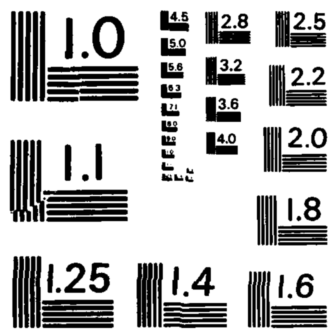

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EALIZED SIGNAL TO NOISE RATIO WITH AN ESTIMATED DISCRIMINANT FUNCTION

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Key Words and Phrases: Confluent hypergeometric distribution; discriminant function; signal to noise ratio.

ABSTRACT

Percentage points of a new distribution involving a confluent-hypergeometric distribution obtained by Khatri and Rao (1985) are tabulated. The use of the tabulated values in obtaining a lower confidence bound for the realized signal to noise ratio based on an estimated discriminant function for signal detection is explained.

1. INTRODUCTION

Let X be a random variable having one of two possible p-variate normal distributions $N_p(0,\Sigma)$ and $N_p(\delta,\Sigma)$ when X is real (or $\tilde{N}_p(0,\Sigma)$ and $\tilde{N}_p(\delta,\Sigma)$ when X is complex). In the sequel, we follow the practice of giving the results for the real case first, and then for the complex case within brackets. For exact expressions of the p-variate normal density in the real and complex cases reference may be made to Khatri and Rao (1985). We consider the case where δ is known and Σ is unknown but an estimate $f^{-1}S$ of Σ

is available, where S has the Wishart distribution $W_p(f,\Sigma)$ (or $\tilde{W}_p(f,\Sigma)$) on f degrees of freedom. In such a case, the estimated linear discriminant function is proportional to $\delta' S^{-1} X$ (or $\delta^* S^{-1} X$). The discriminatory power or the probability of correct classification when the estimated discriminant function is used on future samples is a monotonic function of the signal to noise ratio

$$\rho(S,\Sigma) = \frac{(\delta' S^{-1}\delta)^2}{\delta' S^{-1}\Sigma S^{-1}\delta}, \text{ (or } \tilde{\rho}(S,\Sigma) = \frac{(\delta^* S^{-1}\delta)^2}{\delta^* S^{-1}\Sigma S^{-1}\delta}) \tag{1.1}$$

which involves the unknown Σ. By the Cauchy-Schwartz inequality, (1.1) is $\leq \delta'\Sigma^{-1}\delta$ (or $\delta^*\Sigma^{-1}\delta$), the signal to noise ratio when Σ is known so that there is some loss of information in using an estimated discriminant function. The problem is to make an inferential statement on the realized ratio (1.1) in terms of known quantities δ, S and f. This can be done in several ways as shown in the next section.

2. INFERENCE ON SIGNAL TO NOISE RATIO

The key result for this purpose is Theorem 1 in Khatri and Rao (1985) where it is shown that in the real case

$$B = \frac{(\delta' S^{-1}\delta)^2}{(\delta'\Sigma^{-1}\delta)(\delta' S^{-1}\Sigma S^{-1}\delta)} \quad \text{and} \quad G = \frac{\delta'\Sigma^{-1}\delta}{\delta' S^{-1}\delta} \tag{2.1}$$

are independently distributed with the p.d.f. (probability density function) of B as

$$\frac{\Gamma(\frac{f+1}{2})}{\Gamma(\frac{f-p+2}{2})\Gamma(\frac{p-1}{2})} b^{(f-p)/2} (1-b)^{(p-3)/2} \tag{2.2}$$

and that of G as

$$\frac{1}{2^{(f-p+1)/2}\Gamma(\frac{f-p+1}{2})} e^{-g/2} g^{(f-p+1)/2}. \tag{2.3}$$

In the complex case, defining $\tilde{B}$ and $\tilde{G}$ with δ' replaced by δ^* in (2.1), it is shown that $\tilde{B}$ and $\tilde{G}$ are independently distributed, with the p.d.f. of $\tilde{B}$ as

$$\frac{\Gamma(f+1)}{\Gamma(f-p+2)\Gamma(p-1)} \tilde{b}^{(f-p+1)} (1-\tilde{b})^{p-2} \tag{2.4}$$

and that of $\tilde{G}$ as

$$\frac{1}{\Gamma(f-p+1)} e^{-g} g^{f-p}. \tag{2.5}$$

The distribution (2.4) was earlier obtained by Reed, Mallet and Brennan (1974). The distributions (2.3)-(2.5) are independent of the unknown parameters which enables inferences to be drawn on (1.1) through the <u>pivotal statistics</u> B and G (or $\tilde{B}$ and $\tilde{G}$).

1) Using the expressions for the moments of the beta distribution (Rao, 1973, p. 168)

$$E[B(\text{or } \tilde{B})] = \frac{f-p+2}{f+1}$$

which gives

$$E[\rho(S,\Sigma)(\text{or } \tilde{\rho}(S,\Sigma))] = \frac{f-p+2}{f+1}\{\delta'\Sigma^{-1}\delta(\text{or } \delta^*\Sigma^{-1}\delta)\}. \tag{2.6}$$

If the average efficiency is to be maintained at about half the optimal efficiency, then from (2.6) we have

$$\frac{f-p+2}{f+1} \simeq \frac{1}{2} \text{ or } f \simeq 2p \tag{2.7}$$

for both the real and complex cases. The result for the complex case is mentioned in Reed, Mallet and Brennan (1974). Similarly we can equate the ratio in (2.7) to any desired ratio other than (½) and obtain an expression for the degrees of freedom f needed for the estimation of Σ.

2) Perhaps a more satisfactory way of using the distributions (2.2) and (2.4) is as follows. Let b_α (or $\tilde{b}_\alpha$) be the lower α% point of

the distribution (2.2) (or (2.4)). Then we can make the confidence statement that

$$\rho(S,\Sigma) \geq b_\alpha \delta'\Sigma^{-1}\delta \ (\text{or } \tilde{\rho}(S,\Sigma) \geq \tilde{b}_\alpha \delta^*\Sigma^{-1}\delta) \tag{2.8}$$

with a confidence coefficient of $(1-\alpha)\%$.

3) The results (2.7) and (2.8) still involve the unknown quantity Σ. We raise the question whether the actual magnitude of $\rho(S,\Sigma)$ for given S can be assessed through known quantities. Using the joint distribution of B and G as in (2.2) and (2.3), or $\tilde{B}$ and $\tilde{G}$ as in (2.4) and (2.5), it is shown in Khatri and Rao (1985) that the estimate

$$\hat{\rho}(S,\Sigma) = \frac{(f-p+2)(f-p-3)}{f(f+1)} D_p^2 \ \left(\text{or } \hat{\tilde{\rho}}(S,\Sigma) = \frac{(f-p+2)(f-p-1)}{f(f+1)} D_p^2\right)$$

where $D_p^2 = f\delta'S^{-1}\delta$ (or $f\delta^*S^{-1}\delta$) has the property $\underset{g}{\text{Min}}\ E[\rho(S,\Sigma) - gD_p^2]^2$

$$= E\,[\rho(S,\Sigma) \ - \ \rho(S,\Sigma)]^2 \ (\text{or} = E\,[\tilde{\rho}(S,\Sigma) \ - \ \hat{\tilde{\rho}}(S,\Sigma)]^2).$$

so that $\hat{\rho}(S,\Sigma)$ (or $\hat{\tilde{\rho}}(S,\Sigma)$) is a predictor of $\rho(S,\Sigma)$ (or $\tilde{\rho}(S,\Sigma)$)

4) Khatri and Rao (1985) also obtained an exact confidence bound for $\rho(S,\Sigma)$ for the computation of which we provide extensive tables in this paper.

We define the random variable

$$Z = \frac{1}{2}BG = \frac{f}{2}\frac{\rho(S,\Sigma)}{D_p^2} \ \left(\text{or } \tilde{Z} = f\,\frac{\tilde{\rho}(S,\Sigma)}{D_p^2}\right)$$

which has the confluent hypergeometric distribution with the probability density function

$$\frac{e^{-z}\,z^{m-1}}{\Gamma(m)}\,\frac{\Gamma(a+b)}{\Gamma(a)}\,\Psi(b,m-a+1;z).$$

where $m = (f-p+1)/2$, $a = (f-p+2)/2$, $b = (p-1)/2$, $m-a+1 = 1/2$

(or $m = f-p+1$, $a = f-p+2$, $b = p-1$, $m-a+1 = 0$), and

$$\Psi(b,c;z) = \frac{1}{\Gamma(b)} \int_0^\infty t^{b-1}(1+t)^{c-b-1}\exp(-zt)dt$$

as described in Khatri and Rao (1985, Theorem 3 and Remark 5). If z_α (or $\tilde{z}_\alpha$) is the lower $\alpha\%$ point of this distribution, then

$$\rho(S,\Sigma) \geq \frac{2z_\alpha}{f} D_p^2, \text{ (or } \tilde{\rho}(S,\Sigma) \geq \frac{\tilde{z}_\alpha}{f} D_p^2)$$

provides the lower confidence bound to the signal to noise ratio $\rho(S,\Sigma)$ (or $\tilde{\rho}(S,\Sigma)$) with a confidence coefficient of $(1-\alpha)\%$. Tables 1-5 given in the next section give the values of $2z_\alpha$ (or $\tilde{z}_\alpha$) for various combinations of p and f, and $\alpha = 0.05, 0.25, 0.50, 0.75$ and 0.95.

3. PERCENTAGE POINTS AND SOME APPROXIMATIONS

Tables 1-5 give the lower $\alpha\%$ (for $\alpha\% = 5, 25, 50, 75$ and 95) points of the distributions of 2z and $\tilde{z}$ for different-values of p and f. Actually z_α is obtained as a solution to the equation

$$\alpha = \int_0^1 \frac{\Gamma(\frac{f+1}{2})}{\Gamma(\frac{p-1}{2})\Gamma(\frac{f-p+2}{2})} y^{(f-p)/2}(1-y)^{(p-3)/2}dy \int_0^{z_\alpha/y} \frac{1}{\Gamma(\frac{f-p+1}{2})} e^{-x} x^{(f-p-1)/2}dx$$

and $\tilde{z}_\alpha$ to the equation

$$\alpha = \int_0^1 \frac{\Gamma(f+1)}{\Gamma(p-1)\Gamma(f-p+2)} y^{f-p+1}(1-y)^{p-2}dy \int_0^{\tilde{z}_\alpha/y} \frac{1}{\Gamma(f-p+1)} e^{-x} x^{f-p}dx.$$

To obtain $(1-\alpha)\%$ lower confidence bound, we multiply the Mahalanobis distance $D_p^2 = f\delta'S^{-1}\delta$ (or $f\delta^*S^{-1}\delta$) by $2z_\alpha/f$ (or $\tilde{z}_\alpha/f$).

We give several approximations to the distributions of z (or $\tilde{z}$) from which fairly approximate values of z_α (or $\tilde{z}_\alpha$) can be easily obtained if p/f is not large.

(i) Gamma approximation

(a) We consider the statistic

$$g(p,f)z \quad (\text{or } \tilde{g}(p,f)\tilde{z}) \tag{3.1}$$

and approximate its distribution by a gamma distribution $G(1,\nu)$ or $G(1,\tilde{\nu})$).

To determine $g(p,f)$ and ν we equate the first two moments of $g(p,f)z$ with those of $G(1,\nu)$. The equations are

$$E\,[\,g(p,f)z] = g(p,f)E(z) = \nu$$

$$V\,[\,g(p,f)z] = [\,g(p,f)\,]^2 V(z) = \nu$$

which give

$$g(p,f) = \frac{E(z)}{V(z)} = \frac{(f+1)(f+3)}{(f+1)(f+3)-p(p-1)}$$

$$\nu = \frac{[\,E(z)\,]^2}{V(z)} = \frac{f-p+1}{2}\,\frac{(f+3)(f-p+2)}{(f+1)(f+3)-p(p-1)}\,.$$

Similarly

$$\tilde{g}(p,f) = \frac{E(\tilde{z})}{V(\tilde{z})} = \frac{(f+1)(f+2)}{(f+1)(f+2)-p(p-1)}$$

$$\tilde{\nu} = \frac{[\,E(\tilde{z})\,]^2}{V(\tilde{z})} = \frac{(f+2)(f-p+1)(f-p+2)}{(f+1)(f+2)-p(p-1)}$$

(b) We consider the statistic

$$g(p,f)z^c \quad (\text{or } \tilde{g}(p,f)\tilde{z}^c \tag{3.2}$$

and approximate its distribution by a gamma distribution $G(1,\nu)$ (or $G(1,\tilde{\nu})$). To determine $g(p,f)$, c and ν, we equate the first three moments of $g(p,f)z^c$ with those of $G(1,\nu)$. Taking $m = (f-p+1)/2$, $a = (f-p+2)/2$ and $b = (p-1)/2$, the equations are

$$\nu = g(p,f)\Gamma(m+c)\Gamma(a+c)\Gamma(a+b)/\{\Gamma(m)\Gamma(a)\Gamma(a+b+c)\},$$

$$\nu+1 = g(p,f)\Gamma(m+2c)\Gamma(a+2c)\Gamma(a+b+c)/\{\Gamma(m+c)\Gamma(a+c)\Gamma(a+b+2c)\}$$

$$\nu+2 = g(p,f)\Gamma(m+3c)\Gamma(a+3c)\Gamma(a+b+2c)/\{\Gamma(m+2c)\Gamma(a+2c)\Gamma(a+b+3c)\}.$$

These equations give

$$\frac{\Gamma(m+3c)\Gamma(a+3c)\Gamma(a+b+2c)}{\Gamma(m+2c)\Gamma(a+2c)\Gamma(a+b+3c)} + \frac{\Gamma(m+c)\Gamma(a+c)\Gamma(a+b)}{\Gamma(m)\Gamma(a)\Gamma(a+b+c)}$$

$$= 2\,\frac{\Gamma(m+2c)\Gamma(a+2c)\Gamma(a+b2c)}{\Gamma(m+c)\Gamma(a+c)\Gamma(a+b+2c)}$$

from which a solution for c is computed using an appropriate computer program.

Similarly, for the complex situation, we take $m = f-p+1$, $a = f-p+2$ and $b = p-1$, and determine the values of $\tilde{c}$, $\tilde{g}(p,f)$ and $\tilde{\nu}$ as above. The values of these constants for the real and complex cases are given in Table 6 for some values of p and f.

We find by comparing with actual values that the approximation given by (3.2) is more accurate than that given by (3.1), and the approximation is fairly accurate even for small values of f and p.

(ii) Normal approximation

(a) Using the Wilson-Hilferty's approximation as modified by Konishi (1981), we have

$$(9\nu)^{\frac{1}{2}}\{(\frac{g(p,f)z}{2\nu})^{1/3} - 1+ \frac{1}{9\nu}\} \sim N(0,1)$$

$$[\text{ or } (9\tilde{\nu})^{\frac{1}{2}}\{(\frac{\tilde{g}(p,f)\tilde{z}}{2\tilde{\nu}})^{1/3} - 1 + \frac{1}{9\tilde{\nu}}\} \sim N(0,1)].$$

where $g(p,f)$ and ν (or $\tilde{g}(p,f)$ and $\tilde{\nu}$) are defined in (3.1).

(b) As in (a), we can take

$$(9\nu)^{\frac{1}{2}}\{(\frac{g(p,f)z^{c}}{2\nu})^{1/3} - 1 + \frac{1}{9\nu}\} \sim N(0,1)$$

$$[\text{ or } (9\tilde{\nu})^{\frac{1}{2}}\{(\frac{\tilde{g}(p,f)\tilde{z}^{\tilde{c}}}{2\tilde{\nu}})^{1/3} - 1 + \frac{1}{9\tilde{\nu}}\} \sim N(0,1)]$$

where $g(p,f)$, ν and c (or $\tilde{g}(p,f)$, $\tilde{\nu}$ and $\tilde{c}$) are defined in (3.2). The normal approximation is fairly accurate even for small values of f and p.

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TABLE I. Values of $2z_\alpha$ and $\tilde{z}_\alpha$ for $\alpha = 0.05$ and various combinations of p (number of variables) and f (degrees of freedom). ($2z_\alpha$ is the upper value and $\tilde{z}_\alpha$ is the lower value)

p \ f	5	10	15	20	25	30	35	40	45	50	60	70	80	90	100
2 :	0.512	2.876	5.952	9.232	13.150	16.976	20.900	24.900	29.026	33.100	41.524	50.048	58.678	67.394	76.184
:	1.050	4.098	7.813	11.725	15.775	19.950	24.206	28.507	32.854	37.239	46.099	55.056	64.088	73.181	82.325
3 :	0.172	2.026	4.900	8.214	11.788	15.550	19.450	23.426	27.450	31.550	39.896	48.386	56.992	65.688	74.458
:	0.456	3.064	6.488	10.288	14.275	18.413	22.604	26.872	31.193	35.555	44.382	53.312	62.324	71.401	80.531
4 :	0.030	1.338	3.912	7.052	10.500	14.176	18.026	21.950	25.950	30.026	38.304	46.758	55.332	64.004	72.756
:	0.131	2.164	5.325	8.975	12.875	16.925	21.063	25.291	29.579	33.915	42.699	51.599	60.587	69.644	78.758
5 :	---	0.832	3.076	6.028	9.326	12.900	16.650	20.500	24.500	28.500	36.748	45.160	53.700	62.346	71.074
:	---	1.450	4.288	7.750	11.525	15.475	19.581	23.762	28.011	32.316	41.053	49.917	58.876	67.911	77.007
6 :	---	0.460	2.350	5.078	8.226	11.700	15.350	19.150	23.050	27.050	35.230	43.592	52.096	60.710	69.414
:	---	0.894	3.375	6.625	10.275	14.150	18.159	22.285	26.491	30.760	39.441	48.264	57.192	66.202	75.277
7 :	---	0.216	1.726	4.214	7.176	10.550	14.100	17.850	21.700	25.650	33.746	42.054	50.518	59.100	67.776
:	---	0.488	2.575	5.613	9.088	12.850	16.796	20.860	25.017	29.245	37.864	46.641	55.534	64.516	73.568
8 :	---	0.076	1.226	3.450	6.226	9.450	12.950	16.576	20.376	24.300	37.296	40.548	48.966	57.512	66.160
:	---	0.217	1.906	4.675	7.994	11.650	15.491	19.487	23.589	27.773	36.322	45.049	53.902	62.853	71.879
9 :	---	0.014	0.826	2.776	5.350	8.450	11.826	15.376	19.100	22.950	30.884	39.072	47.442	55.948	64.564
:	---	0.064	1.338	3.838	6.975	10.488	14.244	18.164	22.206	26.341	34.815	43.486	52.297	61.213	70.212
10 :	---	---	0.512	2.176	4.550	7.476	10.750	14.226	17.900	21.650	29.506	37.626	45.944	54.408	62.990
:	---	---	0.884	3.094	6.038	9.413	13.054	16.893	20.869	24.951	33.343	41.953	50.717	59.597	68.566
11 :	---	---	0.288	1.676	3.812	6.588	9.726	13.100	16.700	20.426	28.162	36.210	44.472	52.892	61.436
:	---	---	0.531	2.438	5.175	8.388	11.921	15.671	19.577	23.601	31.904	40.449	49.164	58.004	66.940
12 :	---	---	0.136	1.238	3.150	5.776	8.776	12.050	15.550	19.226	26.854	34.824	43.026	51.398	59.906
:	---	---	0.278	1.863	4.375	7.425	10.844	14.500	18.330	22.292	30.500	38.976	47.636	56.433	65.335
13 :	---	---	0.048	0.882	2.576	5.012	7.876	11.050	14.450	18.050	25.580	33.468	41.606	49.930	58.394
:	---	---	0.113	1.375	3.650	6.538	9.822	13.378	17.128	21.024	29.130	37.531	46.134	54.886	63.752
14 :	---	---	0.008	0.594	2.050	4.300	7.026	10.100	13.426	16.900	24.340	32.142	40.214	48.482	56.904
:	---	---	0.026	0.969	3.006	5.713	8.856	12.305	15.969	19.796	27.794	36.116	44.659	53.362	62.188
15 :	---	---	---	0.376	1.594	3.650	6.250	9.188	12.426	15.850	23.134	30.844	38.846	47.060	55.436
:	---	---	---	0.644	2.425	4.938	7.944	11.280	14.855	18.607	26.492	34.730	43.208	51.861	60.646
20 :	---	---	---	---	0.228	1.288	3.050	5.350	8.026	11.026	17.606	24.796	32.396	40.290	48.404
:	---	---	---	---	0.503	1.988	4.179	6.874	9.931	13.257	20.483	28.234	36.340	44.697	53.242
25 :	---	---	---	---	---	0.184	1.076	2.626	4.676	7.126	12.896	19.464	26.580	34.088	41.888
:	---	---	---	---	---	0.413	1.681	3.621	6.059	8.870	15.295	22.454	30.103	38.098	46.349
30 :	---	---	---	---	---	---	0.152	0.926	2.300	4.150	8.976	14.830	21.386	28.446	35.880
:	---	---	---	---	---	---	0.348	1.458	3.195	5.416	10.911	17.375	24.488	32.057	39.961
35 :	---	---	---	---	---	---	---	0.130	0.806	2.038	5.820	10.872	16.800	23.354	30.372
:	---	---	---	---	---	---	---	0.302	1.288	2.858	7.309	12.985	19.486	26.567	34.073
40 :	---	---	---	---	---	---	---	---	0.114	0.718	3.396	7.576	12.808	18.800	25.356
:	---	---	---	---	---	---	---	---	0.266	1.153	4.468	9.271	15.088	21.622	28.680
45 :	---	---	---	---	---	---	---	---	---	0.100	1.666	4.916	9.398	14.776	20.826
:	---	---	---	---	---	---	---	---	---	0.239	2.360	6.215	11.282	17.213	23.777
50 :	---	---	---	---	---	---	---	---	---	---	0.582	2.870	6.552	11.272	16.772
:	---	---	---	---	---	---	---	---	---	---	0.953	3.802	8.059	13.333	19.357

TABLE II. Same as in Table I for $\alpha = 0.25$

p \ f	5	10	15	20	25	30	35	40	45	50	60	70	80	90	100
2	1.488	5.238	9.348	13.416	18.212	22.726	27.276	31.876	36.476	41.126	50.458	59.848	69.282	78.754	88.254
	2.028	6.070	10.544	15.075	19.688	24.338	29.024	33.732	38.460	43.205	52.733	62.302	71.902	81.528	91.175
3	0.676	3.950	7.938	12.180	16.562	21.026	25.550	30.126	34.726	39.326	48.650	58.022	67.440	76.900	86.390
	1.047	4.733	8.981	13.438	17.988	22.600	27.254	31.941	36.652	41.382	50.888	60.440	70.028	79.644	89.283
4	0.204	2.856	6.600	10.704	15.012	19.426	23.900	28.426	33.000	37.600	46.878	56.224	65.626	75.070	84.548
	0.402	3.527	7.556	11.888	16.363	20.919	25.543	30.200	34.888	39.600	49.077	58.609	68.180	77.784	87.412
5	---	1.950	5.388	9.342	13.538	17.876	22.300	26.776	31.326	35.888	45.140	54.456	63.836	73.262	82.726
	---	2.507	6.256	10.438	14.819	19.313	23.889	28.511	33.170	37.858	47.300	56.806	66.358	75.945	85.561
6	---	1.234	4.312	8.116	12.138	16.388	20.756	25.200	29.700	34.238	43.436	52.720	62.072	71.478	80.926
	---	1.669	5.088	9.088	13.356	17.775	22.293	26.872	31.497	36.158	45.557	55.032	64.561	74.130	83.731
7	---	0.692	3.362	6.892	10.826	14.976	19.276	23.662	28.126	32.626	41.766	51.012	60.334	69.716	79.146
	---	1.013	4.044	7.838	11.975	16.300	20.755	25.283	29.869	34.498	43.849	53.288	62.789	72.337	81.920
8	---	0.318	2.538	5.814	9.588	13.626	17.850	22.162	26.588	31.050	40.132	49.332	58.622	67.978	77.386
	---	0.525	3.125	6.681	10.669	14.900	19.274	23.745	28.286	32.878	42.174	51.573	61.043	70.567	80.131
9	---	0.098	1.838	4.838	8.426	12.350	16.476	20.726	25.088	29.526	38.530	47.684	56.936	66.262	75.646
	---	0.203	2.328	5.625	9.444	13.563	17.851	22.258	26.748	31.299	40.532	49.886	59.323	68.819	78.361
10	---	---	1.262	3.950	7.344	11.138	15.162	19.350	23.650	28.026	36.964	46.064	55.276	64.570	73.928
	---	---	1.653	4.656	8.300	12.288	16.484	20.820	25.254	29.760	38.925	48.229	57.627	67.094	76.612
11	---	---	0.800	3.168	6.350	9.994	13.912	18.012	22.250	26.576	35.432	44.472	53.640	62.900	72.230
	---	---	1.099	3.788	7.225	11.075	15.173	19.432	23.805	28.262	37.351	46.600	55.957	65.392	74.883
12	---	---	0.450	2.472	5.426	8.912	12.712	16.726	20.888	25.150	33.932	42.910	52.030	61.252	70.552
	---	---	0.659	3.013	6.238	9.931	13.919	18.094	22.400	26.803	35.810	45.001	54.313	63.712	73.175
13	---	---	0.208	1.868	4.578	7.894	11.576	15.488	19.576	23.788	32.468	41.378	50.446	59.628	68.894
	---	---	0.335	2.331	5.319	8.850	12.722	16.805	21.040	25.384	34.304	43.430	52.694	62.054	71.487
14	---	---	0.064	1.356	3.800	6.938	10.488	14.312	18.312	22.450	31.038	39.874	48.888	58.026	67.258
	---	---	0.122	1.738	4.475	7.838	11.580	15.566	19.723	24.006	32.830	41.888	51.099	60.419	69.819
15	---	---	---	0.932	3.106	6.056	9.462	13.162	17.088	21.162	29.640	38.400	47.356	56.448	65.642
	---	---	---	1.234	3.713	6.888	10.494	14.376	18.451	22.667	31.390	40.375	49.531	58.807	68.171
20	---	---	---	---	0.706	2.560	5.150	8.226	11.638	15.300	23.156	31.462	40.072	48.892	57.866
	---	---	---	---	0.984	3.081	5.890	9.157	12.746	16.568	24.690	33.238	42.064	51.081	60.236
25	---	---	---	---	---	0.580	2.176	4.476	7.268	10.426	17.502	25.240	33.418	41.898	50.598
	---	---	---	---	---	0.817	2.640	5.143	8.123	11.449	18.816	26.815	35.223	43.913	52.804
30	---	---	---	---	---	---	0.492	1.894	3.956	6.512	12.666	19.728	27.390	35.462	43.834
	---	---	---	---	---	---	0.702	2.306	4.564	7.299	13.760	21.099	29.005	37.300	45.873
35	---	---	---	---	---	---	---	0.428	1.676	3.550	8.638	14.918	21.980	29.582	37.572
	---	---	---	---	---	---	---	0.614	2.048	4.102	9.513	16.084	23.406	31.240	39.442
40	---	---	---	---	---	---	---	---	0.378	1.500	5.402	10.800	17.186	24.250	31.806
	---	---	---	---	---	---	---	---	0.545	1.841	6.068	11.766	18.421	25.730	33.507
45	---	---	---	---	---	---	---	---	---	0.340	2.946	7.368	12.998	19.466	26.536
	---	---	---	---	---	---	---	---	---	0.490	3.412	8.138	14.046	20.766	28.067
50	---	---	---	---	---	---	---	---	---	---	1.250	4.610	9.414	15.224	21.758
	---	---	---	---	---	---	---	---	---	---	1.532	5.192	10.277	16.346	23.120

TABLE III. Same as Table I for $\alpha = 0.50$

p \ f		5	10	15	20	25	30	35	40	45	50	60?	70	80	90	100
2	:	2.712	7.526	12.374	17.010	22.418	27.400	32.400	37.376	42.376	47.376	57.370	67.364	77.360	87.358	97.354
	:	3.016	7.774	12.794	17.763	22.750	27.731	32.724	37.717	42.711	47.707	57.700	67.696	77.692	87.689	97.687
3	:	1.448	5.906	10.720	15.606	20.576	25.526	30.500	35.476	40.476	45.450	55.436	65.420	75.410	85.402	95.394
	:	1.713	6.222	11.050	15.963	20.900	25.863	30.836	35.815	40.799	45.786	55.766	65.752	[illegible]	85.733	95.727
4	:	0.574	4.476	9.100	13.912	18.806	23.726	28.676	33.638	38.594	43.576	53.534	63.506	[illegible]	83.468	93.454
	:	0.769	4.781	9.431	14.250	19.138	24.056	29.004	33.963	38.930	43.904	53.865	63.837	73.816	83.799	93.786
5	:	---	3.238	7.620	12.306	17.126	22.000	26.906	31.826	36.776	41.738	51.668	61.620	71.584	81.556	91.534
	:	---	3.522	7.944	12.631	17.444	22.325	27.229	32.159	37.106	42.062	51.997	61.950	71.915	81.888	91.866
6	:	---	2.204	6.270	10.894	15.512	20.326	25.182	30.076	35.000	39.938	49.834	59.762	69.708	79.668	89.634
	:	---	2.457	6.581	11.119	15.831	20.650	25.509	30.405	35.324	40.260	50.162	[illegible]	[illegible]	79.998	89.965
7	:	---	1.364	5.038	9.382	13.988	18.726	23.526	28.376	33.262	38.176	48.032	57.932	67.858	77.800	87.754
	:	---	1.581	5.347	9.700	14.300	19.038	23.845	28.700	33.587	38.496	48.360	58.262	68.188	78.130	88.084
8	:	---	0.724	3.944	8.050	12.544	17.176	21.926	26.726	31.562	36.438	46.264	56.132	66.032	75.956	85.894
	:	---	0.895	4.238	8.375	12.850	17.494	22.238	27.045	31.894	36.773	46.590	56.460	66.361	76.285	86.223
9	:	---	0.284	2.988	6.836	11.176	15.712	20.376	25.112	29.926	34.776	44.530	54.360	64.232	74.132	84.052
	:	---	0.401	3.259	7.150	11.469	16.013	20.686	25.438	30.244	35.088	44.854	54.686	64.560	74.461	84.382
10	:	---	---	2.160	5.720	9.882	14.300	18.888	23.562	28.326	33.126	42.828	52.616	62.456	72.332	82.232
	:	---	---	2.406	6.019	10.175	14.600	19.191	23.881	28.638	33.444	43.151	52.941	62.783	72.660	82.561
11	:	---	---	1.466	4.700	8.676	12.956	17.450	22.062	26.762	31.526	41.160	50.900	60.704	70.552	80.432
	:	---	---	1.680	4.984	8.950	13.250	17.751	22.372	27.076	31.838	41.480	51.224	61.031	70.880	80.760
12	:	---	---	0.908	3.778	7.538	11.688	16.062	20.600	25.238	29.962	39.524	49.212	58.978	68.796	78.650
	:	---	---	1.083	4.050	7.806	11.969	16.368	20.913	25.558	30.273	39.843	49.535	59.304	69.123	78.978
13	:	---	---	0.480	2.956	6.484	10.468	14.750	19.200	23.768	28.438	37.922	47.554	57.278	67.062	76.890
	:	---	---	0.613	3.209	6.744	10.756	15.041	19.503	24.083	28.746	38.239	47.875	57.601	67.388	77.217
14	:	---	---	0.190	2.238	5.506	9.326	13.482	17.838	22.350	26.950	36.352	45.924	55.600	65.350	75.148
	:	---	---	0.272	2.469	5.756	9.600	13.770	18.143	22.653	27.259	36.667	46.243	55.923	65.674	75.475
15	:	---	---	---	1.616	4.612	8.250	12.276	16.538	20.962	25.512	34.816	44.322	53.950	63.660	73.428
	:	---	---	---	1.822	4.844	8.513	12.555	16.831	21.266	25.812	35.129	44.639	54.270	63.983	73.753
20	:	---	---	---	---	1.310	3.848	7.082	10.750	14.712	18.876	27.634	36.738	46.066	55.542	65.122
	:	---	---	---	---	1.466	4.053	7.320	11.011	14.988	19.167	27.931	37.044	46.378	55.858	65.442
25	:	---	---	---	---	---	1.092	3.300	6.200	9.562	13.250	21.278	29.866	38.804	47.976	57.314
	:	---	---	---	---	---	1.225	3.487	6.419	9.805	13.508	21.557	30.157	39.105	48.285	57.627
30	:	---	---	---	---	---	---	0.934	2.888	5.512	8.612	15.752	23.702	32.164	40.966	50.006
	:	---	---	---	---	---	---	1.055	3.057	5.716	8.837	16.007	23.978	32.452	41.262	50.309
35	:	---	---	---	---	---	---	---	0.818	2.568	4.962	11.054	18.252	26.146	34.508	43.194
	:	---	---	---	---	---	---	---	0.925	2.722	5.152	11.281	18.506	26.419	34.792	43.488
40	:	---	---	---	---	---	---	---	---	0.726	2.312	7.184	13.510	20.752	28.602	36.882
	:	---	---	---	---	---	---	---	---	0.823	2.453	7.380	13.741	21.005	28.872	37.163
45	:	---	---	---	---	---	---	---	---	---	0.654	4.144	9.482	15.978	23.252	31.068
	:	---	---	---	---	---	---	---	---	---	0.742	4.302	9.685	16.212	23.505	31.335
50	:	---	---	---	---	---	---	---	---	---	---	1.932	6.162	11.828	18.454	25.752
	:	---	---	---	---	---	---	---	---	---	---	2.049	6.335	12.038	18.688	26.004

TABLE IV. Same as in Table I for α = 0.75

p \ f	5	10	15	20	25	30	35	40	45	50	60	70	80	90	100
2	4.506	10.400	16.002	21.180	27.232	32.676	38.126	43.526	48.876	54.226	64.884	75.484	86.042	96.566	107.060
	4.297	9.771	15.344	20.750	26.100	31.425	36.726	42.004	47.264	52.511	62.970	73.392	83.785	94.153	104.502
3	2.730	8.456	14.094	19.646	25.182	30.626	36.050	41.450	46.826	52.176	62.824	73.424	83.984	94.508	105.004
	2.644	8.010	13.413	18.788	24.125	29.438	34.719	39.991	45.248	50.492	60.947	71.366	81.757	92.125	102.473
4	1.326	6.662	12.200	17.702	23.194	28.650	34.050	39.450	44.820	50.150	60.794	71.392	81.948	92.470	102.966
	1.347	6.326	11.600	16.900	22.213	27.494	32.765	38.026	43.273	48.510	58.955	69.368	79.754	90.118	100.463
5	---	5.062	10.432	15.858	21.300	26.726	32.094	37.476	42.826	48.162	58.796	69.386	79.936	90.454	100.946
	---	4.810	9.906	15.119	20.369	25.613	30.866	36.107	41.341	46.566	56.995	67.396	77.774	88.132	98.473
6	---	3.654	8.794	14.346	19.488	24.850	30.206	35.550	40.900	46.212	56.830	67.406	77.948	88.460	98.946
	---	3.491	8.344	13.431	18.606	23.813	29.020	34.236	39.450	44.660	55.066	65.452	75.819	86.168	96.502
7	---	2.452	7.250	12.434	17.738	23.050	28.376	33.676	39.012	44.300	54.896	65.454	75.982	86.486	96.964
	---	2.366	6.903	11.838	16.913	22.063	27.228	32.412	37.602	42.793	53.170	63.535	73.887	84.225	94.550
8	---	1.458	5.856	10.844	16.068	21.300	26.576	31.876	37.138	42.426	52.992	63.528	74.042	84.532	95.002
	---	1.442	5.594	10.338	15.300	20.369	25.490	30.636	35.796	40.963	51.305	61.646	71.980	82.304	92.617
9	---	0.698	4.600	9.386	14.462	19.626	24.850	30.088	35.350	40.600	51.120	61.632	72.126	82.602	93.060
	---	0.727	4.409	8.931	13.766	18.750	23.807	28.908	34.033	39.172	49.473	59.784	70.096	80.404	90.704
10	---	---	3.484	8.000	12.944	18.012	23.162	28.350	33.576	38.800	49.280	59.760	70.232	80.692	91.136
	---	---	3.356	7.625	12.313	17.194	22.179	27.228	32.312	37.419	47.672	57.950	68.237	78.525	88.810
11	---	---	2.504	6.726	11.512	16.456	21.538	26.688	31.862	37.050	47.472	57.918	68.364	78.802	89.232
	---	---	2.442	6.409	10.925	15.700	20.606	25.595	30.634	35.705	45.904	56.144	66.402	76.668	86.936
12	---	---	1.676	5.548	10.150	14.988	19.962	25.050	30.162	35.338	45.696	56.102	66.518	76.936	87.346
	---	---	1.661	5.300	9.625	14.275	19.089	24.012	28.999	34.029	44.168	54.364	64.591	74.832	85.081
13	---	---	0.990	4.476	8.860	13.568	18.450	23.462	28.544	33.662	43.954	54.314	64.698	75.090	85.480
	---	---	1.018	4.291	8.400	12.906	17.626	22.476	27.407	32.392	42.464	52.613	62.804	73.018	83.245
14	---	---	0.470	3.506	7.668	12.212	17.006	21.926	26.938	32.000	42.242	52.554	62.902	73.264	83.634
	---	---	0.519	3.381	7.259	11.613	16.220	20.989	25.858	30.794	40.792	50.889	61.042	71.226	81.429
15	---	---	---	2.648	6.544	10.926	15.600	20.438	25.388	30.412	40.562	50.820	61.128	71.462	81.806
	---	---	---	2.572	6.194	10.375	14.869	19.551	24.353	29.235	39.153	49.193	59.303	69.455	79.632
20	---	---	---	---	2.198	5.510	9.444	13.756	18.288	22.976	32.652	42.572	52.628	62.768	72.960
	---	---	---	---	2.084	5.209	8.964	13.098	17.478	22.025	31.443	41.130	50.977	60.925	70.939
25	---	---	---	---	---	1.848	4.762	8.326	12.288	16.538	25.562	35.022	44.738	54.616	64.600
	---	---	---	---	---	1.753	4.497	7.891	11.703	15.801	24.552	33.767	43.262	52.938	62.736
30	---	---	---	---	---	---	1.590	4.188	7.438	11.112	19.302	28.178	37.464	47.010	56.730
	---	---	---	---	---	---	1.509	3.956	7.047	10.577	18.487	27.108	36.162	45.496	55.022
35	---	---	---	---	---	---	---	1.398	3.738	6.718	13.884	22.048	30.808	39.950	49.352
	---	---	---	---	---	---	---	1.326	3.531	6.366	13.256	21.159	29.680	38.603	47.802
40	---	---	---	---	---	---	---	---	1.248	3.376	9.320	16.640	24.780	33.446	42.470
	---	---	---	---	---	---	---	---	1.183	3.189	8.868	15.926	23.821	32.261	41.076
45	---	---	---	---	---	---	---	---	---	1.126	5.628	11.964	19.382	27.498	36.086
	---	---	---	---	---	---	---	---	---	1.068	5.335	11.416	18.589	26.474	34.848
50	---	---	---	---	---	---	---	---	---	---	2.824	8.028	14.624	22.110	30.204
	---	---	---	---	---	---	---	---	---	---	2.670	7.635	13.988	21.244	29.119

TABLE V. Same as in Table 1 for α = 0.95

p \ f	5	10	15	20	25	30	35	40	45	50	60	70	80	90	100
2	8.256	15.700	22.354	28.300	35.282	41.376	47.426	53.426	59.376	65.176	76.806	88.274	99.638	110.918	122.124
	6.710	13.206	19.569	25.600	31.475	37.275	43.036	48.723	54.366	59.973	71.101	82.135	93.097	103.998	114.851
3	5.704	13.232	20.070	26.548	32.882	39.126	45.150	51.150	57.126	62.926	74.562	86.046	97.422	108.710	119.928
	4.519	11.136	17.363	23.413	29.275	35.088	40.857	46.550	52.199	57.812	68.948	79.990	90.958	101.866	112.723
4	3.426	10.962	17.752	24.204	30.644	36.800	42.900	48.950	54.870	60.700	72.348	83.842	95.226	106.524	117.748
	2.672	9.102	15.250	21.250	27.163	32.969	38.727	44.420	50.071	55.685	66.825	77.871	88.842	99.753	110.614
5	---	8.762	15.582	22.110	28.400	34.626	40.644	46.726	52.676	58.512	70.160	81.662	93.054	104.356	115.586
	---	7.185	13.256	19.244	25.119	30.888	36.645	42.334	47.981	53.594	64.732	75.777	86.749	97.661	108.522
6	---	6.804	13.544	21.398	26.238	32.426	38.506	44.500	50.450	56.312	68.000	79.506	90.902	102.208	113.442
	---	5.479	11.394	17.281	23.131	28.888	34.614	40.292	45.931	51.538	62.668	73.709	84.679	95.589	106.450
7	---	5.002	11.500	17.986	24.188	30.350	36.426	42.376	48.312	54.200	65.870	77.376	88.772	100.080	111.314
	---	3.953	9.628	15.413	21.188	26.913	32.634	38.294	43.921	49.517	60.635	71.667	82.631	93.537	104.395
8	---	3.358	9.682	15.944	22.168	28.276	34.326	40.376	46.238	52.126	63.770	75.270	86.664	97.972	109.206
	---	2.630	7.981	13.663	19.350	25.019	30.784	36.341	41.950	47.533	58.631	69.651	80.606	91.506	102.359
9	---	1.960	7.926	14.086	20.212	26.326	32.350	38.338	44.250	50.100	61.698	73.190	84.578	95.882	107.114
	---	1.521	6.484	11.981	17.591	23.200	28.826	34.433	40.019	45.585	56.658	67.661	78.604	89.495	100.341
10	---	---	6.334	12.352	18.368	24.362	30.388	36.300	42.226	48.100	59.654	71.134	82.514	93.812	105.040
	---	---	5.106	10.375	15.863	21.444	27.000	32.571	38.129	43.674	54.715	65.697	76.625	87.504	98.342
11	---	---	4.854	10.626	16.562	22.506	28.438	34.388	40.212	46.100	57.642	69.104	80.474	91.764	102.986
	---	---	3.880	8.884	14.250	19.725	25.227	30.754	36.280	41.799	52.803	63.759	74.668	85.534	96.361
12	---	---	3.602	9.048	14.850	20.688	26.612	32.500	38.312	44.088	55.658	67.100	78.454	89.734	100.948
	---	---	2.786	7.513	12.700	18.050	23.508	28.984	34.472	39.961	50.922	61.848	72.735	83.585	94.399
13	---	---	2.366	7.576	13.210	18.994	24.800	30.612	36.444	42.162	53.704	65.122	76.458	87.724	98.928
	---	---	1.856	6.216	11.225	16.481	21.842	27.261	32.705	38.160	49.071	59.963	70.826	81.656	92.456
14	---	---	1.358	6.206	11.618	17.312	23.056	28.826	34.588	40.300	51.782	63.168	74.484	85.736	96.928
	---	---	1.075	5.044	9.834	14.963	20.230	25.585	30.980	36.397	47.252	58.105	68.939	79.748	90.531
15	---	---	---	4.922	10.168	15.676	21.350	27.062	32.738	38.512	49.888	61.242	72.534	83.766	94.944
	---	---	---	3.966	8.519	13.500	18.674	23.956	29.297	34.671	45.464	56.274	67.076	77.861	88.625
20	---	---	---	---	4.048	8.660	13.694	18.932	24.388	29.826	40.888	52.004	63.122	74.226	85.304
	---	---	---	---	3.234	7,234	11.736	16.541	21.522	26.614	36.996	47.520	58.113	68.738	79.376
25	---	---	---	---	---	3.424	7.512	12.138	17.038	22.162	32.680	43.436	54.296	65.202	76.128
	---	---	---	---	---	2.741	6.267	10.380	14.843	19.533	29.330	39.448	49.744	60.142	70.601
30	---	---	---	---	---	---	2.966	6.638	10.888	15.488	25.288	35.556	46.064	56.706	67.424
	---	---	---	---	---	---	2.367	5.534	9.303	13.459	22.485	32.072	41.979	52.080	62.306
35	---	---	---	---	---	---	---	2.624	5.962	9.868	18.744	28.384	38.442	48.744	59.198
	---	---	---	---	---	---	---	2.086	4.954	8.428	16.481	25.404	34.827	44.559	54.495
40	---	---	---	---	---	---	---	---	2.348	5.400	13.078	21.940	31.442	41.328	51.458
	---	---	---	---	---	---	---	---	1.865	4.484	11.342	19.459	28.297	37.587	47.173
45	---	---	---	---	---	---	---	---	---	2.132	8.328	16.244	25.080	34.470	44.214
	---	---	---	---	---	---	---	---	---	1.687	7.093	14.251	22.401	31.169	40.347
50	---	---	---	---	---	---	---	---	---	---	4.540	11.320	19.370	28.176	37.470
	---	---	---	---	---	---	---	---	---	---	3.768	9.799	17.148	25.314	34.022

TABLE VI. Values of c, g, f (real case) and $\tilde{c}$, $\tilde{g}$, $\tilde{f}$ (complex case) for the chi-square approximation (3.2)

f \ p		2	3	4	5	6	7	8	9	10	11	12	13	14	15	20
5	c :	0.952	0.878	0.790	---	---	---	---	---	---	---	---	---	---	---	---
	g :	1.194	1.549	2.078	---	---	---	---	---	---	---	---	---	---	---	---
	f :	1.920	1.488	1.089	---	---	---	---	---	---	---	---	---	---	---	---
	$\tilde{c}$:	0.943	0.862	0.772	---	---	---	---	---	---	---	---	---	---	---	---
	$\tilde{g}$:	1.272	1.775	2.517	---	---	---	---	---	---	---	---	---	---	---	---
	$\tilde{f}$:	3.934	3.150	2.381	---	---	---	---	---	---	---	---	---	---	---	---
10	c :	0.980	0.948	0.907	0.861	0.811	0.759	0.706	0.653	---	---	---	---	---	---	---
	g :	1.087	1.244	1.468	1.765	2.140	2.597	3.133	3.732	---	---	---	---	---	---	---
	f :	4.316	3.801	3.379	3.005	2.644	2.270	1.854	1.366	---	---	---	---	---	---	---
	$\tilde{c}$:	0.978	0.942	0.898	0.850	0.798	0.746	0.695	0.648	---	---	---	---	---	---	---
	$\tilde{g}$:	1.112	1.321	1.622	2.030	2.554	3.201	3.958	4.773	---	---	---	---	---	---	---
	$\tilde{f}$:	8.682	7.726	6.945	6.250	5.567	4.838	3.996	2.969	---	---	---	---	---	---	---
15	c :	0.989	0.971	0.947	0.919	0.887	0.854	0.819	0.783	0.747	0.711	0.674	0.638	0.602	---	---
	g :	1.050	1.142	1.272	1.442	1.653	1.907	2.208	2.559	2.961	3.413	3.908	4.438	4.978	---	---
	f :	6.751	6.162	5.675	5.257	4.881	4.529	4.185	3.835	3.468	3.069	2.623	2.115	1.525	---	---
	$\tilde{c}$:	0.990	0.968	0.940	0.914	0.878	0.846	0.810	0.772	0.737	0.701	0.666	0.633	0.603	---	---
	$\tilde{g}$:	1.053	1.181	1.359	1.555	1.856	2.184	2.597	3.104	3.658	4.311	5.013	5.763	6.477	---	---
	$\tilde{f}$:	13.479	12.412	11.537	10.659	10.025	9.319	8.665	8.023	7.276	6.490	5.576	4.527	3.276	---	---
20	c :	0.993	0.982	0.965	0.946	0.924	0.901	0.876	0.850	0.823	0.796	0.768	0.741	0.713	0.685	---
	g :	1.036	1.093	1.182	1.293	1.433	1.595	1.790	2.013	2.271	2.559	2.883	3.246	3.643	4.074	---
	f :	9.219	8.555	8.021	7.547	7.135	6.749	6.397	6.055	5.724	5.382	5.034	4.670	4.280	3.856	---
	$\tilde{c}$:	1.000	0.984	0.962	0.941	0.924	0.894	0.869	0.842	0.817	0.787	0.760	0.731	0.706	0.678	---
	$\tilde{g}$:	1.002	1.096	1.229	1.376	1.512	1.765	2.015	2.313	2.641	3.066	3.501	4.034	4.553	5.198	---
	$\tilde{f}$:	18.154	17.048	16.160	15.290	14.320	13.768	13.077	12.427	11.736	11.146	10.430	9.750	8.899	8.083	---
25	c :	0.996	0.987	0.975	0.963	0.946	0.928	0.910	0.890	0.868	0.848	0.825	0.804	0.781	0.759	0.647
	g :	1.021	1.066	1.132	1.204	1.307	1.426	1.562	1.720	1.902	2.099	2.326	2.575	2.850	3.155	5.066
	f :	11.662	10.980	10.401	9.849	9.408	9.002	8.615	8.260	7.926	7.584	7.260	6.926	6.589	6.242	4.153
	$\tilde{c}$:	0.988	0.998	0.990	0.958	0.931	0.929	0.911	0.881	0.859	0.846	0.825	0.796	0.773	0.752	0.642
	$\tilde{g}$:	1.066	1.018	1.069	1.269	1.466	1.497	1.654	1.931	2.173	2.348	2.637	3.053	3.433	3.834	6.582
	$\tilde{f}$:	23.700	21.488	20.197	19.936	19.488	18.025	17.235	16.891	16.259	15.304	14.657	14.241	13.581	12.853	8.683

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Percentage points of a new distribution involving a confluent-hypergeometric distribution obtained by Khatri and Rao (1985) are tabulated. The use of the tabulated values in obtaining a lower confidence bound for the realized signal to noise ratio based on an estimated discriminant function for signal detection is explained.

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