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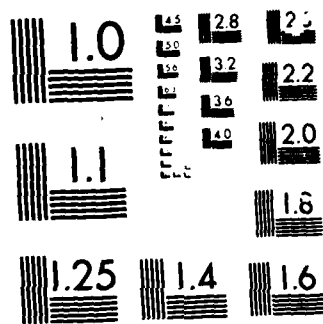
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MICROCOPY RESOLUTION TEST CHART
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Final report for Grant AFOSR 82-0213

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Basic work on the numerical solution of the Navier-Stokes equations for incompressible flows was undertaken. Attention was focussed on finite element methods, and on questions of their stability and efficiency. A general theory of stability has been constructed. This theory solved one of the outstanding problems in the application of finite element methods to incompressible flows.

The problem is basically that of the compatability of the finite element space used to approximate the velocity field with the finite element space used to approximate the pressure field. The need for such compatability is expressed in one of the abstract mathematical conditions guaranteeing convergence. As is typical with such conditions, verifying that it is satisfied is a problem of difficulty comparable to solving the original problem. In our work, we separated the problem into two parts. The first part required that the abstract mathematical condition be verified once and for all for a specific (but computationally inefficient) finite element scheme. In fact, the verification had been carried out already by other researchers. In the second part, the actual finite element space to be used is subjected to a simple test which requires, at most, solution

of a small system of linear equations whose size is not dependent of the mesh size of the computational scheme. An inspection of the solution to this linear system immediately reveals whether or not the proposed scheme is stable.

When we applied our test to one of the widely used finite element schemes for the Navier-Stokes equations, for which stability had never been proved, it failed our test. We were able to show, by constructing explicit data for which convergence did not occur that the failure was real, and not an artifact of our test. Since then, although some researchers have tried to repair the scheme, it has been abandoned for the most part. This is a tangible result of our theory. The theory has been subsequently applied by others to viscoelastic computations, where similar difficulties arise and to the design of accurate finite element schemes.

A second area of activity was an investigation of the applicability of the stream function formulation of the Navier-Stokes equations. The hope was to achieve greater efficiency in the overall solution process than is possible with the Navier-Stokes equations. Initially, we analyzed a stream function-vorticity model, and established its rate of convergence, improving an already known result in the literature. In spite of this improvement, it was felt that still greater improvements would come from going to a full stream function formulation. Here, we implemented conforming finite element schemes quite

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Chief, Technical Information Division

successfully. Effort was then devoted to optimal pressure computation. It is well known that pressure computation from stream function or stream function vorticity solutions is full of pitfalls, usually associated with obtaining correct boundary conditions for the pressure. Our initial attempt, modelled on the finite difference ideas was not successful. Eventually, we found a completely new treatment, naturally suited to finite elements. For this scheme, we were able to give general guidelines for picking pressure spaces and to prove rigourously that the convergence rates were the best possible. Interestingly enough, the stability theory outlined above turned out to be essential for this purpose.

The third area of work was in the serial and parallel processing of the algebraic systems arising from discretization of the Navier-Stokes equations. Several algorithms were developed and tested on different computers. The results generally were not conclusive. Machines with rather different architectures (FEM at NASA Langley, TRAC,...) were disappointing for a variety of reasons. Nevertheless, valuable experience was gained with both the hardware and software of these machines. It is expected to put this experience to good use in the future on the CMU-AP system.



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19. ABSTRACT (Continue on reverse if necessary and identify by block number) This effort was focused on basic work on the numerical solution of the Navier-Stokes equations for incompressible flows. A basic problem in this theory is the compatibility of the finite element space used to approximate the pressure field. The investigator was able to devise a simple test of the finite element space. An inspection of the solution to this linear system immediately reveals whether the proposed scheme is stable. A second area of activity was an investigation of the applicability of the stream function formulation of the Navier-Stokes equations. This effort resulted in a number of publications in the scientific literature.			
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