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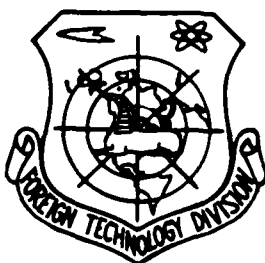
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ON THE ISOMORPHISM BETWEEN GAUGE GROUPS BEFORE AND AFTER RENORMALIZATION
IN THE PRESENCE OF ABEL SUBGROUPS AND HIGGS FIELDS

by

Rong Wang



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On The Isomorphism Between Gauge Groups Before And After
Renormalization In The Presence Of Abel Subgroups And Higgs Fields

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ABSTRACT

We give a rigorous proof on the isomorphism between gauge groups before and after renormalization, in the presence of Abel subgroups and Higgs fields.

I.

The gauge groups which contain Abel subgroups (The W-S model is a special case of these groups) are more complicated than the gauge groups which do not contain Abel subgroups. This complicated case was never discussed by 't Hooft and Veltman [1], and by Lee and Zinn-Justin [2] in their early renormalization studies. Later, although Ross and Taylor [3] studied the renormalization of the W-S model and the gauge invariance after the renormalization, their discussion was not rigorous. In fact, their results were accurate only within two orders of perturbation. By using the identical equations of Slavnov, Julve and Tonin [4] found out that gauge groups before and after renormalization were isomorphic even in the presence of Abel subgroups. Their method is very similar to Zinn-Justin's discussion where the Abel subgroups were not presented.[5] But this

method, as pointed out by Zinn-Justin [5] and Taylor [6], requires a strict definition of S_n^0 . Namely, for each order (i.e. each n , n represents n loops), $\tilde{S}_n^0 * \tilde{S}_n^0 = 0$ must be satisfied. To achieve this requirement, \tilde{S}_n^0 after n loops of renormalization and \tilde{S}_{n+1}^0 after $n+1$ loops of renormalization should satisfy the following relation:

$$\tilde{S}_{n+1}^0 = \tilde{S}_n^0 - \Gamma_{(n+1)}^{div}(S_n^0) + o(\gamma^{n+2}).$$

Here, $(\frac{1}{\eta} S + J.A. + \xi\mu + \bar{u}.S.)$, which replaces $(S + J.A. + \xi\mu + \bar{u}.S.)$, is used to calculate Γ and γ^n , where η is the loop number parameter. $o(\gamma^{n+2})$ represents the high order correction with order higher than $(n+2)$. However, how to define \tilde{S}_n^0 of each order n and how to determine the high order correction $o(\gamma^{n+2})$ were not very clear in this method. This also was not clearly described in the paper of Julve and Tonin. This vagueness was later cleared up by Lee [7] who was able to determine the relation between \tilde{S}_{n+1}^0 and \tilde{S}_n^0 . In his proof, it was clearly shown that, for each order of n , \tilde{S}_n^0 rigorously satisfies

$$\tilde{S}_n^0 * \tilde{S}_n^0 = 0$$

In this paper, we will modify and improve Lee's method. We will give a rigorous and clear proof that the gauge groups before and after renormalization are isomorphic in the presence of Abel subgroups, and that for each n , \tilde{S}_n^0 rigorously satisfies $\tilde{S}_n^0 * \tilde{S}_n^0 = 0$.

II.

In this section the Slavnov's identical equations in the presence of Abel subgroups and Higgs fields will be derived. The gauge group can be expressed as $G = A \otimes B$,

where A is not an Abel subgroup and B is an Abel subgroup. Without fermion field, the Lagrangians which are invariant under the gauge

transformation can be expressed as

$$\mathcal{L}_{inv}[A, B_\mu, S_i] = \mathcal{L}_{inv}[A_i] + \mathcal{L}_{inv}[B_\mu] - \frac{1}{2} (D_\mu s_i)^2 - \Delta V(s_i^*),$$

$$\mathcal{L}_{inv}[A_i] = -\frac{1}{4} (\partial_\mu A_i^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c)^2,$$

$$\mathcal{L}_{inv}[B_\mu] = -\frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2,$$

$$(D_\mu s_i)^2 = \left(\partial_\mu - \frac{i}{2} g \vec{\tau}^a \cdot A_\mu^a - \frac{i}{2} g' B_\mu \right) s_i \cdot s_i,$$

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where A_i is the non-Abel gauge field, B_μ is the Abelian gauge field, s_a and s_a^* are the Higgs fields. We also choose

$$V(s_i^*) = (s_i^* s_i)^2.$$

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In the W-S model, for example, we have

$$s_1 = \frac{1}{\sqrt{2}} (\varphi_1 + i\varphi_2), \quad s_2 = \frac{1}{\sqrt{2}} (\varphi_1 - i\varphi_2),$$

$$s_1^* = \frac{1}{\sqrt{2}} (\varphi_1 - i\varphi_2), \quad s_2^* = \frac{1}{\sqrt{2}} (\varphi_1 + i\varphi_2),$$

$\vec{\tau}^1, \vec{\tau}^2, \vec{\tau}^3$ are Pauli matrices.

The gauge fixed term of the R_ξ gauge is

$$-\frac{1}{2} \xi_a (C^a)^2 - \frac{1}{2} \xi_b (C^b)^2,$$

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where

$$C^a = \partial_\mu s_i^* + \frac{1}{\xi_a} \frac{g^a}{2} \frac{1}{\sqrt{2}} (\partial_\mu \vec{\tau}^a_{ik} s_k - s_k^* \vec{\tau}^a_{ki} \partial_\mu s_i) \quad (a = 1, 2, 3)$$

$$C^b = \partial_\mu B_\mu - \frac{1}{\xi_b} \frac{g^b}{2} \frac{1}{\sqrt{2}} (\partial_\mu \vec{\tau}^b_{ik} s_k - s_k^* \vec{\tau}^b_{ki} \partial_\mu s_i).$$

These expressions can be rewritten if the gauge is expanded into the general case of one gauge only (includes the R_ξ gauge)

$$C^a = F_\mu^a A_\mu + c_{ik}^a s_k + c_k^{a*} s_i^*,$$

4'

$$C^b = F_\mu^b B_\mu + c_{ik}^b s_k + c_k^{b*} s_i^*.$$

Since this is the more general case, it is not necessary to be confined in SU(2) and a is not necessary to be 1, 2, or 3.

The gauge transformations are:

$$A_\mu^a \rightarrow A_\mu^a - \frac{1}{g} (\partial_\mu \theta^a - g f_{abc} \theta^b A_\mu^c),$$

$$B_\mu \rightarrow B_\mu - \frac{1}{g'} \partial_\mu \theta,$$

and

$$s_i \rightarrow s'_i = s_i + \left(-\frac{1}{2} \tilde{r}_{i,m}^a \theta^a - \frac{1}{2} \theta^a \delta_{i,m} \right) s_m,$$

$$s'_i \rightarrow s''_i = s'_i + s'_m \theta^a \tilde{r}_{m,i}^a \frac{1}{2} + s'_m \theta^a \frac{1}{2}.$$

If $\theta^a = g \lambda^a$, $\theta = g' \lambda$, and the simplified functionals are used, then the above expressions can be rewritten as:

$$A_i \rightarrow A'_i = A_i + (\Delta_i^a + g t_{ik}^a A_k) \lambda^a,$$

$$B_a \rightarrow B'_a = B_a + \Delta_a^b \lambda_b,$$

$$s_i \rightarrow s'_i = s_i - \frac{1}{2} (g r_{im}^a \lambda^a s_m + g' \lambda^a s_i),$$

$$s'_i \rightarrow s''_i = s'_i + \frac{1}{2} (g s_m^a r_{mi}^a \lambda^a + g' s'_i \lambda).$$

Some symbols are simplified as below (α, β are the space-time indices; a, b, c are the group indices of the non-Abel subgroup; l, m are the group representation indices of the non-Abel subgroup).

$$A_i = A_i^a, \quad \Delta_i^a = -\partial_a \delta^a(x_i - x_j) \delta_{ab},$$

where the i in the left hand side represents a, α , and x_i in the right hand side; the b in the left hand side represents b and x_j in the right hand side.

$$t_{ik}^a = f_{abc} \delta^a(x_i - x_k) \delta^c(x_i - x_j) \delta_{ab},$$

where the i in the left hand side represents a, α , and x_i in the right hand side; the k in the left hand side represents c, β , and x_k in the right hand side; the b in the left hand side represents b and x_j in the right hand side.

Similar to the previous expressions, we have

$$\Delta_a^b = -\partial_a \delta^a(x_i - x_j), \quad r_{im}^a = \tilde{r}_{im}^a \delta^a(x_i - x_k) \delta^c(x_i - x_j).$$

Same as in Ref.7, there are the following relations:

$$[r^a, r^b] = f_{abc} r^c,$$

$$t_{ik}^a \Delta_i^b - t_{ik}^b \Delta_i^a = f_{abc} \Delta_i^c,$$

$$\left[-\frac{1}{2} r^a, -\frac{1}{2} r^b \right] = f_{abc} \left(-\frac{1}{2} r^c \right).$$

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These relations do not appear in the Abel subgroups because the structure constant is zero. Now, from Eqs. (4) and (5) we have

$$\begin{aligned}
 -M_{\dot{a}} &= \frac{\delta c^{Aa}}{\delta \lambda^b} = F_i^{Aa}(\Delta_i^{Aa} + g_i^{Aa} A_i) + c_i^{Aa} \left(-\frac{i}{2} g_i^{\dagger k} r_{ki} \right) + c_i^{Aa\dagger} \left(\frac{i}{2} g_i^{\dagger k} r_{ki} \right), \\
 -M_{\dot{a}} &= \frac{\delta c^{Aa}}{\delta \lambda} = c_i^{Aa} \left(-\frac{i}{2} g_i^{\dagger i} \right) + c_i^{Aa\dagger} \left(\frac{i}{2} g_i^{\dagger i} \right), \\
 -M_{\dot{a}} &= \frac{\delta c^B}{\delta \lambda} = F_i^B \Delta_i^B + c_i^B \left(-\frac{i}{2} g_i^{\dagger i} \right) + c_i^{B\dagger} \left(\frac{i}{2} g_i^{\dagger i} \right), \\
 -M_{\dot{a}} &= \frac{\delta c^b}{\delta \lambda^b} = c_i^b \left(-\frac{i}{2} g_i^{\dagger k} r_{ki} \right) + c_i^{b\dagger} \left(\frac{i}{2} g_i^{\dagger k} r_{ki} \right).
 \end{aligned}
 \tag{7}$$

There are two sets of F-P fields needed to be chosen. One set is \bar{u}_a, u_a which is corresponding to the non-Abel subgroup; the other set is \bar{u}, u which is corresponding to Abel subgroup. The gauge compensated term should be

$$\begin{aligned}
 \mathcal{L}_{F-P} &= \bar{u}_a M_{\dot{a}} u_a + \bar{u}_a M_{\dot{a}} u_a + \bar{u} M_{\dot{a}} u + \bar{u} M_{\dot{a}} u \\
 &= -\bar{u}_a F_i^{Aa} (\Delta_i^{Aa} + g_i^{Aa} A_i) u_a \\
 &\quad - c_i^{Aa} u_a \left(-\frac{i}{2} g_i^{\dagger k} r_{ki} \right) u_a - c_i^{Aa\dagger} \bar{u}_a \left(\frac{i}{2} g_i^{\dagger k} r_{ki} \right) u_a - \bar{u} F_i^B \Delta_i^B u \\
 &\quad - c_i^B u \left(-\frac{i}{2} g_i^{\dagger i} \right) u - c_i^{B\dagger} \bar{u} \left(\frac{i}{2} g_i^{\dagger i} \right) u \\
 &\quad - c_i^{Aa} \bar{u}_a \left(-\frac{i}{2} g_i^{\dagger i} \right) u - c_i^{Aa\dagger} \bar{u}_a \left(\frac{i}{2} g_i^{\dagger i} \right) u \\
 &\quad - c_i^b u \left(-\frac{i}{2} g_i^{\dagger k} r_{ki} \right) u_b - c_i^{b\dagger} \bar{u} \left(\frac{i}{2} g_i^{\dagger k} r_{ki} \right) u_b.
 \end{aligned}
 \tag{8}$$

Let's take

$$\begin{aligned}
 s^A &= \mathcal{L}_{inv}[A_i] + \mathcal{L}_{inv}[B_a] - \frac{1}{2} (D_i s_a)^2 - \Lambda V(s^{\dagger a}) \\
 &\quad - \frac{1}{2} \xi_A (C^{Aa})^2 - \frac{1}{2} \xi_B (C^B)^2 \\
 &\quad + (K_i^A - F_i^{Aa} \bar{u}_a) (\Delta_i^{Aa} + g_i^{Aa} A_i) u_a + (K_i^B - F_i^B \bar{u}) \Delta_i^B u \\
 &\quad + (K_i^{\dagger} - c_i^{Aa} \bar{u}_a - c_i^B u) \left(-\frac{i}{2} g_i^{\dagger k} r_{ki} u_b - \frac{i}{2} g_i^{\dagger i} u \right) \\
 &\quad + (K_i^{\dagger\dagger} - c_i^{Aa\dagger} \bar{u}_a - c_i^{B\dagger} \bar{u}) \left(\frac{i}{2} g_i^{\dagger k} r_{ki} u_b + \frac{i}{2} g_i^{\dagger i} u \right) \\
 &\quad + \frac{1}{2} g_{ab} s_a s_b u_c,
 \end{aligned}
 \tag{9}$$

where $K_i^A, K_i^B, K_i^{\dagger}, K_i^{\dagger\dagger}, \bar{u}_a, u_a, \bar{u}, u$ are mutually anticommuting. In this case, the B.R.S. transformation should be ($\delta \lambda$ is also anticommuting

with respect to K, \bar{u}, u, \dots , etc)

$$\delta A_i = (\Delta_i^{\mu\nu} + g_i^{\mu\nu} A_i) u_\nu \delta \lambda, \quad \delta B_\mu = (\Delta_\mu^{\nu\lambda} \delta \lambda,$$

$$\delta u_\nu = -\frac{i}{2} (g_i^{\mu\nu} A_i u_\mu + g_i^{\nu\mu} u) \delta \lambda,$$

$$\delta \bar{u}_i = \frac{i}{2} (g_i^{\mu\nu} A_i u_\mu + g_i^{\nu\mu} u) \delta \lambda,$$

$$\delta u_\mu = -\frac{1}{2} g_{\mu\nu} u_\nu \delta \lambda,$$

$$\delta u_\nu = \xi_\nu (F_i^{\mu\alpha} A_i + c_i^{\mu\alpha} \partial_\mu + c_i^{\mu\alpha} \partial_\mu^+) \delta \lambda,$$

$$\delta u = 0$$

$$\delta \bar{u} = \xi_\nu (F_i^{\mu\alpha} B_\mu + c_i^{\mu\alpha} \partial_\mu + c_i^{\mu\alpha} \partial_\mu^+) \delta \lambda.$$

According to the calculation, the S^R in Eq.(9) is invariant under the B.R.S. transformation of Eq.(10). From this, the Slavnov identical equations which are satisfied by S^R can be derived

$$\frac{\delta \bar{S}^R}{\delta K_i^\mu} \frac{\delta \bar{S}^R}{\delta A_i} + \frac{\delta \bar{S}^R}{\delta K_\mu^\nu} \frac{\delta \bar{S}^R}{\delta B_\nu} + \frac{\delta \bar{S}^R}{\delta K_i^\mu} \frac{\delta \bar{S}^R}{\delta s_i} + \frac{\delta \bar{S}^R}{\delta K_i^{\mu+}} \frac{\delta \bar{S}^R}{\delta s_i^+} + \frac{\delta \bar{S}^R}{\delta L_\nu} \frac{\delta \bar{S}^R}{\delta u_\nu} = 0, \quad 11a$$

$$- F_i^{\mu\alpha} \frac{\delta \bar{S}^R}{\delta K_i^\mu} - c_i^{\mu\alpha} \frac{\delta \bar{S}^R}{\delta K_i^\mu} - c_i^{\mu\alpha+} \frac{\delta \bar{S}^R}{\delta K_i^{\mu+}} - \frac{\delta \bar{S}^R}{\delta u_\nu} = 0, \quad 11b$$

$$- F_\mu^{\nu\alpha} \frac{\delta \bar{S}^R}{\delta K_\mu^\nu} - c_\mu^{\nu\alpha} \frac{\delta \bar{S}^R}{\delta K_\mu^\nu} - c_\mu^{\nu\alpha+} \frac{\delta \bar{S}^R}{\delta K_\mu^{\nu+}} - \frac{\delta \bar{S}^R}{\delta u_\nu} = 0, \quad 11c$$

$$\bar{S}^R = S^R + \frac{1}{2} \xi_\nu (C^{\nu\alpha})^2 + \frac{1}{2} \xi_\nu (C^{\nu\beta})^2. \quad 11d$$

S^R can be used to derive the (perturbation) vertex generating functional $\bar{\Gamma}$. Since the integral volume element $d(A_i) d(B_\mu) d(\bar{u}_\nu) d(u_\nu) d(\bar{u}) d(u)$ is invariant under the B.R.S. transformation, the Slavnov identical equations of $\bar{\Gamma}$ can be obtained

$$\frac{\delta \bar{\Gamma}}{\delta K_i^\mu} \frac{\delta \bar{\Gamma}}{\delta A_i} + \frac{\delta \bar{\Gamma}}{\delta K_\mu^\nu} \frac{\delta \bar{\Gamma}}{\delta B_\nu} + \frac{\delta \bar{\Gamma}}{\delta K_i^\mu} \frac{\delta \bar{\Gamma}}{\delta s_i} + \frac{\delta \bar{\Gamma}}{\delta K_i^{\mu+}} \frac{\delta \bar{\Gamma}}{\delta s_i^+} + \frac{\delta \bar{\Gamma}}{\delta L_\nu} \frac{\delta \bar{\Gamma}}{\delta u_\nu} = 0, \quad 12a$$

$$- F_i^{\mu\alpha} \frac{\delta \bar{\Gamma}}{\delta K_i^\mu} - c_i^{\mu\alpha} \frac{\delta \bar{\Gamma}}{\delta K_i^\mu} - c_i^{\mu\alpha+} \frac{\delta \bar{\Gamma}}{\delta K_i^{\mu+}} - \frac{\delta \bar{\Gamma}}{\delta u_\nu} = 0, \quad 12b$$

$$- F_\mu^{\nu\alpha} \frac{\delta \bar{\Gamma}}{\delta K_\mu^\nu} - c_\mu^{\nu\alpha} \frac{\delta \bar{\Gamma}}{\delta K_\mu^\nu} - c_\mu^{\nu\alpha+} \frac{\delta \bar{\Gamma}}{\delta K_\mu^{\nu+}} - \frac{\delta \bar{\Gamma}}{\delta u_\nu} = 0, \quad 12c$$

$$\bar{\Gamma} = \Gamma + \frac{1}{2} \xi_\nu (C^{\nu\alpha})^2 + \frac{1}{2} \xi_\nu (C^{\nu\beta})^2. \quad 12d$$

Eqs.(11a) and (12a) can be simplified into

$$\bar{S}^R * \bar{S}^R = 0, \quad \bar{\Gamma} * \bar{\Gamma} = 0.$$

III.

Corresponding to the Slavnov identical equations (11) and (12), the operators \mathcal{G} and \mathcal{G}_1 are defined by

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_1,$$

$$\mathcal{G}_0 = \frac{\delta \bar{S}^n}{\delta K_i^a} \frac{\delta}{\delta A_i} + \frac{\delta \bar{S}^n}{\delta K_\mu^b} \frac{\delta}{\delta B_\mu} + \frac{\delta \bar{S}^n}{\delta K_i^c} \frac{\delta}{\delta s_i} + \frac{\delta \bar{S}^n}{\delta K_i^{\prime c}} \frac{\delta}{\delta s_i^{\prime}} + \frac{\delta \bar{S}^n}{\delta L_a} \frac{\delta}{\delta u_a},$$

$$= (\Delta_i^{\prime ab} + g_i^{\prime ab} A_i) u_b \frac{\delta}{\delta A_i} + \Delta_{\mu}^{\prime ab} \frac{\delta}{\delta B_\mu}$$

$$+ \left(-\frac{i}{2} g_i^{\prime ab} \tau_{ab}^c u_c - \frac{i}{2} g_i^{\prime ab} u_c \right) \frac{\delta}{\delta s_i}$$

$$+ \left(\frac{i}{2} g_i^{\prime ab} \tau_{ab}^c u_c + \frac{i}{2} g_i^{\prime ab} u_c \right) \frac{\delta}{\delta s_i^{\prime}} + \frac{1}{2} g_i^{\prime abc} u_a u_b \frac{\delta}{\delta u_c},$$

$$\mathcal{G}_1 = \frac{\delta \bar{S}^n}{\delta A_i} \frac{\delta}{\delta K_i^a} + \frac{\delta \bar{S}^n}{\delta B_\mu} \frac{\delta}{\delta K_\mu^b} + \frac{\delta \bar{S}^n}{\delta s_i} \frac{\delta}{\delta K_i^c} + \frac{\delta \bar{S}^n}{\delta s_i^{\prime}} \frac{\delta}{\delta K_i^{\prime c}} + \frac{\delta \bar{S}^n}{\delta u_a} \frac{\delta}{\delta L_a}.$$

They also satisfy the following relations

$$\mathcal{G}_0 \cdot \mathcal{G}_0 = 0, \quad \mathcal{G}_0 \cdot \mathcal{G}_1 + \mathcal{G}_1 \cdot \mathcal{G}_0 = 0,$$

So,

$$\mathcal{G} \cdot \mathcal{G} = 0.$$

(Similar to the case of Ref.7, $\frac{\delta}{\delta u}$ does not appear in the above equations. The only difference is that some u terms appear in our expression but not in Ref.7. Since they always show up in the forms of $u_a u + u u_a (=0)$ and $u u (=0)$, they can be eliminated.)

Now let's start our proof by the method of induction. First of all, we assume that n loops of renormalization has been done. The bare \bar{S}^0 after n loops of renormalization is defined by \bar{S}_n^0 (equal to S_n^0 of Ref.6). It rigorously satisfies the following equations

$$\bar{S}_n^0 * \bar{S}_n^0 = 0,$$

$$- F_i^{\prime ab} \frac{\delta \bar{S}_n^0}{\delta K_i^a} - c_i^{\prime ab} \frac{\delta \bar{S}_n^0}{\delta K_i^b} - c_i^{\prime abc} \frac{\delta \bar{S}_n^0}{\delta K_i^c} - \frac{\delta \bar{S}_n^0}{\delta u_a} = 0,$$

$$- F_\mu^{\prime ab} \frac{\delta \bar{S}_n^0}{\delta K_\mu^a} - c_\mu^{\prime ab} \frac{\delta \bar{S}_n^0}{\delta K_\mu^b} - c_\mu^{\prime abc} \frac{\delta \bar{S}_n^0}{\delta K_\mu^c} - \frac{\delta \bar{S}_n^0}{\delta u} = 0.$$

The first equation means that \bar{S}_n^0 , after n loops of renormalization, is

invariant under the B.R.S. transformations (This is true for $n=0$, where $\tilde{S}_{n+1}^0 = \tilde{S}^R$). After n loops of renormalization, the B.R.S. transformations are the four K 's and \bar{u}_n, \bar{u}_{n+1} are all mutually anticommuting

$$\begin{aligned} \delta A_i &= \frac{\delta S_n^0}{\delta K_i^1} \delta \lambda, & \delta B_{i^*} &= \frac{\delta S_n^0}{\delta K_i^2} \delta \lambda, \\ \delta \psi_i &= \frac{\delta S_n^0}{\delta K_i^3} \delta \lambda, & \delta \psi_{i^*} &= \frac{\delta S_n^0}{\delta K_i^4} \delta \lambda, \\ \delta u_i &= -\frac{\delta S_n^0}{\delta L_i} \delta \lambda, & \delta u_{i^*} &= \xi_i C^{i^*} \delta \lambda, \\ \delta u &= 0, & \delta \bar{u} &= \xi_u C^u \delta \lambda. \end{aligned} \quad (15)$$

Since S_n^0 is invariant under the B.R.S. transformation of Eq.(16), the calculated $\tilde{\Gamma}$ by using S_n^0 instead of S^R must satisfy the Slavnov identical equations. (There is one condition that the integral volume element must be invariant under the B.R.S. transformation of Eq. (16).

It will be proved later that, after the S_n^0, S_{n+1}^0, \dots etc. are determined, the sum of the Jacobian diagonal terms is still zero. So the integral volume element is invariant under the B.R.S transformation.) This implies that

$$\begin{aligned} \tilde{\Gamma} * \tilde{I} &= 0, \\ -F_i^{i^*} \frac{\delta \tilde{\Gamma}}{\delta K_i^1} - c_i^{i^*} \frac{\delta \tilde{\Gamma}}{\delta K_i^2} - c_i^{i^*} \frac{\delta \tilde{\Gamma}}{\delta K_i^3} - \frac{\delta \tilde{\Gamma}}{\delta u_i} &= 0, \\ -F_i^i \frac{\delta \tilde{\Gamma}}{\delta K_i^4} - c_i^i \frac{\delta \tilde{\Gamma}}{\delta K_i^1} - c_i^i \frac{\delta \tilde{\Gamma}}{\delta K_i^2} - \frac{\delta \tilde{\Gamma}}{\delta u_i} &= 0. \end{aligned} \quad (17)$$

Here $\tilde{\Gamma}$ can be expanded about the loop number (at first S^R is replaced by $\frac{1}{\gamma} S^R$):

$$\tilde{\Gamma} = \sum_{l=0}^{\infty} \gamma^l \tilde{\Gamma}_l. \quad (18)$$

According to the assumption of induction, $\tilde{\Gamma}_1, \tilde{\Gamma}_2, \dots, \tilde{\Gamma}_n$ are all limited, and $\tilde{\Gamma}_{n+1}$ starts to become divergent. This divergence is a general divergence of $n+1$ loops. The divergences of all subloops (less than $n+1$ loops) have been eliminated. According to the discussion in Ref. 5, the divergent part of $\tilde{\Gamma}_{n+1}$ is expressed by

$\tilde{F}_{(n+1)} \text{div}(S_n^0)$ and must satisfy:

$$\begin{aligned} \tilde{F}_{(n+1)}(S_n^0) * \tilde{S}^n + \tilde{S}^n * \tilde{F}_{(n+1)} \text{div}(S_n^0) &= (\mathcal{G}_0 + \mathcal{G}_1) \tilde{F}_{(n+1)} \text{div}(S_n^0) = 0, \\ (-F_{ij}^a \frac{\delta}{\delta K_i^a} - c_i^a \frac{\delta}{\delta K_j^a} - c_i^a \frac{\delta}{\delta K_j^{a*}} - \frac{\delta}{\delta u_a}) \tilde{F}_{(n+1)} \text{div}(S_n^0) &= 0, \\ (-F_{ij}^a \frac{\delta}{\delta K_i^a} - c_i^a \frac{\delta}{\delta K_j^a} - c_i^a \frac{\delta}{\delta K_j^{a*}} - \frac{\delta}{\delta u_a}) \tilde{F}_{(n+1)} \text{div}(S_n^0) &= 0. \end{aligned} \quad 19$$

In order to eliminate this divergence, we must find out the complete solutions of $\tilde{F}_{(n+1)} \text{div}(S_n^0)$ which satisfies Eq. (19).

It should be noted that $S/\delta u$ does not show up both in the Slavnov identical equations and in \mathcal{G}_0 and \mathcal{G}_1 . Therefore, there are three kinds of solutions. The first kind of solution is without F-P field; the second kind of solution has u_a and \bar{u}_a (also can have u and \bar{u}); the third kind of solution has u but not \bar{u}_a . For the first two kinds of solutions, the proof of Jaglekar and Lee (8) is still applicable. In other words, the local complete solutions without F-P field and with u_a and \bar{u}_a should have the form like:

$$G[A, B, s, s^*] + \mathcal{G} \mathcal{F}[A, B, s, s^*, u_a, \bar{u}_a, u, K, L]. \quad 20$$

Although expression (20) may contain the third kind of solution (see F of Eq. (30)), it can not contain all the third kind of solutions. The complete solution of the third kind will be discussed later. Let's first study $G[A, B, s, s^*]$ which is a gauge invariant functional. Since \tilde{F} is a local functional with dimension 4, and only $\mathcal{L}_{inv}[A_i]$, $\mathcal{L}_{inv}[B_\mu]$, $-\frac{1}{2}|Ds_a|^2$, and $V(s^*s)$ are the local gauge invariant functionals with dimension 4, so the general form of $G[A, B, s, s^*]$ should be like:

$$\begin{aligned} G[A, B, s, s^*] &= \alpha(\theta) \mathcal{L}_{inv}[A_i] + \alpha'(\theta) \mathcal{L}_{inv}[B_\mu] \\ &\quad - \alpha''(\theta) \left(\frac{1}{2} |Ds_a|^2 \right) - \zeta(\theta) V(s^*s). \end{aligned} \quad 21$$

(use the minimum renormalization, $e=n-4$)

For $\mathcal{G} \mathcal{F}$, \mathcal{G} is the operator of Eq. (13a), and \mathcal{F} must satisfy the following requirements:

(a) The F-P charge of \mathcal{G} is +1, the F-P charge of $\bar{\Gamma}$ is zero, so the F-P charge of \mathcal{F} should be -1. ($L_a: -2; K, \bar{u}_a, \bar{u}: -1; A, B, s: 0; u_a, u: +1$)

(b) The dimension of \mathcal{G} is +1, the dimension of $\bar{\Gamma}$ is 4, so the dimension of \mathcal{F} should be 3 ($K, L_a: +2; C^{Aa}, C^{Aa*}, C^B, C^{B*}: +1$).

(c) All the indices of \mathcal{G} are contracted, all the indices of $\bar{\Gamma}$ are also contracted, so all the indices of \mathcal{F} should be contracted.

From these three requirements, \mathcal{F} only has seven terms $K^A A_i, K^B B_\mu, K_i^+ s_i^+, K_i^+ (d_{im}^a c_m^{Aa*} + c c_i^{B*}), K_i^+ (d_{im}^b c_m^{Bb*} + c^+ c_i^+), L_\mu$, which contain K and L_a . No single term in \mathcal{F} contains both K (or L_a) and u because of the violation of the above three requirements. $K_i^A t_{ij}^b t_{jk}^b A_k$ will come back to $K_i^A A_i$ because of $t_{ij}^b t_{jk}^b \sim C(G) \delta_{ik}$. $K_i^+ t_{im}^+ t_{mk}^+$ will come back to $K_i^+ s_i^+$ because of $t_{im}^+ t_{mk}^+ \sim C(R) \delta_{ik}$. Since the propagators satisfy $\overline{s_R^+ s_R^+} = 0, \overline{s_R s_R} = 0, \overline{s_R s_R^+} \neq 0$, both $K_i^+ s_i^+$ and $K_i^{++} s_i^+$ are excluded from \mathcal{F} .

(d) Due to the last two of the Slavnov equations (19), $K_i^A A_i, K_\mu^B B_\mu, K_i^+ s_i^+$, and $K_i^{++} s_i^+$ have to show up with \bar{u}_a and \bar{u} in the following forms:

$$(K_i^A - \bar{u}_a F_i^{Aa}) A_i, \quad 22$$

$$(K_\mu^B - u F_\mu^B) B_\mu, \quad 23$$

$$(K_i^+ - c_i^{Aa} \bar{u}_a - c_i^B u) s_i^+, \quad 24$$

$$(K_i^{++} - c_i^{Aa*} \bar{u}_a - c_i^{B*} u) s_i^+. \quad 25$$

Besides, there are also some other terms which satisfy (a), (b), (c), (d) and contain $K_i^+ s_i^+$ and $K_i^{++} s_i^+$. They are:

$$(K_i^+ - c_i^{Aa} \bar{u}_a - c_i^B u) (d_{im}^a c_m^{Aa*} + c c_i^{B*}), \quad 26$$

$$(K_i^{++} - c_i^{Aa*} \bar{u}_a - c_i^{B*} u) (d_{im}^b c_m^{Bb*} + c^+ c_i^+). \quad 27$$

It can be seen from the perturbation theory that d^{b^+} and d^b are the group covariant constants, e^+ and e are constants. Since the

propagators satisfy $\overline{s_i} s_i^* \neq 0$, $\overline{s_i} s_i = \overline{s_i^*} s_i^* = 0$, K_i^i only multiplies with $c_m^{A^*}$ and $c_i^{B^*}$, and $K_i^{i^*}$ only multiplies with c_m^A and c_i^B .

The term, which contains L_a , is not affected by requirement (d) and is still

$$L_a u_a. \quad (22)$$

If \mathcal{G} is operated on (22), and (24) to (28), then as mentioned before only the second kind of solution (contains u_a and \bar{u}_a or u and \bar{u}) will be obtained.

If \mathcal{G} is operated on (23), then the third kind of solution which contains B_μ will be obtained. But, as mentioned before, expression (20) does not contain all the third kind of solutions (i.e., for the Abel subgroup, the solution of Joglekar and Lee is not completed). Therefore, the other method must be used in order to find out the third kind of solutions.

If the perturbation is on S^A , $\tilde{\Gamma}$ must have the term of $\sim \bar{u} F_\mu^B \Delta_\mu^B u$ (i.e., the $\bar{u}u$ self-energy term). According to the third one of the Slavnov identical equations (19), this term of $\tilde{\Gamma}$ should have the form like

$$\sim (K_\mu^B - \bar{u} F_\mu^B) \Delta_\mu^B u$$

At the same time, if the perturbation is on $c_i^B \bar{u}_s, u$, $\tilde{\Gamma}$ must have the term of $\sim c_i^B \bar{u}_s, u$ (the \bar{u}_s, u vertex of the renormalization). According to the last two of Eq. (19), this term should have the form like

$$\sim (K_i^i - c^{A^*} \bar{u}_s - c_i^B \bar{u}) s_i u.$$

Similarly, $\tilde{\Gamma}$ should also have the term like

$$\sim (K_i^{i^*} - c^{A^*} \bar{u}_s - c_i^{B^*} \bar{u}) s_i^* u.$$

If \mathcal{G} is operated on all these terms, they will become

$$\begin{aligned} \mathcal{G}(K_\mu^B - \bar{u}F_\mu^B)\Delta_\mu^B u &= \frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu} \Delta_\mu^B u, \\ \mathcal{G} \left[(K_l^i - c_l^{i\alpha} \bar{u}_\alpha - c_l^i \bar{u}) \left(\frac{-1}{2} \right) g'_{ij} u \right. \\ &\quad \left. + (K_l^{i*} - c_l^{i*\alpha} \bar{u}_\alpha - c_l^{i*} \bar{u}) \left(\frac{1}{2} \right) g'_{ij} u \right] \\ &= \frac{\delta \mathcal{L}'_{inv}}{\delta s_l} \left(\frac{-1}{2} g'_{ij} u \right) + \frac{\delta \mathcal{L}'_{inv}}{\delta s_l^*} \left(\frac{1}{2} g'_{ij} u \right). \end{aligned}$$

Since \mathcal{L}'_{inv} is invariant under the $\nu(1)$ gauge transformation, it is zero if \mathcal{G} is operated on

$$+ (K_l^i - c_l^{i\alpha} \bar{u}_\alpha - c_l^i \bar{u}) \left(\frac{-1}{2} \right) g'_{ij} u + (K_l^{i*} - c_l^{i*\alpha} \bar{u}_\alpha - c_l^{i*} \bar{u}) \left(\frac{1}{2} \right) g'_{ij} u \quad 29$$

Therefore, Eq. (29) is also a solution and should be the third kind of solution. However, it can not be written as in the form of (20).

It also should be mentioned that $\mathcal{G} \times (23)$ is the only third kind of solution which contains B_μ , and that (29) is the only third kind of solution which does not contains B_μ . For the former case, since only

$$\mathcal{G} \frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu} B_\mu = \frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu} \Delta_\mu^B u \quad \text{can cancel out}$$

$$\mathcal{G}(K_\mu^B - \bar{u}F_\mu^B)\Delta_\mu^B u = \frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu} \Delta_\mu^B u$$

, but not the other functional of B_μ .

$$\mathcal{G} \times (23) = \frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu} B_\mu - (K_\mu^B - \bar{u}F_\mu^B)\Delta_\mu^B u$$

is the only third kind of solution* which contains B_μ . For the latter case, since the F-P charge of u is +1, when it shows up in \tilde{F} it must be the combination of $K_\mu^B u$, $K_l^j u$, $K_l^{j*} u$, $\bar{u}u$, $\bar{u}_\alpha u$, and $L_\alpha u_\alpha u$. The dimension of \tilde{F} is 4. Although the dimension of $L_\alpha u_\alpha u$ is also 4, it should be excluded. This is because the form of $L_\alpha u_\alpha \dots$ can not be obtained from the perturbation on $L_\alpha f_{\alpha\beta} u_\beta u_\alpha$. $\bar{u}u\bar{u}u$ and $\bar{u}_\alpha u \bar{u}_\alpha u$ which have dimension 4 also have to be excluded. This is because $uu=0$ and $\bar{u}_\alpha \bar{u}_\alpha=0$. In fact, according to the last two of equations (19), $\bar{u}_\alpha u_\beta \bar{u}u$ only can show up in the forms of $(K_l^j - c_l^{j\alpha} \bar{u}_\alpha - c_l^j \bar{u})u_\alpha$, $(K_l^{j*} - c_l^{j*\alpha} \bar{u}_\alpha - c_l^{j*} \bar{u})u_\alpha$, or $(K_l^A - \bar{u}_\alpha F_l^{A\alpha})u_\beta$ multiplying $(K_l^j - c_l^{j\alpha} \bar{u}_\alpha - c_l^j \bar{u})u$, $(K_l^{j*} - c_l^{j*\alpha} \bar{u}_\alpha - c_l^{j*} \bar{u})u$, or

* see F of Eq. (30)

$(K_i^A - \bar{u}_\mu F_i^{A\mu})u$. Because of this, the dimension will be more than 4. So they should be excluded (here there is no perturbation for the vacuum spontaneous symmetry breaking). Therefore the solutions, which satisfy the last two of the Slavnov equations (19), have F-P charge of zero and dimension of 4, are consistent with the perturbation, have all their indices contracted, and do not contain B_μ and u_μ , but contain u , only can have the following three combinations (A_i has a different index and can not be included):

$$\begin{aligned} &\sim (K_i^B - \bar{u} F_i^B) \Delta_i^B u, \\ &\sim (K_i^C - c_i^{A\mu} \bar{u}_\mu - c_i^B \bar{u}) s_i u, \\ &\sim (K_i^{C^*} - c_i^{A\mu*} \bar{u}_\mu - c_i^{B^*} \bar{u}) s_i^* u. \end{aligned}$$

(The reason why $K_i^D s_i^*$ and $K_i^{D^*} s_i$ do not show up here has been discussed before.) If the three terms above also want to satisfy the first one of the Slavnov equations (19), the only possibility is that these three terms must combine themselves into the form of (29). So, (29) is the only third kind of solution without containing B_μ .

IV.

All the independent solutions which satisfy the requirements of $\tilde{r} \text{div}_{(n^*)} (S_n^0)$ have been found. They are (from (21) to (29)):

$$\begin{aligned} A &= \mathcal{L}_{1n^*}[A_i], & B &= \mathcal{L}_{1n^*}[B_\mu], \\ C &= -\frac{1}{2}|D s_i|^2, & D &= V(s_i^* s_i), \\ E &= \mathcal{L}(K_i^A - \bar{u}_\mu F_i^{A\mu})A_i \\ &= \frac{\delta \mathcal{L}_{1n^*}}{\delta A_i} A_i - (K_i^A - \bar{u}_\mu F_i^{A\mu}) \Delta_i^A u, \\ F &= \mathcal{L}(K_i^B - \bar{u} F_i^B) B_\mu = \frac{\delta \mathcal{L}_{1n^*}}{\delta B_\mu} B_\mu - (K_i^B - \bar{u} F_i^B) \Delta_i^B u, \\ G &= \mathcal{L}(K_i^C - c_i^{A\mu} \bar{u}_\mu - c_i^B \bar{u}) s_i \\ &= \frac{\delta \mathcal{L}_{1n^*}}{\delta s_i} s_i - (K_i^C - c_i^{A\mu} \bar{u}_\mu - c_i^B \bar{u}) \left(-\frac{i}{2} g_{ik}^A s_k u_\mu - \frac{i}{2} g' s_i u \right), \\ H &= \mathcal{L}(K_i^{C^*} - c_i^{A\mu*} \bar{u}_\mu - c_i^{B^*} \bar{u}) s_i^* \\ &= \frac{\delta \mathcal{L}_{1n^*}}{\delta s_i^*} s_i^* - (K_i^{C^*} - c_i^{A\mu*} \bar{u}_\mu - c_i^{B^*} \bar{u}) \left(\frac{i}{2} g_{ik}^{A^*} s_k^* u_\mu + \frac{i}{2} g' s_i^* u \right), \\ I &= \mathcal{L}(K_i^D - c_i^{A\mu} \bar{u}_\mu - c_i^B \bar{u}) (d_{im}^D c_m^{A^*} + c c_i^{B^*}) \end{aligned}$$

$$\begin{aligned}
&= \frac{\delta \bar{S}^K}{\delta s_i} (d_{i\mu}^{\mu+} c_{\mu+}^{\mu+} + c_i^{\mu+}), \\
J &= \mathcal{G}(K_i^{\mu+} - c_i^{\mu+} \bar{u}_\mu - c_i^{\mu+} \bar{u})(d_{i\mu}^{\mu+} c_{\mu+}^{\mu+} + c_i^{\mu+}) \\
&= \frac{\delta \bar{S}^K}{\delta s_i} (d_{i\mu}^{\mu+} c_{\mu+}^{\mu+} + c_i^{\mu+}),
\end{aligned}$$

$$\begin{aligned}
K &= -\mathcal{G}(L_{\mu s}) = -\frac{g}{2} L_{\mu s} u_{\mu s} - \frac{\delta \bar{S}^K}{\delta u_s} u_s, \\
L &= u \frac{\delta \bar{S}^K}{\delta u}.
\end{aligned}$$

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For convenience, these twelve independent solutions are rearranged. They become:

$$\begin{aligned}
A - K &= \frac{A_i}{2} \frac{\delta \bar{S}^K}{\delta A_i} - \frac{g}{2} \frac{\partial \bar{S}^K}{\partial g} + \frac{L_s}{2} \frac{\delta \bar{S}^K}{\delta L_s} - \frac{1}{2} \left(K_i^{\mu+} \frac{\delta}{\delta K_i^{\mu+}} + u_s \frac{\delta}{\delta u_s} + \bar{u}_s \frac{\delta}{\delta \bar{u}_s} \right) \bar{S}^K \\
&\quad + \frac{1}{2} c_i^{\mu+} \frac{\delta \bar{S}^K}{\delta c_i^{\mu+}} + \frac{1}{2} c_i^{\mu+} \frac{\delta \bar{S}^K}{\delta c_i^{\mu+}}.
\end{aligned} \tag{31a}$$

(It will be zero if $\frac{A_i}{2} \frac{\delta}{\delta A_i} - \frac{g}{2} \frac{\partial}{\partial g}$ operates on the terms containing gA_i)

$$\begin{aligned}
B - L &= \frac{B_\mu}{2} \frac{\delta \bar{S}^K}{\delta B_\mu} - \frac{g'}{2} \frac{\partial \bar{S}^K}{\partial g'} - \frac{1}{2} \left(K_\mu^{\mu+} \frac{\delta}{\delta K_\mu^{\mu+}} + u \frac{\delta}{\delta u} + \bar{u} \frac{\delta}{\delta \bar{u}} \right) \bar{S}^K \\
&\quad + \frac{1}{2} c_i^{\mu+} \frac{\delta \bar{S}^K}{\delta c_i^{\mu+}} + \frac{1}{2} c_i^{\mu+} \frac{\delta \bar{S}^K}{\delta c_i^{\mu+}}.
\end{aligned} \tag{31b}$$

(For all those terms containing $g'B_\mu$ will become zero if they are operated by $\frac{B_\mu}{2} \frac{\delta}{\delta B_\mu} - \frac{g'}{2} \frac{\partial}{\partial g'}$)

$$C + 2D = \frac{s_\mu}{2} \frac{\delta \bar{S}^K}{\delta s_\mu} + \frac{s_\mu^+}{2} \frac{\delta \bar{S}^K}{\delta s_\mu^+} - \frac{1}{2} \left(K_i^{\mu+} \frac{\delta}{\delta K_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} \right) \bar{S}^K \tag{31c}$$

$$- \frac{1}{2} \left(K_i^{\mu+} \frac{\delta}{\delta K_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} \right) \bar{S}^K, \tag{31d}$$

$$D = V(s^+ s),$$

$$\begin{aligned}
-E + K &= -A_i \frac{\delta \bar{S}^K}{\delta A_i} + \left(K_i^{\mu+} \frac{\delta}{\delta K_i^{\mu+}} + u_s \frac{\delta}{\delta u_s} + \bar{u}_s \frac{\delta}{\delta \bar{u}_s} \right) \bar{S}^K \\
&\quad - \left(c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} \right) \bar{S}^K - L_s \frac{\delta \bar{S}^K}{\delta L_s},
\end{aligned} \tag{31c}$$

$$-F + L = -B_\mu \frac{\delta \bar{S}^K}{\delta B_\mu} + \left(K_\mu^{\mu+} \frac{\delta}{\delta K_\mu^{\mu+}} + u \frac{\delta}{\delta u} + \bar{u} \frac{\delta}{\delta \bar{u}} \right) \bar{S}^K \tag{31f}$$

$$- \left(c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} \right) \bar{S}^K, \tag{31g}$$

$$-G = \left(K_i^{\mu+} \frac{\delta}{\delta K_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} - s_i \frac{\delta}{\delta s_i} \right) \bar{S}^K, \tag{31h}$$

$$-H = \left(K_i^{\mu+} \frac{\delta}{\delta K_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} + c_i^{\mu+} \frac{\delta}{\delta c_i^{\mu+}} - s_i^+ \frac{\delta}{\delta s_i^+} \right) \bar{S}^K, \tag{31h}$$

$$I = (d_{i\mu}^{\mu+} c_{\mu+}^{\mu+} + c_i^{\mu+}) \frac{\delta \bar{S}^K}{\delta s_i}, \tag{31i}$$

$$J = (d_{\mu\nu}^{\dagger} c_{\mu}^{\nu} + c^{\dagger} c_{\mu}^{\nu}) \frac{\delta S^{\mu}}{\delta s_{\mu}^{\dagger}}, \quad 31j$$

$$K + L = \frac{1}{2} \left(K_1^{\dagger} \frac{\delta}{\delta K_1^{\dagger}} + u_{\mu} \frac{\delta}{\delta u_{\mu}} + \bar{u}_{\mu} \frac{\delta}{\delta \bar{u}_{\mu}} + K_{\mu}^{\dagger} \frac{\delta}{\delta K_{\mu}^{\dagger}} + u \frac{\delta}{\delta u} + \bar{u} \frac{\delta}{\delta \bar{u}} \right. \quad 31k$$

$$\left. + K_1^{\dagger} \frac{\delta}{\delta K_1^{\dagger}} + K_1^{\dagger} \frac{\delta}{\delta K_1^{\dagger}} \right) S^{\mu},$$

$$L = u \frac{\delta S^{\mu}}{\delta u}. \quad 31l$$

So, the general form of $\bar{\Gamma}^{\text{div}}(S_{\mu}^{\nu})$ is

$$\bar{\Gamma}^{\text{div}}(S_{\mu}^{\nu}) = \alpha(\epsilon)(31a) + \alpha'(\epsilon)(31b) + \alpha''(\epsilon)(31c) + \zeta(\epsilon)(31d).$$

$$+ \beta(\epsilon)(31e) + \beta'(\epsilon)(31f) + \beta_1''(\epsilon)(31g)$$

$$+ \beta_2''(\epsilon)(31h) + \delta_1(\epsilon)(31i) + \delta_2(\epsilon)(31j)$$

$$- \gamma(\epsilon)(31k) + \eta(\epsilon)(31l)$$

$$\begin{aligned} \Rightarrow & \left[\left(\frac{\alpha(\epsilon)}{2} - \beta(\epsilon) \right) \left(A_1 \frac{\delta}{\delta A_1} + L_{\mu} \frac{\delta}{\delta L_{\mu}} \right) + \left(\frac{\alpha'(\epsilon)}{2} - \beta'(\epsilon) \right) B_{\mu} \frac{\delta}{\delta B_{\mu}} \right. \\ & + \left(\frac{\alpha''(\epsilon)}{2} - \beta''(\epsilon) \right) \left(s_1 \frac{\delta}{\delta s_1} + s_1^{\dagger} \frac{\delta}{\delta s_1^{\dagger}} \right) - \frac{\alpha(\epsilon)}{2} g \frac{\partial}{\partial g} - \frac{\alpha'(\epsilon)}{2} g' \frac{\partial}{\partial g'} \\ & + \left(-\frac{\alpha(\epsilon)}{2} + \beta(\epsilon) - \frac{\gamma(\epsilon)}{2} \right) \left(K_1^{\dagger} \frac{\delta}{\delta K_1^{\dagger}} + u_{\mu} \frac{\delta}{\delta u_{\mu}} + \bar{u}_{\mu} \frac{\delta}{\delta \bar{u}_{\mu}} \right) \\ & + \left(-\frac{\alpha'(\epsilon)}{2} + \beta'(\epsilon) - \frac{\gamma(\epsilon)}{2} \right) \left(K_{\mu}^{\dagger} \frac{\delta}{\delta K_{\mu}^{\dagger}} + u \frac{\delta}{\delta u} + \bar{u} \frac{\delta}{\delta \bar{u}} \right) \\ & + \left(-\left(\frac{\alpha''(\epsilon)}{2} - \beta''(\epsilon) \right) + \left(-\frac{\alpha(\epsilon)}{2} + \beta(\epsilon) - \frac{\gamma(\epsilon)}{2} \right) \right. \\ & + \left. \left(\frac{\alpha(\epsilon)}{2} - \beta(\epsilon) \right) \right] \cdot \left(K_1^{\dagger} \frac{\delta}{\delta K_1^{\dagger}} + K_1^{\dagger} \frac{\delta}{\delta K_1^{\dagger}} \right) \\ & + \left(\left(\frac{\alpha(\epsilon)}{2} - \beta(\epsilon) \right) - \left(\frac{\alpha''(\epsilon)}{2} - \beta''(\epsilon) \right) \right) \cdot \left(c_1^{\mu} \frac{\delta}{\delta c_1^{\mu}} + c_1^{\mu\dagger} \frac{\delta}{\delta c_1^{\mu\dagger}} \right) \\ & + \left(\left(\frac{\alpha'(\epsilon)}{2} - \beta'(\epsilon) \right) - \left(\frac{\alpha''(\epsilon)}{2} - \beta''(\epsilon) \right) \right) \cdot \left(c_1^{\mu} \frac{\delta}{\delta c_1^{\mu}} + c_1^{\mu\dagger} \frac{\delta}{\delta c_1^{\mu\dagger}} \right) \\ & \left. - \nu_{1(\mu+\nu)} \frac{\delta}{\delta s_1} - \nu_{1(\mu+\nu)}^{\dagger} \frac{\delta}{\delta s_1^{\dagger}} \right] \bar{S}^{\mu} + \zeta(\epsilon) V(s^{\dagger} s). \end{aligned} \quad 32$$

Some points should be explained. They are:

(a) Since u and \bar{u} have common renormalization constant \bar{Z}_3 , so let $\eta(\epsilon) = 0$.

(b) Since s_1 and s_1^{\dagger} have common renormalization constant Z_5 (i.e., the Z_5 of $\overline{s_1 s_1^{\dagger}}$ propagator), so let

$$\beta_1''(\epsilon) = \beta_2''(\epsilon) = \beta_3''(\epsilon).$$

(c) $\zeta(\epsilon)V(s^{\dagger} s)$ can be eliminated by the renormalization of Λ in

Eq. (2) (For example, we can choose $\Lambda_{n+1} = \Lambda_n + \eta^{n+1}\zeta(\theta)$). There is an independent Z_n .

$$\begin{aligned} v_{i(n+1)} &= -(d_{in}^A c_{in}^{A*} + c c_i^{B*}) \delta_i(\theta), \\ v_{i(n+1)} &= -(d_{in}^B c_{in}^B + c^* c_i^B) \delta_i(\theta). \end{aligned} \quad 33a$$

Let

$$\begin{aligned} s_{i(n+1)}^0 &= (Z_i)_{n+1}^{1/2} (\bar{s}_i + (v_i)_{n+1}), \\ s_{i(n)}^0 &= (Z_i)_n^{1/2} (\bar{s}_i + (v_i)_n), \end{aligned} \quad 33b$$

where

$$\begin{aligned} (Z_i)_n &= 1 + \eta z_{i(n)} + \eta^2 z_{i(n)}^2 + \dots + \eta^n z_{i(n)}^n, \\ (v_i)_n &= v_{i(n)} + \eta v_{i(n-1)} + \eta^2 v_{i(n-2)} + \dots + \eta^n v_{i(1)}. \end{aligned} \quad 33c$$

Here the zeroth order does not have spontaneous breaking, and $v_{i(n)} = 0$.

$v_{i(1)}, \dots, v_{i(n)}, v_{i(n+1)}$ are gauge related (related to $c_L^{A^*}, c_L^B, c_L^{A^*}, c_L^{B^*}$) cancelling terms of the polliwog diagram. They can be eliminated by renormalization as following:

$$\begin{aligned} s_{i(n+1)}^0 - s_{i(n)}^0 &= ((Z_i)_{n+1}^{1/2} (\bar{s}_i + (v_i)_{n+1}) - (Z_i)_n^{1/2} (\bar{s}_i + (v_i)_n)) \\ &= \left(\left(1 + \frac{\eta^{n+1} z_{i(n+1)}}{1 + \eta z_{i(n)} + \dots + \eta^n z_{i(n)}^n} \right)^{1/2} - 1 \right) (Z_i)_n^{1/2} \bar{s}_i \\ &\quad + \left(\left(1 + \frac{\eta^{n+1} z_{i(n+1)}}{1 + \eta z_{i(n)} + \dots + \eta^n z_{i(n)}^n} \right)^{1/2} (\eta v_{i(n)} + \dots \right. \\ &\quad \left. + \eta^{n+1} v_{i(n+1)}) - (\eta v_{i(n)} + \dots + \eta^n v_{i(1)}) \right) (Z_i)_n^{1/2} \\ &= \frac{1}{2} \eta^{n+1} z_{i(n+1)} \bar{s}_i + \eta^{n+1} v_{i(n+1)} + o(\eta^{n+2}). \end{aligned} \quad 34a$$

Similarly, we have

$$s_{i(n+1)}^0 - s_{i(n)}^0 = \frac{1}{2} \eta^{n+1} z_{i(n+1)} \bar{s}_i + \eta^{n+1} v_{i(n+1)} + o(\eta^{n+2}). \quad 34b$$

Again, let

$$\begin{aligned} A_{i(n)}^0 &= (Z_i)_n^{1/2} A_i, & u_{i(n)}^0 &= (\tilde{Z}_i)_n^{1/2} u_i, & \bar{u}_{i(n)}^0 &= (\tilde{Z}_i)_n^{1/2} \bar{u}_i, \\ B_{i(n)}^0 &= (Z_i)_n^{1/2} B_i, & u_{i(n)}^0 &= (\tilde{Z}_i)_n^{1/2} u_i, & \bar{u}_{i(n)}^0 &= (\tilde{Z}_i)_n^{1/2} \bar{u}_i. \end{aligned} \quad 34c$$

If every $(Z)_n$ is expanded by

$$(Z)_n = 1 + \eta z_{i(n)} + \dots + \eta^n z_{i(n)}^n. \quad 34d$$

then we have

$$A_{i(n+1)}^0 - A_{i(n)}^0 = \frac{1}{2} \eta^{n+1} z_{i(n+1)} A_i + o(\eta^{n+2})$$

We also can choose neglect ()_n symbol)

$$\begin{aligned} \xi &= \frac{\bar{Z}_1}{\bar{Z}_1 Z_1^{1/2}} \xi, & \xi^{10} &= \frac{\bar{Z}_1}{\bar{Z}_1 Z_1^{1/2}} \xi', & \xi_4^0 &= \frac{\bar{Z}_4}{Z_4}, & \xi_5^0 &= \frac{\bar{Z}_5}{Z_5}, \\ K_1^0 &= (\bar{Z}_1)^{1/2} K_1^0, & K_1^{10} &= (\bar{Z}_1)^{1/2} K_1^{10}, & L_1^0 &= (Z_1)^{1/2} L_1^0, \\ K_1^{10} &= \frac{\bar{Z}_1^{1/2} Z_1^{1/2}}{Z_1^{1/2}} K_1^{10} = \frac{\bar{Z}_1^{1/2} Z_1^{1/2}}{Z_1^{1/2}} K_1^{10}, \\ K_1^{10} &= \frac{\bar{Z}_1^{1/2} Z_1^{1/2}}{Z_1^{1/2}}, & K_1^{10} &= \frac{\bar{Z}_1^{1/2} Z_1^{1/2}}{Z_1^{1/2}} K_1^{10}, \\ c_1^{10} &= \frac{Z_1^{1/2}}{Z_1^{1/2}} c_1^{10}, & c_1^{10} &= \frac{Z_1^{1/2}}{Z_1^{1/2}} c_1^{10}, \\ c_1^{10} &= \frac{Z_1^{1/2}}{Z_1^{1/2}} c_1^{10}, & c_1^{10} &= \frac{Z_1^{1/2}}{Z_1^{1/2}} c_1^{10}. \end{aligned}$$

34e

Comparing (32), then the $z_{(n+1)}$'s terms

$$\begin{aligned} \frac{\alpha(\epsilon)}{2} - \beta(\epsilon) &= -\frac{z_{3(n+1)}}{2}, & -\frac{\alpha(\epsilon)}{2} + \beta(\epsilon) - \frac{\gamma(\epsilon)}{2} &= -\frac{\bar{z}_{3(n+1)}}{2}, \\ \frac{\alpha'(\epsilon)}{2} - \beta'(\epsilon) &= -\frac{z_{1(n+1)}}{2}, & -\frac{\alpha'(\epsilon)}{2} + \beta'(\epsilon) - \frac{\gamma(\epsilon)}{2} &= -\frac{\bar{z}'_{1(n+1)}}{2}, \\ \frac{\alpha''(\epsilon)}{2} - \beta''(\epsilon) &= -\frac{z_{1(n+1)}}{2}, \\ -\frac{\alpha(\epsilon)}{2} &= -\left(\bar{z}_{1(n+1)} - \bar{z}'_{1(n+1)} - \frac{1}{2} z_{3(n+1)}\right), \\ -\frac{\alpha'(\epsilon)}{2} &= -\left(\bar{z}'_{1(n+1)} - \bar{z}_{1(n+1)} - \frac{1}{2} z'_{1(n+1)}\right). \end{aligned}$$

35

If the $A, B, s, s', K, L, c, g, g', \xi, \dots, \bar{u}$, and u of S^R are replaced by $A_{(n+1)}^0, B_{(n+1)}^0, s_{(n+1)}^0, s'_{(n+1)}^0, K_{(n+1)}^0, \dots, \bar{u}_{(n+1)}^0$, and $u_{(n+1)}^0$, then S^R is redefined as S_{n+1}^0 . Similarly, if they are replaced by $A_{(n)}^0, B_{(n)}^0, s_{(n)}^0, s'_{(n)}^0, K_{(n)}^0, \dots, \bar{u}_{(n)}^0$, and $u_{(n)}^0$, then S^R is redefined as S_n^0 . After these replacements, we have

$$\tilde{F}_{(n+1)}(S_{n+1}^0) = \tilde{F}_{(n+1)}(S_n^0) + (\tilde{S}_{n+1}^0 - \tilde{S}_n^0) + o(\gamma^{n+2}).$$

36

Because of Eqs. (32) and (35), $(\tilde{S}_{n+1}^0 - \tilde{S}_n^0) (-o(\gamma^{n+2}))$ will cancel out all the divergent terms $\gamma^{n+2} \tilde{F}_{(n+1)} \text{div}(s_n^0) (-o(\gamma^{n+2}))$; this is a general divergence of (n+1) order) of $\tilde{F}_{(n+1)}(s_n^0)$. But this will not affect those terms with an order less than γ^{n+2} . So, for $\tilde{F}_{(n+1)}(s_{n+1}^0)$, not only those limited terms $\gamma^1, \gamma^2, \dots, \gamma^n$ are still limited, γ^{n+2} also becomes limited. This is the result we want for the renormalization.

Since $s_{\mu\nu}^0$ and s_{μ}^0 are obtained from the variant transformation of s^{μ} , the Slavnov identical equations should hold after the variant transformation.

$$\begin{aligned} & \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu\nu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta J_{\mu\nu}^0} + \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta B_{\mu}^0} + \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\nu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta B_{\nu}^0} \\ & + \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta J_{\mu}^0} + \frac{\delta \tilde{S}_{n+1}^0}{\delta L_{\mu\nu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta u_{\mu\nu}^0} = 0, \\ & - F_{\mu\nu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu\nu}^0} - c_{\mu\nu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu}^0} - c_{\mu\nu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\nu}^0} - \frac{\delta \tilde{S}_{n+1}^0}{\delta u_{\mu\nu}^0} = 0, \\ & - F_{\mu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu}^0} - c_{\mu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\nu}^0} - c_{\mu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\nu}^0} - \frac{\delta \tilde{S}_{n+1}^0}{\delta u_{\mu}^0} = 0, \\ & \tilde{S}_{n+1}^0 = S_{n+1}^0 + \frac{1}{2} \xi_{\mu\nu}^0 (C^{\mu\nu})_{n+1}^2 + \frac{1}{2} \xi_{\mu}^0 (C^{\mu})_{n+1}^2. \end{aligned}$$

From the multiplication relation (Eq. 34) of the renormalization, we have (see expression (4))

$$\frac{1}{2} \xi_{\mu\nu}^0 (C^{\mu\nu})^2 + \frac{1}{2} \xi_{\mu}^0 (C^{\mu})^2 = \frac{1}{2} \xi_{\mu\nu} (C^{\mu\nu})^2 + \frac{1}{2} \xi_{\mu} (C^{\mu})^2.$$

So the gauge fixed term is invariant under renormalization. Because of the following equation (from Eq. 35)

$$\frac{1}{2} z_{3(n+1)} + \frac{1}{2} \tilde{z}_{3(n+1)} = \frac{1}{2} z_{3(n+1)} + \frac{1}{2} \tilde{z}_{3(n+1)} = \frac{\gamma(e)}{2}.$$

we have (the equation above is true for each order)

$$Z_3 \tilde{Z}_3 = Z_3' Z_3'.$$

So, after eliminating the common constant factor, Eq. (37) can be rewritten as

$$\begin{aligned} & \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu\nu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta J_{\mu\nu}^0} + \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta B_{\mu}^0} + \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\nu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta B_{\nu}^0} + \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta J_{\mu}^0} + \frac{\delta \tilde{S}_{n+1}^0}{\delta L_{\mu\nu}^0} \frac{\delta \tilde{S}_{n+1}^0}{\delta u_{\mu\nu}^0} = 0, \\ & - F_{\mu\nu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu\nu}^0} - c_{\mu\nu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu}^0} - c_{\mu\nu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\nu}^0} - \frac{\delta \tilde{S}_{n+1}^0}{\delta u_{\mu\nu}^0} = 0, \\ & - F_{\mu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\mu}^0} - c_{\mu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\nu}^0} - c_{\mu}^0 \frac{\delta \tilde{S}_{n+1}^0}{\delta K_{\nu}^0} - \frac{\delta \tilde{S}_{n+1}^0}{\delta u_{\mu}^0} = 0. \end{aligned}$$

It is obvious that Eq. (39) can be proved rigorously, and is consistent with Eq. (15). So, it can be used as a starting point for the higher order renormalization.

It should be noted that the first one of the equations (37) implies that \tilde{S}_{n+1}^0 is invariant under the following B.R.S. transformations (Eq.

10 is under the variant transformation):

$$\begin{aligned}
\delta A_{i(n+1)}^0 &= (\Delta_i^{A_0} + g_{n+1}^0 A_{i(n+1)}^0) u_{i(n+1)}^0 \delta \lambda^0, \\
\delta B_{i(n+1)}^0 &= (\Delta_i^{B_0} u_{i(n+1)}^0) \delta \lambda^0, \\
\delta s_{i(n+1)}^0 &= \frac{-i}{2} (g_{n+1}^0 \tau_{ik}^0 A_{i(n+1)}^0 u_{i(n+1)}^0 + g'_{n+1}^0 s_{i(n+1)}^0 u_{i(n+1)}^0) \delta \lambda^0, \\
\delta s_{i(n+1)}^{\dagger} &= \frac{i}{2} (g_{n+1}^0 \tau_{ik}^{\dagger} A_{i(n+1)}^{\dagger} u_{i(n+1)}^{\dagger} + g'_{n+1}^0 s_{i(n+1)}^{\dagger} u_{i(n+1)}^{\dagger}) \delta \lambda^0, \\
\delta u_{i(n+1)}^0 &= -\frac{1}{2} g_{n+1}^0 \tau_{ab}^0 u_{i(n+1)}^0 u_{i(n+1)}^0 \delta \lambda^0, \\
\delta u_{i(n+1)}^{\dagger} &= \xi_A^0 (F_i^{A_0} A_{i(n+1)}^{\dagger} + c_{ik}^{A_0} s_{i(n+1)}^{\dagger} + c_{ik}^{\dagger A_0} s_{i(n+1)}^0) \delta \lambda^0, \\
\delta u_{i(n+1)}^0 &= 0, \\
\delta u_{i(n+1)}^{\dagger} &= \xi_B^0 (F_i^{B_0} B_{i(n+1)}^{\dagger} + c_{ik}^{B_0} s_{i(n+1)}^{\dagger} + c_{ik}^{\dagger B_0} s_{i(n+1)}^0) \delta \lambda^0.
\end{aligned}$$

Trying variant transformation again, $A_{i(n+1)}^0, B_{i(n+1)}^0, \dots$ are transformed into A_i, B_i, \dots according to Eq. (34). Among those gauge fixed terms, $\xi_A^{1/2} F_i^{A_0} A_i$ and $\xi_B^{1/2} F_i^{B_0} B_i$ are not renormalized. For the B.R.S. transformation which is related to δu_i and $\delta \bar{u}_i$, $\xi_A F_i^{A_0} A_i$ and $\xi_B F_i^{B_0} B_i$ are invariant before and after renormalization. Because of these, we should choose

$$\delta \lambda^0 = Z_1^{1/2} \bar{Z}_1^{1/2} \delta \lambda = Z_1^{1/2} \bar{Z}_1^{1/2} \delta \lambda.$$

Then we have:

$$\begin{aligned}
\delta A_i &= ((\bar{Z}_1)_{n+1} \Delta_i^{A_0} + g(\bar{Z}_1)_{n+1} A_{i(n+1)}^0) u_i \delta \lambda, \\
\delta B_i &= ((\bar{Z}_1)_{n+1} \Delta_i^{B_0} u_i) \delta \lambda, \\
\delta s_i &= \frac{-i}{2} (g(\bar{Z}_1)_{n+1} \tau_{ik}^0 A_{i(n+1)}^0 u_i + g'(\bar{Z}_1)_{i(n+1)} s_i u_i) \delta \lambda, \\
\delta s_i^{\dagger} &= \frac{i}{2} (g s_i^{\dagger} (\bar{Z}_1)_{n+1} \tau_{ik}^{\dagger} u_i + g'(\bar{Z}_1)_{i(n+1)} s_i^{\dagger} u_i) \delta \lambda, \\
\delta u_i &= -\frac{1}{2} g(\bar{Z}_1)_{n+1} \tau_{ab}^0 u_i u_i \delta \lambda, \\
\delta u_i &= \xi_A (F_i^{A_0} A_i + c_{ik}^{A_0} s_k + c_{ik}^{\dagger A_0} s_k^{\dagger}) \delta \lambda, \\
\delta u_i &= 0, \\
\delta \bar{u}_i &= \xi_B (F_i^{B_0} B_i + c_{ik}^{B_0} s_k + c_{ik}^{\dagger B_0} s_k^{\dagger}) \delta \lambda,
\end{aligned}$$

where

$$s_i = \bar{s}_i + (v_i)_{n+1}, \quad s_i^{\dagger} = \bar{s}_i^{\dagger} + (v_i^{\dagger})_{n+1}.$$

Here, (42) is the B.R.S. transformation related to Eq. (39) (s_{n+1}^0 is invariant under the B.R.S. transformation), and is also the general

form of the B.R.S. transformation of (16). Replacing $n+1$ by n , (42) will become (16).

Comparison between (42) and (10):

(a) In Eq. (10), $s_l = \bar{s}_l$, $s_l^* = \bar{s}_l^*$, but in Eq. (42), s_l and s_l^* are translated by Eq. (43). This is necessary for the elimination of the divergent terms ν_l and ν_l^* (gauge related vacuum expectation value) which appeared during the renormalization process.

(b) Between Eqs. (10) and (42), the following relations hold:

$$\begin{aligned}\Delta_i^{A^0} &\rightarrow \Delta_{i(n+1)}^{A^0} = (\bar{Z}_3)_{n+1} \Delta_i^{A^0}, \\ t_{ij}^0 &\rightarrow t_{ij(n+1)}^0 = (\bar{Z}_2)_{n+1} t_{ij}^0, \\ \tau_{im}^0 &\rightarrow \tau_{im(n+1)}^0 = (\bar{Z}_4)_{n+1} \tau_{im}^0, \\ f_{abc} &\rightarrow f_{abc(n+1)}^0 = (\bar{Z}_1)_{n+1} f_{abc}.\end{aligned}\tag{44}$$

In other words, $\Delta_i^{A^0}$ will become bare $\Delta_{i(n+1)}^{A^0}$, ..., etc. after the correction of $(n+1)$ order perturbation. These bare Δ , t^0 , τ^0 , and f^0 still satisfy (from Eqs. (44) and (6))

$$\begin{aligned}t_{ij(n+1)}^0 t_{kl(n+1)}^0 - t_{il(n+1)}^0 t_{kj(n+1)}^0 &= f_{abc(n+1)}^0 t_{ij(n+1)}^0, \\ \left(-\frac{i}{2}\right) \tau_{kl(n+1)}^0 \left(-\frac{i}{2}\right) \tau_{im(n+1)}^0 - \left(-\frac{i}{2}\right) \tau_{kl(n+1)}^0 \left(-\frac{i}{2}\right) \tau_{im(n+1)}^0 \\ &= f_{abc(n+1)}^0 \left(-\frac{i}{2}\right) \tau_{kl(n+1)}^0, \\ t_{im(n+1)}^0 \Delta_{m(n+1)}^{A^0} - t_{im(n+1)}^{B^0} \Delta_{m(n+1)}^{A^0} &= f_{abc(n+1)}^0 \Delta_{i(n+1)}^{A^0}.\end{aligned}\tag{45}$$

This is the same as Eq. (6).

(c) Exchanging i and l in Eq. (45), we have

$$t_{il(n+1)}^0 t_{ij(n+1)}^0 - t_{il(n+1)}^{B^0} t_{ij(n+1)}^0 = f_{abc(n+1)}^0 t_{il(n+1)}^0 = 0.$$

$f_{abc} = M_{ab}^c$ can be treated as a matrix. For different c , M^c 's are mutually independent. So, we have

$$t_{il(n+1)}^0 = 0.\tag{46}$$

Since $f_{abc} \sim \mathcal{E}_{abc}$, so $f_{abc(n+1)}^0 \sim \mathcal{E}_{abc}$. This implies

$$f_{abc(n+1)}^0 = 0.\tag{47}$$

Therefore, under the B.R.S. transformations of Eq. (42), the sum of

the Jacobian diagonal of the functional integral volume element is zero and the Jacobian is 1. The Slavnov identical equations of $\tilde{\Gamma}$ at (n+1) order can thus be obtained. (i.e., Eq.(17), $\tilde{\Gamma}$ is derived by s_{μ}^c ,)

This will guarantee that the same renormalization process can be continued for the next higher orders.

(d) The gauge transformations which are corresponding to Eq. (42)

are (let $u, \delta\lambda = \frac{1}{(\tilde{Z}_1)_{n+1}} \theta^a, u\delta\lambda = \frac{1}{(\tilde{Z}')_{n+1}} \theta$)

$$\delta A_i = \Delta_i^a + \frac{(\tilde{Z}_1)_{n+1}}{(\tilde{Z}_2)_{n+1}} g_i^a A_i \theta^a,$$

$$\delta B_\mu = (\Delta_\mu^a) \theta,$$

$$\delta s_i = \frac{-1}{2} \left(\frac{(\tilde{Z}_1)_{n+1}}{(\tilde{Z}_3)_{n+1}} g_i^a \tau_{ik}^a \theta^b + \frac{(\tilde{Z}')_{n+1}}{(\tilde{Z}_3)_{n+1}} g_i^a \theta \right),$$

$$\delta s_j = \frac{1}{2} \left(\frac{(\tilde{Z}_1)_{n+1}}{(\tilde{Z}_3)_{n+1}} g_j^a \tau_{kj}^a \theta^b + \frac{(\tilde{Z}')_{n+1}}{(\tilde{Z}_3)_{n+1}} g_j^a \theta \right). \quad 48$$

It can be seen from (b), (c), and (d) that the structure constants of gauge groups before and after renormalization are one to one correspondence, and that the Abel subgroups are still Abel subgroups. This is the proof of the isomorphism between gauge groups before and after renormalization.

In the presence of fermion fields ψ_L and ψ_R , since gA_i and $g'B_\mu$ appear in \mathcal{L}_L and \mathcal{L}_R , they are zero after the operation by $\frac{1}{2} \frac{\delta}{\delta A_i}$, $-\frac{g}{2} \frac{\partial}{\partial g}$, $\frac{B_\mu}{2} \frac{\delta}{\delta B_\mu}$, $-\frac{g'}{2} \frac{\partial}{\partial g'}$. This will only give new renormalization constants Z_L and Z_R , but will not affect the overall renormalization. The fermion field coupling with the Higgs field will give new coupling constants and then new renormalization constants. But again this will not affect the overall renormalization.

If the vacuum spontaneous symmetry is broken, then the discussion of Zinn-Justin and Lee [2] is applicable. No discussion will be given here.

The method in this paper also can be applied to the case with several subgroups.

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ON THE ISOMORPHISM BETWEEN GAUGE GROUPS BEFORE AND AFTER RENORMALIZATION, IN THE PRESENCE OF ABEL SUBGROUPS AND HIGGS FIELDS

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ABSTRACT

We give a rigorous proof on the isomorphism between gauge groups before and after renormalization, in the presence of Abelian subgroups and Higgs fields.

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