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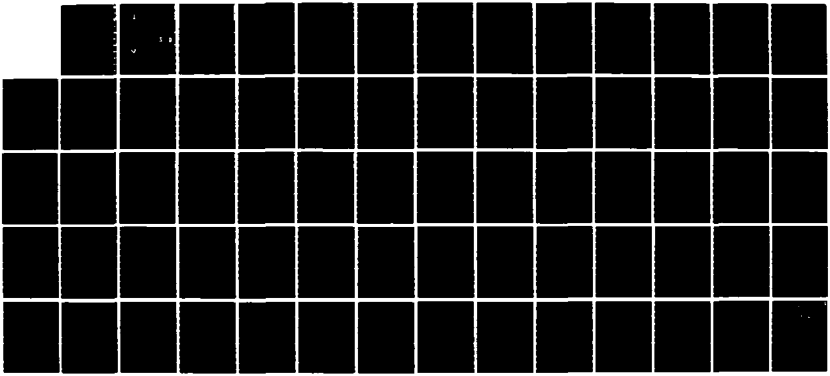

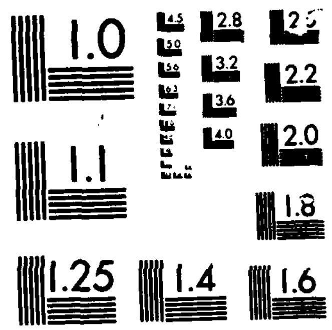

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TECHNICAL REPORT RL-85-4

A VISCOELASTIC/DAMAGE SIMULATION MODEL FOR FILAMENT-WOUND PRESSURE VESSELS

Johnny L. Prater
Structures Directorate
Research, Development, and Engineering Center

MAY 1985

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35898-5000

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A model to predict the viscoelastic/damage response of a filament-wound composite cylindrical pressure vessel to a proof-test loading is developed. The matrix material of the composite system is assumed to be isotropic and linearly viscoelastic. A damage model is proposed which produces a quadratic relationship between the transverse modulus and the circumferential strains. A nonlinear model results and is solved by iterative solution based on the damage response. The elastic-viscoelastic correspondence principle is used to (over)

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20. ABSTRACT (Continued)

produce, in the Laplace domain, an associated elastic solution for the circumferential strains.

The associated elastic solution is inverted by using the method of collocation to yield the time-dependent circumferential strain. A numerical example to demonstrate the simulation model is included. A proof-test loading of 5.5 MPa is applied for one minute. Results indicate that the damage as modeled reduces the instantaneous failure load from 9.977 MPa to 7.771 MPa, or 22%. The creep that occurs during the constant load phase of the proof-test loading induces a 3.65% increase in the damage. After removal of the proof-test loading, the strain recovers to zero. However, the transverse modulus does not recover. This emphasizes the need to consider the effects of proof-test loading when the in-service load limits are determined.

TABLE OF CONTENTS

LIST OF FIGURES

PAGE NO.

LIST OF SYMBOLS

A	Constant
a_{ij}	Function of laminate stiffness ($i,j = 1,2,3$)
B	Constant
B_j	Constant ($j = 1,2,\ldots,6$)
b_{ij}	Function of a_{ij} ($i,j = 1,2,3$)
C	Constant
C_{ij}	Stiffness term
$\underline{\underline{C}}$	Lamina stiffness matrix in local coordinates
$\underline{\underline{C}}'$	Lamina stiffness matrix in global coordinates
$\underline{\underline{C}}_L$	Laminate stiffness matrix
D	Constant
E_L	Lamina longitudinal modulus
E_T	Lamina transverse modulus
E_{T_I}	Initial transverse modulus
E_f	Young's modulus of the fiber
E_m	Relaxation modulus of the matrix
E_{m_O}	Relaxation modulus of the matrix at time zero
e_θ, e_z, e_r	Vector components in the r,θ,z coordinate system
e_L', e_T', e_r'	Vector components in the L,T,r coordinate system
F	Instantaneous failure load
F_f	Fiber fracture strength
F_i	Constant

LIST OF SYMBOLS (Continued)

G	Constant
G_{LT}	Lamina shear modulus for L-T plane
G_f	Shear modulus of the fiber
G_m	Shear modulus of the matrix
g_v	Constant in solution approximation for linearly varying load
H	Constant
h_v	Constant in solution approximation for step load
J_m	Creep compliance of matrix
K	Bulk modulus of matrix
K_θ	Composite ultimate failure strain
L,T,r	Lamina coordinates
ℓ_i, m_i, n_i	Direction cosines ($i = 1,2,3$)
P	Pressure
r	Radius
r_i	Cylinder inner radius
r_o	Cylinder outer radius
s	Laplace parameter
T_i	Surface traction
$\underline{\underline{T}}$	Transformation matrix from L,T,r to r,θ,z
t	Time
t_i	Thickness of i^{th} lamina
t_L	Total laminate thickness
u	Radial displacement
u_r, u_θ, u_z	Respectively, radial, circumferential, and meridional displacement

α	Constant
a_v, α_v	Constant in exponent of approximate solution
β	Constant
γ	Fiber wrap angle
Δ	Determinant of a_{ij} matrix
δ	Constant = $(1 + \nu_{TT})(1 - \nu_{TT}) - 2\ \nu_{LT}\ \nu_{LT}$
ϵ_{ij}	Tensor strain
ϵ'	Circumferential strain response to step load
ϵ''	Circumferential strain response to linearly varying load
ϵ_f	Failure strain
$\underline{\epsilon}$	Strain vector
λ_i	Constant in approximate solution
ν_{TT}, ν_{LT}, ν_{TL}	Lamina Poisson's ratio
ν_f	Poisson's ratio of fiber
ν_m	Poisson's ratio of matrix
σ_{ij}	Stress tensor
$\underline{\sigma}$	Stress vector
ϕ_v	Constant in general solution approximation
ϕ, ϕ', ϕ''	Constants in general solution approximation
ω	Damage

I. INTRODUCTION

Filamentary organic matrix composites, hereafter called composites, are efficient structural material, coupling low weight with high strength. They have been used for many years in various secondary and primary structural roles both in commercial products and in military applications. A primary use is for pressure containment structures such as rocket motor cases, launch tubes, and cold gas storage vessels. Some design philosophies require that such pressure containment structures be 100% proof tested, i.e., proof test each item produced. These proof tests, to be effective in eliminating defective hardware and assuring safety during the operational life, are usually conducted at significant load levels. It is a well-known and frequently observed phenomenon that such proof tests induce a form of damage called crazing in most composites. Other forms of damage may also occur such as matrix/fiber debonding and ply delamination. Hereafter the term "damage" will be used to denote the internal damage resulting from a proof-test loading. In order to fully utilize the high specific strength of composites, a mechanism to include the effect of this proof-test-induced damage is needed.

There has been a significant amount of work in the area of composite damage with a large part being devoted to fatigue damage. A compilation of some of these efforts can be found in References [1], [2], [3], [4], and [5]. The following discussion will point out some of the different approaches used to include damage accumulation in materials.

Smith and Huang [6] and Lee [7] have used linearly elastic finite element analyses to model damage accumulation in composites. Both use a failure criterion to define the matrix damage zone. The stiffness is then reduced in the damaged elements. The loading is incremental until a global failure condition is met.

Reifsnider and Highsmith [8], [9] have investigated stiffness reduction in laminated composites. They reference an earlier work by Reifsnider [10] where a characteristic damage state of crack patterns in transverse plies was predicted. The crack patterns were shown to be a laminate property, determined only by lamina properties and the orientations and stacking of the laminae. In References [8] and [9], a linear fracture mechanics approach was taken to predict crack growth and subsequent failure where the moduli were reduced as a function of the crack geometry.

Nuismer and Tan [11] included the effects of matrix cracking through a constitutive theory based upon simple physical models of ply damage, where certain parameters relating to the growth of ply damage and subsequent loss of ply stiffness are predicted using a Griffith-type energy balance. Several investigators [12-15] have used what they termed "continuum damage mechanics" to study the accumulation of damage in an elastic homogeneous body. This approach is based on earlier work by Kachanov [16] where the process of failure was considered as a process of crack formation against a background of creep deformation. Kachanov defines a quantity called the "continuity" which is a measure of the damage or cracking in the body. Murakami [12] elucidates the notion of continuum damage mechanics which hypothesizes that the effects of damage to a homogeneous body can be described by appropriate mechanical variables which he termed damage variables. The continuity function defined by Kachanov is extended to tensor form. Controlled experiments are conducted with good agreement between results and theory. Krajcinovic [13, 15] and Krajcinovic and Fonseka [14] have also investigated damage in homogeneous elastic materials using continuum damage mechanics with similar success. All of the efforts discussed used either a linear elastic or linear fracture mechanics approach to incorporate the internal damage of a material. Schapery [17] approached the problem of damage in composites from a viscoelastic point of view. Schapery developed relationships of the hereditary integral form to include not only damage, but also a "healing," i.e., decrease in damage, effect.

In view of the well-known viscoelastic behavior of organic matrix composites, it seems appropriate that this behavior be included in the simulation model discussed here. Well-established classical viscoelastic procedures are used with modifications to account for damage and its accumulation. The inclusion of damage will be shown to produce a nonlinear model.

As noted earlier, Kachanov [16] used a parameter termed the continuity function to define the state of partial cracking, i.e., partial damage, of a material. Appealing to this idea, a damage function, ω, is defined. It is assumed that this function has zero value when no damage is present and increases monotonically to the value of one at failure. One may think of ω as an index of the internal damage.

A viscoelastic material may initially contain minute voids or other defects. As the material is loaded and creep occurs, these voids will tend to grow in size. New voids will form at the defects and grow also. At some time, these voids will coalesce into microcracks and continue to grow as a result of the creep of the material until critical cracks are formed and failure occurs. Other forms of damage may also be aggravated by the creep. The creep of this damaged material produces a change in its deformation state. It is well known that a change in displacement state is related to a change in the state of strain in the material. This implies that strain and the damage that has accumulated are related. Therefore, it is assumed that the damage is some function of the state of strain that exists.

O'Brien [18] and Highsmith and Reifsnider [9] have shown that damage accumulation reduces the stiffness of a composite. Highsmith and Reifsnider modeled the overall laminate response by reducing the transverse modulus of the laminae as a result of accumulated damage. This idea is central to the development here and is the basis for the model that is constructed. Here, as in Highsmith's and Reifsnider's work, a laminate analysis using the damage-reduced transverse modulus of the laminae is employed to predict the laminate stiffness values. These resulting stiffnesses are then used in a simulation model to predict the viscoelastic response of a composite cylinder which includes the damage effect. The simulation uses both linear elastic and linear viscoelastic responses modified to account for the dependence of the transverse modulus on the strain via the damage function. The matrix is assumed to be linearly viscoelastic and isotropic with the fiber being linearly elastic and isotropic. The well-known elastic-viscoelastic correspondence principle is employed in the solution. A discussion of the correspondence principle is found in Section V. Basically, it allows one to obtain the Laplace transform of a time-dependent solution by simple substitutions of various Laplace transformed parameters into an elastic solution of the same geometry and constraints. This Laplace transformed solution is called the "associated elastic" solution. Due to the complexity of the associated elastic solution, a closed-form inversion is not sought. Rather, the associated elastic solution is inverted by a numerical procedure to yield the time-dependent response. The numerical procedure requires that the associated elastic solution be known only for discreet values of the Laplace parameter.

The remaining sections are arranged in the order that the respective elements are used in the solution process. Section II develops the governing equations for an elastic solution for a cylindrical body under uniform internal pressure. These equations are then solved to yield the radial displacement, meridional strain, radial strain, and circumferential strain. Section III develops the elastic material relationships property that is required in the solution. The transformation from local lamina to global laminate properties is defined. The various micromechanical models for the lamina constitutive properties are also included for completeness. In Section IV, a short discussion of the time-dependent material properties is included. Section V discusses the viscoelastic solution procedure. The correspondence principle is defined and the parameters for substitution into the elastic solution to produce the associated elastic solution are noted. The numerical inversion technique is discussed. The approximate solutions for both the linearly varying load and a step load are developed and the method for determining the unknown solution constants are shown. Section VI contains a discussion of damage in a viscoelastic material. In Section VII, implementation of the solution is discussed. The results of a numerical example are included in Section VIII and a listing of the program developed is contained in the Appendix.

II. GOVERNING EQUATIONS FOR ELASTIC SOLUTION

A. Equilibrium Equations

The model to be developed is for a composite cylinder. Choosing a point sufficiently far from the ends and neglecting body forces, the general three-dimensional equilibrium equations are given as [19]:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta r}}{\partial \theta} + \frac{\partial \sigma_{zr}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \qquad (1a)$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{z\theta}}{\partial z} + \frac{2}{r}\sigma_{r\theta} = 0 \qquad (1b)$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{1}{r}\sigma_{rz} = 0 \qquad (1c)$$

where the σ_{ij} are stresses in the θ,r,z cylindrical coordinate system. For uniform loading, all the shear stress components vanish, i.e., $\sigma_{r\theta} = \sigma_{rz} = \sigma_{\theta z} = \sigma_{\theta r} = \sigma_{z\theta} = \sigma_{zr} = 0$. Another consequence of the uniform loading is that there is no variation of the stress in the θ- and z-directions at a point sufficiently far from the ends of the cylinder. Therefore Eqs. (1) reduce to:

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \qquad (2)$$

where the partial differentiation becomes total differentiation since r is the only remaining variable.

B. Kinematic Equations

Assuming small deformations and linear elasticity, the kinematic equations for small deformation, linear elasticity for cylindrical coordinates are given by [19]:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} \qquad (3a)$$

$$\varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \tag{3b}$$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} \tag{3c}$$

$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) \tag{3d}$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) \tag{3e}$$

$$\varepsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) \tag{3f}$$

where the ε_{ij} are tensor strain components and the u_i are displacements in the θ,r,z coordinate system. For uniform loading, the shear strain components vanish, and $u_\theta = 0$. At a point sufficiently far from the ends of the cylinder, ε_{zz} is assumed to be constant. Thus Eqs. (3) reduce to:

$$\varepsilon_{rr} = \frac{du_r}{dr} \tag{4a}$$

$$\varepsilon_{\theta\theta} = \frac{u_r}{r} \tag{4b}$$

$$\varepsilon_{zz} = \text{constant} \tag{4c}$$

C. Constitutive Equations

For a uniformly loaded cylinder, the shear stresses and shear strains are zero and the constitutive relationship reduces to [19]:

$$\begin{Bmatrix} \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{rr} \end{Bmatrix} = \begin{bmatrix} C_{\theta\theta} & C_{\theta z} & C_{\theta r} \\ C_{\theta z} & C_{zz} & C_{zr} \\ C_{\theta r} & C_{zr} & C_{rr} \end{bmatrix} \begin{Bmatrix} \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ \varepsilon_{rr} \end{Bmatrix} \tag{5}$$

The elements of the 3x3 matrix of Eq. (5) will be developed in a following section.

D. Boundary Conditions

Boundary conditions for the internally pressurized cylinder are:

$$\sigma_{rr} = -P \text{ (at the inner surface)} \tag{6a}$$

$$\sigma_{rr} = 0 \text{ (at the outer surface)} \tag{6b}$$

where P is the internal pressure.

E. Solution

Combining Eqs. (2), (4) and (5) yields the governing differential equation:

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{1}{r^2}\alpha^2 u = \frac{1}{r}\beta\,\varepsilon_{zz} \tag{7}$$

where

$$\alpha^2 = \frac{C_{\theta\theta}}{C_{rr}} \tag{8a}$$

and

$$\beta = \frac{C_{z\theta} - C_{zr}}{C_{rr}} \tag{8b}$$

The subscript, r, has been dropped on the radial displacement for convenience, i.e., $u_r = u$.

The general solution to Eq. (7) is given as [20]:

$$u = Ar^{\alpha} + Br^{-\alpha} + \frac{\beta \varepsilon_{zz} r}{1-\alpha^2} \; ; \; \alpha \neq 1 \qquad (9a)$$

$$u = Ar + Br^{-1} + \frac{r}{2} \beta \varepsilon_{zz} \left[\ln(r) + \frac{1}{4} \right] \; ; \; \alpha = 1 \qquad (9b)$$

The parameter α is a measure of the anisotropy in the plane of cross section. Normally, $\alpha = 1$ only for unidirectional composites with fiber direction parallel to the meridional axis. For this model, the fibers are not parallel to the meridional axis. Therefore, the expression given by Eq. (9a) will be the only one considered.

From Eqs. (4b) and (9a):

$$\varepsilon_{\theta\theta} = Ar^{(\alpha-1)} + Br^{-(\alpha+1)} + \frac{\beta \varepsilon_{zz}}{1-\alpha^2} \qquad (10)$$

From Eqs. (4a) and (9a):

$$\varepsilon_{rr} = A\alpha r^{(\alpha-1)} - B\alpha r^{-(\alpha+1)} + \frac{\beta \varepsilon_{zz}}{1-\alpha^2} \qquad (11)$$

Combining Eqs. (6), (10) and (11) yields:

$$\sigma_{rr} = A(C_{\theta r} + C_{rr}\alpha) r^{(\alpha-1)} + B(C_{\theta r} - C_{rr}\alpha) r^{-(\alpha+1)} + \varepsilon_{zz} \left[C_{zr} + \frac{\beta}{1-\alpha^2} (C_{\theta r} + C_{rr}) \right] \qquad (12)$$

Equation (12) has three unknowns (A, B, and ε_{zz}) but only two boundary conditions exist for determining them. A third equation is needed and may be developed by considering static equilibriums of the cylinder in the z-direction. Taking a freebody of the cylinder and summing forces in the z-direction yields:

$$P \pi r_i^2 = \int_{r=r_i}^{r_o} \sigma_{zz} (2\pi r) dr \tag{13}$$

which reduces to:

$$\frac{P r_i^2}{2} = \int_{r=r_i}^{r_o} \sigma_{zz} \, r dr \tag{14}$$

where r_i is the inner radius and r_o is the outer radius of the cylinder. Using Eqs. (5) in Eq. (14) gives:

$$\frac{P r_i^2}{2} = \int_{r=r_i}^{r_o} (C_{zz} \varepsilon_{zz} + C_{z\theta} \varepsilon_{\theta\theta} + C_{zr} \varepsilon_{rr}) r dr \tag{15}$$

Substituting Eqs. (10) and (11) into Eq. (15), rearranging, and performing the indicated integration yields:

$$P = A \frac{2}{r_i^2} \left[\frac{r_o^{(1+\alpha)} - r_i^{(1+\alpha)}}{1+\alpha} \right] (C_{z\theta} + \alpha C_{zr})$$

$$+ B \frac{2}{r_i^2} \left[\frac{r_o^{(1-\alpha)} - r_i^{(1-\alpha)}}{1-\alpha} \right] (C_{z\theta} - \alpha C_{zr})$$

$$+ \varepsilon_{zz} \frac{r_o^2 - r_i^2}{r_i^2} \left[C_{zz} + \frac{\beta}{1-\alpha^2} (C_{z\theta} + C_{zr}) \right] \tag{16}$$

Substituting Eqs. (6) into Eq. (16) gives:

$$-P = A(C_{\theta r} + \alpha C_{rr})r_i^{-(1-\alpha)} + B(C_{\theta r} - \alpha C_{rr})r_i^{-(1+\alpha)}$$

$$+ \varepsilon_{zz} \left[C_{zr} + \frac{\beta}{1 - \alpha^2}(C_{\theta r} + C_{rr})\right] \qquad (17)$$

and

$$0 = A(C_{\theta r} + \alpha C_{rr})r_o^{-(1+\alpha)} + B(C_{\theta r} - \alpha C_{rr})r_o^{-(1+\alpha)}$$

$$+ \varepsilon_{zz} \left[C_{zr} + \frac{\beta}{1 - \alpha^2}(C_{\theta r} + C_{rr})\right] \qquad (18)$$

Equations (16), (17) and (18) can now be solved simultaneously to obtain A, B, and ε_{zz} as:

$$A = \frac{P}{\Delta}(b_{11} - b_{21}) \qquad (19a)$$

$$B = \frac{P}{\Delta}(b_{12} - b_{22}) \qquad (19b)$$

$$\varepsilon_{zz} = \frac{P}{\Delta}(b_{13} - b_{23}) \qquad (19c)$$

where

$$b_{11} = a_{22}a_{33} - a_{32}a_{23} \qquad (20a)$$

$$b_{12} = -(a_{21}a_{33} - a_{31}a_{23}) \qquad (20b)$$

$$b_{13} = a_{21}a_{32} - a_{22}a_{31} \qquad (20c)$$

$$b_{21} = -(a_{12}a_{33} - a_{32}a_{13}) \qquad (20d)$$

$$b_{22} = a_{11}a_{33} - a_{31}a_{13} \qquad (20e)$$

$$b_{23} = -(a_{11}a_{32} - a_{31}a_{12}) \qquad (20f)$$

$$\Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$$

$$-a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{32}a_{23} \qquad (20g)$$

with

$$a_{11} = \frac{2}{r_i^2}\left[\frac{r_o^{(1+\alpha)} - r_i^{(1+\alpha)}}{1+\alpha}\right](C_{z\theta} + \alpha\, C_{zr}) \qquad (21a)$$

$$a_{12} = \frac{2}{r_i^2}\left[\frac{r_o^{(1-\alpha)} - r_i^{(1-\alpha)}}{1-\alpha}\right](C_{z\theta} - \alpha\, C_{zr}) \qquad (21b)$$

$$a_{13} \quad \frac{r_o^2 - r_i^2}{r_i^2}\left[C_{zz} + \frac{\beta}{1-\alpha^2}(C_{z\theta} + C_{zr})\right] \qquad (21c)$$

$$a_{21} = (C_{\theta r} + \alpha\, C_{rr})r_i^{-(1-\alpha)} \qquad (21d)$$

$$a_{22} = (C_{\theta r} - \alpha\, C_{rr})r_i^{-(1+\alpha)} \qquad (21e)$$

$$a_{23} = C_{zr} + \frac{\beta}{1-\alpha^2}(C_{\theta r} + C_{rr}) \qquad (21f)$$

$$a_{31} = (C_{\theta r} + \alpha\, C_{rr})r_o^{-(1-\alpha)} \qquad (21g)$$

$$a_{32} = (C_{\theta r} - \alpha\, C_{rr})r_o^{-(1+\alpha)} \qquad (21h)$$

$$a_{33} = C_{zr} + \frac{\beta}{1-\alpha^2}(C_{\theta r} + C_{rr}) \qquad (21i)$$

The radial displacement given by Eq. (9a) may now be determined. The hoop strain, $\varepsilon_{\theta\theta}$, and radial strain, ε_{rr}, may be calculated by using Eqs. (10) and (11), respectively. With the strains (ε_{rr}, $\varepsilon_{\theta\theta}$ and ε_{zz}) known, the stresses may be determined from Eq. (5).

III. MATERIAL PROPERTIES

The cylinder under discussion is assumed to be composed of a laminate made up of several laminae. It is further assumed that the laminae alternate with an angle of $\pm\ \gamma$ to the hoop direction as shown in Figure 1. To produce the laminate stiffness, one must combine the stiffnesses of the various laminae in an appropriate manner. The first step is the transformation of the lamina properties from the local L,T,r coordinate system to the global θ,z,r coordinate system.

A. Lamina Stiffness

Consider the vector transformation from the lamina L,T,r coordinate system to the global θ,z,r system given by:

$$\begin{Bmatrix} e_\theta \\ e_z \\ e_r \end{Bmatrix} = \begin{bmatrix} \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \\ \ell_3 & m_3 & n_3 \end{bmatrix} \begin{Bmatrix} e'_L \\ e'_T \\ e'_r \end{Bmatrix} \tag{22}$$

where the e_i are the respective components and the prime denotes the global θ,r,z system. The direction cosines, ℓ_i, m_i, n_i ($i = 1,2,3$) are given by:

$$\ell_1 = \cos(L,\theta) = \cos\gamma \tag{23a}$$

$$m_1 = \cos(T,\theta) = \sin\gamma \tag{23b}$$

$$n_1 = \cos(r,\theta) = 0 \tag{23c}$$

$$\ell_2 = \cos(L,z) = -\sin\gamma \tag{23d}$$

$$m_2 = \cos(T,z) = \cos\gamma \tag{23e}$$

$$n_2 = \cos(r,z) = 0 \tag{23f}$$

$$\ell_3 = \cos(L,r) = 0 \tag{23g}$$

$$m_3 = \cos(T,r) = 0 \tag{23h}$$

$$n_3 = \cos(r,r) = 1 \tag{23i}$$

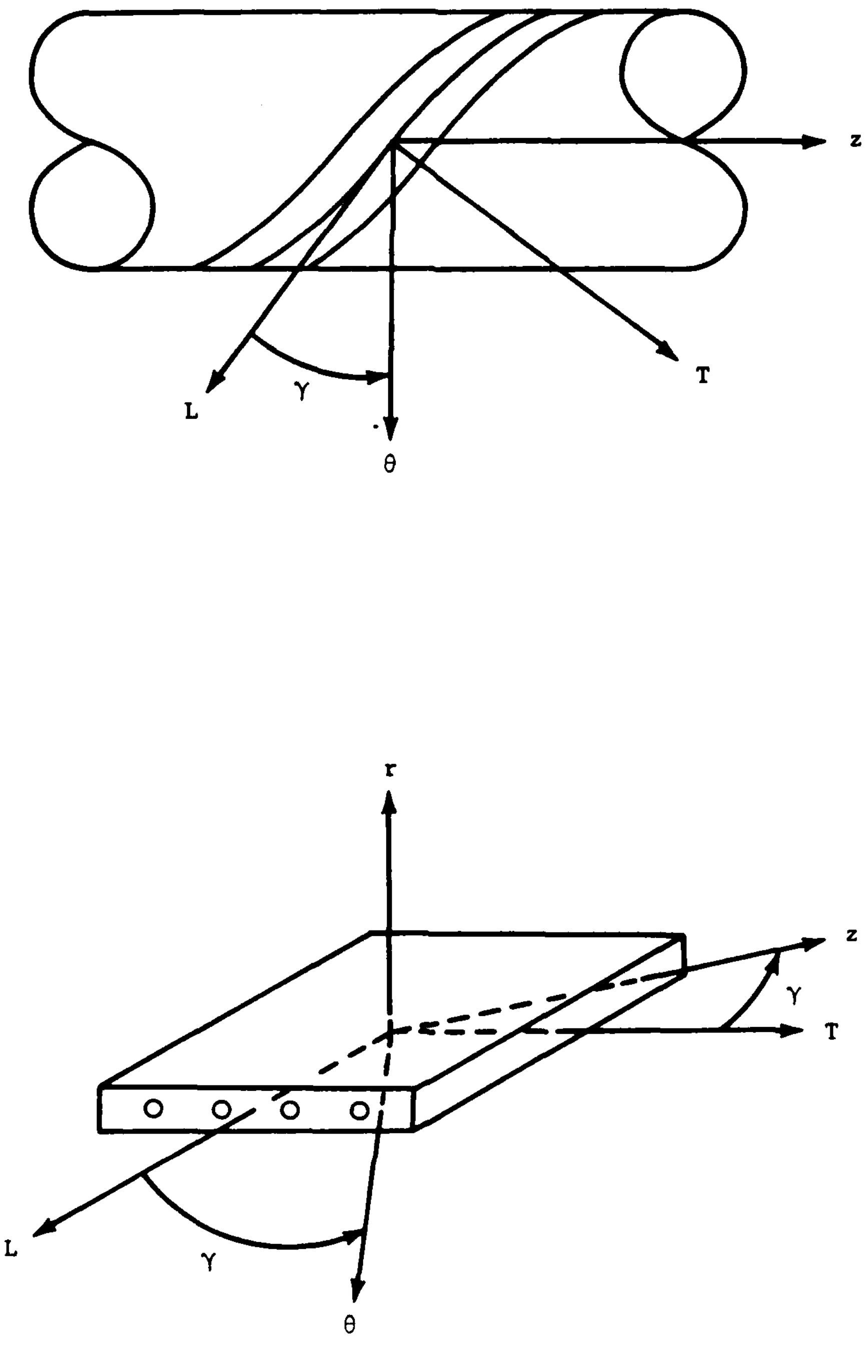


Figure 1. Definition of coordinate system.

It is well known that the stresses transform according to:

$$\underline{\sigma}' = \underline{T}\,\underline{\sigma} \tag{24}$$

where

$$\underline{\sigma}' = \begin{Bmatrix} \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{rr} \\ \sigma_{\theta z} \\ \sigma_{zr} \\ \sigma_{\theta r} \end{Bmatrix} \tag{25}$$

$$\underline{\sigma} = \begin{Bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{rr} \\ \sigma_{LT} \\ \sigma_{Tr} \\ \sigma_{rL} \end{Bmatrix} \tag{26}$$

and

$$\underline{T} = \begin{bmatrix} \ell_1^2 & m_1^2 & n_1^2 & 2\ell_1 m_1 & 2m_1 n_1 & 2n_1\ell_1 \\ \ell_2^2 & m_2^2 & n_2^2 & 2\ell_2 m_2 & 2m_2 n_2 & 2n_2\ell_2 \\ \ell_3^2 & m_3^2 & n_3^2 & 2\ell_3 m_3 & 2m_3 n_3 & 2n_3\ell_3 \\ \ell_1\ell_2 & m_1 m_2 & n_1 n_2 & (\ell_1 m_2 + m_1\ell_2) & (m_1 n_2 + n_1 m_2) & (n_1\ell_2 + \ell_1 n_2) \\ \ell_2\ell_3 & m_2 m_3 & n_2 n_3 & (\ell_2 m_3 + m_2\ell_3) & (m_2 n_3 + n_2 m_3) & (n_2\ell_3 + \ell_2 n_3) \\ \ell_3\ell_1 & m_3 m_1 & n_3 n_1 & (\ell_3 m_1 + m_3\ell_1) & (m_3 n_1 + n_3 m_1) & (n_3\ell_1 + \ell_3 n_1) \end{bmatrix} \tag{27}$$

For the local system, the constitutive relationship is given by:

$$\underline{\sigma} = \underline{C}\ \underline{\varepsilon} \tag{28}$$

where $\underline{C}$ is a 6x6 matrix of effective lamina properties and is given by [21]:

$$\underline{C} = \begin{bmatrix} C_{LL} & C_{LT} & C_{Lr} & 0 & 0 & 0 \\ C_{LT} & C_{TT} & C_{Tr} & 0 & 0 & 0 \\ C_{Lr} & C_{Tr} & C_{rr} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\ G_{LT} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\ G_{Tr} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\ G_{Lr} \end{bmatrix} \tag{29}$$

and tensor strain, $\underline{\varepsilon}$, given by,

$$\underline{\varepsilon} = \begin{Bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{rr} \\ \varepsilon_{LT} \\ \varepsilon_{Tr} \\ \varepsilon_{Lr} \end{Bmatrix} \tag{30}$$

A single bar under a variable is used to denote a vector or first-order tensor. A double bar under a variable is used to denote a second-order tensor quantity.

The effective lamina properties are given by [21]:

$$C_{LL} = \frac{(1 - \nu_{TT}^2 E_L)}{\delta} \quad (31a)$$

$$C_{TT} = \frac{(1 - \nu_{LT} \nu_{TL}) E_T}{\delta} \quad (31b)$$

$$C_{LT} = \frac{(\nu_{TL} 1 + \nu_{TT}) E_L}{\delta} \quad (31c)$$

$$C_{Tr} = \frac{(\nu_{TT} + \nu_{LT}\nu_{TL}) E_L}{\delta} \quad (31d)$$

$$G_{Tr} = \frac{(1 - \nu_{TT} - 2\nu_{LT}\nu_{TL}) E_T}{2\delta} \quad (31e)$$

and, because of isotropy in the T-r plane:

$$C_{Lr} = C_{LT} \quad (31f)$$

$$C_{rr} = C_{TT} \quad (31g)$$

$$G_{Lr} = G_{LT} \quad (31h)$$

with

$$\delta = (1 + \nu_{TT})(1 - \nu_{TT} - 2\nu_{LT}\nu_{TL}) \quad (31i)$$

The constituent property models for E_L, E_T, ν_{TT}, ν_{LT}, ν_{TL} and G_{LT} will be discussed later.

Substituting Eq. (28) into Eq. (24):

$$\underline{\sigma}' = \underline{\underline{T}}\, \underline{\underline{C}}\, \underline{\varepsilon} \quad (32)$$

The tensor strain transforms according to:

$$\underline{\varepsilon}' = \underline{\underline{T}}\, \underline{\varepsilon} \quad (33)$$

Solving for ε in Eq. (33):

$$\underline{\varepsilon}' = \underline{\underline{T}}^{-1} \; \underline{\varepsilon}' \tag{34}$$

where the -1 superscript denotes the matrix inverse. Substituting Eq. (34) into Eq. (32), and noting that for the primed, global θ,r,z coordinate system,

$$\underline{\sigma}' = \underline{\underline{C}}' \; \underline{\varepsilon}' \tag{35}$$

yields:

$$\underline{\underline{C}}' \; \underline{\varepsilon}' = \underline{\underline{T}} \; \underline{\underline{C}} \; \underline{\underline{T}}^{-1} \; \underline{\varepsilon}' \tag{36}$$

Thus it is seen that

$$\underline{\underline{C}}' = \underline{\underline{T}} \; \underline{\underline{C}} \; \underline{\underline{T}}^{-1} \tag{37}$$

where $\underline{\underline{C}}'$ is the stiffness of the lamina transformed to the global θ,r,z system. The task now is to perform the indicated multiplication of Eq. (37) to produce the C values in Eq. (5). After much algebra, one obtains for the reduced lamina stiffness transformed to the global θ,r,z, system:

$$C_{zz} = \sin^4\gamma \, C_{LL} + \cos^4\gamma \, C_{TT} + 2 \cos^2\gamma \, \sin^2\gamma \, (C_{LT} + 2 \, G_{LT}) \tag{38a}$$

$$C_{z\theta} = C_{LT} \, (\sin^4\gamma + \cos^4\gamma) + \sin^2\gamma \, \cos^2\gamma \, (C_{LL} + C_{TT} - 4 \, G_{LT}) \tag{38b}$$

$$C_{zr} = \sin^2\gamma \, C_{Lr} + \cos^2\gamma \, C_{Tr} \tag{38c}$$

$$C_{\theta\theta} = \cos^4\gamma \, C_{LL} + \sin^4\gamma \, C_{TT} + 2 \sin^2\gamma \, \cos^2\gamma \, (C_{LT} + 2 \, G_{LT}) \tag{38d}$$

$$C_{\theta r} = \cos^2\gamma \, C_{Lr} + \sin^2\gamma \, C_{Tr} \tag{38e}$$

$$C_{rr} = C_{TT} \tag{38f}$$

B. Laminate Stiffness

Consider now a laminate composed of several lamina. For this model, a symmetric laminate with N laminae of alternating $\pm$ γ wrap angle is assumed. Gamma (γ) is shown in Figure 1 and is the angle between the fiber direction, L, and the global circumferential direction, θ. For the k^{th} lamina of the laminate, the constitutive relationship is given by:

$$\begin{Bmatrix} \sigma_{zz} \\ \sigma_{\theta\theta} \\ \sigma_{rr} \end{Bmatrix}_k = \begin{bmatrix} C_{zz} & C_{z\theta} & C_{zr} \\ C_{z\theta} & C_{\theta\theta} & C_{\theta r} \\ C_{zr} & C_{\theta r} & C_{rr} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{zz} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{rr} \end{Bmatrix}_k \tag{39}$$

$$k = 1,2,\ldots,N$$

where N is the total number of laminae.

Using the approach taken by Chamis [22], the reduced properties of the laminate may be found from:

$$\underline{\underline{C}}_L = \frac{1}{t_L}\sum_{i=1}^{N} t_i \, \underline{\underline{C}}_i \tag{40}$$

where $\underline{\underline{C}}_i$ is the 3x3 matrix in Eq. (39), t_L is the total laminate thickness, and t_i is the thickness of the $\underline{i^{th}}$ lamina. From Eqs. (38), it is noted that the elements of $\underline{\underline{C}}_i$ are even functions of γ. Equation (40) then reduces to:

$$\underline{\underline{C}}_L = \begin{bmatrix} C_{zz} & C_{z\theta} & C_{zr} \\ C_{z\theta} & C_{\theta\theta} & C_{\theta r} \\ C_{zr} & C_{\theta r} & C_{rr} \end{bmatrix} \tag{41}$$

C. Constituent Properties

The laminate stiffness has now been expressed in terms of the wrap angle and the lamina properties. The lamina properties in turn can be expressed in terms of the constituent properties and volume fractions of the composite.

A filamentary composite is made up of fibers embedded in a matrix. Both the fiber and the matrix are assumed herein to be isotropic. The longitudinal modules, E_L, and the major Poisson's ratio, ν_{LT}, can be expressed by a rule of mixtures formulation [23] as:

$$E_L = v_f E_f + v_m E_m \tag{42}$$

$$\nu_{LT} = v_f \ \nu_f + v_m \ \nu_m \tag{43}$$

where E_m is the modulus of the matrix, E_f is the modulus of the fiber, ν_f is the Poisson's ratio of the fiber, ν_m is the Poisson's ratio of the matrix, v_f is the fiber volume fraction, and v_m is the matrix volume fraction.

Expressions for ν_{TT} are G_{LT} are given by Whitney [24] and Foye [25] as, respectively:

$$\nu_{TT} = v_f \nu_f + v_m \nu_m \left(\frac{1 + \nu_m - \nu_{LT} \frac{E_m}{E_L}}{1 - \nu_m^2 + \nu_m \ \nu_{LT} \frac{E_m}{E_L}} \right) \tag{44}$$

$$G_{LT} = \frac{G_m}{2} \left(\frac{4 - \pi + \pi N}{4} + \frac{4N}{(4-\pi)N + \pi} \right) \tag{45}$$

where

$$N = \frac{G_f(\pi + 4v_f) + G_m(\pi - 4v_f)}{G_f(\pi - 4v_f) + G_m(\pi + 4v_f)} \tag{46}$$

where G_m is the matrix shear modulus and G_f is the fiber shear modulus.

The transverse modulus, E_T, is given by the lower bound expression [21] as:

$$E_T = \frac{1}{\frac{v_f}{E_f} + \frac{v_m}{E_m}} \tag{47}$$

From a consideration of the symmetry of the compliance matrix, the minor Poisson's ratio, ν_{TL}, is expressed as:

$$\nu_{TL} = \frac{E_T}{E_L} \nu_{LT} \tag{48}$$

IV. TIME-DEPENDENT MATERIAL PROPERTIES

In this development, it is assumed that only the matrix exhibits time-dependent properties, these being designated relaxation modulus, E_m, and Poisson's ratio, ν_m. These two quantities may be determined from experimental tests of the matrix material in three different ways [26], [27].

The relaxation modulus may be measured directly in a relaxation test and a method such as least squares used to fit the data to a Prony series of the form:

$$E_m(t) = \sum_{j=1}^{N} B_j \exp(-\lambda_j t) + E_{m_0} \tag{49}$$

where N is the total number of exponential terms included with the λ_j being assigned. The B_j and E_{m_0} are constants determined from a curve fit of the data.

A second method to determine the relaxation modulus is to first obtain, from a creep test, the creep compliance, $J_m(t)$, of the form:

$$J_m(t) = G + Ht + \sum_{i=1}^{N} F_i \exp(-R_i t) \tag{50}$$

where the R_i are assigned and with the F_i, G_i and H being constants determined by a curve fit to the data. An expression relating the Laplace transforms of the creep compliance and relaxation modulus [27] is:

$$\bar{E}_m(s) = \frac{1}{s^2 \bar{J}_m(s)} \tag{51}$$

where the bar over a variable is used to denote the Laplace transform of that function and s in the Laplace parameter. Inversion of Eq. (51) yields an expression for the relaxation modulus of the form given by Eq. (49). A thorough development of this second method is given by Hackett and Dozier in Reference [26].

A third method would yield the relaxation modulus from the complex relaxation modulus. Details of this method are given by Findly in Reference [27].

The time-dependent Poisson's ration is obtained from the well-known expression for the bulk modulus:

$$K = \frac{E_m(t)}{3[1-2\ \nu_m(t)]} \tag{52}$$

Rearranging Eq. (52) gives:

$$\nu_m(t) = \frac{1}{2} - \frac{E_m(t)}{6K} \tag{53}$$

where the bulk modulus is time-independent.

V. VISCOELASTIC SOLUTION

A. Elastic-Viscoelastic Correspondence Principle

The elastic-viscoelastic correspondence principle, hereafter referred to as the correspondence principle, is a well-known method for the solution of viscoelastic problems [26], [29] and will be employed here. Basically, the correspondence principle allows one to convert an elastic solution for a given geometry and nonmoving boundary conditions to a quasi-static "associated elastic" solution in the Laplace domain. This requires that the corresponding substitutions noted in Table 1 be made in the elastic solution.

TABLE 1. Elastic-Viscoelastic Correspondence Principle Parameters

Elastic	Substitution
$\sigma_{ij}(t)$	$\bar{\sigma}_{ij}(s)$
$\varepsilon_{ij}(t)$	$\bar{\varepsilon}_{ij}(s)$
$E_m(t)$	$s\bar{E}_m(s)$
$\nu_m(t)$	$s\,\bar{\nu}_m(s)$
$T_i(t)$	$\bar{T}_i(s)$

All elements of Table 1 have been defined earlier except for $T_i(t)$ which is the time-varying surface traction. This associated elastic solution is then inverted back into the physical domain to yield the time-dependent solution.

If the associated elastic solution has a closed form expression, it may be possible to use an exact inversion technique, e.g., the method of partial fractions, to obtain the time-dependent solution. However, many times the solution may be known only for discrete values of the Laplace parameter. If this is the case, then some numerical Laplace inversion technique may be used. Even in cases where a closed form solution is possible, the complexity of the solution may dictate the use of a numerical inversion procedure. Due to the complexity of this problem, the latter approach is taken.

B. Numerical Laplace Transform version

There are several numerical inversion techniques which may be used [30]. The one used here is called the method of collocation [31] and its application has been demonstrated by numerous researchers [28], [32], [33].

Schapery [31] presented a general expression for a stress or displacement response, $\phi(t)$, as:

$$\phi(t) = \phi' + \phi''t + \sum_{v=1}^{N} \phi_v \exp(-t/\alpha_v) \tag{54}$$

where ϕ', ϕ'', and ϕ_v are constants to be determined, α_v are prescribed positive constants, and N is the number of exponential terms included. As will be discussed later, the simulation model is constructed by decomposing the pressure load into the sum of several loads as shown in Figure 2. Superposition of the several respective responses is then used to determine the total response. From Figure 2, it is noted that only two types of loadings are considered: a linearly varying load and a step load.

For the step load shown in Figure 3a, it has been shown [31] that the circumferential strain, $\varepsilon_{\theta\theta}$, can be approximated by:

$$\varepsilon'_{\theta\theta}(t) = A + \sum_{v=1}^{N} h_v \exp(-t/\alpha_v) \tag{55}$$

where the constant A is given by:

$$A = \varepsilon'_{\theta\theta}(t_0) - \sum_{v=1}^{N} h_v \tag{56}$$

with $\varepsilon_{\theta\theta}(t_0)$ being the elastic response at time t_0 and with the prime denoting a step load.

Substituting Eq. (56) into Eq. (55), taking the Laplace transform and rearranging gives:

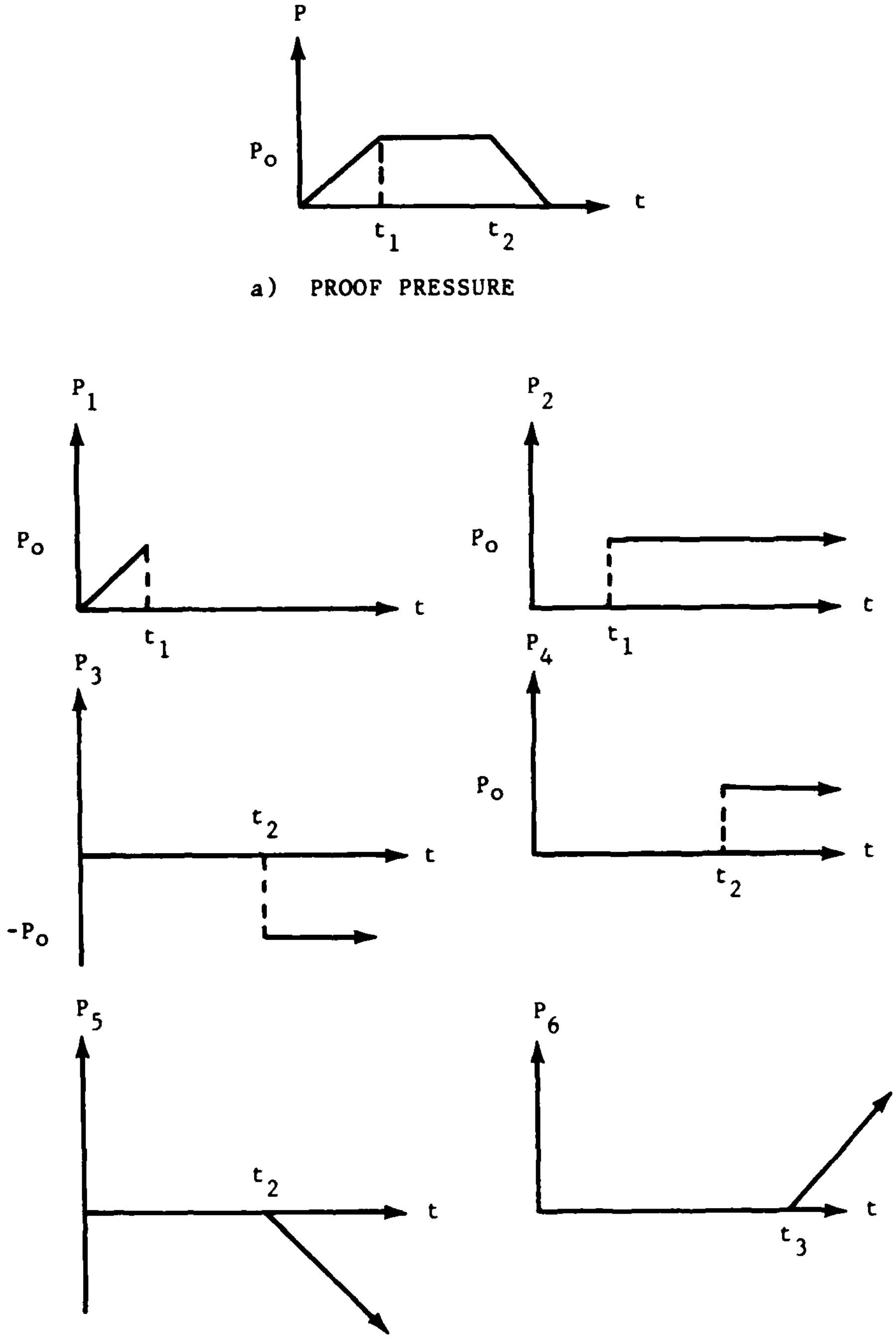
$$\sum_{v=1}^{N} \frac{h_v}{S + \frac{1}{\alpha_v}} = s\,\varepsilon'_{\theta\theta}(t_0) - \bar{\varepsilon}'_{\theta\theta}(s) \tag{57}$$

where $\varepsilon'_{\theta\theta}(s)$ is the associated elastic solution for the circumferential strain found by using the correspondence principle as discussed earlier and s is the Laplace parameter.

For this model, a six-term exponential function is used to define the circumferential strain as a function of time, i.e., N = 6. Therefore, six discrete values of the Laplace parameter, s, are needed to solve for the six unknown constants, h_v. The choice of the discrete Laplace parameters is a matter of judgment and experience. A reasonable expression for α_v and s seems to be:

$$\alpha_v = \exp(7-2v) \tag{58a}$$

$$s = \frac{1}{\alpha_v} \; ; \; v = 1,2,\ldots,6 \tag{58b}$$



a) PROOF PRESSURE

b) PRESSURE DECOMPOSITION

Figure 2. Internal pressure loading.

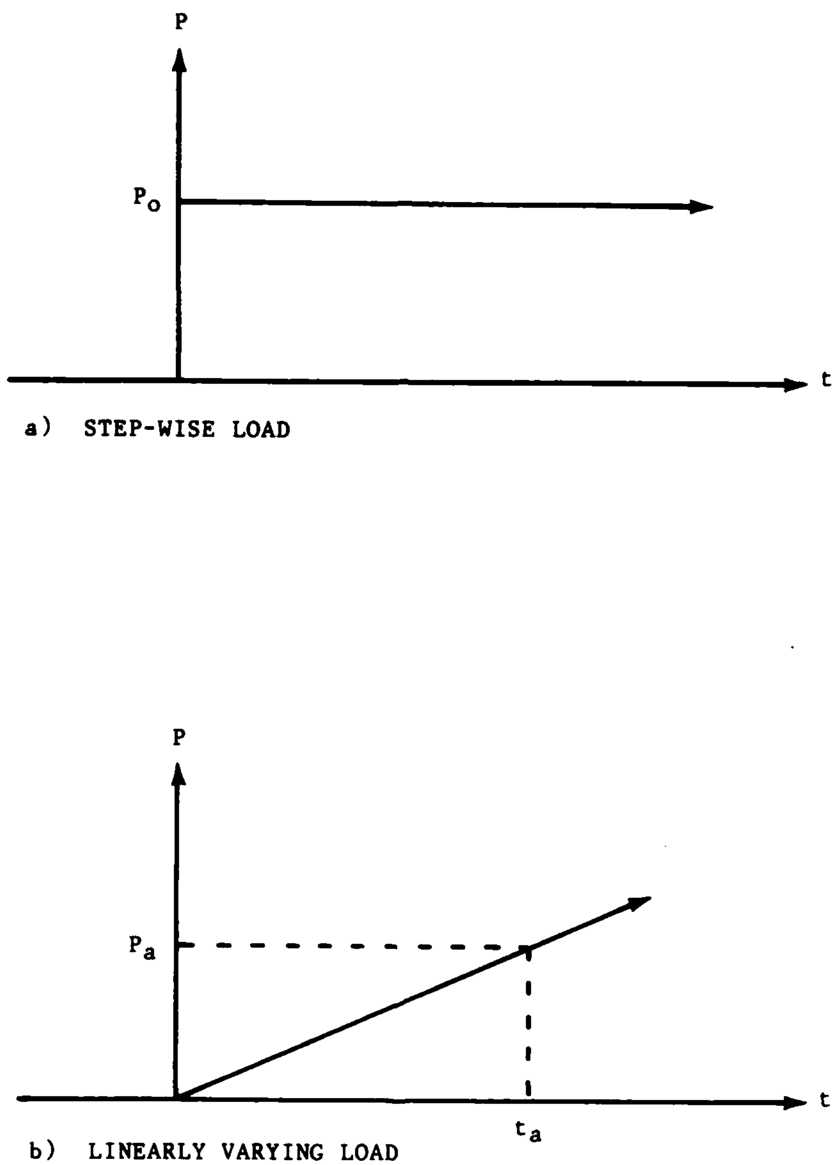


Figure 3. Load types.

Utilizing these values, Eq. (57) may be solved for the h_ν's. The constant A is then calculated using Eq. (56). The time-dependent circumferential strain may then be found from Eq. (55). The results will be the time-dependent circumferential strain due to a stepwise internal pressure load.

The second type of load to be considered is one that varies linearly with time. It is assumed that the circumferential strain may be expressed by:

$$\epsilon''_{\theta\theta}(t) = C + Dt + \sum_{v=1}^{N} g_v \exp(-t/\alpha_v) \tag{59}$$

where the double prime denotes a response to a linearly varying load with C, D and g_v being constants to be determined.

The initial conditions for the linearly varying load of Figure 3b are:

$$\epsilon''_{\theta\theta}(t) = 0 \qquad \text{at } t = 0 \tag{60a}$$

$$\frac{\partial \epsilon''_{\theta\theta}(t)}{\partial t} = \frac{\epsilon''_{\theta\theta}(t_a)}{t_a} \qquad \text{at } t = 0 \tag{60b}$$

where $\epsilon''_{\theta\theta}(t_a)$ is the elastic response to a pressure, P_a, corresponding to some arbitrary time, t_a, as shown in Figure 3b. The time derivative of Eq. (59) is:

$$\frac{\partial \epsilon''_{\theta\theta}(t)}{\partial t} = D - \sum_{v-1}^{N} \frac{g_v}{\alpha_v} \exp(-t/\alpha_v) \tag{61}$$

Substituting Eqs. (60) into Eq. (59) and Eq. (61) and rearranging gives:

$$C = -\sum_{v=1}^{N} g_v \tag{62a}$$

$$D = \frac{\epsilon''_{\theta\theta}(t_a)}{t_a} + \sum_{v=1}^{N} \frac{g_v}{\alpha_v} \tag{62b}$$

Substituting Eqs. (62) into Eq. (59) yields:

$$\ddot{\varepsilon}_{\theta\theta}(t) = -\sum_{v=1}^{N} g_v + \frac{\ddot{\varepsilon}_{\theta\theta}(t_a)}{t_a} t + \sum_{v=1}^{N} \frac{g_v}{\alpha_v} t$$

$$+ \sum_{v=1}^{N} g_v \exp(-t/\alpha_v) \tag{63}$$

Taking the Laplace transform of Eq. (63) gives:

$$\ddot{\varepsilon}_{\theta\theta}(s) = -\frac{1}{s} \sum_{v=1}^{N} g_v + \frac{\ddot{\varepsilon}_{\theta\theta}(t_a)}{s^2 t_a} + \frac{1}{s^2} \sum_{v=1}^{N} \frac{g_v}{\alpha_v}$$

$$+ \sum_{v=1}^{N} g_v \frac{1}{s + \frac{1}{\alpha_v}} \tag{64}$$

where $\ddot{\varepsilon}_{\theta\theta}(s)$, the Laplace transform of $\ddot{\varepsilon}_{\theta\theta}(t)$, is the associated elastic solution determined by using the correspondence principle as discussed earlier. A six-term exponential function is used to define the circumferential strain, i.e., N = 6. Thus, six arbitrary values of the Laplace parameters, s, are needed to solve for the six unknowns, g_v. This choice is, as before, given by Eqs. (58). Substituting Eq. (58b) into Eq. (64) and rearranging gives:

$$\sum_{v=1}^{N} g_v \left(\frac{\alpha_v \alpha_w}{\alpha_v + \alpha_w} + \frac{\alpha_w^2}{\alpha_v} - \alpha_w \right) = \varepsilon_{\theta\theta}(s) \Bigg|_{s = \frac{1}{w}} - \frac{\ddot{\varepsilon}_{\theta\theta}(t_a)}{t_a} \alpha_w^2 \tag{65}$$

from which the g_v's may be found. The constants, C and D, may then be obtained using Eqs. (62). Substitution of C and D into Eq. (59) gives the time-dependent circumferential strain resulting from the linearly varying loads.

VI. DAMAGE IN A VISCOELASTIC MATERIAL

A viscoelastic material may initially contain minute voids or other defects. As the material is loaded and creep occurs, the voids will grow in size. New voids will form at the defects and grow also. At some time, these voids will coalesce into microcracks and continue to grow, again as a result of the creep of the material. This condition progresses until critical cracks are formed and failure occurs. Other forms of damage may also be aggravated by the creep. Kachanov [11] introduced a parameter termed the "continuity" to account for the damage in an elastic material. Borrowing this idea, a damage function, ω, is defined. Assuming the damage to be isotropic, the damage function becomes a scalar. The damage, ω, can be thought of as an index of the damage that exists in the materials. A damage value of zero denotes no damage. The value 1 is arbitrarily assigned to the damage at failure. The deformation of the material is related to the creep that occurs. It is well known that the strain is a function of the deformation. Therefore, it is assumed that the damage may be expressed as a scalar function of the strain. Gerstle [29], using the maximum strain failure theory, has demonstrated good correlation between the circumferential strain and failure of biaxially-loaded quasi-isotropic cylinders. The failure strain, K_θ, being given as, for linearly elastic fibers:

$$K_\theta \approx \frac{3F_f}{4E_f} \qquad (66)$$

where F_f is the fiber fracture strength. In view of this correlation, it is assumed, as a first approximation, that the damage is a function of the circumferential strain only. Many choices exist for this relationship. Curve #1 in Figure 4 represents some general relationship which includes a "healing" or decrease of damage with a decrease in circumferential strain. A first approximation for the circumferential strain-damage relationship is denoted by Curve #2 in Figure 4, where the damage increases monotonically with circumferential strain. This results in the damage remaining constant for a decrease in circumferential strain as shown by the horizontal portion of Curve #2 in Figure 4. The corresponding analytical expression for the linearly varying portion of Curve #2 is:

$$\omega = \frac{\varepsilon_{\theta\theta}(t)}{\varepsilon_f} \qquad (67)$$

where ε_f is the circumferential failure strain and is set equal to K_θ given by Eq. (68). Failure of the cylinder will then occur when the following expression is satisfied:

$$\varepsilon_{\theta\theta}(t) \geq \varepsilon_f \qquad (68)$$

It can be enlightening to consider the response of a simple viscoelastic material. A schematic response diagram for a simple spring-dashpot model is shown in Figure 5. The load shown in Figure 6 is applied to this simple model. A sketch of the resultant response is shown in Figure 7. The straight lines with slopes k_1 and $k_1 + k_2$ are the respective elastic response curves for stiffnesses k_1 and $k_1 + k_2$ where k_1 and k_2 are the respective spring

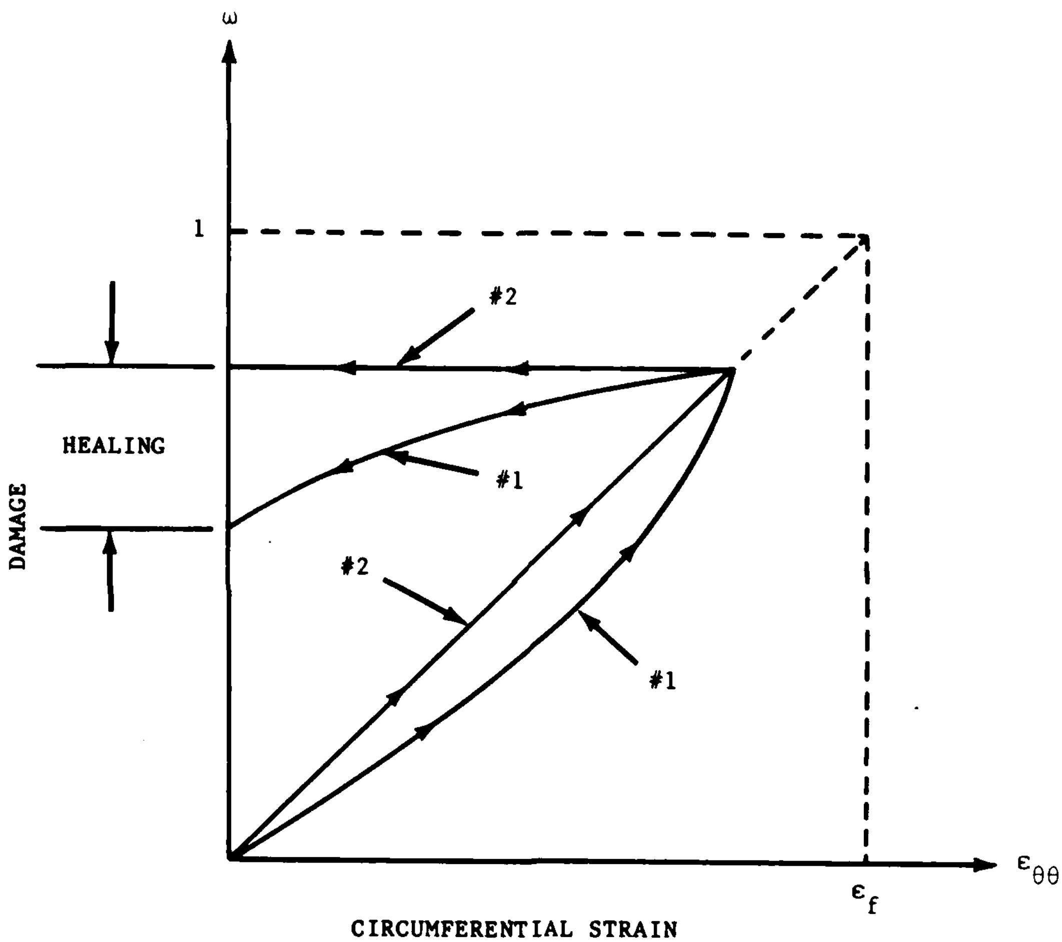


Figure 4. Damage versus circumferential strain.

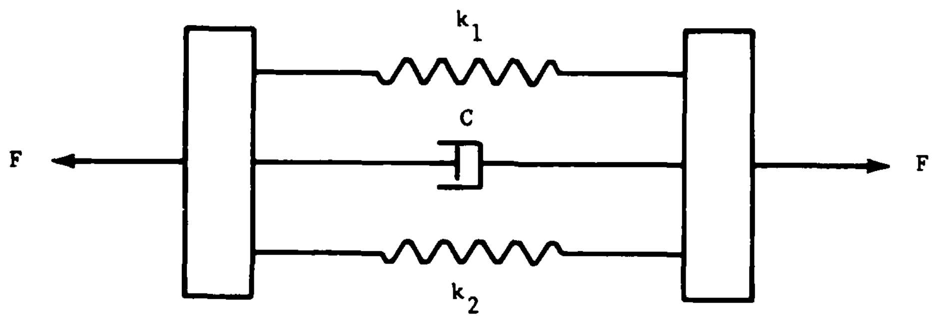


Figure 5. Simple viscoelastic model.

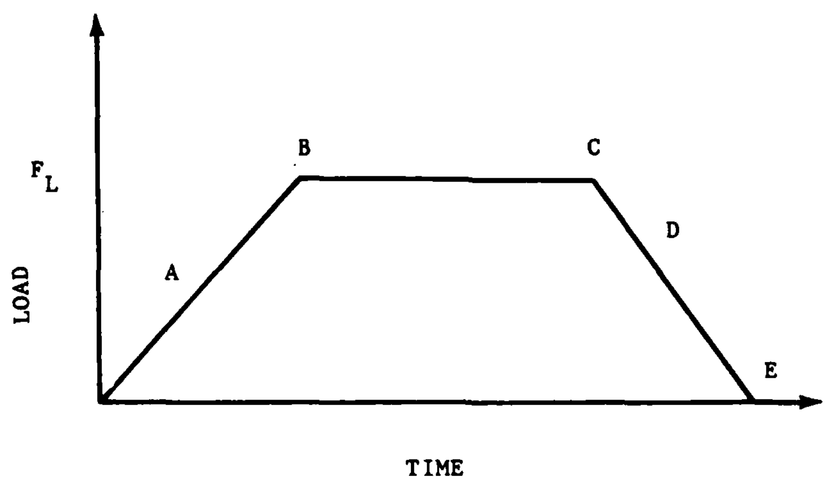


Figure 6. Loading for the simple viscoelatic model.

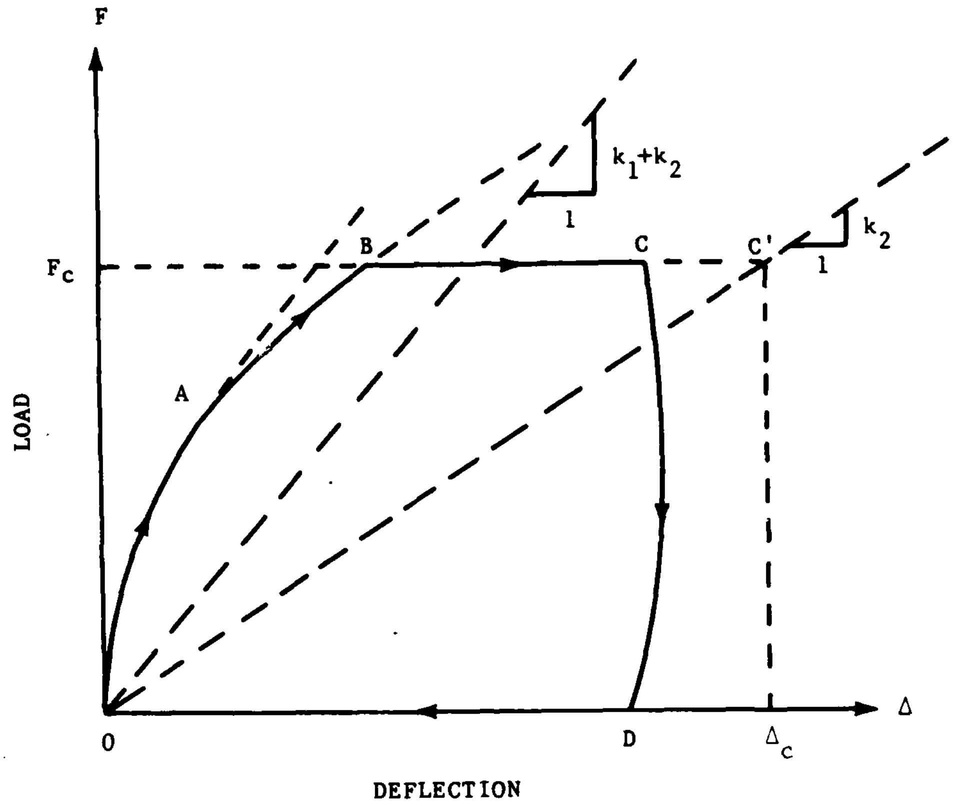


Figure 7. Response curve for the simple viscoelastic model with damage.

constants for springs 1 and 2. As the load increases to point A, the response is defined by segment OA in Figure 7. At point A, damage occurs. This damage is simulated by cutting one of the springs, say k_1. In order for a force balance to be maintained, the force in the dashpot must increase. This results in an increase in the displacement rate which is indicated by the change in slope of the response curve at point A. The load continues to increase until point B is reached. The corresponding response is given by segment AB in Figure 7. At point B, the load is held constant. If held constant long enough, the displacement will increase to point C' which is an equilibrium point and lies on the elastic response curve, k_2. However, at point C the load is decreased and the deformation follows some curve from point C to point D. At point D, the load is zero, but the displacement is not fully recovered. The recovery continues along segment DO with decreasing speed until the deformation reaches zero. After a sufficiently long time, the only evidence that remains of the damage is the reduced stiffness. The response of this or any other model to future loading, e.g., in-service conditions, must be based upon this deduced stiffness. The point to be emphasized here is that damage is modeled as a reduction in stiffness.

Several experimental studies have produced data which relate the stiffness of a composite system to an increase in internal damage [8], [9], [35], [36], [37]. Highsmith and Reifsnider [9] modeled the overall response of a laminate containing damage by reducing the transverse modulus of the laminae. O'Brien [18] used the change in transverse modulus as a predictor of buckling in a damaged composite. This basic idea of reducing the transverse modulus of the laminate as a function of damage is employed here.

Several different forms for the transverse modulus-damage relationship are possible. Highsmith and Reifsnider [9] published data which indicates large stiffness changes early in the life of a composite material undergoing fatigue loading. The pertinent data is reproduced here in Figure 8. As a first approximation, it is assumed that a transverse modulus-damage relationship is similar to the stiffness/number-of-cycles relationship shown in Figure 8. A simple model of the relationship between transverse modulus and damage is shown in Figure 9. The corresponding analytical expression is:

$$E_T = E_{T_I} (1 - \omega)^2 \qquad \text{for } 0 \leq \omega \leq 1 \tag{69}$$

where E_{T_I} is the initial or undamaged transverse modulus. Substituting Eq. (67) into Eq. (69) yields:

$$E_T = E_{T_I} \left(1 - \frac{\varepsilon_{\theta\theta}(t)}{\varepsilon_f}\right)^2 \tag{70}$$

where the transverse modulus now becomes a function of time as a consequence of the circumferential strain being time-dependent. In addition, the formulated problem has become nonlinear because of the interdependence of $\varepsilon_{\theta\theta}$ and E_T. In the simulation model, this nonlinearity is accounted for by an iterative process whereby, the first transverse modulus is determined from Eq. (70), using the circumferential strain from the previous iteration. Convergence is based upon the value of circumferential strain calculated at a given time for the initial linearly varying load and on time to failure for the remaining portion of the simulation.

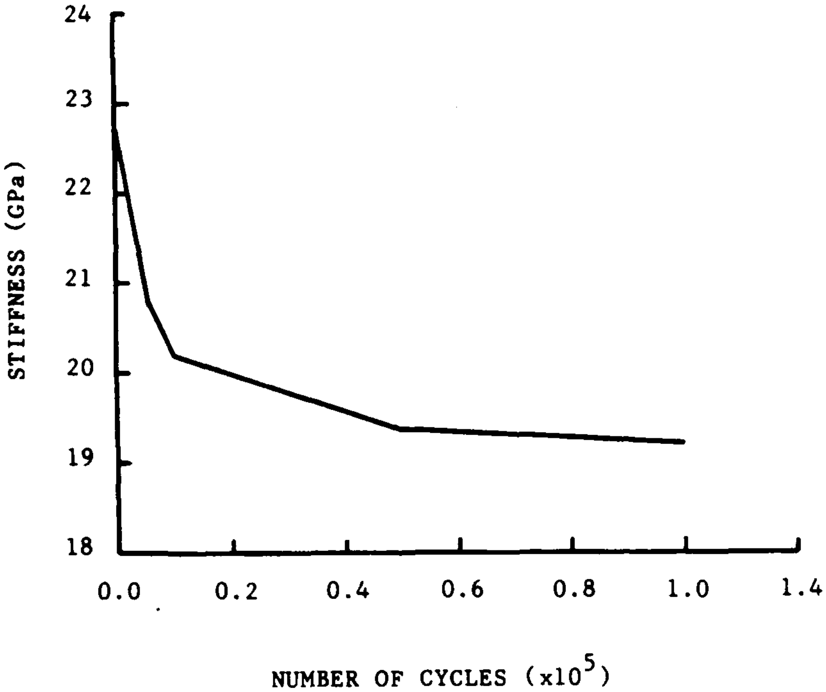


Figure 8. Stiffness versus cycles of loading for a [0,90]s laminate.[1]

[1]Reproduced from data given in Reference [9].

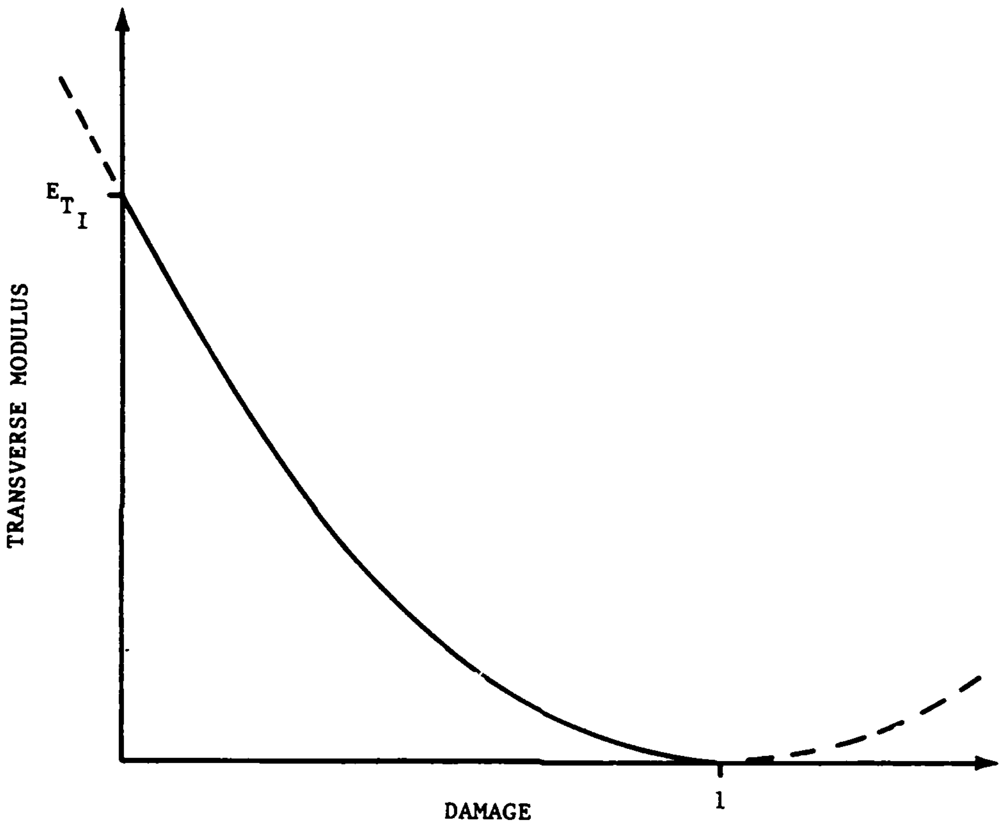


Figure) Transverse modulus versus damage.

VII. SIMULATION MODEL

In this section the strategy used to develop the simulation model is discussed. The simulation model developed predicts the circumferential strain of a filament-wound cylinder having accumulated internal damage. The procedure followed is to first decompose the load into several components as shown in Figure 2. The respective solutions are calculated and superposition used to construct the total solution. For load P_1, $0 \leq t \leq _1$, an elastic solution, which includes the damage effect given by Eq. (69), is computed. This approach is taken for this phase of the loading since there is very little difference between the elastic and viscoelastic response for small time. A comparison of an undamaged elastic and a viscoelastic response is shown in Figure 10 and demonstrates this point. An iterative solution is required as discussed earlier since damage is occurring. All other responses are simulated for a viscoelastic material with accumulated damage. The damage is allowed to accumulate as long as the circumferential strain is increasing. When the circumferential strain decreases, the damage, and hence E_T, is held constant.

In order to implement this constant damage, two loads, P_3 and P_4, are added to load P_2. The response to P_2 is viscoelastic with damage. The initial conditions are found from the state that exists at t_1 due to P_1. At time t_2, the negative of P_2, i.e., P_3, is applied. Assuming a non-aging material, the Boltzman superposition principle gives the response to P_3 as the negative of the response to P_2 delayed by $\tau - \tau_2 - \tau_1$. This in effect "subtracts" out the damage for time greater than t_2. If a step decrease in load to zero at time t_2 is desired, no further loads need be applied. However, the desired loading is a linearly decreasing pressure from time t_2 to t_3. To accomplish this, the load P_4, which is equal to P_2, is applied at time t_2. The response to P_4 is viscoelastic with constant damage, i.e., constant E_T. This is done so that we may "ramp down" the load.

The linearly decreasing phase of the loading, i.e., t_2 to t_3, is accomplished by applying load P_5. The response is viscoelastic with constant damage. Load P_6 is applied at time t_3 to maintain zero load for time greater than t_3. The response to P_6 is the negative of the response to P_5 delayed to $\tau = t_3 - t_2$.

The total solution is constructed from the superposition of the responses discussed above as indicated below:

1. For $0 < t < t_1$

   An elastic solution with damage. An iterative procedure is required.

2. For $t_1 \leq t \leq t_2$

   Viscoelastic analysis with damage for the step load P_2. Initial conditions are determined from the state at $t = t_1$ from 1. Solution form is given by Eq. (55). An iterative procedure is required.

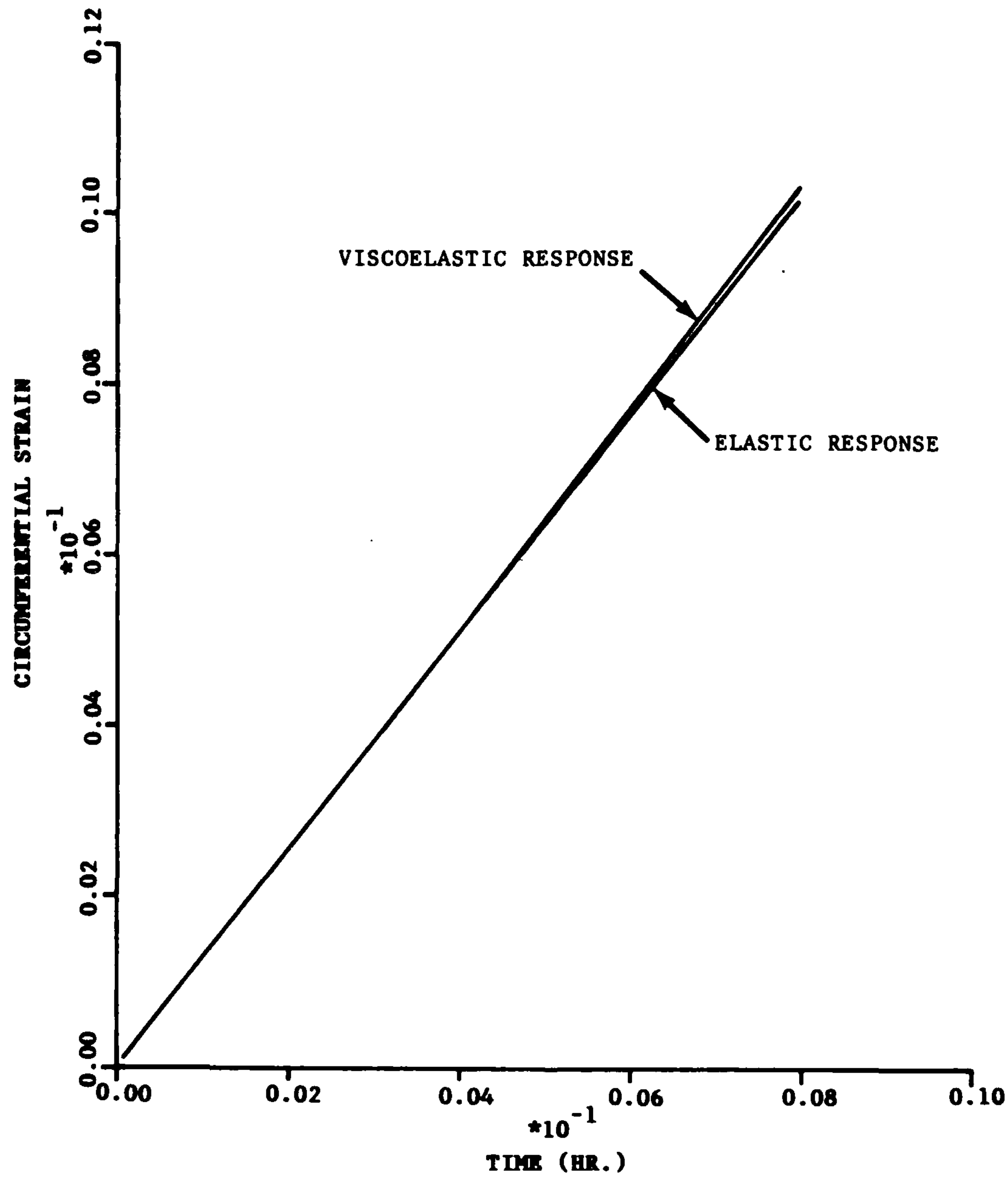


Figure 10. Comparison of elastic and viscoelastic response to a linearly varying load for $o < t < t_1$.

3. For $t_2 \leq t \leq t_3$

   a. The solution from 2., plus

   b. The negative of the solution from 2 shifted by $\tau = t_2 - t_1$ hours, plus

   c. A viscoelastic solution with constant damage for the step load P_4. Initial conditions are determined from the state at $t = t_2$ from 2. Solution form is given by Eq. (55), plus

   d. A viscoelastic solution with constant damage for the linearly varying P_5. Initial conditions are determined from the state at $t = t_2$. Solution form is given by Eq. (59).

4. For $t_3 \leq t$

   a. Solution from 3., plus

   b. The negative of the solution from 3.c. shifted by $\tau = t_3 - t_2$ hours.

VIII. EXAMPLE APPLICATION

A numerical experiment was conducted to demonstrate the model developed. The solution is for the time-dependent circumferential strain of a filament-wound composite cylinder. The internal pressure loading is given in Figure 2 with P_o = 5.516 MP_o, t_1 = 0.00833 hr, t_2 = 0.025 hr, and t_3 = 0.03333 hr.

The material system is a Kevlar 49/HBRF-55A filamentary composite. The Kevlar 49 fibers are considered to be isotropic and linearly elastic. Fiber properties were obtained from the literature [28]. The reported Young's modulus and Poisson's ratio are, respectively, 131 GP_a and 0.20. The matrix material, HBRF-55A, is considered to be isotropic and linearly viscoelastic. The time-dependent matrix properties used here were determined by Hackett and Dozier [28] from creep test data. The expression for the relaxation modulus and time-dependent Poisson's ratio are given as, respectively:

$$E_m(t) = 152.5 \exp(-106.5t) + 40.23 \exp(-10.18t) + 154.6 \exp(-1.070t) - 211.3 \exp(-0.092t) + 2346 \exp(-0.0075t) \quad (71)$$

and

$$\nu_m(t) = 0.5 - E_m(t)/16548 \quad (72)$$

where the units of $E_m(t)$ are MP_a.

The geometry for this example is that of a cylindrical shell having an inner radius of 38.10 mm and an outer radius of 39.62 mm. A fiber volume fraction of 0.65 and a composite failure strain of 0.0192 are used. The fiber wrap angle (γ) is $\pm$ 55°. Positive wrap angle is shown in Figure 1.

A plot of the simulated response with and without damage for time less than t_1 is shown in Figure 11. It is seen that the inclusion of damage induced an increase of approximately 24% over the undamaged response. This is indicative of the reduction in transverse modulus caused by the damage. The strain reaches a given level in smaller time for the damaged composite, implying an earlier failure than without damage.

Figure 12 is a comparison of the modeled response with and without damage. As discussed in Chapter VI, the strain should be totally recovered since only elastic elements in the model are damaged and no plastic behavior is included. Close examination of Figure 12 shows that the response with damage indicates a very small negative residual strain. This is believed to be the result of round-off error and of a strong sensitivity to initial conditions that was observed during model development. Very little viscoelastic-induced strain occurs prior to time t_1. The damage is mainly due to the elastic increase in strain. For time t_1 to t_2, a creep under the constant load is seen. The creep predicted for the damaged case is 2.54 times that predicted for the undamaged case. The damage at times t_1 and t_2 are, respectively, ω_{t_1} = 0.7021 and ω_{t_2} = 0.7386. Thus, the creep has induced an additional increase in the level of damage of approximately 3.65%.

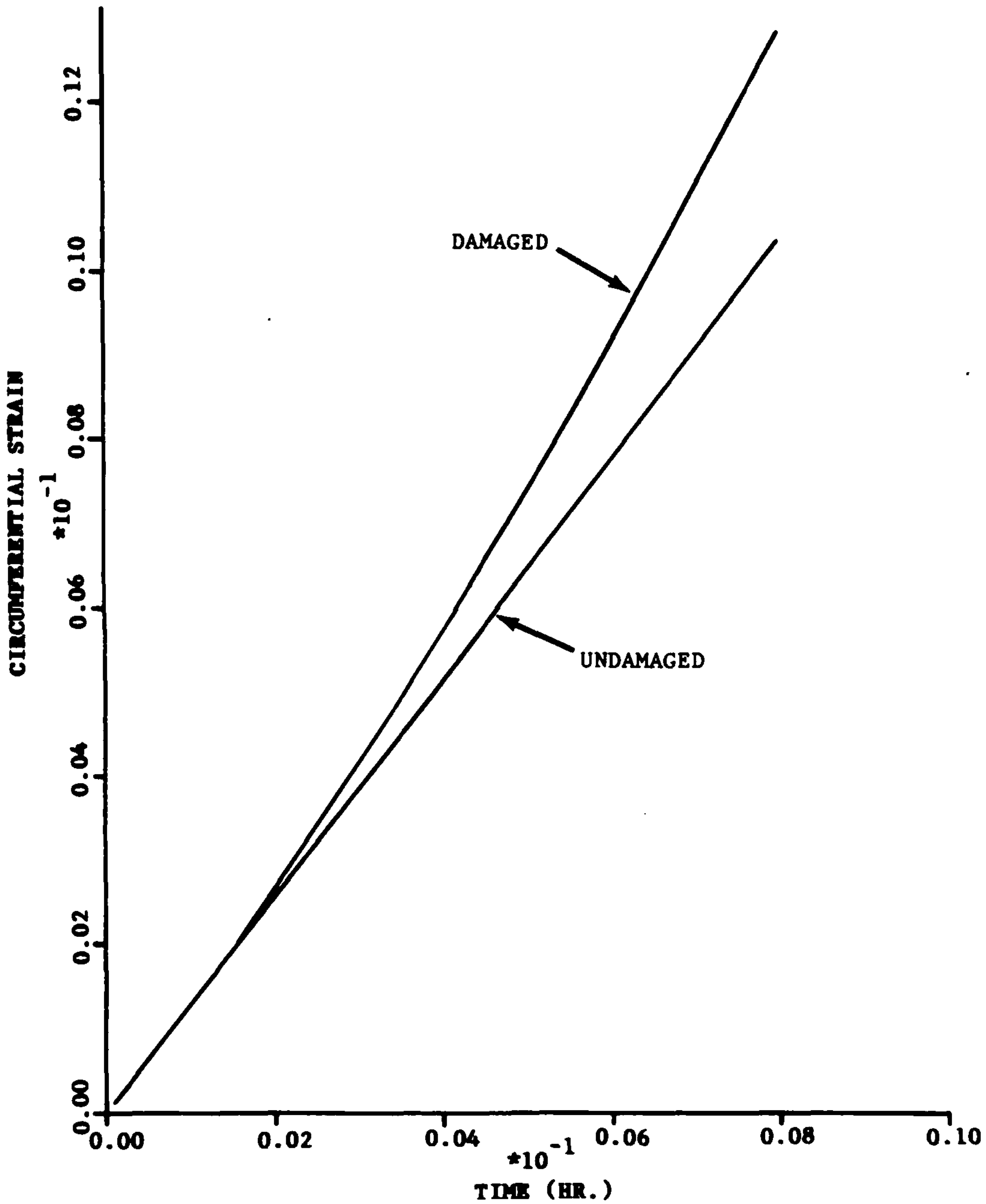


Figure 11. Comparison of damaged and undamaged response for $t < t_1$.

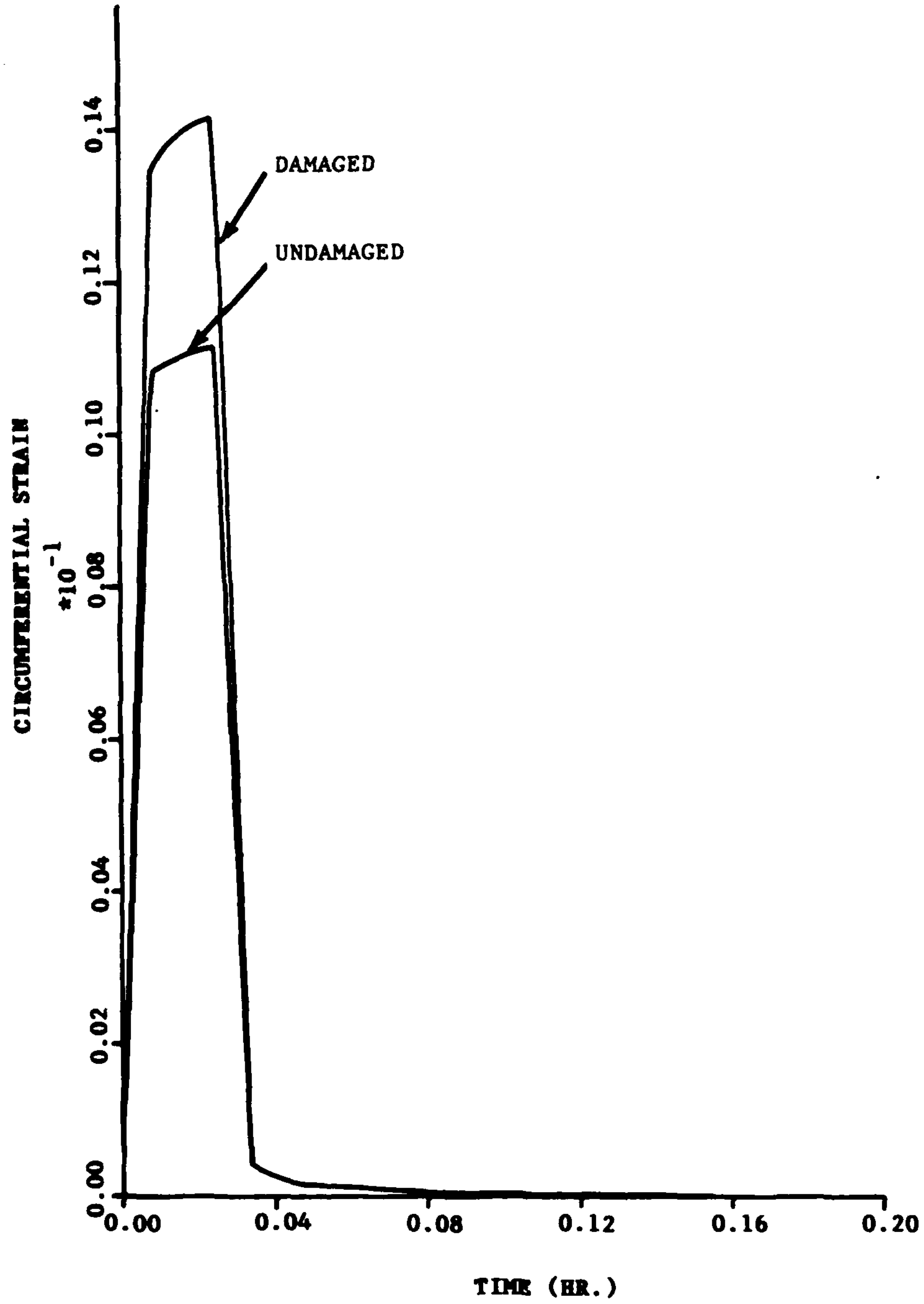


Figure 12. Comparison of damaged and undamaged response through recovery.

After the load has been removed, the strain is seen to recover completely by approximately 0.20 hours. However, the damage is not recovered and the damage present at time t_2 remains. Using Eq. (69), it is calculated that the transverse modulus at time t_2 retains only 6.83% of its initial value. The majority of the damage is seen to occur prior to time t_1, i.e., during the linearly increasing load, with a smaller portion of the total damage occurring during the constant load phase. This is consistent with data published by Hahn and Kim [38] on their study of proof testing of composite materials.

The elastic failure load (based on instantaneous values of the moduli) are calculated at initial time and for maximum damage. Their respective values are: $F_{t=0}$ = 9.977 MP_a and $F_{max\ (\omega)}$ = 7.771 MP_a. This gives a 22.1% decrease in failure load as a result of the accumulated damage. The calculated response of this or any other damaged structure to future loading must be based on the effects of the proof-test-induced damage. This points out the importance of considering the proof-test-induced damage in determining in-service load limits.

Figure 13 contains a plot of the circumferential strain versus pressure. The pressure is plotted as the vertical so that a qualitative comparison between Figure 13 and Figure 7 can be made. Comparison indicates good agreement and gives confidence in the model. The creep at constant load and recovery are noted by the change in strain for constant pressure. The change in stiffness that occurs as a result of the accumulated damage is indicated by the change in slope of the two lines drawn tangent to the response curve. The upper line corresponds to the initial or undamaged stiffness with the lower line corresponding to stiffness after damage has accumulated.

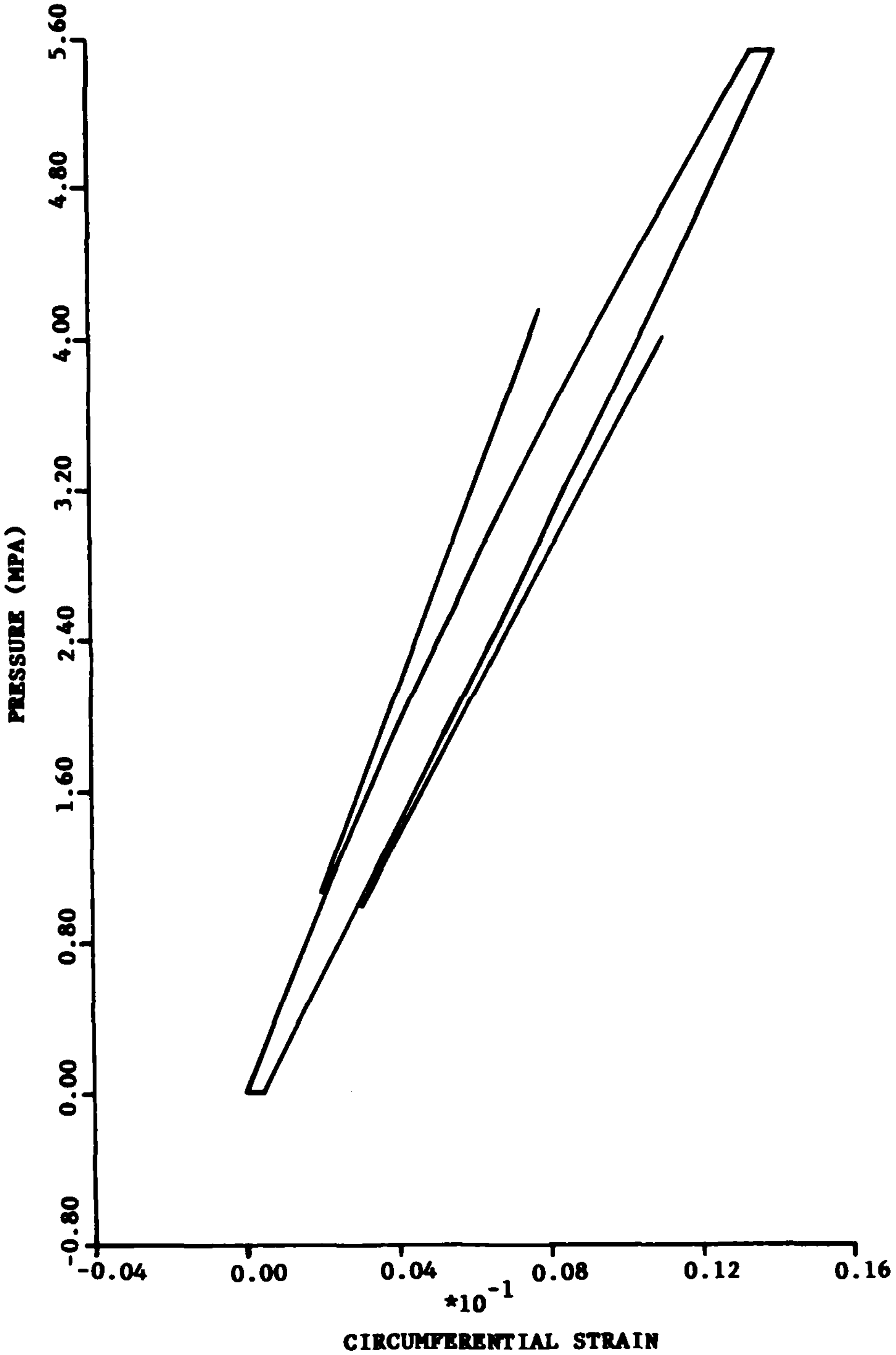


Figure 13. Load-response curve.

IX. CONCLUSION

A model to predict the viscoelastic/damage response of a filament-wound composite cylindrical pressure vessel to a proof-test loading was developed. The matrix material of the composite system was assumed to be isotropic and linearly viscoelastic. The Kevlar 49 fiber was assumed to be isotropic and linearly elastic. A damage model which relates the circumferential strain to a damage index was developed. This damage index was in turn related to the transverse modulus as a first approximation. A quadratic relationship between the transverse modulus and the circumferential strain resulted. An iterative solution based on the damaged elastic response was employed to predict the time-dependent circumferential strain during the linearly increasing phase of the loading. Thereafter, an iterative solution based on the elastic-visco-elastic correspondence principle was employed to predict the time-dependent circumferential strain which is then used to determine the transverse modulus. A new expression for the time-dependent circumferential strain is then found using the previously calculated transverse modulus. This iterative procedure is continued until convergence is achieved. The associated elastic solution resulting from use of the elastic-viscoelastic correspondence principle is inverted by using the method of collocation for each iteration.

The simulation model developed was demonstrated by a numerical example. There is good qualitative agreement between the predicted and anticipated behavior. After unloading, the strain was totally recovered. However, the transverse modulus has retained only 6.83% of its initial value. The response of the cylinder to future loadings, i.e., in-service loading, must include the effects of the reduced transverse modulus.

The simulation is well founded in classical viscoelastic procedures and the damage model produces results which are consistent with reported results and should prove useful in future work. Future efforts should include experiments to verify the model developed, as well as efforts to investigate the effect on the response of different strain-damage-transverse modulus relationships.

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APPENDIX

COMPUTER LISTING OF SOLUTION MODEL

APPENDIX. COMPUTER LISTING OF SOLUTION MODEL

```
C*********************************************************************
C
C           PROGRAM TO CALCULATE THE VISCOELASTIC HOOP STRAIN
C           OF A  FILAMENT WOUND CYLINDRICAL PRESSURE VESSEL
C           INCLUDING A "DAMAGE" EFFECT.
C
C
C*********************************************************************
C
C BY JOHNNY L. PRATER UNDER DIRECTION OF DR. ROBERT M. HACKETT FOR
C PARTIAL FULFILLMENT OF REQUIRMENTS FOR PH.D. DEGREE IN SOLID
C MECHANICS AT THE UNIVERSIT OF ALABAMA IN HUNTSVILLE, 1984-1985.
C
C     LOAD= M*T
C
C PARAMETERS
C       EF = YOUNG'S MODULUS OF THE FIBER (PSI)
C       PF = POISSON'S RATIO OF THE FIBER
C       VF = FIBER VOLUME FRACTION
C       VM = MATRIX VOLUME FRACTION
C       EMI = INITIAL RELAXATION MODULUS OF MATRIX (PSI)
C       PMI = INITIAL POISSON'S RATIO OF MATRIX
C       GAMMA = ANGLE BETWEEN THE FIBER LONGITUDINAL DIRECTION AND THE
C               CYLINDER THETA( HOOP ) DIRECTION (DEG).
C       RIN    = INSIDE RADIUS OF CYLINDER  (INCH)
C       ROUT   = OUTER RADIUS OF CYLINDER  (INCH)
C       XLOAD = INTERNAL PRESSURE (PSI) OF THE CYLINDER
C       THK    = COMPOSITE FAILURE STRAIN
C       FLI    = ELASTIC FAILURE LOAD (PSI)
C       EPTHTH = HOOP STRAIN
C       EPMM   = MERIDIONAL STRAIN
C       EPRR   = RADIAL STRAIN
C       PSTRN  = HOOP STRAIN IN LAPLACE DOMAIN
C       PLOAD  = LAPLACE TRANSFORM OF THE LOAD
C       T      = TIME (HOURS)
C       NLOOP1 = NUMBER OF LOOPS USED FOR CONVERGENCE OF ELASTIC
C                PHASE ( O ( T ( T1 ).
C       NLOOP2 = NUMBER OF LOOPS USED FOR CONVERGENCE OF VISCOELASTIC
C                PRASE ( T1 ( T ).
C

      IMPLICIT REAL*8 (A-H,P-Z)
      REAL*8 LOAD, LAMBDAI, LAMBDA
      DIMENSION STRN(200),PSTRN(6),GV(6),STN(6),S(6),V(6),A(6,6)
      DIMENSION GV1(6),GV2(6),AA(6,6),STN1(6),STN2(6)
      DIMENSION EMP(6),PMP(6),GVCD(6),STNCD(6)
      OPEN(UNIT=7,FILE='VISD.OUT',STATUS='NEW')
      OPEN(UNIT=14,FILE='VISD.PLT',STATUS='NEW')
      OPEN(UNIT=15,FILE='VISD.PRS',STATUS='NEW')
C*********************************************************************
C
      WRITE(6,*)
      WRITE(6,*)' THE SOLUTION WILL BE FOR AN ELASTIC COMPOSITE'
      WRITE(6,*)' CYLINDER WITH A LOAD AS SHOWN BELOW.'
      WRITE(6,*)
```

```
      WRITE(6,*)'       (T1,XLOAD)'
      WRITE(6,*)'    I     ******** (T2,XLOAD)'
      WRITE(6,*)'    I    *          * '
      WRITE(6,*)'    I   *            *'
      WRITE(6,*)'    I *               *'
      WRITE(6,*)'    I*                 *'
      WRITE(6,*)'    *__________________*_____'
      WRITE(6,*)' (0,0)              (T3,0)'
      WRITE(6,*)
      WRITE(6,*)
      WRITE(6,*)
      XLOAD=800.
      T1=.008333333333
      T2=.025
      T3=.033333333333
      GAMMA=55.
      RIN=1.5
      ROUT=1.56
      WRITE(7,*)' LOAD (PSI)     = ',XLOAD
      WRITE(7,*)' T1 (HR)        = ',T1
      WRITE(7,*)' T2 (HR)        = ',T2
      WRITE(7,*)' T3 (HR)        = ',T3
      WRITE(7,*)' GAMMA (DEG)    = ',GAMMA
      WRITE(7,*)' RIN (IN)       = ',RIN
      WRITE(7,*)' ROUT (IN)      = ',ROUT

      WRITE(6,*) ' NUMBER OF LOOPS FOR ELASTIC SOLUTION ???
      READ(5,*)NLOOP1
      WRITE(6,*)' NUMBER OF LOOPS FOR CREEP PHASE ??? '
      READ(5,*)NLOOP2

      W=0.
      ND=0
      SLOPE1=XLOAD/T1

C*****************************************************************

C      FAILURE STRAIN

      THK=.0192

C*********************************************************************

C     TIME-DEPENDENT MATRIX PROPERTIES WERE DETERMINED BY LEAST SQUARES
C     CURVE FIT TO CREEP TEST DATA BY JAN DOZIER.
C     INITIAL VALUE OF RELAXATION MODULUS AND POISSON'S RATIO OF HBRF-55A

      EMI=359935.
      PMI= 0.3501
      WRITE(7,*)' EMI = ',EMI
      WRITE(7,*)' PMI = ',PMI

C     YOUNG'S MODULUS AND POISSON'S RATIO OF KEVLAR 49 FIBER.

      EF= 19000000.
      PF= 0.20
      WRITE(7,*)' EF = ',EF
      WRITE(7,*)' PF = ',PF

C     FIBER VOLUME FRACTION
```

```
      VF= 0.65
      WRITE(7,*)' VF = ',VF

C*******************************************************************

C                  ELASTIC RESPONSE WITH DAMAGE
C                  0 < T < T1

C*******************************************************************
      PI=1.
      PI= 4.*ATAN(PI)

      NTIMES=100
      DELT=T1/NTIMES
      GAMMA=GAMMA*PI/180.

      DO 2000 IX=1,NLOOP1
      T=DELT

      DO 1000 I=1,NTIMES
      LOAD=SLOPE1*T

C       CALCULATE TRANSVERSE MODULUS RESULTING FROM DAMAGE CALCULATED
C       IN LAST LOOP.

      IF(IX .NE. 1 )ET=ETI*(STRN(I)/THK-1)**2
      CALL SOLN(GAMMA,RIN,ROUT,VF,EF,PF,EMI,PMI,THK,LOAD,
     *                FLI,EPTHTH,EPMM,EPRR,U,ND,ET,W)
      STRN(I)=EPTHTH
      TLOG=LOG10(T)
      IF(IX.EQ.NLOOP1)THEN
             PRESS=SLOPE1*T*6895./1000000.
             WRITE(14,101)T,STRN(I)
             WRITE(15,101)STRN(I),PRESS
      ENDIF
 101  FORMAT(2(1X,E14.8))

      T=T+DELT
 1000 CONTINUE
      IF(IX.EQ.1)THEN
             ETI=ET
             WRITE(6,*)' FAILURE LOAD = ',FLI
             WRITE(7,*)' FAILURE LOAD = ',FLI
      ENDIF
      ND=1
      WRITE(6,*)' STRN = ',STRN(100)
 2000 CONTINUE

      STRNI=STRN(NTIMES)
      W1=STRNI/THK

C*******************************************************************

C                  CONSTANT LOAD PHASE
C                  (CREEP WITH DAMAGE)

C*******************************************************************


C     CALCULATE THE LAPLACE PARAMETERS.
```

```
      Q=7.
      RR=2.

      DO 201 I=1,6
      S(I)=EXP(RR*I-Q)
      V(I)=1/S(I)
  201 CONTINUE

       ETP=ETI*(W1-1)**2
      DO 2100 IX=1,NLOOP2
      DO 1100 I=1,6

C****************************************************

C     LAPLACE TRANSFORM OF XLOAD

      PLOAD=XLOAD/S(I)

C     LAPLACE TRANSFORM OF MATRIX PROPERTIES.
      EMP(I)=S(I)*(22120./(S(I)+106.5)+5835./(S(I)+10.18)
     *      +22420./(S(I)+1.07)-30640./(S(I)+.092)
     *      +340200./(S(I)+.0075))

      PMP(I)=.5-EMP(I)/2400000.

C***************************************************************
C    CALCULATE LAPLACE TRANSFORM OF DAMAGED TRANSVERSE MODULUS

      IF(IX .GT. 1)THEN
           ETB=(1/S(I)-2*PSTRN(I)/THK+(AC**2)/S(I)/THK**2)
           DO 310 IJ=1,6
           ETB=ETB+2*AC*GV(IJ)/(S(I)+1/V(IJ))/THK**2
           DO 320 IK=1,6
           ETB=ETB+GV(IJ)*GV(IK)/(S(I)+1/V(IJ)+1/V(IK))/THK**2
 320       CONTINUE
 310       CONTINUE
           ETP=ETB*S(I)*ETI
      ENDIF


      CALL SOLN(GAMMA,RIN,ROUT,VF,EF,PF,EMP(I),PMP(I),THK,PLOAD,
     *          PFL,PSTRN(I),PEPMM,PEPRR,PU,ND,ETP,W1)

C  **** RIGHT HAND SIDE VECTOR TO USE IN SOLN FOR COEFFICIENTS (GV'S)

       STN(I)=(PSTRN(I)- STRNI/S(I))*S(I)
 1100 CONTINUE
      ND=1

C*****************************************************************

C      PERFORM LAPLACE TRANSFORM INVERSION AND SOLVE FOR CONSTANTS
C      TO BE USED IN TIME DEPENDENT MAXIMUM HOOP STRAIN.

      DO 220 I=1,6
      DO 220 J=1,6
      A(I,J)=(V(I)*V(J)/(V(I)+V(J))-V(I))/V(I)
 220  CONTINUE

C     ***** ASSEMBLE MATRIX FO INVERSION.
```

```
      CALL SOLVE(A,STN,6)

      AC=STRNI
      DO 240 I=1,6
      GV(I)=STN(I)
      AC=AC-GV(I)
 240  CONTINUE

C**************************************************************

C             CALCULATE TIME TO FAILURE (USED TO CHECK CONVERGENCE)


      TN1=50.
      DO [illegible] I=1,10
      F1=AC-THK
      DF1=0.
      DO 32 IJ=1,6
      F1=F1+GV(IJ)*EXP(-TN1/V(IJ))
      DF1=DF1-GV(IJ)*EXP(-TN1/V(IJ))/V(IJ)
  32  CONTINUE
      TN2=TN1-F1/DF1
      TERR=TN2-TN1
      TN1=TN2
  31  CONTINUE

      WRITE(6,*)' T FAIL = ',TN1,' TERR = ',TERR
 2100  CONTINUE

C****************************************

C      CALCULATE STRAIN AT TIME T2 (USED AS INITIAL CONDITION FOR

      STRNI1=AC
      T=T2-T1
      DO 312 I=1,6
      STRNI1=STRNI1+GV(I)*EXP(-T*S(I))
 312  CONTINUE

C*******************************************************************

C             ****** ASSOCIATED SOLUTION ******
C                    FOR RAMP DOWN PHASE

C*******************************************************************

C           CALCULATE INITIAL PARAMETERS FOR RAMP DOWN

      ND=0
      W2=STRNI1/THK
      SLOPE=-XLOAD/(T3-T2)
      ET=ETI*(W2-1)**2
      CALL SOLN(GAMMA,RIN,ROUT,VF,EF,PF,EMI,PMI,THK,XLOAD,
     *          FLI,EPTHTH,EPMM,EPRR,U,ND,ET,W2)
      SRATE=-EPTHTH/(T3-T2)

C          "ASSOCIATED ELASTIC" SOLUTION

      DO 1200 I=1,6
```

```
C       LAPLACE TRANSFORM OF THE LOAD.

      PLOAD=SLOPE/S(I)**2
      CALL SOLN(GAMMA,RIN,ROUT,VF,EF,PF,EMP(I),PMP(I),THK,PLOAD,
     *          PFL,PSTN,PEPMM,PEPRR,PU,ND,ET,W2)

C  **** RIGHT-HAND SIDE VECTOR TO USE IN SOLN FOR COEFFICIENTS

       STN1(I)=(PSTN- SRATE*(V(I)**2))*S(I)
       STN2(I)=-STN1(I)
 1200 CONTINUE

C*******************************************************************

C       DEVELOPE MATRIX OF LAPLACE PRAMETERS ( A )

      CALL AMAT(V,0.0,A,6)

C       SOLVE FOR COEFFICIENTS OF EXPONENTIAL TERMS.

      DO 11 I=1,6
      DO 11 J=1,6
      AA(I,J)=A(I,J)
   11 CONTINUE

      CALL SOLVE(A,STN1,6)
      CALL SOLVE(AA,STN2,6)

      AC1=0.
      AC2=0.
      AD1=SRATE
      AD2=-SRATE
      DO 250 I=1,6
      AC1=AC1-STN1(I)
      AC2=AC2-STN2(I)
      AD1=AD1+SIN1(I)/V(I)
      AD2=AD2+STN2(I)/V(I)
      GV1(I)=STN1(I)
      GV2(I)=STN2(I)
  250 CONTINUE

C*******************************************************************

C       ****  CONSTANT DAMAGE CREEP   ***

C*******************************************************************

      ND=1
       ETP=ETI*(W2-1)**2
      CALL SOLN(GAMMA,RIN,ROUT,VF,EF,PF,EMI,PMI,THK,XLOAD,
     *          FLI,EPTHTH,EPMM,EPRR,U,ND,ETP,W2)
      STRNI2=EPTHTH
      DO 1300 I=1,6
      PLOAD=XLOAD/S(I)
      CALL SOLN(GAMMA,RIN,ROUT,VF,EF,PF,EMP(I),PMP(I),THK,PLOAD,
     *          PFL,PSTRN(I),PEPMM,PEPRR,PU,ND,ETP,W2)

C  **** RIGHT HAND SIDE VECTOR TO USE IN SOLN FOR COEFFICIENTS

       STNCD(I)=(PSTRN(I)- STRNI2/S(I))*S(I)
```

```
 1300 CONTINUE
C*********************************************************************
C       PERFORM LAPLACE TRANSFORM INVERSION AND SOLVE FOR CONSTANTS
C       TO BE USED IN TIME DEPENDENT MAXIMUM HOOP STRAIN.

      DO 222 I=1,6
      DO 222 J=1,6
      A(I,J)=(V(I)*V(J)/(V(I)+V(J))-V(I))/V(I)
 222  CONTINUE

C     ***** ASSEMBLE MATRIX FOR INVERSION.

      CALL SOLVE(A,STNCD,6)


      ACCD=STRNI2
      DO 244 I=1,6
      GVCD(I)=STNCD(I)
      ACCD=ACCD-GVCD(I)
 244  CONTINUE
C*********************************************************************

C       CALCULATE CIRCUMFERENTIAL STRAIN IN TIME DOMAIN.

      T=.001
      DO 300 LL=1,125
      STRNT=0.

      DO 350 I=1,6
      STRNT=STRNT+GV(I)*EXP(-T*S(I))
      IF(T.GT.T2-T1)STRNT=STRNT-GV(I)*EXP(-(T-(T2-T1))*S(I))
      IF(T.GT.T2-T1)STRNT=STRNT+GVCD(I)*EXP(-(T-(T2-T1))*S(I))
      IF(T.GT.T2-T1)STRNT=STRNT+GV1(I)*EXP(-(T-(T2-T1))*S(I))
      IF( T .GT. T3-T1)STRNT=STRNT+GV2(I)*EXP(-(T-(T3-T1))*S(I))
 350  CONTINUE

      STRNT=STRNT+AC
      IF(T.GT.T2-T1)STRNT=STRNT-AC
      IF(T.GT.T2-T1)STRNT=STRNT+ACCD
      IF(T.GT.T2-T1)STRNT=STRNT+AC1+AD1*(T-(T2-T1))
      IF(T .GT. T3-T1 ) STRNT=STRNT+AC2 +AD2*(T-(T3-T1))

      PRESS=XLOAD*6895./1000000.
      IF(T.GT.T2-T1)PRESS=PRESS+(SLOPE*(T-(T2-T1)))*6895./1000000.
      IF(T.GT.T3-T1)PRESS=0.
      TT=T+T1
      TLOG=LOG10(TT)
      WRITE(14,101)TT,STRNT
      WRITE(15,101)STRNT,PRESS
      T=T+.0015
 300  CONTINUE
C*********************************************************************

     CALCULATE FAILURE LOAD AFTER PROOF TEST
      ETI=ETI*(1-W2)**2
      ND=1
      CALL SOLN(GAMMA,RIN,ROUT,VF,EF,PF,EMI,PMI,THK,XLOAD,
     *               FLI,EPTHTH,EPMM,EPRR,U,ND,ET,W2)
```

```
WRITE(6,*)' DAMAGE = ',W2
WRITE(6,*)' FAILURE LOAD FOR DAMAGE (PSI) = ',FLI
WRITE(6,*) ' OUTPUT FILE = ED1.OUT'
WRITE(6,*) ' HOOP STRAIN VS. TIME (IN HOURS) IS ON '
WRITE(6,*)' ELD2.PLT'
END
```

```
      SUBROUTINE SOLN(GAMMA,RIN,ROUT,VF,EF,PF,EMI,PMI,THK,LOAD,
     *                FLI,EPTHTH,EPMM,EPRR,U,ND,ETI,W)
      IMPLICIT REAL*8 (A-H,P-Z)
      REAL*8 LAMBDA,LOAD,LAMBDAI
      PI=1.
      PI=4.*ATAN(PI)
      S1=SIN(GAMMA)
      C1=COS(GAMMA)
      S2=S1**2
      C2=C1**2
      S4=S1**4
      C4=C1**4
      VM= 1.0 - VF
      GF= EF/(2.*(1.+PF))
      GMI= EMI/(2.*(1.+PMI))
      ELI= VF*EF+VM*EMI
      IF (ND .EQ. 1)GO TO 2
      ETI=(1.-W)**2/(VF/EF+VM/EMI)
 2    PLTI= VF*PF+VM*PMI
      PTLI= PLTI*ETI/ELI
      PTTI= VF*PF+VM*PMI*((1+PMI-PLTI*(EMI/ELI))/
     *   (1-PMI**2+PMI*PLTI*(EMI/ELI)))
      BNI= (GF*(PI+4*VF)+GMI*(PI-4*VF))/
     *   (GF*(PI-4*VF)+GMI*(PI+4*VF))
      GLTI= (GMI/2)*(((4-PI)+PI*BNI)/4+4*BNI/((4-PI)*BNI+PI))

      LAMBDAI= 1/((1+PTTI)*(1-PTTI-2*PLTI*PTLI))
      CLLI= (1-PTTI**2)*LAMBDAI*ELI
      CTTI= (1-PLTI*PTLI)*LAMBDAI*ETI
      CLTI= PTLI*(1+PTTI)*LAMBDAI*ELI
      CTRI= (PTTI+PLTI*PTLI)*LAMBDAI*ETI
      GTRI= (1-PTTI-2*PLTI*PTLI)*LAMBDAI*ETI/2

      CRRI= CTTI
      CMMI=S4*CLLI+C4*CTTI+2*S2*C2*(CLTI+2*GLTI)
      CTHTHI=C4*CLLI+S4*CTTI+2*C2*S2*(CLTI+2*GLTI)
      CMTHI=(S4+C4)*CLTI+(CLLI+CTTI-4*GLTI)*S2*C2
      CMRI=S2*CLTI+C2*CTRI
      CTHRI=C2*CLTI+S2*CTRI

      ALPHA2=CTHTHI/CRRI
      ALPHA=SQRT(ALPHA2)
      BETA=(CMTHI-CMRI)/CRRI/(1-ALPHA2)

C*****************************************

C       CALCULATE CAPA, CAPB, AND EPMM , NEEDED TO CALCULATE U (DISPLACEMENT)

      A11=2*(ROUT**(1+ALPHA)-RIN**(1+ALPHA))*(CMTHI+ALPHA*CMRI)
     *     /(1+ALPHA)/RIN**2
      A12=2*(ROUT**(1-ALPHA)-RIN**(1-ALPHA))*(CMTHI-ALPHA*CMRI)
     *     /(1-ALPHA)/RIN**2
      A13=(ROUT**2-RIN**2)*(CMMI+BETA*(CMTHI+CMRI))/RIN**2
      A21=(CTHRI+ALPHA*CRRI)/RIN**(1-ALPHA)
      A22=(CTHRI-ALPHA*CRRI)/RIN**(1+ALPHA)
      A23=CMRI+BETA*(CTHRI+CRRI)
      A31=(CTHRI+ALPHA*CRRI)/ROUT**(1-ALPHA)
      A32=(CTHRI-ALPHA*CRRI)/ROUT**(1+ALPHA)
      A33=CMRI+BETA*(CTHRI+CRRI)
```

```
      B11=A22*A33-A32*A23
      B12=-(A21*A33-A31*A23)
      B13=A21*A32-A22*A31
      B21=-(A12*A33-A32*A13)
      B22=A11*A33-A31*A13
      B23=-(A11*A32-A31*A12)
      B31=A12*A23-A22*A13
      B32=-(A11*A23-A21*A13)
      B33=A11*A22-A12*A21
      DELTA=A11*A22*A33+A12*A23*A31+A13*A21*A32
     *      -A13*A22*A31-A12*A21*A33-A11*A32*A23

      CAPA=LOAD*(B11-B21)/DELTA
      CAPB=LOAD*(B12-B22)/DELTA
      EPMM=LOAD*(B13-B23)/DELTA

C***************************************************************

C     CALCULATE RADIAL DISPLACEMENT ( U ), HOOP STRAIN ( EPTHTH ),
C     AND RADIAL STRAIN ( EPRR ).

      R=RIN

      U=CAPA*R**ALPHA+CAPB/R**ALPHA+BETA*EPMM*R

C******************************************************************

C      CALCULATE THE FAILURE LOAD

       FLI=LOAD*THK*R/U
      EPTHTH=U/R
      EPRR=CAPA*ALPHA*R**(ALPHA-1)-CAPB*ALPHA/R**(ALPHA+1)
     *     +BETA*EPMM
      RETURN
      END
```

```
      SUBROUTINE SOLVE(E,G,N)
      IMPLICIT REAL*8 (A-H,P-Z)
      DIMENSION E(N,N),G(N)

C     SIMPLE GAUSS ELIMINATION
C     FORWARD ELIMINATION

      DO 200 J=1,N-1
      IF(E(J,J).NE.O.)GO TO 205
      WRITE(6,*)'******  E(J,J) IS ZERO FOR J = ',J
      RETURN
  205 EJJ= E(J,J)
      DO 201 I=J,N
      E(J,I)=E(J,I)/EJJ
  201 CONTINUE
      G(J)=G(J)/EJJ
      DO 220 JJ=J+1,N
      CJ=E(JJ,J)
      DO 230 I=J,N
      E(JJ,I)=E(JJ,I)-E(J,I)*CJ
  230 CONTINUE
      G(JJ)=G(JJ)-G(J)*CJ
  220 CONTINUE
  200 CONTINUE

C     BACK SUBSTITUTION

      G(N)=G(N)/E(N,N)
      DO 240 J=N-1,1,-1
      DO 250 K=J+1,N
      G(J)=G(J)-G(K)*E(J,K)
  250 CONTINUE
  240 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AMAT(V,TD,A,N)
      REAL*8 V,TD,A
      DIMENSION V(N),A(N,N)
      DO 220 IW=1,6
      DO 220 IV=1,6
      A(IW,IV)=(V(IV)*V(IW)*EXP(TD/V(IV))/(V(IV)+V(IW))
     *         +V(IW)**2/V(IV)-V(IW)
     *         -V(IW)*TD/V(IV))/V(IW)
  220 CONTINUE
      RETURN
      END
```

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