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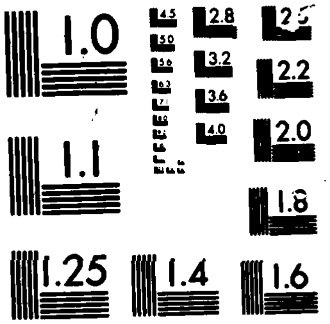

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THE WORLD GEODETIC SYSTEM 1984
EARTH GRAVITATIONAL MODEL

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SUMMARY

The World Geodetic System 1984 (WGS 84) Earth Gravitational Model (EGM) consists of a set of normalized geopotential coefficients complete through degree (n) and order (m) 180. The first part of the EGM, through degree and order 41, was developed as a weighted least squares combination solution from mean free-air gravity anomalies; geoid undulations derived from satellite radar altimetry; laser, Doppler and NAVSTAR Global Positioning System (GPS) satellite tracking data and "lumped coefficient" data. Procedures used in the EGM development, testing and evaluation are discussed with particular emphasis on orbital analysis results as they apply to Doppler point positioning. Keywords:

1.0 INTRODUCTION

The procedure used in the development of the WGS 84 EGM through degree and order 41 was to form normal equations for each of the various data sets; mean free-air gravity anomalies, mean geoid undulations, Doppler or laser tracking data from each satellite used and "lumped coefficient" data. These normal equations were then combined one at a time to obtain preliminary or intermediate solutions. All of these solutions were evaluated by comparing differences between observed and computed mean gravity anomalies, mean geoid undulations and the Doppler residuals from selected satellite orbit reductions. The magnitude of and changes in the gravity anomaly degree variances computed from these intermediate EGMs were also carefully monitored. This procedure, although time comsuming, ensured prompt identification of any problems associated with a particular data set. Upon completion of the degree and order 41 portion of the WGS 84 EGM using least squares techniques, a residual 1° X 1° (equiangular) mean gravity anomaly field was developed by subtracting the contribution of this EGM from the observed 1° X 1° field. This residual mean gravity anomaly field was then used in a spherical harmonic analysis to extend the final WGS 84

EGM to degree and order 180. This expanded WGS 84 EGM produces significant gravity anomaly and geoid undulation differences when compared to similar values from the degree and order 41 EGM in areas containing short wavelength high frequency information. On the other hand, very little difference occurs in areas devoid of trenches and/or ridges or in areas with a relatively smooth surface gravity field. For moderate to high altitude satellite orbit computations, the degree and order 41 portion of the WGS 84 EGM is adequate. Further discussions in this paper deal primarily with the tests, evaluations and applications of the least squares derived portion of the WGS 84 EGM.

2.0 DATA USED FOR GEOPOTENTIAL MODEL DEVELOPMENT

The data used in the development of the degree and order 41 portion of the WGS 84 EGM consists of 3° X 3° equal-area mean free-air gravity anomalies, 3° X 3° equal-area mean altimetric geoid undulations, data from five modern and two historical Doppler tracked satellites, laser tracking data from two satellites, GPS satellite tracking data (processed as range difference data) and "lumped geopotential coefficient" information. Each of these data types and the processing of the data into normal equations are discussed separately.

2.1 3° X 3° EQUAL-AREA MEAN FREE-AIR GRAVITY ANOMALY DATA

The June 1984 Department of Defense (DoD) Gravity Library equiangular 1° X 1° mean free-air gravity anomaly file, balanced with respect to the WGS 84 Ellipsoidal Gravity Formula and with terrain corrections applied (where available), along with their error estimates was used as basic input to compute 4584 3° X 3° approximately equal-area mean free-air gravity anomalies and their corresponding sigmas. These gravity anomalies were defined in terms of 3° latitude bands subdivided into whole degree longitude increments. The sigmas used as input to the weighting scheme for each observed 3° X 3° mean gravity anomaly were developed directly from the corresponding sigmas of the 1° X 1° data.

The mean gravity anomaly normal equations were formed from the observation equations which included all nonzero geopotential coefficients below degree and order 41 as well as a bias parameter for the geopotential. The WGS 84 constants for the earth's rotation rate (ω), the ellipsoid semimajor axis (a) and flattening (f), and the product of the earth's mass and the universal gravitational constant (GM) were used in these equations. Other required constants, such as those necessary to define normal or theoretical gravity, were computed using appropriate WGS 84 constants. The weighting scheme used for each observed 3° X 3° equal-area mean gravity anomaly (i) is

$$W_i = 1.0/(\sigma_i^2 + \sigma_m^2)$$

where

W_i = weight of the i^{th} 3°X 3° equal-area mean gravity anomaly

σ_i = sigma for the i^{th} 3°X 3° equal-area mean gravity anomaly developed from the 1° X 1° sigmas

σ_m = an a priori estimate of the model error assumed for each 3° X 3° equal-area observation. This represents the error of omission resulting from truncating the EGM at degree 41. The value assumed was ±10 milligals (one sigma).

2.2 3° X 3° EQUAL-AREA ALTIMETRIC MEAN GEOID UNDULATION DATA

A 1° X 1° *equiangular mean geoid undulation data* file developed from SEASAT radar altimeter measurements was merged into 3° x 3° approximately equal-area mean geoid undulations. Since this data file was latitude bounded as well as restricted to oceanic coverage, it consisted of 2918 3° x 3° mean geoid undulations. An additional requirement when forming a 3° X 3° mean value was that at least two thirds of each 3° X 3° mean geoid undulation be oceanic in ocean/land interface areas. Error estimates for these interface areas were significantly larger than those for the "fully observed" broad ocean area geoid undulations. As in the mean gravity anomaly normal equations, all nonzero geopotential coefficients below degree and order 41 and a bias parameter for the geopotential were included as parameters. Similarly, WGS 84 constants were used throughout when computing the observation equations. The model error assumed for weighting purposes for the 3° X 3° equal-area geoid undulations was ±1 meter (one sigma).

2.3 DOPPLER DATA

Significant improvements in the Doppler satellite tracking network were made in 1971 when the collection of continuous count Doppler data was begun. This led to the categorization of Doppler tracking data collected before and after 1971 as historical and modern Doppler data, respectively. Other equipment changes such as the installation in 1975 of rubidium oscillators in the fixed Doppler receivers represent an additional improvement of the modern data. Doppler data from seven satellites, including five in the modern era, was included in the WGS 84 EGM development. The common names and orbital characteristics of these satellites are tabulated in Table 1. Two six-day data spans were selected and processed for each of the seven Doppler satellites except HILAT. Due to time limitations, only one six-day span was processed for this satellite.

TABLE 1

DOPPLER SATELLITE ORBITAL DATA

SATELLITE NAME	SEMIMAJOR AXIS (KM)	PERIGEE HEIGHT (KM)	INCLINATION (DEGREES)
GEOS 3	7216	818	115
SEASAT	7159	778	108
NNS 68	7457	949	90
HILAT	7179	768	82
DB 14	7487	971	63
*GEOS 1	8072	1113	59
*BEACON C	7503	941	41

*HISTORICAL SATELLITES

These data spans were selected, as much as possible, to satisfy the need for dense Doppler tracking, good balance between northern and southern hemisphere tracking stations and variation in the arguments of perigee. These basic data spans were very carefully edited on a points-within-a-pass and on a pass basis. After the data editing was completed, normal equations were formed for each six-day arc consisting of coordinates of all data contributing tracking stations as well as for all geopotential coefficients through degree and order 41. Other parameters included in the normal equations were those for satellite initial conditions, drag multipliers, radiation pressure and solid earth tidal forces, time correction parameters (when required) and a small collection of analysis parameters. Bias parameters were mathematically eliminated as part of the normal equation formation procedure. The Doppler normal equations were tested individually in EGM tuning solutions and in various combinations as part of an extensive validation effort prior to their incorporation into a combined set of normal equations supporting a final WGS 84 EGM solution.

2.4 LASER DATA

Two laser observed satellites, Starlette and LAGEOS, were selected for WGS 84 EGM exploitation purposes. The laser data processing and normal equation development was performed by the Center for Space Research, The University of Texas at Austin, under a Defense Mapping Agency/Naval Surface Weapons Center contract. The Starlette satellite with its 50 degree inclination and 805 km perigee height supplemented the Doppler geodetic satellites for the determination of the geopotential. The data span selected for processing was the 93-day observational phase (August through October 1980) of the MERIT* Short Campaign. During this period, an average of eight passes per day were collected from 18 stations. The entire 93-day data span was treated as a single dynamical arc. The dynamical model used included conventional gravitational and solid earth tidal forces, a 60 term ocean tide force described by Dr. Schwiderski of the Naval Surface Weapons Center, solar radiation pressure and the Jacchia 1971 drag model. Estimable geodetic parameters included the same geopotential coefficients as the Doppler medium altitude satellite equations together with a few additional resonant terms. Other parameters included in the laser normal equations were tracking station coordinates, satellite initial conditions and multipliers for the radiation, drag and solid earth tidal forces. Bias parameters were mathematically eliminated in the process of forming the normal equations.

The laser tracking data available for the LAGEOS satellite was sufficient for a two year data span to be processed. This data span, covering 1980 and 1981, overlapped the MERIT Short Campaign data set chosen for Starlette. The LAGEOS observations represent over 4000 passes of data collected by 32 laser tracking stations. As with Starlette, the entire data span was treated as a single dynamical arc.

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*MERIT = Monitor Earth Rotation and Intercompare the Techniques of Observation and Analysis

Estimable geodetic parameters included tracking station coordinates and a reduced set of geopotential coefficients. However, the reduced geopotential coefficient set was selected to include all coefficients producing orbital perturbations at the one cm level. The selection process was based on the use of estimated coefficients an order of magnitude larger than Kaula's rule ($10^{-5}/n^2$). LAGEOS, because of its high altitude, contributes only to the determination of the lower degree and order harmonic coefficients.

2.5 NAVSTAR GLOBAL POSITIONING SYSTEM (GPS) DATA

Four continuous weeks of simultaneous tracking data from five GPS satellites was selected as the GPS data set. The data set was characterized by the absence of irregularities in the atomic clock time histories and the avoidance of eclipses (the entry of the satellites into the earth's shadow causing force modelling problems). Separate sets of normal equations were formed for each week of data. The normal equations included 50 geopotential coefficients as parameters and allowed for the adjustment of the universal gravitational constant, a systematic Z-axis shift and a scale correction for the tracking network. Other incidental parameters included a clock frequency and aging parameter for each satellite and station in addition to some pass and station bias parameters. Due to their altitude, the GPS satellites were not expected to contribute significantly to the WGS 84 EGM. However, this data was included because it could possibly result in a slight improvement of the WGS 84 EGM for GPS orbit applications.

2.6 LUMPED GEOPOTENTIAL COEFFICIENT DATA

Lumped coefficients refer to certain linear combinations of zonal, low degree and resonant geopotential coefficients which are responsible for rather large satellite orbital perturbations. Analyses of fitted satellite orbital element histories yield estimates of these lumped coefficients which may be used as "observational" data in determining the applicable geopotential coefficients themselves. Numerous papers have been published on this subject by D.G. King-Hele and C.A. Wagner.

An extensive literature search at the Naval Surface Weapons Center produced a total of 426 unique observations of lumped coefficients.

Approximately 20 of these equations were deleted initially on the basis of the author's own remarks. Normal equations were formed with the remaining observational values and standard errors taken from the literature. The parameters of the normal equations generally consisted of all relevant geopotential coefficients through the 41st degree. An exception was the synchronous satellite data which was truncated above the 6th degree. Preliminary validation tests were not completely successful in terms of duplicating published results for preliminary test solutions. However, preliminary test solutions did produce some residuals ranging from five to 15 times the standard error. This resulted in the deletion of an additional 20 observations. Finally, the remaining observation equations were combined into two sets of normal equations, one representing the observations of synchronous satellite and zonal lumped coefficients and another representing the higher order resonant tesseral geopotential coefficients. Unfortunately test solutions developed with the higher order normal equations slightly degraded rather than improved the WGS 84 EGM. As a result, only synchronous satellite and zonal lumped coefficient observations were incorporated into the final least squares portion of the WGS 84 EGM. The problem with the higher order tesseral harmonic resonant data set has not been identified.

3.0 WGS 84 EARTH GRAVITATIONAL MODEL DEVELOPMENT AND EVALUATION

The WGS 84 EGM through degree and order 41 represents the solution of a set of normal equations formed by combining the individual normal equations developed from the previously described data sets. All of the individual normal equations were combined on a one to one basis without any scaling. The initial step was to combine the mean gravity anomaly and geoid undulation normal equations and then add to these combined equations the satellite normal equations one at a time. Each time a new set of normal equations was added a preliminary EGM solution was obtained, tested and evaluated. Evaluation was accomplished by computing gravity anomaly degree variances, determining the differences between computed and observed 3° X 3° equal-area mean gravity anomalies, determining the differences between computed and observed 3° X 3° equal-area mean altimetric geoid undulations and by analyzing the Doppler residuals obtained in orbit reductions of a six-day GEOS 3, a four-day NNS 68 and a four-day SEASAT data span. All of the data spans used in the orbit reduction analysis were distinctly different from the data spans used in the WGS 84 EGM development. Two-day orbit reductions were also accomplished at the Naval Surface Weapons Center.

3.1 DEGREE VARIANCES

Gravity anomaly degree variances were computed for each intermediate EGM as the first step in its evaluation. This relatively simple computation serves as a problem indicator when larger than expected coefficient magnitudes occur. Gravity anomaly degree variances are computed by the equation

$$\sigma_n{}^2 = \gamma(n-1)^2 \sum_{m=0}^{n} (\bar{C}_{n,m}{}^2 + \bar{S}_{n,m}{}^2)$$

where

$\sigma_n{}^2$ = gravity anomaly degree variance in $mgal^2$ for degree n

γ = the average value of theoretical gravity

$\bar{C}_{n,m}$, $\bar{S}_{n,m}$ = normalized geopotential coefficients of degree *n and order m*

Gravity anomaly degree variances for selected geopotential models are tabulated in Table 2. The first of these models, GAGH (n=m=41), was obtained from a combination of the 3° X 3° equal-area mean gravity anomaly and the 3° X 3° equal-area mean geoid undulation normal equations. Degree variances for this model are quite similar to those obtained for the WGS 84 (n=m=41) EGM. The other models given in Table 2 -- WGS 84 (n=m=36), WGS 84 (n=m=30), and WGS 84 (n=m=24) -- represent least squares test solutions of the WGS 84 EGM combined normal equations with the parameter set limited to degree and order 36, 30 and 24, respectively. The degree variances for these test solutions agree quite well with the WGS 84 (n=m=41) EGM degree variances.

3.2 MEAN GRAVITY ANOMALY COMPARISONS

One method of evaluating an earth gravitational model is to compute the mean square difference between mean gravity anomalies developed from geopotential coefficients (Δg_h) and mean gravity anomalies developed from observed terrestrial data (Δg_t). The terrestrial field used for this comparison was developed from observed data only and categorized

TABLE 2

GRAVITY ANOMALY DEGREE VARIANCES ($\sigma_n{}^2$)
UNITS = MILLIGALS^2

DEGREE	GAGH* (n=m=41)	WGS 84 (n=m=41)	WGS 84* (n=m=36)	WGS 84* (n=m=30)	WGS 84* (n=m=24)
2	7.6	7.6	7.6	7.6	7.6
3	33.6	33.9	33.9	34.0	34.0
4	19.2	19.2	19.2	19.2	19.2
5	20.8	20.9	21.0	20.9	20.9
6	17.6	19.4	19.4	19.4	19.4
7	20.6	19.3	19.4	19.5	19.5
8	10.2	10.9	10.9	10.9	11.1
9	10.0	11.5	11.5	11.4	11.4
10	9.6	9.7	9.7	9.7	9.5
11	8.7	6.4	6.3	6.3	6.1
12	3.8	2.6	2.6	2.5	2.4
13	7.0	7.4	7.4	7.3	7.3
14	3.2	3.2	3.2	3.2	3.4
15	3.2	3.4	3.4	3.4	3.3
16	5.3	3.9	3.9	4.1	4.3
17	4.0	3.6	3.6	3.6	4.3
18	3.0	3.6	3.5	3.6	3.7
19	3.2	3.3	3.4	3.5	3.6
20	2.4	3.1	3.2	3.2	3.5
21	2.4	3.2	3.3	3.3	3.5
22	3.4	3.5	3.6	3.9	4.3
23	2.5	2.7	2.8	3.1	2.8
24	2.3	2.6	2.6	2.8	2.2
25	2.9	2.9	2.9	2.9	
26	1.9	2.4	2.6	2.9	
27	2.0	1.9	2.0	2.5	
28	2.7	2.4	2.6	3.4	
29	2.5	2.4	2.6	2.7	
30	3.0	2.8	3.0	3.2	
31	1.7	2.9	2.9		
32	2.6	4.1	4.1		
33	2.9	3.4	3.9		
34	4.0	5.0	4.6		
35	4.2	4.4	4.4		
36	2.9	3.6	2.8		
37	2.8	3.4			
38	2.5	2.8			
39	3.1	3.5			
40	3.0	3.6			
41	2.5	2.8			

*TEST MODELS FROM LEAST SQUARES SOLUTIONS in terms of percentage of observed data available. For example, 100 percent observed requires all nine of the 1° X 1° equal-area mean gravity anomalies within a 3° X 3° equal-area boundary to be observed. Similarly, six out of nine implies 67 percent observed, etc. Equivalent logic applies to the percentage of observed values used in forming the 5° X 5° equal-area mean gravity anomalies.

WGS 84 EGM through degree and order 41 was used to compute 3° X 3° and 5° X 5° equal-area mean gravity anomalies. These computed means were then compared to their observed counterparts. Comparisons were also made at different truncation levels to study the additional informational content obtained by increasing the degree and order of the EGM. Other models evaluated were the degree and order 24, 30 and 36 EGMs developed as test solutions of the WGS 84 EGM data set, the WGS 84 (n=m=41) EGM without the geoid undulation data, and an EGM developed from a mean gravity anomaly/mean geoid undulation only solution. The 3° X 3° and the 5° X 5° mean gravity anomaly comparisons are given in Tables 3 and 4, respectively. The results of these comparisons can be summarized as follows:

a. There is no appreciable difference between the results obtained by truncating the WGS 84 (n=m=41) EGM and the results obtained from the truncated solution models. This indicates that the aliasing of higher degree and order information into lower degree and order coefficients is insignificant for the WGS 84 EGM.

b. The difference between the computed and observed mean gravity anomalies decreases as the degree and order of the model increases indicating the validity of the higher degree and order coefficients of the WGS 84 EGM.

c. The WGS 84 (n=m=41) EGM appears to be almost as good for representing the observed mean gravity anomaly field as the model developed from the mean gravity anomaly/mean geoid undulation data only.

d. The inclusion of mean geoid undulation data in the WGS 84 EGM produced a model that agrees better with the observed mean gravity anomaly field than the model developed with the WGS 84 EGM data set less the geoid undulation data.

TABLE 3

COMPARISON OF 3°X 3° EQUAL-AREA MEAN GRAVITY ANOMALIES COMPUTED FROM EARTH GRAVITATIONAL MODELS WITH THOSE DERIVED FROM TERRESTRIAL DATA

EARTH GRAVITATIONAL MODEL	DEGREE OF TRUNCATION	$[<(\Delta g_t - \Delta g_h)^2>]^{1/2}$		
		33% OBS n = 4007	67% OBS n = 3679	100% OBS n = 3190
WGS 84	41	±9.31	±8.44	±7.53
	36	9.78	8.99	8.15
	30	10.44	9.73	8.93
	24	10.90	10.26	9.47
WGS 84 EXPERIMENTAL EARTH GRAVITATIONAL MODELS				
WGS 84 TRUNCATED MODELS*	36	±9.85	±9.03	±8.15
	30	10.56	9.84	8.95
	24	11.03	10.37	9.50
WGS 84 LESS GEOID UNDULATION DATA	41	9.32	8.52	7.79
	36	9.82	9.08	8.33
	30	10.51	9.84	9.10
	24	10.97	10.36	9.60
WGS 84 GRAVITY ANOMALY AND GEOID UNDULATION DATA ONLY	41	8.80	8.02	7.34
	36	9.35	8.62	7.97
	30	10.12	9.45	8.79
	24	10.67	10.07	9.38

UNITS = MILLIGALS

n IS THE NUMBER OF SQUARES INCLUDED IN THE SAMPLE OUT OF A POSSIBLE 4584 FOR WORLDWIDE COVERAGE.

*LEAST SQUARES SOLUTIONS OBTAINED TO THE DEGREE AND ORDER SPECIFIED USING THE WGS 84 EGM DATA SET.

TABLE 4

COMPARISON OF 5°X 5° EQUAL-AREA MEAN GRAVITY ANOMALIES COMPUTED FROM EARTH GRAVITATIONAL MODELS WITH THOSE DERIVED FROM TERRESTRIAL DATA

EARTH GRAVITATIONAL MODEL	DEGREE OF TRUNCATION	$[\langle(\Delta g_t - \Delta g_h)^2\rangle]^{\frac{1}{2}}$		
		40% OBS n = 1421	80% OBS n = 1238	100% OBS n = 1036
WGS 84	41	±5.72	±4.23	±3.66
	36	6.11	4.66	4.14
	30	6.67	5.24	4.63
	24	7.27	5.89	5.20
WGS 84 EXPERIMENTAL EARTH GRAVITATIONAL MODELS				
WGS 84 TRUNCATED MODELS*	36	±6.15	±4.65	±4.08
	30	6.74	5.33	4.66
	24	7.41	5.98	5.21
WGS 84 LESS GEOID UNDULATION DATA	41	5.83	4.54	4.13
	36	6.20	4.89	4.47
	30	6.76	5.46	4.94
	24	7.34	6.07	5.44
WGS 84 GRAVITY ANOMALY AND GEOID UNDULATION DATA ONLY	41	5.11	3.98	3.52
	36	5.53	4.41	3.94
	30	6.21	5.03	4.48
	24	6.92	5.77	5.13

UNITS = MILLIGALS

n IS THE NUMBER OF SQUARES INCLUDED IN THE SAMPLE OUT OF A POSSIBLE 1654 FOR WORLDWIDE COVERAGE.

*LEAST SQUARES SOLUTIONS OBTAINED TO THE DEGREE AND ORDER SPECIFIED USING THE WGS 84 EGM DATA SET.

3.3 GEOID UNDULATION COMPARISONS

The principles involved in earth gravitational model and observed mean gravity anomaly comparisons can be extended to geoid undulations. However, such comparisons are limited to the oceanic geoid determined from satellite radar altimetry. The basic data set is 1° X 1° equiangular mean geoid undulations developed into 3° X 3° and 5° X 5° equal-area mean geoid undulations in much the same way as the equal-area mean gravity anomalies were formed. In the geoid undulation comparisons shown in Tables 5 and 6, N_t is an observed mean geoid undulation developed from altimetry and N_h is a mean geoid undulation computed using the various EGMs being evaluated. The results of these comparisons can be summarized as follows:

a. As in the mean gravity anomaly comparisons there is no significant difference between the results obtained by truncating the WGS 84 (n=m=41) EGM and the truncated models obtained from the WGS 84 EGM data set.

b. The difference between the observed and computed mean geoid undulations decreases as the degree and order of the model used to compute the geoid undulations increases.

c. The models developed with the geoid undulation data included in the solution produce better agreement with the observed geoid undulations than the WGS 84 EGM developed without the geoid undulation data.

d. The WGS 84 (n=m=41) EGM is almost as good for representing geoid undulations as the model developed from mean gravity anomaly/mean geoid undulation data only.

3.4 SATELLITE ORBIT ANALYSIS

A good general purpose earth gravitational model is expected to provide a means for computing not only gravity anomalies and geoid undulations at points on or near the earth's surface but also to serve as adequate gravitational force model for precise satellite orbit computations. An objective in the development of the WGS 84 EGM was

TABLE 5

COMPARISON OF 3° X 3° EQUAL-AREA GEOID UNDULATIONS COMPUTED FROM EARTH GRAVITATIONAL MODELS WITH THOSE DERIVED FROM SEASAT GEOID UNDULATION DATA

EARTH GRAVITATIONAL MODEL	DEGREE OF TRUNCATION	$[\langle (N_t - N_h)^2 \rangle]^{1/2}$		
		33% OBS n = 3101	67% OBS n = 2918	100% OBS n = 2672
WGS 84	41	±1.55	±1.28	±1.05
	36	1.64	1.38	1.16
	30	1.76	1.53	1.30
	24	1.91	1.70	1.46
WGS 84 EXPERIMENTAL EARTH GRAVITATIONAL MODELS				
WGS 84 TRUNCATED MODELS*	36	±1.62	±1.35	±1.12
	30	1.74	1.49	1.25
	24	1.88	1.67	1.42
WGS 84 LESS GEOID UNDULATION DATA	41	1.99	1.79	1.59
	36	2.03	1.84	1.63
	30	2.12	1.93	1.71
	24	2.22	2.04	1.82
WGS 84 GRAVITY ANOMALY AND GEOID UNDULATION DATA ONLY	41	1.47	1.22	0.96
	36	1.55	1.31	1.07
	30	1.67	1.45	1.21
	24	1.83	1.62	1.38

UNITS = METERS

n IS THE NUMBER OF SQUARES INCLUDED IN THE SAMPLE OUT OF A POSSIBLE 4584 FOR WORLDWIDE COVERAGE.

*LEAST SQUARES SOLUTIONS OBTAINED TO THE DEGREE AND ORDER SPECIFIED USING THE WGS 84 EGM DATA SET.

TABLE 6

COMPARISON OF 5° X 5° EQUAL-AREA GEOID UNDULATIONS COMPUTED FROM EARTH GRAVITATIONAL MODELS WITH THOSE DERIVED FROM SEASAT GEOID UNDULATION DATA

EARTH GRAVITATIONAL MODEL	DEGREE OF TRUNCATION	$[\langle(N_t - N_h)^2\rangle]^{\frac{1}{2}}$		
		40% OBS n = 1108	80% OBS n = 993	100% OBS n = 887
WGS 84	41	±2.34	±1.24	±0.89
	36	2.35	1.28	0.94
	30	2.41	1.36	1.04
	24	2.45	1.46	1.15
WGS 84 EXPERIMENTAL EARTH GRAVITATIONAL MODELS				
WGS 84 TRUNCATED MODELS*	36	±2.35	±1.26	±0.92
	30	2.41	1.34	1.00
	24	2.50	1.45	1.12
WGS 84 LESS GEOID UNDULATION DATA	41	2.65	1.67	1.38
	36	2.65	1.69	1.40
	30	2.68	1.74	1.46
	24	2.71	1.80	1.53
WGS 84 GRAVITY ANOMALY AND GEOID UNDULATION DATA ONLY	41	2.29	1.20	0.80
	36	2.31	1.23	0.85
	30	2.35	1.31	0.94
	24	2.40	1.41	1.07

UNITS = METERS

n IS THE NUMBER OF SQUARES INCLUDED IN THE SAMPLE OUT OF A POSSIBLE 1654 FOR WORLDWIDE COVERAGE.

*LEAST SQUARES SOLUTIONS OBTAINED TO THE DEGREE AND ORDER SPECIFIED USING THE WGS 84 EGM DATA SET.

that it be as good as a "tuned" model for orbit computations where a tuned EGM is one based on extensive use of tracking data from a given satellite in the EGM development. Within this context, a good earth gravitational model for orbit computations is one that has been extensively tuned by including satellite tracking information from as large a variety of satellites as possible. However, the inclusion in the WGS 84 EGM of data from all satellites for which good Doppler tracking data was available limits orbit analysis as an independent method for evaluating the WGS 84 EGM. A second consideration is that not all EGM coefficients produce measurable orbit perturbations. Sensitivity of the WGS 84 EGM coefficients through degree and order 41 at or above the 0.5 meter level is shown in Table 7 for GEOS 3, in Table 8 for NNS 68 and in Table 9 for all ten satellites providing data for the WGS 84 EGM development. The blank areas in Table 9 indicate those coefficients that are being determined primarily from mean gravity anomaly and geoid undulation data. Conversely, these coefficients are not being evaluated in the respective orbit analysis tests to any degree of reliability.

Data spans for most of the satellites used in the WGS 84 EGM development were used to test and evaluate the WGS 84 EGM. Orbit reduction results for two of these satellites -- NNS 68 and NOVA -- are presented in Table 10. Data spans longer than the period of the first order resonance terms were selected so that the resonant coefficients would be fully tested. The tuned EGM for both the NNS 68 and NOVA satellites (NWL 10E-1) was used for the initial orbit reduction and data editing process. These data sets were then used in orbit reductions with the WGS 84 (n=m=41) EGM without any further editing. Since the purpose of these tests was to evaluate the geopotential coefficients, other constants such as GM and tracking station coordinates are common to both the tuned model and the WGS 84 EGM reductions. One-day drag segments were used for NNS 68. The NOVA satellite has a drag compensation system that in effect makes it drag free. The tabular results show the RWS (the square root of the sum of the squares of the weighted residuals divided by the sum

TABLE 7

WGS 84 GEOS 3 ORBITAL PERTURBATIONS $\geq$0.5 METER

DEGREE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
2		X																																							
3	X	X	X																																						
4	X	X	X	X																																					
5	X	X	X	X	X																																				
6	X	X	X	X	X	X																																			
7	X	X	X	X	X	X	X																																		
8	X	X	X	X	X	X	X	X																																	
9	X	X	X	X	X		X	X	X																																
10	X	X	X	X	X	X		X	X																																
11	X		X				X	X	X	X	X																														
12	X	X	X	X	X	X	X	X		X	X																														
13	X	X								X	X	X	X																												
14	X	X	X	X		X	X					X	X	X																											
15	X	X							X			X	X	X	X																										
16	X	X	X	X		X		X					X	X	X																										
17	X										X	X	X	X	X	X																									
18	X		X		X									X	X	X																									
19													X	X	X	X	X	X																							
20	X	X													X	X																									
21														X	X	X		X	X																						
22		X												X																											
23														X	X	X																									
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36																													X												
37																																									
38																													X												
39																																									
40																																									
41																																									

ORDER

a = 1.1314 a_e
a_e = 6378137 meters
e = 0.0028
i = 2.0071 radians
$\dot{\psi}$ = -0.3460 degrees/day
Ω = 2.7341 degrees/day

TABLE 8

WGS 84 NNS 68 ORBITAL PERTURBATIONS >0.5 METER

DEGREE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
2	X																																								
3	X	X	X																																						
4	X	X	X	X																																					
5	X	X	X	X	X																																				
6	X	X	X	X	X	X																																			
7		X		X		X	X																																		
8	X	X	X	X	X	X	X	X																																	
9		X				X	X	X	X																																
10	X	X	X	X	X	X				X																															
11		X				X		X	X	X	X																														
12	X	X	X	X		X				X																															
13										X	X	X	X																												
14	X	X	X	X		X						X	X	X																											
15												X	X	X	X																										
16	X	X		X									X	X																											
17												X	X	X	X	X																									
18	X	X										X	X																												
19												X	X	X	X	X																									
20													X																												
21												X	X	X																											
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36																											X														
37																																									
38																											X														
39																																									
40																											X														
41																																									

ORDER

a = 1.1692 a_e
a_e = 6378137 meters
e = 0.0040
i = 1.5743 radians
$\dot{\omega}$ = -2.8844 degrees/day
Ω = 0.0201 degrees/day

TABLE 9

NUMBER OF SATELLITES USED IN EGM DEVELOPMENT THAT ARE SENSITIVE TO WGS 84 GEOPOTENTIAL COEFFICIENTS EXCLUDING LUMPED DATA SETS
(ORBITAL PERTURBATIONS ≥0.5 METER)

DEGREE																																									
2		10																																							
3	9	10	9																																						
4	9	10	9	10																																					
5	8	9	8	8	7																																				
6	8	8	8	9	8	8																																			
7	7	9	6	8	6	9	6																																		
8	8	8	8	8	8	8	7	4																																	
9	7	6	5	6	5	6	8	7	6																																
10	8	8	8	7	8	7		2	4	3																															
11	6	3	4	1		2	6	4	8	7	6																														
12	8	8	7	8	5	5	5	2		3	5	4																													
13	5	3				1		2	5	6	7	7	8																												
14	8	7	6	5	3	5	4	1			1	7	8	6																											
15	4	1						2	3	3	3	7	8	7	5																										
16	7	6	6	4		1		1				3	8	6	3	2																									
17	3									1	2	7	8	8	6	5	3																								
18	7	5	3	3	1							2	6	7	3	2																									
19												4	8	6	5	6	5	2																							
20	4	2	1	2								1	3	7	4																										
21												1	7	7	7	3	1	3	1	1																					
22	1	2										1	2	5	1																										
23												1	3	5	4	3	2	2																							
24		1										1	1	4	1																										
25												1	2	6	3	2	1																								
26	2											1	1	4																											
27												1	2	6										1																	
28													1	3	1									1		1	2	2													
29													1	3	2									1			1	1													
30												1	1	2												1	3	3	2	1											
31												1	1	4													2		1												
32												1												1			3	3	3	1											
33													1	2																											
34													1														3	2	3	1											
35														3	1												1														
36																											2	1	3												
37															1																										
38																											2		3												
39																																									
40																											1		1												
41														1																											
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41

ORDER

TABLE 10

EVALUATION OF EARTH GRAVITATIONAL MODEL COEFFICIENTS IN ORBIT REDUCTION APPLICATIONS (IDENTICAL DATA SETS, STATION COORDINATES AND GM)

SATELLITE NAME	EARTH GRAVITATIONAL MODEL	RWS (METERS)	
		RAD	TANG
NNS 68 DAYS 205-208, 1982	NWL 10E-1*	1.8	2.8
	WGS 84	1.5	2.6
NOVA DAYS 135-138, 1984	NWL 10E-1*	1.7	1.9
	WGS 84	1.4	1.9
NOVA DAYS 141-144, 1984	NWL 10E-1*	1.8	2.0
	WGS 84	1.6	2.0

* MODEL TUNED TO THESE RESPECTIVE SATELLITES

TABLE 11

COMPARISON OF ORBIT REDUCTION RESULTS BETWEEN WGS 84 AND THE NSWC 9Z-2 GEODETIC SYSTEMS FOR THE NOVA SATELLITE

GEODETIC SYSTEM	DATE		RWS (METERS)	
	DAYS	YEAR	RAD	TANG
NSWC 9Z-2	135-136	1984	1.6	1.6
WGS 84			1.5	1.1
NSWC 9Z-2	137-138	1984	1.5	1.5
WGS 84			1.2	1.2
NSWC 9Z-2	141-142	1984	1.6	1.7
WGS 84			1.6	1.5
NSWC 9Z-2	143-144	1984	1.4	1.4
WGS 84			1.3	1.2

of the weights) of the slant range (RAD) component (station-to-satellite) and the tangential (TANG) or intrack component of the Doppler residuals resolved at TCA (Time of Closest Approach). As can be seen from the tabular data, the WGS 84 EGM produced Doppler residuals as low as and sometimes lower than the tuned models. The total RWS could have been reduced in all cases by using a more stringent editing criteria, improved tracking station coordinates, etc.

Orbit computations were also made using NOVA Doppler tracking data for two-day data spans comparable to those used to compute precise ephemerides for Doppler point positioning. In these computations, WGS 84 parameters (EGM, GM and station coordinates) were used for the WGS 84 reductions. The NSWC 9Z-2 reductions used the NWL 10E-1 EGM and GM, and NSWC 9Z-2 station coordinates. The data set was selected on the basis that any edited pass must produce large residuals in both the WGS 84 and NSWC 9Z-2 reductions. The results of these computations are given in Table 11. In all cases, the Doppler residuals are either equal to or smaller for WGS 84 than for the NSWC 9Z-2 geodetic system. As in the case of the longer data spans, extensive editing was not required since the objective of these tests was to evaluate the two systems in terms of the relative magnitude of the Doppler residuals rather than in an absolute sense.

3.5 WGS 84 EGM CORRELATION MATRIX

The correlation matrix for the WGS 84 EGM is summarized in Table 12. This table indicates those coefficient pairs ($\bar{C}_{n,m}$ and $\bar{S}_{n,m}$ for a given degree and order) which are correlated with at least one other coefficient pair at a level ≥ 0.5 and also ≥ 0.7. Comparison of Tables 9 and 12 indicates that those coefficient pairs that produce significant perturbations of the satellite orbits are also the ones with correlation coefficients ≥ 0.5. This relatively high correlation is as expected between the satellite sensitive coefficients because of the similarity of the period of the orbital perturbations of a given satellite orbit for coefficient pairs of the same order. This higher than desired correlation does not necessarily present a problem in terms of some EGM applications, particularly orbit determination, because the total

TABLE 12

CORRELATION MATRIX
WGS 84 EARTH GRAVITATIONAL MODEL
X=CORRELATION COEFFICIENTS ≥ 0.5
+=CORRELATION COEFFICIENTS ≥ 0.7

DEGREE	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
2																																										
3	+	+	X																																							
4	+	+	+	X																																						
5	+	+	+	X																																						
6	+	+	+	+	X																																					
7	+	+	+	X	X	X																																				
8	+	+	+	+	+		X																																			
9	+	+	+	+	X	X																																				
10	+	+	+	+	+	X	X																																			
11	+	+	+	+	X	X	X																																			
12	+	+	+	X	X	X	X	X																																		
13	+	+	+	+	X	X	X						+	+																												
14	+	+	+	X	X	X	X	X						X																												
15	+	X	+	X	X		X						+	X	+	+																										
16	+	+	X	X	X									X	X	X																										
17	+	X	X										+	+	+	+																										
18	+	X	X											X	X			X																								
19	+	X												+	X	+	+	X	X																							
20	X	X	X												+	X		X																								
21	+														X	+			X																							
22	X		X											X	+	X																										
23	X														X	X	X	X																								
24	X													X	+	X																										
25	X														X	X																										
26	X													X	+	X									X																	
27															X	X	X										+															
28	X													X	+										X		X	+														
29															X	X											+	X														
30															X												X	+	+													
31															X												X		+													
32																											X		+													
33																												X	+													
34																											X	X	+	+												
35	X																										X	X														
36																											X	X	X													
37																											X															
38																											X	X														
39																																										
40															X																											
41																																										

ORDER

or "lumped" effect is of primary importance rather than the singular effect of a particular coefficient pair. The test comparisons and results presented for both mean gravity anomaly and orbit computations indicate that the WGS 84 EGM is not significantly affected by these higher than desired correlations.

4.0 CONCLUSION

During the decade since the development of WGS 72, significant improvements have been made in the mean gravity anomaly data available for gravity modelling. In addition, satellite radar altimeter data for determining the oceanic geoid has also become available. Other improvements such as accurate Doppler surveys, improved Doppler tracking equipment and laser tracking technology represent significant advances that have been utilized in the development of WGS 84 and its associated EGM. The test results and comparisons presented here demonstrate that the WGS 84 EGM is a superior EGM for DoD applications. Orbit reduction tests show that the NNS 68 and NOVA orbits used for Doppler surveying can be improved by using the WGS 84 EGM. This improvement, although small, demonstrates that a properly developed general purpose EGM such as WGS 84 is as good or better than the "tuned" EGM previously used by DMA for NAVSAT orbit computations.

ACKNOWLEDGEMENT

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