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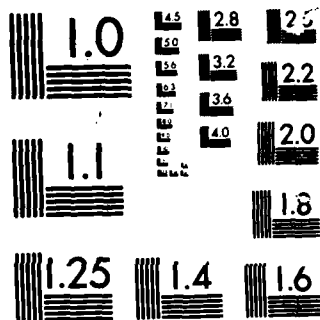   ARFSD-CR-86004 DAAG29-81-D-0100

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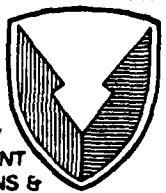
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FUNDAMENTAL RELATIVISTIC SOLUTION FOR THE RAILGUN

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A fully relativistic analysis is made of the dynamics of a railgun based on three assumptions: (1) Ohm's law is valid in the rest frame of the plasma, (2) total electron momentum is transferred to the projectile, and (3) motion of the projectile is constrained to one direction. With these assumptions, a relativistic equation for the velocity of the projectile is obtained whose solution monotonically increases to one of two values depending on field strengths. For $B > E$, the maximum velocity is cE/B whereas for $E > B$ it is c where c is the speed of light, and E and B are applied electric and magnetic fields, respectively.

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INTRODUCTION

Attention has recently been given to dynamical properties of electromagnetic propulsion. In the rail-gun device,[1-5] a non-conducting projectile is propelled by a current-carrying plasma driven by the Lorentz force. Plasma dynamics is more difficult due to ablation of the projectile and for the most part previous studies have attempted to incorporate this effect.

In the present study, we return to a more elementary configuration for the purpose of developing a fully relativistic study of this problem. Thus, for example, it is assumed that the rest mass of the projectile is constant, and that total electron momentum from the plasma is transferred to the projectile. Furthermore, it is assumed that the projectile is constrained to move in one direction. Our remaining assumption is that Ohm's law is valid in the rest frame of the projectile.[6]

With these assumptions at hand, a relativistic equation is constructed for the projectile velocity. Solution to this equation reveals two asymptotic velocities which depends on initial field strengths. Thus, for example, for the case $E > B$, the velocity is c, the speed of light, whereas for $B > E$, the velocity is cE/B. It is further demonstrated that for initial velocities less than respective asymptotic values, velocities monotonically approach their respective limiting values. For the case $B > E$, starting velocities greater than cE/B are found to decay to this asymptotic value.

ANALYSIS

Our starting equation is Ohm's law, which in the rest frame of the projectile (primed coordinates) is written

$$\underset{\sim}{J}' = \sigma \underset{\sim}{E}'$$ (1)

where σ is conductivity. Transforming back to the lab frame (see Figure 1) we find

$$\gamma(J_x - c\beta\rho) = \sigma E_x$$

$$J_y = \sigma\gamma(E_y - \beta B_z)$$ (2)

$$J_z = \sigma\gamma(E_z - \beta B_y)$$

where $\beta \equiv v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$. The charge density ρ, for a charge-neutral plasma, is equal to zero. In the lab frame we take

$$\underset{\sim}{E} = E\underset{\sim}{\hat{y}}$$ (3)

$$\underset{\sim}{B} = B\underset{\sim}{\hat{z}}$$

where hatted variables denote unit vectors. Inserting these values into (2) gives

$$\underset{\sim}{J} = \sigma\gamma(E - \beta B)\underset{\sim}{\hat{y}}$$ (4)

with $\underset{\sim}{\xi}$ denoting microscopic electron velocity we write

$$\underset{\sim}{J} = qn<\underset{\sim}{\xi}>$$ (5)

Electron charge and density are q and n respectively. We further recall that the density transforms as

$$n = \gamma n'$$ (6)

Combining the latter four equations gives

$$\langle \underset{\sim}{\xi} \rangle = \mu (E - \beta B) \hat{\underset{\sim}{\chi}}$$ (7)

where

$$\mu \equiv \frac{\sigma}{qn'}$$ (8)

represents mobility in the rest frame.

Taking the average of the Lorentz force on electrons we obtain

$$\frac{d}{dt} \langle p_y \rangle = \frac{q}{c} \langle \xi_y \rangle B$$ (9)

where we have recalled the vector property given by (7). We assume a total transfer of electron momentum to the projectile which gives

$$N \frac{d}{dt} \langle p_y \rangle \hat{\underset{\sim}{y}} = \frac{d}{dt} \underset{\sim}{P}$$ (10)

where N is total number of current-carrying electrons in the column. The momentum of the projectile, $\underset{\sim}{P}$, is given by

$$\underset{\sim}{P} = M \gamma \underset{\sim}{v}$$ (11)

where M is the mass of the projectile.

Combining (7), (9), (10), and (11) gives the desired equation of motion

$$\frac{d}{dt} \gamma M v = \frac{qI}{c} \left(1 - \frac{v}{w} \right) B$$ (12)

where a is the length of the conducting column, I is the current

$$aI = Nq\mu E \tag{13}$$

and

$$v \equiv \frac{cE}{B} \tag{14}$$

Note in particular that from (13) we may write

$$E = \frac{I}{A\sigma} \tag{15}$$

where aA represents the volume of the conducting column in the rest frame.

Integrating (12) gives

$$\eta - \eta_o = \frac{1}{c} \int_{v_o}^{v} \frac{du}{\left[1 - (\frac{u}{c})^2\right]^{3/2} (1 - \frac{u}{w})} \tag{15}$$

where η is the dimensionless time

$$\eta \equiv \frac{aIBt}{Mc^2} \tag{16}$$

and $v = v_o$ at $\eta = \eta_o$. From (15) we see that $\eta \to \infty$ at the singular points $u = c$ and $u = w$ which represent asymptotic velocities. It will be shown below that these asymptotic velocities are approached monotonically. With this property we may conclude that for zero starting velocities maximum values are given by

$$E > B \qquad v_{max} = c \tag{17}$$

$$E < B \qquad v_{max} = w$$

A sketch of these findings is shown in Figure 2.

To examine the monotonicity of v(t) we differentiate (12) to obtain

$$\frac{1}{c} \frac{dv}{d\eta} = \left[1 - (\frac{v}{c})^2\right]^{3/2} \left[1 - \frac{v}{w}\right] \tag{18}$$

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We conclude that for $v \leq c$ and $v \leq w$, $dv/d\eta \geq o$. Furthermore, with $v = o$ at $t = o$, (18) gives the starting acceleration (in dimensional form)

$$\frac{dv}{dt}\bigg|_o = \frac{aIB}{Mc} \tag{19}$$

Note that for the case $E < B$, an initial velocity $v_o > w$, decays to $v = w$ as is evident from (18). Furthermore, as is clear from (15), asymptotic values (17) are independent of initial velocities.

Characteristic times corresponding to the maximum velocities (17) are as follows. In the limit $w >> c$, (12) gives the characteristic time

$$\tau_1 = \frac{Mc^2}{aIB} \tag{20}$$

with maximum velocity

$$v_{max}^{(1)} = c \tag{20a}$$

In the limit $w << c$, (12) has the solution (with $v = o$ at $t = o$).

$$v = w\left[1 - \exp(-t/\tau_2)\right] \tag{21}$$

where

$$\tau_2 = \frac{E}{B}\tau_1 \tag{22}$$

and

$$v_{max}^{(2)} = w \tag{22a}$$

We note that although $\tau_2 << \tau_1$, accelerations

$$\frac{v_{max}^{(1)}}{\tau_1} = \frac{v_{max}^{(2)}}{\tau_2}$$

are the same.

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APPLICATION

In applying the preceding results to experimental values, first we rewrite w in practical units. Setting

$$E = \frac{V}{a} \tag{19}$$

where V is the potential across the rails, permits (14) to be rewritten

$$w = \frac{cV}{aB} \tag{20}$$

In practical units this expression becomes

$$w\left(\frac{km}{s}\right) = \frac{10^3 V(kV)}{B(kgauss)a(cm)} \tag{21}$$

Typical experimental values are[5,7]: V = 1kV, a ≃ 1cm, B ≃ 200 kgauss which gives w ≃ 5km/s. This value agrees in order-of-magnitude with observed maximum velocities.

CONCLUSIONS

We have examined the relativistic solution to the rail-gun configuration. Incorporating some simplifying assumptions we found that the projectile velocity goes monotonically to the minimum of the two velocities, c and cE/B. The asymptotic value c corresponds to E < B whereas the value cE/B corresponds to the limit B > E. It should be emphasized that this present study does not take into account thermodynamic effects such as momentum imparted to the projectile from the exploding "fuse."
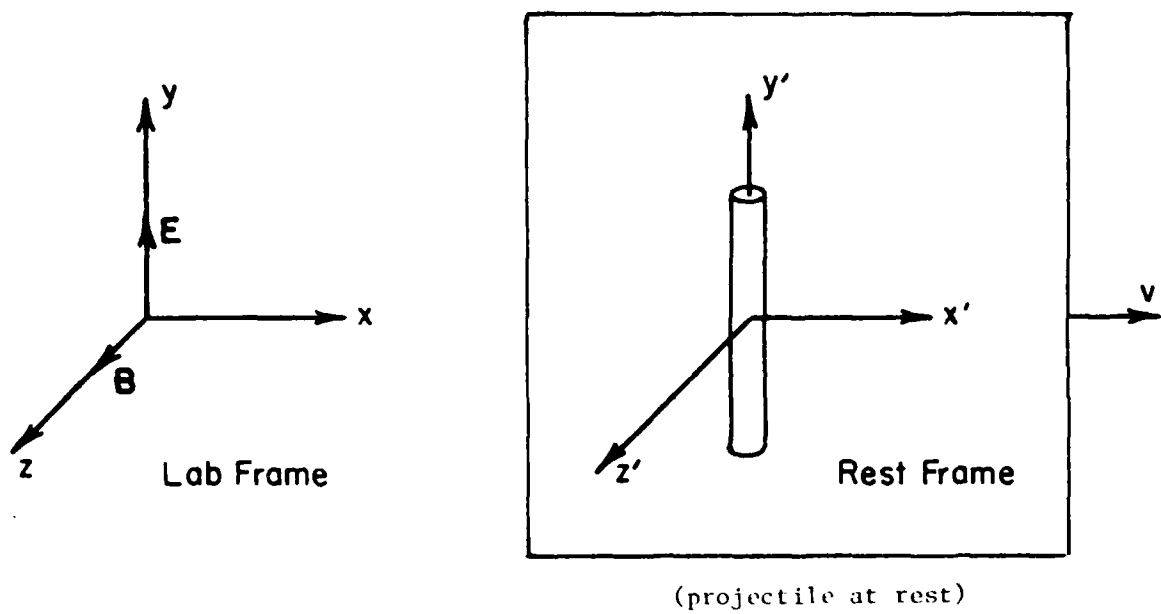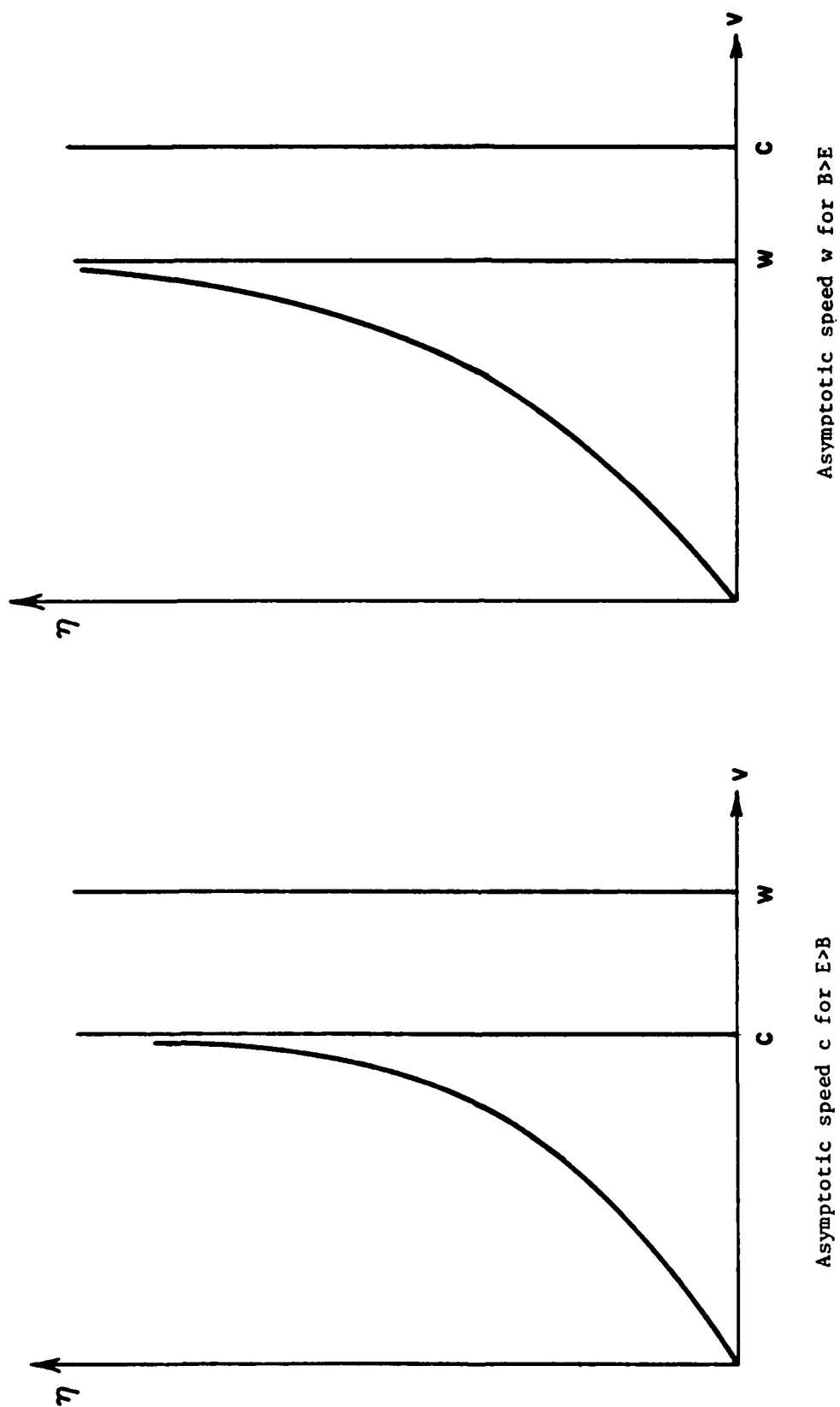
Lab Frame

Rest Frame

(projectile at rest)

Figure 1. Lab frame and rest frame

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Asymptotic speed w for B>E

Asymptotic speed c for E>B

Figure 2. Sketches of dimensionless time, η, as a function of projectile velocity v, with v = o at t = o

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REFERENCES

1. I. R. McNab, _Journal of Applied Physics_, vol 51, 1980, p 2549.

2. J. D. Powell and J. H. Batteh, _Journal of Applied Physics_, vol 52, 1981, p 2717.

3. J. D. Powell and J. H. Batteh, _Journal of Applied Physics_, vol 54, 1983, p 2242.

4. J. H. Batteh, _Journal of Applied Physics_, vol 56, 1984, p 3182.

5. Collection of papers on electromagnetic propulsion, _IEEE Trans. on Magnetics_, mag 20, March 1984.

6. J. D. Jackson, _Classical Electrodynamics_, 2nd ed., John Wiley, New York, 1975.

7. G. Ferrentino, personal communication.

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