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NRL Memorandum Report 5755

Interactions Between Ocean Surface Waves and Currents

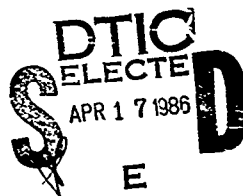
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<i>Attention:</i> A model is developed to describe the interaction between random ocean surface waves and the wake produced by a surface ship. Calculations are presented for the mean square free surface displacement energy for the short wave length components of the wave field and its modification due to refraction by the surface wake. It is demonstrated that a significant reduction of the short wavelength contribution to the surface energy can result from this interaction. A procedure to calculate the spectral composition of the free surface and the spatial realization of the free surface displacement is outlined. Free surface realizations will be presented in a forthcoming report. Keywords: ocean models,					
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INTERACTIONS BETWEEN OCEAN SURFACE WAVES AND CURRENTS

1. INTRODUCTION

A surface wave field may be significantly altered by interaction with currents. These currents can be induced by many mechanisms in the ocean. A potentially important wave-current interaction results from the interaction of ocean surface gravity waves with the wake produced by a moving object or vessel. Visual observations suggest that in the wake behind a surface ship there exists a "dead-water" region which is characterized by the relative absence of short wavelength waves. This region often extends to very large distances behind the body. Relative to the ambient sea, the wake region appears to be very smooth. This is further displayed by some of the synthetic aperture radar (SAR) images made of surface ship wakes.

In this report we will evaluate a potential model to predict the far-field development of this quiescent region. We will consider the possibility that it is the interaction of the ambient and Kelvin wave fields with the momentum wake of the vessel which is responsible for the generation of this region. In conjunction with the conceptual model of the total wake flow field previously developed by Skop [1], and by Cooper and Skop [2] we will assume that the leading order approximation to this interaction problem is obtained from the interaction of the turbulent momentum wake as superimposed on an otherwise quiescent background and the actual background wave field. The results of this phase of the study will provide some initial estimates of the effectiveness of this mechanism for the generation of the dead water region. Since our interest is in the short waves, we will further assume that the wave-field of interest is composed totally of short wavelength waves. The deep water dispersion relation may be utilized throughout and the vertical variations in wake velocity may be neglected. The surface velocity field produced by the wake may then be considered as a current field in its interaction with the surface waves.

Some of the earliest work on the problem of the interaction of regular, linear, surface waves and currents is that of Longuet-Higgins and Stewart [3,4]. This work as well as other work on regular wave interactions is summarized in the comprehensive review article by Peregrine [5]. Peregrine [6] also developed a model to estimate the overall effect of the wake velocity field of a ship upon the Kelvin waves generated at the ship's stern and was able to demonstrate that for a sufficiently strong "effective" wake, the envelope of the stern wave is significantly reduced from its ideal non-interactive value. More recently, Peregrine and Thomas [7] have considered the interaction for finite-amplitude symmetric waves. The finite-amplitude wave properties are computed using the method of Longuet-Higgins [8] for predicting the energy and wave action densities and fluxes. Studies of irregular or random wave field interactions were conducted by numerous workers [9, 10, 11, 12, 13]. The work of Tayfun et al. [10] and James et al. [11] are most relevant here as they form the basis for the work to be presented in this report.

II. ANALYSIS

A. Wake Velocity Field

Far downstream of the trailing edge of a moving object we expect the wake to possess only horizontal velocity components u and v . The vertical velocity component $w = 0$ and the variations of u and v will be predominantly in the transverse direction, y . Very slow variation in the downstream coordinate x is expected relative to the y variation. We therefore consider the fundamental problem a wake velocity field given by

$$\mathbf{V} = (u(y), v(y), 0)$$

where y is transverse to the direction of motion of the body, $-x$. If $\mathbf{V}$ is to be a legitimate incompressible wake velocity field, then $\nabla \cdot \mathbf{V} = 0$ from continuity, implies that $v(y) \equiv 0^*$ or $\mathbf{V} = (u(y), 0, 0)$. That is, far downstream of the trailing edge the wake is expected to be described by parabolic equations with a single velocity component $u(y)$ being predominant. However, if the body is not infinite in the vertical direction or, equivalently, if there is a finite velocity component in the vertical direction then a legitimate wake field is $\mathbf{V} = (u(y), v(y), w(y, z))$. Then, from continuity,

$$\frac{\partial v}{\partial y} = - \frac{\partial w}{\partial z}.$$

It is expected further, Garrett [14], that in the far wake, $v/u \ll 1$. Therefore, the simplest interaction wake field and one which appears to bear close connection to observation is a current profile $\mathbf{V} = u(y)$. The effects of a finite transverse velocity component, v , will also be addressed in some of this work.

B. Wave Kinematic Relations

In this section we will derive the kinematic relations which are satisfied by the individual Fourier modes of the wave field spectrum in their interaction with the surface current. Each mode is approximated as a nearly plane, slowly varying, small amplitude wave field. Subsequently, the results of this section will be utilized to develop the relevant equations to describe continuous random wave fields which are appropriate for real ocean waves

Consider a nearly plane, slowly varying wave field. This field may be approximately described by an amplitude function $A(x, t)$ and a phase function $S(x, t)$. These two functions satisfy

$$\phi = A(x, t) \exp i(S(x, t)).$$

In analogy with plane waves where $S \equiv (\mathbf{k} \cdot \mathbf{x} - \omega t)$, we draw the following definitions for wave number and frequency

$$\mathbf{k} = \nabla S \quad (1)$$

$$\text{and } \omega = - \frac{\partial S}{\partial t} \quad (2)$$

From Eq. (1) we have $\nabla \times \mathbf{k} = 0$ and by eliminating S from (1) and (2) we obtain

$$\frac{\partial \mathbf{k}}{\partial t} + \nabla \omega = 0. \quad (3)$$

*The scale of v is determined from the slow variation $\partial u / \partial x$ and is negligible to lowest order

This is the well known conservation of crests equation. To complete the description, a dispersion relation is required which also relates ω and $\mathbf{k}$. For traveling waves which propagate relative to a coordinate system which is fixed in the fluid we have

$$\omega = \sigma(\mathbf{k}). \quad (4)$$

Here σ denotes the intrinsic frequency. If the medium possesses a slowly varying current $\mathbf{U}(\mathbf{x}, t)$ then with respect to a stationary coordinate system we have

$$\omega = \sigma(\mathbf{k}) + \mathbf{k} \cdot \mathbf{U}(\mathbf{x}, t). \quad (5)$$

If we take the time derivative of Eq. (5) we obtain

$$\frac{\partial \omega}{\partial t} = \frac{\partial \sigma}{\partial \mathbf{k}} \cdot \frac{\partial \mathbf{k}}{\partial t} + \frac{\partial \mathbf{k}}{\partial t} \cdot \mathbf{U} + \mathbf{k} \cdot \frac{\partial \mathbf{U}}{\partial t}, \quad (6)$$

or from Eq. (3) we have

$$\frac{\partial \omega}{\partial t} + \left(\frac{\partial \sigma}{\partial \mathbf{k}} + \mathbf{U} \right) \cdot \nabla \omega = \mathbf{k} \cdot \frac{\partial \mathbf{U}}{\partial t} \quad (7)$$

where $\frac{\partial \sigma}{\partial \mathbf{k}}$ is recognized as the group velocity. That is,

$$\mathbf{c}_g \equiv \frac{\partial \sigma}{\partial \mathbf{k}}. \quad (8)$$

Equation (7) is expressed as

$$\frac{\partial \omega}{\partial t} + (\mathbf{c}_g + \mathbf{U}) \cdot \nabla \omega = \mathbf{k} \cdot \frac{\partial \mathbf{U}}{\partial t} \quad (9)$$

or

$$\frac{d}{dt} \omega = \mathbf{k} \cdot \frac{\partial \mathbf{U}}{\partial t} \quad (10)$$

where $\frac{d}{dt}$ represents a time derivative along rays. Equation (10) therefore indicates that the total frequency ω is conserved along rays for a steady current or when observed in a coordinate system where the current is steady. The wake field is steady when referred to a body-fixed coordinate system. The transformation to a body-fixed coordinate system is therefore advantageous since the total frequency ω of the waves will then be invariant during the interaction.

The ambient wave field also must be characterized with respect to the same body-fixed coordinate system. Consider an ambient traveling wave at a frequency ω_∞ and wave number $\mathbf{k}_\infty$ where

$$\omega_\infty = \sigma(\mathbf{k}_\infty) \quad (11)$$

If the body which is generating the wake is translating at a uniform velocity $\mathbf{U}_b$ then with respect to a body-fixed observer the frequency would be given as

$$\begin{aligned} \omega &= \omega_\infty - \mathbf{k}_\infty \cdot \mathbf{U}_b \\ \omega &= \sigma(\mathbf{k}_\infty) - \mathbf{k}_\infty \cdot \mathbf{U}_b \end{aligned} \quad (12)$$

and the wake velocity field $\mathbf{U} = -\mathbf{U}_b + \bar{\mathbf{u}}$ where $\bar{\mathbf{u}}$ is the wake field relative to a fixed observer. In Equation (12) we have used the fact that $\mathbf{k}_\infty$ is unchanged in the Galilean transformation. The frequency ω given by (12) is the total frequency which is invariant through the interaction. That is, by combining (12) and (5) for a steady wake field we obtain

$$\omega = \sigma(\mathbf{k}_\infty) - \mathbf{k}_\infty \cdot \mathbf{U}_b = \sigma(\mathbf{k}) + \mathbf{k} \cdot \mathbf{U}. \quad (13)$$

If we specialize to linear deep water waves then we may write

$$\sigma(k) = (kg)^{1/2} \quad (14)$$

and Eq. (13) becomes

$$(k_{\infty}g)^{1/2} - k_{\infty x} U_b = (kg)^{1/2} + k \cdot U = (kg)^{1/2} - k_x U_b + k \cdot \bar{u} \quad (15)$$

where we have taken U_b in the negative x direction.

By combining Eqs. (3) and (5) and eliminating ω we obtain the ray equation for the wave number k ,

$$\frac{\partial k_i}{\partial t} + (U_j + c_{ij}) \frac{\partial k_i}{\partial x_j} = -k_j \frac{\partial U_j}{\partial x_i} \quad (16)$$

It is informative to examine the two component equations of Eq. (16). In the x direction we have

$$\frac{\partial k_x}{\partial t} + (u + c_{xx}) \frac{\partial k_x}{\partial x} + (v + c_{xy}) \frac{\partial k_x}{\partial y} = -k_x \frac{\partial u}{\partial x} - k_y \frac{\partial v}{\partial x} \quad (16a)$$

and in the y direction

$$\frac{\partial k_y}{\partial t} + (u + c_{xy}) \frac{\partial k_y}{\partial x} + (v + c_{yy}) \frac{\partial k_y}{\partial y} = -k_x \frac{\partial u}{\partial y} - k_y \frac{\partial v}{\partial y} \quad (16b)$$

From Eq. (16a) we note that for $U = (u(y), v(y), w)$, the right side vanishes and we have $\frac{dk_x}{dt} = 0$ or k_x is conserved. The variation of k_y is described by Eq. (16b). The coordinate y is a horizontal direction transverse to the wake axis (x).

For $k_x = \text{constant}$, the terms involving U_b cancel from both sides of Eq. (15) and we find that Eq. (15) provides

$$(k_{\infty}g)^{1/2} = (kg)^{1/2} + k \cdot \bar{u} \quad (17)$$

This equation indicates that the wave refraction is governed by the viscous or turbulent velocity for the case where $k_x = \text{constant}$. The kinematics of the refraction process are therefore governed by

$$(k_{\infty}g)^{1/2} = (kg)^{1/2} + k_x \bar{u} + k_y \bar{v} \quad (18)$$

and

$$k_x \approx k_{\infty x} = \text{constant} \quad (19)$$

From Eq. (19) we have

$$\begin{aligned} k_x &= k_{\infty x} = k_{\infty} \sin \theta_{\infty} = k \sin \theta \\ k_y &= k \cos \theta = \frac{k_{\infty} \sin \theta_{\infty} \cos \theta}{\sin \theta} \end{aligned} \quad (20)$$

Equation (18) provides

$$(k_{\infty}g)^{1/2} = \left(\frac{k_{\infty} \sin \theta_{\infty}}{\sin \theta} k \right)^{1/2} + \bar{u} k_{\infty} \sin \theta_{\infty} - \bar{v} \frac{k_{\infty} \sin \theta_{\infty}}{\sin \theta} \cos \theta \quad (21)$$

where θ represents the angle between k and the y -axis and for $\bar{v} > 0$ we consider only $k_y < 0$ as indicated by the minus sign preceding the $\bar{v}$ terms. Therefore, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, $k_y < 0$ and Eq. (21) may be written as

$$1 = \left(\frac{\sin \theta_\infty}{\sin \theta} \right)^{1/2} + \sin \theta_\infty \left\{ u^* - v^* \frac{\cos \theta}{\sin \theta} \right\} \quad (22)$$

where

$$\begin{pmatrix} u^* \\ v^* \end{pmatrix} = \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} / (\sqrt{g/k_\infty}). \quad (23)$$

If we define

$$\xi \equiv \left(\frac{\sin \theta}{\sin \theta_\infty} \right)^{1/2}, \quad (24)$$

then Eq. (22) may be expressed as

$$\xi^2 (1 - \sin \theta_\infty u^*) = \xi - v^* \sqrt{1 - \xi^4 \sin \theta_\infty}. \quad (25)$$

We note that in the far field where u^* and v^* vanish, $\sin \theta \rightarrow \sin \theta_\infty$ or $\xi \rightarrow 1$. Also, in the limit where $v^* \rightarrow 0$ we recover Eq. (2.151) of Peregrine [5] which is consistent with $v = 0$.

Equation (25) is valid for refraction of the wave field by a general two component current ($u(y), v(y)$). In what follows, we will neglect v in comparison to u . In future work on this problem the effects of finite v variation may be addressed. The basic problem described by Eq. (25) with $v = 0$ is the interaction of a wave field with a parallel surface current shear $u(y)$.

The variation of wave amplitude due to the refraction may be determined by the conservation of wave action. For the steady case considered here this is expressed as

$$\nabla \cdot \left\{ (\bar{u} + c_x) \frac{E}{\sigma} \right\} = 0, \quad (26)$$

where $E \equiv \frac{1}{2} \rho g a^2$ represents the wave energy density for linear waves of amplitude a , and σ once again represents the intrinsic frequency relative to the fluid. For the parallel shear flow considered, $u(y)$, Eq. (26) implies that

$$\left\{ c_x \frac{E}{\sigma} \right\} = \text{constant} \quad (27)$$

or

$$\left\{ c_x \cos \theta \frac{E}{\sigma} \right\} = c_x \cos \theta \left\{ \frac{1}{2} \frac{\rho g a^2}{\sigma} \right\} = \text{constant}.$$

For linear deep water waves $\frac{c_x}{\sigma} = \frac{1}{2k}$, so that

$$\frac{c_x E}{\sigma} = \frac{\cos \theta}{4} \frac{\rho g a^2}{k} = \frac{\sin \theta \cos \theta \rho g a^2}{4k_x}$$

and therefore

$$\frac{a^2 \rho g \sin 2\theta}{8k_x} = \text{const ant.}$$

It can be seen that the wave amplitude becomes unbounded when $\sin 2\theta \rightarrow 0$ or $\theta \rightarrow \pm \frac{\pi}{2}$, 0. As indicated by Peregrine [5], $\theta = \pm \frac{\pi}{2}$ correspond to a caustic where the plane-wave assumption breaks down. For $\theta = 0$, wave steepness becomes unbounded.

C. Random Wave Field

The above results and discussion have been associated with the behavior of the individual Fourier components of a wave field. In practical problems, however, we are usually confronted by a continuous spectrum of random waves. The refraction of wave fields of this type has been the subject of a great deal of recent work [9, 10, 11, 12, 13]. These random fields are typically described by the inhomogeneous wave number spectral density $\psi(\mathbf{k}; \mathbf{x})$ which is defined by

$$dE/\rho g \equiv \psi(\mathbf{k}; \mathbf{x}) d^2\mathbf{k} = \psi(\mathbf{k}; \mathbf{x}) dk_1 dk_2 = \psi k dk d\theta \quad (28)$$

where dE represents the wave energy at wave number $\mathbf{k}$. The total energy associated with the local mean square value of the free surface displacement is obtained by integrating dE over all of $\mathbf{k}$ space. That is,

$$\bar{E} = \int_{k_1} \int_{k_2} \psi(\mathbf{k}, \mathbf{x}) dk_1 dk_2 = \langle \zeta^2(\mathbf{x}) \rangle. \quad (29)$$

where $\bar{E}$ represents the mean square surface elevation and is defined by $\bar{E} = E/\rho g$. But the energy can also be defined in terms of a frequency-direction spectral density $\Phi(\omega, \theta; \mathbf{x})$ as

$$\bar{E} = \int_0^\infty \int_0^{2\pi} \Phi(\omega, \theta; \mathbf{x}) \omega d\omega d\theta. \quad (30)$$

Therefore from Eqs. (29) and (30)

$$\psi(\mathbf{k}, \mathbf{x}) = \frac{\omega}{k} \frac{d\omega}{dk} \Phi(\omega, \theta; \mathbf{x}). \quad (31)$$

As previously demonstrated for steady wave-current interactions, ω is constant and therefore the frequency direction spectral density $\Phi(\omega, \theta; \mathbf{x})$ is the more convenient variable.

From Eq. (28) we can derive the consequences of conservation of wave action (for each wavelet). That is for

$$dE/\rho g = \psi(\mathbf{k}, \mathbf{x}) d^2\mathbf{k}$$

the wave action is written as $(\psi d^2\mathbf{k})/\sigma$ and therefore conservation of wave action demands that

$$\frac{\partial}{\partial t} \left(\frac{\psi}{\sigma} d^2\mathbf{k} \right) + \frac{\partial}{\partial x_i} \left((U_i + c_{xi}) \frac{\psi}{\sigma} d^2\mathbf{k} \right) = 0 \quad (32)$$

It can be shown, Le Blond and Mysak [5] and James et al. [11], that

$$\frac{\partial}{\partial t} (d^2\mathbf{k}) + \frac{\partial}{\partial x_i} ((U_i + c_{xi}) d^2\mathbf{k}) = 0. \quad (33)$$

Therefore, combining this result with Eq. (32) we obtain a conservation law for the spectral action density $\frac{\psi}{\sigma}$ which is given as

$$\frac{\partial}{\partial t} \left(\frac{\psi}{\sigma} \right) + (c_{xi} + U_i) \frac{\partial}{\partial x_i} \left(\frac{\psi}{\sigma} \right) = 0 \quad (34)$$

Equation (34) indicates that the spectral density of wave action ψ/σ is conserved along rays. The reader should note the difference between Eqs. (32) and (34). For steady interactions where ω is also conserved along rays it is much more convenient to study the spectral modifications of Φ , the frequency-direction spectral density. From Eq. (31) we find that

$$\frac{\psi}{\sigma} = \frac{\omega}{k} \frac{d\omega}{dk} \frac{\Phi}{\sigma} (\omega, \theta; \mathbf{x}). \quad (35)$$

Therefore for steady interactions where ω and $\frac{\psi}{\sigma}$ are conserved along rays, Eq. (35) implies that $\frac{1}{k} \frac{d\omega}{dk} \frac{\Phi}{\sigma}$ is conserved.

These properties were utilized by James et al. [11] in their studies of the refraction of a random wave field by a parallel surface shear current. They included the effects of wave breaking and wave reflection and generated the resulting mean square surface displacement given by Eqs. (29) or (30). Their computations were for some rather drastic and hence non-physical assumptions relating to the ambient frequency-direction spectral density. These assumptions simplified the computations. It would appear however, that it is possible to utilize more realistic ambient spectral densities within a similarly-based formulation of the problem. Furthermore, it is possible to extract specific information about the surface energy contributions from particular wavelength ranges in the interaction zone and thereby begin to address the question of the origins of the dead water region. Most importantly, it is possible to evaluate from the spectral density modifications, free surface spatial realizations corresponding to various random sea state models.

The techniques developed by James et al. [11] enable the calculation of the modification of the energy associated with the mean-square free surface elevation of the incoming wave field due to the refraction caused by a surface current. They assumed a simplified incoming spectral density distribution which was taken to be independent of frequency and direction. That is, they assumed

$$\Phi_{\infty}(\omega_{\infty}, \theta_{\infty}) = 1.$$

This approximates the incoming wave spectrum as one similar to that of white noise. In addition, all the kinematics of the refraction process were referred to the initial plane $(\omega_{\infty}, \theta_{\infty})$ where the incoming energy for a given wavelet is proportional to the polar area in that plane:

$$dE_{\infty} = \Phi_{\infty}(\omega_{\infty}, \theta_{\infty}) \omega_{\infty} d\omega_{\infty} d\theta_{\infty} = \omega_{\infty} d\omega_{\infty} d\theta_{\infty} \quad (36)$$

when $\Phi_{\infty} = 1$. From Eqs. (34) and (35) we can express the ratio of the local frequency-direction spectral density Φ to its original undisturbed value Φ_{∞} along a ray. That ratio is given by

$$A \equiv \frac{\Phi(\omega, \theta; \mathbf{x})}{\Phi_{\infty}(\omega_{\infty}, \theta_{\infty}; \mathbf{x}_{\infty})} = \frac{k}{k_{\infty}} \frac{\left(\frac{d\omega}{dk} \right)_{\infty}}{\left(\frac{d\omega}{dk} \right)} \frac{\sigma}{\sigma_{\infty}} \quad (37)$$

where $\mathbf{x}$ and $\mathbf{x}_{\infty}$ lie on the same wave ray. From Eq. (18), we have

$$\omega = \sigma(k) + k_l \bar{u}_l = \sigma_{\infty}. \quad (38)$$

For linear deep water waves, $\sigma(k) = (kg)^{1/2}$. Therefore, combining Eqs. (38) and (37) we obtain

$$A = \left(\frac{k}{k_{\infty}} \right)^2 \left[\frac{1 + \frac{k_{\infty} \bar{u}_l}{\omega}}{1 + \frac{k_l \bar{u}_l}{\omega}} \right] \left[\frac{1 - \frac{k_l \bar{u}_l}{\omega}}{1 - \frac{k_{\infty} \bar{u}_l}{\omega}} \right] \quad (39)$$

or, since $\bar{u}_\infty = 0$ by definition, we have

$$A \equiv \left(\frac{k}{k_\infty} \right)^2 \left[\frac{1 - \frac{k_i \bar{u}_i}{\omega}}{1 + \frac{k_i \bar{u}_i}{\omega}} \right] \quad (40)$$

Furthermore, for the case where the transverse component of $\bar{u}_i$ is negligible ($\bar{v} = 0$) we find that

$$A = \left[1 - \frac{k_{\infty x} \bar{u}}{\omega} \right]^2 / \left[1 + \frac{k_{\infty x} \bar{u}}{\omega} \right] \quad (41)$$

Following James et al. [11] we adopt the following normalization for k and ω based upon the maximum current, V_m

$$\omega = \frac{g}{V_m} \omega^*, \quad k = \frac{g}{V_m^2} k^*,$$

and we define a velocity scale Λ by

$$\Lambda \equiv \Lambda(y) = \frac{\bar{u}(y)}{V_m}.$$

As a consequence, we find from Eqs. (20) and (25) that

$$\frac{\sin \theta_\infty}{\sin \theta} = \frac{k^*}{k_{\infty}^*} = (1 - \Lambda \omega^* \sin \theta_\infty)^2. \quad (42)$$

Defining $\eta = \omega^* \sin \theta$ and $\eta_\infty = \omega^* \sin \theta_\infty$ we have

$$\frac{\eta}{\eta_\infty} = \frac{1}{(1 - \Lambda \eta_\infty)^2} \quad (43)$$

which expresses the local wave direction in terms of the initial direction. We can also obtain the inverse solution by solving the quadratic for η_∞ . This yields

$$\eta_\infty = \frac{1}{2\Lambda^2\eta} \left[1 + 2\Lambda\eta - \sqrt{1 + 4\Lambda\eta} \right] \quad (44)$$

where we have selected that root which provides $\eta \rightarrow \eta_\infty$ at $\Lambda \rightarrow 0$.

In terms of the above normalization, Eq. (41) for the frequency-direction spectral density ratio becomes

$$\frac{\Phi}{\Phi_\infty} \equiv A = \frac{(1 - \Lambda \eta_\infty)^5}{(1 + \Lambda \eta_\infty)}, \quad (45)$$

and if we substitute the inverse refraction Eq. (44) this ratio then becomes

$$A = \left[\frac{\sqrt{1 + 4\Lambda\eta} - 1}{2\Lambda\eta} \right]^4 \frac{1}{\sqrt{1 + 4\Lambda\eta}}. \quad (46)$$

Equations (45) and (46) are extremely useful for defining the Fourier decomposition of the surface elevation with respect to the initial and the physical frequency-direction planes respectively.

D. Fourier Spectrum

As discussed above, a random wave field may be characterized by means of the spectral density. To describe the refraction of these waves by a steady current, the frequency-direction spectral density is most convenient since the total frequency is conserved in such an interaction as demonstrated above. From Eq. (30) we have

$$d\bar{E} = \Phi(\omega, \theta; \mathbf{x}) \omega d\omega d\theta$$

where $d\bar{E}$ represents the mean square surface displacement energy contribution for the region between θ and $\theta + d\theta$ and ω and $\omega + d\omega$. The total energy is obtained by integration over all directions $-\pi \leq \theta \leq \pi$ and over all frequencies $0 \leq \omega < \infty$. That is, the mean-square free surface elevation is

$$\langle \zeta^2 \rangle = \int_{-\pi}^{\pi} \int_0^{\infty} \Phi(\omega, \theta; \mathbf{x}) \omega d\omega d\theta \quad (47a)$$

James et al. [11] choose to define the interaction in the initial frequency-direction space $(\omega_{\infty}, \theta_{\infty})$ and therefore compute $\langle \zeta^2 \rangle$ as

$$\langle \zeta^2 \rangle = \int_{-\pi}^{\pi} \int_{\omega_{\infty}}^{\infty} \Phi_{\infty} A \omega_{\infty} d\omega_{\infty} \left[\frac{\partial \theta}{\partial \theta_{\infty}} \right] d\theta_{\infty} \quad (47b)$$

employing Eq. (45) for A . In this work we will, at times, find it more convenient to utilize Eq. (47a) for $\langle \zeta^2 \rangle$ and to evaluate A by Eq. (46), thereby evaluating the energy integral in the physical plane (ω, θ) . Eq. (47b) will be utilized to calculate of the mean-square surface elevation energy while we will find it more convenient to utilize Eq. (47a) in physical space to evaluate the Fourier decomposition of the surface elevation to be discussed below.

It therefore follows that the contribution to surface elevation corresponding to frequencies of $\Delta\omega$ about ω_j , and directions $\Delta\theta$ about θ_j is given by [9, 17, 18, 20]

$$\zeta_{ij} = \sqrt{2} \Phi(\omega_j, \theta_j; \mathbf{x}) \omega_j \Delta\omega \Delta\theta \cdot \cos \left\{ k_{ij} (x \sin \theta_j + y \cos \theta_j) - \omega_j t + \phi_{ij} \right\} \quad (48a)$$

From Eqs. (43) and (44) we have $k = k_{\infty}/(k_{\infty}/k) = \frac{\omega^2}{g} \left[\frac{\sqrt{1+4\Lambda\eta} - 1}{2\Lambda\eta} \right]^2$. Therefore in Eq. (48) we have

$$k_{ij} = \frac{\omega_j^2}{g} \left[\frac{\sqrt{1+4\Lambda\eta} - 1}{2\Lambda\eta} \right]^2 \quad (48b)$$

where

$$\eta = \omega_j^* \sin \theta_j$$

and ϕ_{ij} represents a random phase shift uniformly distributed between 0 and 2π .

The above expressions indicate that the Fourier coefficients for the surface displacement can be determined from the local value of the frequency-direction spectral density $\Phi(\omega, \theta)$. The two expressions derived for A , Eqs. (45) and (46), representing the ratio of spectral density along a ray can be utilized to construct the local spectral density from its initial value due to the interaction with a current distribution. The surface energy can be obtained by means of integration in either initial space (45) or physical space (46). Hence, from Eq. (46) we have in physical space

$$\langle \zeta^2 \rangle = \int_{-\pi}^{\pi} \int_0^{\infty} \Phi_{\infty}(\omega_{\infty}, \theta_{\infty}; \mathbf{x}) \left[\left(\frac{\sqrt{1+4\Lambda\eta} - 1}{2\Lambda\eta} \right)^4 \frac{1}{\sqrt{1+4\Lambda\eta}} \right] \omega d\omega d\theta \quad (49)$$

or

$$\zeta_{ij} = \left[2\Phi_{\infty}(\omega_{\infty}, \theta_{\infty}(\omega, \theta), x_{\infty}) \left\{ \frac{\sqrt{1+4\lambda\eta}-1}{2\lambda\eta} \right\} \frac{1}{\sqrt{1+4\lambda\eta}} \right] \omega \Delta\omega \Delta\theta \left[\cos\{k_{ij}(x \sin\theta_i + y \cos\theta_i) - \omega_i t + \phi_{ij}\} \right]^{1/2} \quad (50)$$

where $\theta_{\infty}(\omega^*, \theta)$ is determined from Equation (44) as

$$\theta_{\infty} = \sin^{-1} \left\{ \frac{1}{2\omega^* \lambda^2 \eta} \left[1 + 2\lambda\eta - \sqrt{1+4\lambda\eta} \right] \right\}$$

and the full free surface realization or displacement at a given time

$$\zeta(x, y) = \sum_i \sum_j \zeta_{ij}(\omega_i^*, \theta_i, \lambda, y) \quad (51)$$

In Eq. (51), we have evaluated all ζ_{ij} at $t = 0$. Equations (50) and (51) form the basis of a numerical procedure in physical space (ω, θ) to determine the surface elevation

Various criteria can be utilized for the inclusion of wave breaking in the model. The one selected by James et al. [11] represents a maximum condition. They define the wave breaking boundary as the condition that the local component of the current in the direction of the wave motion is equal but opposite to the local wave group velocity. In dimensionless terms this is equivalent to

$$\eta_{\infty} = -\frac{1}{\lambda} \quad (52a)$$

or, in the physical plane

$$\eta = -\frac{1}{\lambda} \quad (52b)$$

Clearly, these conditions are conservative and they are independent of the precise value of the wave amplitude. The wave amplitude defined by these conditions is infinite. In a continuous refraction process, waves break well before these boundaries are reached. Therefore, this represents a conservative condition in terms of the unbroken wave energy. More energy is lost to wave breaking than is indicated by these conditions. It may be more realistic to establish the wave breaking boundary based on the local wave steepness [19]. This represents a criteria which depends both on the initial spectral distribution and the integrated growth of the particular spectral component. For simplicity in this paper we will adopt the conservative criterion established above. However, we recognize the limitations of the condition and will reconsider this at a future time.

The wave-breaking boundary is utilized to establish limits of integration for mean square surface elevation energy $\langle \zeta^2 \rangle$ or, equivalently, limits for the contributions to the Fourier coefficients for the surface elevation, ζ . Utilizing Equation (52b) as the definition of the wave breaking boundary (θ_b), we obtain

$$\sin \theta_c = -\frac{1}{4\lambda\omega^*} \quad (53)$$

This establishes the lower integration limit for θ . For waves with components in the direction of the current, $\theta > 0$, interaction with the current can produce reflected waves. Waves which are about to reflect are characterized in the physical plane at $\theta = \pi/2$. In the initial plane, these were characterized by James et al. as $\theta_{\infty} = \theta_{\text{min}}$. Therefore the θ limits for incoming waves in Eqs. (49), (50) and (51) are given as $\theta_{\text{min}} \leq \theta \leq \pi/2$. θ_{min} is the minimum of θ_b and $\sin^{-1}(-1/(1+\Lambda\omega^*)^2)$ obtained from equation (42) at $\theta_{\infty} = \frac{-\pi}{2}$. The outgoing waves can be shown to obey the same limits. These are waves originating on the other side of the wake and those which have been reflected from regions beyond the

point in question. These results may be utilized to generate free surface realizations within the interaction zone. Furthermore, it is possible to evaluate other surface properties which may be important in interpreting observations. For example, the surface slope distribution which is important in the remote sensing of ocean surfaces is expressible from a knowledge of the surface elevation Fourier decomposition.

Included here has been an outline of procedures to obtain the spectral density distribution within the interaction zone of a wave field and a current distribution and thereby to obtain the Fourier spectrum and a spatial realization of the surface elevation. This work is currently underway and further results will be available shortly both for the simplified, $\Phi_\infty = 1$, and for more comprehensive incoming spectra such as Pierson - Moskowitz or Jonswap [17]. More realistic Φ_∞ does not greatly complicate the procedure. The basic required relations have been derived in this report.

E. Short Wavelength Contribution to the Surface Energy

Short of solving for the complete Fourier spectral distribution throughout the interaction zone we can also characterize the contributions from various ranges of wavelengths to the surface energy. This necessitates establishing modified limits of integration for the energy integral previously developed. That is, we must find the locus of constant wavelength within the interaction zone in either the initial or physical plane. Equation (47a) or (47b) may then be used to calculate the surface energy contribution from the wavelength ranges of interest. We have performed this calculation in the initial plan utilizing Eq. (47b). In general, there are two types of wavelength contours in initial space: closed and open. The major differences in these cases arise in that portion of frequency-direction space where the waves are propagating against the current. The evaluation of the dimensional wavelength contour proceeds from Eqs. (42) and (43). These can be combined as

$$k = \frac{k_\infty}{\left[\frac{k_\infty}{k} \right]} = k_\infty (1 - \Lambda \eta_\infty)^2 \quad (54)$$

and therefore

$$k = \frac{\omega_\infty^2}{g} \left(1 - \Lambda \frac{V_m \omega_\infty}{g} \sin \theta_\infty \right)^2. \quad (55)$$

Solving for $\sin \theta_\infty$ we obtain

$$\sin \theta_\infty = \frac{g}{\Lambda V_m \omega_\infty} \left(\frac{\omega_\infty - \sqrt{kg}}{\omega_\infty} \right) \equiv \sin \theta_k. \quad (56)$$

This equation provides the value of θ_∞ which produces from a frequency ω_∞ a wave number of k at a position y where the local current is given by $V_m \Lambda(y)$, and it therefore defines the required locus in the initial plane.

Equation (56) can be used to establish the required limits of integration for the surface energy as discussed above. Of course, the limitations imposed by Eq. (56) have to be incorporated with respect to those limitations already associated with wave breaking, $\theta_{\infty b}$ and wave reflection, $\theta_{\infty r}$. In those cases where θ_k from Eq. (56) for the wave number boundary are intermediate of $\theta_{\infty b}$ and $\theta_{\infty r}$, for wave breaking and wave reflection, the integration is from θ_k to $\theta_{\infty r}$, to characterize contributions for those wavelength smaller than $2\pi/k$ only. This sort of wave length decomposition allows a partial extraction of the spectral density distribution. The full procedures established above for the numerical evaluation of the spectral density will provide accurate estimates of the Fourier coefficients of the free surface elevation. These modifications are now in progress and the results of this capability will be reported soon. The remainder of this report will concentrate on the partial spectral knowledge associated with

the k contours and $\Phi_\infty(\omega_\infty, \theta_\infty) = 1$. That is, we will determine the surface energy contributions for the small wavelength range. Subsequently, these computations will be repeated for more realistic incident spectral density distributions.

III. RESULTS

In this section we will discuss briefly some of the results that have been obtained from our studies of the refraction of a random wave field by a simple surface current shear $u(y)$. This is expected to be the dominant behavior of the wake velocity field of a surface vessel. These results have been selected to provide some initial estimates for the effectiveness of this wave-current interaction mechanism for the elimination of short wavelength components in the downstream region. Short wave components may have their origin in the incoming wave spectrum as short waves or they may be generated from longer waves by the refraction of the incoming wave field. It is useful to distinguish between and to quantify these two origins.

Figures 1 to 4 display the variation of mean-square surface elevation energy for wavelengths less than λ_L (greater than k_L) as a function of position within different strength parabolic wake structures. $\{u(y)/V_m = 1 - (y/y_{max})^2\}$. λ_1 and λ_2 represent wave length limits for the uniform spectral density of the incoming waves and λ_L represents the maximum wavelength (minimum wave number $k_L = 2\pi/\lambda_L$) which is included in the calculated wave field energy, E_λ . V_m presents the maximum of the current velocity profile. It is seen from those cases where $\lambda_L < \lambda_2 < \lambda_1$ or $k_L > k_2 > k_1$ that the short wavelength components may be created from longer wavelengths. Also from the other cases it is seen that the short wavelength components originally present are reduced by the refraction process.

Figure 1 shows a comparison for fixed values of λ_L , λ_1 and λ_2 of the variation of the root mean square surface energy for wavelengths less than $\lambda_L = 6.28$ cm produced from oncoming wavelengths between 628 cm and 12.56 cm with $\Phi_\infty = 1$ for wakes with various maximum velocities. For these cases, the short wavelength components must be generated by the refraction process. More energetic short wave components are generated for the wake with a maximum velocity of 40 cm/sec then for that with a maximum of 30 cm/sec. Both the 30 and 40 cm/sec maxima are not strong enough to produce any wave breaking according to the conservative wave breaking criteria given by Eqs. (52a) and (52b). The wakes with maximum velocity of 100 or 200 cm/sec do cause wave breaking. This is seen for the near centerline energy which is reduced in those cases while the production of short waves at the current structure edges is higher. Also included in the figure is the variation of the full energy over all wavelengths E_T for the four maximum currents cases. The strongest currents produce a larger reduction in surface mean square elevation energy. Note that in those cases where wave breaking occurs E_T/E_{INC} is less than unity at the wake boundaries, $y/y_{max} = \pm 1$.

The same energy variation with position in the current is shown in Fig. 2 for the cases where λ_L is intermediate between λ_1 and λ_2 and for two values of maximum current both of which do not induce wave breaking. The balancing effects of destruction and production of short wave length components due to refraction is apparent. Figure 3 shows these effects for a case where wave breaking is present. The parameters of Fig. 3 are $\lambda_L = 6.28$ cm, $\lambda_1 = 628$ cm and $\lambda_2 = 6.28$ cm. The maximum current strength is $V_m = 70$ cm/sec. The upper curve represents the variation of the full surface energy while the lower curve represents the variation of the surface energy for wavelengths less than 6.28 cm. For the majority of the structure, the short wave energy is approximately one-half the full energy.

In Fig 4 we display the same variations for a maximum current strength of $V_m = 20$ cm/sec with $\lambda_1 = 628$ cm and $\lambda_2 = \lambda_L = 6.28$ cm. No breaking occurs for this case. The short wave length energy does not have a depression near the maximum velocity at the centerline of the current, $y = 0$.

IV. SUMMARY

A model has been outlined in this report for characterizing the modification of an incoming random ocean wave field by a surface wake. At the most fundamental level, we can compute, for a white noise-like incoming spectrum, the mean square surface elevation energy and its variation throughout the wake structure. On the next level of complication, we can subdivide the results into various wavelength regions and begin to describe some of the characteristics of the spectral changes. Then, more realistic incoming spectral distributions such as Pierson-Moskowitz, Jonswap, etc. can be incorporated into the model. Lastly and most importantly we will be able to determine for each of the cases above the Fourier description of the surface elevation field.

From the representative cases which have been considered here, it is suggested that a significant reduction in the short wavelength components of the incident wave field is possible from the interaction between ocean surface waves and currents. This is particularly true for the waves with components in the direction of the current. Those wave components oppositely directed to the current are reduced in wavelength by the interaction. However, they are refracted so that they are turned away from the current direction. This will show more clearly when we are able to determine the relative amplitudes of the components of the frequency-direction spectral density. The relative absence of components with angles close to $\theta \approx -\frac{\pi}{2}$ (opposite to current) may be important in analyzing the response of a particular remote sensing system. This directional distribution may be more important than the relative population of short wave lengths.

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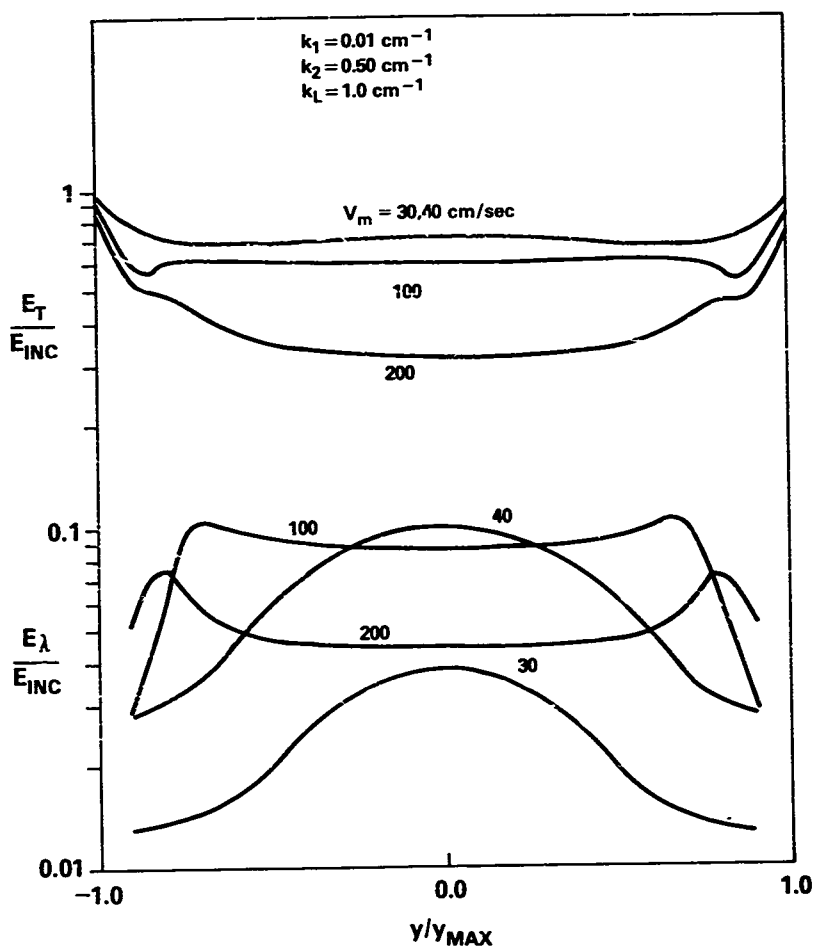


Fig. 1 — Variation of normalized mean square surface displacement energy as a function of position in a parabolic wake structure for different strength wakes, V_m . E_λ represents the surface energy for wavelengths less than λ_L . E_T represents the surface energy for all wavelengths considered and E_{INC} represents the total surface energy for the incoming spectrum. Incoming spectrum — $\Phi_\infty = 1$ for $12.56 \text{ cm} \leq \lambda \leq 628 \text{ cm}$.

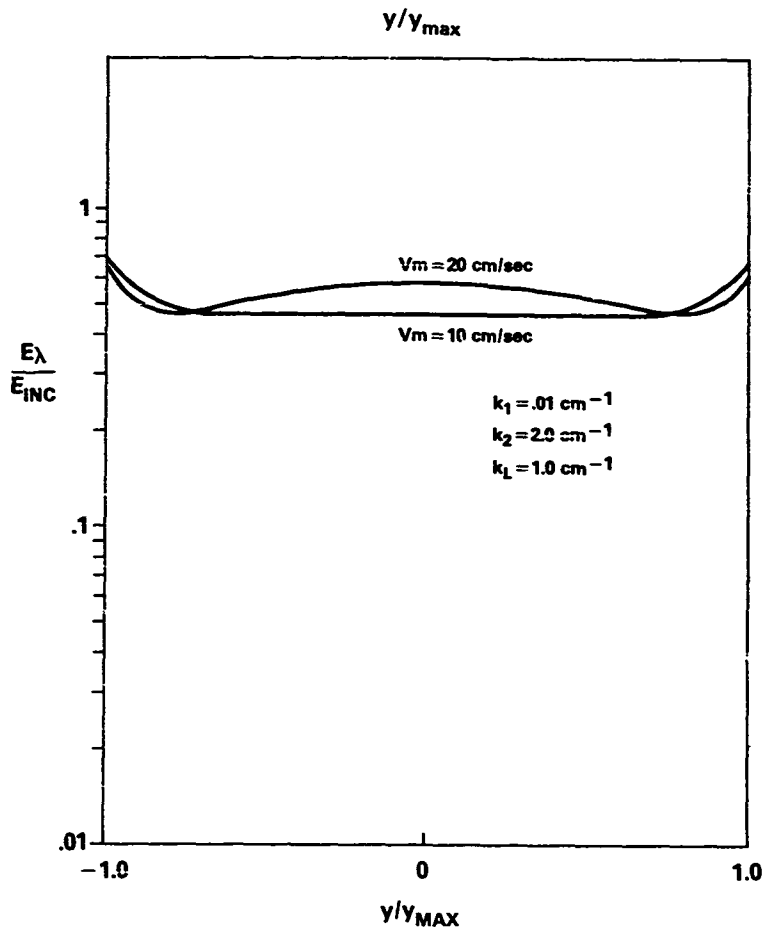


Fig. 2 — Variation of normalized mean square surface displacement energy for wavelengths less than 6.28 cm as a function of position in the parabolic wake structure. Incoming spectrum — $\Phi_\infty = 1$ for $3.14 \text{ cm} \leq \lambda \leq 628 \text{ cm}$.

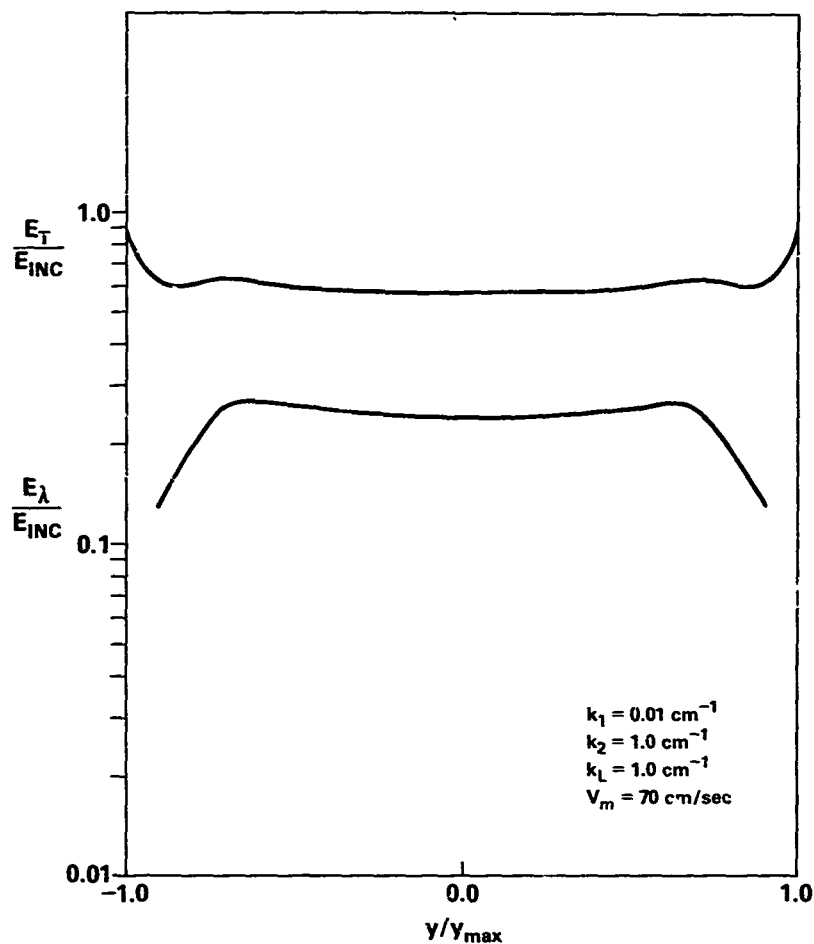


Fig. 3 — Variation of normalized mean square surface displacement energy as a function of position in the parabolic wake structure. E_λ represents the energy for wavelengths less than 6.28 cm and E_T represents surface energy for all wavelengths considered. Incoming spectrum — $\Phi_\infty = 1$ for $6.28 \text{ cm} \leq \lambda \leq 628 \text{ cm}$.

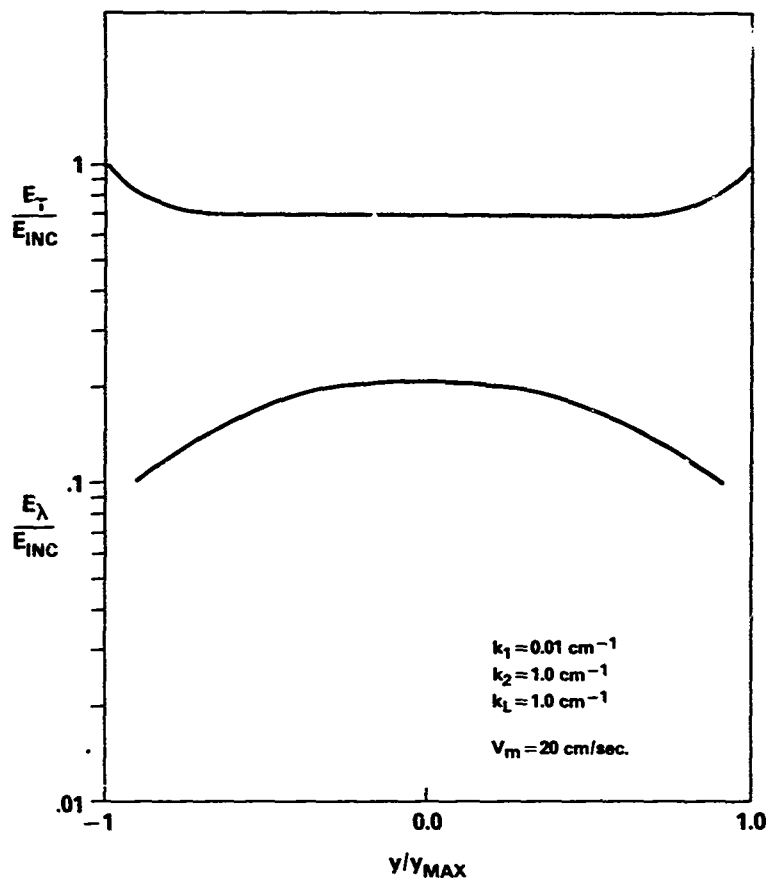


Fig. 4 — Variation of normalized mean square surface displacement energy as a function of position in the parabolic wake structure for wavelengths less than 6.28 cm. Incoming spectrum $\sim \Phi_\omega = 1$ for $6.28 \text{ cm} \leq \lambda \leq 628 \text{ cm}$.