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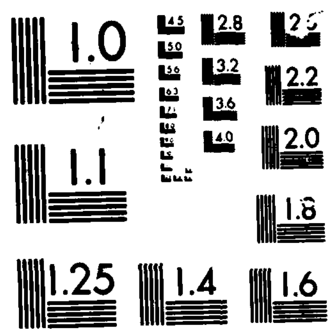

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AERIAL ASTROGEODETIC - GRADIOMETRIC VERTICAL DEFLECTION DETERMINATION

by

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ABSTRACT

Following introduction, the paper addresses vertical deflection estimation methods developed by Jordan and by White and Goldstein, presents the development of two independent linear least squares estimation methods with one having optimal characteristics as to accuracy, economy, and versatility, and arrives at significant conclusions. → p 10

1. INTRODUCTION

Bell Aerospace completed development of the ball-bearing rotating accelerometer gravity gradiometer instrument (GGI) in 1981. During that year, Bell also concluded investigations concerning gimbal and survey vehicle self gradients. GGI tests on board a ship have been encouraging and have yielded data for the establishment of operational self-noise power spectral densities. Essentialy during the period 1983-84, Bell accomplished some GGI modifications for airborne and terrestrial applications. A variety of corresponding tests will be conducted during 1985 and 1986. Self-noise power spectral densities for the GGI mounted at the umbrella angle (spin axis 35^{0} from horizontal) have been approximated in the open literature by $\phi_s(f) = = Af^{-2} + B$ where f denotes frequency in cycles per second, A is a constant relating to low frequency or red noise, and B is a constant indicating high frequency or white noise. White (1980) listed $A = 2.10^{-6}\ E^{2}\cdot Hz$ and $16.10^{-6}\ E^{2}\cdot Hz$ and $B = 81\ E^{2}\cdot Hz^{-1}$ and $86\ E^{2}\ Hz^{-1}$ for configurations with vertical and horizontal spin axis, respectively where E is Eötvös unit = $10^{-9}\ sec^{-2}$, and H is Hertz = $1\ sec^{-1}$. As to environmental sensitivities, linear acceleration influences are the most severe, particularly at multiples of the nominal rotation frequency $\Omega = 0.25$ Hz. The low-frequency error may be essentially interpreted as random drift. Non-spectral data processing methods require a different analytical representation of along track and cross track self-noise power spectra in order to obtain covariance functions by the Fourier cosine transform. Essentially different moving GGI information becomes available every two seconds. Gradient filters are nominally fourth order Butterworth filters to attenuate 1Ω, 3Ω and other noise manifestations. Signal information is obtained as a ten second moving window average. Elimination of systematic drifts and biases has to be accomplished at the start of a survey mission. In this respect, initial gravity tensor information would be valuable. Other preprocessing includes transformation of data from umbrella to local level frame, residual compensations, prefiltering, removal of spikes where indicated, and subtraction of normal gravity gradients. Provided that sufficient regional gravity anomaly and vertical deflection data are available, regional means may be removed and variances and a distance correlation parameter may be adjusted at the outset. Regionally improved covariance functions should, however, only apply to a gradiometer data generation domain sufficently embedded in the region. This restriction applies also if gravity anomalies and vertical deflections are estimated by means of standard covariance functions and first estimates of components of the disturbance gravity vector are utilized for covariance parameter improvements. The restriction of vertical deflection or gravity anomaly estimation to a domain somewhat smaller than the data coverage area is further based on estimation analysis.

Following the above preliminaries, this paper addresses successively in the following three sections Jordan's spectral estimation method and significant conclusions, the two stage estimation method developed by White and Goldstein which includes variable template averages during the first stage, limitations associated with the use of $\partial^2T/\partial z^2$-measurements, and two independent linear least squares estimation methods. The second has optimal characteristics as to accuracy, economy, and versatility both to vertical deflection and gravity anomaly determination. As a result of this research, several important conclusions can be drawn.

2. JORDAN'S SPECTRAL ESTIMATION METHOD AND GENERAL FINDINGS.

Jordan's (1982) spectral estimation method is confined to gravity anomaly and vertical deflection error estimation. Because of the assumption of steady state and the associated availability of measured gradients in an infinite plane, first derivatives of the disturbing potential T and not differences thereof may be estimated by measured second derivatives of T all of which contain a noise component. Under the assumption that all gravity-related quantities are stationary random variables it is, for example, with the functional $g = \partial T/\partial x$ and the along track measurement $x = \partial^2T/\partial x^2 + n = s + n$,

$$H\,(j\omega) = S_{gx}\,S_{xx}^{-1} = S_{gs}\,S_{xx}^{-1} \tag{1}$$

In eq. (1), H is the systems function, Sgs is the signal cross-spectrum, and $S_{xx} = S_{gs} + S_{nn}$ is the message or signal plus noise spectrum. The power spectral density of the estimation error is

$$S_{ee}(j\omega) = S_{gg} - H\,S_{gx} = S_{gg} - HS_{gs} \tag{2}$$

The variance of the estimation error is computed by integrating under S_{ee}, using ω as an independent variable. In this respect, Jordan excluded wavelength exceeding 500 Km. In the case of two independent gravity gradients, $H_1\,(j\omega)$ and $H_2\,(j\omega)$ would have to be determined from

$$S_{gx_1}\,(j\omega) = S_{x_1x_1}\,(\omega)\,H_1\,(j\omega) + S_{x_1x_2}H_2\,(j\omega) \tag{3}$$

$$S_{gx_2}\,(j\omega) = S_{x_1x_2}\,(\omega)\,H_1\,(j\omega) + S_{x_2x_2}\,H_2\,(j\omega) \tag{4}$$

In order to solve eqs. (3) and (4) for H_1 and H_2, noise cross-spectra would be required as well; however, this tends to be neglected for simplicity. Jordan established the signal and cross-signal spectra from a three-dimensional algebraic gravity model (STAG). The STAG model was fitted to a third-order Markov model and covariance functions involving T and first and second derivatives of T were established by Jordan, Moonan and Weiss (1981). Because the STAG model permits the use in three dimensions, Jordan estimated errors at ground level. In accordance with the above analysis, a matrix of dimension 5 N X 5 N has to be inverted if five independent gradient measurements and N parallel tracks are employed for error estimation. Jordan used $A = 17.10^{-5}$ E^2 Hz and B values from 30 to 630 E^2 Hz^{-1} for the computation of standard errors under inclusion of analytical downward continuation. Table 1 shows single track gravity component standard errors for a track length of 500 Km, an aircraft speed of 300 Km hr^{-1}, an aircraft altitude of 600 m, and a self-noise characterized by $B = 160$ E^2 Hz^{-1}.

	Standard Errors (mgal)
along-track	1.9
cross-track	2.2
vertical	2.0

TABLE 1. SINGLE TRACK STANDARD ERRORS

The relatively high single track standard error of about 2 mgal or 0.4 arc sec is due to the wavelength restriction of 500 Km. For this reason, gradiometer measurements generated along parallel and cross-tracks, i.e., a greater sample size, is required to achieve smaller standard errors. To assure strong signal correlations, the track spacing should be small and economical. Utilization of cross-track data for the same number of parallel and cross-tracks results in an error reduction by about $(\sqrt{2})^{-1}$. Table 2 shows vertical deflection and gravity anomaly standard errors due to the utilization of five north-south tracks and five east-west tracks and a track spacing of 5 Km under otherwise unchanged conditions.

	Standard Errors
north vertical deflection	0.12 arcsec
east vertical deflection	0.12 arcsec
gravity anomaly	0.63 m gal

TABLE 2. MULTIPLE TRACK STANDARD ERRORS
(5 PARALLEL TRACKS, 5 PARALLEL CROSS-TRACKS)

Smaller variations in self-noise (B), aircraft speed, aircraft altitude, and track spacing from the parameters used do not significantly affect the standard errors shown in Table 2. It must, however, be emphasized that these standard errors are not uniformly valid in the survey area and increase towards the boundary. The necessity of using at least two vertical deflections and one gravity anomaly in a limited domain has also an impact on nonuniform error estimation.

3. ESTIMATION METHOD OF WHITE AND GOLDSTEIN

A square survey area containing 61 flight paths parallel to one side with a path length of 300 Km and a spacing of 5 Km and 61 orthogonal flight paths with equal length and spacing provides for a total track length of 36,600 Km. If five independent preprocessed gradient measurements are available at 1 Km intervals, the measurement vector would comprise 183,000 numbers, requiring the inversion of a prohibitively large positive definite matrix. For this reason, White and Goldstein (1984) proposed the utilization of aggregated gradients obtained by averaging measurements along peripheries of squares centered above the point of estimation, evident from Figure 1.

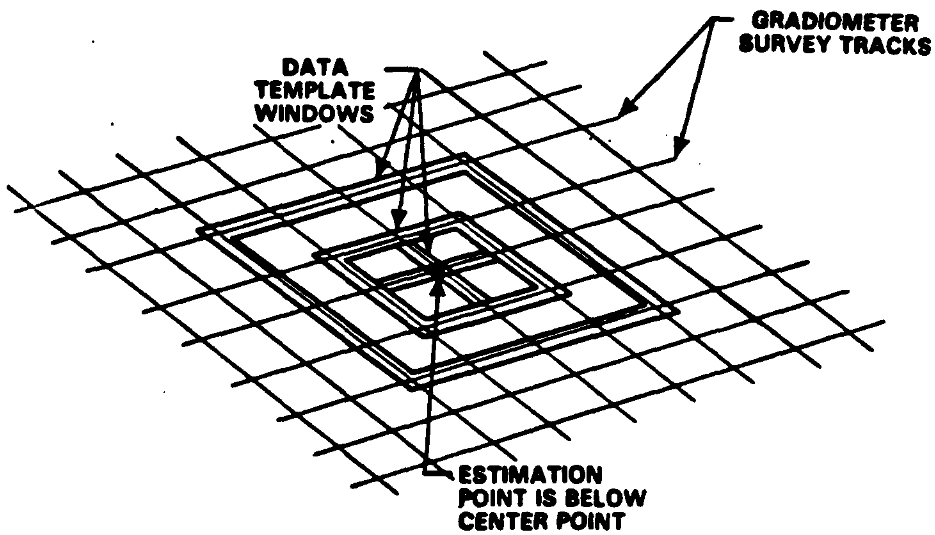

FIGURE 1. DATA TEMPLATE WINDOWS FOR DATE AVERAGING

As an illustration, the authors define a truth vector under restriction to T_{zz} - gravity gradiometer measurements and 12 template zones as:

$$\underline{X} = \begin{bmatrix} \delta\,(\underline{r}) \\ \bar{\underline{\delta}}_m(\bar{\underline{r}}) \\ T_{zz}^{(1)} \\ \vdots \\ T_{zz}^{(12)} \end{bmatrix} \qquad (5)$$

In eq. (5), $\underline{r}$ designates position vector, $\underline{\delta}$ is the gravity disturbance vector consisting of three components, and $\underline{\delta}_m$ is the local mean disturbance vector.

The residual gravity disturbance vector may then be estimated in the form

$$\underline{d}(\underline{r}) = \underline{\delta}(\underline{r}) - \underline{\delta}_m(\underline{r}) = B\,\underline{x} \tag{6}$$

where the 3 x n output matrix B selects those linear combinations of the elements of $\underline{x}$ that are being estimated.

A solution of eq. (6) can be achieved by recursive Kalman estimation algorithm under limitations associated therewith. In this respect, the authors entertain the possibility of reducing red noise covariances by utilization of gradiometer measurements at flight traverse crossing points without an analysis to achieve such reduction. The quadratic template method requires a modification in the neighborhood of the survey boundary. This restriction applies to some extent to any estimation method.

Although eq. (5), augmented by additional averaged gradiometer measurements, represents a convenient truth vector, the aggregate gradient estimators are neither best, complete, or suboptiomal estimators. As discussed in the following section, highly accurate, economic, and versatile determinations of $\underline{d}(\underline{r})$ under consideration of measurements of $\partial^2T/\partial x^2$, $\partial^2T/\partial x\partial y$, $\partial^2T/\partial y^2$, $\partial^2T/\partial x\partial z$, and $\partial^2T/\partial x\partial y$ require the utilization of at least one measured gravity disturbance vector.

4. DEVELOPMENT OF TWO INDEPENDENT TWO-STAGE LINEAR LEAST-SQUARES ESTIMATION METHODS

The utilization of $\partial^2T/\partial z^2$ - measurements without knowledge of the components of the disturbance vector on the boundary does not contribute much to gravity vector component determination with desired rms errors of about 0.2 arcsec. As shown by Baussus von Luetzow (1977), vertical deflections ξ and η can be found as the solution of a Poisson equation in planar coordinates from measurements in the interior of a region, preferably a rectangle or a circle, and boundary deflections. In the case of a quadrangle with a side of 300 Km, the omission of boundary values leads to a computation error of approximately 1.3 arcsec for the center vertical deflection component. For an extended side of 1,000 Km, the error would amount to about 0.7 arcsec. The errors tend to increase towards the boundary. In the context of a parallel track estimation scheme, the emphasis, therefore, has to be on the determination of $\partial T/\partial x$ from $\partial^2T/\partial x^2$ - and $\partial^2T/\partial x\partial y$ measurements and on a similar determination of $\partial T/\partial y$ and $\partial T/\partial z$. This requires the use of at least one set of measured first order derivatives of T. In this respect, it should be emphasized that the use of a limited number of astrogeodetic vertical deflections with rms errors of 0.10 arcsec and of gravity anomalies is not uneconomic and is expected to improve estimations in regions close to the survey boundary. A utilization of more than the above gradiometer measurements for the estimation of only one first order derivative of T does not significantly reduce the estimation error and is uneconomic. It may, however, be possible to consider $\partial^2T/\partial z^2$-measurements for an independent computation of ξ and η after their initial estimation on the boundary. A statistical estimation of ξ and η would only slightly reduce the above errors $\partial^2T/\partial x^2$ - and $\partial^2T/\partial x\partial y$ measurements along both "horizontal" and "vertical" traverses are to be used for estimation; and data relating to about seven tracks would have to be employed to reduce the estimation error, as indicated in Figure 2.

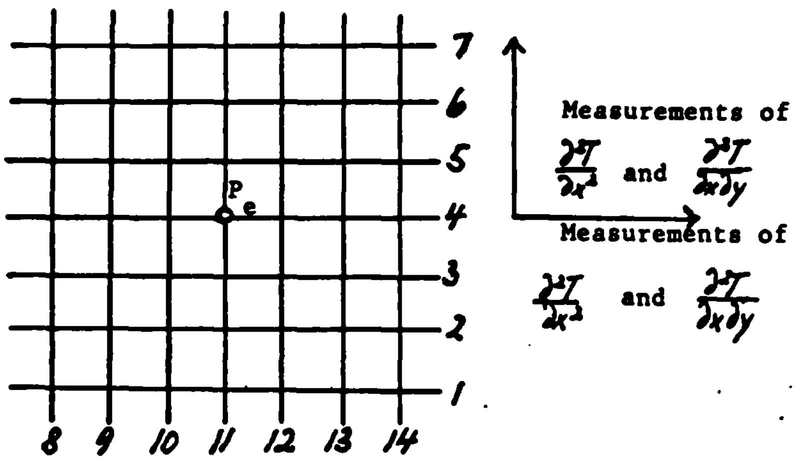

FIGURE 2. TRACK SYSTEM FOR ESTIMATION OF $\frac{\partial T}{\partial x}$ AT P_e FROM $\frac{\partial^2 T}{\partial x^2}$ - and $\frac{\partial^2 T}{\partial x \partial y}$ - MEASUREMENTS

In a two-stage optimization, with gradiometer measurements suitably arranged over a grid length of 5 Km to avoid formidable matrix inversions, the 14 measurement matrices may be written in the compact form

$$\underline{X}_i = \begin{bmatrix} \frac{\partial^2 \hat{T}_{i_1 k}}{\partial x^2} \\ \frac{\partial^2 \hat{T}_{i_1 k}}{\partial x \partial y} \\ \frac{\partial^2 \hat{T}_{i_2 k}}{\partial x^2} \\ \frac{\partial^2 \hat{T}_{i_2 k}}{\partial x \partial y} \end{bmatrix} \tag{7}$$

with i_1 = 1, 2, ..., 7; i_2 = 8, 9, ..., 14; k = 1, 2, ..., 60 for a survey guadrangle with a side of 300 Km. The symbol ^ indicates averaged gradient.

Fourteen first-stage solutions may be obtained from covariance equations of the structure

$$\overline{\frac{\partial Te}{\partial x}\underline{X}_i^{(1)}} = \Delta_i = A_i \overline{\underline{X}_i^{(1)}\underline{X}_i^{(1)T}} \tag{8}$$

where $\underline{X}_i^{(1)}$ refers to the first 2 matrix elements of eq. (7), A_i is the matrix of 120 regression coefficients for a specific i, the bar symbolizes the covariance operator, and T indicates the transpose operator.

The second-stage solution can then be found from the regression equation

$$\frac{\partial Te}{\partial x} = b_1\Delta_1 + b_2\Delta_2 + \ldots + b_{14}\Delta_{14} = B\underline{\Delta}i \tag{9}$$

Because of a track length of 300 km instead of the track length of 500 km used by Jordan, the standard errors listed in Table 2 will be exceeded. For some points, the estimation might be performed under utilization of quasi-diagonal parallel lines longer than measurement tracks. This involves, however, a re-arrangement of suitably averaged gradiometer measurements. The use of an ergodic estimation procedure including the use of covariance functions for long distances further contributes to an rms error increase to 0.20-0.25 arcsec. The corresponding error in the boundary region with width 15 Km will be somewhat greater.

In view of the above and due to the fact that initial gravity anomaly errors are small, the following alternative estimation method is presented which would simultaneously offer estimates essentially independent of the Wiener-Kolmogorov optimization characterized by eqs. (8) and (9).

The alternative two-stage estimation method presupposes the availability of highly accurate vertical deflection components with rms errors of 0.1 arcsec and of a $\partial T/\partial_z$ -component with an rms error of 0.1 mgal at the center of the survey square with tracks 300 Km long. A straight line is drawn between the center or initial point P_0 and the estimation point P_e, for convenience to coincide with a track intersection point. As evident from Figure 3, the connecting line intersects with at most 30 grid squares.

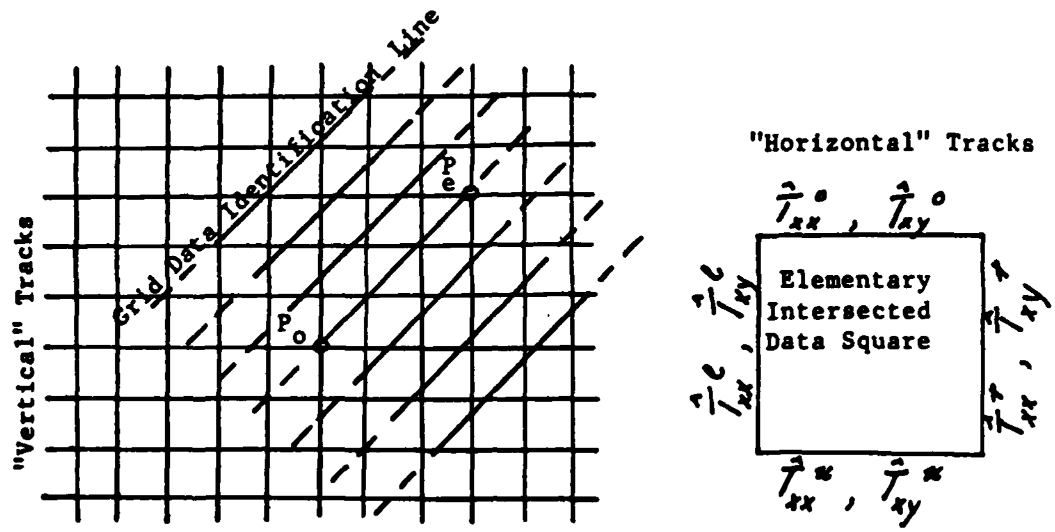

FIGURE 3. INTERSECTED SQUARES CONTAINING
8 AVERAGE MEASUREMENTS PER SQUARE.

The measurement vector for a square identified by the superscript i may be written as

$$\underline{X}_k = \begin{bmatrix} \hat{T}_{xx}^{o} \\ \hat{T}_{xy}^{o} \\ \hat{T}_{xx}^{u} \\ \hat{T}_{xy}^{u} \\ \hat{T}_{xx}^{r} \\ \hat{T}_{xy}^{r} \\ \hat{T}_{xx}^{l} \\ \hat{T}_{xy}^{l} \end{bmatrix}_k = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \end{bmatrix}_k \qquad (10)$$

In eq. (10), the symbol ^ indicates an average measurement over a square grid length, and o,u,l, and r upper, lower, left, and right sides of a square grid, respectively.

There are i measurement vectors relating to the line $\overline{P_o P_e}$, and there are 71 measurement vectors if six parallel lines with a separation of 5 Km are drawn, also evident from Figure 3. Except for the boundary region, this is possible.

As to first-stage estimation, the regression equation

$$(T_{x_e} - T_{x_o})_i = \Delta_i = \sum_{i=1,\,k=1}^{i=n,\,k=8} a_{ik}\, m_{ik} \qquad (11)$$

holds with maximally 240 coefficients to be determined. For greater accuracy, $\overline{P_o P_e}$ may be extended to include 6 or more additional measurement vectors.

Equation (11) is applied with respect to the 6 lines parallel to $\overline{P_o P_e}$, so that altogether the solutions $(T_{x_e} - T_{x_o})\,\mu = \Delta\mu$ are obtained, where $\mu = 1, 2, \cdots 7$ except for the survey boundary region.

The second-stage estimation yields the final estimate

$$(T_{x_e} - T_{x_o}) = b_1\Delta_1 + b_2\Delta_2 + \ldots. + b_7\Delta_7 \qquad (12)$$

where the regression coefficients $b\mu$ are computed in the usual manner. The estimation by means of eqs. (11) and (12) is optimal as to accuracy and efficiency, and the error associated with the second-stage estimation of T_{x_e} is expected not to exceed the rms error of 0.18 arcsec with the exception of the boundary region. Further, this estimation is relatively less dependent on long distance covariances.

To meet a requirement of 0.18 arcsec rms for the boundary region, highly accurate disturbance vector components would be required at the survey area corners, or the survey area would have to be enlarged to 315 X 315 Km^2.

The second two-stage estimation method developed would also facilitate the computation of survey area mean values $\partial T/\partial x$, $\partial T/\partial y$, $\partial T/\partial_z$ and the improvement of a priori covariance functions. Gravity anomaly coverage in 9 adjacent survey areas and utilization of a spherical harmonics representation of Δg, resulting in a modified Stokes' function, would allow an independent determination of ξ_o and η_o in the central survey area.

As already stressed by White and Goldstein, a considerable effort has to be made as to data identification and organization. The computational effort involved requires the use of standardized survey areas and track separations. Regression coefficients need only be computed once under exploitation of symmetries.

Immediate analytical downward continuation has not been considered since the estimation process is simpler in a horizontal plane, and is thus not dependent on the Sperry three-dimensional algebraic gravity model (STAG). Downward continuation can be linearly applied by means of representative vertical gradients determined from gradiometer measurements for low altitudes.

5. CONCLUSIONS.

Jordan's analyses are basic as to error estimation of gravity vector components exclusively from measurements of gravity gradients under consideration of important parameters. Error estimates become, ~~however,~~ greater with a decreasing quadratic survey area for which the spectral estimation approach is not suitable. In addition, the ergodic estimation structure employed is less promising ~~for the~~ estimation of point vector components than for the estimation of vector ~~component~~ differences because of the existence of an underlying initial value problem. White and Goldstein attempt to use weakly and strongly averaged aggregates of all gradiometer measurements, covering the whole survey area, for the estimation of point vector components, starting with the consideration of T_{zz} -measurements. As to the utilization of other tensor measurements and estimation of vector components at points outside an interior survey area region, they would have to address the problem of quasi-optimal data aggregation. The utilization of T_{zz} -measurements is insufficient for an accurate estimation of vertical deflections. A two-stage estimation process similar to Jordan's spectral approach but applicable to a track length of 300 Km and requiring a relatively simple data averaging effort has been outlined. A developed two-stage estimation of gravity vector component differences, to be augmented by a highly accurate gravity vector in the center of the survey area, is most promising from the standpoint of accuracy, economy, and versatility. Four additional gravity vector components at the corners of the quadratic survey area would alleviate the problem of accuracy deterioration close to the survey area boundary. The second method solves the problem of optimal data aggregation and provides estimates essentially independent of those obtained by the first method. It can provide an estimate of gravity component mean values relating to the survey area and a possible improvement of a priori covariance functions. Under certain conditions, it permits the computation of the vertical deflection components at the survey area center. Both estimation methods do not use spatial covariance functions, a useful simplification, since representative analytical downward continuation of gravity vector components can be accomplished by means of gradiometer measurements without appreciable error for altitudes of the order 600 m.

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