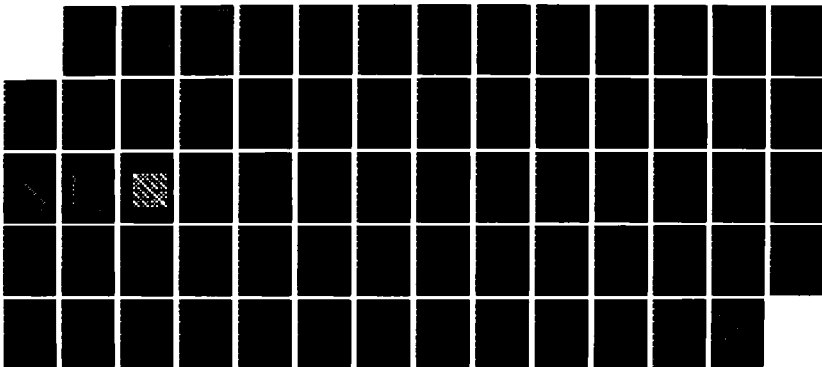
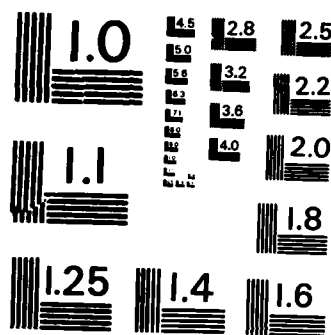


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TECHNICAL REPORT
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AN ECONOMICAL MULTIFACTOR WITHIN-
SUBJECT DESIGN ROBUST AGAINST
TREND AND CARRYOVER EFFECTS

Charles W. Simon

Contract No. N61339-81-C-0105

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When the same subject is tested serially across the conditions of an experimental design, unless precautions are taken, there is a large chance that the experimental results will be biased by sequence effects. In human performance research, two types of sequence effects are quite common, i.e., trends through the data, such as learning (and even systematic variations in the equipment and/or environment) and carryover effects, when the characteristics of one condition influence the performance on the condition that follows. Unless these effects are removed or neutralized, the information obtained from the experiment will be distorted and incorrect conclusions may be drawn. Whether 2^k factors or 2^{k+p} fractional factorials are used with the within-subject approach, sequence effects must be properly handled. The					
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→ traditional way of counterbalancing or assigning conditions at random can be inadequate and certainly uneconomical. When large-scale multifactor experiments are performed, concern with economy is paramount. *to the 5th power*

to the 4th power - 1/2
In this report, a within-subject design constructed from 2^5 factorial plan is evolved into a 2^{4+2} fractional factorial, Resolution IV, design. The novelty of this design is that with limited but reasonable restrictions, direct effects are robust against linear, quadratic, and cubic trend effects and additive and interactive one-trial carryover effects. Tables are provided that will help an investigator make the decisions necessary to properly use the design for screening purposes. The principles by which larger designs can be constructed are provided. Thus, we can screen up to seven factors with 32 serially-tested experimental conditions, 15 factors with 64 conditions, and 31 factors with 128 conditions without serious concern for sequence effects. In some cases, additional conditions may be required in accordance with the holistic approach. Several plans for analyzing the data are suggested.

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PREFACE

The contributions of the following persons are gratefully acknowledged:

Dr. Daniel P. Westra noted and clarified a serious logico-statistical error that I had made in an early version of this report. He also developed some of the computer programs used in the analyses.

Mr. Ronald Mauk wrote and ran some of the computer programs used in the analyses.

Dr. E. Christopher Poulton, Applied Psychology Unit, Medical Research Council, Cambridge, England, made many valuable suggestions regarding the contents of this report. His comments regarding the inadequate handling of carryover effects in an earlier report of mine was a primary stimulant leading to this investigation.

Mrs. Patricia Patrizzi materially assisted in the preparation of this report.

SECTION I

INTRODUCTION

Too frequently, textbooks on experimental design treat separately the spatial and the temporal assignments of data collection points. Consequently, experimenters more attentive to the former task often fail to reduce or isolate sources of unwanted sequence effects that may severely bias experimental results (Poulton, 1975). In an analysis of 239 experiments published in Human Factors from 1958 to 1972, Simon (1976) found that 31% used a form of the "within-subject" design, in which each subject is tested serially over many experimental conditions. Few of these, however, were designed or analyzed in a way that would offset sequence effects likely to be engendered by the design.

Of the various types of sequence effects that are likely to occur, psychologists have shown the greatest concern for trends associated with operator learning and fatigue. When such effects are unwanted, investigators have traditionally tried to offset them by randomizing or systematically counterbalancing the order in which the experimental conditions are presented to the subject. Unfortunately, these methods have often been inadequate and/or improperly employed. Furthermore, when large scale multifactor experiments are to be designed, these conventional means of handling sequence effects require more replications of the basic design than ordinarily would be economically acceptable.

As an alternative, designs are needed that are economical, yet able to minimize the effects of selected sequence effects. Simon (1977a) describes an economical multifactor 2^{k-p} design of Resolution IV suitable for screening studies that is robust against linear, quadratic, and cubic trend effects with no replicated conditions in the design. In this report, that design is analyzed to determine to what extent it is also robust against carryover effects. A modified design is proposed that is robust against trend and one-trial carryover effects.

WHAT IS A CARRYOVER EFFECT?

A "carryover" effect is said to occur when experience on one experimental condition affects the performance on a different condition that follows it (Simon, 1974). Carryover effects may be simple or complex and may last for one or many

trials. In the statistical and psychological literature, such terms as "change over", "residual", and "transfer" have been used synonymously with "carryover", the term that will be used in this report.

SECTION II

BRIEF REVIEW OF TREND-ROBUST HOLISTIC DESIGNS [1]*

In Table 1, an example of a 2^{16-11} Resolution IV trend-robust holistic design is shown. The basic design is made up of 32 experimental conditions. If fully saturated, it is theoretically possible to examine the effects of 16 different factors (which may include the "trend factors"), and to obtain some indication of the existence of critical two-factor interaction effects. Main effects can be estimated independently of all other main effects and of all two-factor interaction effects, but are confounded with strings of three-factor interactions (Appendix A). Two-factor interactions are aliased in sub-sets of independent strings (Appendix B). As with all fractional factorial designs, the indicated effects are actually aliased with still higher-order effects. Specifically in this design, which is a one-two-thousand-and-forty-eighth fraction of the full factorial design for 16 factors, the main effects are aliased with three-, five-, and all other odd-numbered interactions and the two-factor interactions are aliased with themselves and with all even-numbered interactions.

This aliasing of sources of variance [2] can be tolerated in most experiments because of certain characteristics of human performance data and of the paradigm associated with the holistic approach that differentiates it from the traditional fractional factorial design. Critical intrinsic three-factor interactions are found infrequently in human performance data, at least when the variables are quantitative, and finding a critical intrinsic four-factor interaction effect would be an extremely rare event (Simon, 1971; 1973; 1976). Therefore, these designs can be used with impunity as long as the assumption that higher-order interactions are not likely to exist is employed only as a tentative "working hypothesis" that eventually will be tested as prescribed by the holistic approach.

Economy is an important feature of the holistic approach. Data for the full factorial will probably never be collected because to do so adds no new information beyond a certain level

*All future footnotes are presented collectively beginning on page 42. They are noted by the numbers in brackets, e.g., [1].

of complexity. Instead, data is systematically collected a little at a time according to a special paradigm, each new step being taken only when tests indicate that additional data is likely to produce additional information. Some features employed to support the successful application of this heirarchical approach are:

(a) Data are collected only to satisfy the requirements of each step of an informational heirarchy. Major plateaus in the data collection process include: (1) screening many potentially critical factors, (2) developing a response surface based on the critical ones, and (3) obtaining more precise performance data for selected configuration (see Simon, 1977b; 1979).

(b) An elaborate preexperimental analysis is used to try to rationally spotlight potentially critical interactions prior to data collection and to place them within the fractional factorial experimental design where they may best be detected if the assumptions are valid.

(c) Special techniques are employed to detect the lack of fit of linear and/or quadratic models (when continuous data are involved). Data is not collected if these tests fail.

(d) Transformation techniques are employed to try to eliminate higher-order effects. If the model can be linearized through proper scaling, the requirement for additional data may be eliminated.

(e) During screening, critical interactions are important only to the extent they may involve factors that individually did not prove to be critical. When critical interaction strings are detected it may be necessary to collect additional data in a systematic modular fashion to identify which interactions in the string are the critical ones (see Daniel, 1976).

In practice, one seldom uses a saturated experimental design. There are practical reasons why the design in Table 1 -- with its potential for handling 16 factors -- would only be used to investigate no more than approximately 12 factors. One needs the extra degrees of freedom to represent sequence effects and possibly some three-factor interactions.

Note that in Table 1, two sets of labels are provided: (1) new screening design labels, and (2) original factor labels. The first set of labels is used in an economical, multifactor experiment that takes into consideration the fractional nature of the design, while the second set -- the traditional one -- is more useful when thinking about and manipulating the design itself. Thus, the sign matrix for a 2^{k-p} fractional factorial is the same as a 2^n factorial when $(k-p) = n$. However, certain advantages can be realized if the columns are shifted about as shown in Table 1 for reasons to be explained below.

The columns and row of the traditional factorial design are ordinarily written in Yates' standard order. When labelled, this order for the columns would be an expansion of the form (original labels): A, B, AB, C, AC, BC, ABC, D, AD... and so forth, depending on how many factors are involved and a similar pattern for rows: (1), a, b, ab, c, ac ... and so forth. In this standard order, the pattern of minus and plus signs in the factor A column, for example, would be ordered: - + - + - + - + and so forth, where the plus and minus signs are associated with experimental conditions containing the low and high levels of Factor A, respectively (or any two categorical levels). Except for the need to maintain the validity of the generalized interactions, which columns were selected for main effects had never been an issue until Daniel and Wilcoxin (1966) showed how it mattered and how it could be used to reduce the bias from trend effects. Simon (1977a) developed this procedure still further and arrived at the holistic trend-robust design shown in Table 1.

To optimize the fractional factorial so that it will be effectively robust against trend effects, the design in Table 1 is arranged as follows:

(1) All columns with letter A in the original factor labels are used for the main effects. When there are not enough main effects to fill the A-columns, those columns left over represent independent strings of three-factor interactions (Appendix A).

(2) For both main and interaction effects, the columns are ordered (using original factor labels) so that the highest interactions come first, followed by smaller and smaller interactions until the main effects are reached.

(3) Within a series of interactions of the same degree, the interactions are listed alphabetically using the original factorial labels.

The degree to which each column of the design is affected by trend effects is shown at the bottom of Table 1 in the rows labelled: "Percent Trend/Effect Overlap." This percent overlap is the proportion of variance (times 100) of the factor effect that could be attributed to the trend effect. The column rearrangement per se does not affect the degree of trend-robustness of each column, but does facilitate the use of the overall design. For all practical purposes, no main effect (new factor labels) will be seriously affected by a linear, quadratic, and/or cubic trend should any exist. Only five of the 15 two-factor interaction strings will be confounded with trends to any notable effect and of these, only three appear serious enough to avoid. Of course, the importance of this confounding also depends on the relative strength of the trend effects themselves. For example, a 71% trend-factor overlap may be irrelevant if the actual trend effect is trivial, while a

TABLE 1. SIGN MATRIX FOR A 2^{15-11} TREND RESISTANT SCREENING DESIGN (N = 32)

TEST ORDER	EXPERIMENTAL CONDITION	NEW SCREENING DESIGN LABELS*															(Two-Factor Interaction Strings)**																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																						
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		(1)	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	A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modest 10% overlap may be more serious if the trend itself is very large. Rather than speculate too much on the relative magnitudes of these effects, use of the holistic design makes it possible to minimize any serious confounding trends by:

(1) Using the results of a preexperiment analysis to assign factors to the columns of the design in such a way -- to the extent possible -- that potentially large main and generalized interaction effects will be least confounded with trends.

(2) Employing statistical techniques to remove the trend components when they cannot otherwise be avoided (Simon, 1977a).

The importance of this trend-robust design is in its power and economy when used for screening purposes. The most likely sources of critical effects are kept relatively free from trend bias. It is not necessary to replicate and counterbalance the complete design to achieve this as with the conventional approach, which handles only linear trends well. Thus, a single design provides trend-robust data for a large number of factors and with a reliability that compares favorably with the more conventional few-factor-at-a-time experiments. These benefits increase as the size of the design increases (Simon, 1979).

The biggest drawback of the design is that all two-factor interactions are aliased, a maximum of eight in a single interaction string in our 2^{16-11} design (Appendix B). Other two-level Resolution IV fractional factorial designs can be found that may have less aliasing among the two-factor interactions or "aberrations patterns" (Fries and Hunter, 1978), but often show some serious trend confounding. As in the design of any experiment, the investigator must compare the advantages and disadvantages of alternative data collection plans for his particular problem at the particular stage of his investigation. The trend-robust design is intended first as an economical means of screening a very large number of factors to find the truly important ones and, as such, compares most favorably with other two-level fractional factorials [3].

If and when the empirical evidence suggests that critical interaction effects exist, the holistic approach provides an economical data collection strategy for isolating them. It also provides a means of determining whether or not the linear model of these 2^{k-p} designs adequately fits the data.

Simon (1977a, 1977b, 1981) has discussed the design, analysis, and interpretation of these trend-robust designs. Westra, Simon, Collyer, and Chambers (1981) successfully employed the holistic design to evaluate the relative importance

of 11 features of a pilot training simulator at the Naval Training Systems Center, Orlando, Florida. The basic design was used subsequently in other experiments at that facility.

SECTION III

CONCERN WITH CARRYOVER EFFECTS

E.C.Poulton at the Applied Psychology Unit, Medical Research Council, Cambridge, England, has identified a number of sources of bias in experimental data when a subject is tested across experimental conditions. Among these -- in addition to trend effects -- are range effects (1973), asymmetric transfer (1969), and others (1982) found in cognitive, perceptual-motor, vigilance, and psychophysical judgment tasks. Of particular interest here is his discussion of effects that transfer, or carryover, from one condition to another.

Unlike trend effects that tend to affect performance across conditions over the term of the experiment, carryover effects occur between adjacent conditions being studied sequentially. Their existence depends more on the nature of the task and the characteristics of the conditions being tested than on the overall amount of practice. When a subject is being tested serially across conditions, performance on one condition may affect performance on the conditions that follow. Characteristics of these specific conditions determine the nature of the carryover effect, which may be positive or negative, additive, proportional, or interactive, and may persist beyond a single trial (Finney, 1956).

Carryover effects are task specific and preexperiment analysis is needed to determine whether a particular design can adequately handle them. There are several types [4], the characteristics of which affect their degree of persistence. For example, the carryover effect may persist for only a trial or two when conditions differ in the required magnitude of a response, as would be the case when there are variations in the gear ratio of a steering wheel. Carryover may extend over a longer period when one condition involves a "natural response" (e.g., right movement of stick results in a right movement of the display indicator, which a lifetime of experience has reinforced) that is qualitatively different from the other condition, an "unnatural response" (e.g., a right movement of the stick results in a left movement of the display indicator). Principles of transfer of training frequently apply in these two types of carryover. On the other hand, there are situations in which the carryover may never dissipate once a new strategy or cognitive principle has been learned while performing on one

condition. Instead, it will persist and affect performance on all conditions for the remainder of the experiment.

Counterbalanced within-subject Latin square designs have been used by psychologists who wish to reduce the presence of individual differences in the error variance and to compensate for learning and other sequence effects. As the most primitive case, the 2 x 2 balanced Latin square has been employed, involving two groups of subjects, two conditions, and two trials (with a different condition on each trial and each group beginning with a different condition). Thus:

	Trial 1	Trial 2
Group I	A	B + <u>a</u>
Group II	B	A + <u>b</u>

The capital letters, A and B, represent direct effects while the small letters, a and b, represent the amount carried over from the previous trial and condition.

As originally conceived, a 2 x 2 Latin square is actually a two-dimensional representation of a three-dimensional space. The three independent factors are identified with the columns, rows, and diagonals. The 2 x 2 Latin square of our example is actually a fractional-factorial design, a 2^{3-2} plan with the defining contrast: [I] = ABC (see Simon, 1973 for further explanation of this symbology). The sources associated with the three degrees of freedom show the following aliasing structure: (A + BC), (B + AC), and (C + AB). Interaction ABC is lost due to the fractionation. To obtain unbiased estimates of A, B, and C, all interaction effects must be -- not merely assumed to be -- negligible.

In deciding whether it is appropriate or not to use the 2 x 2 Latin square as a within-group design as psychologists do, it helps to identify what has been confounded and to decide whether these sources are likely to exist in our experiment. If the rows (A), columns (B), and diagonals (C) are labelled Groups, Trials, and Conditions, respectively, then:

<u>Designated Source</u>	<u>Actual Source of Variance</u>
Conditions	[Conditions + (Groups x Trials)]
Groups	[Groups + (Conditions x Trials)]
Trials	[Trials + (Groups x Conditions)]

The sources within the brackets are totally aliased and cannot be isolated within the context of this design. In many

human performance experiments, these interaction terms represent the following sources of variance:

Groups x Trials: Groups with different learning rates

Conditions x Trials: Conditions which are learned at different rates

Groups x Conditions: Groups with different amounts of prior experience on a condition

While experiments do differ, still it is believed that interactions of these types are found too frequently in human performance experiments to just assume them out of existence. Without the support of directly relevant empirical evidence, it would be risky to assume the additive model is the correct one. If it is not, this 2 x 2 Latin square design should not be used.

But problems with this design do not stop with these potential interactions. There are also carryover effects that can appear when a within-subject design is used. In the 2 x 2 Latin square design shown above, it can be seen that the two carryover effects, a and b, are not balanced in the design. Each does not occur with all conditions of the other dimensions; ergo, carryover effects are not orthogonal to the other sources of variance. Should there be critical carryover effects, this situation will bias the results. For screening purposes, it would be unacceptable if direct condition effects were biased.

Poulton and Freeman (1966; Poulton, 1969) noted that if amounts carried over from conditions A and B were equal -- they called this "symmetrical conditions carryover" -- then the Condition effect would not be biased. While symmetrical carryover would change the means of the two conditions, the difference between their means (i.e., the Condition effect) would be unaffected since the same amount is being added or subtracted. However, Poulton collected evidence to show that in many experiments, the size of the carryover effect differs depending on whether B follows A, or A follows B. This "asymmetrical transfer," they postulate, biases our estimate of the condition effect and seriously jeopardizes the effectiveness of within-subject designs. This, they contend, argues for the use of between-subject designs as the primary data collection method for most human performance experiments.

It helps our understanding of the problem if we identify Poulton's symmetrical and asymmetrical carryover in terms of the sources of variance in the Latin square design. Symmetrical carryover is equivalent to trial effects and asymmetrical carryover is an interactive carryover effect. In other words, when conditions change the same amount on the second trial, regardless of the conditions involved, that is a trial effect (if positive, possibly a learning effect). When, however, the

conditions show different amounts of carryover, we may have either an "additive" or an "interactive" carryover effect. In the former case, condition A -- while having a different amount of carryover than condition B -- always shows the same amount regardless of what condition follows it, while in the latter case, how much carryover A shows depends on which condition follows it. These distinctions become somewhat blurred in the oversimplified 2 x 2 Latin square, but are important to make when larger Latin squares or other counterbalanced designs are involved.

While the evidence that Poulton has accumulated shows that the 2 x 2 Latin square within-subject design as used by psychologists definitely should not be used, it does not follow that all within-subject designs should be avoided in the presence of learning and carryover.

ALTERNATIVE WITHIN-SUBJECT DESIGNS

There are different Latin squares and other balanced arrangements that can be used to offset certain carryover effects, although some are not necessarily economical nor satisfactory for our purposes (Simon, 1974). Of particular interest in this discussion, however, is the "extra-period Latin square" design (Simon, 1974, p. 43-50; 1979, p. 93-98). If an investigator wished to correct the lack of orthogonality in the 2 x 2 Latin square relevant to carryover effects, an extra-period Latin square design might be employed. This is done by giving each group a third trial, and repeating the condition in trial two, thus:

		Trials		
		<u>1</u>	<u>2</u>	<u>3</u>
Groups	1)	A	B + a	B + <u>b</u>
	2)	B	A + b	A + <u>a</u>

In the basic Latin square design, only one condition, B, follows condition A, and vice versa. In this design, however, conditions A and B both follow conditions A and B. Consequently, each carryover condition is paired with each direct condition an equal number of times, making these two sources of variance orthogonal. The extra-degrees of freedom obtained from the extra trials enables the additive carryover effect to be isolated. The design assumes only one-trial carryover and is not suitable if direct and carryover effects interact or, for that matter, if any of the main effects interact.

A more powerful and cleaner design when using a single subject to study the difference in performance between two conditions without concern for trend and carryover effects is

between the groups. To ensure a reasonable small standard error for the individual differences, you have to use large numbers of people. You are likely to need more people in each group than you would need for a complete within-subject design. But once you have accepted the need for more people, your statistical tests take care of the differences between groups. A rejection of the null hypothesis indicates that the obtained differences are not likely due to the chance allocation of people to groups."

Using many subjects, however, increases the cost of the experiment and when large, multifactor experiments are to be performed, one may run out of time, money, and available subjects. It may not be logistically possible to use a between-subject design when a great many factors are to be screened.

Furthermore, when a large number of replications (i.e., number of subjects within cells in the between-subject designs) are required to obtain a small enough standard error to overcome the effects of individual differences, reducing that problem creates another. If an investigator can afford the luxury of a large number of subjects, although in practice he may not, he will find himself in the paradoxical situation noted by Meehl (1967), that the process of replicating to increase the precision of the experimental data decreases the confidence with which one can generalize the results to the operational situation. While using many subjects per group does decrease the error variance by a factor of the square root of the quantity four over the number of replications (Simon and Westra, 1984), it also increases the chance of finding the experimental data "statistically significant" when in fact the results in practice are negligible (Bakan, 1966).

Nor can we assume that a between-subject design is devoid of sequence effects. Those created by the equipment and the environment can still be present and they can distort operator performance. If, for example, temperature shifts affect the sensitivity of some electronic sensing device that feeds information to the operator, the visual signals may vary in a systematic fashion throughout the day if proper controls are not taken, causing a comparable shift in performance among operators. Similarly, failure to allow for enough recycling time for certain devices may result in an equipment carryover effect that confounds the performance of a subject tested too soon after the previous subject has finished. Sequence effects may even be introduced by the experimenter who tests all of the subjects, e.g., as he becomes fatigued or as he gradually alters the instructions as the experiments drag on.

A more subtle weakness of the between-subject design stems from the "situation specific" nature of all human performance experiments. Results from these experiments are a function of

the characteristics present at the time data is being collected, whether studied or not, whether known or not. If the experimental subjects are not representative of the population to which the results are to be generalized, the information, e.g., which factors critically influence the performance under investigation, can be incorrect. Subject skill level is one of the more difficult experimental factors to measure accurately (Simon and Westra, 1984) and frequently little effort is made to measure it at all. The random assignment of individuals to the cells of a between-subject design does not resolve this problem and assigning all subjects at a specific skill level is difficult to do or for that matter even to ascertain that it has been done correctly. Introducing skill level as a dimension can reduce this problem but increases experimental costs considerably in large multifactor investigations. The problem is handled systematically and economically when within-subject designs are used, since a single naive subject can be tested repeatedly across all the experimental conditions, enabling the investigator to see the shift in results that takes place as ability improves with the practice.

In practice, the choice of using a within-subject or between-subjects design will probably seldom rest solely on the existence of sequence effects. Logistical considerations, such as subject availability and time and money, are also common limiting factors. Still, as Poulton has implied, the overriding principle in any experiment should be to maintain the integrity of the data. This should mean minimizing sequence effects and all other sources of irrelevant variance. It should mean including enough subjects to overcome otherwise insurmountable sampling problems. At times, however, this may best be achieved by developing new procedures that will satisfy these multiple criteria, i.e., to obtain clean, accurate, and generalizable information at minimal costs. When a large multifactor experiment is being conducted, the opportunities for meeting these criteria increase over what is possible with a few-factor experiment. A data collection plan for that purpose will be described in the next section.

SECTION IV

CARRYOVER CHARACTERISTICS OF HOLISTIC DESIGNS

To determine how resistant elements of the trend-robust holistic design are to one-trial carryover effects, correlations must be made among the main and interaction columns for direct and for additive and interactive carryover effects as well as for linear, quadratic, and cubic trends. For this analysis a 2^5 factorial design was used as the test vehicle. [7]

DIRECT AND CARRYOVER SIGN PATTERNS OF AN EXTRA PERIOD 2^k DESIGN

The 96 columns (with 32 signs to a column) required to analyze the 2^5 factorial for its trend-carryover robust characteristics are created in four sections as follows:

(1) Direct Effect Columns. In the conventional way, create the 32-row by 31-column sign matrix of estimable main and interaction direct effects for the 2^5 factorial. Columns (effects) and rows (conditions) will remain in Yates' standard order for this analysis. No Identity column is included. Plus or minus signs are used to indicate which of the two levels of each factor are found in each experimental condition (row). These arithmetic symbols, + and -, are the conventional abbreviated forms of the labels +1 and -1, respectively.

(2) Additive Carryover Columns. Create a second 32-row by 31-column sign matrix representing the additive carryover columns, one for each of the direct effects created previously. With the experimental conditions (rows) of the basic design arranged in Yates' standard order, additive carryover columns are formed by associating with each sign in the direct effects column the sign that immediately precedes it in the columnar sequence.

Since a preceding condition is needed to create a carryover effect, and since the first row of a 2^k matrix has no preceding condition, it is necessary to add an extra condition before the first row of the design, a "preconditioner." This extra condition should be the same as the one in the last row of the basic 2^5 design.

(3) Interactive Carryover Columns. Create a 32-row by 31-column sign matrix representing the interactive carryover columns. In each row, multiply the signs -- using the Rule of

Signs -- in corresponding direct and carryover columns obtained in the two previous steps.

(4) Trend Columns. Create still another 32-row by 3-column sign matrix made up of the integer Tchebycheff orthogonal polynomials (found in many statistical tables, e.g., in Beyer, 1966) representing the linear, quadratic, and cubic trends effects for $N = 2^k$ conditions of the basic design.

To illustrate the above procedures, the table below was created for just two factors, A and B, each at two levels. With two, two-level factors, a 4-row by 12-column matrix would be required, plus the "preconditioner," P, thus:

Direct			Additive Carryover			Interactive Carryover			Trends		
A	B	AB	A'	B'	AB'	A''	B''	AB''	L	Q	C
P)[+	+	+	]								
a) -	-	+	+	+	+	-	-	+	-3	+1	-1
b) +	-	-	-	-	+	-	+	-	-1	-1	+3
c) -	+	-	+	-	-	-	-	+	+1	-1	-3
d) +	+	+	-	+	-	-	+	-	+3	+1	+1

The extra-condition, i.e., the preconditioner, can be thought of as a warm-up trial and will not be included in the analysis. The fact that the number of conditions is not enough to provide the degrees of freedom needed to estimate all twelve effects is not relevant at this time. For the moment, we wish only to illustrate how such a table is constructed.

CORRELATIONS AMONG THE DIRECT AND CARRYOVER COLUMNS

Correlations were calculated among all 96 main and interaction columns for the 31 direct, 31 additive carryover, 31 interactive carryover, and 3 trend effects. From these correlations, the main effect columns can be selected that will create the best trend/carryover-robust design available without collecting additional data.

The actual intercolumn correlations are given in Appendix C. However, to facilitate both the discussion and interpretation of this rather large matrix, the full 93-row by 96-column intercorrelation matrix was divided into 31 x 31

submatrices and three 31 x 3 submatrices, these sections being identified by letters of the alphabet in Table 2.

TABLE 2. SECTIONING OF THE COMPLETE INTERCORRELATION MATRIX

	31 Direct	31 Additive Carryover	31 Interactive Carryover	3 Trend
R O W S	31 Direct	31 A. Carryover	31 I. Carryover	
	A	B	C	J
	(D)	E	F	K
	(G)	(H)	I	L

The correlations in all sections of Table 2 need not be shown since certain relationships are known on logical or other grounds. For example, correlations in Sections D, G, and H are the same as those in their counterparts B, C, and F, respectively. Only the latter will be discussed.

In Section A, i.e., the intercorrelation matrix among direct main and interaction effects of a basic 2^5 design, all correlations on the matrix diagonal are 1.00, since each direct effect is correlated with itself. All off-diagonal correlations are zero, meaning there is no confounding among the direct effects, main or interaction, a result to be expected in an orthogonal factorial design. This section will not be shown.

In Section E, i.e., the intercorrelation matrix among additive carryover main and interaction effect columns, the correlations again form a unit matrix, with a diagonal of ones and all off-diagonals zero. Since the sign patterns of the additive carryover columns are the same as those of the direct columns, except that they are shifted down one row and the last row cycled to the top, this shift will not change the correlations from those found in Section A. Thus, the additive carryover effects in Section E are orthogonal to one another. This section will not be shown nor discussed.

The correlations among linear, quadratic, and cubic trend effects are not included in Table 2 since the orthogonal polynomials used to represent trend are, by definition, uncorrelated. However, how trends correlate with the direct and the additive and interactive carryover columns are shown in Table 2 and will be discussed.

CODING THE SYMBOLIC MATRICES. Omitting Sections A, D, E, G, and H for the reasons cited above leaves only Sections B, C, F, I,

J, K, and L to be considered. To facilitate the study of this rather large matrix of numbers, the correlations in the matrices were assigned to one of five categories according to size and estimated impact on the interpretation of experimental results and shown symbolically. These categories -- qualitative judgments on the part of the investigator -- were selected to represent how much a correlation within each size range is likely to distort the interpretation of experimental results prior to any effort to partial out unwanted effects. The correctness of these judgments depend, of course, not only on the size of the correlation but also on the actual and relative sizes of the effects being correlated and the practical consequences of erroneous conclusions. The distortions expected, therefore, tend to represent the worst case. It is the responsibility of the investigator to determine during the preexperiment analysis phase what conditions are likely to be expected and if they can be tolerated.

The five categories as defined below are represented by the indicated geometric symbol:

<u>Correlation*</u>	<u>Proportion Overlap</u>	<u>Symbol</u>	<u>Distortion</u>
0	.0000		None (N)
.0+ to .31	.00 to .09	□	Trivial (T)
.32 to .54	.10 to .29	▢	Marginal (M)
.55 to .70	.30 to .49	▣	Dangerous (D)
> .71	> .50	■	Unacceptable (U)

* Correlations with 30 degrees of freedom (N = 32) must exceed .349 to be statistically significant at the $p = .05$ level. In this report, correlation signs are ignored.

Symbolic representation of the intercorrelation matrices for Sections B and K, Sections C and J, and Sections I and L, are given in Tables 3-A, 3-B, and 3-C respectively. Because the intercorrelation matrix for Section F proved to be identical to that for Section C -- in size, if not in sign -- it is not printed separately. It is obtained from Table 3-B when primes are added to the direct column labels, identifying them also as additive carryover labels.

The columns in Tables 3-A, 3-B, and 3-C are identified both by their original factor labels and the new labels of a trend-robust design. The former represent the labels for all main and interaction columns of a 2^5 factorial design. The latter are the labels for the main and two-factor interaction strings of a 2^{16-11} fractional factorial design used to create the trend-robust holistic design in Table 1. Direct, additive

TABLE 3. SYMBOLIC MATRICES OF INTERCORRELATIONS AMONG DIRECT, CARRYOVER AND TREND EFFECT COLUMNS. (2^5 factorial or equivalent fractional factorial design)

A. DIRECT vs ADDITIVE CARRYOVER (Section B = D, with K)

		ADDITIVE CARRYOVER																														
Original Factor Labels	New Factor Labels	A'	B'	AB'	C'	AC'	BC'	ABC'	D'	AD'	BD'	ABD'	CD'	ACD'	BCD'	ABCD'	E'	AE'	BE'	ABE'	CE'	ACE'	BCE'	ABCE'	DE'	ADE'	BDE'	ABDE'	CDE'	ACDE'	BCDE'	ABCDE'
		P	AE+	L	AD+	M	AK+	F	AC+	N	AJ+	G	AH+	I	AO+	B	AB+	O	AI+	H	AG+	J	AN+	C	AF+	K	AM+	D	AL+	E	AP+	A
A	P																															
B	AE+																															
AB	L																															
C	AD+																															
AC	M																															
BC	AK+																															
ABC	F																															
D	AC+																															
AD	N																															
BD	AJ+																															
ABD	G																															
CD	AH+																															
ACD	I																															
BCD	AO+																															
ABCD	B																															
E	AB+																															
AE	O																															
BE	AI+																															
ABE	H																															
CE	AG+																															
ACE	J																															
BCE	AN+																															
ABCE	C																															
DE	AF+																															
ADE	K																															
BDE	AM+																															
ABDE	D																															
CDE	AL+																															
ACDE	E																															
BCDE	AP+																															
ABCDE	A																															
TREND	L																															
	Q																															
	C																															

Notations:

$r = 0$ (N)

$r = .00+ \text{ to } .31$ (T)

$r = .32 \text{ to } .54$ (M)

$r = .55 \text{ to } .70$ (D)

$r = >.70$ (U)

(CONTINUED)

B. INTERACTIVE CARRYOVER vs DIRECT* (Section C = G, with J)

* The correlations between interactive carryover and additive carryover effect columns (Section F=H) correspond to those shown in this table of correlations between interactive carryover and direct effect columns, respectively, except for occasional differences in signs.

[illegible]

Notations:

• $r = .00$ to $.31$ (T)

☒ $r = .55$ to $.70$ (D)

 $\square \quad r = 0 \{N\}$

B $r = .32 \text{ to } .54 \text{ (M)}$

■ $r = >.70$ (U)

C. INTERACTIVE CARRYOVER vs INTERACTIVE CARRYOVER (Section I, with L)

Notations:

	$r = 0$ (N)		$r = .00+ \text{ to } .31$ (T)		$r = .55 \text{ to } .70$ (D)
			$r = .32 \text{ to } .54$ (M)		$r = >.70$ (U)

carryover, and interactive carryover columns are distinguished by the use of primes, none for direct, one for additive carryover, and two for interactive carryover. For example, with either original or new factor labels, the letter A would represent the direct column of factor A, A' the additive carryover effect column, and A'' the interactive carryover column. The same use of primes applies to interaction effect columns. When the nominal labels of an interaction column actually represents a string of interactions, the label is followed by a plus sign, e.g., AB+. The 2- and 3-factor interaction aliases for each interaction column of Table 1 are given in Appendices A and B.

Summary discussions of these tables will be given in the next section when the best columns are selected in order to create a viable trend-carryover robust (TCR) design.

SECTION V

DEVELOPING THE TREND-CARRYOVER ROBUST DESIGN

The correlation data presented in the previous section were used to develop a viable TCR design. The rational employed and the results obtained are described below. Columns were sought which would result in a reasonably orthogonal relationships among direct, additive carryover, interactive carryover, and trend effect columns. In cases where columns remained correlated to a degree, it was necessary to select the ones that would maximize the number of trivial relationships, accept as few as possible marginal relationships, and avoid dangerous and unacceptable ones. In making compromises regarding these selections, priorities were given to those favoring the orthogonality of main effects over interaction effects, direct effects over additive and interactive carryover effects, and direct with their own additive and interactive carryover effects over direct with other additive and interactive carryover effects. Tables 3-A, 3-B, and 3-C were inspected with these criteria in mind.

CORRELATIONS AMONG DIRECT COLUMNS (SECTION A) AND AMONG ADDITIVE CARRYOVER COLUMNS (SECTION E)

As indicated earlier, the intercorrelation matrix among direct columns as well as the intercorrelation matrix among additive carryover columns are both unit diagonal matrices. Each column is orthogonal to all other columns within the same matrix.

CORRELATIONS BETWEEN DIRECT AND ADDITIVE CARRYOVER COLUMNS (SECTION B)

Inspection of Table 3-A (Section B) shows that the greatest amount of nonorthogonality lies along the diagonal of the matrix. This diagonal contains the correlations between each direct column, whether main or interaction effect, and its own additive carryover counterpart. The search was made for columns that were most trend robust and had the smallest correlations between themselves and their own carryover columns.

It has been established that direct columns with an A-term in their original factor label are the most trend resistant. These columns have new factor labels A through P (see Table 1). Inspecting the diagonal values for these columns in Table 3-A,

we find that 13 out of the 31 correlations between direct and its own additive carryover column are dangerously or unacceptably high. Nine others of these diagonal pairs are correlated marginally and nine trivially, or not at all.

If we remove from consideration all direct main effect columns that show dangerously or unacceptably high correlations with their own additive carryover columns (i.e., E, I, J, K, N, O, and P), nine columns remain that tend to be relatively robust against additive carryover effects. These are direct, main effect columns with new factor labels: A, B, C, D, F, G, H, L, and M. To include their generalized interactions in the design, we must also select interaction-string columns labeled: AB+, AC+, AD+, AF+, AG+, AH+, AL+, and AM+ (see Table 1).

Further inspection of Table 3-A shows that four of these selected direct main effect columns (i.e., A, C, F, and M) are marginally correlated with their own main and certain interaction-string additive carryover columns. Direct effects A and C overlap their additive carryover counterparts only 14%; F and M overlap theirs 25%. Off-diagonal correlations are all trivial or zero except for those between M and F', M and AD+', F and AD+, and F and M', all of which are marginally correlated. Also, pairs D and AM+' and AM+ and D' are correlated at unacceptably high levels.

A more serious problem exists, however, with the direct interaction-string columns. Six out of eight of these correlate dangerously or unacceptably high with their own additive carryover interaction-string columns. What appears to be a good selection for main effects is poor insofar as their generalized interaction-strings are concerned.

Under these circumstances, if a useful TCR design is to be developed, we must be satisfied that the following statement is viable:

Direct interaction-strings have no critical additive carryover effects.

If this hypothesis is in fact invalid, then there would be no way to determine whether a critical interaction-string effect is a consequence of some interaction within the string or the string's additive carryover effect. In practice, however, this dilemma is probably a moot one, particularly if the design is used in a screening experiment. For one thing, critical interaction-strings in a large multifactor experiment are infrequent (though not impossible). That their carryover effects might be critical seems to be even less likely in view of their composite nature. More pertinent is the fact that in screening experiments, precise estimations of interaction effects are of less concern than the detection that they are likely to have critical effects on performance. For screening

purposes, we are interested in critical interactions only to the extent that they contain a factor that was not found to be an important main effect. This condition occurs with intrinsic interactions.

A casual check on the validity of the hypothesis may be made. One can see how the estimated effects change before and after the additive carryover effects are partialled out of the direct main and interaction effects. If the issue is still not resolved, the proper procedure would be to collect more data, a necessary step anyway to determine which interactions within a string are actually contributing to any critical string effects.

It is interesting to note that with the exception of column M, the other remaining eight columns all have an AB-term in their original factor label. Column M, new label, is column AC, original label. Furthermore, these columns (i.e., A, B, C, D, F, G, H, and L, but not M) form a complete 2 factorial.

CORRELATIONS BETWEEN DIRECT AND INTERACTIVE CARRYOVER COLUMNS (SECTION C) AND BETWEEN ADDITIVE CARRYOVER AND INTERACTIVE CARRYOVER COLUMNS (SECTION F)

The correlation matrices for Sections C and F are identical except for certain sign reversals so only one matrix is given. When developing the TCR design, magnitude rather than sign is the critical concern regarding correlations; therefore, signs are only shown in the complete correlation table in Appendix C.

Inspection of Table 3-B (representing both Sections C and F) along the diagonal reveals that between any direct column and its own interactive carryover column, the correlation is zero. This is understandable, since the cross-product between a direct column and its own interactive carryover column produces the sign pattern of their corresponding additive carryover column which has a mean of zero. Off-diagonal sign patterns reveal an interesting nonsymmetrical distribution of correlation values. Ninety-two percent of the possible 900 off-diagonal correlations are trivially related or not at all. Less than 3% are at a dangerous or unacceptable level. The remaining 5% are at marginal levels.

Off-diagonal correlations between the interactive carryover columns and the nine main effect columns under consideration, along with their eight generalized interaction-string columns, are for the most part trivial or zero. Column pairs F and F', M and M', and AD+ and AD+' are each marginally correlated (14% overlap) with two interactive carryover interaction columns, AC+ and AH+; so are columns F', M', and AD+' of the corresponding matrix (not shown) of Section F. Only column L of Table 3-B and column L' of the corresponding matrix of Section F show an uncomfortably large number of marginal or

dangerous correlations with nine interactive carryover main or interaction effect columns.

CORRELATIONS AMONG INTERACTIVE CARRYOVER COLUMNS (SECTION I)

A special situation exists in Table 3-C due to the fact that interactive carryover column P" (new label -- the product of columns P and P' -- contains only the minus level condition, i.e., $P'' = [-I]$, the identity column. Consequently, the "P" effect cannot be estimated. Furthermore, when an interactive carryover column containing an A-term in its original factor label is correlated with some other interactive carryover column, the correlation obtained will be the same as the corresponding column without an A-term in the label. For example, the size of the correlation between any column and ABC" will be the same as with that same column and BC".

Inspection of the selected columns $\leftrightarrow$ A,B,C,D,F,G,H,L, and M, and the columns of their generalized interactions $\leftrightarrow$ in Table 3-C (Section I) shows that overall, intercorrelations among the interactive carryover main effect columns are generally dangerously or unacceptably high. This is not too surprising since the two levels (plus and minus) of a great many of these interactive carryover columns are not equally represented. This less than optimum feature exists because, we must remember, we are examining the characteristics of an existing design, not one that has been created specifically to study carryover effects.

For the select group, most interactive carryover main effect columns are trivially or marginally correlated with interactive carryover interaction columns. There are exceptions. Column AM+" is correlated dangerously or unacceptably high with seven out of the nine interactive carryover main effect columns and D" is dangerously correlated with column AL+",

Too many correlations among the interactive carryover interaction columns, as a group, are dangerously or unacceptably high.

Because of the serious confounding among interactive carryover main effect columns, another limitation must be placed on the use of this TCR design, i.e.:

Interactive carryover effects will not be estimated with this TCR design.

This does not seriously affect the use of this design as a screening tool where the primary purpose is to determine which factors are having critical effects on performance. The original reason for developing a TCR design was not to estimate carryover effects but to isolate them from direct effects as

much as possible. It does tend to accomplish this to a greater degree than more conventional designs tend to do.

CORRELATIONS OF DIRECT AND CARRYOVER COLUMNS WITH TRENDS (SECTIONS J, K, AND L).

The degree of correlation between trends and direct, additive carryover, and interactive carryover effect columns are found at the bottom of Tables 3-B, 3-A, and 3-C, respectively. Inspection reveals that while the direct main effects are at most only trivially correlated with linear, quadratic, and cubic trends, the direct interaction-strings have some very high and also marginal relationships distributed among the trends. This had already been shown in Table 1. This means that in practice one must judiciously assign the factors to the appropriate columns in a way that is likely to avoid serious confounding with trends when such effects are expected to be large. A careful preexperiment analysis will help determine if and how this can best be accomplished. Difficulties of this type can be reduced if the size of the design is increased, thereby increasing the number of available trend-robust columns. Additive carryover columns correlate with trend effects in essentially the same patterns shown by the direct columns. Interactive carryover columns are all trivially correlated with trends.

SELECTING THE FINAL TCR DESIGN

Up to this point, we have tentatively been considering the relationships among columns with new factor labels: A, B, C, D, F, G, H, L, and M to be used for direct main effects. However, in a number of sections, dangerous and unacceptable correlations occurred too frequently when columns L and M were part of the new factor label.

That fact, along with severe limitations in the total available degrees of freedom in the $N = 32$ design, suggests that columns L and M should not be considered main effect columns. This would leave us with a seven factor, Resolution IV design, with the experimental conditions being defined by the columns (new factor labels): A, B, C, D, F, G, and H. Each of these direct main effect columns are all ones with an AB-term in its original factor label.

The original factor label column AB itself is not included as a main effect column; it is a new factor label column L. The relevant columns for the direct interaction-string effects are: AB+, AC+, AD+, AF+, AG+, AH+, and AL+. [8]

This design is viable only if the two restrictions cited earlier are applicable, namely: (1) additive carryover interaction effects are not critical, and (2) the design is not used to estimate interactive carryover effects.

The pattern of nontrivial correlations for this seven factor ($N = 32$) Resolution IV, trend-carryover robust design is provided in Tables 4-A, 4-B, and 4-C. Letters M, D, and U are used to indicate the nontrivial levels -- Marginal, Dangerous, and Unacceptable, respectively. An empty space indicates that the correlation is either trivial or zero. The percent overlap for each marginal correlation follows. An investigator would use these tables to decide how best to assign experimental factors to the columns.

No table is provided for the intercorrelations among the direct columns and among the interactive carryover columns since both are diagonal matrices, i.e., with ones along the diagonal when a column is correlated with itself and zeroes off the diagonal.

SUMMARY TO TCR DESIGN CONSTRUCTION PROCEDURES

The following steps will produce a 2^{k-p} trend-carryover robust design of Resolution IV:

(1) To construct a design capable of studying k factors, each at two levels, one must use the least number of independent conditions required to satisfy this equation:

$$N = [2^x > 4(k+1)]$$

where k equals the number of factors to be studied and x equals $(k-p)$. The reciprocal of 2^p indicates what proportion the size of this design is to the full 2^k factorial.

Example 1. To determine the size of a design needed to study seven factors, we substitute 7 for k and get $4(7+1) = 32$. That makes $x = 5$ since 32 is equal to two to the fifth power. With $k = 7$, and $x = 5$, then $5 = (7-p)$ and p equals 2. Thus, this design would be of a size $1/2^2$ or one-fourth of the full 2^7 factorial. This is true, of course, since the design requires 32 conditions and a 2^7 design requires 128.

Example 2. To determine the size of a design needed to study 10 factors, we substitute 10 for k and get $4(10+1) = 44$ for which the nearest power of two equal to or greater than that number is $2^6 = 64$. With $k = 10$ and $x = 6$, then $p = 4$. The reciprocal of 2^4 , therefore, indicates that the design is a one-sixteenth fraction of the full 2^{10} factorial.

(2) Construct in Yates' standard order the sign matrix for the 2^x factorial with the value of x being determined in Step 1.

(3) Sort on A-- (original labels). That is, reorganize the columns as described earlier and illustrated in Table 1 to build a trend-robust design. At this point, the A-columns would be used for main effects and the non-A-columns for the two-factor

TABLE 4. CRITICAL RELATIONSHIPS AMONG DIRECT, CARRYOVER AND TREND EFFECT COLUMNS IN A SEVEN FACTOR, QUARTER REPLICATE, RESOLUTION IV TREND-CARRYOVER ROBUST SCREENING DESIGN*

NOTES:

1. Direct vs Direct (Section A of Table 2) is not shown here since all direct columns are orthogonal to one another.
2. Additive Carryover vs Additive Carryover (Section E of Table 2) is not shown here since additive carryover columns are all orthogonal to one another.
3. Interaction Carryover columns in Table 4-B correlate the same (except occasionally for the sign) with both Direct and Additive Carryover columns.

NOTATIONS:

Blank space: No effect or trivial correlations ($r = .00$ to $.31$).

M: Marginal correlations ($r = .32$ to $.54$). Number with M is the percent overlap, i.e., r squared, of the two variables.

D: Dangerous correlations ($r = .55$ to $.70$).

U: Unacceptable correlations ($r = > .71$).

ASSUMPTIONS TO BE MET:

When this design is used for screening purposes,

1. Direct interaction strings have no critical additive carryover effects.
2. Interactive carryover effects will not be estimated.

4-A		ADDITIVE CARRYOVER								TREND			
		A'	B'	C'	D'	F'	G'	H'	AB+'AC+'AD+'AF+'AG+'AH+'AL+'		L	Q	C
DIRECT	A	M14							M14				
	B												
	C	M14							M14				
	D												
	F	M25							M25				
	G												
	H												
	AB+								U		U	M11	
	AC+								U		M19	M21	
	AD+	M25							M25				
AF+								U			U		
AG+	M14							D			M18		
AH+								U					
AL+	M14								D			D	
	L								U M19				
	Q								U M17				
	C								M21				

*All labels in Tables 4-A, 4-B, and 4-C are "new factor labels".

TABLE 4. CRITICAL RELATIONSHIPS (cont'd)

4-B		INTERACTIVE CARRYOVER									
		A"	B"	C"	D"	F"	G"	H"	AB+"AC+"AD+"AF+"AG+"AH+"AL+"		
DIRECT	A										
	B										
	C										
	D										
	F								M14	M14	
	G										
	H										
	AB+										
	AC+										
	AD+								M14	M14	
	AF+										
	AG+										
	AH+										
	AL+										
TREND	L										
	Q										
	C										

4-C		INTERACTIVE CARRYOVER									
		A"	B"	C"	D"	F"	G"	H"	AB+"AC+"AD+"AF+"AG+"AH+"AL+"		
INTERACTIVE CARRYOVER	A"	U	U	U	M	U	D	U	M15 M10		
	B"	U	U	U	D	U	M	D	M11 M24	M11	
	C"	U	U	U	U	U	D	M	M15	M10	
	D"	M	D	U	U	D	U	U	M13	M18 D	
	F"	U	U	U	D	U	U	D	M11		
	G"	D	M	D	U	U	U	U		M24	
	H"	U	D	M	U	D	U	U	M13	M18	
	AB+"	M11 M15							U D M	M	
	AC+"	M24							D U D D		
	AD+"	M13 M11 M13							M D U M U D U		
	AF+"	M15 M11							D M U D		
	AG+"	M10							U D U U D		
	AH+"	M18 M24 M18							D U U U		
	AL+"	M10 D							M U D U U		

interaction strings. As in any fractional factorial, all odd-order effects are confounded with main effects and all even-order effects are confounded with two-factor interaction strings. Any unravelling required would follow the procedures used for any screening study (Simon, 1977a).

(4) To make a design also carryover robust, sort the A-- columns into AB-- and non-AB-- groups, with the exception of column AB itself, which is to remain with the non-AB-- columns. The columns in the AB-- group can be used as main effect columns in the trend-carryover robust design.

(5) Identify the relevant two-factor interaction-string columns by finding those in which new factor label A interacts with the labels used as main effects, as well as all other columns in which the generalized interactions among main effects appear. (Column aliases will have to be worked out or derived from existing tables; see Simon, 1977a).

(6) All unused AB-- and A-- columns may be used to estimate three-factor interaction effects. All unused non-A-- columns may be used to estimate four-factor interaction effects. However, as will be discussed in the next section, we may wish to use the degrees of freedom from these unused columns to partial out trend and carryover effects, as well as to provide an estimate of internal error variance.

Just how trend and carryover robust this N = 32, Resolution IV TCR design is is summarized in Table 5. This table shows the type and frequency of nontrivial correlations among the direct columns and all other columns. The number in parenthesis associated with each cell in the table indicates the total number of correlations considered in that cell. The size of nontrivial correlations among columns are indicated by the M, D, U coding used earlier. Correlations with onesself and correlations between direct and additive carryover interactions (which our working hypothesis has excluded) are not counted in this table. Out of the total 700 correlations considered in the table, 17 (2.4%) are marginal, 3 (0.4%) are dangerous or unacceptable, and 97.2% are trivial or absent.

TABLE 5. NUMBER OF NONTRIVIAL CORRELATIONS BETWEEN
DIRECT EFFECTS AND ALL SOURCES OF VARIANCE IN THIS
SEVEN FACTOR TCR RESOLUTION IV DESIGN

	Direct		Additive		Interactive		Trend		C
	MAIN	INT	MAIN	INT	MAIN	INT	L	Q	
Main Direct (N)	0 (42)	0 (49)	3-M (49)	3-M (49)	0 (49)	2-M (49)	0 (21)	0 (21)	0 (21)
Interaction Direct (N)	0 (49)	0 (42)	3-M (49)	0 * (49)	0 (49)	2-M (49)	1-M 1-U (21)	1-M 1-U (21)	2-M 1-D (21)

* Correlations between direct and additive carryover interaction columns are not included, as per the limitation stated earlier.

Since the design is not completely orthogonal to sequence effects and is a Resolution IV design with two-factor interactions in strings, it will be necessary for the investigator to take some reasonable steps to optimize its effectiveness. First, through preexperiment analysis, he must decide whether the strengths and weaknesses of the TCR design fit the characteristics of his particular task. This frequently is not a black or white decision and involves compromise among a number of criteria. It can be done best by comparing the information, distortion, and costs associated with this design against those of alternative data collection plans. How effectively the design will be used also depends on the investigator's skill in assigning the experimental factors to the columns. Preexperiment analysis combined with the information in Table 4 and the tables in Appendix C can greatly facilitate this. It must be remembered that assigning a factor known to have a small or no carryover effect to a highly confounded column, and vice versa, tends to neutralize considerably the weaknesses from the lack of orthogonality. Finally, the design will be most effective if the investigator employs the appropriate analysis (to be discussed in the next section), properly interprets the results, and uses them to iterate the process and supplement the data, if necessary.

In an actual experiment, following the screening phase, it is likely that the above design would be amended by collecting additional data as prescribed by the holistic approach (Simon, 1977a; 1979). The purpose of adding more data might be to add new factors, shift the range of old ones, obtain more precise

information on the more important ones, isolate potentially critical effects from their aliased sources, and/or improve the fit of the response surface (if one exists).

SECTION VI

ANALYSES AND INTERPRETATIONS

Theoretically, $N-1$ effects can be isolated from a design composed of N independent experimental conditions. In the case of the $N = 32$, TCR design, 31 effects can be isolated. Because the design is in fact oversaturated with identifiable sources of variance (i.e., there are more known sources of variance than there are degrees of freedom) the experimenter must decide in which way he will cut the pie, which sources he will try to isolate. As in any oversaturated design, the remaining sources are confounded with those being isolated, introducing the specter of possible bias should they prove to have significant effects of their own. In the previous sections, a TCR design was selected that would reduce the risk of this danger. Still, the experimenter has a number of options when he analyzes the data from this design and must select the appropriate one(s) taking into consideration the information demanded by the experiment, the capability of the design to provide it, characteristics of the real world that provide a priori guesses as to what might be important, and the cost of drawing wrong conclusions. If the design is primarily used as a screening design, precise estimates of the effects are less important at this stage, and ordinarily the investigator may collect additional data to resolve some ambiguities. Furthermore, the investigator is not limited to a single analysis, and in fact, if the term "research" is to be taken seriously, he will "play" with his data as a Daniel or a Tukey might do. Still, a word of caution is in order. In general, selective examination of nonorthogonal data can be dangerous. This general principle applies here and the investigator should be cautious in his interpretation.

Below, some alternative analyses are suggested for the $N = 32$, TCR Resolution IV design just developed. The investigator can probably think of others. The direct effect labels apply to the columns in Table 1.

ANALYSIS 1

The 31 degrees of freedom might be partitioned in the following manner:

7 MAIN EFFECTS:

Original factor labels:	All with an AB-- term in label except AB
New factor labels	A, B, C, D, F, G, H

7 TWO-FACTOR INTERACTION STRINGS:

Original factor labels	All without a B-- term in label
New factor labels	AB+, AC+, AD+, AF+, AG+, AH+, AL+

9 THREE-FACTOR INTERACTION STRINGS:

Original factor labels	Unused columns with an A-- term in label
New factor labels	E, I, J, K, L, M, N, O, P

8 FOUR-FACTOR INTERACTION STRINGS:

Original factor labels	Remaining columns, all with a B-- term in label
New factor labels	AE+, AI+, AJ+, AK+, AM+, AN+, AO+, AP+

In this partitioning, no error variance is estimated with which to do a conventional test of statistical significance. This in itself is not a serious deficiency since it is preferable to use a normal-order plot (Simon, 1977a; Daniel, 1976) to estimate the criticalness of the effects in a 2^{k-p} multifactor design of this size. Should an external estimate of error still be desired -- to use and compare with the one derived from the normal-order plot as well as to use as a test of the goodness of fit of the data -- then data might be collected at the center of the experimental space (Simon, 1977a) if the factors are quantitative and continuous and have a center point. Some investigators have been satisfied with an estimate of error variance obtained by combining the data on the four-factor interaction strings based on the assumption that it is highly unlikely that four-factor interaction effects are more than a chance occurrence.

This analysis plan follows the traditional approach which fails to recognize the existence of sequence effects during the analysis of a within-subject design. It differs, however, to the extent that the TCR design is likely to be less confounded with trend and carryover effects than most available fractional factorial designs. Still, failing to remove some of the marginally confounded carryover and trend effects may create a bias and is risky, considering it is not necessary. One might wish to contrast the results from this analysis with those from the other analyses presented here insofar as the direct effects are concerned. Do the estimates of the direct effects change

markedly when sequence effects are removed? Do the changes -- if they exist -- crucially affect the interpretation of the results for screening purposes?

ANALYSIS 2

We may be content only to remove the trend effects statistically and leave whatever carryover effects that might be present confounded with the direct effects (see Table 4). We may partial trends from the experimental effects using the equations suggested by Daniel and Wilcoxon (1966) to handle linear and quadratic trends and supplemented by Simon (1977a, p. 121-128) to handle cubic trends. Since the removal of three trend effects require three additional degrees of freedom, we must sacrifice three of the 31 columns -- those which are least likely to be statistically significant -- and use them to partial out the trends. The column selected must correlate with the trend effect it represents.

Since four-factor interactions are least likely to be critical, then we might select columns D, E, and DE (original labels) to represent linear, cubic, and quadratic trends, respectively.

ANALYSIS 3

If one wished to estimate the effects of all direct (main and two-factor interaction strings), carryover (additive and interactive), and trend effects in this $N = 32$, TCR design, 45 degrees of freedom would be the minimum number required. Since only 31 degrees of freedom are available, some kind of compromise must be made. The goal of such a compromise would be to estimate the 31 effects most likely to be significant and ignore, i.e., allow to remain confounded, effects believed on some a priori basis unlikely to be critical.

This plan allows us to estimate direct effects, both main and two-factor interaction strings, and also additive carryover main effects. This might be useful where there is outside empirical evidence that interactive carryover effects are negligible.

The following (not totally independent) sources of variance would be selected to be terms in a regression analysis:

7 MAIN-DIRECT EFFECTS:

A, B, C, D, F, G, H

7 TWO-FACTOR INTERACTION DIRECT STRINGS:

AB+, AC+, AD+, AF+, AG+, AH+, AL+

7 MAIN-ADDITIVE CARRYOVER EFFECTS:

A', B', C', D', F', G', H'

9 THREE-FACTOR INTERACTION STRINGS:

E, I, J, K, L, M, N, O, P

1 FOUR-FACTOR INTERACTION STRING:

AE+

If this analysis is used, the following conditions must be valid:

1. Interactive carryover effects are negligible.
2. Interactions among additive carryover effects are negligible.
3. Direct and carryover effects do not interact critically with one another.
4. Higher-order interactions (not included in the strings listed above) are negligible.

Whatever degrees of freedom are available from noncritical three-factor interaction strings can be used to isolate trend effects or serve as an internal estimate of error variance. Some three-factor interaction strings are confounded with main effects. If from the preexperiment analysis the investigator suspects the possibility of a critical three-factor interaction, care should be taken to assign the factors so that the interaction will be located in a string that is isolated from main effects. While main effects will be robust against trend and interactive carryover, failure to isolate any component of these sequential effects that is critical will inflate the error variance and reduce the sensitivity of a significance test. Column AE+ might be selected from among the unused columns representing four-factor interaction strings to isolate the linear and cubic trends.

ANALYSIS 4

An analysis might be performed with the same breakdown as shown above, except that interactive carryover columns are used instead of the additive ones. This might be the case when there is some evidence that they are likely to have the more severe effects.

However, because of the high degree of intercorrelation among interactive carryover effects, the analysis of their individual terms should not be taken seriously. Instead, their composite variance should be evaluated for statistical significance to decide whether further isolation is needed.

ANALYSES 5A AND 5B

One may be highly selective regarding the terms (columns) that will be included in an analyses in a way that will maximize the isolation of direct effects and sacrifice the information regarding carryover effects per se. Information regarding carryover would be used solely to partial these effects from the direct main and interaction effects. When the TCR design is used for screening purposes, this is probably the soundest analysis of them all.

Inspection of Table 4 shows the sources of variance most likely to bias direct main and interaction effects in the N = 32, TCR design. This information is summarized here (new factor labels):

Direct Effects	Confounded Sources
A	A', AL+'
B	
C	C', AG+'
D	
F	F', AD+', AC+", AH+"
G	
H	
AB+	AB+', LT, CT
AC+	AC+', LT, CT
AD+	AD+', F', AC+", AH+"
AF+	AF+', QT
AG+	AG+', C', QT
AH+	AH+'
AL+	AL+', A', CT

The sources to serve as terms in a regression analysis are: .

- 7 Direct main effects: A, B, C, D, F, G, H
- 7 Direct interaction effects: AB+, AC+, AD+, AF+, AG+, AH+, AL+
- 10 Additive carryover effects: A', C', F', AB+', AC+', AD+', AF+', AG+', AH+', AL+'
- 2 Interactive carryover effects: AC+", AH+"
- 3 Trend effects: LT, QT, CT
- 2 Residual

This plan uses 29 of the available 31 degrees of freedom for identifiable effects, leaving two for whatever purpose the experimenter deems important to isolate.

Given that plan, the investigator may approach the analysis in several ways:

A. DEVELOP A PREDICTION EQUATION. Prepare a matrix composed only of the columns for the above terms along with the performance associated with each experimental condition. Perform a multiple (possibly stepwise) regression analysis.

B. ESTIMATE DIRECT EFFECTS. Partial from each direct effect the source(s) specifically confounded with it to obtain a less confounded estimate of the particular effect (see Daniel, 1976, p. 226-232, for ways of examining partially confounded data).

The exact strategies for these analyses are not supplied since circumstances will differ and the lines to follow are best left up to the ingenuity of the investigator. There may be other more appropriate analyses for particular purposes. Those readers who have used a "cookbook" approach to experimentation in the past -- canned designs from a book and the results from a computerized analysis of variance -- may be uncomfortable with the task of having to think about the analysis and interpretation and to have to work with effects that are only quasi-orthogonal. If so, as a general model, one may find Daniel's book (1976) a breath of fresh air in this regard, as he illustrates the thought processes employed to extract useful information from slightly marred data. Where limitations on subjects and data collection time permit, a larger design might be employed to provide more trend-carryover robust columns from which to select. Here the investigator must decide whether that approach is more cost-effective than accepting some uncertainty in the first stage of the screening process and using their resources in a second stage once the location of ambiguities have been better identified.

For those who question the value of a design that requires such exercises, were they not to use a TCR design in the presence of possible sequence effects, then they must suggest an alternative plan that can examine (screen) a great many factors as economically while keeping their estimated effects robust against the possible biases. Certainly ignoring or overlooking the problem, the historical "solution" for the past 100 years, is not the answer.

SECTION VII

CONCLUSIONS

When a single subject is tested serially on all conditions of an experiment, a 2^{k-p}_{IV} holistic design can be modified in a way that will make the experimental effects robust against linear, quadratic, and cubic trends as well as additive and interactive carryover effects that may exist. Robustness relates more to the effect -- the difference between means -- than to the means themselves. This makes this design particularly useful as a screening device with which the more important factors out of a large number of potentially critical ones can be identified. Valid mean values are better obtained after the number of conditions have been markedly reduced to a relatively few, selected by the screening process. At that time, each configuration would be evaluated under operational conditions using different groups of representative subjects.

$N = 64$ TCR designs should be particularly attractive since they not only would allow 10 to 12 factors to be examined using a single subject with minimum concern for sequence effects, but the size is enough to provide reasonably precise and generalizable answers at marginal costs. Additional data might be required once the initial block has been collected in accordance with the principles associated with the holistic approach and screening studies (Simon, 1977a, 1977b, 1979).

The design presented in this report demonstrates that it is possible to have a practical, cost-effective TCR 2^{k-p}_{IV} holistic design. If one wished to estimate both direct and carryover effects, then totally new designs for that purpose must be developed instead of modifying existing ones as was done in this report. While such designs are likely to require that more data be collected, the additional information gained would ordinarily justify the cost.

FOOTNOTES

- [1] A holistic approach to behavioral research which emphasizes the importance of accounting for as many potentially critical variables as possible, whether equipment, environment, subject, or temporal, controlled or uncontrolled, whether generated in the real world for a specific task or within the experimental setting. This approach involves a sequential strategy and bundle of techniques that enables the response surface of a large multidimensional space of practical importance to be estimated while reducing irrelevant sources of variance, all at minimum data collection costs.
- [2] When factors are aliased in 2^k or 2^{k-p} designs, the experimental conditions being compared to determine each factor's effects are identical, except for a possible reversal of signs. When it has been determined that the effect of a string composed of aliased two-factor interactions is significantly large, it is not possible to determine, without additional data collection, which interactions in the string are responsible for the observed effect. For example, the column labelled AB+ in Table 1 would actually be measuring the combined effects of a string of two-factor interactions, specifically, the combined effects of (AB + CE + DF + GH), with each term contributing an unknown amount (Appendix II). Fractional factorial designs of the same size and resolution can differ as to the number of two-factor interactions that will be confounded (Fries and Hunter, 1978). The experimenter must decide which designs best suits his needs.
- [3] Some published fractional factorial designs (e.g., by the U.S. Department of Commerce, 1957) may be trend robust and have a better aberration pattern than the holistic design described here. However, some may not be trend robust and there is usually no indication either way to help the user decide. They may be used, of course, provided the experimenter first identifies what the original factor labels of the columns are (using the letters and patterns of Yates' standard order and by referencing Table 1) and does not use those too seriously confounded with trend effects.
- [4] No taxonomy has ever been developed that relates tasks or task characteristics to the types and strengths of carryover and other sequence effects that are likely to exist. That would be a worthy project and one of considerable value for optimizing the selection and effective use of an experimental design.

- [5] This sequence is the AB sign pattern in a conventional 2^5 factorial design arranged in Yates' standard order. Repeating the ABBA pattern for any within-subject plan of length $N = 2^k$ is the best alternative for sequencing two conditions when trends and additive and interactive carryover effects are considered potentially critical. As in the case when any experimental design is selected, the investigator must determine whether the choice is suitable for the particular task being studied. For example, the design would not be appropriate if the carryover from any early condition resulted in a permanent change throughout the remainder of the experiment since the plan is to be used only when the effects last for essentially one trial. Designs for examining more than one trial carryover effect do exist (Simon, 1974).
- [6] Actually, the sources of variance are not fully isolatable. While the direct and carryover patterns for the ABBA-- pattern are orthogonal, intercorrelations between direct or carryover patterns and linear, quadratic, or cubic trends are not totally independent. There are small correlations which for all practical purposes are inconsequential and do not destroy the effectiveness of the data collection plan. Specifically, the direct pattern correlates 0.0131 with quadratic trend; this is a 0.02% overlap. The additive carryover pattern for this sequence correlates -0.1083 with linear trend and -0.1634 with cubic trend; these are 1.17% and 2.67% overlaps respectively. The interactive carryover pattern for this sequence correlates -0.0542 with linear trend and -0.0829 with cubic trend; these are 0.29% and 0.69% overlaps respectively.
- [7] When the experimental conditions of a 2^k factorial design are arranged in Yates' standard order, the sign patterns for the columns of a 2^k factorial design, for example, are the same as those for any 2^k condition, two-level fractional factorial design. For example, the sign patterns of the columns of a 2^5 factorial are the same as those for a 2^{16-11} , 2^{6-1} , or 2^{13-8} fractional factorial design. The correlations among the columns are the same for all two-level factorial-like designs with 32 conditions. Rearranging the columns of one of these designs to form the trend-robust holistic design does not affect the correlation once the proper labels are identified.
- [8] Note that AL+ is the only one of the interaction-string columns in which both letters in its label are not found represented among the main effect labels; L is not there. Actually, the visible letters of a string's label are only the name of the string, not necessarily the interactions of interest within the string. In the present design, adapted from that shown in Table 1, the 21 relevant two-factor interactions are aliased three to a column, including the column designated new label AL+ (see Appendix B).

NOTATIONS*

2^{k-p} design

A fractional factorial design capable of isolating the effects of K factors at two levels each using a $1/2^p$ fraction of the total design. If p equal 0, the design is a full factorial. If a subscript, IV, is added, that indicates that the design is of Resolution IV.

A, A', A'', and other letters from A through P

Used to label the columns of the experimental design. No superscript indicates a column that represents a direct effect; a single one ('), an additive carryover effect; and a double one ("), an interactive carryover effect.

A--, AB--

Represents a main or interaction term with an A or an AB in its label. For example, ACDE, AF, or ABC, ABDF, ABE, etc.

AB+, AB+', AB+", and other letters from A through P

The plus sign indicates that the designated interaction is the label of a column containing a string of interactions.

D

When referring to a correlation, indicates that its magnitude for a TCR design is "dangerously" high, i.e., from .55 and .70, making the percent overlap equal to 38% to 49%. Its symbol in Table 3 is $\boxtimes$.

* Letters A to P are used as new factor labels for main effects in the 2^{7-2} experimental design discussed in this paper. Some of these letters are also used to represent other things. The distinction should be clear from the context.

[I]

The identity column of a 2^k factorial or fractional factorial design. All signs are plus. $-[I]$ would be used if all signs in the column are minus.

M

When referring to a correlation, indicates that its magnitude for a TCR design is "moderately" high, i.e., from .32 to .54, making the percent overlap equal to 10% to 29%. Its symbol in Table 3 is $\boxminus$.

T

When referring to a correlation, indicates that its magnitude for a TCR design is "trivial," i.e., from 0+ to .31, making the percent overlap equal to .00+ to .09. Its symbol in Table 3 is $\boxdot$.

TCR design

Design that is robust against trends and carryover effects.

U

When referring to a correlation, indicates that its magnitude for TCR design is "unacceptably" high, i.e., from .71 or higher, making the percent overlap equal to 50% or higher. Its symbol in Table 3 is $\blacksquare$.

GLOSSARY

aliases

Term used when sources of variance in an experimental design are 100% confounded and their effects cannot be independently estimated. This aliasing occurs when, in a fractional factorial design, a contrast that estimates one effect also estimates one or more other effects.

between-subject design

Design in which a different subject or subjects are tested on each experimental condition.

carryover effect

When the characteristics of a condition affects the performance on a different condition that follows it, carryover is said to have taken place. In the literature, the terms "carryover," "crossover," and "transfer" have been used synonymously. Additive carryover occurs when the size of the carryover effect is the same for any condition whatever the condition that follows it might be. Interactive carryover occurs when the size of a carryover effect for a condition differs depending on what condition follows it.

holistic approach

A unique sequential process for ultimately mapping a large multifactor-multivariate space. The approach is composed of a philosophy, a strategy, and a bundle of techniques that require and enable an investigator to economically study under controlled conditions "all" of the relevant factors affecting performance on a specific task. Factors to be considered are from sources associated with equipment, environment, subject, or temporal conditions, as well as unwanted biases that must be reduced.

new factor labels

The labels for the experimental designs used when the sign matrix is perceived as a fractional factorial. This would be the case when it is used as a screening design.

original factor labels

The labels used when the sign matrix is perceived as a full factorial in Yates' standard order. These labels are most useful when one must mentally manipulate the design.

percent overlap

This refers to the proportion (times 100) of confounding between two columns or factors in an experimental design. Percent overlap equals the correlation squared.

Resolution IV

Term used to indicate the extent and nature of the aliasing in a fractional factorial design. The IV indicate that all main effects are independent of one another and of all two-factor interactions, and the two-factor interactions are aliased within independent two-factor interaction strings.

screening design

A fractional factorial design capable of studying the effects of a large number of factors economically under controlled conditions in order to discover which few are primarily responsible for the performance on a particular task. It is used most effectively when employed with the holistic approach.

string, interaction

A group of totally aliased interactions, the effects of which cannot be independently estimated. In this report, an interaction string is designated by a plus sign following the interaction label, e.g., AB+.

trend

An effect that progresses systematically from trial to trial through the experiment. Of interest in this report are linear (L or LT), quadratic (Q or QT), and cubic (C or CT) trends.

within-subject design

Design in which a subject is tested serially on all the conditions of an experiment.

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APPENDIX A

MAIN AND THREE-FACTOR INTERACTION EFFECTS ALIASED
IN THE 2^{16-11}_{IV} TREND RESISTANT SCREENING DESIGN IN
TABLE 1.

NEW MAIN EFFECTS	ORIGINAL FACTOR LABELS	ALIASED THREE-FACTOR INTERACTION STRINGS														
O	AE	ABP CFP HLP	ACK CGI IJN	ADJ CHJ IKM	AEH CLM JMP	AFN DFI KNP	AGM DGP	AIL DHK	BCN DLN	BDM EFG	BEL EIP	BFK EJK	BGJ EMN	BHI FHM	CDE FJL	GKL GHM
K	ADE	ABN CFN HLN	ACO CGM IJP	ADE CIL IMO	AFP DFM JMN	AGI DGN NOP	AHJ DHO	ALM DLP	BCP EFL	BDI EIN	BEG EJO	BFO EMP	BHM FGJ	BJL FHI	CDJ GHP	CEH GLO
N	AD	ABK CFK HKL	ACP CGJ IJO	ADI CHI IMP	AEG DFJ JKM	AFO DGK KOP	AHM DHP	AJL DLO	BCO EFH	BDE EIK	BFP EJP	BGI EMO	BHJ FGM	BLM FIL	CDM GHO	CEL GLP
J	ACE	ABM CGN HLM	ACE CHO IKP	ADO CLP INO	AFI DEH KMN	AGP DFN MOP	AHK DGM	ALN DIL	BCI EGL	BOP EIM	BEF EKO	BGO ENP	BHN FGK	BKL FHP	CDK FLO	CFM GHI
F	ABC	ABC CHL HJP	ADL CJM HMO	AEM CKN ILN	AGH COP JLO	AIJ DEP LMP	AKP DIO	ANO DJN	BDH DKM	BEJ EGO	BGL EHN	BIM EKL	BKO GIP	BNP GJK	CDG GMN	CEI HIK
L	AB	ABH CIK GKO	ACG CJP GNP	ADF CMO HJM	AEP DEM HKN	AIO DGH HOP	AJN DIJ	AKM DKP	BCD DNO	BEO EFK	BFG EGJ	BIP EHI	BJK FIN	BMN FJO	CEN FMP	CFH GIM
P	A	ABO CFO HLO	ACN CHM IJK	ADM CJL IMN	AEL DEF JMO	AFK DGO KNO	AGJ DHN	AHI DKL	BCK EIO	BDJ EJN	BEH EKM	BFN FGI	BGM FHJ	BIL FLM	CDI GHK	CEG GLN
E	ACDE	ABI CGP IOP	ACJ CHK JKO	ADK CLN JNP	AFM DFP KMP	AGN DGI MNO	AHO DHJ	ALP DLM	BCM FGO	BDN FHN	BFJ FKL	BGK GHM	BHP GJL	BLO HIL	CDO IJM	CFI IKN
I	ACD	ABE CGO GLM	ACM CHN JKP	ADN CKL JNO	AFJ DEG KMO	AGK DFO MNP	AHP DHM	ALO DJL	BCJ EHL	BDK EJM	BFM EKN	BGN EOP	BHO FGP	BLP FHK	CDP FLN	CEF GHJ
M	AC	ABJ CGK HJL	ACI CHP IKO	ADP CLO INP	AEF DEL JKN	AGO DFK JOP	AHN DGJ	AKL DHI	BCE EGH	BDO EIJ	BFI EKP	BGP ENO	BHK FGN	BLN FHO	CDN FLP	CFJ GIL
H	ABE	ABL CIN GKP	ACD CJO GNO	AEO CMP JLM	AFG DEJ KLN	AIP DGL LOP	AJK DIM	AMN DKO	BCG DNP	BDF EFN	BEP EGM	BIO EIL	BJN FIK	BKM FJP	CEK FMO	CFL GIJ
D	ABDE	ABG CIP HKP	ACH CJK HNO	AEK CMN ILM	AFL EFP KLO	AIN EGI LNP	AJO EHJ	AMP ELM	BCL FIO	BEN FJN	BFH FKM	BIK GHL	BJP GJM	BMO GKN	CEO GOP	CFG HIM
C	ABCE	ABF DIP HJO	ADH DJK HMP	AEJ DMN IKL	AGL EFI JLP	AIM EGP LMO	AKO EHK	ANP ELN	BDL FHL	BEM FJM	BGH FKN	BIJ FOP	BKP GIO	BNO GJN	DEO GKM	DFG HIN
A	ABCDE	BCF DIN HJK	B DG DJO HMN	BEI DMP ILO	BHL EFM JLN	BJM EGN KLM	BKN EHO	BOP ELP	CDH FGH	CEJ FIJ	CGL FKP	CIM FNO	CKO GIK	CNP GJP	DEK GMO	DFL HIP
B	ABCD	ACF DIK HJN	ADG DJP HKM	AEI DMO ILP	AHL EFJ JKL	AJM EGK LMN	AKN EHP	AOP ELO	CDL FGL	CEM FIM	CGH FKO	CIJ FNP	CKP GIN	CNO CJO	DEN GMP	DFH HIO
G	ABD	ABD CIO HKP	ACL CJN HNO	AEN CKM ILM	AFH DEI KLO	AIK DHL LNP	AIP DJM	AMO DKN	BCH DOP	BEK EFO	BFI EHM	BIN EJL	BJO FIP	BMP FJK	CDF FMN	CEP HIJ

APPENDIX B

TWO-FACTOR INTERACTION EFFECTS ALIASED IN THE 2^{16-11}_{IV}
TREND RESISTANT SCREENING DESIGN IN TABLE 1.

	ORIGINAL FACTOR LABELS														
	E	DE	D	CE	CDE	CD	C	BE	BDE	BD	BCE	BCDE	BCD	BC	B
STRINGS OF ALIASED TWO-FACTOR INTERACTIONS (NEW LABELS)	AB	AF	AC	AG	AL	AH	AD	AI	AM	AJ	AN	AP	AO	AK	AE
	CF	BC	BF	BD	BH	BL	BG	BE	BJ	BM	BK	BO	BP	BN	BI
	DG	DL	DH	CL	CG	CD	CH	CM	CI	CE	CP	CN	CK	CO	CJ
	EI	EM	EJ	EN	DF	EO	EK	DN	DP	DO	DI	DM	DJ	DE	DK
	HL	GH	GL	FH	EP	FG	FL	FJ	EF	FI	EG	EL	EH	FP	FM
	JM	IJ	IM	IK	IO	IP	IN	GK	GO	GP	FO	FK	FN	GI	GN
	KN	KP	KO	JP	JN	JK	JO	HP	HN	HK	HM	GJ	GM	HJ	HO
	OP	NO	NP	MO	KM	MN	MP	LO	KL	LN	JL	HI	IL	LM	LP

APPENDIX C

INTERCORRELATIONS AMONG DIRECT, CARRYOVER AND TREND EFFECT COLUMNS (2^5 factorial or 2^{16-11} fractional factorial)

A complete table of intercorrelations for a five-factor, two-levels per factor, factorial with direct, additive carryover, and interactive carryover columns would involve a 93×93 matrix, symmetrical about the diagonal. This matrix can be conveniently sectioned into nine 31×31 submatrices as shown in Table 2. However, as discussed in the body of this report, because some sections are duplicates of others and because some are unit matrices, it is only necessary to show three sections to allow a user to know the intercorrelations among all of the 93×93 combinations in the complete matrix. To examine the robustness of the columns against linear, quadratic, and cubic trend effects, three additional rows of correlations are added to each matrix, making it a 34×31 intercorrelation table. These tables -- the numerical equivalents of those in Table 3 -- are shown in this appendix.

Since the sign patterns in the columns and the relationships among the columns remain the same for equivalent-sized fractional-factorial designs, the information provided here applies to a 2^{16-11} fractional factorial design and comparable designs. The column labels are changed for that purpose and aliasing occurs. These relationships are discussed in the body of the text and by Simon (1973; 1977a; 1981).

Employing the labels and notations used in Tables 1, 3, and 4, the new factor label equivalents of the original factor labels found in this appendix are as follows:

Labels		Labels		Labels		Labels	
Orig.	New	Orig.	New	Orig.	New	Orig.	New
A	P	D	AC+	E	AB+	DE	AF+
B	AE+	AD	N	AE	O	ADE	K
AB	L	BD	AJ+	BE	AI+	BDE	AM+
C	AD+	ABD	G	ABE	H	ABDE	C
AC	M	CD	AH+	CE	AG+	CDE	AL+
BC	AK+	ACD	I	ACE	J	ACDE	E
ABC	F	BCD	AO+	BCE	AN+	BCDE	AP+
		ABCD	B	ABCE	C	ABCDE	A

The plus sign after each new label two-factor interaction indicates that it is the label of a string of aliased two-factor interactions (see Appendix B). Nothing, an apostrophe, or a quotation mark would follow each label to indicate whether the labels apply to direct, additive carryover, or interactive carryover columns, respectively.

MATRIX C-1

ADDITIVE CARRYOVER

	A'	B'	AB'	C'	AC'	BC'	ABC'	D'
DIRECT	A	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000
	AB	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C	0.0000	0.0000	0.0000	0.5000	-0.5000	-0.5000	0.0000
	AC	0.0000	0.0000	0.0000	0.5000	-0.5000	0.5000	0.0000
	BC	0.0000	0.0000	0.0000	0.5000	0.5000	-0.5000	0.0000
	ABC	0.0000	0.0000	0.0000	-0.5000	-0.5000	0.5000	0.0000
	D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7500
	AD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500
	BD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500
	ABD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2500
	CD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500
	ACD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2500
	BCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2500
	ABCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500
	E	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	DE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ADE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCD E	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TREND	LINEAR	-0.0542	0.0000	-0.1083	0.1083	-0.1083	-0.1083	0.3249
	QUAD	0.0000	0.0131	0.0000	0.0394	0.0131	0.0394	0.0919
	CUBIC	-0.0829	0.0000	-0.1634	0.1488	-0.1585	-0.1488	0.3097

ADDITIVE CARRYOVER

	AD'	BD'	ABD'	CD'	ACD'	BCD'	ABCD'	E'
DIRECT	A	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D	-0.2500	-0.2500	-0.2500	-0.2500	-0.2500	-0.2500	0.0000
	AD	-0.7500	0.2500	0.2500	0.2500	0.2500	0.2500	0.0000
	BD	0.2500	0.2500	-0.7500	0.2500	0.2500	0.2500	0.0000
	ABD	-0.2500	0.7500	-0.2500	-0.2500	-0.2500	-0.2500	0.0000
	CD	0.2500	0.2500	0.2500	0.7500	-0.2500	-0.2500	0.0000
	ACD	-0.2500	-0.2500	-0.2500	0.2500	-0.7500	0.2500	0.0000
	BCD	-0.2500	-0.2500	-0.2500	0.2500	0.2500	-0.7500	0.0000
	ABCD	0.2500	0.2500	0.2500	-0.2500	-0.2500	0.7500	0.0000
	E	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7500
	AE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	BE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	ABE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	CE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	ACE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	BCE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	ABCE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	DE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	ADE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	BDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	ABDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	CDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
	ACDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	BCDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250
	ABCDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.1250
TREND	LINEAR	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	0.7591
	QUAD	0.0394	0.0919	-0.0131	0.1970	-0.0131	-0.0131	0.1970
	CUBIC	-0.1488	-0.1293	-0.1488	-0.0902	-0.1293	-0.0902	-0.4610

ADDITIVE CARRYOVER

	AE'	BE'	ABE'	CE'	ACE'	BCE'	ABCE'	DE'
DIRECT	A	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	E	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	AE	-0.0750	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	BE	0.1250	0.1250	-0.0750	0.1250	0.1250	0.1250	0.1250
	ABE	-0.1250	0.0750	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	CE	0.1250	0.1250	0.1250	0.6250	-0.3750	-0.3750	-0.1250
	ACE	-0.1250	-0.1250	-0.1250	0.3750	-0.6250	0.3750	-0.1250
	BCE	-0.1250	-0.1250	-0.1250	0.3750	0.3750	-0.6250	-0.1250
	ABCE	0.1250	0.1250	0.1250	-0.3750	-0.3750	-0.3750	0.1250
	DE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.8750
	ADE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	0.1250
	BDE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	0.1250
	ABDE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	-0.1250
	CDE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	0.1250
	ACDE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	-0.1250
	BCDE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	-0.1250
	ABCDE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	0.1250
TREND	LINEAR	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083
	QUAD	0.0919	0.1970	-0.0131	0.4072	-0.0131	-0.0131	0.4274
	CUBIC	-0.1293	-0.0902	-0.1293	-0.0122	-0.0902	-0.0122	0.1439

ADDITIVE CARRYOVER

	ADE'	BDE'	ABDE'	CDE'	ACDE'	BCDE'	ABCDE'
DIRECT	A	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	E	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	AE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	BE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	ABE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	CE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	ACE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	BCE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	ABCE	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
	DE	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250	-0.1250
	ADE	-0.0750	0.1250	0.1250	0.1250	0.1250	0.1250
	BDE	0.1250	0.1250	-0.0750	0.1250	0.1250	0.1250
	ABDE	-0.1250	0.0750	-0.1250	-0.1250	-0.1250	-0.1250
	CDE	0.1250	0.1250	0.1250	0.6250	-0.3750	-0.3750
	ACDE	-0.1250	-0.1250	-0.1250	0.3750	-0.6250	0.3750
	BCDE	-0.1250	-0.1250	-0.1250	0.3750	0.3750	-0.6250
	ABCDE	0.1250	0.1250	0.1250	-0.3750	-0.3750	-0.3750
TREND	LINEAR	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083	-0.1083
	QUAD	-0.0131	-0.0131	-0.0131	-0.0131	-0.0131	-0.0131
	CUBIC	-0.0122	0.1439	-0.1083	0.4561	-0.1683	-0.1683

MATRIX C-2* (see footnote)

	DIRECT							
	A	B	AB	C	AC	BC	ABC	D
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB*	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C*	0.5774	0.5774	-0.5774	0.0000	0.0000	0.0000	0.0000
	AC*	-0.5774	-0.5774	0.5774	0.0000	0.0000	0.0000	0.0000
	BC*	0.5774	-0.5774	0.5774	0.0000	0.0000	0.0000	0.0000
	ABC*	-0.5774	0.5774	-0.5774	0.0000	0.0000	0.0000	0.0000
	D*	0.3780	0.3780	-0.3780	0.3780	-0.3780	0.3780	0.0000
	AD*	-0.3780	-0.3780	0.3780	-0.3780	0.3780	-0.3780	0.0000
	BD*	0.7746	-0.2582	0.2582	-0.2582	0.2582	-0.2582	0.0000
	ABD*	-0.7746	0.2582	-0.2582	0.2582	-0.2582	0.2582	0.0000
	CD*	0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780	0.0000
	ACD*	-0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780	0.0000
	BCD*	0.7746	-0.2582	0.2582	-0.2582	0.2582	-0.2582	0.0000
	ABCD*	-0.7746	0.2582	-0.2582	0.2582	-0.2582	0.2582	0.0000
	E*	0.2582	0.2582	-0.2582	0.2582	0.2582	-0.2582	-0.2582
	AE*	-0.2582	-0.2582	0.2582	-0.2582	0.2582	-0.2582	-0.2582
	BE*	0.8819	-0.1260	0.1260	-0.1260	0.1260	-0.1260	-0.1260
	ABE*	-0.8819	0.1260	-0.1260	0.1260	-0.1260	0.1260	-0.1260
	CE*	0.4804	0.4804	-0.4804	-0.1601	0.1601	-0.1601	-0.1601
	ACE*	-0.4804	-0.4804	0.4804	0.1601	-0.1601	0.1601	-0.1601
	BCE*	0.6742	-0.4045	0.4045	0.1348	-0.1348	0.1348	-0.1348
	ABCE*	-0.6742	0.4045	-0.4045	-0.1348	0.1348	-0.1348	-0.1348
	DE*	0.2582	0.2582	-0.2582	0.2582	0.2582	-0.2582	-0.2582
	ADE*	-0.2582	-0.2582	0.2582	-0.2582	0.2582	-0.2582	-0.2582
	BDE*	0.8819	-0.1260	0.1260	-0.1260	0.1260	-0.1260	-0.1260
	ABDE*	-0.8819	0.1260	-0.1260	0.1260	-0.1260	0.1260	-0.1260
	CDE*	0.4804	0.4804	-0.4804	-0.1601	0.1601	-0.1601	-0.1601
	ACDE*	-0.4804	-0.4804	0.4804	0.1601	-0.1601	0.1601	-0.1601
	BCDE*	0.6742	-0.4045	0.4045	0.1348	-0.1348	0.1348	-0.1348
	ABCDE*	-0.6742	0.4045	-0.4045	-0.1348	0.1348	-0.1348	-0.1348
TREND	LINEAR	0.0542	0.1083	0.0000	0.2166	0.0000	0.0000	0.4332
	QUAD	0.0000	0.0000	0.0131	0.0000	0.0263	0.0000	0.0000
	CUBIC	0.0029	0.1634	0.0000	0.3073	0.0000	0.0098	0.4585

	DIRECT							
	AD	BD	ABD	CD	ACD	BCD	ABCD	E
INTERACTIVE CARRYOVER	A*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AB*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	C*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABC*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	D*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ACD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BCD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	ABCD*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	E*	-0.2582	-0.2582	0.2582	-0.2582	0.2582	-0.2582	0.0000
	AE*	0.2582	0.2582	-0.2582	0.2582	-0.2582	0.2582	0.0000
	BE*	0.1260	0.1260	-0.1260	0.1260	-0.1260	0.1260	0.0000
	ABE*	-0.1260	-0.1260	0.1260	-0.1260	0.1260	-0.1260	0.0000
	CE*	0.1601	0.1601	-0.1601	0.1601	-0.1601	0.1601	0.0000
	ACE*	-0.1601	-0.1601	0.1601	-0.1601	0.1601	-0.1601	0.0000
	BCE*	-0.1348	-0.1348	0.1348	-0.1348	0.1348	-0.1348	0.0000
	ABCE*	0.1348	0.1348	-0.1348	0.1348	-0.1348	0.1348	0.0000
	DE*	0.2582	0.2582	-0.2582	0.2582	0.2582	-0.2582	0.0000
	ADE*	-0.2582	-0.2582	0.2582	-0.2582	0.2582	-0.2582	0.0000
	BDE*	-0.1260	-0.1260	0.1260	-0.1260	0.1260	-0.1260	0.0000
	ABDE*	0.1260	0.1260	-0.1260	0.1260	-0.1260	0.1260	0.0000
	CDE*	-0.1601	-0.1601	0.1601	-0.1601	0.1601	-0.1601	0.0000
	ACDE*	0.1601	0.1601	-0.1601	0.1601	-0.1601	0.1601	0.0000
	BCDE*	0.1348	0.1348	-0.1348	0.1348	-0.1348	0.1348	0.0000
	ABCDE*	-0.1348	-0.1348	0.1348	-0.1348	0.1348	-0.1348	0.0000
TREND	LINEAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8664
	QUAD	0.0525	0.1051	0.0000	0.2101	0.0000	0.0000	0.0000
	CUBIC	0.0000	0.0000	0.0195	0.0000	0.0390	0.0780	-0.3317

* The Additive vs Interactive Carryover matrix is the same as Matrix C-2 except for reversed signs in all columns with an odd number of letters in the label.

MATRIX C-3

INTERACTIVE CARRYOVER

	A"	B"	AB"	C"	AC"	BC"	ABC"	D"
INTERACTIVE CARRYOVER	A"	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B"	0.0000	1.0000	-1.0000	0.5774	-0.5774	0.5774	-0.5774
	AB"	0.0000	-1.0000	1.0000	-0.5774	0.5774	-0.5774	0.5774
	C"	0.0000	0.5774	-0.5774	1.0000	-1.0000	0.3333	-0.3333
	AC"	0.0000	-0.5774	0.5774	-1.0000	1.0000	0.3333	-0.3333
	BC"	0.0000	0.5774	-0.5774	0.3333	0.3333	-1.0000	1.0000
	ABC"	0.0000	-0.5774	0.5774	0.3333	-0.3333	-1.0000	1.0000
	D"	0.0000	0.3780	-0.3780	0.6547	-0.6547	0.2182	-0.2182
	AD"	0.0000	-0.3780	0.3780	-0.6547	0.6547	0.2182	-0.2182
	BD"	0.0000	0.7746	-0.7746	0.1491	-0.1491	0.7454	-0.7454
	ABD"	0.0000	-0.7746	0.7746	-0.1491	0.1491	-0.7454	0.7454
	CD"	0.0000	0.3780	-0.3780	0.6547	-0.6547	0.2182	-0.2182
	ACD"	0.0000	-0.3780	0.3780	-0.6547	0.6547	0.2182	-0.2182
	BCD"	0.0000	0.7746	-0.7746	0.1491	-0.1491	0.7454	-0.7454
	ABCD"	0.0000	-0.7746	0.7746	-0.1491	0.1491	-0.7454	0.7454
	E"	0.0000	0.2582	-0.2582	0.4472	-0.4472	0.1491	-0.1491
	AE"	0.0000	-0.2582	0.2582	-0.4472	0.4472	0.1491	-0.1491
	BE"	0.0000	0.8819	-0.8819	0.3637	-0.3637	0.6547	-0.6547
	ABE"	0.0000	-0.8819	0.8819	-0.3637	0.3637	-0.6547	0.6547
	CE"	0.0000	0.4804	-0.4804	0.8321	-0.8321	0.2774	-0.2774
	ACE"	0.0000	-0.4804	0.4804	-0.8321	0.8321	0.2774	-0.2774
	BCE"	0.0000	0.6742	-0.6742	0.0778	-0.0778	0.8563	-0.8563
	ABCE"	0.0000	-0.6742	0.6742	-0.0778	0.0778	-0.8563	0.8563
	DE"	0.0000	0.2582	-0.2582	0.4472	-0.4472	0.1491	-0.1491
	ADE"	0.0000	-0.2582	0.2582	-0.4472	0.4472	0.1491	-0.1491
	BDE"	0.0000	0.8819	-0.8819	0.3637	-0.3637	0.6547	-0.6547
	ABDE"	0.0000	-0.8819	0.8819	-0.3637	0.3637	-0.6547	0.6547
	CDE"	0.0000	0.4804	-0.4804	0.8321	-0.8321	0.2774	-0.2774
	ACDE"	0.0000	-0.4804	0.4804	-0.8321	0.8321	0.2774	-0.2774
	BCDE"	0.0000	0.6742	-0.6742	0.0778	-0.0778	0.8563	-0.8563
	ABCDE"	0.0000	-0.6742	0.6742	-0.0778	0.0778	-0.8563	0.8563
TREND	LINEAR	0.0000	0.0542	-0.0542	0.0938	-0.0938	0.0313	-0.0313
	QUAD	0.0000	0.0000	0.0000	-0.0076	0.0076	-0.0076	0.0076
	CUBIC	0.0000	0.0829	-0.0829	0.1422	-0.1422	0.0465	-0.0465

INTERACTIVE CARRYOVER

	AD"	BD"	ABD"	CC"	ACD"	BCD"	ABCD"	E"
INTERACTIVE CARRYOVER	A"	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B"	-0.3780	0.7746	-0.7746	0.3780	-0.3780	0.7746	-0.7746
	AB"	0.3780	-0.7746	0.7746	-0.3780	0.3780	-0.7746	0.7746
	C"	-0.6547	0.1491	-0.1491	0.6547	-0.6547	0.1491	-0.1491
	AC"	0.6547	-0.1491	0.1491	-0.6547	0.6547	-0.1491	0.1491
	BC"	0.2182	0.7454	-0.7454	-0.2182	0.2182	0.7454	-0.7454
	ABC"	-0.2182	-0.7454	0.7454	0.2182	-0.2182	-0.7454	0.7454
	D"	-1.0000	-0.2928	0.2928	-0.1429	0.1429	0.4880	-0.4880
	AD"	1.0000	0.2928	-0.2928	0.1429	-0.1429	-0.4880	0.4880
	BD"	0.2928	1.0000	-1.0000	0.4880	-0.4880	0.4667	-0.4667
	ABD"	-0.2928	-1.0000	1.0000	-0.4880	0.4880	-0.4667	0.4667
	CD"	0.1429	0.4880	-0.4880	1.0000	-1.0000	0.2928	-0.2928
	ACD"	-0.1429	-0.4880	0.4880	-1.0000	1.0000	0.2928	-0.2928
	BCD"	-0.4880	0.4667	-0.4667	-0.2928	0.2928	1.0000	-1.0000
	ABCD"	0.4880	-0.4667	0.4667	0.2928	-0.2928	-1.0000	1.0000
	E"	-0.6831	-0.2000	0.2000	-0.0976	0.0976	0.3333	-0.3333
	AE"	0.6831	0.2000	-0.2000	0.0976	-0.0976	-0.3333	0.3333
	BE"	-0.0476	0.8783	-0.8783	0.4286	-0.4286	0.6181	-0.6181
	ABE"	0.0476	-0.8783	0.8783	-0.4286	0.4286	-0.6181	0.6181
	CE"	-0.3026	0.2894	-0.2894	0.7868	-0.7868	0.0413	-0.0413
	ACE"	0.3026	-0.2894	0.2894	-0.7868	0.7868	-0.0413	0.0413
	BCE"	-0.1529	0.5919	-0.5919	-0.2548	0.2548	0.8704	-0.8704
	ABCE"	0.1529	-0.5919	0.5919	0.2548	-0.2548	-0.8704	0.8704
	DE"	-0.6831	-0.2000	0.2000	-0.0976	0.0976	0.3333	-0.3333
	ADE"	0.6831	0.2000	-0.2000	0.0976	-0.0976	-0.3333	0.3333
	BDE"	-0.0476	0.8783	-0.8783	0.4286	-0.4286	0.6181	-0.6181
	ABDE"	0.0476	-0.8783	0.8783	-0.4286	0.4286	-0.6181	0.6181
	CDE"	-0.3026	0.2894	-0.2894	0.7868	-0.7868	0.0413	-0.0413
	ACDE"	0.3026	-0.2894	0.2894	-0.7868	0.7868	-0.0413	0.0413
	BCDE"	-0.1529	0.5919	-0.5919	-0.2548	0.2548	0.8704	-0.8704
	ABCDE"	0.1529	-0.5919	0.5919	0.2548	-0.2548	-0.8704	0.8704
TREND	LINEAR	-0.1433	-0.0419	0.0419	-0.0205	0.0205	0.0699	-0.0699
	QUAD	0.0347	0.0237	-0.0237	0.0248	-0.0248	-0.0170	0.0170
	CUBIC	-0.2129	-0.0598	0.0598	-0.0267	0.0267	0.1039	-0.1039

INTERACTIVE CARRYOVER								
	AE"	BE"	ABE"	CE"	ACE"	BCE"	ABCE"	DE"
INTERACTIVE CARRYOVER	A"	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B"	-0.2582	0.8819	-0.8819	0.4804	-0.4804	0.6742	-0.6742
	AB"	0.2582	-0.8819	0.8819	-0.4804	0.4804	-0.6742	0.6742
	C"	-0.4472	0.3637	-0.3637	0.8321	-0.8321	0.0778	-0.0778
	AC"	0.4472	-0.3637	0.3637	-0.8321	0.8321	-0.0778	0.0778
	BC"	0.1491	0.6547	-0.6547	0.2774	-0.2774	0.8563	-0.8563
	ABC"	-0.1491	-0.6547	0.6547	0.2774	-0.2774	-0.8563	0.8563
	D"	-0.6831	0.0476	-0.0476	0.3026	-0.3026	0.1529	-0.1529
	AD"	0.6831	-0.0476	0.0476	-0.3026	0.3026	-0.1529	0.1529
	BD"	0.2000	0.8783	-0.8783	0.2894	-0.2894	0.5919	-0.5919
	ABD"	-0.2000	-0.8783	0.8783	-0.2894	0.2894	-0.5919	0.5919
	CD"	0.0976	0.4286	-0.4286	0.7868	-0.7868	0.2548	-0.2548
	ACD"	-0.0976	-0.4286	0.4286	-0.7868	0.7868	-0.2548	0.2548
	BCD"	-0.3333	0.6181	-0.6181	0.0413	-0.0413	0.8704	-0.8704
	ABCD"	0.3333	-0.6181	0.6181	0.0413	-0.0413	-0.8704	0.8704
	E"	-1.0000	-0.2277	0.2277	-0.1240	0.1240	0.3830	-0.3830
	AE"	1.0000	0.2277	-0.2277	0.1240	-0.1240	-0.3830	0.3830
	BE"	0.2277	1.0000	-1.0000	0.5447	-0.5447	0.4927	-0.4927
	ABE"	-0.2277	-1.0000	1.0000	-0.5447	0.5447	-0.4927	0.4927
	CE"	0.1240	0.5447	-0.5447	1.0000	-1.0000	0.3239	-0.3239
	ACE"	-0.1240	-0.5447	0.5447	-1.0000	1.0000	-0.3239	0.3239
	BCE"	0.3830	0.4927	-0.4927	0.3239	-0.3239	1.0000	-1.0000
	ABCE"	0.3830	-0.4927	0.4927	0.3239	-0.3239	-1.0000	1.0000
	DE"	0.0667	0.2928	-0.2928	0.5375	-0.5375	0.1741	-0.1741
	ADE"	-0.0667	-0.2928	0.2928	-0.5375	0.5375	-0.1741	0.1741
	BDE"	-0.2928	0.7460	-0.7460	0.2219	-0.2219	0.7645	-0.7645
	ABDE"	0.2928	-0.7460	0.7460	-0.2219	0.2219	-0.7645	0.7645
TREND	CDE"	-0.5375	0.2219	-0.2219	0.5897	-0.5897	0.0216	-0.0216
	ACDE"	0.5375	-0.2219	0.2219	-0.5897	0.5897	-0.0216	0.0216
	BCDE"	0.1741	0.7645	-0.7645	0.0216	-0.0216	0.7091	-0.7091
	ABCE"	-0.1741	-0.7645	0.7645	-0.0216	0.0216	-0.7091	0.7091
	LINEAR	-0.2097	-0.0478	0.0478	-0.0260	0.0260	0.0803	-0.0803
	QUAD	0.1187	0.0579	-0.0579	0.0652	-0.0652	-0.0549	0.0549
	CUBIC	-0.2991	-0.0624	0.0624	-0.0277	0.0277	0.1128	-0.1128

INTERACTIVE CARRYOVER							
	ADE"	BDE"	ABDE"	CDE"	ACDE"	BCDE"	ABCDE"
INTERACTIVE CARRYOVER	A"	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	B"	-0.2582	0.8819	-0.8819	0.4804	-0.4804	0.6742
	AB"	0.2582	-0.8819	0.8819	-0.4804	0.4804	-0.6742
	C"	-0.4472	0.3637	-0.3637	0.8321	-0.8321	0.0778
	AC"	0.4472	-0.3637	0.3637	-0.8321	0.8321	-0.0778
	BC"	0.1491	0.6547	-0.6547	0.2774	-0.2774	0.8563
	ABC"	-0.1491	-0.6547	0.6547	0.2774	-0.2774	-0.8563
	D"	-0.6831	0.0476	-0.0476	0.3026	-0.3026	0.1529
	AD"	0.6831	-0.0476	0.0476	-0.3026	0.3026	-0.1529
	BD"	0.2000	0.8783	-0.8783	0.2894	-0.2894	0.5919
	ABD"	-0.2000	-0.8783	0.8783	-0.2894	0.2894	-0.5919
	CD"	0.0976	0.4286	-0.4286	0.7868	-0.7868	0.2548
	ACD"	-0.0976	-0.4286	0.4286	-0.7868	0.7868	-0.2548
	BCD"	-0.3333	0.6181	-0.6181	0.0413	-0.0413	0.8704
	ABCD"	0.3333	-0.6181	0.6181	0.0413	-0.0413	-0.8704
	E"	0.0667	0.2928	-0.2928	0.5375	-0.5375	0.1741
	AE"	-0.0667	-0.2928	0.2928	-0.5375	0.5375	-0.1741
	BE"	-0.2928	0.7460	-0.7460	0.2219	-0.2219	0.7645
	ABE"	0.2928	-0.7460	0.7460	-0.2219	0.2219	-0.7645
	CE"	-0.5375	0.2219	-0.2219	0.5897	-0.5897	0.0216
	ACE"	0.5375	-0.2219	0.2219	-0.5897	0.5897	-0.0216
	BCE"	0.1741	0.7645	-0.7645	0.0216	-0.0216	0.7091
	ABCE"	-0.1741	-0.7645	0.7645	-0.0216	0.0216	-0.7091
	DE"	-1.0000	-0.2277	0.2277	-0.1240	0.1240	0.3830
	ADE"	1.0000	0.2277	-0.2277	0.1240	-0.1240	-0.3830
	BDE"	0.2277	1.0000	-1.0000	0.5447	-0.5447	0.4927
	ABDE"	-0.2277	-1.0000	1.0000	-0.5447	0.5447	-0.4927
TREND	CDE"	0.1240	0.5447	-0.5447	1.0000	-1.0000	0.3239
	ACDE"	-0.1240	-0.5447	0.5447	-1.0000	1.0000	-0.3239
	BCDE"	0.3830	0.4927	-0.4927	0.3239	-0.3239	1.0000
	ABCDE"	0.3830	-0.4927	0.4927	0.3239	-0.3239	-1.0000
	LINEAR	0.0140	0.0614	-0.0614	0.1127	-0.1127	0.0365
	QUAD	-0.0712	-0.0347	0.0347	-0.0526	0.0526	-0.0443
	CUBIC	0.0082	0.0876	-0.0876	0.1629	-0.1629	0.0477

END

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