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Bethesda, Maryland 20084



RELATIVE BOW MOTION AND FREQUENCY  
OF SLAMMING OF SWATH CROSS-STRUCTURE

by

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and  
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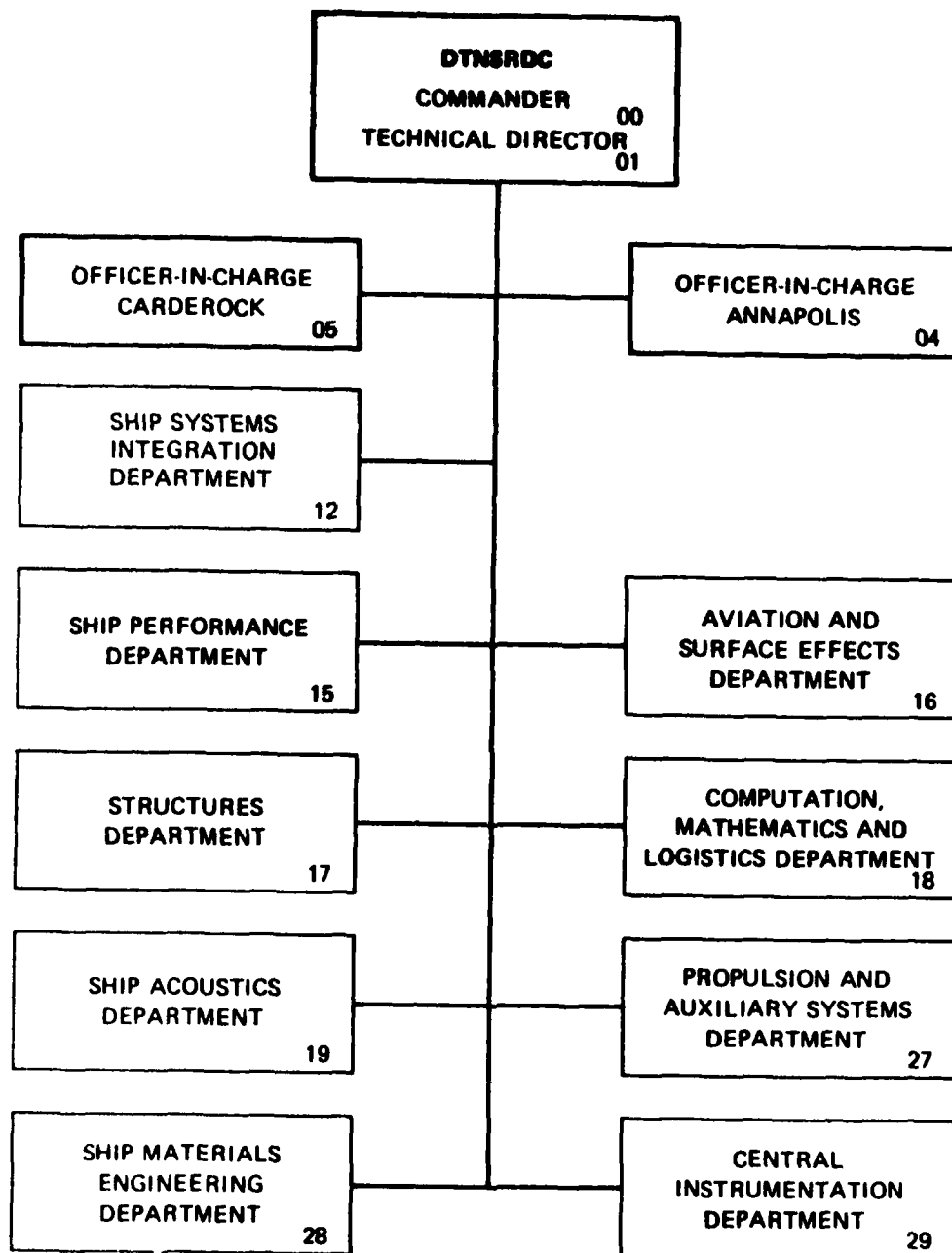
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# NOTATION

|                      |                                                                                                                                           |
|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| A                    | Amplitude of incident wave                                                                                                                |
| g                    | Gravitational constant                                                                                                                    |
| i                    | $\sqrt{-1}$ Imaginary number                                                                                                              |
| k                    | Incident wave number                                                                                                                      |
| n                    | Three dimensional unit normal vector                                                                                                      |
| $G(x, z)$            | Two dimensional Green's Function                                                                                                          |
| Im                   | Imaginary part of complex function                                                                                                        |
| Re                   | Real part of complex function                                                                                                             |
| U                    | Forward velocity of ship                                                                                                                  |
| $\xi_j$              | Displacement of ship in jth mode from its mean position; j = 1 for surge, 2 for sway, 3 for heave, 5 for pitch, and 6 for yaw             |
| $\xi$                | Relative angle between cross-structure and free surface (alternate definition)                                                            |
| $\beta$              | Heading of ship with respect to x axis; $\beta = 0$ for following waves and 180 for head waves                                            |
| $\rho$               | Density of water                                                                                                                          |
| $\phi(x, y, z, t)$   | Velocity potential which represents the fluid disturbance due to waves and motion                                                         |
| $\phi_I(x, y, z, t)$ | Velocity potential for incident wave                                                                                                      |
| $\phi_j(x, y, z)$    | Complex velocity potential for body per unit motion<br>j = 1 for surge, 2 for sway, 3 for heave, 4 for roll<br>5 for pitch, and 6 for yaw |
| $\omega$             | Wave encounter frequency ( $\omega = \omega_0 - \omega_0^2/g \cos \beta$ )                                                                |
| $\omega_0$           | Incident wave frequency in radians per second                                                                                             |
| $\theta$             | Wave slope                                                                                                                                |

|                    |                                                                                       |
|--------------------|---------------------------------------------------------------------------------------|
| $V$                | Velocity normal to wave surface                                                       |
| $S$                | Sea energy spectrum                                                                   |
| $K_1$              | Complex Response Operator for relative motion                                         |
| $K_3$              | Complex Response Operator for relative angle between cross-structure and free surface |
| $f(x_1, x_2, x_3)$ | Multi-dimensional probability distribution                                            |
| $f$                | Expected number of impacts per unit time                                              |
| $Z_r$              | Relative motion between cross-structure and free surface                              |
| $\dot{Z}_r$        | Relative velocity between cross-structure and free surface                            |

## ABSTRACT

A study was made of methods for improving the means of estimating the expected number of wave impacts per unit time of the SWATH ship cross-structure. Two avenues were explored: (1) improvement of relative motion estimates by adding the components of ship-generated wave and diffracted wave to the incident wave in describing the free surface; and, (2) including a limiting impact angle in the criteria defining the occurrence of a slam in the formulation of the level-crossing definition from which the expected number of impacts is derived. The results of this study show that including the ship-generated wave and diffracted wave does not improve the correlation of the computed relative motion with results obtained from experiments. Imposing a limiting impact angle on the definition of cross-structure slamming, as expected, reduces the estimated frequency of slamming. Additional model experiments are recommended to obtain a more definitive estimate of threshold velocity and limiting impact angle for estimating SWATH cross-structure slamming frequencies of occurrence.

## ADMINISTRATIVE INFORMATION

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## INTRODUCTION

A potentially serious problem inherent in the SWATH ship concept is its propensity for sustaining wave impacts on the underside of the structure connecting the twin hulls while operating in a heavy seaway. The impacts not only can impose large tertiary loads on the local cross-structure, increasing ship structure and weight, but can also induce large vibrations and accelerations resulting in serious structural fatigue problems. Recent experience suggests that methods developed for monohull slamming are inadequate for SWATH ships. This should come as no surprise since the monohull method was developed to predict slams on the ship's bottom; whereas, SWATH impacts take place on the upper structure connecting the two struts. Obviously, new tools are needed by the designer to establish conditions under which wave impacts occur, and to determine design parameters to ameliorate and/or avoid such occurrences.

An important indicator of a ship's susceptibility to slamming is the number of occurrences of slamming per unit time. This not only provides a relative measure of the merit of different SWATH designs from the perspective of slamming, but also helps to identify those factors influencing the ship's slamming characteristics. The occurrence of a cross-structure slam is dependent upon at least three conditions. They are:

1. Entry of the cross structure into the water (An obvious requirement.)
2. An entry velocity exceeding some threshold velocity.
3. A small angle between the cross-structure and free surface at point of impact.

As can be seen from the above criteria, the relative motion between the point of impact and the free surface is an important parameter in determining the occurrence of a cross-structure slam. If it is assumed that the relative motion and the angle between the cross-structure and free surface at the point of impact are stationary Gaussian processes, then the number of slams per unit of time can be determined from the computed statistical properties of these variables. The above assumption is reasonable in the context of linear ship motion theory even though in reality the impacts produce sharp nonlinear peaks in the acceleration. These impacts occur in a short interval of time and the effect upon the ship displacement and velocity are minimal.

Since it is reasonable to expect that a more precise determination of the relative motion would provide a more accurate estimate of the occurrence of slamming, a procedure was developed for including the ship-generated waves and the diffracted waves in the computations of the relative motions. Routine computations only consider the incident wave as part of the free surface and neglect ship-generated waves and corresponding diffracted waves. Computations were also made to determine the effect of assuming that impacts occur for a limited range of angles between the deck and the free surface at the point of impact. This also has not been considered in routine computations.

#### BACKGROUND

A method for estimating the expected number of slams per second of a mono-hull or conventional ship's bottom was developed by Tick<sup>1</sup>, based upon the relative bow motion and angle between wave and keel at the contact point. It was

assumed that the slam occurred when the relative velocity between the bow and the sea surface exceeded a critical amount at the time of contact, the bow came out of the water previous to contact, and the angle between the wave and keel at some chosen contact point was small. In addition, the relative motion and angle between keel and wave were assumed to be stationary random Gaussian processes. Ochi<sup>2</sup> arrived at the same formulation, excluding the effects of limiting the angle between wave and keel, by assuming a more restricting narrow band process, and was able to obtain other important statistical properties of the slamming phenomenon. Ochi was also able to derive empirically a threshold velocity of 12 ft/sec (3.7 m/sec) for a 520 foot (158 meters) Mariner Class ship from model experiments in irregular waves. Ochi's data are shown in Figure 1. In a subsequent paper, Ochi<sup>3</sup> proposed that the threshold velocity of 12 ft/sec (3.7 m/sec) found for the Mariner Class be Froude scaled for ships of different lengths. This is the general practice currently in use for computing the expected number of occurrences of monohull bottom slamming.

Application of the above criteria to the bottom of the deck structure connecting the twin hulls of a SWATH ship has not been verified and a procedure for scaling the threshold velocity from the bottom slamming of a Mariner Class hull to the cross-structure of a SWATH ship is not readily apparent. Figure 2 shows a plot of the cross-structure slamming pressure variation with respect to the relative velocity obtained from model experiments with a 1/32 scale SWATH T-AGOS in waves. These data show slams occurring at relative velocities as low as 1/2 ft/sec (.15 m/sec) at a ship speed of 3 knots in head waves and as low as 2 ft/sec (.60 m/sec) at a ship speed of 8 knots. The threshold velocity obtained by Froude scaling the value for the Mariner Class (based upon length) is larger than the velocities at which slams were recorded for the T-AGOS model. Another paradox is that the expected number of occurrences of slamming computed from the relative motions and the observed threshold velocities do not coincide with the actual measured values. It is quite evident that the conventional formulation used in estimating the expected number of occurrences for monohulls is not directly applicable to the SWATH cross-structure problem, and the influence of other parameters such as the angle between the structure and free surface needs to be investigated.

# PROBABILITY OF SLAMMING INCLUDING ANGLE OF IMPACT

Chuang<sup>4</sup> developed a relationship between pressure and velocity for estimating maximum slamming loads on high speed craft which is given by

$$\text{Max } p_1 = k\rho V$$

where  $k$  is an arbitrary constant

$\rho$  is the mass density of the fluid and

$V$  is the velocity normal to the wave surface

The impact pressure  $p_1$  is that part due to the velocity component of the craft normal to the wave surface. The total impact pressure includes a contribution due to the forward velocity of the craft. Of particular interest is the fact that the constant  $k$  is actually a function of the impact angle, i.e., the angle between the structure and the free surface at the point of contact. Figure 3 presents the relationship between the constant  $k$  and the impact angle  $\xi$  as established by Chuang. Chuang<sup>5</sup> has applied this method to the cross-structure slamming of a catamaran with good results. In this case, the slamming pressure due to the horizontal velocity component of the ship could be neglected without serious error because of the relatively low speed of the ship. Since the cross-structure of the catamaran is very nearly the same as that of a SWATH ship, it can be reasonably assumed that the expected number of occurrences of slamming per unit time is also dependent upon the angle of impact.

As previously indicated, Tick developed an analytical expression for computing the expected number of slams per unit of time of a ship's bottom based on the conditions that at the time of impact: the relative motion  $Z_r(t)$  passes through the value  $-k$ ; the relative velocity  $\dot{Z}_r(t)$  exceeds some threshold velocity  $v > 0$ ; and  $|\xi| < \xi_0$ , where  $\xi_0$  is the difference between the angle of the keel and the slope of the wave at the bow. It is assumed that the relative motion  $Z_r(t)$  and the angle  $\xi(t)$  are stationary Gaussian processes. The development is a generalization of the method of Rice<sup>6</sup> and establishes the probability of an  $x_0$  level crossing in the interval between times  $t$ , and  $t + dt$ , under the specified conditions. In the following notation,  $x_1$ ,  $x_2$ ,  $x_3$ , correspond to the relative motion, relative velocity, and relative angle, i.e.,  $x_1 = Z_r(t)$ ,  $x_2 = \dot{Z}_r(t)$ ,  $x_3 = \xi(t)$ .

$x_3 = \xi(t)$ . For  $dt$  sufficiently small  $x_1(t)$  can be considered as a straight line in the interval and

$$x_1(t) = x_1(t_0) + x_2(t_0)(t - t_0)$$

Then, if  $p(x_1, x_2, x_3)$  is the joint probability of the random variables, the probability of a  $k$ -crossing with the required properties in the interval  $dt$  is given by

$$\bar{f}(dt) = dt \int_{\xi_1}^{\xi_2} dx_3 \int_{v_0}^{\infty} dx_2 \int_{x_0 - x_2 dt}^{x_0} dx_1 f(x_1, x_2, x_3)$$

and since the integrand is continuous this reduces to

$$\bar{f}(dt) = dt \int_{\xi_1}^{\xi_2} dx_3 \int_{v_0}^{\infty} dx_2 x_2 f(x_0, x_2, x_3)$$

for Gaussian variables

$$f(x_1, x_2, x_3) = \frac{1}{(2\pi)^{3/2} D^{1/2}} \exp(-1/2Q)$$

where

$$Q = \sum_{ij} \sigma_{ij} x_i x_j,$$

is the element of matrix inverse to the matrix  $(\sigma_{ij})$  of the covariances and  $D$  is the Determinant of the matrix  $(\sigma_{ij})$ .

Integrating the above equation for the case  $\xi_1 = -\xi_0, \xi_2 = +\xi_0$  over the interval from 0 to  $T$  gives the expected number of slams in the interval.

Dividing by  $T$  gives the expected number of slams per unit time

$$f_s = \frac{1}{2\pi} \left[ \frac{\sigma_{22}}{\sigma_{11}} \right]^{1/2} \exp \left[ -1/2 \left( \frac{k^2}{\sigma_{11}} + \frac{v_0^2}{\sigma_{22}} \right) \right] \{ \Phi(\alpha_2) - \Phi(\alpha_1) \}$$

$$\frac{+\sigma_{23}}{N} \exp \left[ -\frac{1}{2} \frac{(\sigma_{11} \xi_2 + 2\sigma_{11} \xi_k - \sigma_{33} k_2)}{N} \right] \{ \Phi(\beta_2) - \Phi(\beta_1) \}$$

$$\alpha_1 = \sqrt{\sigma_{33}} \left\{ \xi_1 - \frac{\sigma_{13}}{\sigma_{11}} k - \frac{\sigma_{23}}{\sigma_{22}} v_0 \right\}$$

$$\beta_1 = \sqrt{\sigma_{22}} \left\{ v_0 - \frac{\sigma_{23} \sigma_{11}}{N} \xi_1 + \frac{\sigma_{23} \sigma_{13}}{N} \right\}$$

where  $\xi_1 = \xi_0$ ,  $\xi_2 = -\xi_0$

$$N = \sigma_{11} \sigma_{33} - (\sigma_{13})^2$$

$$\sigma_{22} = \frac{N}{\sigma_{22} N - (\sigma_{23})^2 \sigma_{11}}$$

$$\sigma_{33} = \frac{\sigma_{11} \sigma_{22}}{\sigma_{22} N - (\sigma_{23})^2 \sigma_{11}}$$

$\sigma_{11}$  = variance of relative motion  $Z_r(t)$

$$= \int_0^{\infty} |K_1(\omega)|^2 S(\omega) d\omega$$

$\sigma_{22}$  = variance of the relative velocity  $\dot{Z}_r(t)$

$$= \int_0^{\infty} \omega^2 |K_1(\omega)|^2 S(\omega) d\omega$$

$\sigma_{33}$  = variance of the impact angle

$$= \int_0^{\infty} |K_3(\omega)|^2 S(\omega) d\omega$$

$\sigma_{13}$  = covariance between  $Z_r(t)$  and  $\xi(t)$

$$= \int_0^{\infty} \text{Re}\{H\} S(\omega) d\omega$$

$\sigma_{23}$  = covariance between  $p(t)$  and  $(t)$

$$= \int_0^{\infty} \text{Im}\{H\} S(\omega) d\omega$$

$S(\omega)$  = Wave Spectrum

$$H(\omega) = K_1(\omega) K_3^*(\omega)$$

where \* denotes complex potential

$K_1$  = Complex Response Operator for relative motion

$K_3$  = Complex Response Operator for impact angle

and

$$\phi(x) = \frac{1}{2\pi} \int_{-\infty}^x e^{-y^2/2} dy$$

If the condition of small angle is ignored the above equation reduces to the formulation currently in use for monohulls

$$\bar{f}(t) = \frac{1}{2\pi} \left[ \frac{\sigma_{22}}{\sigma_{11}} \right]^{1/2} e^{-1/2 \left[ \frac{k^2}{\sigma_{11}} + \frac{v^2}{\sigma_{22}} \right]}$$

This is derived by letting  $\xi_0 \rightarrow \infty$ . The formulation including the influence of the impact angle is directly applicable to the cross-structure slamming of a SWATH ship under analogous specified conditions, i.e., the cross-structure enters the free surface with a velocity exceeding some critical velocity and the absolute value of the angle between the structure and free surface at the point of contact is less than some value  $\xi_0$ .

Computations of the expected number of impacts per unit of time were made for a T-AGOS SWATH at the forward end of the cross-structure, in a State 7 sea, at a ship speed of 3 knots. These computations were made for a threshold velocity of 1/2 feet/sec (.15 m/sec) (full scale) which was predicted from model experiments and a much higher threshold velocity of 5 feet/sec (1.5 m/sec). The impact angle in head seas is given by

$$\xi = \theta - v_\ell$$

where  $\theta$  is the SWATH pitch angle and

$v_\ell$  is the wave slope at the point of impact.

Transforming the wave slope,  $v$ , at the point of reference to the point of impact is accomplished by the operator

$$v_\ell = v \exp(\omega^2 l/g)$$

where  $l$  is the distance along the longitudinal axis from the reference point to the point of impact and  $\omega$  is the wave frequency. The wave spectrum used in these calculations, presented in Figure 4, was that obtained from Program A in the Maneuvering and Seakeeping Facility during the experiments. The corresponding Relative Motion and Pitch Motion Response Operators used in these calculations are shown in Figures 5 and 6.

Figure 7 shows the variation of the expected number of impacts per hour as a function of the limiting impact angle. As can be seen in the figure, the effect of imposing a bound on the range of impact angles over which a slam can occur reduces the expected number of impacts as the range becomes smaller. Paradoxically, the number of slams actually measured in irregular waves for the same conditions (4 per hour) was considerably less than that predicted by the formula for any reasonable values of the parameters. The reason for this is not known and, unfortunately, there is insufficient data available to thoroughly investigate this discrepancy.

### RELATIVE MOTION COMPUTATIONS

#### Analytical Development

Since relative motion is an important variable in the determination of the expected number of impacts per unit of time, an analytical procedure was developed to include ship-generated waves and diffracted wave components in defining the free surface for relative motion computations. Computational procedures currently in use take the free surface as being the same as that of the incident wave.

Lee<sup>7</sup>, et al., have conducted extensive studies of the influence of ship-generated waves and ship-diffracted waves in the case of monohulls for points contiguous to the hull surface. A similar analytical development for SWATH ships for points slightly away from the wetted hull surface, i.e., along the cross-structure is presented below:

If we express the total velocity potential with

$$\Phi(x,y,z,t) = \text{Re} [\phi(x,y,z)e^{-i\omega t}] \quad (1)$$

Then, the total wave height is computed by the following

$$\eta_t = -\frac{1}{g} \frac{\partial \Phi}{\partial t} = \text{Re} \left[ \frac{i\omega\phi(x,y,0)}{g} e^{-i\omega t} \right] \quad (2)$$

The time-independent velocity potential can be given by

$$\begin{aligned}\phi(x,y,z) &= \phi_I + \phi_B \\ &= \phi_I + \psi_7 + \sum_{j=1}^6 \xi_j \psi_j\end{aligned}\quad (3)$$

where  $\phi_I$  is the potential of the incoming wave and is given by

$$\phi_I = \frac{1gA}{\omega_0} \exp [kz + ikx \cos\beta - iky \sin\beta] \quad (4)$$

and  $\psi_j$  ( $j=1,2,\dots,6$ ) is the velocity potential arising from the motion of SWATH ship with unit amplitude in each of six degrees of freedom, and  $\xi_j$  ( $j=1,2,\dots,6$ ) the amplitude of motion in each of six degrees of freedom. The diffraction potential is expressed by  $\psi_7$ . In Equation (4),  $A$  is the amplitude of the incoming wave,  $\omega_0$  its frequency,  $g$  is gravitational acceleration,  $\beta$  is the angle of the incoming wave relative to the  $OX$ -axis ( $\beta=0$ ) following sea and  $\beta=180$ , head sea),  $k$  is the wave number, and  $i$  is  $\sqrt{-1}$ . By substitution of Equation (3) into Equation (2), the total wave height is expressed by

$$\eta_T = R_e \left[ \frac{i\omega}{g} (\phi_I + \psi_7 + \sum_{j=1}^6 \xi_j \psi_j) e^{-i\omega t} \right] \quad (5)$$

The potential  $\psi_j$  is determined as the solution of the Laplace equation with appropriate boundary conditions. Using the strip theory,  $\psi_j$  is solved for  $j=2, 3, 4$ , and  $7$  (sway, heave, roll motion, and diffraction potential). Potentials for pitch and yaw motion are given by

$$\psi_5 = (-x + \frac{iU}{\omega})\psi_3 \quad (6)$$

$$\psi_6 = (x - \frac{iU}{\omega})\psi_2 \quad (7)$$

where  $U$  is the forward speed of the SWATH ship and  $\omega$  is the encounter frequency.

The surge potential,  $\psi_1$ , is assumed to be zero.

The solution of  $\psi_j$  ( $j = 1, 2, 3, 4$ , and  $7$ ) has been presented by Frank<sup>8</sup>

$$\psi_j(y, z) = \sigma_j(n, \zeta) G(y, z; n, \zeta) dl \quad j=2, 3, 4, 7 \quad (8)$$

where  $\sigma_j$  is the source strength distributed on the SWATH ship's contour and  $G$  is two-dimensional Green function due to a unit source on the contour. The Green function,  $G$ , is given by Wehausen and Laitone<sup>9</sup>

$$\begin{aligned} G(y, z; n, \zeta) = & \operatorname{Re} \left\{ \frac{1}{2\pi} [\log(y+z-n-i\zeta) - \log(y+iz-n+i\zeta) \right. \\ & \left. + 2pv \int \frac{e^{-ik(y+iz-n+i\zeta)} dk}{K-k} \right\} \\ & - i \operatorname{Re} [e^{-ik(y+iz-n+i\zeta)}] \end{aligned} \quad (9)$$

where  $(y, z)$  is the point where the potential is sought and  $(n, \zeta)$  is a source point at the SWATH ship's contour. The source strength,  $\sigma_j$  is determined by the body boundary conditions as follows

$$\begin{aligned} \frac{\partial \psi_j}{\partial n} &= -i\omega n; , \text{ for } j = 2, 3, 4 \\ &= -\frac{\partial \phi_I}{\partial n} , \text{ for } j = 7 \end{aligned} \quad (10)$$

where  $n_j$  is the component of the unit normal vector directed into the fluid.

The absolute vertical motion of the SWATH ship is computed by

$$\xi_v = \text{Re}[(\xi_3 + y\xi_4 - x\xi_5)e^{-i\omega t}] \quad (11)$$

The relative bow motion (RBM) is simply the difference between Equations (5) and (11)

$$\begin{aligned} Z_r &= \xi_v - n_t \\ &= \text{Re} \left\{ [\xi_3 + y\xi_4 - x\xi_5] - \frac{i\omega(\phi_I + \psi_7 + \sum \xi_j \psi_j)}{g} e^{-i\omega t} \right\} \end{aligned}$$

In the numerical computation of Equation (12), the amplitude of motion,  $\xi_1$ , is computed with SWATH motion program and the velocity potential,  $\psi_1$ , is computed with HYDRO2 (ship motion program) at a given point (y,z) located outside the wetted body.

#### Comparison with Experiments

A comparison of the computed relative motions with experimental results for three locations on T-AGOS SWATH is presented in Figures 8 through 10. The locations correspond to the forward, amidship and after portion of the cross-structure along the center line of the ship. Figure 8 shows the computations of relative motion in head waves at a ship speed of 3 knots with only the incident wave representing the free surface and again with all three components: incident wave, ship generated wave, and diffracted wave included. These results show that the computations with just incident waves are very close to the experimental results and paradoxically, including the ship-generated wave and diffracted waves, in general, degrades the correlation between the computed and experimental results. The same trend is evident at a heading angle of 135 degrees and a speed of 3 knots as shown in Figure 9. In beam waves at a speed of 3 knots, Figure 10, the incident wave was the dominant free surface factor and the ship-generated waves and diffracted wave components had an insignificant effect upon the computed results.

It is interesting to note that Lee, et al.,<sup>7</sup> in their more extensive studies of the relative motion computations of monohulls concluded that there is no conclusive evidence that the inclusion of diffracted and motion-generated waves, as computed by strip theory, improves the results. Apparently, the same conclusion can be made in regard to SWATH-type ships.

#### SUMMARY AND CONCLUSIONS

An investigation was made to improve methods for estimating the expected number of occurrences of SWATH ship cross-structure slamming per unit time. Slamming of the cross structure is an important consideration in the assessment of the operability of SWATH ships in waves and the number of occurrences per unit time provides a quantitative measure of the slamming characteristics of SWATH ships. A method for computing the number of occurrences of slams for monohulls from relative ship motion has been in use by the ship designer for many years. It is essentially a level-crossing problem based upon the assumption that a slam occurs when the keel at the point of impact, enters the water with a velocity greater than some limiting threshold velocity. The threshold velocity has been determined from model experiments for a Mariner Class ship in waves. This threshold velocity is Froude scaled for ships of different lengths.

The same approach is directly applicable to the cross-structure slamming of SWATH ships. However, it is not readily apparent how to scale a threshold velocity value for keel slamming of a Mariner Class ship to the cross-structure slamming of a SWATH.

In addition, it is believed that other parameters such as the impact angle may also govern the occurrence of a slam on the SWATH cross structure. Calculation of the expected number of slams for a SWATH T-AGOS from the relative motion using the lowest impact velocity recorded, as the threshold velocity resulted in a value much higher than the observed value. It was concluded that other parameters, such as the limiting impact angle, contributed to this discrepancy. Calculations were made showing the extent to which the limiting impact angle reduces the estimated number of occurrences of slams, but there was insufficient experimental data to definitively define a limiting impact angle and, therefore, the results of this aspect are inconclusive.

Since the relative motion is an important parameter for estimating the frequency of slamming, an examination was made to determine the effects of including ship-generated waves and diffracted waves in the computation of the relative motion. Strip theory was used to compute the ship-generated wave and diffracted wave which was added to the incident wave in describing the free surface away from the hull at the point of impact of the cross structure. Computations normally include only the incident wave.

The results of this investigation showed that there was no improvement in the computed relative motion (when compared to experimental results) by the inclusion of the ship-generated wave and diffracted wave in the free surface elevation. In the cases examined, the use of the incident wave alone gave better results or the same as that obtained by including the ship-generated wave and diffracted wave.

In summary, the following conclusions can be made based on these studies:

1. The frequency of slamming of SWATH cross-structure is not only dependent upon the relative motion, relative velocity, and a threshold velocity as in the case of monohull keel slamming, but is also dependent upon some limiting impact angle.
2. There is insufficient experimental data to establish limiting impact angles for SWATH ships or other parameters influencing the frequency of slamming.
3. The inclusion of ship-generated wave and diffracted wave in the computation of relative motion, which is needed for the estimation of the frequency of slamming, does not improve the correlation with model experiment results.

It can be concluded from the above that additional model experiments are required on a SWATH ship to obtain data to specifically address the problem of estimating the expected number of impacts per unit of time by obtaining, in addition to the customary measurements, other important measurements, such as the angle of impact.

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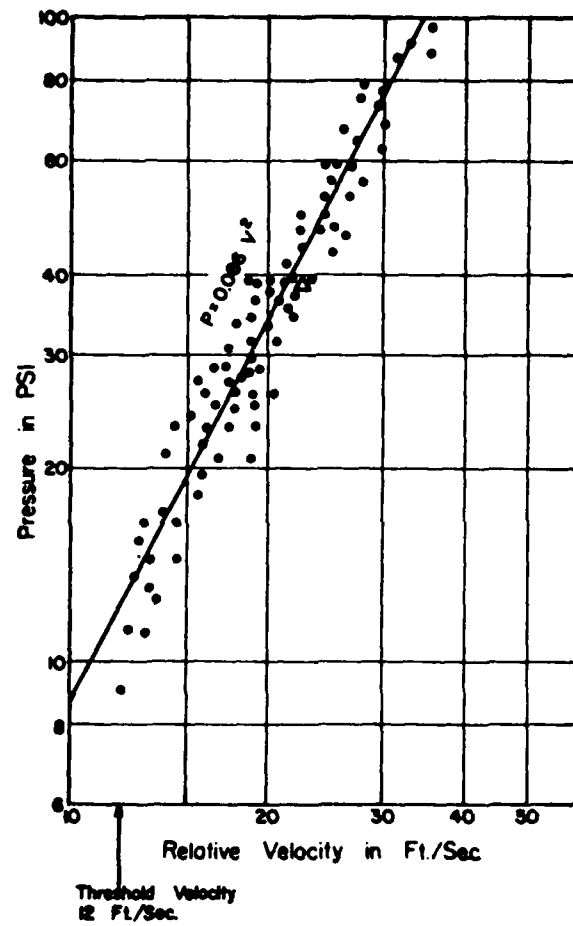


Figure 1 - Mariner Class Slam Data ( $Ochi^2$ )

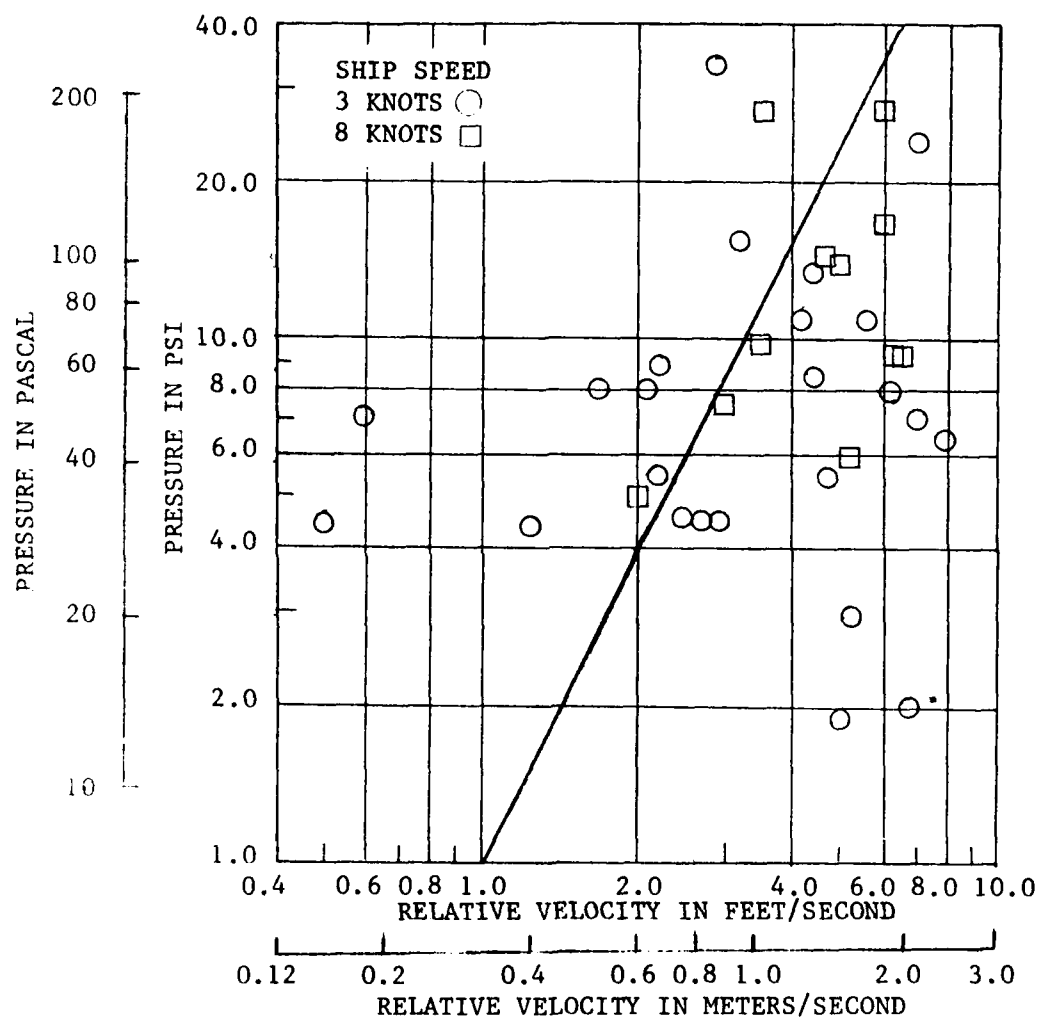


Figure 2 - Cross-Structure Slam Data for T-AGOS SWATH

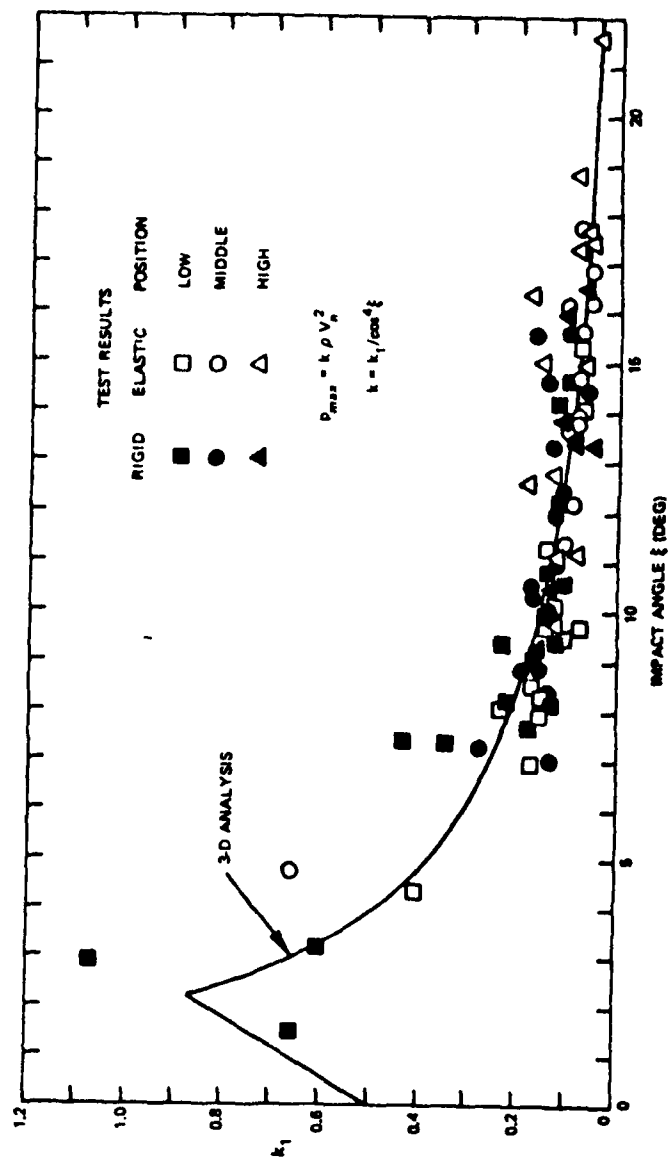
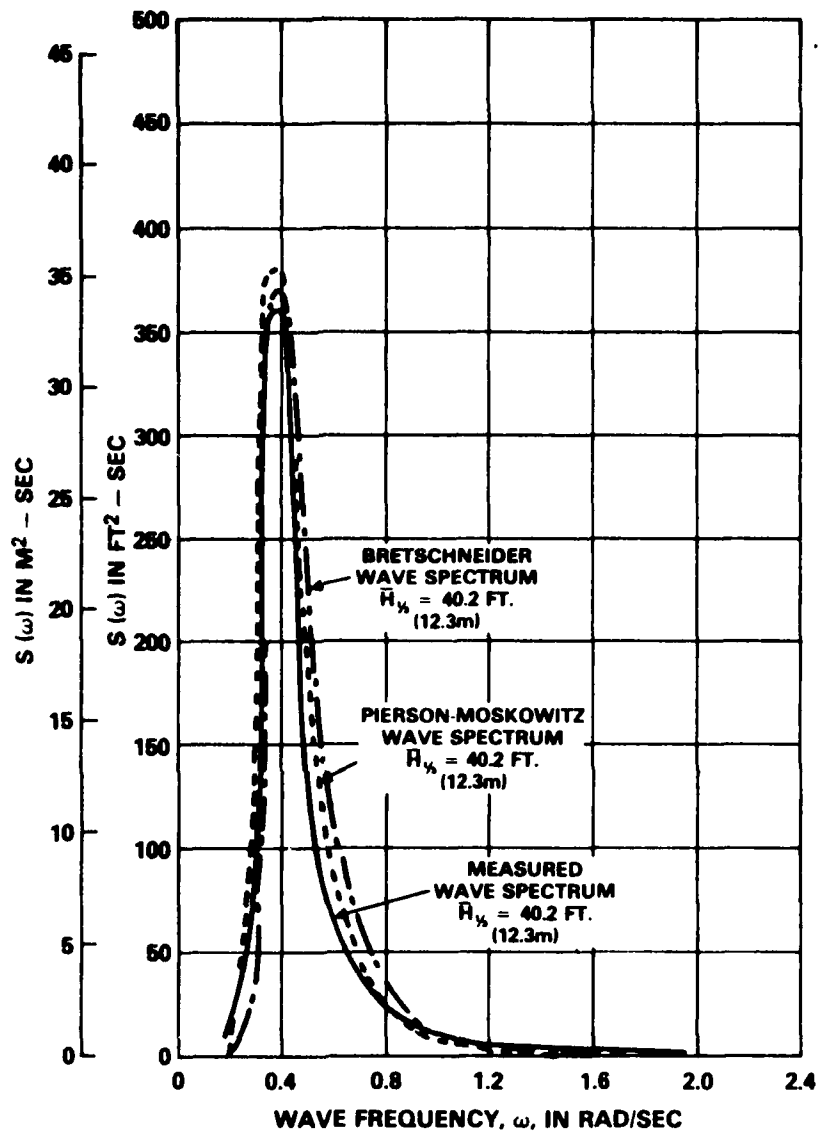


Figure 3 - Slam Pressure Coefficient (Chuang<sup>4</sup>)



#### SEA STATE 7, PROGRAM A

Figure 4 - Wave Spectrum for Relative Motion Calculations

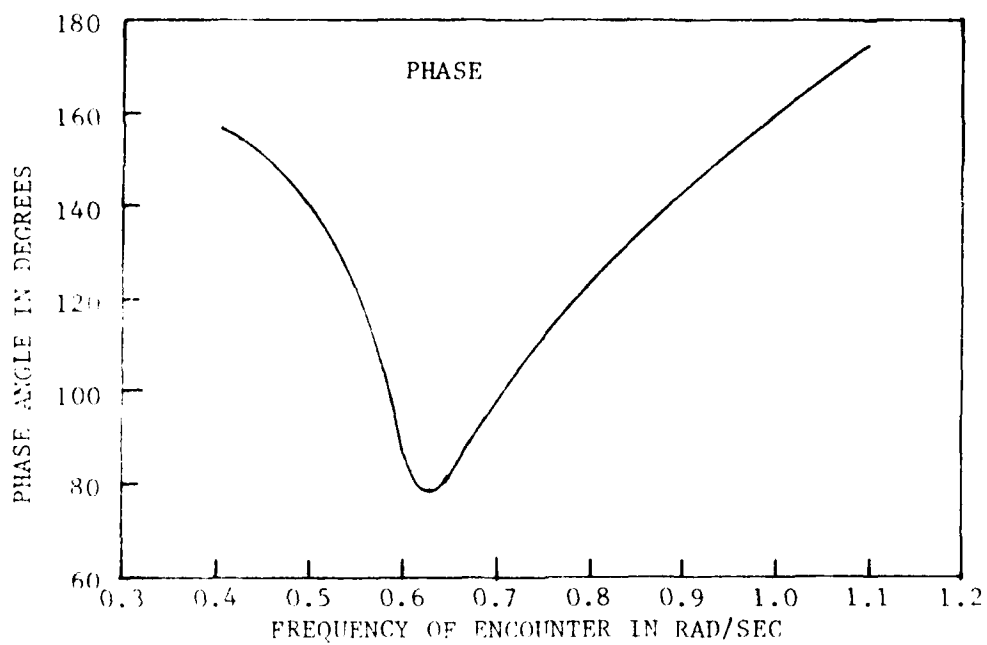
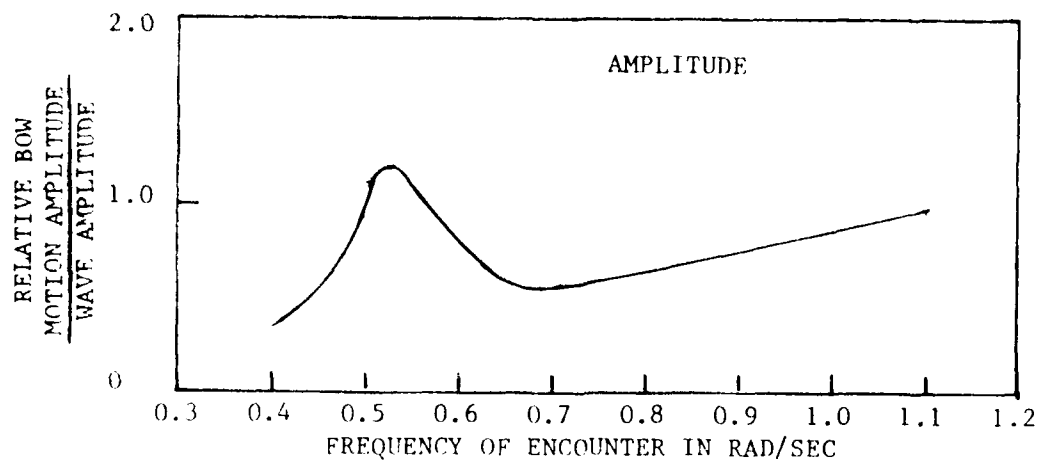


Figure 5 - Relative Motion Response Operator for T-AGOS at 3 Knots in Head Waves

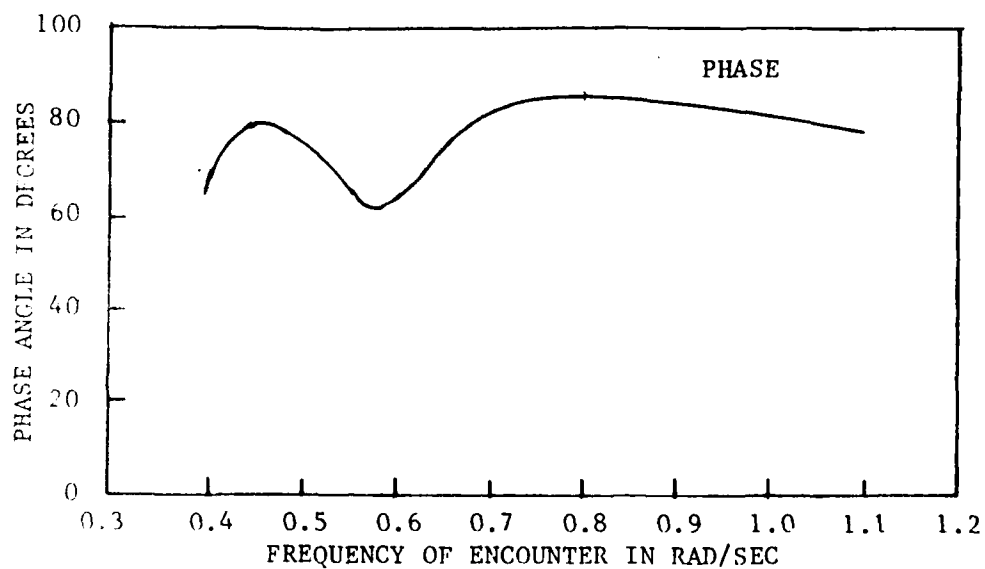
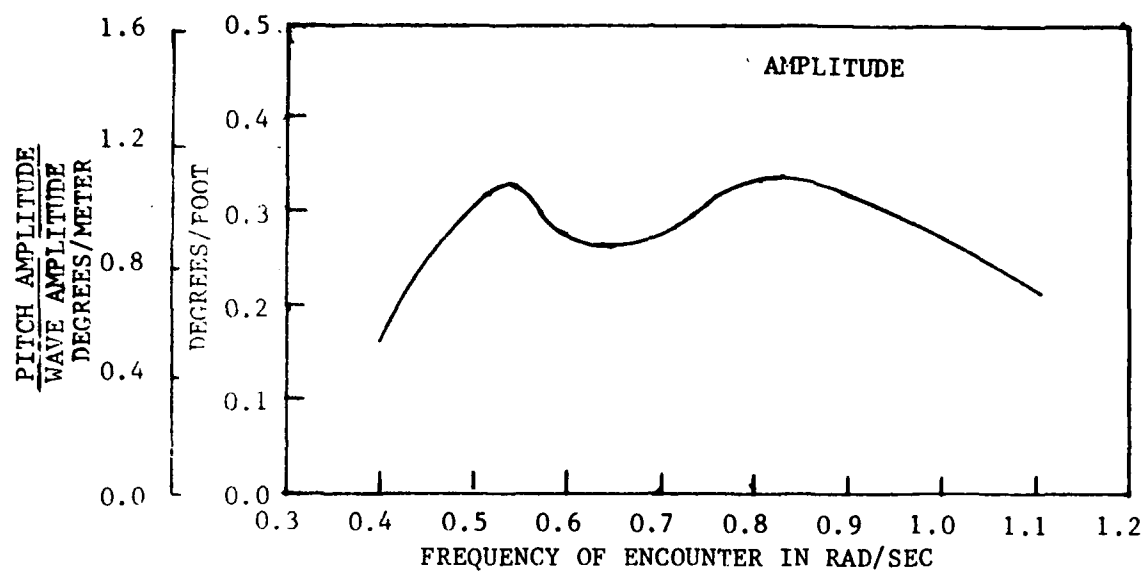


Figure 6 - Pitch Motion Response Operator for T-AGOS at 3 Knots in Head Wave

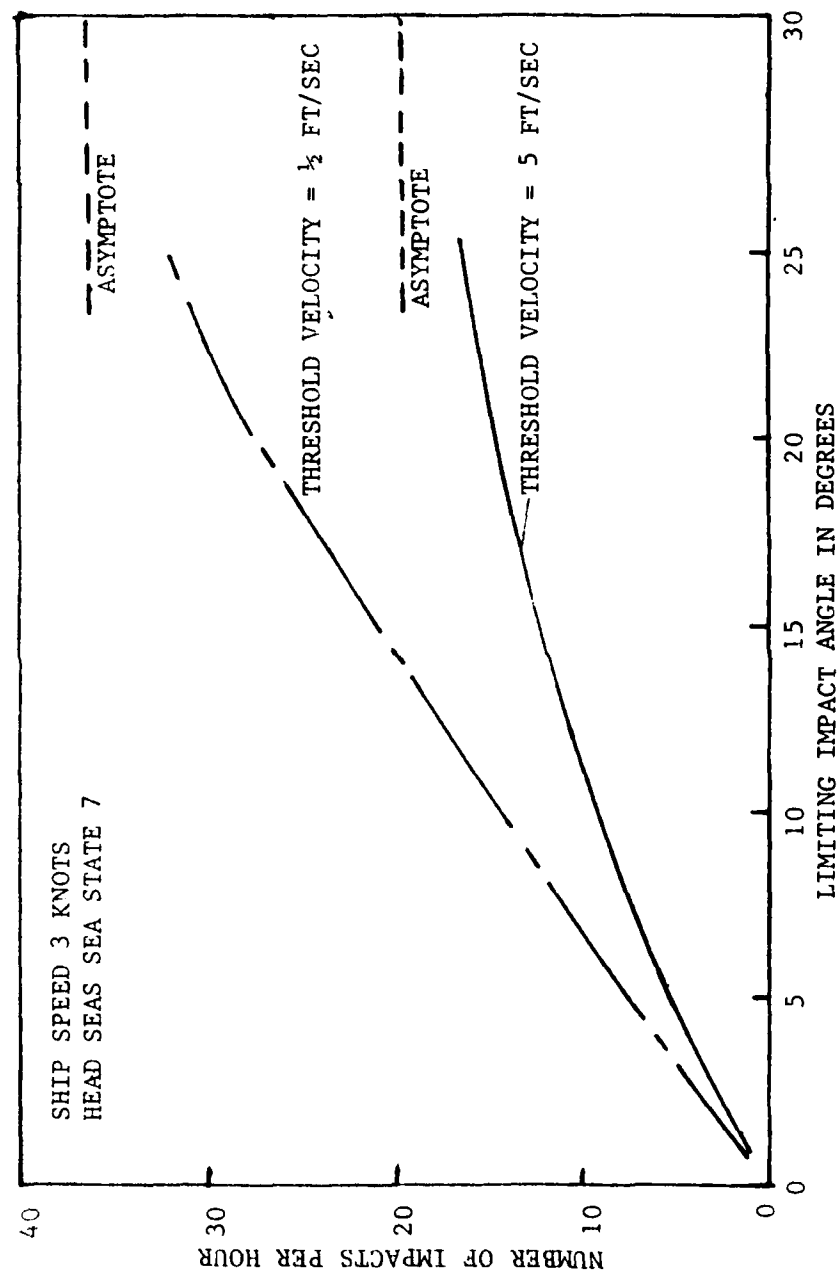


Figure 7 - Variation in Slams per Hour with Limiting Impact Angle for T-AGOS

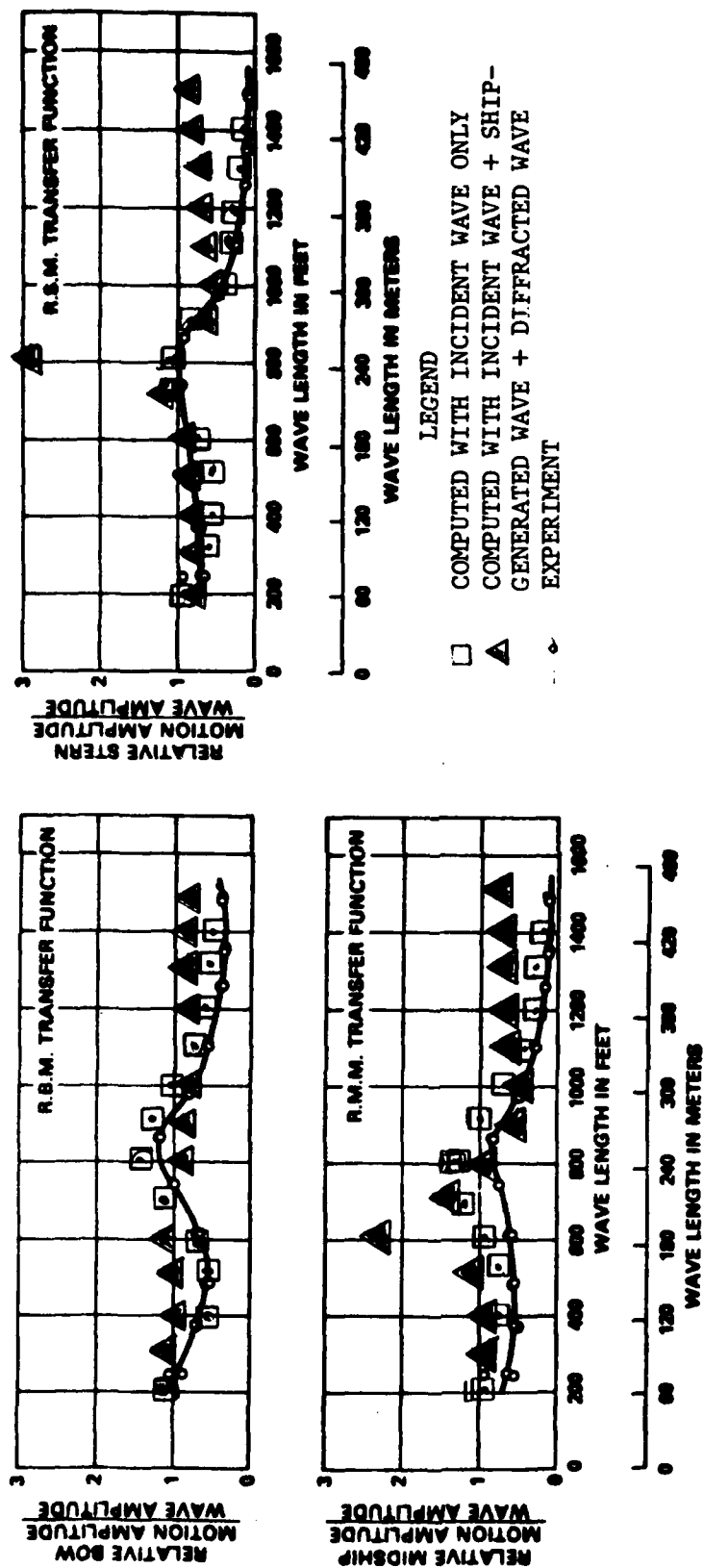


Figure 8 - Comparison of Relative Motion Computations with Model Experiment for T-AGOS in Head Waves (180 degrees) at Speed of 3 Knots

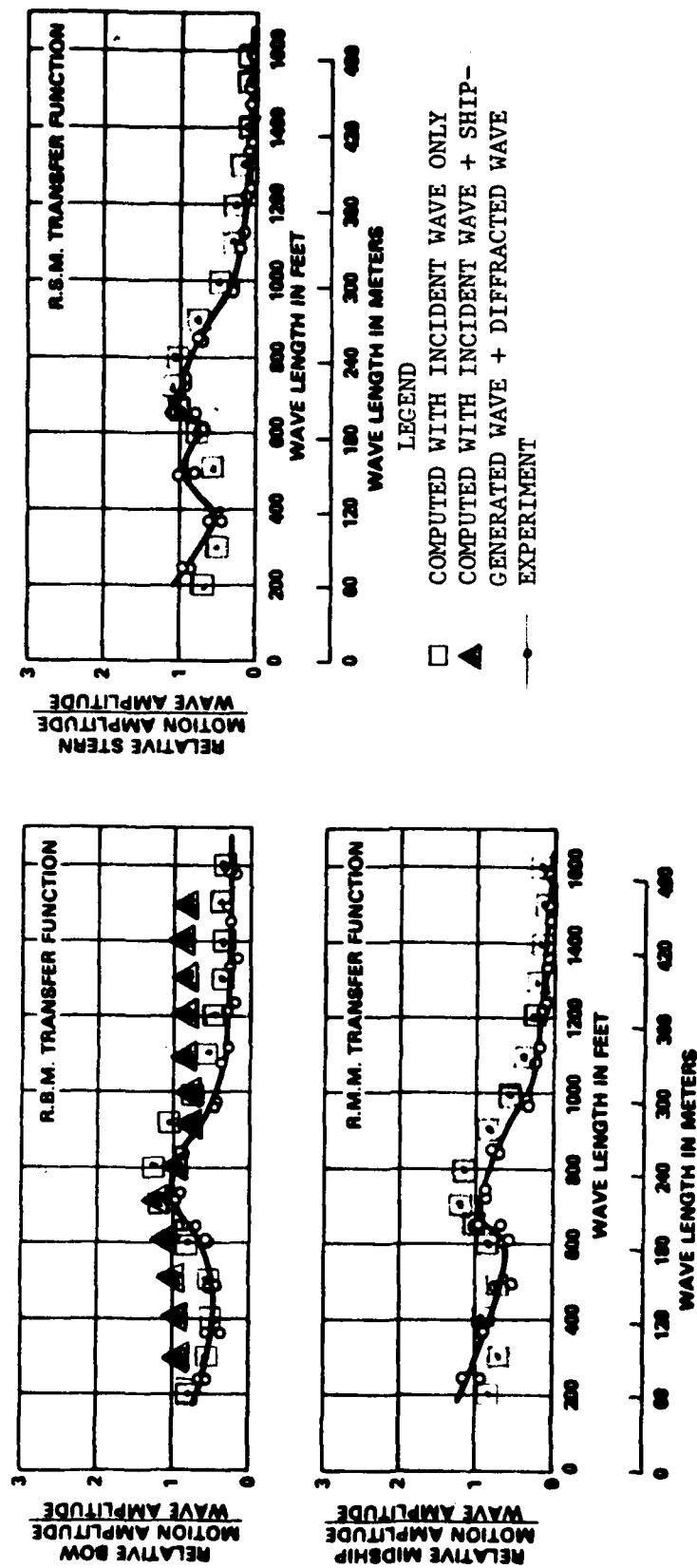


Figure 9 - Comparison of Relative Motion Computations with Model Experiment for T-AGOS in Bow Quartering Waves (135 Degrees) at Speed of 3 Knots

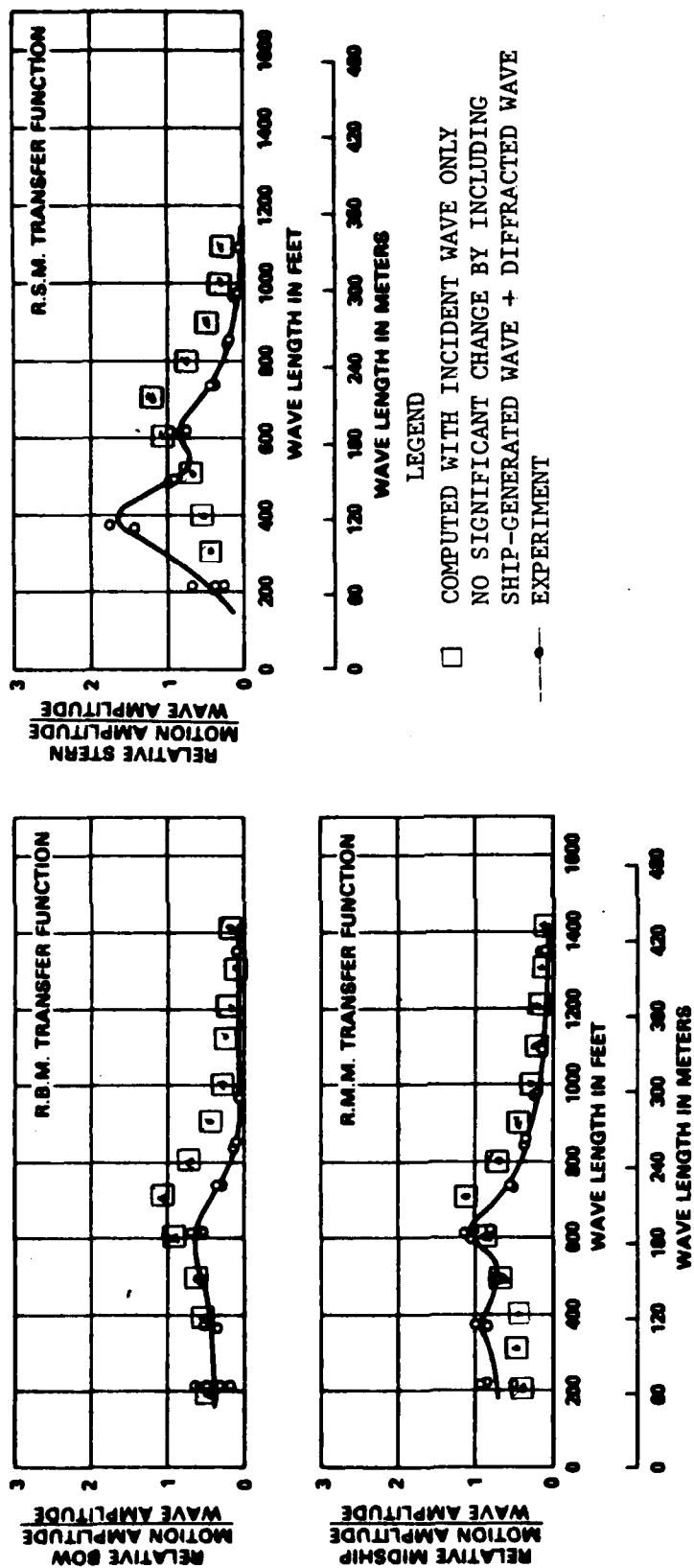


Figure 10 - Comparison of Relative Motion Computations with Model Experiment for T-AGOS in Beam Waves (90 Degrees) at Speed of 3 Knots

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