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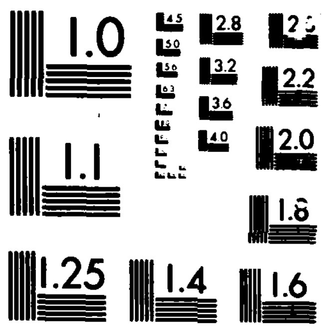


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Center for Multivariate Analysis
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Fifth Floor, Thackeray Hall
University of Pittsburgh
Pittsburgh, PA 15260

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[2]Invited paper presented at the Second International Symposium on Advances in Pattern Recognition and Digital Techniques held at the Indian Statistical Institute, January 6-9, 1986.

# SIGNAL PROCESSING USING MODEL SELECTION METHODS

Z.D. Bai, P.R. Krishnaiah and L.C. Zhao
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ABSTRACT

In this paper, the authors gave a review of some recent developments on multivariate statistical techniques for detection of the number of signals using eigenvalues. The main emphasis of the review is on EFFICIENT DETECTION CRITERION (EDC) procedures proposed by the authors recently. These procedures are strongly consistent.

Key words ~~and phrases~~: Signal processing, eigenvalues, additive model, strong consistency, information theoretic criteria.

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In this paper, the authors gave a review of some recent developments on multivariate statistical techniques for detection of the number of signals using eigenvalues. The main emphasis of the review is on EFFICIENT DETECTION CRITERION (EDC) procedures proposed by the authors recently. These procedures are strongly consistent.


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# 1. INTRODUCTION

In the area of signal processing, eigenstructure methods are becoming popular in determination of the number of signals and estimation of the parameters of the signals when noise is present. Problems of signal detection in presence of noise are quite complex and they arise in many situations. For example, in the area of active sonar detection, signals are transmitted and the echoes from the targets are recorded. The observations are corrupted by disturbances in the sea and we are interested in identification of the targets. In radar detection, pulses are sent and the reflections from the targets are recorded. Here, the observations may be subject to atmospheric disturbance and disturbances due to jammer and clutter. Eigenstructure methods were used in the past by Haykins (1985), Kumaresan and Tufts (1980), Ligget (1973), Owsley ((1977),(1985)), Pisarenko (1973), Reddi (1979), Schmidt (1979), Tufts and Kumaresan (1982) and Wax, Shan and Kailath (1984). Recently, Wax and Kailath ((1984),(1985)), Zhao, Krishnaiah and Bai ((1985a), (1985b)), Bai, Krishnaiah and Zhao (1985) and Paulraj and Kailath (1986) considered the problem of determination of the number of signals using model selection approach. The object of this paper is to give a review of the work on model selection approach to determine the number of signals in presence of noise.

In the model considered in this paper, the observations from the sensors are expressed as the sum of noise and linear combinations of the wavefronts of the signals from the sources. When the noise is spatially white, the number of signals is related to the multiplicity of the smallest eigenvalue of the covariance matrix $\Sigma_2$ of the observation vectors. Various test procedures are available in the literature (e.g., see Anderson (1963), Bartlett (1954), Krishnaiah (1976) and Rao (1973)) for testing the hypothesis that the multiplicity of the smallest eigenvalue of the covariance matrix is equal to a specified value when the underlying distribution is multivariate normal. The above tests can be used to test the hypothesis that the number of signals is equal to a specified value. When the noise is spatially colored, the number of signals is related to the multiplicity of the smallest eigenvalue of $\Sigma_2\Sigma_1^{-1}$ where $\Sigma_1$ is the covariance matrix of the noise vector. Rao (1983) considered the following related problem. Let $S_1$ and $S_2$ be distributed independently as central Wishart matrices with $n_1$ and $n_2$ degrees of freedom and $E(S_i) = n_i\Sigma_i$ for $i = 1,2$. Also, let $\Sigma_2 = \Gamma + \lambda\Sigma_1$ where $\lambda$ is a known or unknown scalar, and $\Gamma$ is nonnegative definite matrix. Rao (1983) derived the likelihood ratio test statistic for testing the hypothesis on the rank of $\Gamma$ when $\lambda$ is unknown; when $\lambda$ is known, he derived modified likelihood ratio test statistic. The results of Rao can be used to test the hypothesis that the number of signals is equal to a specified value in the case of spatially colored noise.

There are a number of situations where the experimenter is confronted with the problem of making one of the various possible decisions instead of testing one specified hypothesis. In these situations, one can formulate the problem within the framework of model selection. In our case, we have to choose one of the several possible models where each model corresponds to the statement that the number of signals is equal to a particular value. In the model selection approaches discussed in this paper, we do not need any threshold value (critical value). In tests of hypotheses, we need a critical value for implementation of the tests and it is quite complicated to compute the critical values in some situations. The model selection approaches discussed here involve using certain information theoretic criteria. Of course, model selection methods should have optimum properties to have an appeal. One such desirable property in our case is the consistency of the estimate of the number of signals. Consistency property of the model selection methods is also discussed in this paper. A description of the organization of the paper is given below.

In Section 2, we give some preliminaries. In Section 3, we give a review of the literature on detection of the number of signals in presence of white noise. Analogous problems are reviewed in Section 4 when the noise is spatially colored. In Section 5, we review the literature on detection of the number of signals when the subarrays are located at wide distances apart. The main emphasis of this review is on the Efficient Detection Criteria (EDC) procedures developed by Zhao, Krishnaiah and Bai ((1985a), (1985b)) very recently. These procedures are strongly consistent. In the case of white noise, Bai Krishnaiah and Zhao (1985a) gave exponential type bounds on the probability of wrong detection of the number of signals if the EDC procedures are used. These bounds tend to zero rapidly as the sample size increases. Similar bounds can be derived in the case of spatially colored noise.

## 2. PRELIMINARIES

We now define complex multivariate normal and Wishart distributions since these definitions are needed in the sequel.

Let $\underset{\sim}{x} = \underset{\sim}{y}_1 + i\underset{\sim}{y}_2$ where $(\underset{\sim}{y}_1', \underset{\sim}{y}_2')$ is distributed as 2p–variate normal with mean vector $\underset{\sim}{0}$ and covariance matrix

$$\begin{pmatrix} \Sigma_1 & \Sigma_2 \\ -\Sigma_2 & \Sigma_1 \end{pmatrix} \tag{2.1}$$

$i = \sqrt{-1}$ and $\Sigma_1$ and $\Sigma_2$ are of order $p \times p$. Then $\underset{\sim}{x} : p \times 1$ is known to be distributed as complex multivariate normal with mean vector $\underset{\sim}{0}$ and covariance matrix $\Sigma = 2(\Sigma_1 + i\Sigma_2)$. The density of $\underset{\sim}{x}$ is of the form

$$f(\underset{\sim}{x}) = \{1/\pi^p|\Sigma|\}\exp(-\bar{\underset{\sim}{x}}'\Sigma^{-1}\underset{\sim}{x}) \tag{2.2}$$

where $\bar{\underset{\sim}{a}}$ and $\bar{A}'$ respectively denote the complex conjugate of $\underset{\sim}{a}$ and transpose of $\bar{A}$. Next, let $\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_n$ be distributed independently and identically as complex multivariate normal with mean vector $\underset{\sim}{0}$ and covariance matrix $\Sigma$. Then $S = \sum_{j=1}^{n} \underset{\sim}{x}_j \bar{\underset{\sim}{x}}_j'$ is known to be distributed as the central complex Wishart distribution with n degrees of freedom and $E(S) = n\Sigma$. The complex multivariate normal distribution was considered by Wooding (1956) whereas the complex Wishart distribution was investigated in Goodman (1963). Complex multivariate normal distribution belongs to the family of complex elliptically symmetric distributions studied by Krishnaiah and Lin (1984). For a review of the literature on complex multivariate distributions, the reader is referred to Krishnaiah (1976).

## 3. DETECTION OF NUMBER OF SIGNALS IN PRESENCE OF WHITE NOISE

In this section, we discuss certain methods of detection of number of signals in presence of white noise. The model that is considered is given by

$$\underset{\sim}{x}(t) = A\underset{\sim}{s}(t) + \underset{\sim}{n}(t) \tag{3.1}$$

where $A = [A(\phi_1),\dots,A(\phi_q)]$, $x(t) = (x_1(t),\dots,x_p(t))'$, $s'(t) = (s_1(t),\dots,s_q(t))$, and $n'(t) = (n_1(t),\dots,n_p(t))$. Here the elements of j-th column of A depend upon the parameters of j-th signal. Also, $n(t)$ and $s(t)$ are distributed independently as complex multivariate normal with $En(t) = 0$, $Es(t) = 0$, $E\{n(t)\overline{n(t)}'\} = \sigma^2 I_p$, $E\{s(t)\overline{s(t)}'\} = \psi$. The covariance matrix $\Sigma_2$ of $x(t)$ is given by

$$\Sigma_2 = A\psi\bar{A}' + \sigma^2 I_p. \tag{3.2}$$

In the model (3.1), $x_j(t)$ denotes the observation on j-th sensor, $n_j(t)$ denotes the noise component associated with j-th sensor, and $s_j(t)$ is the signal due to j-th wavefront. Also, we assume that a uniform linear array of p identical sensors is used and q is unknown. In addition, we assume that $x(t_1),\dots,x(t_n)$ are n independent observations. One special case (e.g., see Shan, Wax and Kailath (1985)) of the A matrix is

$$A(\phi_j) = (1, \exp(-i\omega_0\tau_j),\dots,\exp(-i\omega_0(p-1)\tau_j)) \tag{3.3}$$

where $\tau_j = (d \sin\theta_j)/c$, d is the distance between the sensors, c is the propagation speed of wavefront, and $\theta_j$ denotes the angle of arrival of j-th signal.

One important problem in signal processing is to estimate q, the number of unknown signals. We will now discuss this problem from the point of view of model selection.

Let $\lambda_1 \geq \dots \geq \lambda_p$ denote the eigenvalues of $\Sigma$ and let $\ell_1 \geq \dots \geq \ell_p$ denote the eigenvalues of $\hat{\Sigma}$ where $n\hat{\Sigma} = \sum_{j=1}^{n} x(t_j)\overline{x(t_j)}'$. Here we assume that $x(t_1),\dots,x(t_n)$ are distributed independent of each other. Now, let $H_k : \lambda_k > \lambda_{k+1} = \dots = \lambda_p = \sigma^2$ where $\sigma^2$ is unknown. Then $H_k$ denotes the hypothesis that the number of signals is equal to k. It is known that the likelihood ratio test statistic for testing $H_k$ is given by

$$L_k = \{\prod_{i=k+1}^{p} \ell_i^n / (1/(p-k) \sum_{i=k+1}^{p} \ell_i)^{n(p-k)}\}. \tag{3.4}$$

Since $-2\log L_k$ is distributed asymptotically as Chi-square with $(p-k)(p-k+1)-1$ degrees of freedom, we can use the above statistic for testing $H_k$ for large samples. In most of the situations, we do not know as for what values of k we should test $H_k$. Also, if $H_k$ is rejected, we should test for $H_{k+1}$ conditional upon the rejection of $H_k$. The distribution problem in this case is complicated. We now discuss a model selection approach proposed by Zhao, Krishnaiah and Bai (1985a) for determination of the number of signals.

Let

$$EDC(k,C_n) = -\log L_k + C_n\gamma(k,p) \tag{3.5}$$

where $\gamma(k,p) = k(2p-k+1)+1$ is the number of free parameters that have to be estimated under $H_k$ and $C_n$ is chosen such that

$$\lim_{n\to\infty} \{C_n/n\} = 0 \tag{3.6}$$

$$\lim_{n\to\infty} \{C_n/\log\log n\} = \infty. \tag{3.7}$$

Also, let

$$EDC(\hat{q},C_n) = \min\{EDC(0,C_n),\ldots,EDC(p-1,C_n)\}. \tag{3.8}$$

Zhao, Krishnaiah and Bai (1985a) showed that $\hat{q}$ is a strongly consistent estimate of q. This criterion is known to be Efficient Detection Criterion (EDC). When $C_n = 1$ in (3.5), $EDC(\hat{q},C_n)$ reduces to Akaike's AIC Criterion but $C_n$ does not satisfy condition (3.7) in this case. When $C_n = 1/2\log n$, $EDC(\hat{q},C_n)$ reduces to Schwartz-Rissanen's minimum description length (MDL) criterion and it satisfies conditions (3.6) and (3.7). Wax and Kailath (1984) used AIC and MDL criteria for determination of the number of signals. They have pointed out that the AIC criterion is not consistent whereas the MDL criterion is consistent; their proofs are incomplete (see Zhao, Krishnaiah and Bai (1985c)). Bai, Krishnaiah and Zhao (1985) gave an exponential bound, under certain conditions, on the probability of wrong detection and this bound tends to zero rapidly as $n\to\infty$.

Let $H_k^*: \lambda_k > \lambda_{k+1} = \ldots = \lambda_p = 1$. Zhao, Krishnaiah and Bai (1985a) derived the following expression for the likelihood ratio test statistic $L_k^*$ for $H_k^*$:

$$\log L_k^* = n \sum_{i=1+\min(\tilde{q},k)}^{p} (\log \ell_i + 1 - \ell_i) \tag{3.9}$$

where $\tau$ denotes the number of eigenvalues $\ell_i$ which are greater than one. They also showed that $\hat{q}$ is a strongly consistent estimate of $q$ where $\hat{q}$ is chosen such that

$$\mathrm{EDC}(\hat{q},C_n) = -\log L_k^* + C_n\gamma(k,p) \tag{3.10}$$

where $C_n$ satisfies the conditions (3.6) and (3.7) and $\gamma(k,p) = k(2p-k+1)$.

In the EDC procedure described above, it was assumed that $\underset{\sim}{x}(t_1),\ldots,\underset{\sim}{x}(t_n)$ are distributed independently as complex multivariate Gaussian. But, these assumptions need not be valid in a number of situations. But, the proofs of the strong consistency of $\hat{q}$ given by Zhao, Krishnaiah and Bai for the cases of unknown $\sigma^2$ and $\sigma^2 = 1$ are valid under the following weak conditions:

(i) $\{\underset{\sim}{x}(t_i),\ i = 1,2,\ldots\}$ is a stationary and $\phi$ - mixing sample sequence with $\phi$ (3.11) decreasing and $\sum_{j=1}^{\infty} \phi^{1/2}(j) < \infty$.

(ii) $E(\underset{\sim}{x}(t_1)) = \underset{\sim}{0}$, $E(\{\bar{\underset{\sim}{x}}'(t_1)\underset{\sim}{x}(t_1)\}^{2+\epsilon}) < \infty$ for some $\varepsilon > 0$ and $i = 1,2,\ldots$ (3.12)

If $\underset{\sim}{x}(t_1),\ldots,\underset{\sim}{x}(t_n)$ are not distributed as independent complex multivariate normal, $L_k$ and $L_k^*$ are not likelihood ratio test statistics for $H_k$ and $H_k^*$ but the procedures described in this section can be still applied if the conditions (3.11) and (3.12) are satisfied.

## 4. DETECTION OF NUMBER OF SIGNALS IN PRESENCE OF SPATIALLY COLORED NOISE

Consider the model

$$\underset{\sim}{x}(t) = A\underset{\sim}{s}(t) + \lambda^{1/2}\underset{\sim}{n}(t) \tag{4.1}$$

where $\lambda$ is known or unknown. Also, $\underset{\sim}{x}(t)$ : $p\times 1$, A, $\underset{\sim}{s}(t)$ $q\times 1$ and $\underset{\sim}{n}(t)$ are as defined in Section 3 with the following difference. The covariance matrix of $\underset{\sim}{n}(t)$ is $\Sigma_1$, that is, noise is spatially correlated. Here, the covariance matrix $\Sigma_2$ of $\underset{\sim}{x}(t)$ is related to $\Sigma_1$ as

$$\Sigma_2 = A\psi\bar{A}' + \lambda\Sigma_1 \tag{4.2}$$

Then, we are interested in finding out the number of signals q which is unknown. This problem is equivalent to the problem of finding the number of eigenvalues of $\Sigma_2\Sigma_1^{-1}$ which are different from its smallest eigenvalue.

Rao (1983) considered the following problem which is very closely related to the above problem. Let $S_1$ and $S_2$ be distributed independetly as central Wishart matrices with $n_1$ and $n_2$ degrees of freedom respectively and $E(S_i) = n_i\Sigma_i$ for $i = 1,2$. Also, let $\Sigma_2 = \Gamma + \lambda\Sigma_1$ where $\lambda$ is known or unknown and $\Gamma$ is of rank $q(<p)$ and is nonnegative definite. Rao (1983) derived the likelihood ratio test and modified likelihood ratio test for the rank of $\Gamma$ when $\lambda$ is unknown and known respectively. The above procedures, with minor modifications, can be applied to the case when $S_1$ and $S_2$ are distributed as complex Wishart matrices. So, the results of Rao can be applied for testing the hypothesis that the number of signals is equal to a specified value. We will now describe certain model selection procedures proposed by Zhao, Krishnaiah and Bai (1985b) for determination of the number of signals.

When the data is collected at widely spaced time points $t_1,\ldots,t_{n_2}$, it is realistic to assume that $\underset{\sim}{x}(t_1),\ldots,\underset{\sim}{x}(t_{n_2})$, the observations on the sensors, are distributed independently. In this case, we can estimate $\Sigma_2$ with $S_2/n_2$ where $S_2 = \sum_{j=1}^{n_2}\underset{\sim}{x}(t_j)\overline{\underset{\sim}{x}(t_j)}'$. Now, let $S_1/n_1$ denote an unbiased biased estimate of $\Sigma_1$ obtained from an independent set of $n_1$ observations which consist of noise only and no signals. In this case $S_1$ and $S_2$ are distributed independently as complex Wishart matrices with $n_1$ and $n_2$ degrees of freedom and $E(S_i) = n_i\Sigma_i$ for $i = 1,2$ where the relationship between $\Sigma_1$ and $\Sigma_2$ is given by (3.2) with $\Sigma_{\Gamma}$

instead of $\sigma^2 I$. Now, let $\lambda_1 \geq \dots \geq \lambda_p$ denote the eigenvalues of $\Sigma_2 \Sigma_1^{-1}$ and let $\delta_1 \geq \dots \geq \delta_p$ denote the eigenvalues of $S_2 S_1^{-1} n_1/n_2$. Also let

$$H_q^* : \lambda_q > \lambda_{q+1} = \dots = \lambda_p = 1. \tag{4.3}$$

Zhao, Krishnaiah and Bai (1985b) showed that the LRT statistic for $H_q^*$ is given by

$$L_q^* = \prod_{i=1+\min(q,\tau)}^{p} (\alpha_n + \beta_n \delta_i)^{-n/2} \delta_i^{-n\beta_n/2} \tag{4.4}$$

where $n = n_1 + n_2$, $\alpha_n = n_1/n$, $\beta_n = n_2/n$ and $\tau$ denotes the number of $\delta_i$'s greater than one. Now, let

$$EDC^*(k, C_n) = -\log L_k^* + \gamma^*(k,p) C_n \tag{4.5}$$

where $\gamma^*(k,p) = k(2p-k)$ and $C_n$ satistfies the following conditions:

$$\text{(i)} \lim_{n \to \infty} (C_n/n) = 0 \tag{4.6}$$

$$\text{(ii)} \lim_{n \to \infty} (C_n/\log\log n) = \infty. \tag{4.7}$$

An estimate of $\hat{q}$ of $q$ is given by

$$EDC^*(\hat{q}, C_n) = \min\{EDC^*(0, C_n), \dots, EDC^*(p-1, C_n)\}. \tag{4.8}$$

Zhao, Krishnaiah and Bai (1985b) proposed the above estimate and proved its strong consistency.

Next, let $H_q : \lambda_q > \lambda_{q+1} = \dots = \lambda_p = \lambda$ where $\lambda$ is unknown. In this case, the likelihood ratio test statistic is known (see Rao (1983)) to be

$$L_q = \prod_{i=q+1}^{p} \left[ ((n_2 \delta_i + n_i \hat{\lambda}_{k0})/n)^n / \delta_1^{n_2} \hat{\lambda}_{k0}^{n_1} \right] \tag{4.9}$$

where $\hat{\lambda}_{k,0}$ satisfies the equation

$$(p-k) = \sum_{j=k+1}^{p} \{\hat{\lambda}_{k,0} / (\alpha_n \hat{\lambda}_{k,0} + \beta_n \delta_j)\}. \tag{4.10}$$

Now, let

$$\mathrm{EDC}(k,C_n) = -\log L_k + C_n\gamma(k,p) \tag{4.11}$$

where $\gamma(k,p) = k(2p-k)+1$. Then, an estimate of $\hat{q}$ of q is given by

$$\mathrm{EDC}(q,C_n) = \min\{\mathrm{EDC}(0,C_n),\ldots,\mathrm{EDC}(p-1,C_n)\}. \tag{4.12}$$

Zhao, Krishnaiah and Bai (1985b) proposed the above estimate and established its strong consistency. The property of strong consistency holds good even if the assumptions of independence and normality are violated provided the conditions (4.6) and (4.7) are satisfied and (3.11) and (3.12) are true for the two sets of observations used to estimate $\Sigma_1$ and $\Sigma_2$.

We will now discuss the method proposed by Paulraj and Kailath (1986) for the determination of the number of signals when the noise is spatially colored. They considered the situations when two estimates of the array covariances are available by displacing the array. The two-measurement model considered by them is given by

$$\underset{\sim}{x}_i(t) = A_i\underset{\sim}{s}_i(t) + \underset{\sim}{n}(t) \quad i = 1,2 \tag{4.13}$$

where $\underset{\sim}{x}_i(t)$ and $\underset{\sim}{s}_i(t)$ are respectively the observation vector and vector of wavefront of i-th measurement. The covariance matrices of the observation vectors for the measurements is given by

$$\Sigma_{2i} = A_i\psi_i\bar{A}_i' + \Sigma_1. \tag{4.14}$$

Also, $\Sigma_{2i}$ is estimated with $S_{2i}/n$ where

$$nS_{2i} = \sum_{j=1}^{n}\underset{\sim}{x}_i(t_j)\overline{\underset{\sim}{x}_i(t_j)}'.$$

In addition, let $P = \Sigma_{21} - \Sigma_{22}$ and $R = (1/n)(S_{21} - S_{22})$. Now, let $\lambda_1 \geq \ldots \geq \lambda_p$ denote the eigenvalues of P and let $\ell_1 \geq \ldots \geq \ell_p$ denote the eigenvalues of R. If the columns of $[A_1\ A_2]$ are distinct and $p \geq 2q$, then

$$\lambda_{q+1} = \ldots = \lambda_{p-q} = 0. \tag{4.15}$$

If $\psi_1$ and $\psi_2$ are positive definite and $\psi_1 - \psi_2$ is nonsingular and $A_1 = A_2$, then

$$\lambda_{q+1} = \dots = \lambda_p = 0. \tag{4.16}$$

Taking advantage of (4.15) and (4.16), Paulraj and Kailath proposed using methods similar to those used in Wax and Kailath (1984). But, unfortunately, the distribution of $S_{22} - S_{21}$ is very complicated since it is the difference between two complex Wishart matrices. So, we feel that the derivation of the likelihood ratio test statistics for testing the hypothesis that some of the eigenvalues are zero and the establishment of the consistency of the information theoretic procedures are difficult and may not be feasible. When $A_1 = A_2$, one may use some suitable function of $\ell_{k+1},\dots,\ell_p$ to test the hypothesis that $\lambda_{k+1} = \dots = \lambda_p = 0$. One possible test statistic is $\ell_{k+1}$. But, unfortunately, the distributions of test statistics based upon $\ell_{k+1},\dots,\ell_p$ involve $\lambda_1,\dots,\lambda_k$ as nuisance parameters. But, asymptotic distributions of the test statistics may not involve nuisance parameters. Remarks similar to the above may be made for testing the hypothesis that $\lambda_{k+1} = \dots = \lambda_{p-q} = 0$ when $\psi_1$ and $\psi_2$ are positive definite and the columns of $[A_1\ A_2]$ are distinct.

## 5. DETECTION OF NUMBER OF SIGNALS IN DECENTRALIZED PROCESSING

In some situations, the arrays of sensors are located in locations which are wide apart. In these cases, the model can be written as

$$\underset{\sim}{x}_h(t) = A_h \underset{\sim}{s}(t) + \underset{\sim}{n}_h(t) \tag{5.1}$$

for $h = 1,2,\dots,M$ where $\underset{\sim}{x}_h(t) : p_h \times 1$ denotes the vector of observation at t-th time point using h-th group of arrays, $\underset{\sim}{s}(t) : q \times 1$ is the signal vector, $\underset{\sim}{n}_h(t)$ is the noise vector on h-th subarray and $A_h : p_h \times q$ is a matrix whose elements depend upon the parameters of the signals. We also assume that $\underset{\sim}{s}(t)$ and $\underset{\sim}{n}_h(t)$ are distributed independently as complex multivariate normal with $E(\underset{\sim}{s}(t)) = \underset{\sim}{0}$, $E(\underset{\sim}{n}_h(t)) = \underset{\sim}{0}$, $E(\underset{\sim}{s}(t)\overline{\underset{\sim}{s}(t)'}) = \psi$ and $E(\underset{\sim}{n}_h(t)\overline{\underset{\sim}{n}_h(t)'}) = \sigma^2 I$. We also assume that $\underset{\sim}{x}_1(t),\dots,\underset{\sim}{x}_M(t)$ are distributed independent of each other. The covariance matrix $\Sigma_{2h}$ of $\underset{\sim}{x}_h$ is given by

$$\Sigma_{2h} = A_h \psi \bar{A}_h' + \sigma^2 I_{p_h} \tag{5.2}$$

Now let $S_{2h} = \sum_{j=1}^{n} x_h(t_j)\bar{x}_h(t_j)$. Also, let $\ell_{1h} \geq \dots \geq \ell_{p_h h}$ denote the eigenvalues of $S_{2h}/n$. Under the model (5.1) Wax and Kailath (1985) proposed to estimate q, the number of sources, using AIC and MDL criteria. The same method can be used even if the noise variance differs from subarray to subarray. We now propose a method of estimation of the number of signals using the EDC criterion when the covariance matrix of $\underset{\sim}{n}_h(t)$ is $\sigma_h^2$.

$$\mathrm{EDC}(C_{nh};p_h) = -\log L_{kh} + C_{nh}\gamma(k,p_h) \tag{5.3}$$

where $C_{nh}$ satisfies the conditions

$$\text{(i)} \lim_{n\to\infty} \{C_{nh}/n\} = 0$$

$$\text{(ii)} \lim_{n\to\infty} \{C_{nh}/\log\log n\} = \infty. \tag{5.4}$$

Also, $\gamma(k,p_h) = k(2p_h-k+1)+1$, and

$$L_{kh} = \{ \prod_{i=k+1}^{p_h} \ell_{ih}^n / (\sum_{i=k+1}^{p_h} \ell_{ih}/(p_h-k))^{n(p_k-k)} \} \tag{5.5}$$

In addition, let

$$\mathrm{EDC}(k) = \sum_{h=1}^{M} \mathrm{EDC}(k,C_{Nh};p_h) \tag{5.6}$$

Then, an estimate $\hat{q}$ of q is given by

$$\mathrm{EDC}(\hat{q}) = \min\{\mathrm{EDC}(0),\dots,\mathrm{EDC}(p-1)\} \tag{5.7}$$

where $p = \min(p_1,\dots,p_M)$. We may choose $C_{nh}$'s to be equal to $C_n$. The above procedure may be applied even if $\sigma_1^2 = \dots = \sigma_M^2$. Alternatively, we can use the procedure described below. Let $L_k$ denote the likelihood ratio test statistics for testing the hypothesis that

$$\lambda_{k+1,h} = \dots = \lambda_{p_h} = \sigma^2 \quad h = 1,2,\dots,M.$$

In this case

$$L_k = \{\prod_{h=1}^{M} \prod_{i=k+1}^{p_h} \ell_{ih}^{n} / (\sum_{h=1}^{M} \sum_{i=p+1}^{p_h} \ell_{ih}/(p_0-kM))^{n(p_0-kM)} \tag{5.8}$$

where $p_0 = p_1 + \ldots + p_M$. Now, let

$$EDC(k,C_n) = -\log L_k + C_n \gamma(k,p_0) \tag{5.9}$$

where $\gamma(k,p_0) = k(2p_0-kM+M) + M$ and $C_n$ satisfies the condition

$$\text{(i)} \lim_{n\to\infty} (C_n/n) = 0$$
$$\text{(ii)} \lim_{n\to\infty} (C_n/\log\log n) = \infty.$$

Then, the number of signals q can be estimated with $\hat{q}$ where $\hat{q}$ is given by

$$EDC(\hat{q},C_n) = \min\{EDC(0,C_n),\ldots,EDC(p-1,C_n)\} \tag{5.10}$$

where $p = \min(p_1,\ldots,p_M)$ as before. Here, we note that $\underset{\sim}{x}_1(t),\ldots,\underset{\sim}{x}_M(t)$ are not distributed independent of each other. If we assume that $\underset{\sim}{n}_1(t),\ldots,\underset{\sim}{n}_M(t)$ are distributed independent of each other, then $E\{\underset{\sim}{x}_i(t)\overline{\underset{\sim}{x}_j(t)}'\} = A_i\psi A_j^{-1}$ So, $L_k$ is not the likelihood ratio test statistic unless $A_i\psi\bar{A}_j' = 0$ for $i \neq j$.

Next, let the covariance matrix of $\underset{\sim}{n}_h(t)$ be $\Sigma_h$ and other assumptions on the model (5.1) remain the same. Also, let $\hat{\Sigma}_{1h}$ denote an unbiased estimate of $\Sigma_{1h}$ obtained from an independent sample of observations which consist of noise without signal. We assume that the size of this sample is n, and the observations in this sample are independent. We now propose an EDC procedure for the determination of the number of signals in this case.

Let $\lambda_{1h} \geq \ldots \geq \lambda_{p_h h}$ denote the eigenvalues of $\Sigma_{2h}\Sigma_{1h}^{-1}$ and let $\delta_{1h} \geq \ldots \geq \delta_{p_h h}$ denote the eigenvalues of $\hat{\Sigma}_{2h}\hat{\Sigma}_{1h}^{-1}$. If the number of signals is q, then

$$\lambda_{q+1,h} = \ldots = \lambda_{p_h,h} = 1 \tag{5.11}$$

for $h = 1,2,\ldots,M$. Now, let

$$EDC(k,C_{nh};p_h) = -\log L_{kh} + \gamma_h(k,p_h)C_{nh} \tag{5.12}$$

where $\gamma_h(k,p_h) =$

$$L_{kh} = \prod_{i=1+\min(k,\tau_h)}^{p_h} (\alpha_n + \beta_n \delta_{ih})^{-n/2} \delta^{-nb_n/2} \tag{5.13}$$

and $\tau_h$ denotes the number of eigenvalues of $\hat{\Sigma}_{2h}\hat{\Sigma}_{1h}^{-1}$ which are greater than one. Also, $C_{nh}$ satisfies the following conditions:

$$\text{(i)} \lim_{n\to\infty} \{C_{nh}/n\} = 0$$

$$\text{(ii)} \lim_{n\to\infty} \{C_{nh}/\log\log n\} = \infty. \tag{5.14}$$

Here, we note that $L_{kh}$ denotes the likelihood ratio test statistic for testing the hypothesis that $\lambda_{q+1,h} = \ldots = \lambda_{p_h,h} = 1$. Now,

$$EDC(k) = \sum_{h=1}^{M} EDC(k, C_{nh}; p_h). \tag{5.15}$$

Then, an estimate $\hat{q}$ of q is given by

$$EDC(\hat{q}) = \min\{EDC(0), \ldots, EDC(p-1)\} \tag{5.16}$$

where $p = \min(p_1, \ldots, p_M)$ as before. By following the same lines as in Zhao, Bai and Krishnaiah (1985b), the consistency of the above procedure can be established.

Now, let $\Sigma_{11} = \ldots = \Sigma_{1M}$ and let $L_k$ denote the likelihood ratio test statistic for testing the hypothesis

$$\lambda_{k+1,h} = \ldots = \lambda_{p,h} = 1 \quad \text{for } h = 1,2,\ldots,M.$$

In addition, let

$$EDC(k, C_n) = -\log L_k + C_n \gamma(k)$$

where $\gamma(k)$ denotes the number of free parameters that have to be estimated and $C_n$ satisfies the following conditions:

$$\text{(i)} \lim_{n\to\infty} \{C_n/n\} = 0$$

$$\text{(ii)} \lim_{n\to\infty} \{C_n/\log\log n\} = \infty.$$

Then, we can estimate q with $\hat{q}$ where $\hat{q}$ is given by

$$EDC(\hat{q}, C_n) = \min\{EDC(0, C_n), \ldots, EDC(p-1, C_n)\}$$

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