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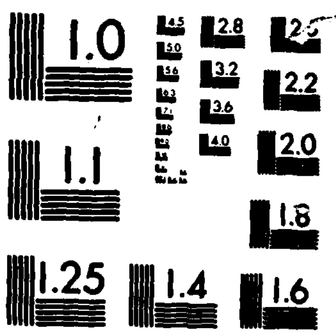

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ON THE STABILITY OF THE PROBLEM OF TARGET TRACKING WITH THE FLIR

IN-HOUSE LABORATORY INDEPENDENT RESEARCH

Final Report for Research Undertaken During Calendar Years 1984 and 1985

BY

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I. Problem

The problem considered in this research is as follows:

Consider stochastic differential equations of Ito type:

(*) $dx_t = f(x,t) + \sigma(x,t)\ dw_t$

where $x \in$ R**m, $t \in$ R,

f:R**m+1 --> R**m, σ:R**m+1 --> $M_{m,n}$ (R)

(**) f and σ are C^{∞},

$M_{m,n}$ (R) is the set of all matrices with entries in R

(R is the set of real numbers, and for any integer p>o, R**p is the set of ordered p-tuples of real numbers), and w is a Wiener process with values in R**n. Find a sufficient condition for the existence of a Liapunov function for an equation of the form of (*) and given stability region R' in R**m. This condition should have the following properties:

1) It should be the conjunction of finitely many equations or/and inequalities involving f and σ, valid for all m and n > 1,

2) The set of ordered pairs (f,σ) satisfying this condition should be a non-empty open subset of the space of all ordered pairs (f,σ) such that f and σ satisfy condition (**), with the C^{∞} topology,

3) The condition should be computationally practical to verify for a large subset of this space of ordered pairs (f,σ) for moderately large m and n (in particular, for cases where the components of f and are polynomials in (x,t) of small degree).

Further, find a general expression for a function V which is a Liapunov function for (*) when this condition is satisfied. This expression should be one which can easily be used to obtain estimates for derivatives of V with respect to (x,t) in a bounded region of (x,t) space.

II. Background and Relevance

→ This document considers stochastic differential equations of Ito type.

This problem is a fundamental one in the field of stochastic control. The theory of stochastic control has been successfully applied to a wide variety of practical problems as demonstrated in Maybeck <M2>, including problems involving the tracking of objects with imaging sensors. The problem of tracking aircraft with infrared sensors has been addressed in this mathematical framework by Kendrick, Maybeck, and Reid <K2> and by Maybeck, Jensen, and Harnley <M1>.

The specific problem which gave impetus to this research involves real time control of the orientation of the optical axis of an infrared imaging sensor.

The problem of choosing a good control for a system modeled in this framework involves the property of stability of solutions of the stochastic differential equation representing the system. Stability of these solutions is established by constructing a Liapunov function for the equation. In this way, the construction of a Liapuinov function for the equation is of great value in choosing controls. This and other background material are explained in Kushner <K1>.

In his recent monograph, Mees <M3> remarked that there is no good general method for constructing Liapunov functions for systems represented by ordinary differential equations.

III. Approach

The condition for a function V to be a Liapunov function with stability region R' in the state space is the condition that both of the following inequalities are satisfied at all points in R' at all times:

V > = Ø and L V < = Ø

where L denotes the infinitessimal generator of the solution of the stochastic differential equation.

The approach involves choosing an appropriate set of candidate functions for V according to the criteria to be described.

Finding V which satisfy the second inequality involves an elementary fact from algebraic geometry:

If f and g are polynomials in n variables over a field, then a sufficient condition that the corresponding hypersurfaces do not intersect is that there exist polynomials h and j in the same variables such that the following polynomial identity holds:

$$h\ f + j\ g = 1.$$

V is chosen so that the restriction of L V to the sphere of radius a and center Ø is a polynomial in the state space variables and time, for an interval of a values, and the restriction of the above equation to each of these spheres is applied with f = L V and g = x.x - a**2. This leads to a form for V as a polynomial combination of state space variables and time with coefficients which are unknown functions of a. The bulk of the work involves finding ways to simplify the equations to obtain conditions for existence of a solution which are reasonably general and at the same time practical to verify for particular cases. A system of linear ordinary differential equations for the unknown functions of a is derived and conditions for putting the system into a form such that there is a solution are found.

The first inequality is satisfied by choosing V to be close to a polynomial combination of state space variables with positive coefficients, on each of the spheres described above.

IV. Main Results

The main result obtained is for the case where the region R' is the region between two concentric spheres of radii a_0 and a_1, $0 < a_1 < a_0$, where the center is a point of equilibrium e of the system of ordinary differential equations obtained by setting the noise terms in the given system to Ø. This result states that for systems with a 2-dimensional state space such that

1) The drift term of the stochastic equation consists only of linear and quadratic terms in the state space variables,

2) There is a bound on the magnitudes of the noise terms of the stochastic equation,

there is a system of inequalities which gives a sufficient condition for existence of a Liapunov function V for the given equation for the region R'. The quantities involved in these inequalities are polynomial combinations of

(P) the coefficients of the linear and quadratic terms in (1), the bound on the noise terms in (2), the variable a which represents distance from the point e, 1/a, the parameters a_o and a_1, other constants

and

quantities $q_i(a)$, 1 <= i <= 14, each of which is a linear combination of entries of the product integral over a of a matrix W, where the entries of W are rational functions of the quantities in (P).

A general expression for V as a polynomial combination of the quantities in (P) and (Q) is given.

V. Future Research and Applications.

The principal investigator intends to generalize the theorem to the case of an n-dimensional state space and polynomial drift terms of arbitrary degree. If this could be achieved with $a_1 = 0$ then the applicability of the result to tracking, over either a finite or an infinite time interval, would be clear. For tracking over a finite time interval for a non-degenerate diffusion model, the difficulty of the restriction $a_1 > 0$ can be overcome via the averaging principle, in particular, by means of an inequality in Freidlin <F1>, and the application of the above theorem to the corresponding system of ordinary differential equations. The principal investigator intends to give a fuller report on present results and generalizations of them in a published paper.

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<M1> Maybeck, Jensen, and Harnley, "An Adaptive Extended Kalman Filter for Target Image Tracking", IEEE Transactions on Aerospace and Electronic Systesm, V. AES-27, No. 2 (1981).

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<M2> Maybeck, P. S., Stochastic Models, Estimation, and Control, VOLS. 1, 2, and 3, Academic Press, New York, 1982.

<M3> Mees, A. I., The Dynamics of Feedback Systems, Wiley, New York, 1982.

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