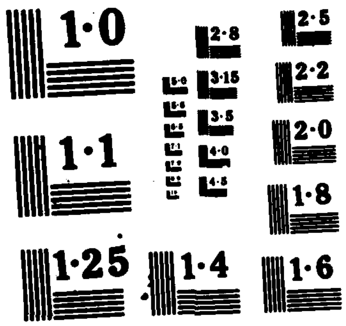
AD-A167 727 ON THE HANDLING QUALITIES OF FLIGHT VEHICLES THROUGH VARIABLE FLIGHT CONDITIONS(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH K C GREEN JAN 86 1/2

UNCLASSIFIED AFIT/CI/NR-86-56T F/G 20/4 NL



NATIONAL BURFAU OF S
MICROCOPY RESOLUT TEST

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFIT/CI/NR 86-56T	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) On The Handling Qualities of Flight Vehicles Through Variable Flight Conditions		5. TYPE OF REPORT & PERIOD COVERED THESIS/~~DISSERTATION~~ 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Kenneth C. Green		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS AFIT STUDENT AT: Massachusetts Institute of Technology		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
. CONTROLLING OFFICE NAME AND ADDRESS AFIT/NR WPAFB OH 45433-6583		12. REPORT DATE 1986 13. NUMBER OF PAGES 98
4. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASS 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

6. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

APPROVED FOR PUBLIC RELEASE: IAW AFR 190-1

Lynn Wolaver

LYNN E. WOLAVER 17 April 86
Dean for Research and
Professional Development
AFIT/NR, WPAFB OH 45433-6583

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

56

ON THE HANDLING QUALITIES OF FLIGHT VEHICLES THROUGH VARIABLE FLIGHT CONDITIONS

by
Kenneth C. Green

Submitted to the Department of Aeronautics and Astronautics on December 20, 1985, in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

(This thesis develops an approach

Specifications of flight vehicle handling qualities are currently based on the solution of linear, constant coefficient equations of motion, even when certain flight vehicles encounter continuously varying conditions in parts or all of their flight regime. ~~An approach is developed here~~ to encompass flight vehicle dynamics that vary during the performance of piloted tasks in the specification of handling qualities. Asymptotic methods, particularly Generalized Multiple Scales, are used to develop approximate analytical solutions to time-varying equations of motion. The solutions are valid throughout the time of variation, with constant flight conditions as a special case. Two classes of flight vehicles are examined to demonstrate the application of asymptotic methods to obtaining parameters pertinent to handling quality analyses. Extensions to existing criteria are suggested. This treatment of variable dynamics can give a clearer understanding of the response characteristics, and therefore of the stability and control requirements, of many piloted vehicles. (Theses)

Thesis Supervisor: Rudrapatna V. Ramnath
Title: Adjunct Professor of Aeronautics and Astronautics, MIT

56

CSDL-T-898

ON THE HANDLING QUALITIES OF FLIGHT VEHICLES THROUGH VARIABLE FLIGHT CONDITIONS

by

Kenneth Green

January 1986

Master of Science Thesis
Massachusetts Institute of Technology

The Charles Stark Draper Laboratory, Inc.
Cambridge, Massachusetts 02139

ON THE HANDLING QUALITIES OF FLIGHT VEHICLES
THROUGH VARIABLE FLIGHT CONDITIONS

by
Kenneth C. Green

B.S., United States Air Force Academy
(1981)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF

MASTER OF SCIENCE
IN AERONAUTICS AND ASTRONAUTICS

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

January 1986

Signature of Author Kenneth C Green
Department of Aeronautics and Astronautics
December 20, 1985

Certified by Rudrapatna V. Ramnath
Rudrapatna V. Ramnath
Adjunct Professor, Thesis Supervisor

Accepted by ____________________
Harold Y. Wachman
Chairman, Departmental Graduate Committee

ON THE HANDLING QUALITIES OF FLIGHT VEHICLES THROUGH VARIABLE FLIGHT CONDITIONS

by
Kenneth C. Green

Submitted to the Department of Aeronautics and Astronautics on December 20, 1985, in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

Specifications of flight vehicle handling qualities are currently based on the solution of linear, constant coefficient equations of motion, even when certain flight vehicles encounter continuously varying conditions in parts or all of their flight regime. An approach is developed here to encompass flight vehicle dynamics that vary during the performance of piloted tasks in the specification of handling qualities. Asymptotic methods, particularly Generalized Multiple Scales, are used to develop approximate analytical solutions to time-varying equations of motion. The solutions are valid throughout the time of variation, with constant flight conditions as a special case. Two classes of flight vehicles are examined to demonstrate the application of asymptotic methods to obtaining parameters pertinent to handling quality analyses. Extensions to existing criteria are suggested. This treatment of variable dynamics can give a clearer understanding of the response characteristics, and therefore of the stability and control requirements, of many piloted vehicles.

Thesis Supervisor: Rudrapatna V. Ramnath
Title: Adjunct Professor of Aeronautics and Astronautics, MIT

Acknowledgements

I am grateful to the Air Force Institute of Technology and Air Force Systems Command for giving me the opportunity to live and learn in Boston. I thank Dr. Rudrapatna Ramnath, my advisor, who has posed many intriguing problems and encouraged me to seek their solutions. A number of people at the C.S. Draper Laboratory have been a great help, especially Diane, Shirley and Dr. Dave Burke. Certain computations could not have been accomplished in time without the assistance of Mr. Jules Gilbert and muSIMP/muMATH computer algebra, c The Soft Warehouse, Honolulu, Hawaii. Special thanks go to my wonderful wife and daughter whom the Lord has given me and who made this time worthwhile. I am pleased to dedicate this work to my expected child.

TABLE OF CONTENTS

List of Figures

List of Figures (cont)

List of Tables

CHAPTER 1

INTRODUCTION

The design process for many piloted flight vehicles includes meeting specifications on their handling characteristics. These characteristics, or handling qualities, are primarily a function of the vehicle size and mission. The specifications are usually given in the form of bounds on parameters of the vehicles' dynamics that can be directly related to pilots' degree of acceptance. Specifications of handling qualities have been written for a wide variety of flight vehicles, including transport, vertical take-off and landing (VTOL) and high performance fighter aircraft, and lifting re-entry vehicles (LRV's). To date all specifications are based on analysis of the dynamic equations of motion at constant flight conditions, and parameters used to define the acceptable handling qualities (natural frequency, damping, bandwidth, etc.) are derived through classical methods of linear constant coefficient differential equation theory. Many aircraft operate primarily at equilibrium conditions, and the current specifications are well suited to such aircraft. But this treatment can, in certain cases, misrepresent design goals or lead to inaccurate interpretations of dynamic behavior. While all flight vehicles encounter a range of flight conditions, certain vehicles have dynamics which vary in a prescribed manner in portions of their flight envelopes. Understanding the variable dynamic behavior as completely as possible aids the design of vehicle and control system configurations. An approach should be developed to encompass flight vehicle dynamics that vary during the performance of piloted tasks, as well as dynamics that are invariant, in the specification of handling qualities. This thesis will examine the specification of handling qualities for flight vehicles whose dynamics are described explicitly by variable coefficient differential equations in parts or all of their flight regimes.

1.1 Classes of Dynamic Systems

The linear, constant coefficient class of problems is imbedded within a framework of analytical treatment that includes many other dynamic characteristics, as shown in Figure 1. The classification used may be due to

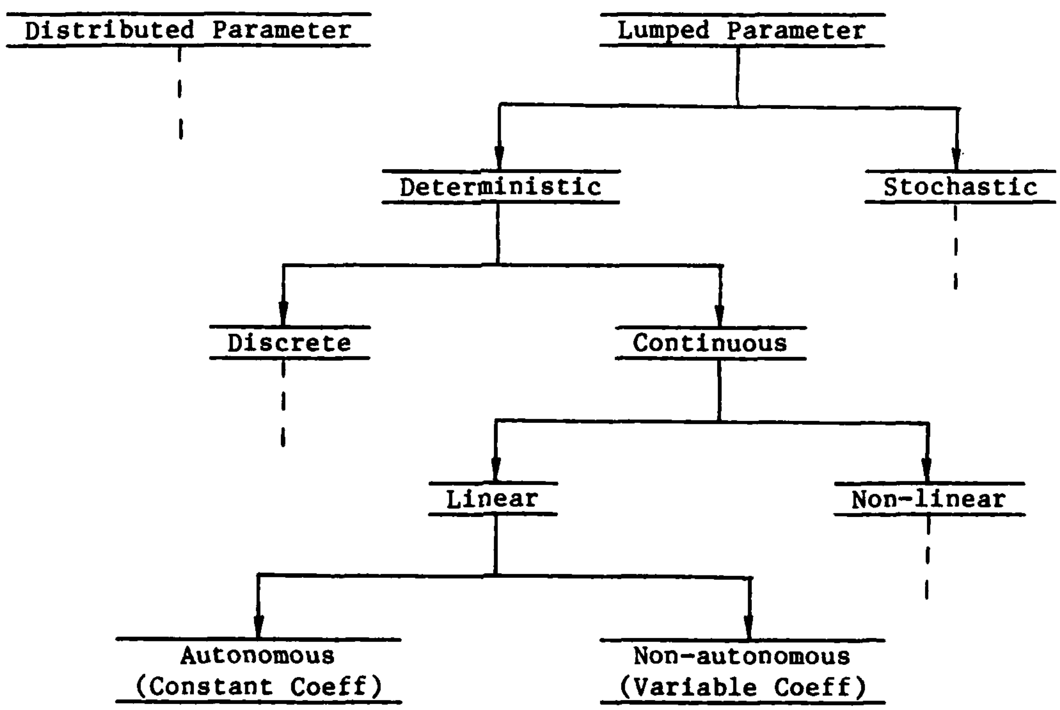


Figure 1

Classes of Dynamic Systems

the inherent nature of the system, or it could result from assumptions made as to its use or bounds on its operation. While such assumptions limit the validity of subsequent designs, they may allow the application of a large body of theory already developed for certain classes of dynamics.

Lumped parameter treatment of flight vehicles is appropriate for rigid body analysis, as will be the case here. Mathematical description of lumped parameter dynamics is in the form of a set of ordinary differential equations. These equations of motion contain a complete description of a flight vehicle's movement in six (rigid body) degrees of freedom. In general these force and moment equations are non-linear, time-varying and coupled.

Linearity is rarely an inherent property of the system, where superposition of inputs and outputs is valid. Many dynamics problems that exhibit "weakly" non-linear behavior are still treated as essentially linear in nature and compensation can counteract the non-linearity. Linearity can be imposed for analysis through simplifying approximations and perturbation techniques. Otherwise, the equations of motion are integrated by numerical techniques for specified initial or boundary conditions. For purposes of this work linear (or linearized) equations of motion will be used.

The dynamics may also be non-autonomous (e.g. time varying) so that an input shifted in time results in a different system response. Variable dynamics are characterized by equations of motion having coefficients which vary as functions of the independent variable(s) of the system. In flight dynamics the variation is typically a function of time, velocity, or atmospheric density. The nominal flight conditions, maneuver, or trajectory must be known explicitly in order to solve the equations. They can be treated either by numerical integration or, to gain some analytical tractability, by "freezing" the coefficients at desired intervals and performing classical linear constant coefficient analysis at each point. This latter approach presumes that the variable behavior is predictable using constant coefficient analysis and it is applied with strong reliance on the designer's experience and intuition. It is not always justified because the variations can lead to counter-intuitive behavior. Ramnath [1] gives examples of such counter-intuitive behavior and of the failure of constant coefficient analysis to correctly predict stability (see Appendix A). Even when a frozen approximation works for short intervals, there is no way to express the magnitude of the error over each interval or to determine the best points for freezing.

While solutions to variable differential equations are rarely known exactly, methods of finding approximate solutions with known bounds on the errors exist and can be widely applied. Pursuing analytical means of solution to variable dynamic problems results in definitive approaches to understanding and specifying their behavior.

1.2 Variable Flight Vehicle Dynamics

Familiar cases of flight vehicles exhibiting variable dynamic behavior are vertical take-off and landing (VTOL) aircraft and lifting re-entry vehicles (LRVs). During its transition from hovering to forward cruising flight a VTOL craft displays unconventional behavior, quite distinct from the standard short period and phugoid longitudinal modes. The equations of motion during this maneuver can be described using stability and control derivatives that vary as functions of forward velocity, which can be given as an explicit function of time [9]. The stability and dynamic response of the aircraft vary continuously through this transition.

A lifting re-entry vehicle encounters flight conditions ranging from zero dynamic pressure and high velocity outside the atmosphere to high dynamic pressure and subsonic speeds within the atmosphere. These extreme variations in velocity, Mach number and dynamic pressure result in continuously varying dynamic response to control commands. The equations of motion can be linearized about a prescribed trajectory to give variable perturbation equations of motion.

Conventional aircraft may also exhibit variable dynamic behavior when performing maneuvers which result in wide variations in flight conditions. Examples of such maneuvers are minimum-time climb to altitude and air-to-ground weapon delivery.

1.3 Approach

This research attempts to provide a link between the rigorous analytical treatment of non-autonomous flight vehicle dynamics and the specification of dynamic responses by handling qualities criteria. The intent is to relate workable methods of accurately computing measures of handling quality from the time-varying dynamics, and to assess the usefulness of existing criteria. The analytical techniques used to find approximate solutions to time-varying equations are described in chapter 2. One of the most general of these asymptotic techniques is Multiple Scales, by which the variable dynamic response of a VTOL aircraft and shuttle type LRV have been studied in [9] and [11]. These two types of vehicles are studied here to demonstrate the application of asymptotic methods to compute different measures of handling quality. Criteria which are currently used for specifying longitudinal handling qualities are described in chapter 3. These criteria are either used directly (e.g. time response envelopes) or modified to admit variable dynamic behavior (e.g. bandwidth histories) in chapter 4. Additional restrictions on the dynamic variation are proposed as extensions to the existing criteria.

CHAPTER 2

ASYMPTOTIC METHODS OF SOLUTION

Approximate solutions to equations of motion can be developed through asymptotic expansions, which generate a sequence of functions of decreasing magnitude [1]. The magnitude of the functions reaches a minimum at which point the asymptotic expansion most accurately represents the solution. Each term in the sequence is smaller than the preceding term by an order of magnitude of the expansion parameter, ε. This parameter, $0 < \varepsilon \ll 1$, arises from the physical characteristics of the dynamic system and may represent various properties depending on the application. It is an inherent parameter of the system if it cannot be removed by a change of variables in the equations of motion. The differential equations are first cast in a perturbation form, where ε appears explicitly, as $\mathcal{L}\{y, t, \varepsilon\} = 0$ (homogeneous case). $\mathcal{L}$ is a linear differential operator and the solution is $y(t,\varepsilon)$. As $\varepsilon \rightarrow 0$ the solution $y(t,0)$, or $y_0(t)$, should be meaningful. Then an asymptotic solution can be written as:

$$y(t,\varepsilon) \cong y_0(t) + \varepsilon y_1(t) + \ldots \tag{2.1}$$

The condition of uniformity must be imposed, in which the error between the function and its approximation is uniform in the entire domain of interest. This may require, for example, that higher order perturbation solutions are no more singular than lower order ones.

A very general method of constructing asymptotic expansions is Multiple Scales [2]. This technique extends the independent variable of the dynamic motion into a higher dimensioned space. In our case, multiple scaling will separate the dynamic response of the flight vehicle into independent scales which stem from intrinsic time constants of the dynamics. Applicability of Multiple Scales depends on slow variation of the coefficients of the equations of motion compared to modes of the vehicle response. The parameter ε arises from the ratio of the magnitudes of these variations. The scales then differ in magnitude by orders of ε. The approximation can be carried to any desired order of ε and therefore bounds the magnitude of the error. The multiple scales technique was generalized by Ramnath and Sandri to include non-linear and complex scales. Further development and extensions of the technique to deal with flight dynamics and control problems can be seen in the works of Ramnath [2-6].

2.1 Time responses

A linear variable dynamic system with state x and input u is represented by the equation

$$\sum_{i=0}^{n} a_i(t) \frac{d^i}{dt^i} x(t) = \sum_{i=0}^{n-1} b_i(t) \frac{d^i}{dt^i} u(t) \qquad (2.2)$$

When the coefficients vary slowly (i.e. on a slow time scale εt) the system equation can be approximated by:

$$\sum_{i=0}^{n} \varepsilon^i a_i(\varepsilon t) \frac{d^i}{d\varepsilon t^i} x(t) = \sum_{i=0}^{n-1} \varepsilon^i b_i(\varepsilon t) \frac{d^i}{d\varepsilon t^i} u(t) \qquad (2.3)$$

A solution to the homogeneous equation is sought first, then a particular solution may be computed by variation of parameters, for example. Equations of any order may be treated. Derivatives are computed in terms of the slow and fast time scales. The lowest order expression (for this type of problem) contains the characteristic equation for the clock function. An n^{th} order system has a characteristic equation of the n^{th} power in $k(t)$. Solutions are then developed in terms of the roots of this equation. The slow scale solution corrects the zeroth order fast scale solution to first order in .

For a state vector $\underline{x}$ and associated state equation

$$\dot{\underline{x}} = A(t)\underline{x}(t) + B(t)u(t) \tag{2.4}$$

the approach is similar. A straightforward way of dealing with a vector problem is to decouple the states into independent, scalar equations. An n^{th} order vector problem can be decoupled into scalar equations of n^{th} order. Decoupling is done by cross-differentiation of each state equation and sequential substitution of variables from equations of $(i-1)^{th}$ order into equations of i^{th} order. Since the dynamics are time varying each state will have a unique characteristic equation. Once decoupled, the time response of each state can be examined in the manner described above for scalar problems. Obviously this becomes unwieldy for high order problems, but the state equations used in handling quality investigations are often not above fourth order. If they are, an approach described in section 2.2 should be taken.

It is not necessary to have functional expressions for the coefficients or the roots (clock functions) in order to construct the asymptotic solution or test the slowness of variation. If tabulated data is the only form available, asymptotic expressions can still be formed symbolically, then tabulated data can be substituted into the symbolic solution. An approximate integration of the clock functions for the fast scale variable will be necessary. This is obviously a different approach than constant coefficient analysis, where a series of solutions at each tabulated time point (or some interval) are computed.

An interesting example first treated by Ramnath [3] using the multiple scales technique is the well-known problem

$$\frac{d^2y}{dt^2} + \omega(t)y = 0 \tag{2.5}$$

where $\omega(t)$ is "slowly" varying. By extending the independent variable

$$t \rightarrow \{\tau_0, \tau_1\}$$

by an appropriate definition of the new scales τ_0 and τ_1, Ramnath [2] derived the well-known WKBJ approximation. This asymptotic solution exhibits a separation of the fast and slow parts of the solution.

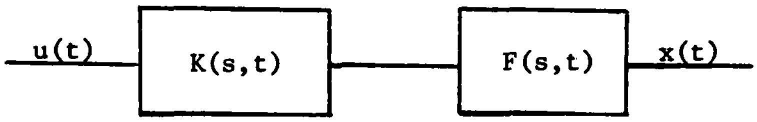
2.2 System Functions

A standard method of analyzing linear time-invariant system response to inputs is by transfer functions. Treatment of open and closed loop systems by well known rules makes design and analysis tractable for even large scale problems. Frequency response methods are very powerful tools for this class of problems and would hopefully have their counterpart in time-varying problems. With a variable system the concept of frequency response is altered. Transfer functions must be generalized to reflect the non-autonomous nature of the dynamics.

The complete description of linear time-varying system behavior is embodied in the impulse response or weighting function $W(t,t-\tau)$, which relates system output at time "t" to an impulse input at time "$t-\tau$". The transform of the weighting function

$$H(j\omega,t) = \int_0^\infty W(t,t-\tau)\, e^{-j\omega\tau}\, d\tau \tag{2.6}$$

is the system function of Zadeh [8], which is a function of frequency and time. For our purposes, $H(j\omega,t)$ is equivalent to $H(s,t)$, where s is the complex Laplace variable. $H(s,t)$ describes the response of a variable system to complex exponential input. A graphical interpretation of the system function might show its magnitude and phase angle as surfaces with ordinates of time and frequency. When a linear system is autonomous, the system function specializes to a transfer function (scalar case), $H(s)$. A useful property of the transfer function description is that cascaded systems are represented by the product of their transfer functions. This property does not generally hold exactly for system functions. For example, a time varying compensator cascaded with a variable system, shown by their system functions K and F,



do not necessarily have an overall system function $K(s,t)F(s,t)$. Likewise, the rules for the description and analysis of linear, constant coefficient systems do not necessarily apply directly when treating variable dynamics. There do exist, however, a set of rules for applying the classical design tools when the system functions vary slowly with time [6]. For instance, from [7],

> If two variable linear sub-systems (each having coefficients which are functions of slow time scale variable ϵt) are cascaded, and if the subsystem nearest the input has a system function possessing an asymptotic expansion of the form
>
> $$G(s,t) \sim G(s) + \epsilon^k G_k(s, t) + o(\epsilon^k) \text{ as } \epsilon \to 0$$
>
> for $k \geq 0$, then the system function of the cascaded system will have an asymptotic expansion equal to the product of the expansions of the system functions of the subsystems, plus terms of order ϵ^k.

A design procedure called Frequency Response by Asymptotic Methods (FRAM) has been developed [6]. By this procedure, design and analysis of variable dynamic systems and control compensation can be accomplished order by order (of ε) until desired tolerances are reached. Under the stated restrictions, conventional frequency domain design tools can be used once the system functions are computed.

2.2.1 Computing the System Function

The system function H(s,t) for a scalar problem satisfies the differential equation [8]

$$\sum_{i=0}^{n} \frac{1}{i!} \frac{\partial^i}{\partial s^i} L(s,t) \frac{\partial H(s,t)}{\partial t^i} = K(s,t) \tag{2.7}$$

where $L(s,t) = \sum_{i=0}^{n} a_i(t)\, s^i$ and $K(s,t) = \sum_{i=0}^{n-1} b_i(t)\, s^i$.

For a vector problem, H(s,t) satisfies the matrix differential equation [3]

$$\dot{H}(s,t) = [A(t) - sI]\, H(s,t) + B(t) \tag{2.8}$$

where A(t) and B(t) are the state dynamics and control input matrices, respectively.

The differential equations for H(s,t) can be solved asymptotically when the coefficients vary slowly. Asymptotic solutions have been developed by Ramnath [2] and by Callaham and Ramnath [6]. Consider an n^{th} order differential equation in the perturbation form of (2.3). Slow and fast scales of motion are defined as

$$\tau_0 = \varepsilon t \tag{2.9}$$

$$\tau_1 = \frac{1}{\varepsilon} \int k(\tau_0) d\tau_0 = \int k(t) dt \tag{2.10}$$

where k(t) is the clock. Following substitution of the multiple time scale derivatives into the perturbation equation, the zeroth order asymptotic equation for x is

$$\sum_{i=0}^{n} k^i a_i \frac{\partial^i x}{\partial \tau_1^i} = \sum_{i=0}^{n-1} k^i b_i \frac{\partial u}{\partial \tau_1^i} \tag{2.11}$$

Taking the Laplace transform of (2.11) with respect to τ_1 gives the transformed equation

$$\left(\sum_{i=0}^{n} k^i s^i a_i\right) X(s,t) = \left(\sum_{i=0}^{n-1} k^i s^i b_i\right) U(s,t) \tag{2.12}$$

The system function to zeroth order is then

$$G(s,t) = \frac{X(s,t)}{U(s,t)} = \frac{b_{n-1}\xi^{n-1} + \dots + b_1\xi + b_0}{\xi^n + \dots + a_1\xi + a_0} \tag{2.13}$$

where $\xi = ks$ is the "Laplace-clock" variable [2]. Improvements to the zeroth order approximation can be made order by order [6].

2.3 Application to Vehicle Dynamics

Analytical studies of the angle of attack (AOA) response of a LRV along an entry trajectory have been accomplished using a unified linear differential equation developed by Vinh and Laitone [10]. This equation has the form

$$\alpha'' + \omega_1(\lambda)\,\alpha' + \omega_0(\lambda)\,\alpha = f(\lambda) \tag{2.14}$$

where the independent variable is transformed from time to scaled lengths traveled along the trajectory. Vinh and Laitone solved two special cases: a steep, straight line entry and a shallow flight path angle entry, using non-elementary functions. Ramnath and Sinha solved the general case using the multiple scales asymptotic technique [11]. The general solution is in terms of elementary (exponential, sine, cosine) functions and is valid for a large class of entry trajectories and vehicles. The only restriction is slowness of variation of the coefficients with respect to vehicle motion, as already discussed. The solution by multiple scaling was compared to solutions by both constant coefficient analysis and numerical integration of the equations of motion. While the constant coefficient approach fails after the first quarter cycle of angle of attack oscillation (homogeneous case), the multiple scales solution is extremely close to the numerically integrated result for both the homogeneous and forced (steered) responses. Using an approach of freezing the coefficients along the trajectory and performing classical analysis at each point would obviously require many points to approach the accuracy of multiple scales. Even then there is no guarantee that it will correctly predict the stability and response of angle of attack motion, let alone be useful for handling qualities study. Having a generally valid approximate solution for angle of attack motion of this non-autonomous system opens the door to handling quality investigations, since angle of attack response is so closely tied to maneuvering ability.

Multiple scaling has also been applied to a calculating time responses of an unaugmented, tilt-wing VTOL aircraft in transition [9] and to pitch control compensation design for the same vehicle [7]. During transition the stability derivatives vary as functions of the thrust vector and elevator deflection. Assuming the aircraft is continuously trimmed along its transition trajectory, the stability derivatives can be expressed as slowly varying functions of flight velocity [9]. Thus, perturbation equations of motion, linearized about the trajectory, have variable coefficients. Given a linear dependence of trimmed velocity on thrust deflection, V(t) can be expressed as a function of time and substituted in the stability derivative expressions. The resulting equations are linear, time-varying and coupled.

Hover and cruise can be considered initial and final (constant flight) conditions for the transition. A typical VTOL aircraft in cruise has the standard short period and phugoid modes, while in hover the modes, and therefore the stability, are entirely different. The time-varying nature of the stability is that the roots must transition from their initial positions in hover to their final positions in forward flight. The roots effectively move in the complex s-plane as a function of time (velocity), which is fundamentally different from the movement of closed loop roots as a function of system gain. The actual stability variation depends on the vehicle.

For the time response calculations, Ramnath decoupled the longitudinal states by cross differentiation and computed multiple scales approximations to zeroth and first order of ϵ. To order (ϵ^{0}) the solution is a function of the fast scale variable only. The fast scale solution is determined by the roots of the characteristic-clock equation, $k_i(t)$. Correction to order (ϵ) includes the slow time scale solution. Results agreed closely with solution by numerical integration [9]. Figure 2 depicts the time-varying root behavior. The two real roots in hover move during transition to make up the short period roots in forward cruising flight. The unstable complex roots in hover become the phugoid roots in cruise.

For the compensator design, Callaham and Ramnath [6] derived an asymptotic approximation to the system function. Following the FRAM design procedure, a zeroth order analysis and compensation were done first. The time-varying compensator stabilized the pitch response throughout transition. Higher order analysis was accomplished, but the zeroth order design met specifications on pitch attitude response. This approach is an alternative to the conventional method of scheduling control system gains, which now become an explicit function of an independent (or dependent) variable of the motion.

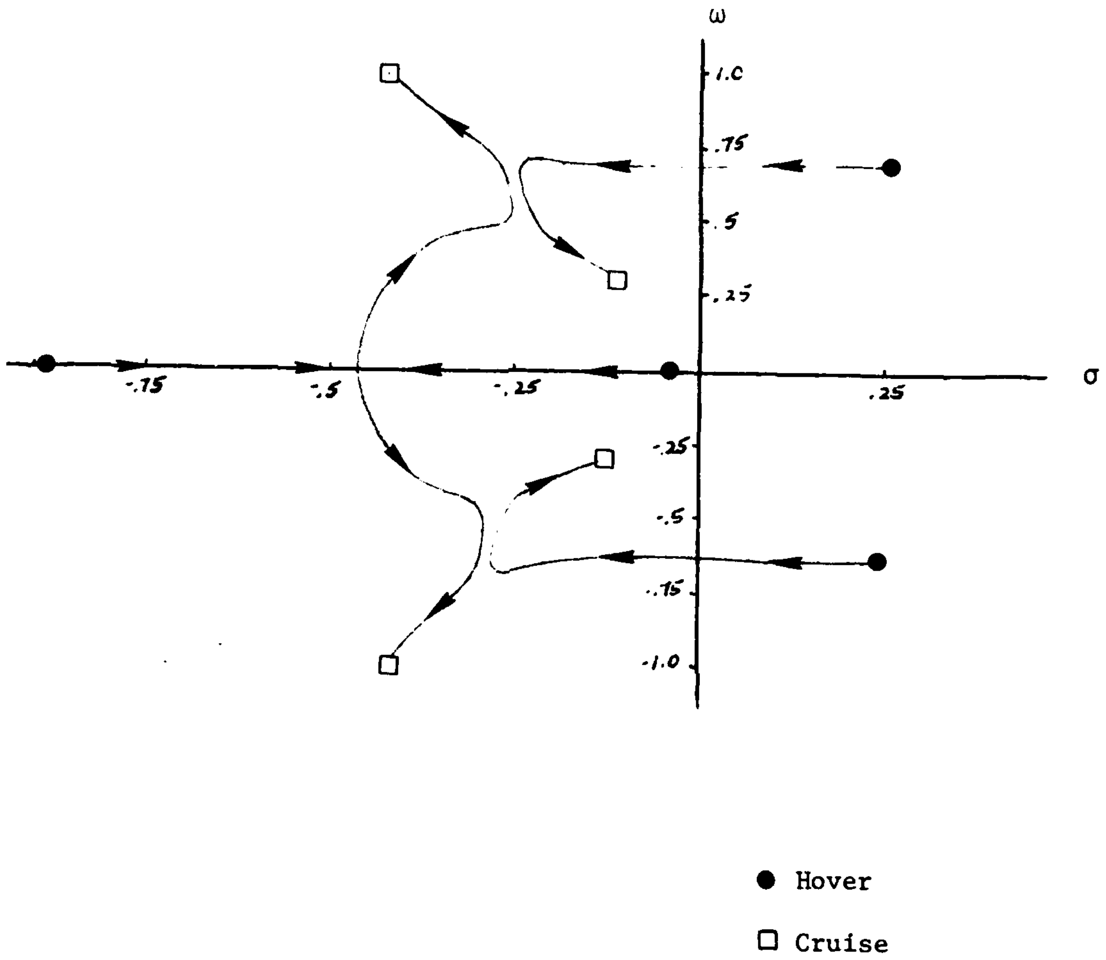


Figure 2

VTOL Root Movement During Transition

CHAPTER 3

HANDLING QUALITIES BACKGROUND

Handling qualities have been defined best as those "characteristics of an aircraft that govern the ease and precision with which a pilot is able to perform the tasks required in support of an aircraft role." [12]. A framework for treating all types of piloted aircraft has evolved that assigns a classification to the vehicle, and defines requirements for its flight phases at each of three levels of handling quality. It is the level rating that links the pilot assessment of vehicle handling characteristics to quantitative bounds on measures of the vehicles dynamic response.

Lifting Re-entry Vehicles are not necessarily in one class, although a space shuttle type vehicle would fall in Class III, large, heavy, low-to-medium maneuverability airplanes. A VTOL aircraft would most likely be either a Class II, medium weight, low-to-medium maneuverability (such as light transport) airplane, or a Class IV, high maneuverability (such as fighter/attack) airplane.

There are categories of Flight Phases into which all segments of an aircraft's mission can logically fit and within which the boundaries of handling parameters may be relaxed or restricted. Table 1 gives representative flight phases for several flight vehicles.

TABLE 1

Non-terminal Flight Phases

Category A - Rapid maneuvering, precision tracking, precise flight path control

Aircraft	LRV	VTOL
Air-to-air combat	AOA transition	
Weapon delivery		
Formation		

Category B - Gradual maneuvers, precise flight path control

Climb	Descent	
Cruise	S-turns	
Descent		

Terminal Flight Phases

Category C - Gradual or Rapid Maneuvering, Precise Flight Path Control

Take-off	Approach	Vertical Take-off
Approach	Landing flare, float, touchdown	Transition - hover to cruise
Landing		

The different levels of flying qualities stem from the goal of subjectively rating the adequacy of an aircraft's handling characteristics to accomplish its mission. The levels are:

Level 1 - Clearly adequate for mission Flight Phase

Level 2 - Adequate to accomplish Flight Phase, but some increase in pilot workload or degraded mission effectiveness exists

Level 3 - Aircraft can be controlled safely, but pilot workload is excessive or mission effectiveness is inadequate. Category A must be terminated, Categories B and C can be completed.

According to Cooper and Harper [12], "pilot evaluation still remains the only method of assessing the interactions between pilot/vehicle performance and total workload in determining suitability of an airplane for the mission." What they built is a pilot opinion rating scale which facilitates sequential decision making by the pilot in assigning a flying qualities rating for a given task. While variability between pilots is inevitable, the Cooper-Harper Pilot Opinion Rating Scale has emerged as the most logical basis for consistent pilot assessments of flying qualities (see Figure 3-1).

Other than the "uncontrollable" rating (10), there is a range of ratings within each of the three levels of handling. The boundaries between the levels are delineated by the necessity of design improvements to adequately accomplish the mission and/or the degree of pilot compensation required to offset design deficiencies.

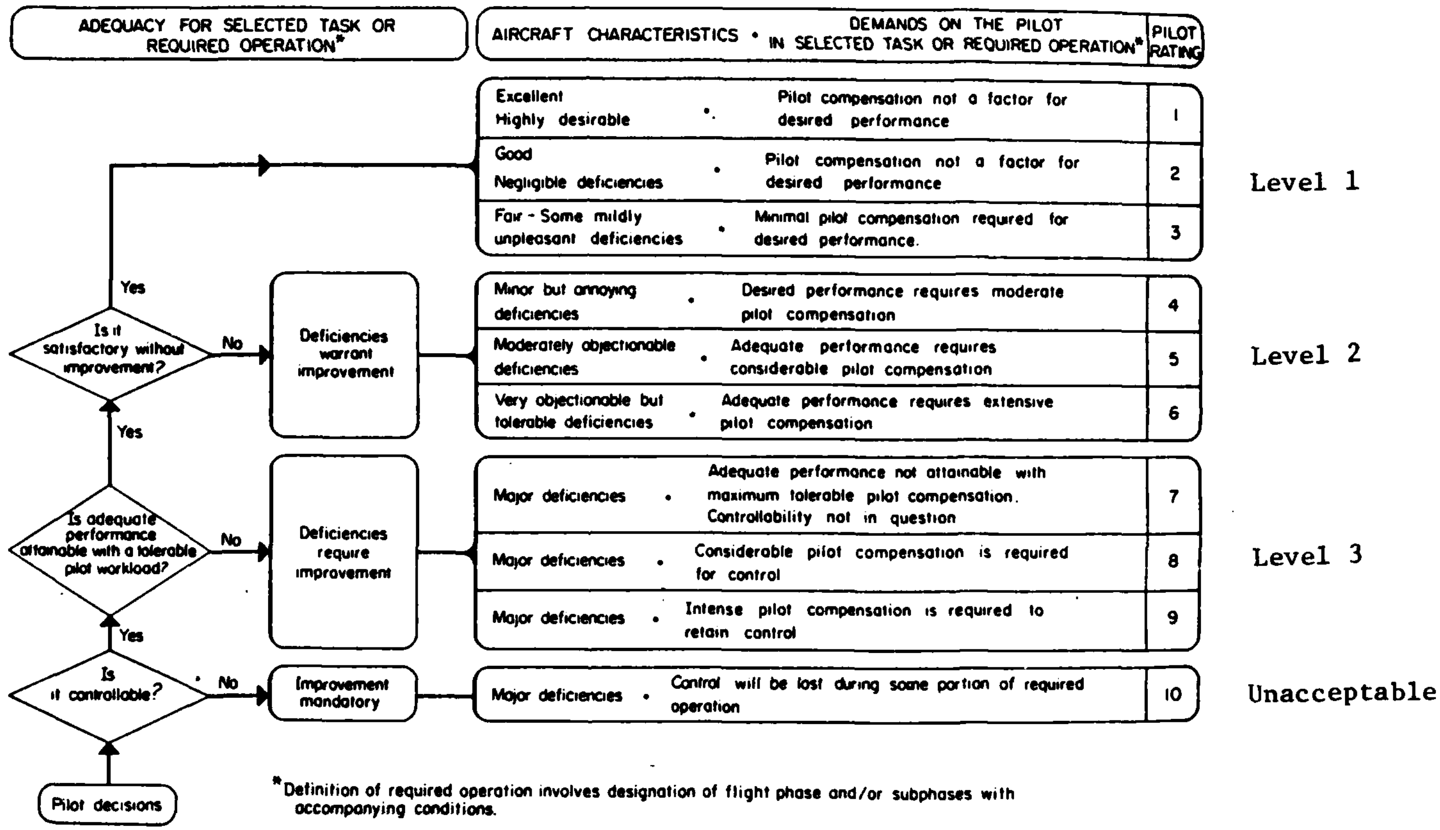


Figure 3-1

Cooper-Harper Pilot Opinion Rating Scale

Reference [12]

3.1 Rationale for Criteria

It is the goal of handling quality specifications to correlate flight vehicle open loop design parameters with flying quality levels for all mission phases of a given aircraft. Criteria which specify vehicle handling qualities should be oriented toward quantifying bounds on desired responses. When possible these bounds can be expressed in terms of commonly used parameters which influence or determine those responses. This allows analysts and pilots to more easily correlate pilot evaluation and dynamic characteristics. Specifying handling qualities is an iterative task. The aircraft and control system designer relies upon existing criteria to create acceptable designs, and handling quality analysts rely on new test data and pilot evaluations to refine or replace current specifications.

Discussion so far has centered on longitudinal vehicle behavior, although the lateral and directional dynamics are no less important. In order to begin defining handling quality specifications for the vehicles under study, longitudinal dynamics will be treated separately (decoupled) from lateral and directional motion. Conditions under which coupling is a factor exist in many flight maneuvers, however. Landing approach, for example, creates coupling effects where lateral corrections at high angle of attack affect longitudinal control.

3.2 State Space Formulation

A comprehensive fromework for treating the dynamics, and therefore the handling qualities, of flight vehicles is in state space. A state vector contains variables of motion which, together, completely describe the motion being studied. They form a set of first order, ordinary, differential equations. The variables used depends on the application and purpose of the analyst. A representative state vector referenced to the body axis of the vehicle is

$$\underline{x}(t) = \begin{bmatrix} u(t) \\ w(t) \\ q(t) \\ \theta(t) \end{bmatrix} \begin{array}{l} \text{axial (x-axis) velocity} \\ \text{normal (z-axis) velocity} \\ \text{pitch rate (about y-axis)} \\ \text{pitch angle (reference local horizon)} \end{array}$$

An alternate state representation referenced to the flight path is

$$\underline{x}(t) = \begin{bmatrix} V(t) \\ \gamma(t) \\ q(t) \\ \alpha(t) \end{bmatrix} \begin{array}{l} \text{total velocity along instantaneous flight path} \\ \text{flight path angle} \\ \text{pitch rate} \\ \text{angle of attack} \end{array}$$

Resulting equations have the non-linear form x(t) = f(x(t), (t),t) where (t) is a control input vector. In linear form the equations are

$$\Delta \dot{\underline{x}} = F(t)\Delta \underline{x} + G(t)\Delta \underline{u} \tag{3.1}$$

where F(t) and G(t) are Jacobian matrices of partial derivatives with respect to states and controls. If the elements of the matrices are evaluated at trim conditions, F and G are constant. F contains stability derivative, gravity and kinematic terms; G contains control derivative terms. From the linearized state equations, transfer functions formed by Laplace transformation relate control inputs to system outputs. When multiple, independent inputs are present, coupling numerators [13] account for their effects on output or state variables.

The ability to form outputs that are meaningful to handling qualities is a useful quality of the state space approach. Generally, outputs

$y(t) = h[x(t),t]$, but often outputs of interest are either states themselves, or linear combinations of state variables. In that case, $y(t) = H(t)x(t)$. Commonly used outputs of this type are pitch rate (q) and normal load factor (Nz).

3.3 Conventional Equations of Motion and Approximations

Classical aircraft longitudinal responses are described by a fourth order characteristic equation. In many cases the responses can be separated into short term and long term behavior according to the characteristic root separation. Short term response, consisting primarily of attitude and angle of attack change, relates to maneuvering characteristics of the aircraft. Long term response has little angle of attack change but relates primarily to flight path and speed stability. This distinction between modes leads to useful approximations of second order.

Commonly used approximations to aircraft dynamics result from reduced order models. Truncation refers to deleting elements of the state vector, thereby reducing system order, and treating only the remaining dynamics. In this way vehicle motion that manifests itself as different modes is separated for analysis. In terms of the dynamics matrix, F, the elements are partitioned into blocks corresponding to the different modes. Elements in F which couple states from one mode into another mode are ignored. Justification for truncated models usually asserts that modes of the response occur on widely separated time scales, with little interaction betewen modes. One way of verifying this is to determine the real axis separation between characteristic roots which correspond to different modes. Interaction between modes is apparent in the numerator terms of transfer functions, formed from

Laplace transformation of the state equation, for an output (state) variable associated with one of the modes. When numerator roots are close to characteristic roots there are small residues associated with those dynamics, meaning little effect on the output. In this way the roots associated with a certain modes may be effectively cancelled.

The natural application for truncation to longitudinal vehicle motion is in separating the phugoid and short period modes. The state vector and dynamics matrix can be set up so that states variables are partitioned by modes they principally describe. Using the state vector $\underline{x} = [V\ \gamma\ q\ \alpha]^T$, correct partitioning is simply $\underline{x} = [V\ \gamma \mid q\ \alpha]^T$ because short term (fast) motion primarily involves pitch rate and angle of attack change, whereas the long term (slow) motion is primarily a combination of velocity and flight path angle oscillations. The truncated F matrix looks like

$$F = \left[\begin{array}{cc|cc} f_{11} & f_{12} & & \\ f_{21} & f_{22} & & \\ \hline & & f_{33} & f_{34} \\ & & f_{43} & f_{44} \end{array}\right]$$

where the upper right and lower left blocks, made up of terms which couple the modes, are ignored.

An extension of the truncated model accounts for steady state effect of the "fast" mode on the "slow" mode; the slow mode is still considered to have neglibible effect on the fast mode. This "residual" of the fast motion can be expressed in terms of blocks of the F matrix. Using the example of longitudinal aircraft motion, the F matrix is partitioned as above but the elements left intact. The steady state fast mode state variables q and α are solved for in terms of slow mode state variables V and γ by setting

$[q \;\; \alpha]^T = \emptyset$. Substituting these expressions for $q_{ss}(V,\Upsilon)$ and $\alpha_{ss}(V,\Upsilon)$ in the differential equations for V and Υ gives

$$\begin{Bmatrix} \dot{V} \\ \dot{\Upsilon} \end{Bmatrix} = \begin{bmatrix} F_1 - F_2F_4F_3 \\ \end{bmatrix} \begin{Bmatrix} V \\ \Upsilon \end{Bmatrix} , \quad \text{from } F = \begin{bmatrix} F_1 & F_2 \\ F_3 & F_4 \end{bmatrix} \tag{3.2}$$

The result in this case is a residualized phugoid model [14].

The conditions under which reduced order modeling is appropriate must be carefully examined. Certain portions of the flight regime will allow accurate modeling with truncated or residualized dynamics, while many flight conditions must be treated using full state models despite the analytical difficulty.

3.4 Criteria and Parameters in Use

Conventional handling qualities criteria are primarily based on modal parameters that are derived from constant coefficient analysis of the linear equations of motion under single input/single output conditions. Transfer functions formed from the state equations provide the significant parameters for this modal analysis. As a result, criteria are frequency domain oriented and specify the natural order of the vehicle response, commonly approximated with first or second order dynamics. When actuators, stability or command augmentation, structural filters or other elements of a control system increase the dynamic order of vehicle response, handling qualities have still been treated within the framework of basic vehicle responses. Therefore much work has been concentrated on developing low order "equivalent" systems to approximate dominant higher order behavior.

Parameters associated with equivalent systems must account for as many charateristics of the higher order response as possible, such as delayed initial response. Handling qualities analysts derive bounds on acceptable values for the parameters and functions of the parameters. To insure that the specificationsare met, the control system designer matches actual vehicle responses from simulation or flight test with appropriate equivalent system responses from the specification. The designer also provides some measure of the closeness of match for various responses, often derived from frequency responses [15].

Another form of criteria is the time history of response envelope. This method specifies upper and lower bounds on the time responses of normalized state or output variables. The envelope may or may not come from an appropriate equivalent system but should nonetheless result in the desired level of handling quality when evaluated by pilots.

The above forms of criteria are based on the assumption of conventional aircraft responses to stick and rudder deflections. Unconventional aircraft (such as VTOL) or aircraft with unconventional control devices (such as direct-lift or other mode decoupling mechanisms) must be treated separately for handling quality purposes. An alternative criterion based on frequency response will be described later in this section.

3.4.1 Conventional Vehicle Responses

Most of the specifications to satisfy aircraft (longitudinal) maneuvering requirements are based on short period dynamics, phugoid damping, static stability and flight path stability. "Equivalent" lower order parameters for pitch responses are most simply of the form shown in the transfer function

$$\frac{\dot{\theta}}{F_s} \cong \frac{K\,(s + 1/T\,)\,e^{-s\tau}}{s + 2\zeta_e\,\omega_e s + \omega_e^{\,2}} \qquad (3.3)$$

where $[\zeta_e, \omega_e]$ are equivalent second order damping and natural frequency, T_θ relates pitch attitude to flight path response and τ_e is an equivalent time delay. This form of transfer function could respresent short term (short period) or long term (phugoid) response using different equivalent parameters. If the pitch response is to be related to a second order system, the short period mode natural frequency must be separated from the phugoid natrual frequency by a factor of 10 or more, as well as from higher order structural and flight control effects. When this is not the case other forms of criteria are necessary [16].

Table 2, taken from [16], shows bounds on damping ratio and time delay for pitch response.

TABLE 2

(2A) Short Period Damping Ratio Limits

Level	Category A and C Flight Phases Min	Category A and C Flight Phases Max	Category B Flight Phases Min	Category B Flight Phases Max
1	0.35	1.30	0.30	2.00
2	0.25	2.00	0.20	2.00
3	Double Time $\geq$ 6 sec		Double Time $\geq$ 6 sec	

(2B) Phugoid Damping Ratio Limits

Level 1	$\zeta_p \geq 0.04$
Level 2	$\zeta_p \geq 0$
Level 3	Double Time $\geq$ 55 sec

TABLE 2 (cont.)

(2C) Limits on Aircraft Response Delay

Level	Alowable Delay (sec)
1	0.10
2	0.20
3	0.25

A principal criterion for pitch axis (maneuvering) response is based on the desire to control both pitch attitude and flight path angle (or normal acceleration) with a single control surface deflection. Standard pitch control is effected by elevator deflection which initially changes pitch attitude and subsequently reorients the flight path. Presuming operation on the front side of the power curve or L/D curve, backward stick will orient attitude and flight path upward and forward stick will do the opposite. The resulting motion is expressed straightforwardly as a "control anticipation parameter" (CAP). When $\dot{q} = \ddot{\theta}$,

$$CAP = \frac{\ddot{\theta}\,|_{t=0^+}}{\Delta\,|_{t_{ss}}} \tag{3.4}$$

where the denominator is the steady state load factor change with a step elevator input. Δn is nearly equivalent to expressing the flight path change. Using a simple 2 degree of freedom transfer function for $\theta(s)/\delta_e$ and $\alpha(s)/\delta_e$:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{M_{\delta_e}(s + L_\alpha/V_0)}{s\left(s^2 + \{L_\alpha/V_0 - M_{\dot{\theta}} - M_{\dot{\alpha}}\} + \{\frac{-L_\alpha M_{\dot{\alpha}}}{V_0} - M_\alpha\}\right)} \tag{3.5}$$

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{M_{\delta_e}}{s^2 + (L_\alpha/V_0 - M_{\dot{\theta}} - M_{\dot{\alpha}})s + (\frac{-L_\alpha M_{\dot{\alpha}}}{V_0} - M_\alpha)} \tag{3.6}$$

For a step input $\delta_e(s) = \frac{1}{s}$,

$$\ddot{\theta}\Big|_{t=0^+} = s\left[\frac{s^2\theta(s)}{\delta_e(s)}\right]\frac{1}{s}\Bigg|_{s\to\infty} = \frac{s\left(s+\frac{1}{T_\theta}\right)M_{\delta_e}}{s^2+2\zeta_{sp}\omega_{sp}s+\omega_{sp}^2}\Bigg|_{s\to\infty} \tag{3.7}$$

$$= M_{\delta_e}$$

$$\Delta n\Big|_{t_{ss}} \cong s\cdot\frac{n}{\alpha}\cdot\frac{\alpha(s)}{\delta_e(s)}\cdot\delta_e(s)\Bigg|_{s=0}$$

$$= \frac{n}{\alpha}\cdot\frac{M_{\delta_e}}{\omega_{sp}^2}$$

$$\therefore\quad CAP = \frac{M_{\delta_e}}{n/\alpha\cdot M_{\delta_e}/\omega_{sp}^2} = \frac{\omega_{sp}^2}{n/\alpha} \tag{3.8}$$

The resulting term, $\omega_{nsp}/(n/\alpha)$, is an approximation to the CAP which is conveniently calculated and plotted to show acceptable bounds. A representative curve is shown in Figure 3-2 for Category C flight phases.

Another term of use is $\omega_{nsp}T_\theta$, representing the phase difference in pitch attitude and path angle [16]. Relative magnitudes of ω_{nsp} relate to the responsiveness of the aircraft in pitch, and T_θ the abruptness of path change according to $\frac{\gamma}{\theta} = \frac{1}{T_\theta s + 1}$. Together, $\omega_{nsp}T_\theta$ should be large enough that flight path change follows pitch attitude with reasonable time for pilot adjustment and trim. When plotted against short period damping ratio, boundaries as shown in Figure 3-3 result.

Flight test data to justify these criteria are presented in [16]. Data were selected based on documentation of vehicle dynamics, control system (actuators, feel, etc.), flight conditions and maneuvers and, especially, pilot comments and opinion rating scale used. An example of data points which validate boundaries in Figure 3-3 with some modification, is shown in Figure 3-4. The discussion in [16] highlights the suitability of low order equivalent system parameters to matching the pitch response of augmented as

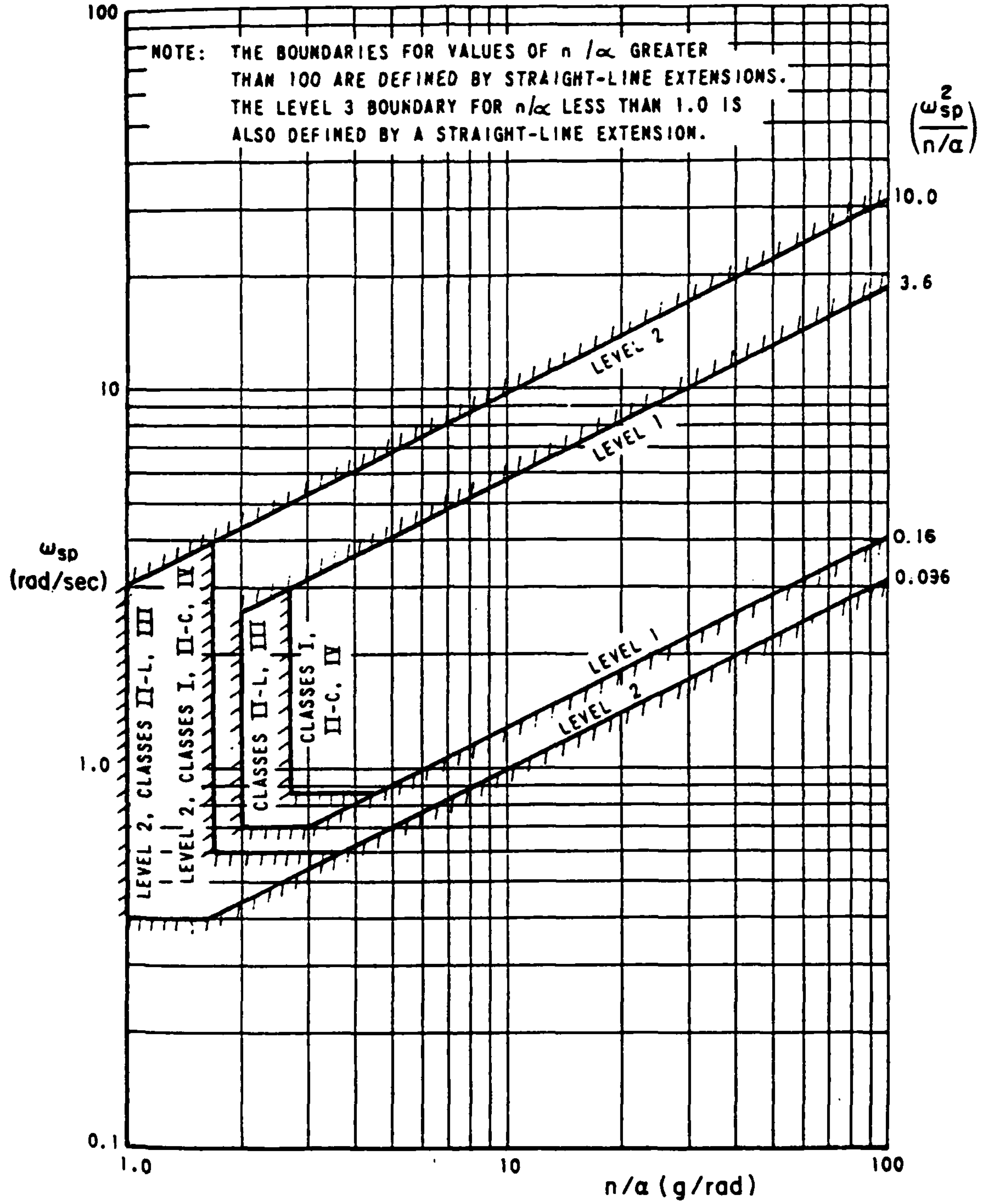


Figure 3-2

Bounds on the Control Anticipation Parameter (CAP)

Category C Flight Phases

Reference [16]

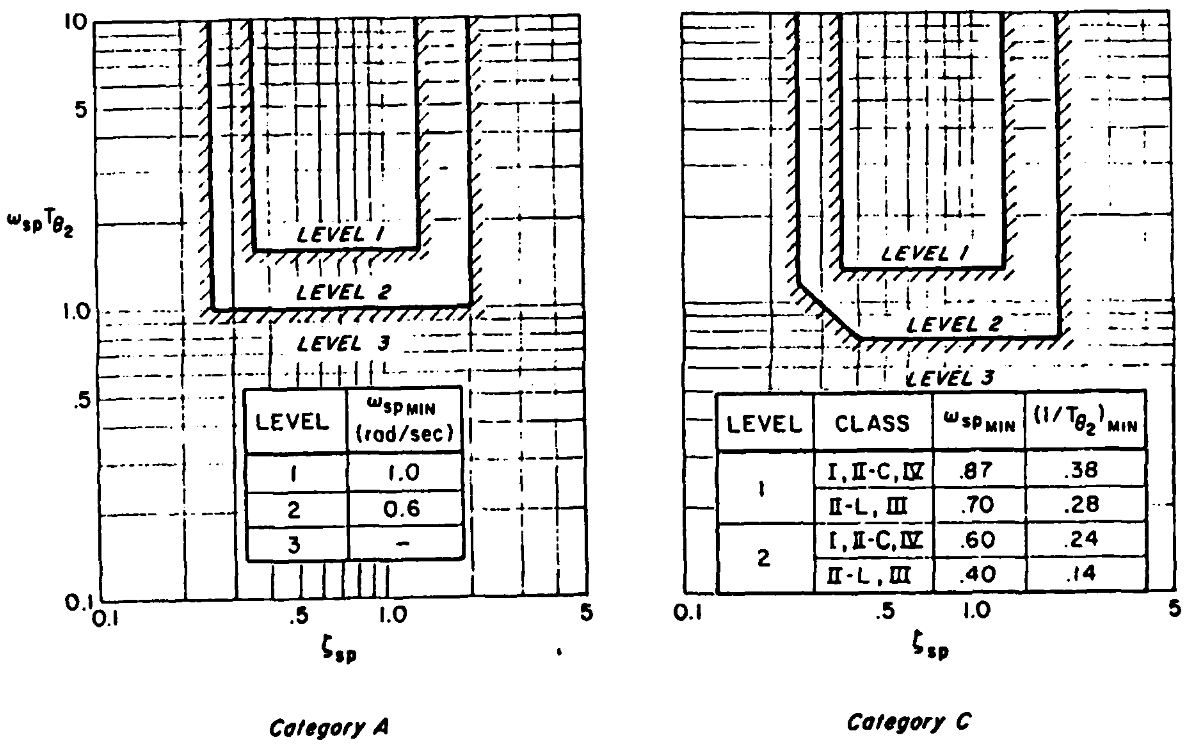
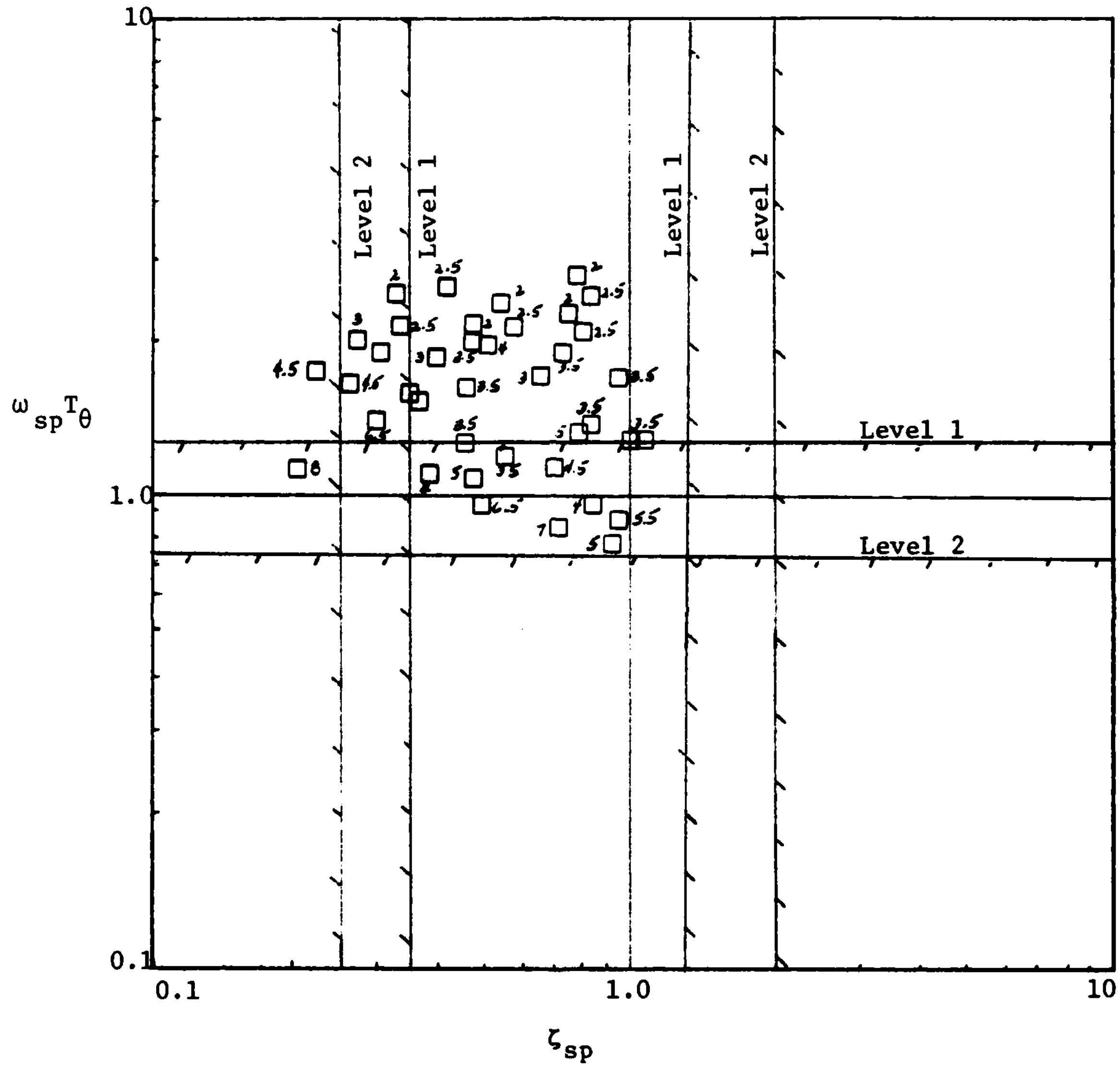


Figure 3-3

Path Response vs Short Period Damping

Reference [16]



(Numbers in the figure are Cooper-Harper ratings)

Figure 3-4

Verification of Figure 3-3 Bounds

Reference [16]

well as unaugmented aircraft. An example given of an augmented airplane whose θ/δ_e response included two well damped second order modes, but was sluggish, yielded equivalent system parameters that explained the Level 3 behavior.

3.4.2 Unconventional Vehicle Responses

Not all responses can be adequately matched without changing the bounds of the criteria, using negative equivalent time delays, or other undesirable means. "When mismatch between lower-order equivalent and higher order systems is large, or when pitch axis augmentation results in unconventional responses" [16], as with direct lift control, a possible alternative is in the form of a frequency response bandwidth criterion. This is essentially a "closed loop describing function of pilot/vehicle (pitch) response." In this case, bandwidth is the lower of the frequencies at which phase margin is 45° or gain margin is 6 dB. Both the value of bandwidth and the shape of the phase versus frequency curve above ω_{bw} determines the stability and quality of the pilot-in-the-loop control. When rapid roll-off in phase occurs past the bandwidth frequency, it can be attributed to time delay in response, which degrades behavior as usual. The delay is estimated linearly by taking the change in phase in the unstable region divided by the frequency past -180° ($\tau_e = -\frac{\Delta\phi}{\omega} = \frac{-\phi_1 + 180}{\omega_1}$). The bandwidth criterion is suited to specifying VTOL vehicle handling qualities during transition because pitch response to elevator command is non-standard until flight velocities approaching cruise. Figure 3-5 shows the definition of bandwidth and flight test results supporting its use.

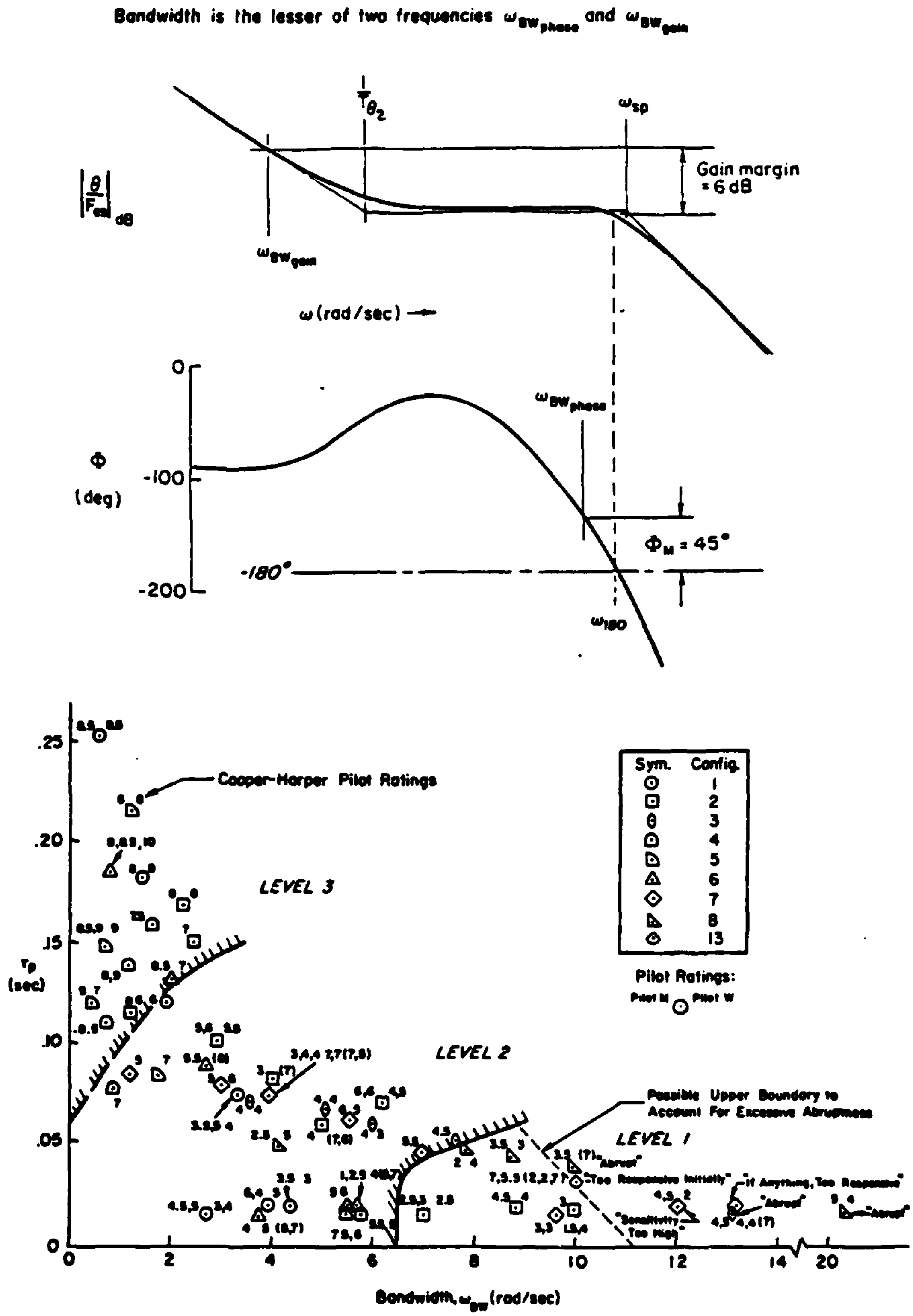


Figure 3-5

Bandwidth Criterion

Reference [16]

CHAPTER 4

SPECIFICATION OF HANDLING QUALITIES FOR VARIABLE FLIGHT DYNAMICS

Having introduced the non-autonomous class of problems in flight vehicle dynamics and a method of obtaining approximate analytical solutions to dynamic response and control compensation design, the treatment of these dynamics relevant to handling qualities specification can be outlined. The vehicles under study will continue to be the LRV and VTOL aircraft, although conventional aircraft performing certain prescribed maneuvers that result in wide variations in flight conditions or stability are potential candidates for separate specification. The piloted tasks in both types of vehicle will be described to clarify the dynamics involved. Then the equations of motion for the vehicles are formed and the time responses or system functions are solved asymptotically for states or outputs relevant to handling qualities. The rationale behind the chosen criteria are described, and extensions to the criteria are suggested.

4.1 Analysis Technique

The first step in examining handling qualities of a flight vehicle is solving the equations of motion. The approximate solutions obtained through multiple scaling consist of elementary transcendental functions which offer ready comparison with conventionally derived solutions. Equations and solutions may be of any order, although the interest presently is in basic, unaugmented vehicle responses. Augmentation which does not increase the natural order of response, as in state feedback, is also examined. When appropriate, second order approximations will be used. "Equivalent" representations are therefore not specifically needed. Homogeneous time responses are derived first; forced (single input) responses are a normal extension by methods such as variation of parameters.

A limitation of straightforward multiple time scaling arises in the asymptotic solution of the equations of motion when characteristic roots coalesce. This happens during VTOL transition, for example. Figure 2 showed that two roots meet on the real axis before branching to become an oscillatory pair. A multiple root condition may also occur during LRV re-entry while at high altitude. For these conditions, called turning points, the asymptotic approximation with simple functions fails and requires the use of more complex functions to describe the behavior [9]. However, traversing quickly through the multiple root condition results primarily in phase angle error; for most purposes of handling quality analysis the accuracy of elementary functions is probably sufficient.

4.2 LRV Handling Qualities

4.2.1 LRV Pilot Tasks

Entry from orbital flight begins at about 400,000 feet altitude and 25,000 feet per second velocity, when air density gives rise to significant aerodynamic effects. A pre-selected angle of attack is established and held while heat and aerodynamic forces build. The angle of attack (AOA) is high, around 40 or 50 degrees, and a transition maneuver to a cruising glide at lower AOA is required. One method of achieving the AOA transition is to program elevator (or other longitudinal control surface) deflection by optimal control to (1) minimize a load factor penalty and time spent in undesirable AOA regions, and (2) achieve terminal conditions of altitude, trim flight path angle, and forward velocity [17]. This "jump" maneuver follows a specified AOA trajectory (depicted in Figure 4-1, for example) and duration of the maneuver directly affects the amount of stabilization required to control disturbances and initial condition errors.

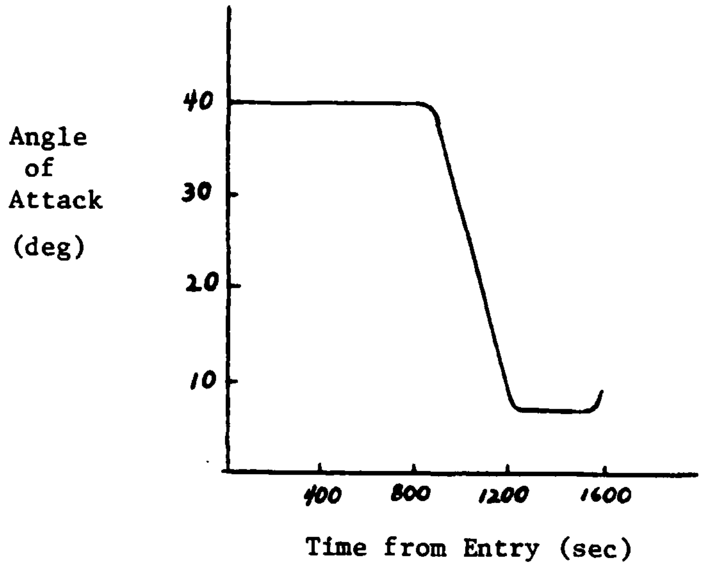


Figure 4-1

Re-entry Angle of Attack Trajectory

Open loop control improves as jump duration decreases, limited by actuator rates and control effectiveness [17]. A NASA simulation study of an orbiter AOA transition maneuver concludes that a pilot can perform the maneuver [18].

Other AOA trajectories result when following entry profiles designed to minimize different costs or achieve certain test objectives. The STS shuttle follows an AOA trajectory which begins at 40 degrees and transitions over a period of 6 minutes to 10 degrees. In any case, angle of attack is a state which is closely controlled throughout re-entry. Angle of attack response to pilot input will certainly influence the analysis of LRV handling qualities.

Once a cruising glide is established, the remaining control tasks are deceleration maneuvers (e.g. S-turns) and terminal area maneuvers including approach and landing. Approach maneuvers may be as simple as a flare and coast on the backside of L/D maximum until landing speed is reached; or they may involve lateral-directional motion such as circling or more conventional downwind, base and final legs.

A major difference from conventional aircraft flight is that the LRV is unpowered and therefore cannot establish an equilibrium flight condition. It can trim aerodynamic moments to zero but cannot maintain constant altitude at constant airspeed. Nevertheless, handling qualities specifications have been formulated as though the equations of motion could be expressed in constant coefficient terms [18]. These handling quality specifications may or may not suffice for the terminal flight phases. But considering the much larger flight regime of a LRV, it is worthwhile to pursue an analytical treatment of the variable dynamics, and to specify handling qualities in terms of variable system responses.

4.2.2 Analysis of Handling Quality

A number of numerical and analytical studies of lifting re-entry dynamics lend preliminary insight into the character of the vehicle responses. Reference [18] states that when dynamic pressure changes become

significant during the period of vehicle motion, "both the character of the vehicle responses and the degree of damping that exists in the various vehicle response variables are affected. The effects appear to be largest for lightly damped responses." A measure given for determining significant dynamic pressure changes is: $\frac{\Delta q}{q_o} = \left(-\beta V_o \sin\gamma_o + \frac{2}{V_o}\frac{dV}{dt}\right)P$ (4.1)
where $\beta = 4.75 \times 10^{-3}/ft$ and P is the time of one cycle of an oscillation. Laitone and Chou [19] provide a survey on studies of phugoid oscillations at hypersonic speeds. In this region aerodynamic coefficients can be linearized independent of mach number, and centrifugal effects increase to the order of gravitational attraction. Important effects arise from altitude changes with phugoid motion: air density variation with altitude decreases phugoid period while gravity gradient has the opposite, but weaker, effect. As orbital speeds are reached, phugoid period approaches orbital period. Etkin [1 9] showed numerically that the period of angle of attack ("short period") oscillations can be greater than the phugoid period at high flight altitude where aerodynamic moments are small. A number of people have studied angle of attack variations during re-entry, usually for trajectories where simplifying assumptions give the equation a specialized form. This approach shows the character of the vehicle response in some practical cases but falls short of being useful to handling qualities work.

Extensive flight tests of lifting bodies in the 1960's examined terminal flight phases of Class IV re-entry type vehicles. A number of results (including trim changes, control surface effectiveness, high effective dihedral, low directional stability and lift loss with pitch control) gave insights relevant to handling qualities for this type of vehicle. Configuration changes on the vehicles remedied certain problems with stability and trim control authority. All of the lifting bodies (M2-F2, HL-10, and X-24A) were equipped with stability augmentation. Pilots felt that inherent stability should be designed into the LRV with minimum reliance on command and stability augmentation [20]. Important longitudinal criteria adopted for the terminal flight phase were bounds on n/α and CAP, given in constant coefficient terms. Figure 4-2, taken from [18], shows flight test results for these criteria over a range of approach and landing conditions.

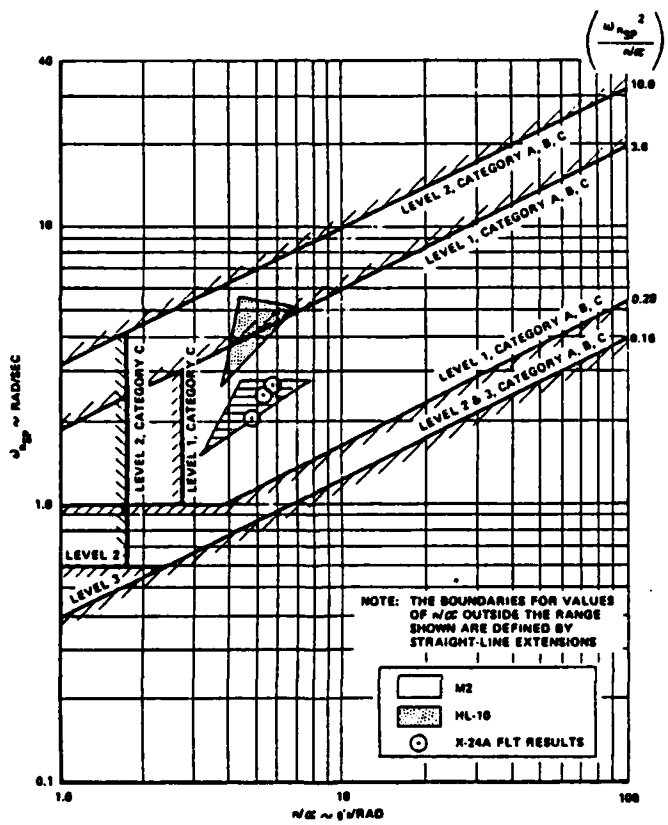


Figure 4-2

Lifting Body Handling Qualities in Terminal Flight

Reference [18]

Actual re-entry flights with the X-15 clarified entry control problems. Entries began from a design altitude of 250,000 feet at 12-20° AOA. The X-15 used an adaptive control system of blended reactive and aerodynamic controls. The controls held angle of attack constant until normal acceleration built to about 4 g's. X-15 entries were from sub-orbital flight and so had a smaller speed and altitude range than orbital re-entry, but the shorter entry duration caused more rapid build-up of pitch rate and load factor. These latter effects created a more severe control problem and higher pilot workload [21]. A comparison of X-15 design entry parameters with those of an orbital LRV is shown in Figure 4-3.

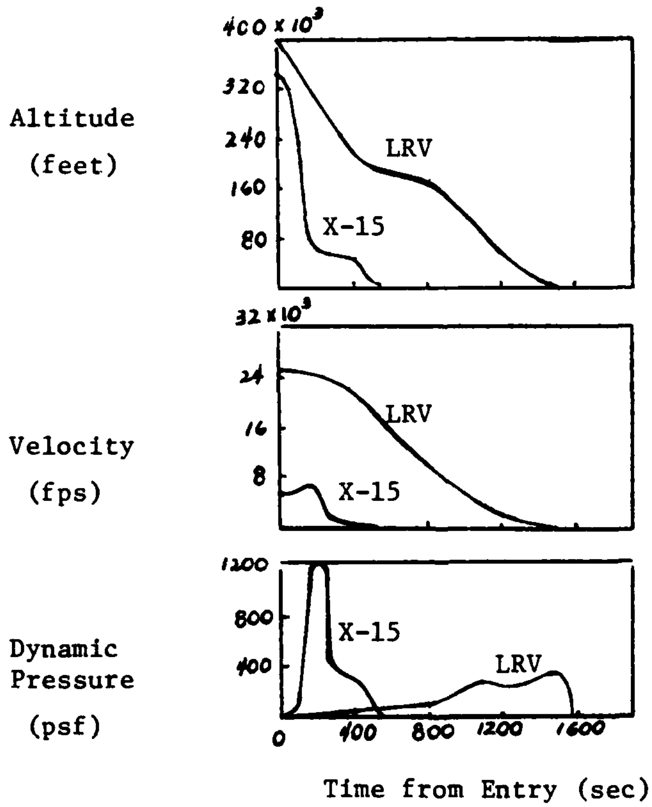


Figure 4-3

X-15 and LRV Entry Parameters

Approach and landing for the X-15 began from a descending glide with a flare at about 1000 feet altitude and 100-180 feet/second vertical velocity. The vehicle was kept on the front (high energy) side of L/D_{max} to increase post-flare time to touchdown. A wide range of landing conditions resulted, depending on vehicle altitude, velocity and flight path angle at flare initiation. While no pilot ratings were recorded, the handling qualities were generally considered good.

Most recently, space shuttle entries from orbital flight have provided measurements of dynamic response of a Class III vehicle over the full flight regime, but few ratings of handling quality. The shuttle dynamics are highly augmented to steer the programmed trajectory, and response to pilot inputs is unconventional. Most manual piloting is during the terminal flight phase, when control by the guidance system ends. Choosing criteria for shuttle handling quality requirements presented a problem since no data base existed. Aerodynamic derivatives determined from wind tunnel experiments, later adjusted by flight data, provided parameters for piloted simulations. These simulations produced measures of expected handling qualities, principally for approach and landing. A pitch rate envelope was chosen as the primary longitudinal criterion, shown in Figure 4-4.

Rynaski [22] supports the use of an angle of attack time history envelope as a longitudinal handling quality criterion, rather than a pitch rate envelope. By his reasoning, angle of attack response directly translates to short term maneuvering ability and flight path control ($\dot{\gamma} = Z_\alpha \alpha$), so criteria based on angle of attack response should give the truest measure of handling quality. He presents data to support his view in [22]. At the initiation of entry, when aerodynamic forces are small and control surfaces ineffective, control is effected primarily by reaction jets. It may be more appropriate at this stage to consider pitch rate and normal acceleration at the pilot station for acceptable handling characteristics. Once aerodynamic controls become primary, however, the angle of attack response should be considered.

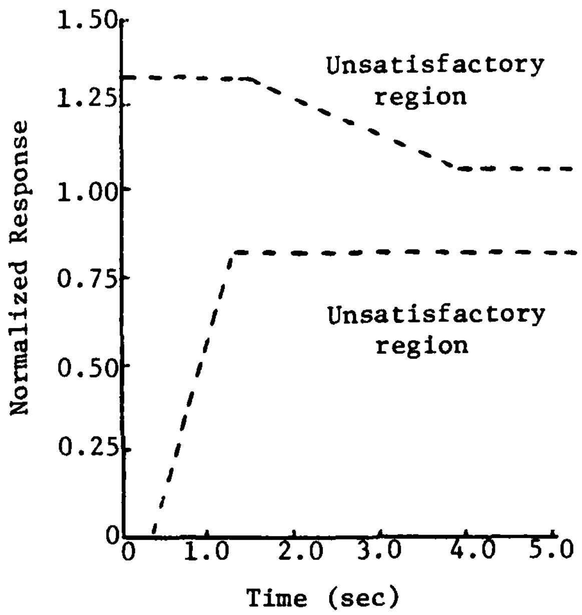


Figure 4-4

Shuttle Pitch Rate Response Envelope

Considering previous flight test results and the development of re-usable manned space flight vehicles, it is useful to examine LRV dynamics in portions of the flight regime subject to pilot control. The vehicle studied here is similar in size and weight to the HL-10 Class IV lifting body. The assumption is that the LRV responds conventionally to control inputs once aerodynamic controls become primary. Our handling quality analysis starts at a time when lower limits on n/α and acceptable CAP values are reached. When CAP is above the upper limit, n/α may be too low or ω_{nsp} too high. If n/α is too low the vehicle cannot generate sufficient lift to re-orient the flight path in response to pilot commands. Therefore, γ/θ has a very long time constant. If ω_{nsp} is too large, given an acceptable n/α , the vehicle may be over-sensitive in its pitch attitude response which will primarily affect pilot workload in his trajectory-following task.

The nature of re-entry allows a fairly predictable qualitative analysis of vehicle dynamics. The variations in flight conditions, although not always linear or even monotonic, indicate that responses will "stiffen" and aerodynamic damping increase as aerodynamic lift and moment generating capabilities increase with penetration into the atmosphere. Basic vehicle instabilities in certain flight regimes (transonic flight or certain AOA regions, for example) should be known beforehand for the trajectory planning stage and flight control design. Augmentation which enhances stability or achieves certain input-output characteristics can be designed over an entire segment, or phase, of the trajectory with analytical expressions for the vehicle dynamic responses in hand. At the very least, asymptotic analysis of the time-varying dynamics will clarify control requirements and identify unusual phenomena early in the design stage.

A plot of CAP at two minute intervals throughout the entry trajectory (using constant coefficient analysis) shows the effective change in acceptability of the handling qualities by aerodynamic control (Figure 4-5). At an altitude of 250,000 feet the flight conditions are

$$\rho = .0000064 \text{ sl/ft}^3 \qquad \gamma = -1.5 \text{ deg}$$

$$V = 24{,}250 \text{ ft/sec} \qquad \alpha = 40 \text{ deg}$$

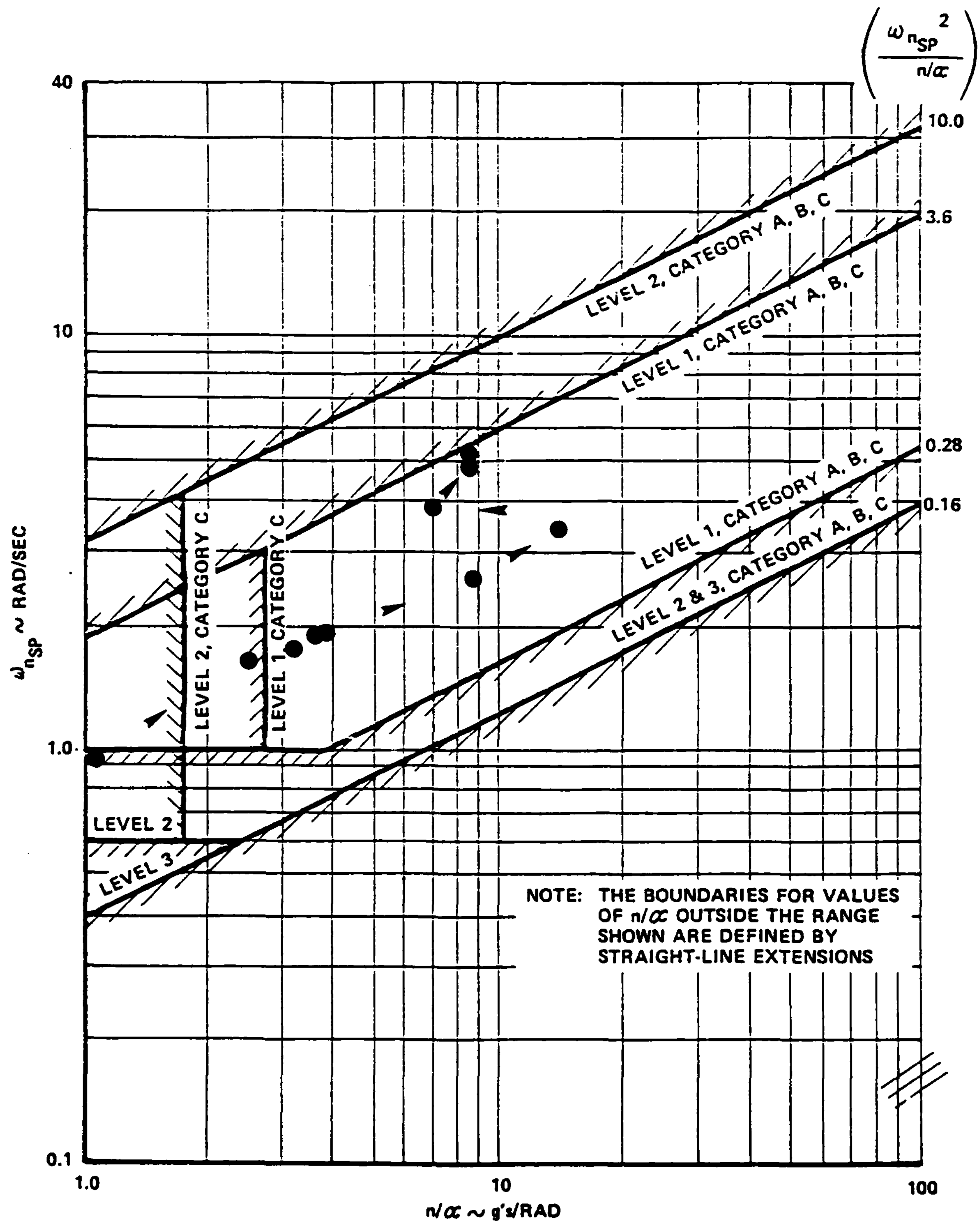


Figure 4-5

Variation of the CAP Through Entry

At these flight conditions

$$n/\alpha = 1.04$$

$$CAP = 0.95$$

which is Level 2 for this Category A Flight Phase. The variation of the CAP shows adequate pitch response from 250,000 feet on down, at least for the points chosen. Pitch response (augmented or unaugmented) was assumed to stable for the entire re-entry. Orbital re-entry will not have the rapid variations in flight conditions which sub-orbital entry has (as the comparison in Figure 4-3 showed), yet it is useful to consider the manner in which the CAP varies and its effect on pilot workload. Even if handling quality parameters remain within Level 1 boundaries, fast variation of the dynamics could mislead the pilot and contribute to pilot induced oscillations, or at least an increase in pilot workload. Changes in direction of the variation may also affect the difficulty of the piloting tasks. The reversal of n/α near Terminal Area Interface, seen in Figure 4-5, takes the CAP closer to the Level 2 boundary. Degrading changes imply an increase in workload to adapt to the dynamics and anticipate the required control inputs. A possible limit on the extent of variation of a handling quality parameter (like the CAP) for a particular flight phase might have the form

$$\frac{\int_{t_o}^{t_f} \omega_o(t)\, dt}{\int_{t_o}^{t_f} \frac{n}{\alpha}(t)\, dt} < c \cdot \frac{\omega_o(t_c)}{\frac{n}{\alpha}(t_c)}$$

where $\omega_n{}^2$ is replaced by $\omega_o(t)$. This would restrict the amount of dynamic variation within a segment of the flight regime.

The angle of attack equation for a general lifting re-entry trajectory permits further evaluation of handling quality. Equations of motion for lifting re-entry without thrust, using a state vector of $\underline{x} = [V \ \gamma \ q \ \alpha]^T$, are:

$$\dot{V} = \frac{-\rho S V^2 C_D}{2m} - g \sin\gamma \qquad (4.2)$$

$$\dot{\gamma} = \frac{\rho S V C_L}{2m} + \left(\frac{V}{r} - \frac{g}{V}\right)\cos\gamma \qquad (4.3)$$

$$\dot{q} = \frac{\rho S c V^2 C_m}{2 I_y} - \frac{3g}{2r}\left(\frac{I_x - I_z}{I_y}\right)\sin 2\theta \qquad (4.4)$$

$$\dot{\alpha} = q + \left(\frac{g}{V}\right)\cos\gamma - \frac{\rho S V C_L}{2m} \qquad (4.5)$$

where,

$$\theta = \gamma + \alpha \qquad (4.6)$$

$$\dot{\theta} = q + \left(\frac{V}{r}\right)\cos\gamma \qquad (4.7)$$

$$\dot{r} = V \sin\gamma \qquad (4.8)$$

This model assumes no lateral or directional motion and accounts for the centrifugal effects of motion around the earth. Aerodynamic terms are linearized about the trajectory and have the forms

$$C_L = C_{L_\alpha} \cdot \alpha \qquad (4.9)$$

$$C_D = C_{D_0} + C_{D_\alpha} \alpha \qquad (4.10)$$

$$C_m = C_{m_\alpha} \alpha + (C_{m_q} + C_{m_{\dot\alpha}})\dot{\alpha} + C_{m_q} \dot{\gamma} \qquad (4.11)$$

where C_{L_α}, C_{D_α}, C_{m_α}, $C_{m_{\dot\alpha}}$, C_{m_q} need not be constant.

The linear, time-varying equation for AOA is:

$$\ddot{\alpha} + \left[\frac{\rho S V}{2m} C_{L_\alpha} - \frac{\rho S c V^2}{2 I_y}\left(C_{m_{\dot\alpha}} + C_{m_q}\right)\right]\dot{\alpha} + \left[\frac{\dot{\rho} S V}{2m} C_{L_\alpha} + \frac{\rho S V}{2m}\dot{C}_{L_\alpha} - \right.$$

$$- \left(\frac{\rho S V}{2m}\right)^2 \left(C_{D_0} C_{L_\alpha}\right) - \frac{\rho S c V^2 C_{m_\alpha}}{2 I_y} - \left(\frac{\rho S V}{2}\right)^2 \frac{V}{m I_y} C_{L_\alpha} C_{m_q}$$

$$\left. - \frac{\rho S C_{D_\alpha}}{2m} g \cos\gamma + \frac{3g}{r}\left(\frac{I_x - I_z}{I_y}\right)\cos 2(\gamma + \alpha)\right]\alpha =$$

$$\frac{\rho S c V^2}{2 I_y} C_{m_q} \left(\frac{g}{V} - \frac{V}{r} \right) \cos \Gamma + \rho \frac{S C_{D_\alpha}}{2m} g \cos \Gamma -$$

$$- \frac{3g}{2r} \left(\frac{I_x - I_z}{I_y} \right) \sin 2(\Gamma + \alpha) + \left(\frac{2g}{V^2} - \frac{3}{r} \right) g \sin \Gamma \cos \Gamma \tag{4.12}$$

Taking the homogeneous part of the equation,

$$\ddot{\alpha} + \omega_1(t) \dot{\alpha} + \omega_0(t) \alpha = 0 \tag{4.13}$$

Through multiple scaling, an asymptotic solution is formed as a function of $\omega_1(t)$ and $\omega_0(t)$. Then the solution is general and specific characteristics of a trajectory can be substituted directly into the solution. Ramnath [11] develops asymptotic solutions to $\alpha(t)$ in the form

$$\alpha(\tau_0, \tau_1) = A(\tau_0) B(\tau_1) = \exp\left\{ \int \frac{\varepsilon \dot{\kappa}}{\sqrt{\omega_1^2 - 4\omega_0}} dt \right\} \exp\left\{ \int \left(-\frac{\omega_1}{2} \pm \sqrt{\frac{\omega_1^2}{4} - \omega_0} \right) dt \right\} \tag{4.14}$$

A few observations about the expression for $\alpha(t,\varepsilon)$ are helpful at this point. First examine the fast part of the solution, $B(\tau_1)$. This contains all the information (asymptotically) of the frequency variations and some of the amplitude variation. If we chose to look at a particular point in time an integrate the expression, corresponding to a constant coefficient analysis, $\omega_1(t_1)$ represents $2\zeta\omega_n$ and $\omega_0(t_1)$ represents ω_n^2. So $\sqrt{\frac{\omega_1^2}{4} - \omega_0}$ becomes ω_d, the damped natural frequency, and

$$\begin{aligned} \exp\left[\int \left(-\frac{\omega_1}{2} \pm \sqrt{\frac{\omega_1^2}{4} - \omega_0} \right) dt \right] &= \exp[(-\zeta\omega_n \pm j\omega_d)t] \\ &= \exp(-\zeta\omega_n t) \exp(\pm j\omega_n \sqrt{1-\zeta^2}\, t) \end{aligned}$$

This expression demonstrates the generalized nature of the multiple scales technique and indicates that these asymptotic solutions subsume the expressions derived from constant coefficient analysis.

Next, the slow part of the solution, $A(\tau_0)$, modifies the fast variation primarily in amplitude. In constant coefficient terms only $e^{-\zeta\omega_n t}$

governs the decay (or rise) in amplitude, so $A(\tau_0)$ is a "new" variable term due to separating the fast and slow system dynamics. From (4.14)

$$\dot{k} = \frac{\partial}{\partial \tau_0}\left(-\frac{\omega_1}{2} \pm \sqrt{\frac{\omega_1^2}{4} - \omega_0}\right)$$

but Ramnath shows in [11] that $A(\tau_0)$ is closely approximated by

$$C(\omega_1^2 - 4\omega_0)^{-1/4}$$

where C is a constant. The effect of this term is to scale the amplitude of response on the slowly varying scale. In a sense the parameters of the fast solution relate straightforwardly to the modal parameters of constant coefficient analysis. Yet the prescribed variation of the coefficients means that dynamic response and pilot compensation are continuously changing. The slow variation contributes to this change and may well be a significant contribution.

Substituting values for the nominal trajectory into $\omega_1(t)$ and $\omega_0(t)$ and computing response to a command or disturbance input using (4.14) shows the angle of attack time response. An input at any point along the trajectory will result in a (asymptotically) true angle of attack response for as long as the state is observed. Since the coefficients of the equation vary "slowly" and monotonically (or at least do not oscillate), a "final" or threshold value of AOA will be reached. Practically, a family of curves at successive initial times should be calculated to represent responses along the trajectory. Figures 4-6 and 4-7 are two such curves beginning at 250,000 feet and 75,000 feet respectively. Time is referenced to t=0 at entry interface (400,000 ft). At 250,000 feet,

$$\alpha(240) = 40^{\circ}, \quad \dot{\alpha}(240) = 0$$

A step response is appropriate for this case since angle of attack is being held at 40^{o} until $t = 840$ seconds. A lower bound on the AOA response for the given value of n/α is also plotted.

At 75,000 feet,

$$\alpha(1140) = 13^{o}, \dot{\alpha}(1140) = -0.1\ ^{o}/sec$$

This response begins while AOA is ramping down; Figure 4-7 shows the response to a disturbance which moves AOA above its reference value.

LRV Angle of Attack Response at High Altitude

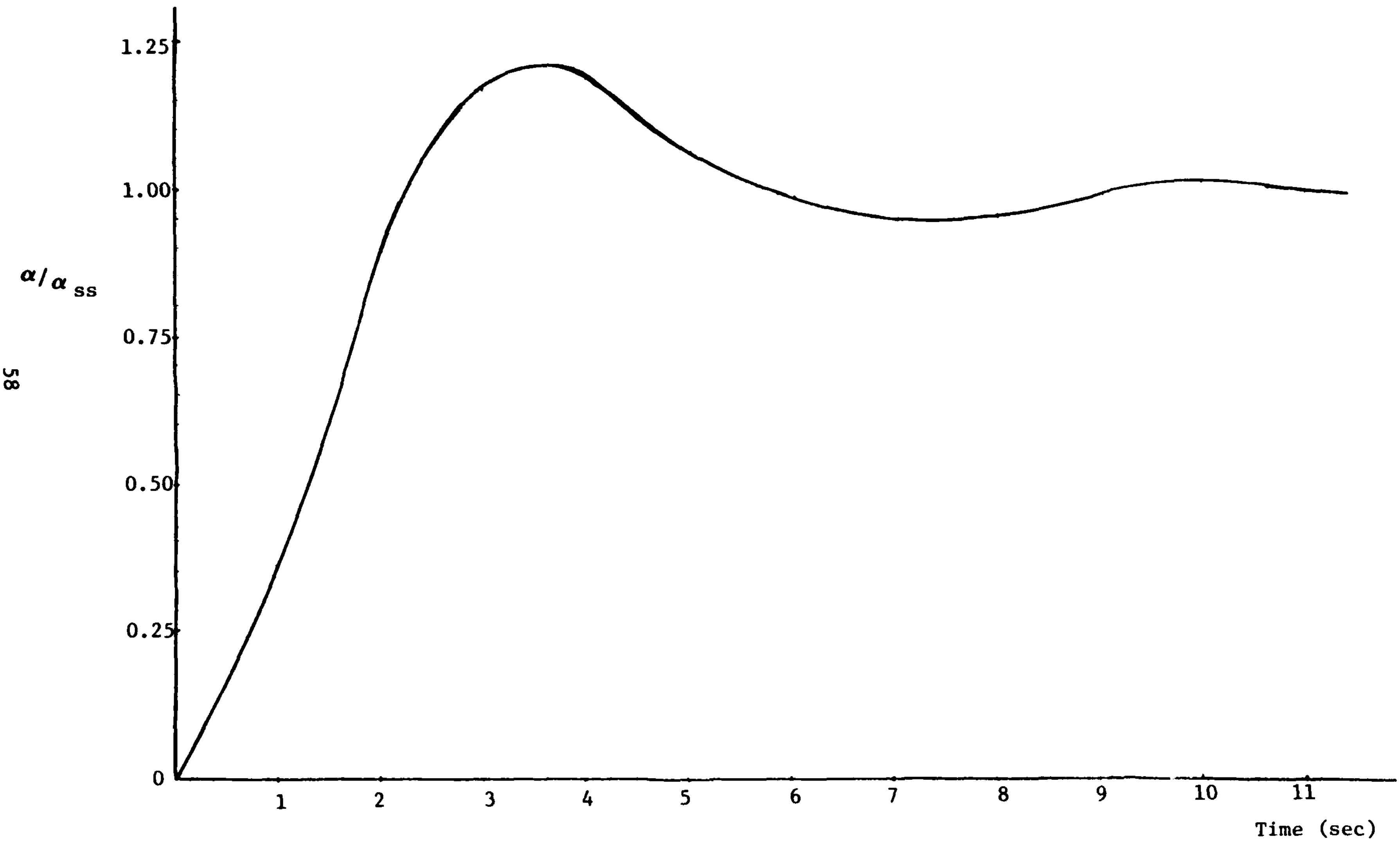


Figure 4-6

LRV Angle of Attack Response at Terminal Area Interface

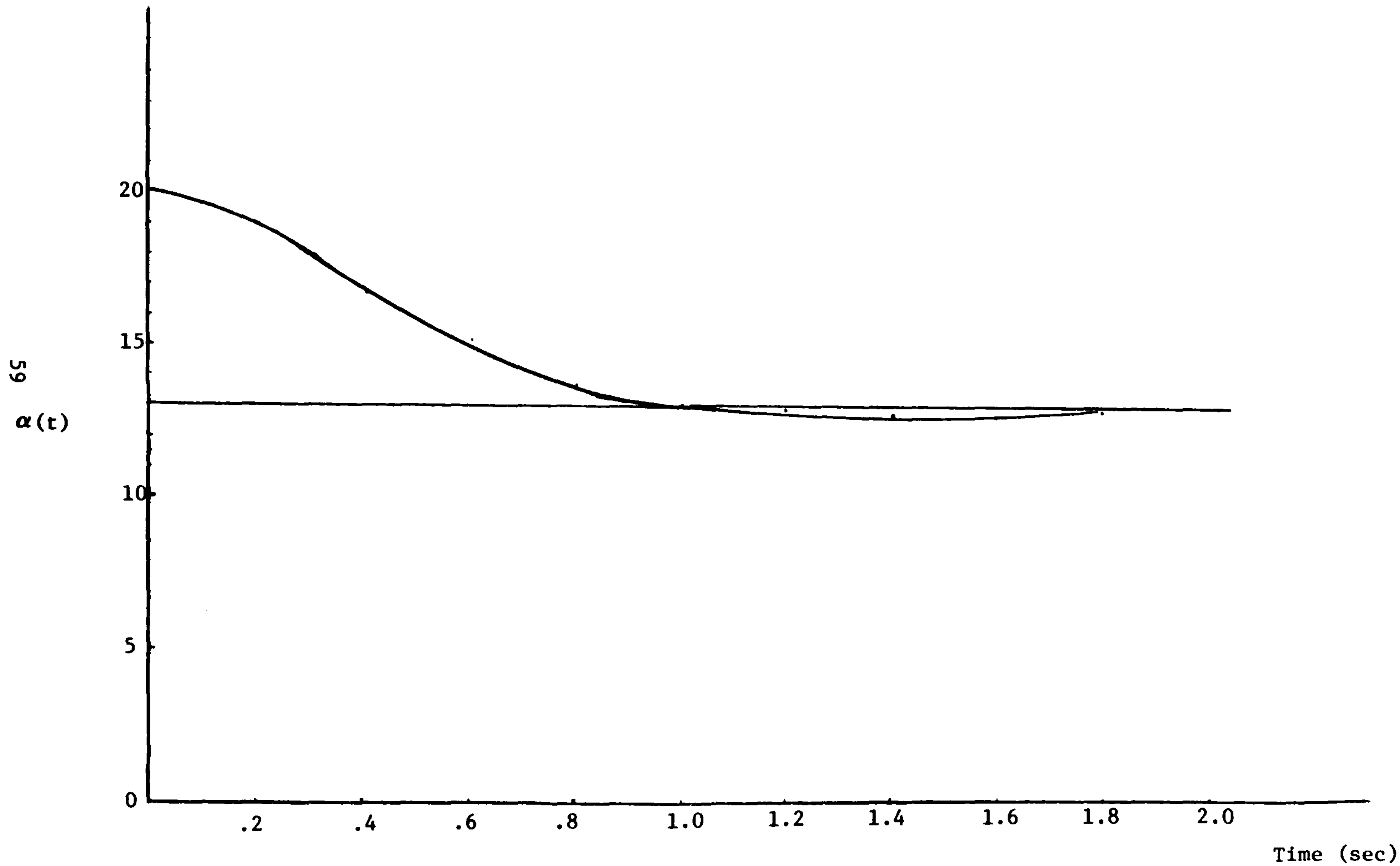


Figure 4-7

Further second order analysis of the LRV handling qualities is possible after linearizing the equations of motion. The form of the state equations is

$$\begin{bmatrix} \Delta\dot{V} \\ \Delta\dot{\gamma} \\ \Delta\dot{q} \\ \Delta\dot{\alpha} \end{bmatrix} = \begin{bmatrix} -D_V & -g\cos\gamma & 0 & D_\alpha \\ \frac{L_V}{V} + \left(\frac{1}{r} + \frac{g}{V^2}\right)\cos\gamma & \left(\frac{g}{V} - \frac{V}{r}\right)\sin\gamma & 0 & \frac{L_\alpha}{V} \\ M_V & \begin{matrix} M_{\dot\alpha}\left(\frac{g}{V}\right)\sin\gamma \\ -\frac{3g}{r}\left(\frac{I_x - I_z}{I_y}\right)\cos 2(\gamma+\alpha) \end{matrix} & M_q + M_{\dot\alpha} & \begin{matrix} M_\alpha \\ -\frac{3g}{r}\left(\frac{I_x - I_z}{I_y}\right)\cos(\cdot) \end{matrix} \\ -\frac{L_V}{V} - \left(\frac{g}{V^2}\right)\cos\gamma & -\frac{g}{V}\sin\gamma & 1 & -\frac{L_\alpha}{V} \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta\gamma \\ \Delta q \\ \Delta\alpha \end{bmatrix} \tag{4.15}$$

where terms in the matrix are evaluated at the reference trajectory conditions.

Following the development of the previous chapter, truncated models for the short term and long term dynamics result. A truncated short period model uses the lower right 2X2 block to give the equations

$$\begin{Bmatrix} \Delta\dot{q} \\ \Delta\dot{\alpha} \end{Bmatrix} = \begin{bmatrix} M_q + M_{\dot\alpha} & M_\alpha - \frac{3g}{r}\left(\frac{I_x - I_z}{I_y}\right)\cos 2(\gamma+\alpha) \\ 1 & -\frac{L_\alpha}{V} \end{bmatrix} \begin{Bmatrix} \Delta q \\ \Delta\alpha \end{Bmatrix} \tag{4.16}$$

Solving for $\Delta\alpha$, differentiate the second equation to give

$$\Delta\ddot{\alpha} = \Delta\dot{q} + \left(-\frac{L_\alpha}{V}\right)\Delta\dot{\alpha} + \left(-\frac{\dot{L}_\alpha}{V}\right)\Delta\alpha$$

$$\text{but,}\quad \Delta\dot{q} = (M_q + M_{\dot\alpha})\Delta q + \left(M_\alpha - \frac{3g}{r}(\cdot)\cos(\cdot)\right)\Delta\alpha$$

$$= (M_q + M_{\dot\alpha})\Delta\dot{\alpha} + \left(M_\alpha - \frac{3g}{r}(\cdot)\cos 2(\gamma+\alpha) + \frac{L_\alpha}{V}\right)\Delta\alpha$$

So,

$$\Delta\ddot{\alpha} + \left[\frac{L_\alpha}{V} - (M_q + M_{\dot\alpha})\right]\Delta\dot{\alpha} + \left[\frac{\dot{L}_\alpha}{V} - M_\alpha - (M_q + M_{\dot\alpha})\frac{L_\alpha}{V} + \frac{3g}{r}(\cdot)\cos 2(\gamma+\alpha)\right]\Delta\alpha = 0 \tag{4.17}$$

Comparing this equation with the complete angle of attack equation, the $\omega_1(t)$ expressions are identical, and $\omega_0(t)$'s are not. This leads to a difference in both the character and damping of the response, but when long term dynamics are not strongly coupled to the short term it is a useful approximation. The difference should be remembered when applying handling qualities criteria.

Another measure of particular interest is the normal acceleration at the pilot's station. This is derived as an output variable from the state equations so that once a solution is determined for the state responses, n_z is readily computed. The complete expression for $n_z(t)$ is $n_z = -(1/g)a_z$, where the acceleration along the aircraft z-axis at the pilot station is derived from Euler's dynamic equations of motion. The cockpit is located in inertial space by the vector

$$\bar{r} = \begin{bmatrix} X + x_p \\ Y + y_p \\ Z + z_p \end{bmatrix} \tag{4.18}$$

where X,Y,Z locates the C.G. of the aircraft in inertial space and x_p, y_p, z_p locates the pilot station relative to the C.G. Then by Euler's equation,

$$\bar{V} = \dot{\bar{r}} + \bar{\omega} \times \bar{r} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} \tag{4.19}$$

gives the derivative of r with respect to inertial space in body coordinates. The time derivatives of X, Y, and Z are u, v, and w (x_p,y_p,z_p are fixed) and x_p, y_p, z_p rotate with the body. Then

$$\bar{A} = \dot{\bar{V}} + \bar{\omega} \times \bar{V} = \begin{bmatrix} \dot{u} + \dot{q}z_p - \dot{r}y_p \\ \dot{v} + \dot{r}x_p - \dot{p}z_p \\ \dot{w} + \dot{p}y_p - \dot{q}x_p \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} u + qz_p - ry_p \\ v + rx_p - pz_p \\ w + py_p - qx_p \end{bmatrix} \tag{4.20}$$

Considering longitudinal motion only and a pilot station at $[x_p,0,0]$,

$$n_z(t) = -(1/g)[w - qu - \dot{q}x_p - g\cos\theta] \tag{4.21}$$

Then since $\dot{w} = \dot{V}\sin\alpha + V\cos\alpha\,\dot{\alpha}$, and $u = V\cos\alpha$,

$$n_z = -(1/g)[\dot{V}\sin\alpha + V\cos\alpha\,\dot{\alpha} - qV\cos\alpha - \dot{q}x_p - g\cos\theta] \tag{4.22}$$

Substituting expressions for $\dot{V}$, $\dot{q}$, and $\dot{\alpha}$ from the LRV equations of motion yields

$$n_z = -\frac{1}{g}\Big[\big(-D_\alpha \sin\alpha - L_\alpha \cos\alpha - x_p M_\alpha + x_p M_{\dot{\alpha}} \frac{L_\alpha}{V}\big)\alpha - x_p(M_q + M_{\dot{\alpha}})q - x_p M_{\dot{\alpha}} \frac{g}{V} \cos\Gamma - \frac{3g}{2r}\left(\frac{I_x - I_z}{I_y}\right)\sin 2(\Gamma + \alpha)\Big] \tag{4.23}$$

Using the truncated short period model and taking partial derivatives of the terms in n_z with respect to q and α, yields:

$$\Delta n_z\big|_{\Delta V, \Delta\Gamma = 0} = \frac{1}{g}\Big[\Big\{L_\alpha \cos\alpha + D_\alpha \sin\alpha - x_p M_{\dot{\alpha}} \frac{L_\alpha}{V} + x_p\Big(M_\alpha - \frac{3g}{r}(\cdot)\cos 2(\Gamma + \alpha)\Big)\Big\} + x_p(M_{\dot{\alpha}} + M_q)\Delta q\Big] \tag{4.24}$$

Since $\Delta q = \Delta\dot{\alpha} + (L_\alpha/V)\,\Delta\alpha$ and $\Delta\alpha$ is known

$$\Delta n_z = \frac{1}{g}\Big[\Big(L_\alpha \cos\alpha + D_\alpha \sin\alpha + x_p M \frac{L_\alpha}{V} + x_p M_\alpha - x_p \frac{3g}{r}(\cdot)\cos 2(\Gamma + \alpha)\Big)\Delta\alpha + x_p(M_{\dot{\alpha}} + M_q)\Delta\dot{\alpha}\Big] \tag{4.25}$$

Once an asymptotic solution for $\Delta\alpha$ is derived, it can be differentiated to give an expression for $\Delta\dot{\alpha}$ and therefore, n_z.

Long Term Response

For the long term modes, truncation with residualization of the short term modes can still apply. But since steady state conditions do not exist and "natural frequency" is continuously changing, the assumptions are interpreted slightly differently. What matters is whether the responses associated with the long term mode (V and Υ) vary (oscillate) on a slow scale compared with the short term responses (q and α). If so, short term deviations from reference trajectory conditions will diminish to zero on the order of $1/\varepsilon$ faster than the long term deviations (where ε is a small, positive number relating the time scales of short term and long term variations). Current specifications restrict the damping of the phugoid mode without any bounds on frequency or mode shapes of the responses. For normal powered, atmospheric flight the damping restrictions insure positive stability of speed and flight path. This benefits pilot workload but the pilot must constantly regulate the low frequency variation of speed and altitude to hold precise flight conditions. An aircraft, or LRV, in high speed flight encounters an additional complication: altitude changes during phugoid motion become large compared with airspeed changes, and resulting air density gradients make the phugoid period shorter [23]. This effect will certainly increase the difficulty of flight path (or trajectory) tracking for the pilot, and it warrants a complete (as possible) study of the long term responses. For flight regimes where the short and long term response time scales are not separated by an order of ϵ, each of the state or output variables should be decoupled from the others and specified individually. This increases the order of the equation describing the response and complicates the solution as well as the task of defining handling quality criteria. But it is a more legitimate approach.

Proceeding with the development of the residualized phugoid equations, we refer to equations (4.15) in this chapter and equations (3.2) in chapter 3. Partitioning the dynamics (F) matrix gives:

$$F_1 = \begin{bmatrix} -D_V & -g\cos\gamma \\ \frac{L_V}{V} + \left(\frac{1}{r} + \frac{g}{V^2}\right)\cos\gamma & \left(\frac{g}{V} - \frac{V}{r}\right)\sin\gamma \end{bmatrix} ; \qquad F_2 = \begin{bmatrix} 0 & -D_\alpha \\ 0 & L_\alpha/V \end{bmatrix}$$

$$F_3 = \begin{bmatrix} M_V & M_{\dot\alpha}\left(\frac{g}{V}\right)\sin\gamma - \frac{3g}{r}(\cdot)\cos 2(\gamma+\alpha) \\ -L_V/V - \left(g/V^2\right)\cos\gamma & -\left(g/V\right)\sin\gamma \end{bmatrix} ; \qquad F_4 = \begin{bmatrix} M_q + M_{\dot\alpha} & M_\alpha - \frac{3g}{r}(\cdot)\cos 2(\gamma+\alpha) \\ 1 & -L_\alpha/V \end{bmatrix} \tag{4.26}$$

The truncated 2X2 state dynamics matrix becomes, following residualization of $[\Delta q \;\Delta\alpha]^T$,

$$\begin{Bmatrix} \Delta\dot{V} \\ \Delta\dot{\gamma} \end{Bmatrix} = \begin{bmatrix} F_1 - F_2F_4\,F_3 \end{bmatrix} \begin{Bmatrix} \Delta V \\ \Delta\gamma \end{Bmatrix} \qquad (4.27)$$

$$F_1 - F_2F_4^{-1}F_3 = \begin{bmatrix} -D_V - D_\alpha\left(\frac{f_1}{\Delta}\right) & -g\cos\gamma - D_\alpha\left(\frac{f_2}{\Delta}\right) \\ \frac{L_V}{V} + \left(\frac{1}{r} + \frac{g}{V^2}\right)\cos\gamma + \frac{L_\alpha}{V}\left(\frac{f_1}{\Delta}\right) & \left(\frac{g}{V} - \frac{V}{r}\right)\sin\gamma + \frac{L_\alpha}{V}\left(\frac{f_2}{\Delta}\right) \end{bmatrix}$$

$$f_1 = M_V + (M_{\dot{\alpha}} + M_q)\left(\frac{L_V}{V} + \frac{g}{V^2}\right)\cos\gamma$$

$$f_2 = M_q\left(\frac{g}{V}\right)\sin\gamma$$

$$\Delta = -(M_q + M_{\dot{\alpha}})\frac{L_\alpha}{V} - M_\alpha + \frac{3g}{r}\left(\frac{I_x - I_z}{I_y}\right)\cos 2(\gamma+\alpha)$$

Decoupling these by cross-differentiation gives:

$$\Delta\ddot{V} - \left(e_{11} + \frac{\dot{e}_{12}}{e_{12}} + e_{22}\right)\Delta\dot{V} - \left(\dot{e}_{11} + e_{12}e_{21} - \frac{\dot{e}_{12}\,e_{11}}{e_{12}} - e_{11}e_{22}\right)\Delta V = 0 \qquad (4.28)$$

$$\Delta\ddot{\gamma} - \left(e_{11} + \frac{\dot{e}_{21}}{e_{21}} + e_{22}\right)\Delta\dot{\gamma} - \left(\dot{e}_{22} + e_{12}e_{21} - \frac{\dot{e}_{21}\,e_{22}}{e_{21}} - e_{11}e_{22}\right)\Delta\gamma = 0 \qquad (4.29)$$

These are linear, second order, time-varying equations describing oscillations in velocity and flight path angle about the reference trajectory. Unlike the negligible effect of short period motion on nominal trajectory conditions, phugoid motion will result in deviations from the trajectory that affect the response, as described above. As a first approximation, the coefficients will be assumed to vary only as functions of the nominal trajectory, unaffected by the oscillations. The form of the solutions to these equations is the same as for angle of attack. Since trajectory data are tabulated and not given as explicit functions of time, the time derivatives in the coefficients must be approximated by fairing through the data. A less accurate but far simpler approach in the absence of functional relationships is to decouple the equations as though the coefficients are constants. This results in identical equations, and thus, identical characteristic roots and time behavior, for V and γ. Validity of the multiple scales approximation again depends on slowness of coefficient variation compared to the time for a cycle of the oscillation.

4.3 VTOL Aircraft Handling Qualities

4.3.1 VTOL Pilot Tasks

A primary pilot control in hover is thrust level. Typical dynamic characteristics are high speed stability and low vertical (heave) damping [24]. Attitude control must be effected by distributing thrust to command moments about the appropriate axis. As the analysis of [9] showed, there may be longitudinal instability in hover requiring pitch attitude control augmentation. Transition involves simultaneous control of thrust level, thrust deflection, and elevator deflection to follow a nominal trajectory toward or away from hover. The speed of transition depends directly on the thrust level and flight path followed. The control task is multiple loop in nature, with pitch attitude an inner loop to flight path (altitude) control. Any cross coupling of control inputs and motion outputs affects the pilot's task, possibly adversely. At the start of a transition from hover all aerodynamic controls are ineffective, so attitude stability augmentation must blend thrust control with surface effectors. Any portion of the VTOL flight regime where there are accelerations toward or away from hover or cruise are of interest here. This dynamic behavior should be treated as explicitly as possible when formulating handling quality specifications.

4.3.2 Analysis of Handling Qualities

The dynamics of VTOL aircraft in transition are not only time varying but unconventional. Using asymptotic methods to derive expressions for the time response of state or output variables may be useful to verify aspects of a particular vehicle or control design, but not directly for handling qualities specification. Truncation of modes is improper so the full order of the natural vehicle response must be examined. Many criteria have been proposed and tested for VTOL hover and transition handling qualities. The criterion discussed in Chapter 3 for unconventional vehicle responses is a form of frequency response bandwidth. A study by Hoh [25] examined the effects of several control and aerodynamic factors on flight path control during transition. A necessary handling quality criterion is interpreted as a path bandwidth, formulated in terms of the altitude-to-throttle transfer function. "Path bandwidth is a measure of how tightly a pilot can close the throttle to the altitude ($\delta_t \rightarrow h$) loop without threatening the stability of the pilot/vehicle system" [25]. Bandwidth is interpreted as the frequency at which phase margin is 45^o or gain margin is 6dB. Among the factors influencing path bandwidth are pitch attitude inner loop bandwidth, aerodynamic heave damping ($-Z_w$), pitch moment due to thrust offset from the center of gravity, and engine lag.

The dynamics examined in [25] have elevator control of pitch attitude with moderate inner loop bandwidth (1.5-4.0 rad/sec). When the inner loop is tightly closed ($\omega_{bw\theta}$ > 4.0 rad/sec) the path bandwidth is primarily dependent on heave damping. For a less tightly closed attitude loop the coupling between throttle control and pitch attitude becomes important. At an intermediate point in transition, for example, thrust is vectored partly upward and partly forward. Positive throttle increment will cause translational acceleration forward and upward; rotational acceleration (if any) depends on the control coupling derivative, M_{δ_T}. Adverse thrust coupling (negative M_{δ_T}) causes downward pitching with thrust increase and therefore degrades the path bandwidth. It is important to quantify the effects of thrust control-to-pitch-attitude coupling on path response for a particular control scheme and vehicle. Engine throttle-to-thrust lag has a minor effect on path control in the range of frequencies for good handling qualities. Its effect is alway degrading, however, and can be easily included in the model. The acceptable path bandwidth chosen in [25] for Level 1 handling quality is $\omega_{bwh} = 0.2$ rad/sec.

The variable dynamics of an aircraft in transition will give it "bandwidth histories", determined from the elements of the system function. The path bandwidth history of a representative VTOL aircraft will be evaluated using an asymptotic approximation to the system function. A good subject for analytically examining handling qualities during transition is the XC-142 tilt-wing VTOL aircraft. Its dynamics are well documented and have been studied by both classical and asymptotic means. Developed in the early 1960's as an experimental vehicle, the XC-142 has four main propellers on the wing and a tail rotor driven by a power takeoff shaft from the main engine [26]. It weighs 40,041 lbs gross, has a cruise speed of 244 knots and maximum speed of 317 knots. At takeoff the wing and engines are tilted 90^o to the ground and thrust-to-weight is 1.05. During transition the wing rotates forward and thrust-to-weight increases to 1.08. Once forward flight is attained with the wing rotated fully forward, the tail rotor can be feathered and pitch attitude controlled by elevator only. A state space description of the longitudinal aircraft dynamics uses the state vector

$$\underline{x} = [u\ w\ q\ \theta]^T \tag{4.30}$$

with equations of motion

$$\underline{\dot{x}} = \begin{bmatrix} X_u & 0 & 0 & -g \\ Z_u & Z_w & Z_q & 0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} X_{\delta_T} \\ Z_{\delta_T} \\ M_{\delta_T} \\ 0 \end{bmatrix} \delta_T \tag{4.31}$$

Thrust is the only control element because the tail rotor will be used in a closed loop to regulate pitch attitude during transition. The intent is to diminish pitch loop gain during transition so that the elevator becomes the attitude controller at the end of transition and the tail rotor may be feathered. For a given thrust setting, attitude regulation will effectively cause only moment generation and no forces in the X or Z direction, since the tail rotor is driven by takeoff from the engines. Reference [9] gives the following relationships for stability derivatives as functions of flight velocity:

$$X_u = -0.2$$

$$Z_u = -V/(40+4V) \qquad Z_w = -0.1-V/250 \qquad Z_q = V$$

$$M_u = .015(1-V/150) \qquad M_w = -.005-.015(V/150)^2 \qquad M_q = -.1-.0034V$$

The asymptotic calculations of vehicle responses in [9] and [7] treated level flight transitions, and therefore used M_{δ_T} as the only control input term, where $M_{\delta_T} = -.000314$ rad/sec/lb-thrust, determined by the pitch moment of inertia and tail rotor offset from the center of gravity. Since the heave response is being examined here,

$$X_{\delta_T} = .000804(V/150)\ \text{ft/sec}^2/\text{lb}$$

$$Z_{\delta_T} = -.000804(1-V/150)\ \text{ft/sec}^2/\text{lb}$$

are used to describe the variable effect of thrust inputs during transition. Two transition velocity profiles are used:

(1) $V(t) = 150t/(20+t)$ ft/sec, $t \geq 0$

which is asymptotic to 150 ft/sec for large t, and

(2) $V(t) = 15t^2/(2+4t)$ ft/sec, $0 \leq t \leq 40$

which is asymptotic to 3.75t ft/sec for large t.

It is assumed that transition initiates at hover and concludes when flight velocity is 150 ft/sec, or 89 knots. The wing is tilted fully forward when transition is complete. Analyses in [9] and [7] used velocity profile 1, which accelerates quickly initially ($V(0)=0$ and $V(1)=7.14$ ft/sec) and very slowly for large t, plotted in Figure 4-8. This profile allows one to check the response calculations against a constant flight condition for large t. The second profile is probably more typical, with slower initial acceleration (2.6 ft/sec^2) and nearly constant acceleration (3.6 ft/sec^2) after 2 seconds, Figure 4-9. A thrust-to-weight setting of 1.08 during transition would cause an acceleration of 2.6 ft/sec^2.

From the state equations, the 2nd element of the system function will describe the path-to-throttle response, and the 3rd element the pitch attitude response. A zeroth order (Poincaré) asymptotic approximation to the system function is

$$H_o(s,t) = [A(t) - sI]^{-1} B(t) \tag{4.32}$$

The pitch compensation design in [7] used $H_o(s,t)$ to find a time-varying feedback gain with pitch angle and pitch rate feedback in the manner shown below.

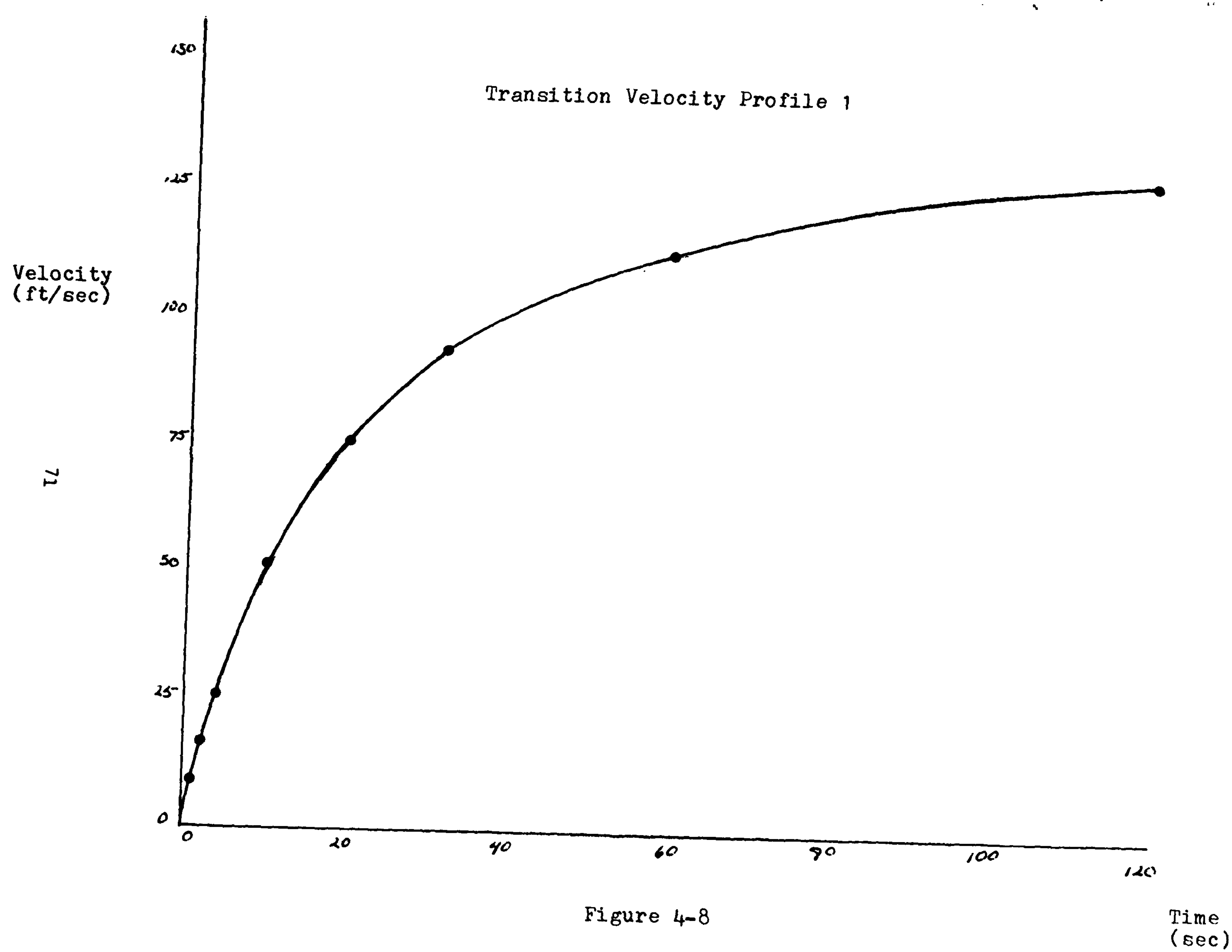

Transition Velocity Profile 1
Velocity
(ft/sec)
150
125
100
75
50
25
0
0
20
40
60
90
100
120
Time
(sec)

Figure 4-8

Transition Velocity Profile 2

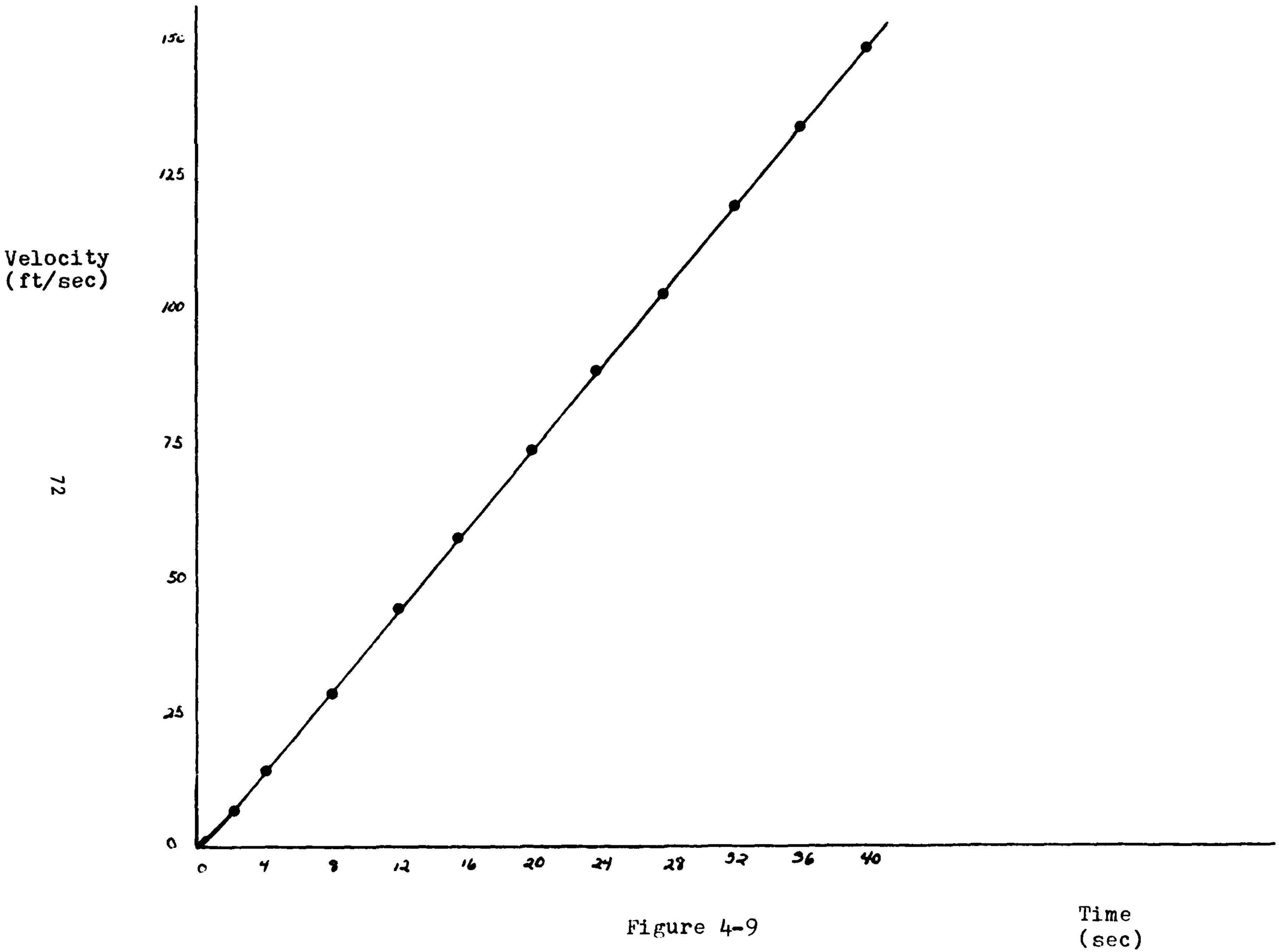
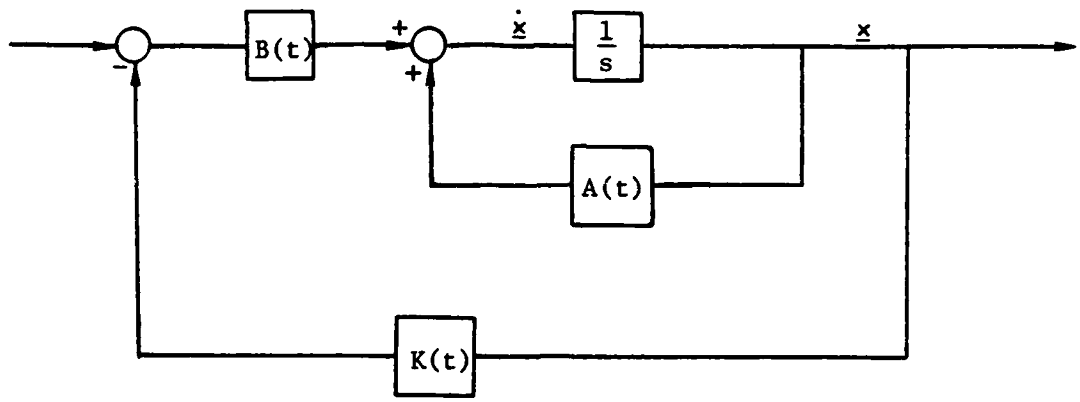


Figure 4-9



where $K(t) = [0\ 0\ 2\ 1]\ (120000/(20 + t))$

The gain $K(0) = 6000$ stabilizes the longitudinal modes in hover while the time variation corresponds to the velocity profile 1 transition, with vanishing gain for large t. Figure 4-10 shows the compensated pitch attitude bandwidth history from this zeroth order asymptotic analysis design. Pitch bandwidth is moderate throughout the transition. A plot of the characteristic root movement is in Figure 4-11. Although the time variation results in unique characteristic roots for each state, $H_o(s,t)$ has a common characteristic equation for all states. Decoupling the pitch attitude equation by cross-differentiation and using the zeroth order by multiple scales equation (2.11) gives an asymptotically equivalent picture of the root movement, shown in Figure 4-12. One real root behaves erratically at about 4 seconds into the transition, crossing the imaginary axis and then returning to branch with the other real root into a complex pair. Near the end of transition (120 seconds) the root positions are identical in Figures 4-11 and 4-12. This is explained by the very slow changes in stability derivatives at 120 seconds for velocity profile 1.

Compensated Pitch Inner Loop Bandwidth History

Velocity Profile 1

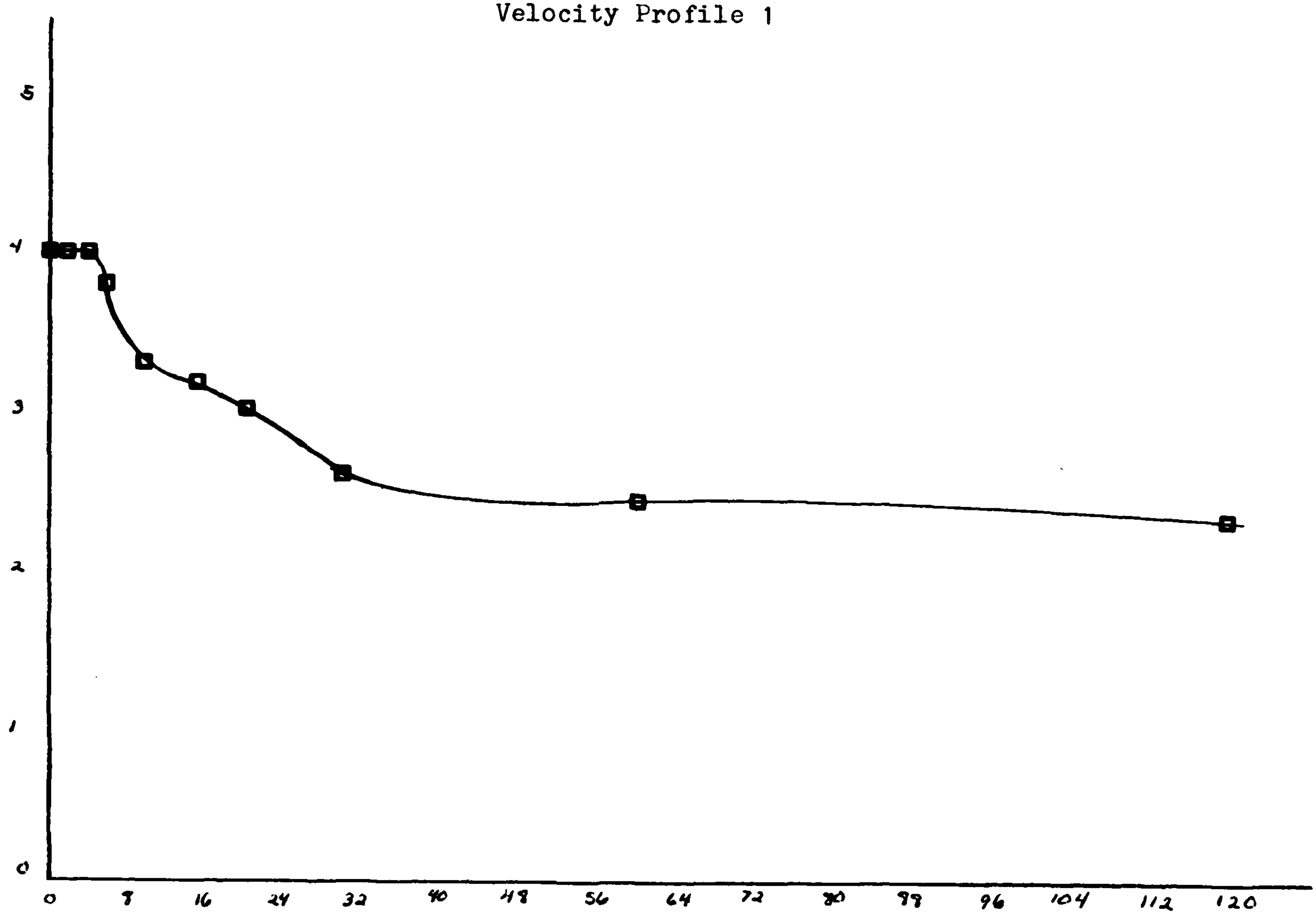


Figure 4-10

Roots of the Zeroth Order Characteristic Equation
For Pitch Attitude

Velocity Profile 1

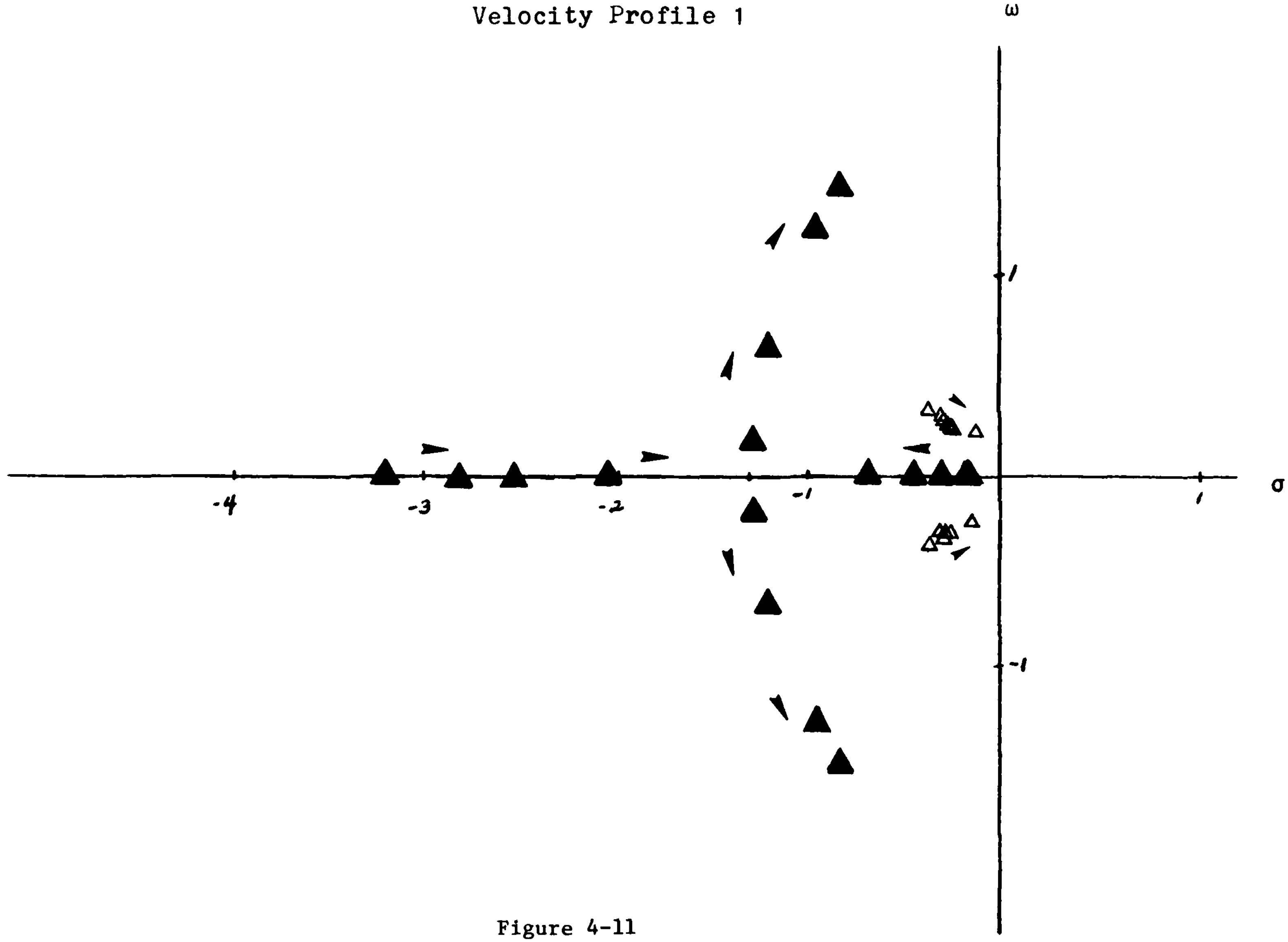


Figure 4-11

Roots of the Decoupled Characteristic Equation
For Pitch Attitude

Velocity Profile 1

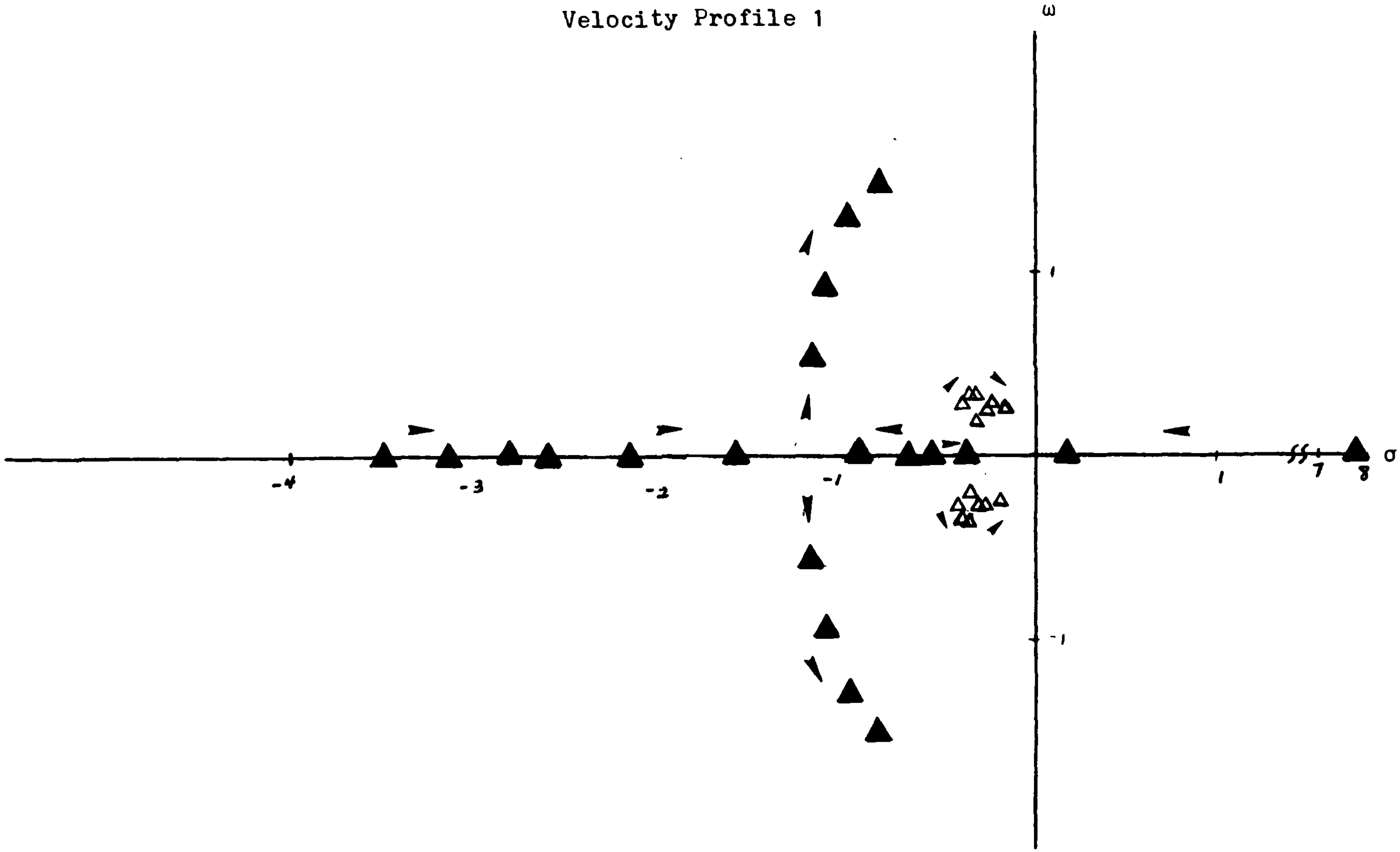


Figure 4-12

A similar pitch compensator design for velocity profile 2 yields

$$K(t) = [0\ 0\ 2\ 1]\ (35000/(5 + t)).$$

Figure 4-13 shows the compensated pitch attitude bandwidth history. It has greater variation than profile 1, changing from a tight loop at hover ($\omega_{bw\theta} > 4$ rad/sec) to moderate closure ($1.5 \leq \omega_{bw\theta} \leq 4.0$) for the rest of the transition, but is very similar. Figure 4-14 shows the characteristic root movement for velocity profile 2.

A check on the accuracy of the asymptotic approximation should be made. Ramnath compared asymptotically derived time responses of the unaugmented aircraft with numerically integrated responses [9], demonstrating the accuracy of zeroth order asymptotic analysis. A rigorous error analysis was carried out by Ramnath [2] including the development of strict and sharp error bounds. Callaham and Ramnath [6] demonstrated the validity of using $H_o(s,t)$ by comparing time responses of $\theta(t)$ to a design incorporating first order correction $H_1(s,t)$. Based on these results, the system function approximation $H_o(s,t)$ can be used for this handling quality analysis.

Compensated Pitch Inner Loop Bandwidth History
Velocity Profile 2

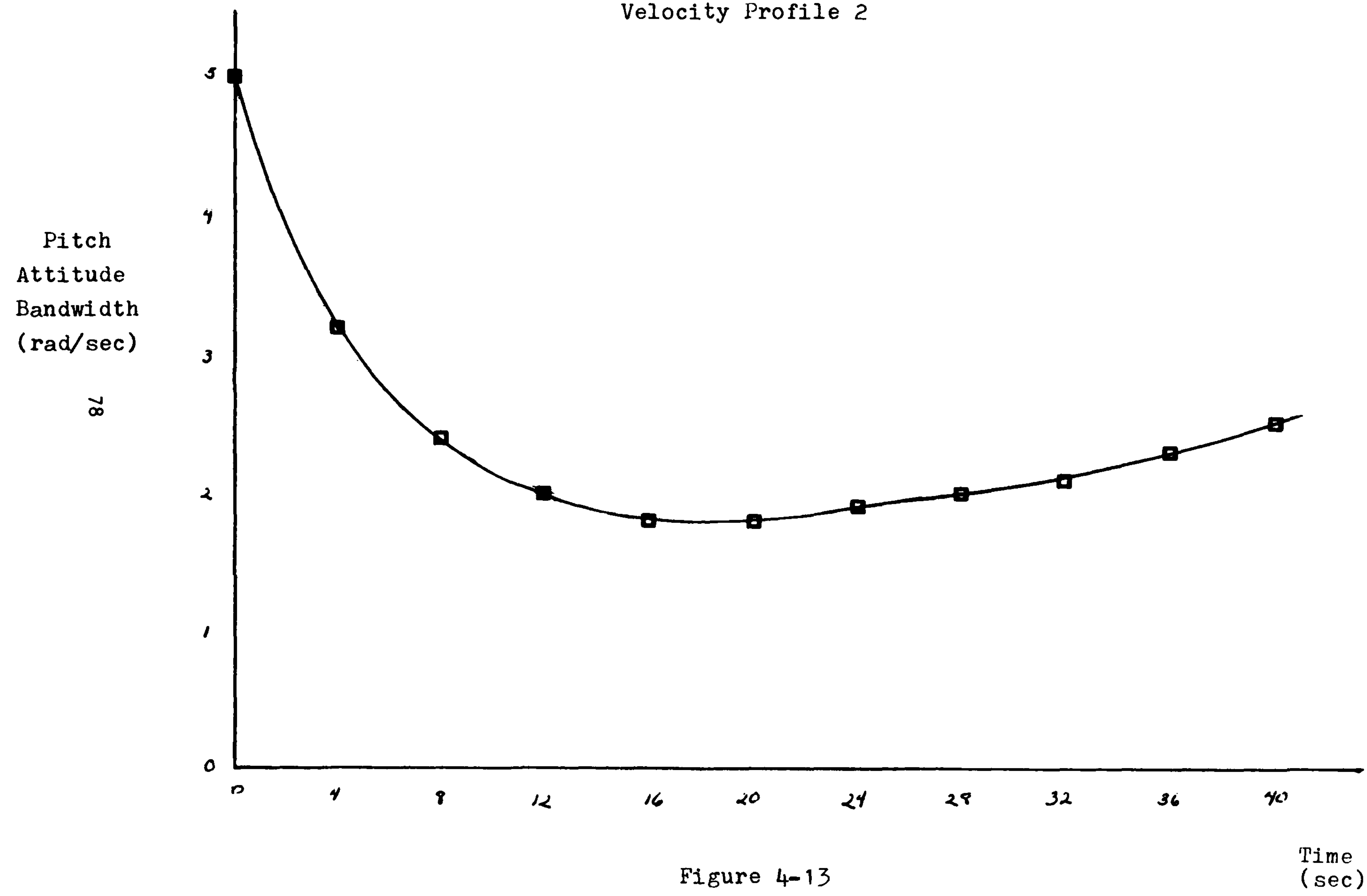


Figure 4-13

Roots of the Zeroth Order Characteristic Equation
For Pitch Attitude

Velocity Profile 2

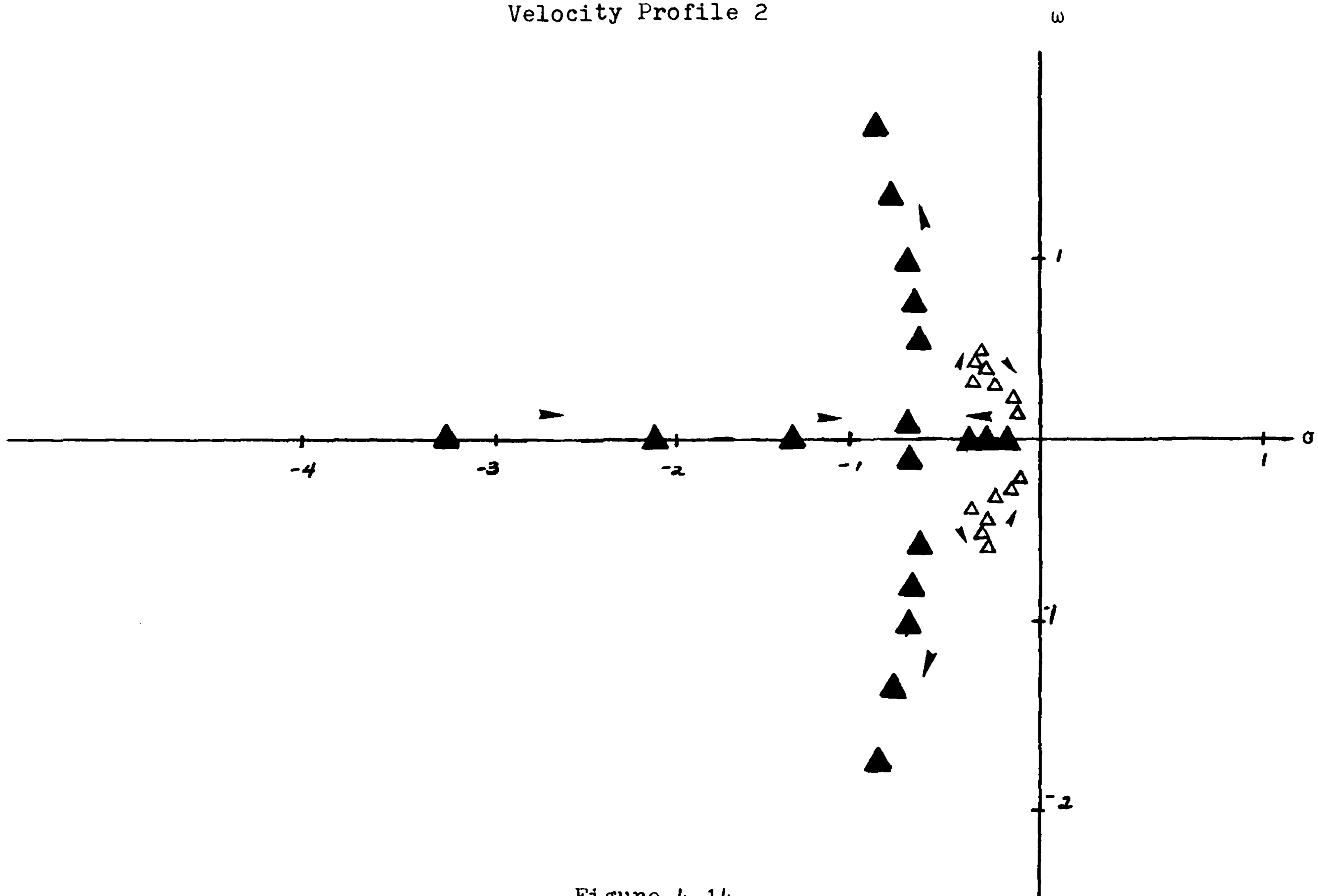


Figure 4-14

Having acceptable designs for pitch compensation, the next step is to evaluate the path bandwidth histories. The magnitude and phase of the second element of $H_O(s,t)$ are shown graphically as functions of frequency and time for profile 1 (Figure 4-15) and profile 2 (Figure 4-16). Since open loop throttle-to-altitude response is being examined, the system function $w(s,t)/\delta_T$ is integrated once to give

$$h(s,t)/\delta_T = (1/s)\ w(s,t)/\delta_T \tag{4.33}$$

Path bandwidth histories based on 45^o phase margin are plotted in Figures 4-17 and 4-18 for profiles 1 and 2, respectively. The dependence of path bandwidth on heave damping ($-Z_w$) early in transition is shown by Figures 4-19 and 4-20. Path bandwidth does not reach the Level 1 minimum until 10 seconds into transition for profile 1, and 8 seconds into transition for profile 2. A jump in the bandwidth seems to occur between 10 and 15 seconds (profile 1) and between 12 and 16 seconds (profile 2). These jumps correspond exactly with the branching of the two real roots into an oscillatory pair, checking Figures 4-11 and 4-14.

Magnitude of the Heave-to-Throttle System Function

Velocity Profile 1

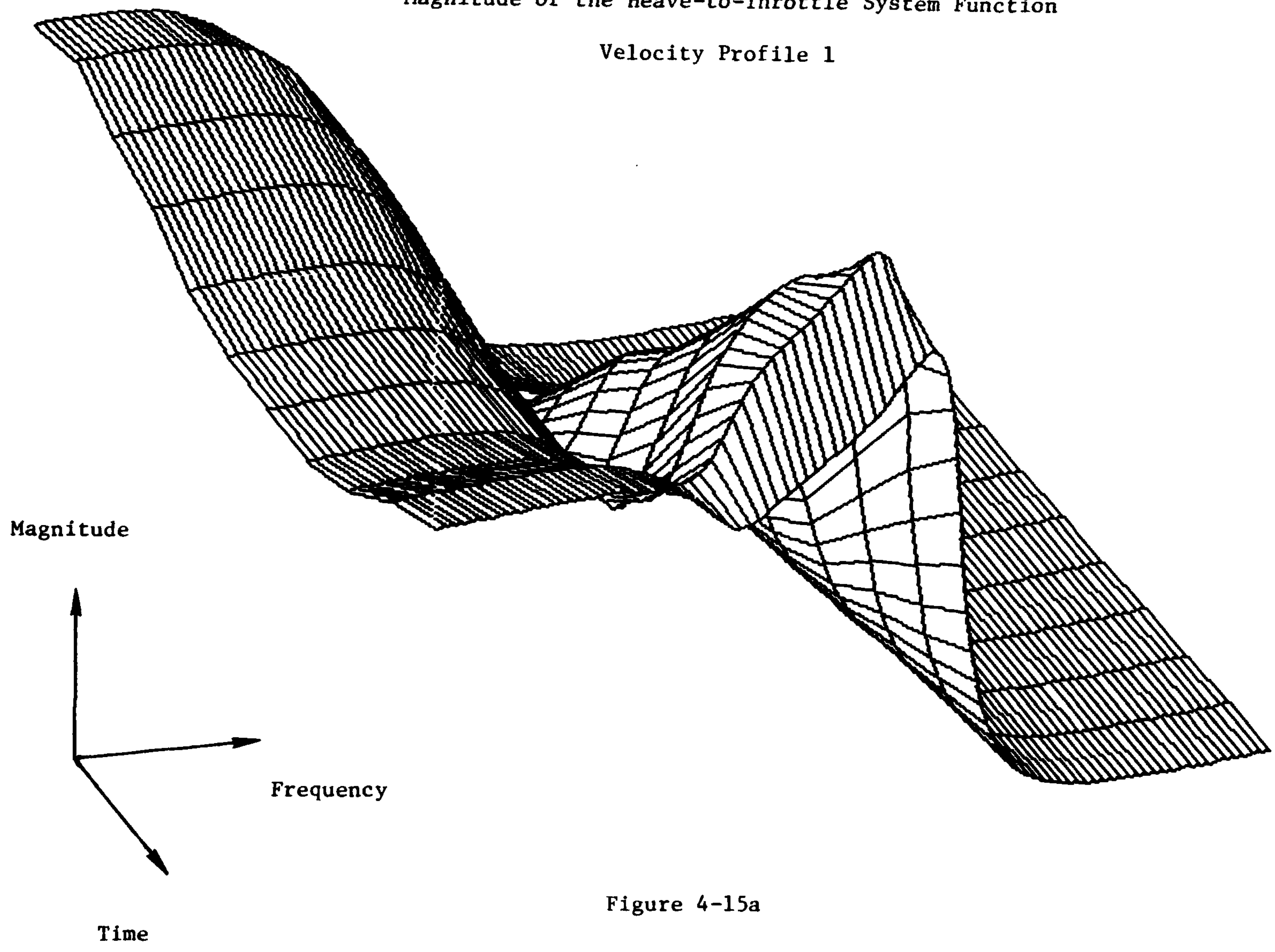


Figure 4-15a

Phase of the Heave-to-Throttle System Function

Velocity Profile 1

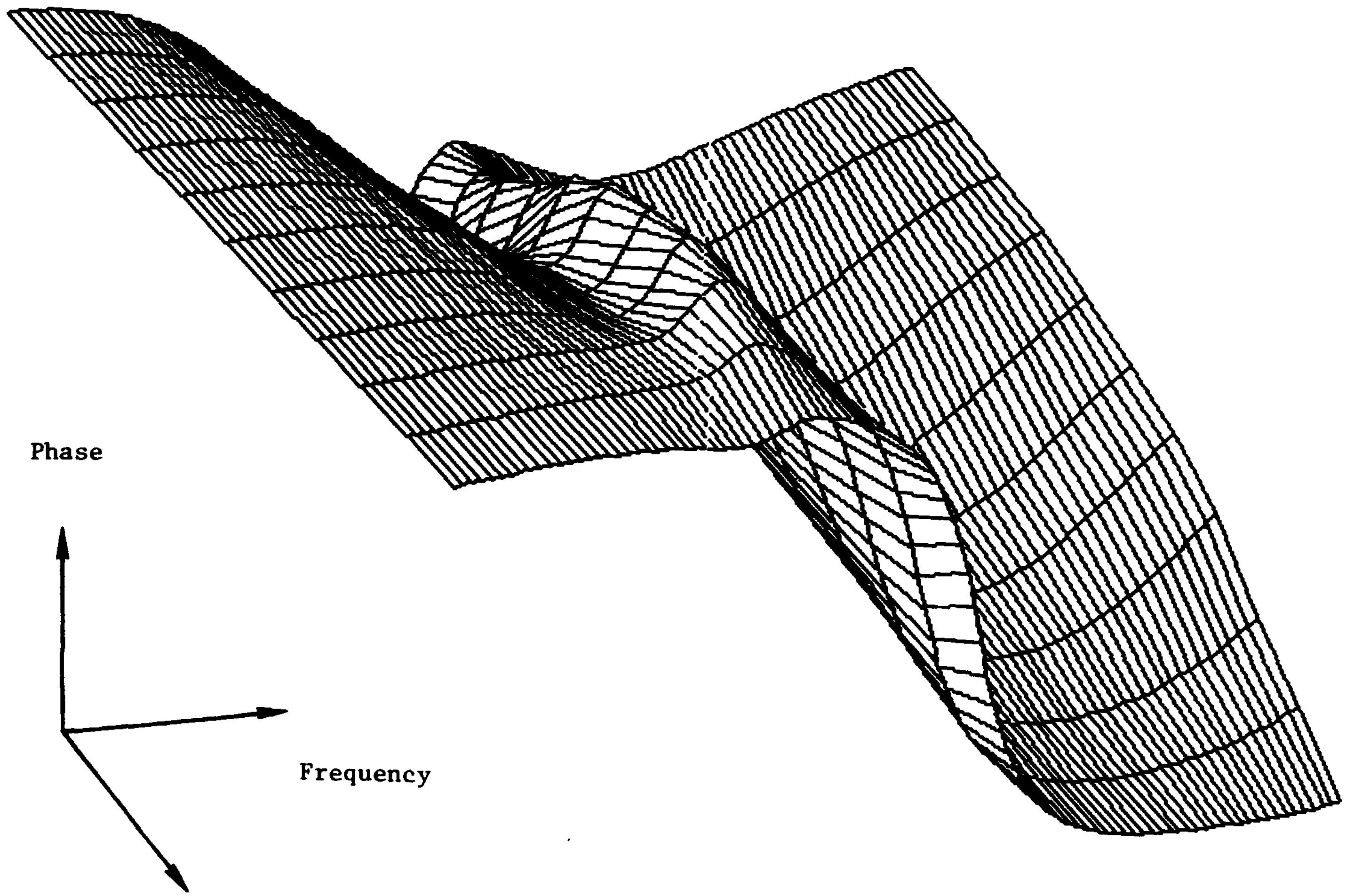


Figure 4-15b

Magnitude of the Heave-to-Throttle System Function

Velocity Profile 2

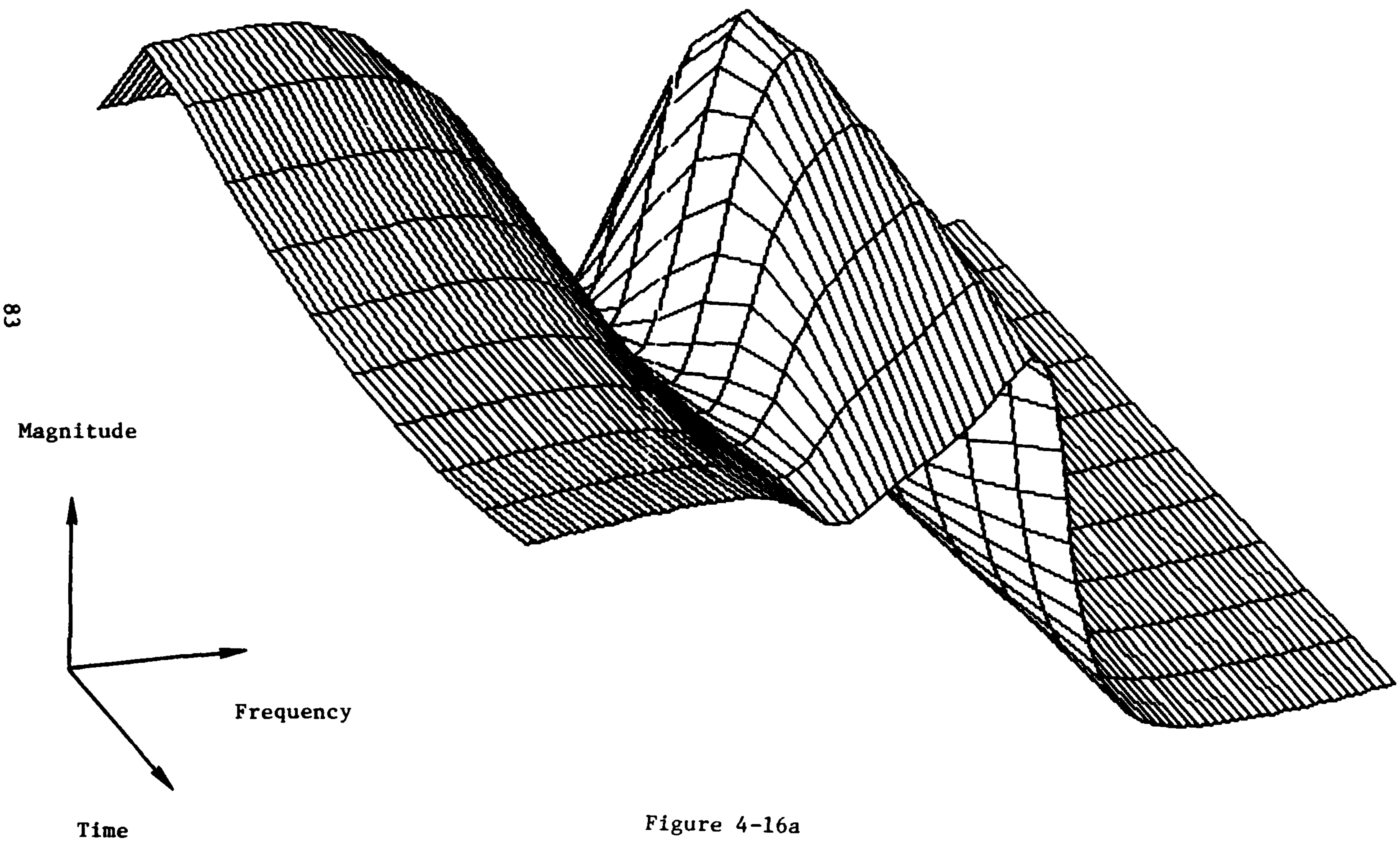


Figure 4-16a

Phase of the Heave-to-Throttle System Function

Velocity Profile 2

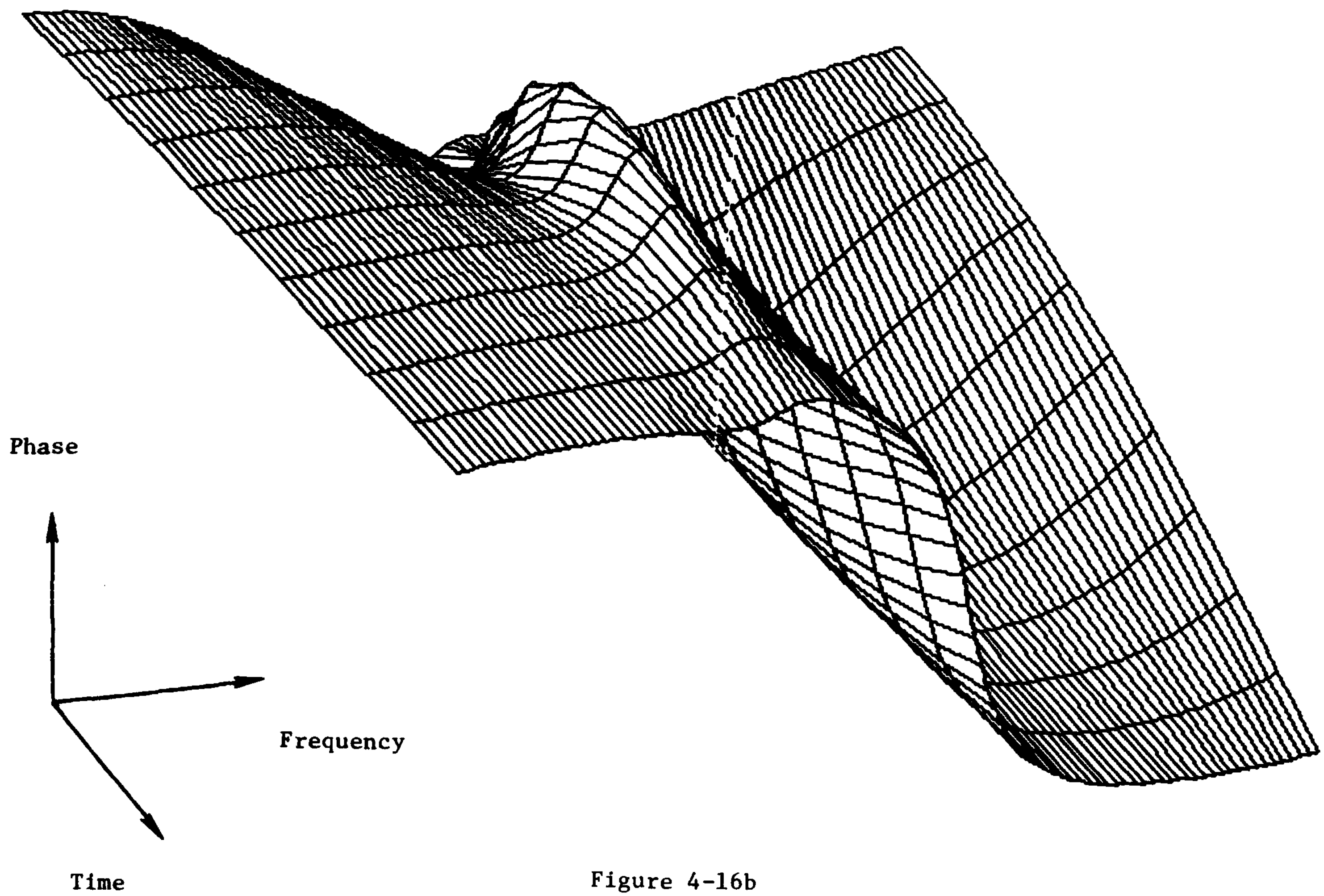


Figure 4-16b

Flight Path Bandwidth History
Velocity Profile 1

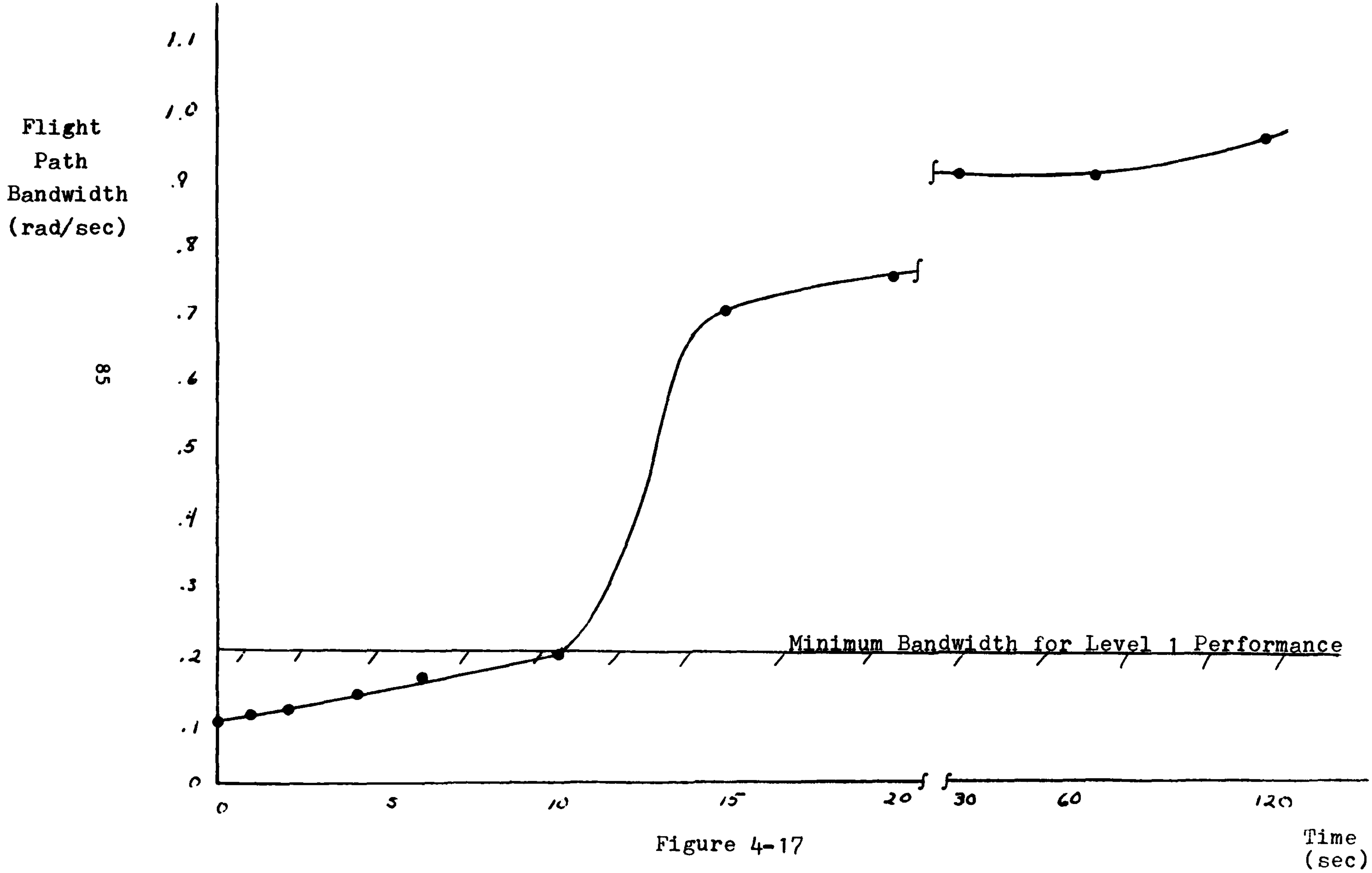


Figure 4-17

Flight Path Bandwidth History
Velocity Profile 2

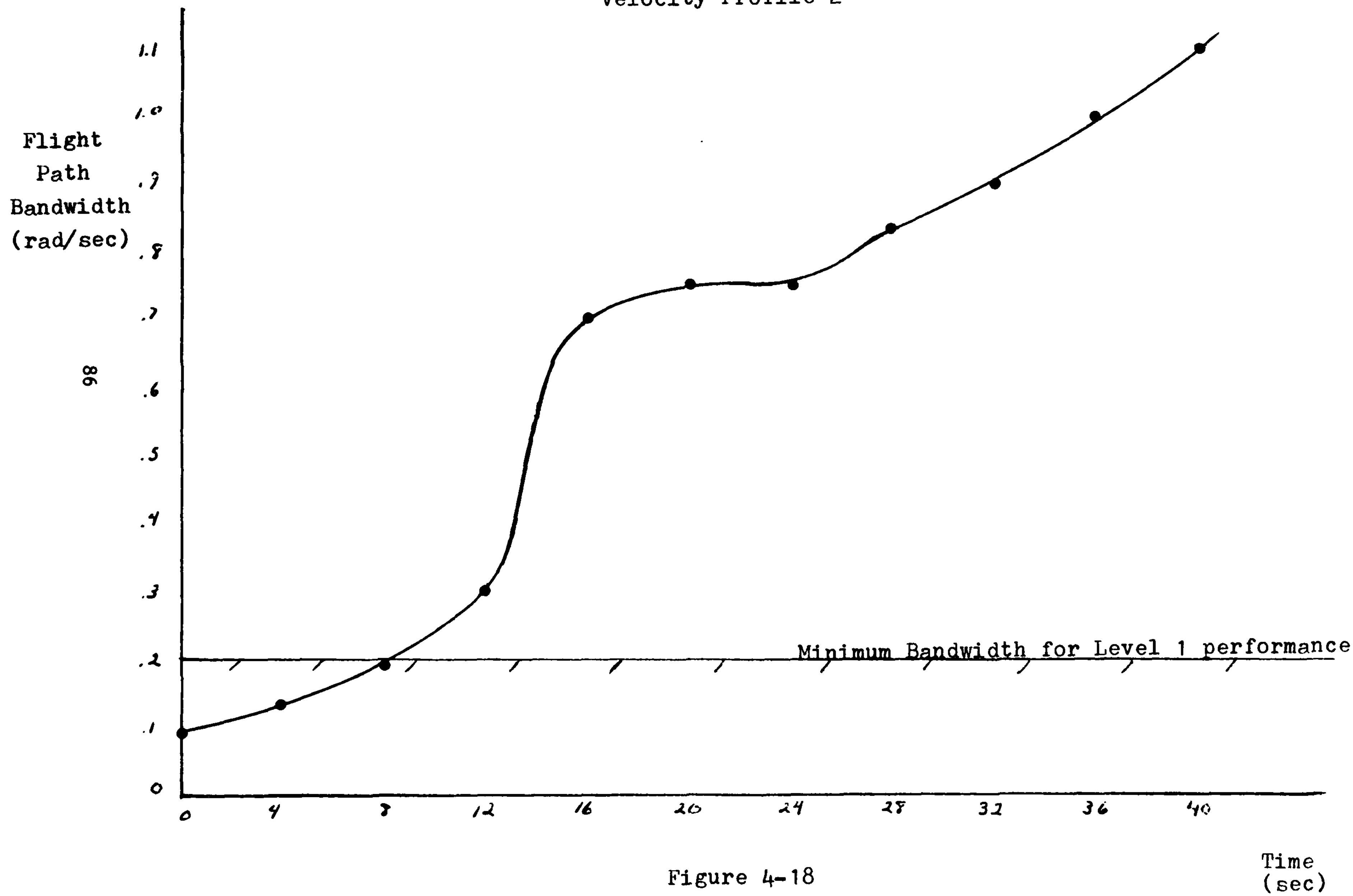
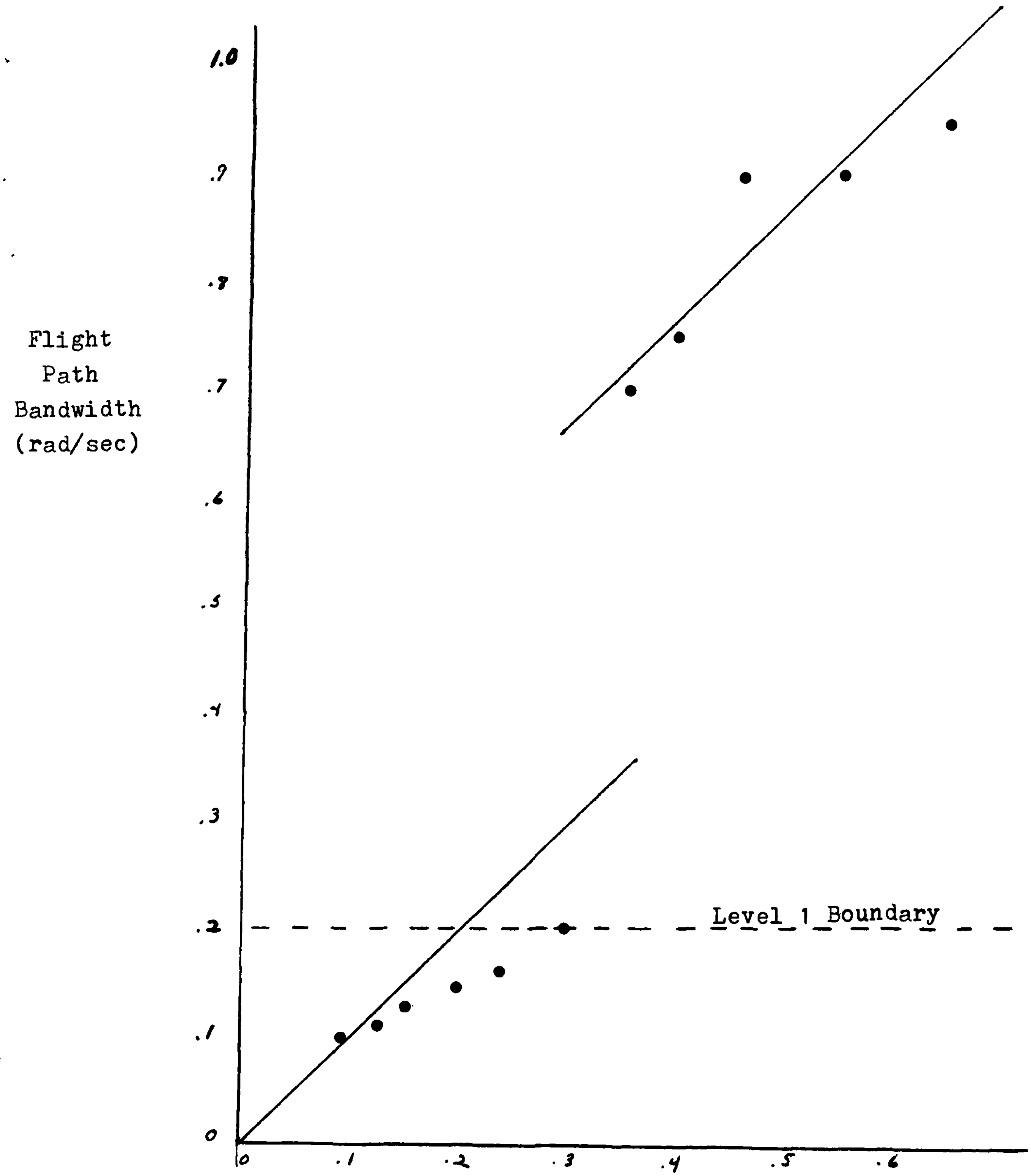
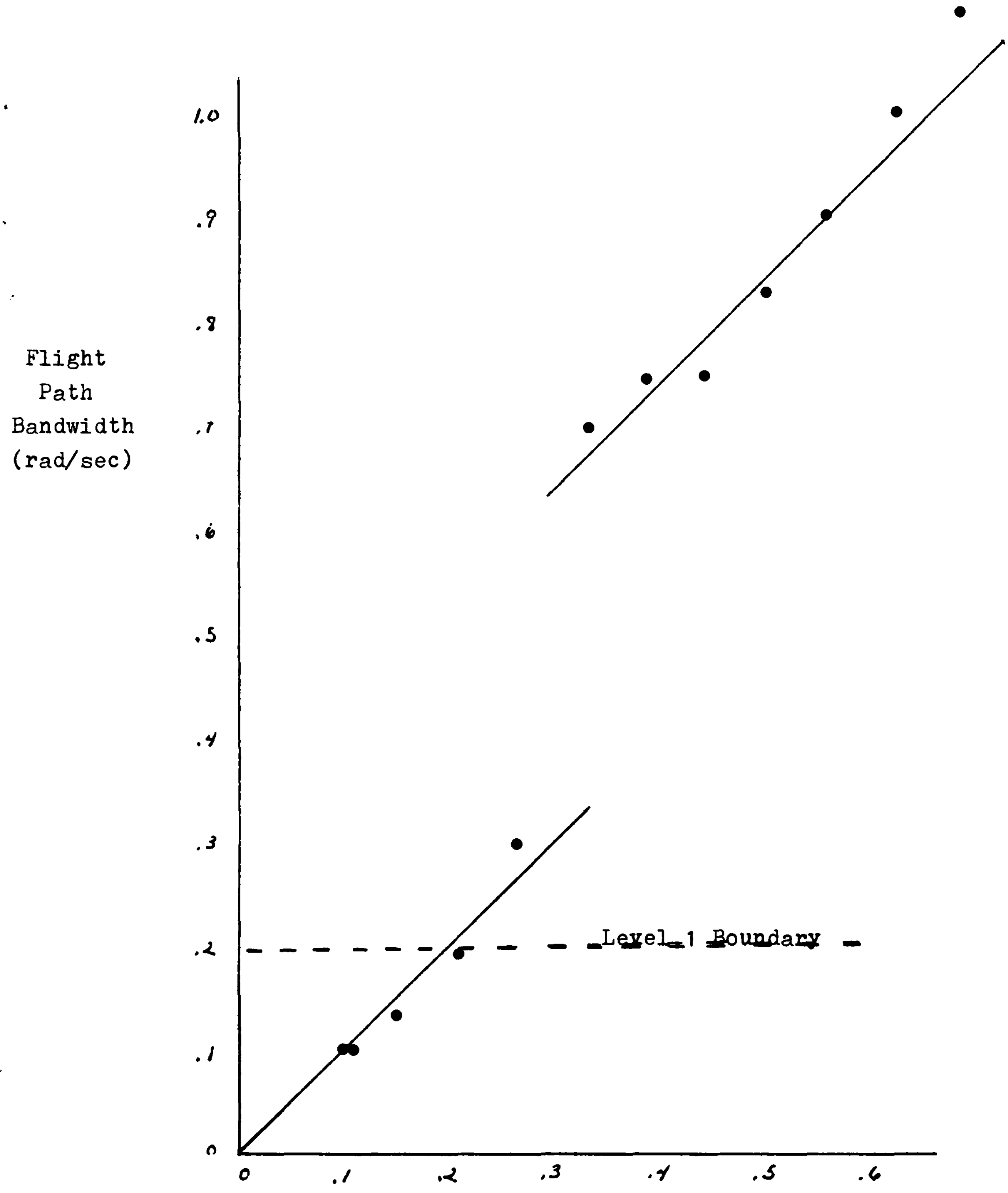


Figure 4-18



Heave Damping ($-Z_w$)
Velocity Profile 1
Figure 4-19



Heave Damping ($-Z_w$)
Velocity Profile 2
Figure 4-20

The study by Hoh [25] gave path bandwidth of an XC-142 at 80 knots (135 ft/sec), with pitch inner loop closure of 2.5 rad/sec by elevator control, as 0.22 rad/sec. This is quite different from the values of ω_{bwh} = 0.95 and 1.0 for the above analyses at times corresponding to the 80 knot flight condition. The difference is certainly due to adverse thrust coupling effect on elevator control of pitch in the first case. When the tail rotor is used in closed loop pitch control, the adverse thrust effect is regulated. Thus, higher path bandwidths with thrust control result.

An aspect of variable flight dynamics which is a significant factor in its handling quality is the rate and direction of variation. As seen in the path bandwidth histories of VTOL transition, the vehicle response does not necessarily change at a constant rate. Since the pilot must anticipate control inputs, a sudden or unpredictable shift in the character o vehicle response may have an adverse impact on controllability and pilot workload. The direction of variation is important: a change in a positive direction (as in increased bandwidth) is beneficial. An observation about th VTOL results is that the jump in bandwidth occurred at a turning point, when real roots became oscillatory (or vice versa). Reversing the direction of transition, the pilot faces a flight region in which path bandwidth decreases sharply over 5 seconds. Such points in the variable dynamics should be identified in the design process when possible. While this adverse change is predictable, it may still cause difficulty. At worst a maneuver or trajector could be modified to slow the rate of change. As always, piloting technique is a critical factor and may also require some modification. A limit of the form

$$\dot{\omega}_{bw} > -c \qquad (4.34$$

where c is a positive constant, could be imposed on bandwidth, for example. This limit may affect flight control design, types of maneuvers permitted, and pilot technique. The value of the above limit would depend on the response parameter being examined. Attitude control might sustain a higher rate of change than path bandwidth, and remain acceptable. Another way of imposing a limit on the rate of change of a response characteristic might be in the form

$$a_j(t)/k_i(t) < ca_j(t) \tag{4.35}$$

where a_j is the most quickly varying coefficient and k_i is the root of smallest magnitude from the characteristic (clock) equation of a multiple scales expansion. The limit above might serve the control designer better than a limit on a response parameter, like equation (4.34). In some cases there may not be a need for such a limit. Certainly the validation of new criteria (or old criteria against new flight vehicles) requires an extensive data base of flight regimes, maneuvers and pilot evaluations.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

A thorough understanding of flight environments and flight vehicle dynamics is essential to designing airframes and control systems which can achieve mission goals with specified handling qualities during piloted tasks. Accurate analytical descriptions of the dynamics are a necessity. Many criteria derived through constant coefficient analysis of equations of motion have already been experimentally validated at certain flight conditions. But in cases where the flight vehicle traverses widely varying flight conditions in a prescribed manner, the actual time-varying equations of motion should be used for analysis. Approximate but accurate solutions to these equations can be constructed by asymptotic methods, of which one of the most general methods is that of Multiple Scales. The scales represent slow and fast behavior of the vehicle responses, defined by linear and non-linear functions (clocks) of the independent variable. Multiple scaling yields useful approximations when the coefficients vary slowly - a condition satisfied in this handling quality analysis of flight vehicles.

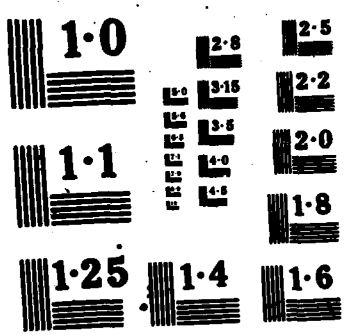
Equations of motion constructed for handling quality analysis should be representative of the variations in flight conditions and vehicle dynamics for typical maneuvers or trajectories. Motion in each degree of freedom will have a unique characteristic equation because of the time dependence. Times at which multiple roots exist (turning points) should be identified since they could affect the accuracy of asymptotic solutions. When a turning point is moved through quickly, a multiple scales solution consisting of elementary functions is probably accurate.

Criteria for measuring handling qualities come in several forms including time response envelopes, frequency response bandwidths and bounds on parameters formed from two or more responses, such as the Control Anticipation Parameter. In variable dynamic cases, multiple scales can be used to solve for time responses or system functions, which are then applied to appropriate criteria. Time response criteria can be applied directly, while other forms require modification. The zeroth order multiple scales equation contains the time-varying characteristic equation, whose roots are the clock functions. The complex clock contains terms which are analogous to parameters of constant coefficient analysis, such as ω_d and $\zeta\omega_n$. These terms may be used to modify criteria based on time invariant dynamics, such as the CAP. The modification is intended to restrict the speed or extent of the variation in a particular response over a flight phase, especially if the variation degrades the response. Of course the instantaneous value of the parameter should remain within the bounds for the level of handling quality desired.

Asymptotic approximations to the system function for a response can be computed in several ways; the multiple scales approximation is accurate and uniformly valid, but can be difficult to evaluate. Alternatively, a Poincaré expansion of the system function can be evaluated order by order rather straightforwardly, but does not necessarily meet boundary conditions (it may be non-uniform). When coefficients vary slowly the zeroth order Poincaré approximation may be sufficiently accurate. Once the system function

AD-A167 727 ON THE HANDLING QUALITIES OF FLIGHT VEHICLES THROUGH VARIABLE FLIGHT CONDITIONS(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH K C GREEN JAN 86 2/2

UNCLASSIFIED AFIT/CI/NR-86-56T F/G 20/4 NL



NATIONAL BUREAU OF S

MICROCOPY RESOLUT TEST

is evaluated, measures of handling quality can be computed for conventional or unconventional vehicle responses.

The longitudinal dynamics of two types of flight vehicle were examined to demonstrate the application of asymptotic methods to solving time-varying equations of motion and to computing measures of handling quality. Both conventional and unconventional dynamic responses were addressed, with their associated criteria. In the example of an unpowered LRV which responds conventionally to aerodynamic controls, the value of the CAP changed significantly from entry interface through the terminal phase of flight. The speed and direction of variation also changed through re-entry. An extension to the CAP was suggested which would limit the extent of variation over a particular flight phase. Angle of attack time responses were computed by the multiple scales method in two flight phases: the first beginning at 250,000 feet altitude when aerodynamic controls become primary, and the second beginning at 75,000 feet altitude in the terminal phase of flight. These responses compared reasonably with specified bounds.

The dynamics of an XC-142 VTOL aircraft during transition from hover to cruise were also examined regarding handling qualities. Based on the results of asymptotic studies by Ramnath and Callaham, who tested the accuracy of a zeroth order Poincare approximation to the system function relating pitch attitude to tail rotor thrust, the zeroth order approximations to the system functions were used. Two transition velocity profiles were used from 0 to 150 ft/sec. In each case the pitch attitude-to-tail rotor thrust loop was closed and the system function of heave, or flight path, to throttle control was calculated by the Poincaré method. Flight path bandwidth histories were computed to compare with current specifications on flight path control using throttle. The treatment of handling qualities and the methods of computing measures of handling quality described above are workable and accurate.

Recommendations

The approach and techniques applied in this work can be extended to other flight vehicles, such as helicopters and short takeoff and landing (STOL) aircraft. Determining specific bounds for handling qualities criteria requires simulator and in-flight experiments with pilot evaluation. The criteria suggested above, and any others formulated for variable flight dynamics, could be examined during the flight control design stage of a new (or modified) flight vehicle when simulator tests are made. Further study of the different vehicle responses is also necessary, and definitive tests for slow system variation must be developed. The effects of perturbations on the reference maneuver or trajectory conditions should be examined, as outlined in section 4.2.2. The effects of turning points on handling quality and on the accuracy of asymptotic approximations needs to be quantified. Lateral and directional dynamics and handling qualities criteria should also be part of another study.

APPENDIX A

COUNTER-INTUITIVE BEHAVIOR OF NON-AUTONOMOUS SYSTEMS

Exact solutions of linear, time-varying systems are contrasted with solutions by "frozen" constant coefficient analysis. Examples are due to Ramnath [1].

(1) The stability of a linear, time varying system cannot, in general, be characterized by the eigenvalues in the same way as the "frozen" time invariant system. Consider the system described by the equation

$$y'' - 0.1y' + (e^{.2t})y = 0 \tag{A-1}$$

Treating the coefficients as constants the characteristic equation is

$$s^2 - 0.1s + e^{.2t} = 0 \tag{A-2}$$

The roots are

$$s_1, s_2 = .05 \pm je^{.1t}\,[1 - (.0025/e^{.2t})] \tag{A-3}$$

The solution according to the above analysis is

$$y(t) = e^{.05t}[A\cos(e^{.1t}[1-.0025/e^{.2t}]) + B\sin(e^{.1t}[1-.0025/e^{.2t}]) \tag{A-4}$$

Treating the coefficients as constant shows an unstable system response. The exact solution is

$$y(t) = \sin(10e^{.1t}) \tag{A-5}$$

which is bounded. Hence the constant coefficient analysis does not correctly predict stability.

(2) Consider the system described by the state equations

$$x(t) = A(t)\,x(t) + B(t)\,u(t) \tag{A-6}$$

and output equation

$$y(t) = C(t)\,x(t) \tag{A-7}$$

where,

$$A(t) = \begin{bmatrix} -.1-.2\cos .5t & -.25-.2\sin .5t \\ .25-.2\sin .5t & -.1+.2\cos .5t \end{bmatrix}$$

$$B(t) = \begin{bmatrix} \cos .25t-\sin .25t \\ \sin .25t+\cos .25t \end{bmatrix} \quad C(t) = B\ (t)$$

Freezing the coefficients of A(t) at any time $t \geq 0$ and computing the characteristic equation shows that the system eigenvalues are invariant and equal to

$$s_1,\ s_2 = -.1 \pm .15j$$

The roots indicate a stable system for all time.

Alternatively, solving asymptotically for the characteristic equation as a function of t (by computing the multiple scales approximation to the system function) yields the exact system roots

$$s_1 = -.3,\ s_2 = .1$$

The actual system roots indicate instability for all time, which was not apparent at all from the constant coefficient analysis. In this case, the time varying system could not be analytically studied by "freezing".

REFERENCES

1. Ramnath, R.V., "Advanced Flight Dynamics and Control," Course Notes, Dept. of Aeronautics and Astronautics, M.I.T., Cambridge, MA, Spring, 1985.

2. Ramnath, R.V., "A Multiple Scales Approach to the Analysis of Linear Systems," AFFDL-TR-68-60, WPAFB, OH, 1968.

3. Ramnath, R.V., and G. Sandri, "A Generalized Multiple Scales Approach to a Class of Linear Differential Equations," JMAA, Vol. 28, No. 2, Nov. 1969, pp.339-364.

4. Ramnath, R.V., "A New Approach to the Design of Time Varying Control Systems with an Application to Space Shuttle Boost," Proc. of the IFAC Coference, Genova, Italy, 1975.

5. Ramnath, R.V., et al (eds), Nonlinear Systems Analysis and Synthesis, Vol. 2, Techniques and Applications, ASME, 1981.

6. Callaham, M.B., and R.V. Ramnath, "Design of Variable Linear Control Systems by Generalized Multiple Scales," Proc. of the JACC, Denver, CO, 1979.

7. Callaham, M.B., "Synthesis of Variable Linear Control Systems using Generalized Multiple Scales," Sc.D. Thesis, Dept. of Aeronautics and Astronautics, M.I.T., Cambridge, MA, January, 1978.

8. Zadeh, L., "Frequency Analysis of Variable Networks," Proc. of the IRE, 1955.

9. Ramnath, R.V., "Transition Dynamics of VTOL Aircraft," AIAA Journal, Vol. 8, No. 7, July 1970, pp. 1214-1221.

10. Vinh, N.X. and E.V. Laitone, "Longitudinal Dynamic Stability of a Shuttle Vehicle," Journal of Astronautical Sciences, Vol. XIX, No. 5, March 1972, pp.337-363.

11. Ramnath, R.V. and P. Sinha, "Dynamics of the Space Shuttle During Entry into Earth's Atmosphere," AIAA Journal, Vol. 13, No. 3, March 1975, pp. 337-342.

12. Cooper, G.E. and R.P. Harper, Jr., "The Use of Pilot Rating in the Evaluation of Aircraft Handling Qualities," NASA-TN-D-5153, April 1969.

13. McRuer, D., I. Ashkenas and D. Graham, Aircraft Dynamics and Automatic Control, Princeton University Press, Princeton, N.J., 1973.

14. Stengel, R.F., "A Unifying Framework for Longitudinal Flying Qualities Criteria," AIAA Paper No. 81-1889, August 1981.

15. Gentry, T.A., "Guidance for the Use of Equivalent Systems with MIL-F-8785C," AIAA Paper No. 82-1355, August 1982.

16. Hoh, R.H., et.al, "Proposed MIL Standard and Handbook - Flying Qualities of Air Vehicles," AFWAL-TR-82-3081, Volume II, WPAFB, OH, November 1982.

17. Stengel, R.F., "Optimal Transition from Entry to Cruising Flight," AIAA Paper No. 70-1018, August 1970.

18. DiFranco, D.A. and J.F. Mitchell, "Preliminary Handling Quality Requirements for Lifting Re-entry Vehicles During Terminal Flight," AFFDL-TR-71-64, WPAFB, OH, 1971.

19. Laitone, E.V. and Y.S. Chou, "Phugoid Oscillations at Hypersonic Speeds," AIAA Journal, Vol. 3, No. 4, April 1965, pp. 732-735.

20. Dana, W.H. and J.R. Gentry, "Pilot Impressions of Lifting Body Vehicles," NASA-TM-X-2101, June 1970, pp. 73-88.

21. Holleman, E.C., "Control Experiences of the X-15 Pertinent to Lifting Entry," NASA-TN-D-3262, February 1966.

22. Rynaski, E.G., "Flight Control to Satisfy Flying Qualities," Proc. of the ACC, Boston, MA, June 1985.

23. Chalk, C.R., et.al., "Background Information and User Guide for MIL-F-8785B(ASG) 'Military Specification - Flying Qualities of Piloted Airplanes,'" AFFDL-TR-69-72, WPAFB, OH, August 1969.

24. Seckel, E., Stability and Control of Airplanes and Helicopters, Academic Press, New York, 1964.

25. Hoh, R.H. and M.B. Tischler, "Handling Qualities Criteria for Flight Path Control of V/STOL Aircraft," AIAA Paper No. 82-1292, August 1982.

26. Ward, J.P., "Lessons Learned from the XC-142 Program," Proceedings of the V/STOL Technology and Planning Conference, Las Vegas, NV, 1969.

END

DTIC

6-86